Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "1 Algebraic functions\1.2 Trinomial products\1.2.1 Quadratic"

Test results for the 143 problems in "1.2.1.1 (a+b x+c x^2)^p.m"

- **Problem 9: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{3 x-4 x^2}\, dx$$

Optimal (type 3, 35 leaves, 3 steps):

$$-\frac{1}{16} (3-8 x) \sqrt{3 x-4 x^2}-\frac{9}{64} \text{ArcSin}\left[1-\frac{8 x}{3}\right]$$

Result (type 3, 72 leaves):

$$\frac{\sqrt{-x (-3+4 x)} \left(2 \sqrt{x} \sqrt{-3+4 x} (-3+8 x)-9 \text{Log}\left[2 \sqrt{x}+\sqrt{-3+4 x}\right]\right)}{32 \sqrt{x} \sqrt{-3+4 x}}$$

- **Problem 11: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{5 x-9 x^2}\, dx$$

Optimal (type 3, 35 leaves, 3 steps):

$$-\frac{1}{36} (5-18 x) \sqrt{5 x-9 x^2}-\frac{25}{216} \text{ArcSin}\left[1-\frac{18 x}{5}\right]$$

Result (type 3, 72 leaves):

$$\frac{\sqrt{-x (-5+9 x)} \left(3 \sqrt{x} \sqrt{-5+9 x} (-5+18 x)-25 \text{Log}\left[3 \sqrt{x}+\sqrt{-5+9 x}\right]\right)}{108 \sqrt{x} \sqrt{-5+9 x}}$$

- **Problem 17: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{3\,i\,x+4\,x^2}}\,dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$\frac{1}{2}\,i\,\text{ArcSin}\left[1-\frac{8\,i\,x}{3}\right]$$

Result (type 3, 50 leaves):

$$\frac{\sqrt{x}\,\sqrt{3\,i+4\,x}\,\text{Log}\left[2\,\sqrt{x}+\sqrt{3\,i+4\,x}\right]}{\sqrt{x\,(3\,i+4\,x)}}$$

- **Problem 21: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{3\,x-4\,x^2}}\,dx$$

Optimal (type 3, 12 leaves, 2 steps):

$$-\frac{1}{2}\,\text{ArcSin}\left[1-\frac{8\,x}{3}\right]$$

Result (type 3, 45 leaves):

$$\frac{\sqrt{x}\,\sqrt{-3+4\,x}\,\text{Log}\left[2\,\sqrt{x}+\sqrt{-3+4\,x}\right]}{\sqrt{-x\,(-3+4\,x)}}$$

- **Problem 25: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{b\,x-b^2\,x^2}}\,dx$$

Optimal (type 3, 12 leaves, 2 steps):

$$-\frac{\text{ArcSin}[1-2\,b\,x]}{b}$$

Result (type 3, 58 leaves):

$$\frac{2\,\sqrt{x}\,\sqrt{-1+b\,x}\,\text{Log}\left[b\,\sqrt{x}+\sqrt{b}\,\sqrt{-1+b\,x}\right]}{\sqrt{b}\,\sqrt{-b\,x\,(-1+b\,x)}}$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{6 x - x^2}} \, dx$$

Optimal (type 3, 10 leaves, 2 steps):

$$-\text{ArcSin}\left[1 - \frac{x}{3}\right]$$

Result (type 3, 38 leaves):

$$\frac{2 \sqrt{-6 + x} \sqrt{x} \,\text{Log}\left[\sqrt{-6 + x} + \sqrt{x}\right]}{\sqrt{-(-6 + x)\, x}}$$

Problem 28: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{4 x + x^2}} \, dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$2 \,\text{ArcTanh}\left[\frac{x}{\sqrt{4 x + x^2}}\right]$$

Result (type 3, 33 leaves):

$$\frac{2 \sqrt{x} \sqrt{4 + x} \,\text{ArcSinh}\left[\frac{\sqrt{x}}{2}\right]}{\sqrt{x\, (4 + x)}}$$

Problem 29: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-2 x + x^2}} \, dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$2 \,\text{ArcTanh}\left[\frac{x}{\sqrt{-2 x + x^2}}\right]$$

Result (type 3, 37 leaves):

$$\frac{2 \sqrt{-2 + x} \sqrt{x} \,\text{Log}\left[\sqrt{-2 + x} + \sqrt{x}\right]}{\sqrt{(-2 + x)\, x}}$$

Problem 30: Result unnecessarily involves higher level functions.

$$\int \left(b x+c x^2\right)^{4/3} \, dx$$

Optimal (type 4, 448 leaves, 6 steps) :

$$\frac{3 \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3} (b+2 c x) \left(b x+c x^2\right)^{4/3}}{55 c \left(-\frac{c \left(b x+c x^2\right)}{b^2}\right)^{4/3}}+\frac{3 \left(-\frac{c x (b+c x)}{b^2}\right)^{4/3} (b+2 c x) \left(b x+c x^2\right)^{4/3}}{22 c \left(-\frac{c \left(b x+c x^2\right)}{b^2}\right)^{4/3}}+$$

$$\left(2^{1/3} 3^{3/4} \sqrt{2-\sqrt{3}}\, b^2 \left(b x+c x^2\right)^{4/3} \left(1-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right) \sqrt{\frac{1+2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}+2\times 2^{1/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right) \Bigg/ \left(55 c (b+2 c x) \left(-\frac{c \left(b x+c x^2\right)}{b^2}\right)^{4/3} \sqrt{-\frac{1-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 94 leaves) :

$$\frac{3 x \left(-2 b^4-b^3 c x+16 b^2 c^2 x^2+25 b c^3 x^3+10 c^4 x^4+2 b^4 \left(1+\frac{c x}{b}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{c x}{b}\right]\right)}{110 c^2 (x (b+c x))^{2/3}}$$

Problem 31: Result unnecessarily involves higher level functions.

$$\int \left(b x+c x^2\right)^{1/3} \, dx$$

Optimal (type 4, 387 leaves, 5 steps) :

$$\frac{3\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}(b+2\,c\,x)\left(b\,x+c\,x^2\right)^{1/3}}{10\,c\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{1/3}}+$$

$$\left(3^{3/4}\sqrt{2-\sqrt{3}}\;b^2\left(b\,x+c\,x^2\right)^{1/3}\left(1-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)\sqrt{\frac{1+2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}\right],\,-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(5\times 2^{2/3}\,c\,(b+2\,c\,x)\left(-\frac{c\,\left(b\,x+c\,x^2\right)}{b^2}\right)^{1/3}\sqrt{-\frac{1-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 70 leaves) :

$$\frac{3\,x\left(b^2+3\,b\,c\,x+2\,c^2\,x^2-b^2\left(1+\frac{c\,x}{b}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{c\,x}{b}\right]\right)}{10\,c\,(x\,(b+c\,x))^{2/3}}$$

Problem 32: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(b\,x+c\,x^2\right)^{2/3}}\,dx$$

Optimal (type 4, 322 leaves, 4 steps) :

$$\left(2^{1/3}\,3^{3/4}\sqrt{2-\sqrt{3}}\;b^2\left(-\frac{c\,\left(b\,x+c\,x^2\right)}{b^2}\right)^{2/3}\left(1-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)\sqrt{\frac{1+2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}\right],\,-7+4\sqrt{3}\right]\right)\Big/\left(c\,(b+2\,c\,x)\left(b\,x+c\,x^2\right)^{2/3}\sqrt{-\frac{1-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 44 leaves) :

$$\frac{3\,x\left(\frac{b+c\,x}{b}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{c\,x}{b}\right]}{(x\,(b+c\,x))^{2/3}}$$

Problem 33: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(b x + c x^2\right)^{5/3}} \, dx$$

Optimal (type 4, 384 leaves, 5 steps):

$$\frac{3 (b + 2 c x) \left(-\frac{c (b x+c x^2)}{b^2}\right)^{5/3}}{2 c \left(-\frac{c x (b+c x)}{b^2}\right)^{2/3} \left(b x + c x^2\right)^{5/3}} + \left(2^{1/3} 3^{3/4} \sqrt{2-\sqrt{3}}\, b^2 \left(-\frac{c (b x + c x^2)}{b^2}\right)^{5/3} \left(1 - 2^{2/3} \left(-\frac{c x (b + c x)}{b^2}\right)^{1/3}\right) \sqrt{\frac{1 + 2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3} + 2 \times 2^{1/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{2/3}}{\left(1 - \sqrt{3} - 2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}} \right.$$
$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{1 + \sqrt{3} - 2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{1 - \sqrt{3} - 2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}\right], -7 + 4 \sqrt{3}\right]\right) \Bigg/ \left(c (b + 2 c x) \left(b x + c x^2\right)^{5/3} \sqrt{-\frac{1 - 2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{\left(1 - \sqrt{3} - 2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 57 leaves):

$$-\frac{3 \left(b + 2 c x + 2 c x \left(1 + \frac{c x}{b}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{c x}{b}\right]\right)}{2 b^2 (x (b + c x))^{2/3}}$$

Problem 34: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(b x + c x^2\right)^{8/3}} \, dx$$

Optimal (type 4, 448 leaves, 6 steps):

$$\frac{3 (b + 2 c x) \left(-\frac{c (b x+c x^2)}{b^2}\right)^{8/3}}{5 c \left(-\frac{c x (b+c x)}{b^2}\right)^{5/3} \left(b x + c x^2\right)^{8/3}} + \frac{21 (b + 2 c x) \left(-\frac{c (b x+c x^2)}{b^2}\right)^{8/3}}{5 c \left(-\frac{c x (b+c x)}{b^2}\right)^{2/3} \left(b x + c x^2\right)^{8/3}} +$$
$$\left(14 \times 2^{1/3} 3^{3/4} \sqrt{2-\sqrt{3}}\, b^2 \left(-\frac{c (b x + c x^2)}{b^2}\right)^{8/3} \left(1 - 2^{2/3} \left(-\frac{c x (b + c x)}{b^2}\right)^{1/3}\right) \sqrt{\frac{1 + 2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3} + 2 \times 2^{1/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{2/3}}{\left(1 - \sqrt{3} - 2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}} \right.$$
$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{1 + \sqrt{3} - 2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{1 - \sqrt{3} - 2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}\right], -7 + 4 \sqrt{3}\right]\right) \Bigg/ \left(5 c (b + 2 c x) \left(b x + c x^2\right)^{8/3} \sqrt{-\frac{1 - 2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{\left(1 - \sqrt{3} - 2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 90 leaves):

$$\frac{-3 b^3+15 b^2 c x+63 b c^2 x^2+42 c^3 x^3+42 c^2 x^2 (b+c x) \left(1+\frac{c x}{b}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{c x}{b}\right]}{5 b^4 (x (b+c x))^{5/3}}$$

- **Problem 35: Result unnecessarily involves higher level functions.**

$$\int \left(b x+c x^2\right)^{5/3} dx$$

Optimal (type 4, 842 leaves, 8 steps):

$$\frac{15 \left(-\frac{c x (b+c x)}{b^2}\right)^{2/3} (b+2 c x) \left(b x+c x^2\right)^{5/3}}{364 c \left(-\frac{c \left(b x+c x^2\right)}{b^2}\right)^{5/3}}+\frac{3 \left(-\frac{c x (b+c x)}{b^2}\right)^{5/3} (b+2 c x) \left(b x+c x^2\right)^{5/3}}{26 c \left(-\frac{c \left(b x+c x^2\right)}{b^2}\right)^{5/3}}-$$

$$\frac{15 (b+2 c x) \left(b x+c x^2\right)^{5/3}}{182\times 2^{1/3} c \left(-\frac{c \left(b x+c x^2\right)}{b^2}\right)^{5/3} \left(1-\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)}-\left(15\times 3^{1/4} \sqrt{2+\sqrt{3}}\, b^2 \left(b x+c x^2\right)^{5/3} \left(1-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)\right.$$

$$\left.\sqrt{\frac{1+2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}+2\times 2^{1/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(364\times 2^{1/3} c (b+2 c x) \left(-\frac{c \left(b x+c x^2\right)}{b^2}\right)^{5/3} \sqrt{-\frac{1-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}\right)+\left(5\times 3^{3/4} b^2 \left(b x+c x^2\right)^{5/3} \left(1-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)\right.$$

$$\left.\sqrt{\frac{1+2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}+2\times 2^{1/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(91\times 2^{5/6} c (b+2 c x) \left(-\frac{c \left(b x+c x^2\right)}{b^2}\right)^{5/3} \sqrt{-\frac{1-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 94 leaves):

$$\frac{3 x \left(-5 b^4-b^3 c x+46 b^2 c^2 x^2+70 b c^3 x^3+28 c^4 x^4+5 b^4 \left(1+\frac{c x}{b}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{c x}{b}\right]\right)}{364 c^2 (x (b+c x))^{1/3}}$$

■ Problem 36: Result unnecessarily involves higher level functions.

$$\int \left(b x+c x^2\right)^{2/3} \, dx$$

Optimal (type 4, 781 leaves, 7 steps):

$$\frac{3 \left(-\frac{c x (b+c x)}{b^2}\right)^{2/3} (b+2 c x) \left(b x+c x^2\right)^{2/3}}{14 c \left(-\frac{c \left(b x+c x^2\right)}{b^2}\right)^{2/3}}-\frac{3 (b+2 c x) \left(b x+c x^2\right)^{2/3}}{7\times 2^{1/3} c \left(-\frac{c \left(b x+c x^2\right)}{b^2}\right)^{2/3} \left(1-\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)}-\left(3\times 3^{1/4} \sqrt{2+\sqrt{3}}\, b^2 \left(b x+c x^2\right)^{2/3} \left(1-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right) \sqrt{\frac{1+2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}+2\times 2^{1/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Big/\left(14\times 2^{1/3} c (b+2 c x) \left(-\frac{c \left(b x+c x^2\right)}{b^2}\right)^{2/3} \sqrt{-\frac{1-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}\right)+\left(2^{1/6}\, 3^{3/4} b^2 \left(b x+c x^2\right)^{2/3} \left(1-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right) \sqrt{\frac{1+2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}+2\times 2^{1/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Big/\left(7 c (b+2 c x) \left(-\frac{c \left(b x+c x^2\right)}{b^2}\right)^{2/3} \sqrt{-\frac{1-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3} \left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 70 leaves):

$$\frac{3 x \left(b^2+3 b c x+2 c^2 x^2-b^2 \left(1+\frac{c x}{b}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{c x}{b}\right]\right)}{14 c (x (b+c x))^{1/3}}$$

■ Problem 37: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(b x+c x^2\right)^{1/3}} \, dx$$

Optimal (type 4, 715 leaves, 6 steps):

$$-\frac{3\,(b+2\,c\,x)\,\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{1/3}}{2^{1/3}\,c\,\left(b\,x+c\,x^2\right)^{1/3}\,\left(1-\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)}-$$

$$\left(3\times 3^{1/4}\,\sqrt{2+\sqrt{3}}\,\,b^2\,\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{1/3}\,\left(1-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)\,\sqrt{\frac{1+2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}\right],\,-7+4\,\sqrt{3}\right]\right)\Bigg/\left(2\times 2^{1/3}\,c\,(b+2\,c\,x)\,\left(b\,x+c\,x^2\right)^{1/3}\,\sqrt{-\frac{1-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right)+$$

$$\left(2^{1/6}\,3^{3/4}\,b^2\,\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{1/3}\,\left(1-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)\,\sqrt{\frac{1+2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}\right],\,-7+4\,\sqrt{3}\right]\right)\Bigg/\left(c\,(b+2\,c\,x)\,\left(b\,x+c\,x^2\right)^{1/3}\,\sqrt{-\frac{1-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 46 leaves) :

$$\frac{3\,x\,\left(\frac{b+c\,x}{b}\right)^{1/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,\frac{2}{3},\,\frac{5}{3},\,-\frac{c\,x}{b}\right]}{2\,(x\,(b+c\,x))^{1/3}}$$

- **Problem 38: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{\left(b\,x+c\,x^2\right)^{4/3}}\,dx$$

Optimal (type 4, 773 leaves, 7 steps) :

$$\frac{3\,(b+2\,c\,x)\,\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{4/3}}{c\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\,(b\,x+c\,x^2)^{4/3}}+\frac{3\times 2^{2/3}\,(b+2\,c\,x)\,\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{4/3}}{c\,(b\,x+c\,x^2)^{4/3}\,\left(1-\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)}+$$

$$\left(3\times 3^{1/4}\,\sqrt{2+\sqrt{3}}\,\,b^2\,\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{4/3}\,\left(1-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)\,\sqrt{\frac{1+2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}\right],\,-7+4\,\sqrt{3}\right]\right)\Bigg/\left(2^{1/3}\,c\,(b+2\,c\,x)\,(b\,x+c\,x^2)^{4/3}\,\sqrt{-\frac{1-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right)-$$

$$\left(2\times 2^{1/6}\,3^{3/4}\,b^2\,\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{4/3}\,\left(1-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)\,\sqrt{\frac{1+2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}\right],\,-7+4\,\sqrt{3}\right]\right)\Bigg/\left(c\,(b+2\,c\,x)\,(b\,x+c\,x^2)^{4/3}\,\sqrt{-\frac{1-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 57 leaves):

$$\frac{-3\,(b+2\,c\,x)+3\,c\,x\,\left(1+\frac{c\,x}{b}\right)^{1/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,\frac{2}{3},\,\frac{5}{3},\,-\frac{c\,x}{b}\right]}{b^2\,(x\,(b+c\,x))^{1/3}}$$

- **Problem 39: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{\left(b\,x+c\,x^2\right)^{7/3}}\,dx$$

Optimal (type 4, 838 leaves, 8 steps):

$$\frac{3\,(b+2\,c\,x)\,\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{7/3}}{4\,c\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{4/3}\,(b\,x+c\,x^2)^{7/3}}+\frac{15\,(b+2\,c\,x)\,\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{7/3}}{2\,c\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\,(b\,x+c\,x^2)^{7/3}}+\frac{15\,(b+2\,c\,x)\,\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{7/3}}{2^{1/3}\,c\,(b\,x+c\,x^2)^{7/3}\,\left(1-\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)}+$$

$$\left(15\times 3^{1/4}\,\sqrt{2+\sqrt{3}}\;b^2\,\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{7/3}\,\left(1-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)\,\sqrt{\frac{1+2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}\right],\,-7+4\,\sqrt{3}\right]\right)\Bigg/\left(2\times 2^{1/3}\,c\,(b+2\,c\,x)\,(b\,x+c\,x^2)^{7/3}\,\sqrt{-\frac{1-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right)-$$

$$\left(5\times 2^{1/6}\,3^{3/4}\,b^2\,\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{7/3}\,\left(1-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)\,\sqrt{\frac{1+2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}\right],\,-7+4\,\sqrt{3}\right]\right)\Bigg/\left(c\,(b+2\,c\,x)\,(b\,x+c\,x^2)^{7/3}\,\sqrt{-\frac{1-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\,\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 90 leaves):

$$\frac{-3\,b^3+24\,b^2\,c\,x+90\,b\,c^2\,x^2+60\,c^3\,x^3-30\,c^2\,x^2\,(b+c\,x)\,\left(1+\frac{c\,x}{b}\right)^{1/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,\frac{2}{3},\,\frac{5}{3},\,-\frac{c\,x}{b}\right]}{4\,b^4\,(x\,(b+c\,x))^{4/3}}$$

- **Problem 40: Result unnecessarily involves higher level functions.**

$$\int (b\,x+c\,x^2)^{5/4}\,dx$$

Optimal (type 4, 119 leaves, 5 steps):

$$-\frac{5\,b^2\,(b+2\,c\,x)\,(b\,x+c\,x^2)^{1/4}}{84\,c^2}+\frac{(b+2\,c\,x)\,(b\,x+c\,x^2)^{5/4}}{7\,c}+\frac{5\,b^5\,\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{3/4}\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcSin}\left[1+\frac{2\,c\,x}{b}\right],\,2\right]}{84\,\sqrt{2}\,c^3\,(b\,x+c\,x^2)^{3/4}}$$

Result (type 5, 94 leaves):

$$\frac{x\,\left(-5\,b^4-3\,b^3\,c\,x+38\,b^2\,c^2\,x^2+60\,b\,c^3\,x^3+24\,c^4\,x^4+5\,b^4\,\left(1+\frac{c\,x}{b}\right)^{3/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{5}{4},\,-\frac{c\,x}{b}\right]\right)}{84\,c^2\,(x\,(b+c\,x))^{3/4}}$$

- **Problem 41: Result unnecessarily involves higher level functions.**

$$\int (b\,x+c\,x^2)^{3/4}\,dx$$

Optimal (type 4, 90 leaves, 4 steps):

$$\frac{(b+2\,c\,x)\,\left(b\,x+c\,x^2\right)^{3/4}}{5\,c}-\frac{3\,b^3\,\left(-\frac{c\,\left(b\,x+c\,x^2\right)}{b^2}\right)^{1/4}\,\text{EllipticE}\left[\frac{1}{2}\,\text{ArcSin}\left[1+\frac{2\,c\,x}{b}\right],\,2\right]}{10\,\sqrt{2}\,c^2\,\left(b\,x+c\,x^2\right)^{1/4}}$$

Result (type 5, 70 leaves):

$$\frac{x\,\left(b^2+3\,b\,c\,x+2\,c^2\,x^2-b^2\,\left(1+\frac{c\,x}{b}\right)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{7}{4},\,-\frac{c\,x}{b}\right]\right)}{5\,c\,(x\,(b+c\,x))^{1/4}}$$

- **Problem 42: Result unnecessarily involves higher level functions.**

$$\int\left(b\,x+c\,x^2\right)^{1/4}\,dx$$

Optimal (type 4, 90 leaves, 4 steps):

$$\frac{(b+2\,c\,x)\,\left(b\,x+c\,x^2\right)^{1/4}}{3\,c}-\frac{b^3\,\left(-\frac{c\,\left(b\,x+c\,x^2\right)}{b^2}\right)^{3/4}\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcSin}\left[1+\frac{2\,c\,x}{b}\right],\,2\right]}{3\,\sqrt{2}\,c^2\,\left(b\,x+c\,x^2\right)^{3/4}}$$

Result (type 5, 70 leaves):

$$\frac{x\,\left(b^2+3\,b\,c\,x+2\,c^2\,x^2-b^2\,\left(1+\frac{c\,x}{b}\right)^{3/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{5}{4},\,-\frac{c\,x}{b}\right]\right)}{3\,c\,(x\,(b+c\,x))^{3/4}}$$

- **Problem 43: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{\left(b\,x+c\,x^2\right)^{1/4}}\,dx$$

Optimal (type 4, 58 leaves, 3 steps):

$$\frac{\sqrt{2}\,b\,\left(-\frac{c\,\left(b\,x+c\,x^2\right)}{b^2}\right)^{1/4}\,\text{EllipticE}\left[\frac{1}{2}\,\text{ArcSin}\left[1+\frac{2\,c\,x}{b}\right],\,2\right]}{c\,\left(b\,x+c\,x^2\right)^{1/4}}$$

Result (type 5, 46 leaves):

$$\frac{4\,x\,\left(\frac{b+c\,x}{b}\right)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{7}{4},\,-\frac{c\,x}{b}\right]}{3\,(x\,(b+c\,x))^{1/4}}$$

- **Problem 44: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{\left(b\,x+c\,x^2\right)^{3/4}}\,dx$$

Optimal (type 4, 59 leaves, 3 steps):

$$\frac{2\sqrt{2}\, b \left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2}\,\text{ArcSin}\left[1+\frac{2\,c\,x}{b}\right],\,2\right]}{c\,\left(b\,x+c\,x^2\right)^{3/4}}$$

Result (type 5, 44 leaves) :

$$\frac{4\,x\,\left(\frac{b+c\,x}{b}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{5}{4},\,-\frac{c\,x}{b}\right]}{(x\,(b+c\,x))^{3/4}}$$

- **Problem 45: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\left(b\,x+c\,x^2\right)^{5/4}}\,dx$$

Optimal (type 4, 83 leaves, 4 steps) :

$$-\frac{4\,(b+2\,c\,x)}{b^2\,\left(b\,x+c\,x^2\right)^{1/4}}+\frac{4\sqrt{2}\,\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2}\,\text{ArcSin}\left[1+\frac{2\,c\,x}{b}\right],\,2\right]}{b\,\left(b\,x+c\,x^2\right)^{1/4}}$$

Result (type 5, 59 leaves) :

$$-\frac{4\,\left(3\,b+6\,c\,x-4\,c\,x\,\left(1+\frac{c\,x}{b}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{7}{4},\,-\frac{c\,x}{b}\right]\right)}{3\,b^2\,(x\,(b+c\,x))^{1/4}}$$

- **Problem 46: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\left(b\,x+c\,x^2\right)^{9/4}}\,dx$$

Optimal (type 4, 115 leaves, 5 steps) :

$$-\frac{4\,(b+2\,c\,x)}{5\,b^2\,\left(b\,x+c\,x^2\right)^{5/4}}+\frac{48\,c\,(b+2\,c\,x)}{5\,b^4\,\left(b\,x+c\,x^2\right)^{1/4}}-\frac{48\sqrt{2}\,c\,\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2}\,\text{ArcSin}\left[1+\frac{2\,c\,x}{b}\right],\,2\right]}{5\,b^3\,\left(b\,x+c\,x^2\right)^{1/4}}$$

Result (type 5, 90 leaves) :

$$\frac{-4\,b^3+40\,b^2\,c\,x+144\,b\,c^2\,x^2+96\,c^3\,x^3-64\,c^2\,x^2\,(b+c\,x)\,\left(1+\frac{c\,x}{b}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{7}{4},\,-\frac{c\,x}{b}\right]}{5\,b^4\,(x\,(b+c\,x))^{5/4}}$$

- **Problem 47: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\left(b\,x+c\,x^2\right)^{13/4}}\,dx$$

Optimal (type 4, 146 leaves, 6 steps) :

$$-\frac{4\,(b+2\,c\,x)}{9\,b^2\,\left(b\,x+c\,x^2\right)^{9/4}}+\frac{112\,c\,(b+2\,c\,x)}{45\,b^4\,\left(b\,x+c\,x^2\right)^{5/4}}-\frac{448\,c^2\,(b+2\,c\,x)}{15\,b^6\,\left(b\,x+c\,x^2\right)^{1/4}}+\frac{448\,\sqrt{2}\,c^2\,\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{1/4}\,\text{EllipticE}\left[\frac{1}{2}\,\text{ArcSin}\left[1+\frac{2\,c\,x}{b}\right],\,2\right]}{15\,b^5\,\left(b\,x+c\,x^2\right)^{1/4}}$$

Result (type 5, 114 leaves) :

$$-\frac{1}{45\,b^6\,(x\,(b+c\,x))^{9/4}}$$
$$4\left(5\,b^5-18\,b^4\,c\,x+252\,b^3\,c^2\,x^2+1288\,b^2\,c^3\,x^3+1680\,b\,c^4\,x^4+672\,c^5\,x^5-448\,c^3\,x^3\,(b+c\,x)^2\,\left(1+\frac{c\,x}{b}\right)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{7}{4},\,-\frac{c\,x}{b}\right]\right)$$

- **Problem 83: Result more than twice size of optimal antiderivative.**

$$\int\frac{1}{3+4\,x+x^2}\,dx$$

Optimal (type 3, 6 leaves, 3 steps) :

$$-\text{ArcTanh}[2+x]$$

Result (type 3, 17 leaves) :

$$\frac{1}{2}\,\text{Log}[1+x]-\frac{1}{2}\,\text{Log}[3+x]$$

- **Problem 102: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{1}{1+x^2+2\,x\,\text{Cos}\left[\frac{\pi}{7}\right]}\,dx$$

Optimal (type 3, 23 leaves, 2 steps) :

$$\text{ArcTan}\left[\text{Cot}\left[\frac{\pi}{7}\right]+x\,\text{Csc}\left[\frac{\pi}{7}\right]\right]\,\text{Csc}\left[\frac{\pi}{7}\right]$$

Result (type 3, 56 leaves) :

$$\frac{2\,\text{ArcTan}\left[\frac{(-1)^{1/7}-(-1)^{6/7}+2\,x}{\sqrt{2-(-1)^{2/7}+(-1)^{5/7}}}\right]}{\sqrt{2-(-1)^{2/7}+(-1)^{5/7}}}$$

- **Problem 133: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\left(3+4\,x+5\,x^2\right)^p\,dx$$

Optimal (type 5, 37 leaves, 2 steps) :

$$5^{-1-p}\,11^p\,(2+5\,x)\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,-p,\,\frac{3}{2},\,-\frac{1}{11}\,(2+5\,x)^2\right]$$

Result (type 5, 93 leaves) :

$$\frac{1}{5(1+p)}11^{p/2}\left(-2 i+\sqrt{11}-5 i x\right)^{-p}\left(2-i\sqrt{11}+5 x\right)\left(6+8 x+10 x^2\right)^p \text{Hypergeometric2F1}\left[-p,1+p,2+p,\frac{2 i+\sqrt{11}+5 i x}{2\sqrt{11}}\right]$$

- **Problem 134: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\left(3+4 x+4 x^2\right)^p dx$$

Optimal (type 5, 32 leaves, 2 steps):

$$2^{-1+p}(1+2 x)\,\text{Hypergeometric2F1}\left[\frac{1}{2},-p,\frac{3}{2},-\frac{1}{2}(1+2 x)^2\right]$$

Result (type 5, 94 leaves):

$$\frac{1}{1+p}2^{-1+\frac{3p}{2}}\left(-i+\sqrt{2}-2 i x\right)^{-p}\left(1-i\sqrt{2}+2 x\right)\left(3+4 x+4 x^2\right)^p \text{Hypergeometric2F1}\left[-p,1+p,2+p,\frac{1}{4}\left(2+i\sqrt{2}+2 i\sqrt{2}\,x\right)\right]$$

- **Problem 135: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\left(3+4 x+3 x^2\right)^p dx$$

Optimal (type 5, 37 leaves, 2 steps):

$$3^{-1-p}5^p(2+3 x)\,\text{Hypergeometric2F1}\left[\frac{1}{2},-p,\frac{3}{2},-\frac{1}{5}(2+3 x)^2\right]$$

Result (type 5, 93 leaves):

$$\frac{1}{3(1+p)}5^{p/2}\left(-2 i+\sqrt{5}-3 i x\right)^{-p}\left(2-i\sqrt{5}+3 x\right)\left(6+8 x+6 x^2\right)^p \text{Hypergeometric2F1}\left[-p,1+p,2+p,\frac{2 i+\sqrt{5}+3 i x}{2\sqrt{5}}\right]$$

- **Problem 136: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\left(3+4 x+2 x^2\right)^p dx$$

Optimal (type 5, 21 leaves, 2 steps):

$$(1+x)\,\text{Hypergeometric2F1}\left[\frac{1}{2},-p,\frac{3}{2},-2(1+x)^2\right]$$

Result (type 5, 92 leaves):

$$\frac{1}{1+p}2^{-1+\frac{3p}{2}}\left(-2 i+\sqrt{2}-2 i x\right)^{-p}\left(2-i\sqrt{2}+2 x\right)\left(3+4 x+2 x^2\right)^p \text{Hypergeometric2F1}\left[-p,1+p,2+p,\frac{2 i+\sqrt{2}+2 i x}{2\sqrt{2}}\right]$$

- **Problem 139: Result more than twice size of optimal antiderivative.**

$$\int\left(3+4 x-x^2\right)^p dx$$

Optimal (type 5, 31 leaves, 2 steps):

$$-7^p\,(2-x)\,\text{Hypergeometric2F1}\left[\frac{1}{2},-p,\frac{3}{2},\frac{1}{7}(2-x)^2\right]$$

Result (type 5, 83 leaves):

$$-\frac{\left(2+\sqrt{7}-x\right)\left(3+4x-x^2\right)^p\left(1+\frac{-2-\sqrt{7}+x}{2\sqrt{7}}\right)^{-p}\text{Hypergeometric2F1}\left[-p,1+p,2+p,-\frac{-2-\sqrt{7}+x}{2\sqrt{7}}\right]}{1+p}$$

- **Problem 140: Result more than twice size of optimal antiderivative.**

$$\int\left(3+4x-2x^2\right)^p\,dx$$

Optimal (type 5, 31 leaves, 2 steps):

$$-5^p\,(1-x)\,\text{Hypergeometric2F1}\left[\frac{1}{2},-p,\frac{3}{2},\frac{2}{5}(1-x)^2\right]$$

Result (type 5, 86 leaves):

$$-\frac{1}{1+p}2^{-1+\frac{3p}{2}}5^{p/2}\left(2+\sqrt{10}-2x\right)\left(-2+\sqrt{10}+2x\right)^{-p}\left(3+4x-2x^2\right)^p\text{Hypergeometric2F1}\left[-p,1+p,2+p,\frac{1}{2}+\frac{1}{\sqrt{10}}-\frac{x}{\sqrt{10}}\right]$$

- **Problem 141: Result more than twice size of optimal antiderivative.**

$$\int\left(3+4x-3x^2\right)^p\,dx$$

Optimal (type 5, 38 leaves, 2 steps):

$$-3^{-1-p}\,13^p\,(2-3x)\,\text{Hypergeometric2F1}\left[\frac{1}{2},-p,\frac{3}{2},\frac{1}{13}(2-3x)^2\right]$$

Result (type 5, 81 leaves):

$$-\frac{13^{p/2}\left(2+\sqrt{13}-3x\right)\left(-2+\sqrt{13}+3x\right)^{-p}\left(6+8x-6x^2\right)^p\text{Hypergeometric2F1}\left[-p,1+p,2+p,\frac{2+\sqrt{13}-3x}{2\sqrt{13}}\right]}{3(1+p)}$$

- **Problem 143: Result more than twice size of optimal antiderivative.**

$$\int\left(3+4x-5x^2\right)^p\,dx$$

Optimal (type 5, 38 leaves, 2 steps):

$$-5^{-1-p}\,19^p\,(2-5x)\,\text{Hypergeometric2F1}\left[\frac{1}{2},-p,\frac{3}{2},\frac{1}{19}(2-5x)^2\right]$$

Result (type 5, 81 leaves):

$$-\frac{1}{5(1+p)}19^{p/2}\left(2+\sqrt{19}-5x\right)\left(-2+\sqrt{19}+5x\right)^{-p}\left(6+8x-10x^2\right)^p\text{Hypergeometric2F1}\left[-p,1+p,2+p,\frac{2+\sqrt{19}-5x}{2\sqrt{19}}\right]$$

Test results for the 2590 problems in "1.2.1.2 (d+e x)^m (a+b x+c x^2)^p.m"

Problem 117: Result more than twice size of optimal antiderivative.

$$\int \frac{(d x)^m}{\left(b x+c x^2\right)^3} \, dx$$

Optimal (type 5, 37 leaves, 2 steps):

$$-\frac{d^2 (d x)^{-2+m} \text{Hypergeometric2F1}\left[3,-2+m,-1+m,-\frac{c x}{b}\right]}{b^3 (2-m)}$$

Result (type 5, 123 leaves):

$$\frac{1}{b^6}(d x)^m \left(\frac{6 b c^2}{m}+\frac{b^3}{(-2+m) x^2}+\frac{3 b^2 c}{x-m x}-\frac{6 c^3 x \text{Hypergeometric2F1}\left[1,1+m,2+m,-\frac{c x}{b}\right]}{1+m}-\frac{3 c^3 x \text{Hypergeometric2F1}\left[2,1+m,2+m,-\frac{c x}{b}\right]}{1+m}-\frac{c^3 x \text{Hypergeometric2F1}\left[3,1+m,2+m,-\frac{c x}{b}\right]}{1+m}\right)$$

Problem 118: Result more than twice size of optimal antiderivative.

$$\int (d x)^m \left(b x+c x^2\right)^{5/2} \, dx$$

Optimal (type 5, 73 leaves, 3 steps):

$$\frac{2 b^2 \left(-\frac{c x}{b}\right)^{-\frac{1}{2}-m} (d x)^m (b+c x) \left(b x+c x^2\right)^{5/2} \text{Hypergeometric2F1}\left[\frac{7}{2},-\frac{5}{2}-m,\frac{9}{2},1+\frac{c x}{b}\right]}{7 c^3 x^2}$$

Result (type 5, 157 leaves):

$$\left(2 x^3 (d x)^m \sqrt{x (b+c x)} \left(b^2 \left(99+40 m+4 m^2\right) \text{Hypergeometric2F1}\left[-\frac{1}{2},\frac{7}{2}+m,\frac{9}{2}+m,-\frac{c x}{b}\right]+c (7+2 m) x \left(2 b (11+2 m) \text{Hypergeometric2F1}\left[-\frac{1}{2},\frac{9}{2}+m,\frac{11}{2}+m,-\frac{c x}{b}\right]+c (9+2 m) x \text{Hypergeometric2F1}\left[-\frac{1}{2},\frac{11}{2}+m,\frac{13}{2}+m,-\frac{c x}{b}\right]\right)\right)\right) \Big/ \left((7+2 m) (9+2 m) (11+2 m) \sqrt{1+\frac{c x}{b}}\right)$$

Problem 175: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a^2+2 a b x+b^2 x^2\right)^{5/2}}{x^7} \, dx$$

Optimal (type 2, 37 leaves, 2 steps):

$$-\frac{(a+b x)^5 \sqrt{a^2+2 a b x+b^2 x^2}}{6 a x^6}$$

Result (type 2, 75 leaves) :

$$-\frac{\sqrt{(a+b x)^2}\left(a^5+6 a^4 b x+15 a^3 b^2 x^2+20 a^2 b^3 x^3+15 a b^4 x^4+6 b^5 x^5\right)}{6 x^6 (a+b x)}$$

- **Problem 311: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{2 x-x^2}}{2-2 x} \, dx$$

Optimal (type 3, 36 leaves, 3 steps) :

$$-\frac{1}{2} \sqrt{2 x-x^2}+\frac{1}{2} \operatorname{ArcTanh}\left[\sqrt{2 x-x^2}\right]$$

Result (type 3, 73 leaves) :

$$\frac{\sqrt{-(-2+x) x}\left(-\sqrt{-2+x} \sqrt{x}+\operatorname{ArcTan}\left[\frac{-2+\sqrt{x}}{\sqrt{-2+x}}\right]+\operatorname{ArcTan}\left[\frac{2+\sqrt{x}}{\sqrt{-2+x}}\right]\right)}{2 \sqrt{-2+x} \sqrt{x}}$$

- **Problem 312: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(2-2 x) \sqrt{2 x-x^2}} \, dx$$

Optimal (type 3, 18 leaves, 2 steps) :

$$\frac{1}{2} \operatorname{ArcTanh}\left[\sqrt{2 x-x^2}\right]$$

Result (type 3, 59 leaves) :

$$-\frac{\sqrt{-2+x} \sqrt{x}\left(\operatorname{ArcTan}\left[\frac{-2+\sqrt{x}}{\sqrt{-2+x}}\right]+\operatorname{ArcTan}\left[\frac{2+\sqrt{x}}{\sqrt{-2+x}}\right]\right)}{2 \sqrt{-(-2+x) x}}$$

- **Problem 313: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(2-2 x)\left(2 x-x^2\right)^{3/2}} \, dx$$

Optimal (type 3, 36 leaves, 3 steps) :

$$-\frac{1}{2\sqrt{2x-x^2}}+\frac{1}{2}\text{ArcTanh}\left[\sqrt{2x-x^2}\right]$$

Result (type 3, 74 leaves) :

$$-\frac{1+\sqrt{-2+x}\,\sqrt{x}\,\text{ArcTan}\left[\frac{-2+\sqrt{x}}{\sqrt{-2+x}}\right]+\sqrt{-2+x}\,\sqrt{x}\,\text{ArcTan}\left[\frac{2+\sqrt{x}}{\sqrt{-2+x}}\right]}{2\sqrt{-(-2+x)\,x}}$$

- **Problem 386: Result unnecessarily involves imaginary or complex numbers.**

$$\int (d+e\,x)^{3/2}\sqrt{b\,x+c\,x^2}\,dx$$

Optimal (type 4, 362 leaves, 9 steps) :

$$\frac{2\sqrt{d+e\,x}\,\left(3c^2d^2+9\,b\,c\,d\,e-4\,b^2e^2+12\,c\,e\,(2\,c\,d-b\,e)\,x\right)\sqrt{b\,x+c\,x^2}}{105\,c^2\,e}+\frac{2\,e\sqrt{d+e\,x}\,\left(b\,x+c\,x^2\right)^{3/2}}{7\,c}-$$

$$\frac{2\sqrt{-b}\,(2\,c\,d-b\,e)\left(3c^2d^2-3\,b\,c\,d\,e+8\,b^2e^2\right)\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{105\,c^{5/2}\,e^2\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}}+$$

$$\frac{4\sqrt{-b}\,d\,(c\,d-b\,e)\left(3c^2d^2-3\,b\,c\,d\,e+2\,b^2e^2\right)\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{105\,c^{5/2}\,e^2\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}$$

Result (type 4, 372 leaves) :

$$\frac{1}{105\,b\,c^2\,e^2\,\sqrt{x}\,(b+c\,x)\,\sqrt{d+e\,x}}\left(2\,b\,e\,x\,(b+c\,x)\,(d+e\,x)\,\left(-4\,b^2\,e^2+3\,b\,c\,e\,(3\,d+e\,x)+3\,c^2\,\left(d^2+8\,d\,e\,x+5\,e^2\,x^2\right)\right)-\sqrt{\frac{b}{c}}\left(\sqrt{\frac{b}{c}}\,\left(6\,c^3\,d^3-9\,b\,c^2\,d^2\,e+19\,b^2\,c\,d\,e^2-8\,b^3\,e^3\right)\,(b+c\,x)\right.\right.$$

$$(d+e\,x)+i\,b\,e\,\left(6\,c^3\,d^3-9\,b\,c^2\,d^2\,e+19\,b^2\,c\,d\,e^2-8\,b^3\,e^3\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]-$$

$$\left.\left.i\,b\,e\,\left(3\,c^3\,d^3-18\,b\,c^2\,d^2\,e+23\,b^2\,c\,d\,e^2-8\,b^3\,e^3\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\right)$$

- **Problem 387: Result unnecessarily involves imaginary or complex numbers.**

$$\int\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}\,dx$$

Optimal (type 4, 308 leaves, 9 steps) :

$$-\frac{2\,(2\,c\,d-b\,e)\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}{15\,c\,e}+\frac{2\,(d+e\,x)^{3/2}\,\sqrt{b\,x+c\,x^2}}{5\,e}-$$

$$\frac{4\,\sqrt{-b}\,\left(c^2\,d^2-b\,c\,d\,e+b^2\,e^2\right)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{15\,c^{3/2}\,e^2\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}}+$$

$$\frac{2\,\sqrt{-b}\,d\,(c\,d-b\,e)\,(2\,c\,d-b\,e)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{15\,c^{3/2}\,e^2\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}$$

Result (type 4, 294 leaves) :

$$\left(2\left(b e x (b+c x) (d+e x) (b e+c (d+3 e x))+\sqrt{\frac{b}{c}}\left(-2\sqrt{\frac{b}{c}}\left(c^2 d^2-b c d e+b^2 e^2\right)(b+c x)(d+e x)-\right.\right.\right.$$

$$2 i b e\left(c^2 d^2-b c d e+b^2 e^2\right)\sqrt{1+\frac{b}{c x}}\sqrt{1+\frac{d}{e x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]+$$

$$\left.\left.\left. i b e\left(c^2 d^2-3 b c d e+2 b^2 e^2\right)\sqrt{1+\frac{b}{c x}}\sqrt{1+\frac{d}{e x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]\right)\right)\right)\Big/\left(15 b c e^2\sqrt{x (b+c x)}\sqrt{d+e x}\right)$$

- **Problem 388: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{b x+c x^2}}{\sqrt{d+e x}}\,dx$$

Optimal (type 4, 246 leaves, 8 steps):

$$\frac{2\sqrt{d+e x}\sqrt{b x+c x^2}}{3 e}-\frac{2\sqrt{-b}\,(2 c d-b e)\sqrt{x}\sqrt{1+\frac{c x}{b}}\sqrt{d+e x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{3\sqrt{c}\,e^2\sqrt{1+\frac{e x}{d}}\sqrt{b x+c x^2}}+$$

$$\frac{4\sqrt{-b}\,d\,(c d-b e)\sqrt{x}\sqrt{1+\frac{c x}{b}}\sqrt{1+\frac{e x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{3\sqrt{c}\,e^2\sqrt{d+e x}\sqrt{b x+c x^2}}$$

Result (type 4, 226 leaves):

$$\left(2\left((b+c x)(d+e x)(-2 c d+b e+c e x)+i\sqrt{\frac{b}{c}}\, c e(-2 c d+b e)\sqrt{1+\frac{b}{c x}}\sqrt{1+\frac{d}{e x}}\, x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]-\right.\right.$$

$$\left.\left. i\sqrt{\frac{b}{c}}\, c e(-c d+b e)\sqrt{1+\frac{b}{c x}}\sqrt{1+\frac{d}{e x}}\, x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]\right)\right)\Big/\left(3 c e^2\sqrt{x(b+c x)}\sqrt{d+e x}\right)$$

- **Problem 389: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{b x+c x^2}}{(d+e x)^{3/2}}\,dx$$

Optimal (type 4, 231 leaves, 8 steps):

$$-\frac{2\sqrt{b x+c x^2}}{e\sqrt{d+e x}}+\frac{4\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{1+\frac{c x}{b}}\sqrt{d+e x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{e^2\sqrt{1+\frac{e x}{d}}\sqrt{b x+c x^2}}-$$

$$\frac{2\sqrt{-b}(2 c d-b e)\sqrt{x}\sqrt{1+\frac{c x}{b}}\sqrt{1+\frac{e x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{\sqrt{c}\, e^2\sqrt{d+e x}\sqrt{b x+c x^2}}$$

Result (type 4, 195 leaves):

$$\left(2\left(\sqrt{\frac{b}{c}}(b+c x)(2 d+e x)+2 i b e\sqrt{1+\frac{b}{c x}}\sqrt{1+\frac{d}{e x}}\, x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]-\right.\right.$$

$$\left.\left. i b e\sqrt{1+\frac{b}{c x}}\sqrt{1+\frac{d}{e x}}\, x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]\right)\right)\Big/\left(\sqrt{\frac{b}{c}}\, e^2\sqrt{x(b+c x)}\sqrt{d+e x}\right)$$

Problem 390: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{b x + c x^2}}{(d + e x)^{5/2}} \, dx$$

Optimal (type 4, 301 leaves, 9 steps):

$$-\frac{2\sqrt{b x + c x^2}}{3 e (d + e x)^{3/2}} + \frac{2 (2 c d - b e) \sqrt{b x + c x^2}}{3 d e (c d - b e) \sqrt{d + e x}} - \frac{2 \sqrt{-b} \sqrt{c} (2 c d - b e) \sqrt{x} \sqrt{1 + \frac{c x}{b}} \sqrt{d + e x} \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]}{3 d e^2 (c d - b e) \sqrt{1 + \frac{e x}{d}} \sqrt{b x + c x^2}} + \frac{4 \sqrt{-b} \sqrt{c} \sqrt{x} \sqrt{1 + \frac{c x}{b}} \sqrt{1 + \frac{e x}{d}} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]}{3 e^2 \sqrt{d + e x} \sqrt{b x + c x^2}}$$

Result (type 4, 265 leaves):

$$-\left(2 \left(e x (b + c x) \left(b e^2 x - c d (d + 2 e x)\right) + (d + e x) \left((2 c d - b e) (b + c x) (d + e x) - i \sqrt{\frac{b}{c}} c e (-2 c d + b e) \sqrt{1 + \frac{b}{c x}} \sqrt{1 + \frac{d}{e x}} x^{3/2} \, \text{EllipticE}\left[i \, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right] + i \sqrt{\frac{b}{c}} c e (-c d + b e) \sqrt{1 + \frac{b}{c x}} \sqrt{1 + \frac{d}{e x}} x^{3/2} \, \text{EllipticF}\left[i \, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right]\right)\right)\right) \Big/ \left(3 d e^2 (c d - b e) \sqrt{x (b + c x)} (d + e x)^{3/2}\right)$$

Problem 391: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{b x + c x^2}}{(d + e x)^{7/2}} \, dx$$

Optimal (type 4, 398 leaves, 10 steps):

$$-\frac{2\sqrt{b x+c x^2}}{5 e (d+e x)^{5/2}}+\frac{2 (2 c d-b e) \sqrt{b x+c x^2}}{15 d e (c d-b e) (d+e x)^{3/2}}+\frac{4 \left(c^2 d^2-b c d e+b^2 e^2\right) \sqrt{b x+c x^2}}{15 d^2 e (c d-b e)^2 \sqrt{d+e x}}-$$

$$\frac{4 \sqrt{-b} \sqrt{c} \left(c^2 d^2-b c d e+b^2 e^2\right) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{d+e x}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{15 d^2 e^2 (c d-b e)^2 \sqrt{1+\frac{e x}{d}} \sqrt{b x+c x^2}}+$$

$$\frac{2 \sqrt{-b} \sqrt{c} (2 c d-b e) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{1+\frac{e x}{d}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{15 d e^2 (c d-b e) \sqrt{d+e x} \sqrt{b x+c x^2}}$$

Result (type 4, 362 leaves):

$$-\frac{1}{15 b d^2 e^2 (c d-b e)^2 \sqrt{x (b+c x)} (d+e x)^{5/2}}$$

$$2 \left(b e x (b+c x) \left(-b^2 e^3 x (5 d+2 e x)-c^2 d^2 \left(d^2+6 d e x+2 e^2 x^2\right)+b c d e \left(-d^2+7 d e x+2 e^2 x^2\right)\right)+\sqrt{\frac{b}{c}}\, c (d+e x)^2\right.$$

$$\left(2 \sqrt{\frac{b}{c}} \left(c^2 d^2-b c d e+b^2 e^2\right) (b+c x) (d+e x)+2 i b e \left(c^2 d^2-b c d e+b^2 e^2\right) \sqrt{1+\frac{b}{c x}} \sqrt{1+\frac{d}{e x}}\, x^{3/2}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\right.\right.$$

$$\left.\left.\left.\frac{c d}{b e}\right]-i b e \left(c^2 d^2-3 b c d e+2 b^2 e^2\right) \sqrt{1+\frac{b}{c x}} \sqrt{1+\frac{d}{e x}}\, x^{3/2}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]\right)\right)$$

- **Problem 392: Result unnecessarily involves imaginary or complex numbers.**

$$\int (d+e x)^{3/2} \left(b x+c x^2\right)^{3/2} dx$$

Optimal (type 4, 521 leaves, 10 steps):

$$\frac{1}{1155\, c^3 e^3} 2 \sqrt{d+e x} \left(8 c^4 d^4-19 b c^3 d^3 e+6 b^2 c^2 d^2 e^2-19 b^3 c d e^3+8 b^4 e^4-3 c e (2 c d-b e) \left(c^2 d^2-b c d e+8 b^2 e^2\right) x\right) \sqrt{b x+c x^2}+$$

$$\frac{2 \sqrt{d+e x} \left(c^2 d^2+13 b c d e-6 b^2 e^2+14 c e (2 c d-b e) x\right) \left(b x+c x^2\right)^{3/2}}{231 c^2 e}+\frac{2 e \sqrt{d+e x} \left(b x+c x^2\right)^{5/2}}{11 c}-$$

$$\left(16 \sqrt{-b} (c d-2 b e) (2 c d-b e) (c d+b e) \left(c^2 d^2-b c d e+b^2 e^2\right) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{d+e x}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/$$

$$\left(1155 c^{7/2} e^4 \sqrt{1+\frac{e x}{d}} \sqrt{b x+c x^2}\right)+$$

$$\left(2 \sqrt{-b}\, d (c d-b e) \left(16 c^4 d^4-32 b c^3 d^3 e+3 b^2 c^2 d^2 e^2+13 b^3 c d e^3-8 b^4 e^4\right) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{1+\frac{e x}{d}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/$$

$$\left(1155 c^{7/2} e^4 \sqrt{d+e x} \sqrt{b x+c x^2}\right)$$

Result (type 4, 559 leaves) :

$$\frac{1}{1155\, b\, c^3 e^4 x^2 (b+c x)^2 \sqrt{d+e x}} 2 (x (b+c x))^{3/2}$$

$$\left(b e x (b+c x) (d+e x) \left(8 b^4 e^4-b^3 c e^3 (19 d+6 e x)+b^2 c^2 e^2 \left(6 d^2+14 d e x+5 e^2 x^2\right)+b c^3 e \left(-19 d^3+14 d^2 e x+205 d e^2 x^2+140 e^3 x^3\right)+\right.\right.$$

$$\left.c^4 \left(8 d^4-6 d^3 e x+5 d^2 e^2 x^2+140 d e^3 x^3+105 e^4 x^4\right)\right)+$$

$$\sqrt{\frac{b}{c}} \left(-8 \sqrt{\frac{b}{c}} \left(2 c^5 d^5-5 b c^4 d^4 e+2 b^2 c^3 d^3 e^2+2 b^3 c^2 d^2 e^3-5 b^4 c d e^4+2 b^5 e^5\right) (b+c x) (d+e x)-8 i b e\right.$$

$$\left(2 c^5 d^5-5 b c^4 d^4 e+2 b^2 c^3 d^3 e^2+2 b^3 c^2 d^2 e^3-5 b^4 c d e^4+2 b^5 e^5\right) \sqrt{1+\frac{b}{c x}} \sqrt{1+\frac{d}{e x}}\, x^{3/2}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right]+i b e$$

$$\left.\left.\left(8 c^5 d^5-21 b c^4 d^4 e+10 b^2 c^3 d^3 e^2+35 b^3 c^2 d^2 e^3-48 b^4 c d e^4+16 b^5 e^5\right) \sqrt{1+\frac{b}{c x}} \sqrt{1+\frac{d}{e x}}\, x^{3/2}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right]\right)\right)$$

- **Problem 393: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{d+e x}\,\left(b x+c x^2\right)^{3/2}\,dx$$

Optimal (type 4, 457 leaves, 10 steps):

$$\frac{2\sqrt{d+e x}\,\left(8 c^3 d^3-15 b c^2 d^2 e+3 b^2 c d e^2-4 b^3 e^3-6 c e\left(c^2 d^2-b c d e+2 b^2 e^2\right) x\right)\sqrt{b x+c x^2}}{315 c^2 e^3}-$$
$$\frac{2 (2 c d-b e)\sqrt{d+e x}\,\left(b x+c x^2\right)^{3/2}}{21 c e}+\frac{2 (d+e x)^{3/2}\left(b x+c x^2\right)^{3/2}}{9 e}-$$
$$\left(2\sqrt{-b}\,\left(16 c^4 d^4-32 b c^3 d^3 e+9 b^2 c^2 d^2 e^2+7 b^3 c d e^3-8 b^4 e^4\right)\sqrt{x}\,\sqrt{1+\frac{c x}{b}}\,\sqrt{d+e x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]\right)\Big/$$
$$\left(315 c^{5/2} e^4\sqrt{1+\frac{e x}{d}}\,\sqrt{b x+c x^2}\right)+$$
$$\left(8\sqrt{-b}\,d (c d-b e) (2 c d-b e)\left(2 c^2 d^2-2 b c d e-b^2 e^2\right)\sqrt{x}\,\sqrt{1+\frac{c x}{b}}\,\sqrt{1+\frac{e x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]\right)\Big/$$
$$\left(315 c^{5/2} e^4\sqrt{d+e x}\,\sqrt{b x+c x^2}\right)$$

Result (type 4, 463 leaves):

$$\frac{1}{315\, b\, c^2\, e^4\, x^2\, (b+c\,x)^2 \sqrt{d+e\,x}} \left(2\,(x\,(b+c\,x))^{3/2} \left(b\,e\,x\,(b+c\,x)\,(d+e\,x)\,\left(-4\,b^3\,e^3+3\,b^2\,c\,e^2\,(d+e\,x)+b\,c^2\,e\,\left(-15\,d^2+11\,d\,e\,x+50\,e^2\,x^2\right)+c^3\,\left(8\,d^3-6\,d^2\,e\,x+5\,d\,e^2\,x^2+35\,e^3\,x^3\right)\right)-\right.\right.$$

$$\sqrt{\frac{b}{c}} \left(\sqrt{\frac{b}{c}}\,\left(16\,c^4\,d^4-32\,b\,c^3\,d^3\,e+9\,b^2\,c^2\,d^2\,e^2+7\,b^3\,c\,d\,e^3-8\,b^4\,e^4\right)\,(b+c\,x)\,(d+e\,x)+\right.$$

$$i\,b\,e\,\left(16\,c^4\,d^4-32\,b\,c^3\,d^3\,e+9\,b^2\,c^2\,d^2\,e^2+7\,b^3\,c\,d\,e^3-8\,b^4\,e^4\right)\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]-$$

$$\left.\left.i\,b\,e\,\left(8\,c^4\,d^4-17\,b\,c^3\,d^3\,e+6\,b^2\,c^2\,d^2\,e^2+11\,b^3\,c\,d\,e^3-8\,b^4\,e^4\right)\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\right)$$

- **Problem 394: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(b\,x+c\,x^2\right)^{3/2}}{\sqrt{d+e\,x}}\,dx$$

Optimal (type 4, 360 leaves, 9 steps):

$$\frac{2\sqrt{d+e\,x}\,\left(8\,c^2\,d^2-11\,b\,c\,d\,e+b^2\,e^2-3\,c\,e\,(2\,c\,d-b\,e)\,x\right)\sqrt{b\,x+c\,x^2}}{35\,c\,e^3}+\frac{2\sqrt{d+e\,x}\,\left(b\,x+c\,x^2\right)^{3/2}}{7\,e}-$$

$$\frac{4\sqrt{-b}\,(2\,c\,d-b\,e)\,\left(4\,c^2\,d^2-4\,b\,c\,d\,e-b^2\,e^2\right)\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{35\,c^{3/2}\,e^4\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}}+$$

$$\left(2\sqrt{-b}\,d\,(c\,d-b\,e)\,\left(16\,c^2\,d^2-16\,b\,c\,d\,e-b^2\,e^2\right)\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]\right)\Big/\left(35\,c^{3/2}\,e^4\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}\right)$$

Result (type 4, 380 leaves):

$$\frac{1}{35\,b\,c\,e^4\,x^2\,(b+c\,x)^2\,\sqrt{d+e\,x}}\,2\,(x\,(b+c\,x))^{3/2}\left(b\,e\,x\,(b+c\,x)\,(d+e\,x)\,\left(b^2\,e^2+b\,c\,e\,(-11\,d+8\,e\,x)+c^2\,\left(8\,d^2-6\,d\,e\,x+5\,e^2\,x^2\right)\right)+\sqrt{\frac{b}{c}}\left(-2\,\sqrt{\frac{b}{c}}\,\left(8\,c^3\,d^3-12\,b\,c^2\,d^2\,e+2\,b^2\,c\,d\,e^2+b^3\,e^3\right)\right.\right.$$

$$(b+c\,x)\,(d+e\,x)-2\,\mathrm{i}\,b\,e\,\left(8\,c^3\,d^3-12\,b\,c^2\,d^2\,e+2\,b^2\,c\,d\,e^2+b^3\,e^3\right)\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]+$$

$$\left.\left.\mathrm{i}\,b\,e\,\left(8\,c^3\,d^3-13\,b\,c^2\,d^2\,e+3\,b^2\,c\,d\,e^2+2\,b^3\,e^3\right)\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\right)$$

- **Problem 395: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(b\,x+c\,x^2\right)^{3/2}}{(d+e\,x)^{3/2}}\,dx$$

Optimal (type 4, 309 leaves, 9 steps):

$$-\frac{2\,\sqrt{d+e\,x}\,(8\,c\,d-7\,b\,e-6\,c\,e\,x)\,\sqrt{b\,x+c\,x^2}}{5\,e^3}-\frac{2\,\left(b\,x+c\,x^2\right)^{3/2}}{e\,\sqrt{d+e\,x}}+$$

$$\frac{2\,\sqrt{-b}\,\left(16\,c^2\,d^2-16\,b\,c\,d\,e+b^2\,e^2\right)\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{5\,\sqrt{c}\,e^4\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}}-$$

$$\frac{16\,\sqrt{-b}\,d\,(c\,d-b\,e)\,(2\,c\,d-b\,e)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{5\,\sqrt{c}\,e^4\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}$$

Result (type 4, 340 leaves):

$$\frac{1}{5\,c\,e^4\,\sqrt{x}\,(b+c\,x)\,\sqrt{d+e\,x}}$$

$$\left(2\left(b^3 e^2 (d+e\,x)+b^2 c\,e\left(-16\,d^2-8\,d\,e\,x+3\,e^2 x^2\right)+c^3 x\left(16\,d^3+8\,d^2 e\,x-2\,d\,e^2 x^2+e^3 x^3\right)+b\,c^2\left(16\,d^3-8\,d^2 e\,x-11\,d\,e^2 x^2+3\,e^3 x^3\right)\right)+\right.$$

$$2\,i\,\sqrt{\frac{b}{c}}\,c\,e\left(16\,c^2 d^2-16\,b\,c\,d\,e+b^2 e^2\right)\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]-$$

$$\left.2\,i\,\sqrt{\frac{b}{c}}\,c\,e\left(8\,c^2 d^2-9\,b\,c\,d\,e+b^2 e^2\right)\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)$$

- **Problem 396: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(b\,x+c\,x^2\right)^{3/2}}{(d+e\,x)^{5/2}}\,dx$$

Optimal (type 4, 298 leaves, 9 steps):

$$\frac{2\,(8\,c\,d-3\,b\,e+2\,c\,e\,x)\sqrt{b\,x+c\,x^2}}{3\,e^3\sqrt{d+e\,x}}-\frac{2\left(b\,x+c\,x^2\right)^{3/2}}{3\,e\,(d+e\,x)^{3/2}}-\frac{16\sqrt{-b}\,\sqrt{c}\,(2\,c\,d-b\,e)\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{3\,e^4\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}}+$$

$$\frac{2\sqrt{-b}\,(4\,c\,d-3\,b\,e)\,(4\,c\,d-b\,e)\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{3\sqrt{c}\,e^4\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}$$

Result (type 4, 279 leaves):

$$\frac{1}{3 e^4 x^2 (b+c x)^2 \sqrt{d+e x}} 2 (x (b+c x))^{3/2} \left(8 (-2 c d+b e) (b+c x) (d+e x)+\frac{e x (b+c x) \left(-b e (3 d+4 e x)+c \left(8 d^2+10 d e x+e^2 x^2\right)\right)}{d+e x}+8 i \sqrt{\frac{b}{c}} c e (-2 c d+b e) \sqrt{1+\frac{b}{c x}} \sqrt{1+\frac{d}{e x}} x^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right]-i \sqrt{\frac{b}{c}} c e (-8 c d+5 b e) \sqrt{1+\frac{b}{c x}} \sqrt{1+\frac{d}{e x}} x^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right]\right)$$

- **Problem 397: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(b x+c x^2\right)^{3/2}}{(d+e x)^{7/2}} \, dx$$

Optimal (type 4, 354 leaves, 9 steps):

$$-\frac{2 \left(c d^2 (8 c d-7 b e)+e \left(10 c^2 d^2-10 b c d e+b^2 e^2\right) x\right) \sqrt{b x+c x^2}}{5 d e^3 (c d-b e) (d+e x)^{3/2}}-\frac{2 \left(b x+c x^2\right)^{3/2}}{5 e (d+e x)^{5/2}}+\frac{2 \sqrt{-b} \sqrt{c} \left(16 c^2 d^2-16 b c d e+b^2 e^2\right) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{d+e x} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]}{5 d e^4 (c d-b e) \sqrt{1+\frac{e x}{d}} \sqrt{b x+c x^2}}-\frac{16 \sqrt{-b} \sqrt{c} (2 c d-b e) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{1+\frac{e x}{d}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]}{5 e^4 \sqrt{d+e x} \sqrt{b x+c x^2}}$$

Result (type 4, 369 leaves):

$$-\frac{1}{5\,b\,d\,e^4\,(c\,d-b\,e)\,x^2\,(b+c\,x)^2\,(d+e\,x)^{5/2}}$$

$$2\,(x\,(b+c\,x))^{3/2}\left(b\,e\,x\,(b+c\,x)\,\left(b^2\,e^4\,x^2-b\,c\,d\,e\,\left(7\,d^2+16\,d\,e\,x+11\,e^2\,x^2\right)+c^2\,d^2\,\left(8\,d^2+18\,d\,e\,x+11\,e^2\,x^2\right)\right)-\right.$$

$$\sqrt{\frac{b}{c}}\,c\,(d+e\,x)^2\left(\sqrt{\frac{b}{c}}\,\left(16\,c^2\,d^2-16\,b\,c\,d\,e+b^2\,e^2\right)\,(b+c\,x)\,(d+e\,x)+i\,b\,e\,\left(16\,c^2\,d^2-16\,b\,c\,d\,e+b^2\,e^2\right)\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\right.$$

$$\left.\left.\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]-i\,b\,e\,\left(8\,c^2\,d^2-9\,b\,c\,d\,e+b^2\,e^2\right)\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\right)$$

- **Problem 398: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(b\,x+c\,x^2\right)^{3/2}}{(d+e\,x)^{9/2}}\,dx$$

Optimal (type 4, 476 leaves, 10 steps):

$$\frac{4\,(2\,c\,d-b\,e)\,\left(4\,c^2\,d^2-4\,b\,c\,d\,e-b^2\,e^2\right)\sqrt{b\,x+c\,x^2}}{35\,d^2\,e^3\,(c\,d-b\,e)^2\,\sqrt{d+e\,x}}-\frac{2\,\left(d\,\left(8\,c^2\,d^2-5\,b\,c\,d\,e-2\,b^2\,e^2\right)+e\,\left(14\,c^2\,d^2-14\,b\,c\,d\,e+b^2\,e^2\right)x\right)\sqrt{b\,x+c\,x^2}}{35\,d\,e^3\,(c\,d-b\,e)\,(d+e\,x)^{5/2}}-$$

$$\frac{2\,\left(b\,x+c\,x^2\right)^{3/2}}{7\,e\,(d+e\,x)^{7/2}}-\left(4\,\sqrt{-b}\,\sqrt{c}\,(2\,c\,d-b\,e)\,\left(4\,c^2\,d^2-4\,b\,c\,d\,e-b^2\,e^2\right)\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]\right)\Big/$$

$$\left(35\,d^2\,e^4\,(c\,d-b\,e)^2\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}\right)+\frac{2\,\sqrt{-b}\,\sqrt{c}\,\left(16\,c^2\,d^2-16\,b\,c\,d\,e-b^2\,e^2\right)\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{35\,d\,e^4\,(c\,d-b\,e)\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}$$

Result (type 4, 479 leaves):

$$-\frac{1}{35\,b\,d^2\,e^4\,(c\,d-b\,e)^2\,x^2\,(b+c\,x)^2\,(d+e\,x)^{7/2}}$$

$$2\,(x\,(b+c\,x))^{3/2}\left(b\,e\,x\,(b+c\,x)\,\left(5\,d^3\,(c\,d-b\,e)^3-8\,d^2\,(c\,d-b\,e)^2\,(2\,c\,d-b\,e)\,(d+e\,x)+d\,(c\,d-b\,e)\,\left(19\,c^2\,d^2-19\,b\,c\,d\,e+b^2\,e^2\right)\,(d+e\,x)^2-\right.\right.$$

$$\left.2\,\left(8\,c^3\,d^3-12\,b\,c^2\,d^2\,e+2\,b^2\,c\,d\,e^2+b^3\,e^3\right)\,(d+e\,x)^3\right)+\sqrt{\frac{b}{c}}\,c\,(d+e\,x)^3\left(2\,\sqrt{\frac{b}{c}}\,\left(8\,c^3\,d^3-12\,b\,c^2\,d^2\,e+2\,b^2\,c\,d\,e^2+b^3\,e^3\right)\,(b+c\,x)\right.$$

$$(d+e\,x)+2\,i\,b\,e\,\left(8\,c^3\,d^3-12\,b\,c^2\,d^2\,e+2\,b^2\,c\,d\,e^2+b^3\,e^3\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]-$$

$$\left.\left.i\,b\,e\,\left(8\,c^3\,d^3-13\,b\,c^2\,d^2\,e+3\,b^2\,c\,d\,e^2+2\,b^3\,e^3\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\right)$$

- **Problem 399: Result unnecessarily involves imaginary or complex numbers.**

$$\int\sqrt{d+e\,x}\,\left(b\,x+c\,x^2\right)^{5/2}\,dx$$

Optimal (type 4, 666 leaves, 11 steps) :

$$\frac{1}{9009\,c^3\,e^5}2\sqrt{d+e\,x}\,\left(128\,c^5\,d^5-368\,b\,c^4\,d^4\,e+303\,b^2\,c^3\,d^3\,e^2-22\,b^3\,c^2\,d^2\,e^3-17\,b^4\,c\,d\,e^4+24\,b^5\,e^5-3\,c\,e\,\left(32\,c^4\,d^4-64\,b\,c^3\,d^3\,e+21\,b^2\,c^2\,d^2\,e^2+11\,b^3\,c\,d\,e^3-24\,b^4\,e^4\right)\,x\right)\,\sqrt{b\,x+c\,x^2}+$$

$$\frac{10\,\sqrt{d+e\,x}\,\left(16\,c^3\,d^3-31\,b\,c^2\,d^2\,e+9\,b^2\,c\,d\,e^2-18\,b^3\,e^3-14\,c\,e\,\left(c^2\,d^2-b\,c\,d\,e+3\,b^2\,e^2\right)\,x\right)\,\left(b\,x+c\,x^2\right)^{3/2}}{9009\,c^2\,e^3}-$$

$$\frac{10\,(2\,c\,d-b\,e)\,\sqrt{d+e\,x}\,\left(b\,x+c\,x^2\right)^{5/2}}{143\,c\,e}+\frac{2\,(d+e\,x)^{3/2}\,\left(b\,x+c\,x^2\right)^{5/2}}{13\,e}-$$

$$\left(4\,\sqrt{-b}\,\left(128\,c^6\,d^6-384\,b\,c^5\,d^5\,e+343\,b^2\,c^4\,d^4\,e^2-46\,b^3\,c^3\,d^3\,e^3-21\,b^4\,c^2\,d^2\,e^4-20\,b^5\,c\,d\,e^5+24\,b^6\,e^6\right)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]\right)\Big/\left(9009\,c^{7/2}\,e^6\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}\right)+$$

$$\left(2\,\sqrt{-b}\,d\,(c\,d-b\,e)\,(2\,c\,d-b\,e)\,\left(128\,c^4\,d^4-256\,b\,c^3\,d^3\,e+79\,b^2\,c^2\,d^2\,e^2+49\,b^3\,c\,d\,e^3+24\,b^4\,e^4\right)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]\right)\Big/\left(9009\,c^{7/2}\,e^6\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}\right)$$

Result (type 4, 663 leaves) :

$$\frac{1}{9009\, b\, c^3\, e^6\, x^3\, (b+c\,x)^3\, \sqrt{d+e\,x}} 2\, (x\,(b+c\,x))^{5/2} \Bigg(b\, e\, x\, (b+c\,x)\, (d+e\,x)$$

$$\Big(24\, b^5\, e^5 - b^4\, c\, e^4\, (17\, d + 18\, e\, x) + b^3\, c^2\, e^3\, \left(-22\, d^2 + 12\, d\, e\, x + 15\, e^2\, x^2\right) + b^2\, c^3\, e^2\, \left(303\, d^3 - 218\, d^2\, e\, x + 178\, d\, e^2\, x^2 + 1113\, e^3\, x^3\right) +$$

$$b\, c^4\, e\, \left(-368\, d^4 + 272\, d^3\, e\, x - 225\, d^2\, e^2\, x^2 + 196\, d\, e^3\, x^3 + 1701\, e^4\, x^4\right) + c^5\, \left(128\, d^5 - 96\, d^4\, e\, x + 80\, d^3\, e^2\, x^2 - 70\, d^2\, e^3\, x^3 + 63\, d\, e^4\, x^4 + 693\, e^5\, x^5\right)\Big) +$$

$$\sqrt{\frac{b}{c}} \Bigg(-2\, \sqrt{\frac{b}{c}}\, \left(128\, c^6\, d^6 - 384\, b\, c^5\, d^5\, e + 343\, b^2\, c^4\, d^4\, e^2 - 46\, b^3\, c^3\, d^3\, e^3 - 21\, b^4\, c^2\, d^2\, e^4 - 20\, b^5\, c\, d\, e^5 + 24\, b^6\, e^6\right)\, (b+c\,x)\, (d+e\,x) -$$

$$2\, i\, b\, e\, \left(128\, c^6\, d^6 - 384\, b\, c^5\, d^5\, e + 343\, b^2\, c^4\, d^4\, e^2 - 46\, b^3\, c^3\, d^3\, e^3 - 21\, b^4\, c^2\, d^2\, e^4 - 20\, b^5\, c\, d\, e^5 + 24\, b^6\, e^6\right) \sqrt{1+\frac{b}{c\,x}}\, \sqrt{1+\frac{d}{e\,x}}\, x^{3/2}$$

$$\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c\,d}{b\,e}\right] + i\, b\, e\, \left(128\, c^6\, d^6 - 400\, b\, c^5\, d^5\, e + 383\, b^2\, c^4\, d^4\, e^2 - 70\, b^3\, c^3\, d^3\, e^3 - 25\, b^4\, c^2\, d^2\, e^4 - 64\, b^5\, c\, d\, e^5 + 48\, b^6\, e^6\right)$$

$$\sqrt{1+\frac{b}{c\,x}}\, \sqrt{1+\frac{d}{e\,x}}\, x^{3/2}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c\,d}{b\,e}\right]\Bigg)\Bigg)$$

- **Problem 400: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(b\, x + c\, x^2\right)^{5/2}}{\sqrt{d+e\,x}}\, dx$$

Optimal (type 4, 537 leaves, 10 steps) :

$$\frac{1}{693 c^2 e^5} 2 \sqrt{d+e x} \left(128 c^4 d^4 - 304 b c^3 d^3 e + 195 b^2 c^2 d^2 e^2 - 7 b^3 c d e^3 - 4 b^4 e^4 - 12 c e (2 c d - b e) \left(4 c^2 d^2 - 4 b c d e - b^2 e^2\right) x\right) \sqrt{b x + c x^2} +$$

$$\frac{10 \sqrt{d+e x} \left(16 c^2 d^2 - 23 b c d e + 3 b^2 e^2 - 7 c e (2 c d - b e) x\right) \left(b x + c x^2\right)^{3/2}}{693 c e^3} + \frac{2 \sqrt{d+e x} \left(b x + c x^2\right)^{5/2}}{11 e} -$$

$$\left(2 \sqrt{-b} (2 c d - b e) \left(128 c^4 d^4 - 256 b c^3 d^3 e + 99 b^2 c^2 d^2 e^2 + 29 b^3 c d e^3 + 8 b^4 e^4\right) \sqrt{x} \sqrt{1 + \frac{c x}{b}} \sqrt{d+e x} \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/$$

$$\left(693 c^{5/2} e^6 \sqrt{1 + \frac{e x}{d}} \sqrt{b x + c x^2}\right) +$$

$$\left(4 \sqrt{-b} d (c d - b e) \left(128 c^4 d^4 - 256 b c^3 d^3 e + 123 b^2 c^2 d^2 e^2 + 5 b^3 c d e^3 + 2 b^4 e^4\right) \sqrt{x} \sqrt{1 + \frac{c x}{b}} \sqrt{1 + \frac{e x}{d}} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/$$

$$\left(693 c^{5/2} e^6 \sqrt{d+e x} \sqrt{b x + c x^2}\right)$$

Result (type 4, 557 leaves) :

$$\frac{1}{693\, b\, c^2\, e^6\, x^3\, (b+c\, x)^3 \sqrt{d+e\, x}}$$

$$\Bigg(2\, (x\, (b+c\, x))^{5/2} \Big(b\, e\, x\, (b+c\, x)\, (d+e\, x)\, \big(-4\, b^4\, e^4 + b^3\, c\, e^3\, (-7\, d+3\, e\, x) + b^2\, c^2\, e^2\, \left(195\, d^2 - 139\, d\, e\, x + 113\, e^2\, x^2\right) + b\, c^3$$

$$e\, \left(-304\, d^3 + 224\, d^2\, e\, x - 185\, d\, e^2\, x^2 + 161\, e^3\, x^3\right) + c^4\, \left(128\, d^4 - 96\, d^3\, e\, x + 80\, d^2\, e^2\, x^2 - 70\, d\, e^3\, x^3 + 63\, e^4\, x^4\right)\big) +$$

$$\sqrt{\frac{b}{c}} \Bigg(\sqrt{\frac{b}{c}}\, \left(-256\, c^5\, d^5 + 640\, b\, c^4\, d^4\, e - 454\, b^2\, c^3\, d^3\, e^2 + 41\, b^3\, c^2\, d^2\, e^3 + 13\, b^4\, c\, d\, e^4 + 8\, b^5\, e^5\right)\, (b+c\, x)\, (d+e\, x) -$$

$$i\, b\, e\, \left(256\, c^5\, d^5 - 640\, b\, c^4\, d^4\, e + 454\, b^2\, c^3\, d^3\, e^2 - 41\, b^3\, c^2\, d^2\, e^3 - 13\, b^4\, c\, d\, e^4 - 8\, b^5\, e^5\right) \sqrt{1+\frac{b}{c\, x}}\, \sqrt{1+\frac{d}{e\, x}}\, x^{3/2}$$

$$\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c\, d}{b\, e}\right] + i\, b\, e\, \left(128\, c^5\, d^5 - 336\, b\, c^4\, d^4\, e + 259\, b^2\, c^3\, d^3\, e^2 - 34\, b^3\, c^2\, d^2\, e^3 - 9\, b^4\, c\, d\, e^4 - 8\, b^5\, e^5\right)$$

$$\sqrt{1+\frac{b}{c\, x}}\, \sqrt{1+\frac{d}{e\, x}}\, x^{3/2}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c\, d}{b\, e}\right]\Bigg)\Bigg)$$

- **Problem 401: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(b\, x + c\, x^2\right)^{5/2}}{(d+e\, x)^{3/2}}\, dx$$

Optimal (type 4, 457 leaves, 10 steps) :

$$-\frac{1}{63 c e^5}2\sqrt{d+e x}\left(128 c^3 d^3-240 b c^2 d^2 e+111 b^2 c d e^2-b^3 e^3-3 c e\left(32 c^2 d^2-32 b c d e+b^2 e^2\right) x\right)\sqrt{b x+c x^2}-$$
$$\frac{10\sqrt{d+e x}\,(16 c d-15 b e-14 c e x)\left(b x+c x^2\right)^{3/2}}{63 e^3}-\frac{2\left(b x+c x^2\right)^{5/2}}{e\sqrt{d+e x}}+$$
$$\left(4\sqrt{-b}\left(128 c^4 d^4-256 b c^3 d^3 e+135 b^2 c^2 d^2 e^2-7 b^3 c d e^3-b^4 e^4\right)\sqrt{x}\sqrt{1+\frac{c x}{b}}\sqrt{d+e x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]\right)\Big/$$
$$\left(63 c^{3/2} e^6\sqrt{1+\frac{e x}{d}}\sqrt{b x+c x^2}\right)-$$
$$\left(2\sqrt{-b}\, d\,(c d-b e)\,(2 c d-b e)\left(128 c^2 d^2-128 b c d e-b^2 e^2\right)\sqrt{x}\sqrt{1+\frac{c x}{b}}\sqrt{1+\frac{e x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]\right)\Big/$$
$$\left(63 c^{3/2} e^6\sqrt{d+e x}\sqrt{b x+c x^2}\right)$$

Result (type 4, 498 leaves) :

$$\frac{1}{63 c e^6 x^{5/2}(b+c x)^3\sqrt{d+e x}}$$
$$2\,(x\,(b+c x))^{5/2}\left(\frac{2\left(128 c^4 d^4-256 b c^3 d^3 e+135 b^2 c^2 d^2 e^2-7 b^3 c d e^3-b^4 e^4\right)(b+c x)\,(d+e x)}{c\sqrt{x}}-e\sqrt{x}\,(b+c x)\left(-b^3 e^3\,(d+e x)+\right.\right.$$
$$\left.3 b^2 c e^2\left(37 d^2+11 d e x-5 e^2 x^2\right)-b c^2 e\left(240 d^3+64 d^2 e x-31 d e^2 x^2+19 e^3 x^3\right)+c^3\left(128 d^4+32 d^3 e x-16 d^2 e^2 x^2+10 d e^3 x^3-7 e^4 x^4\right)\right)-$$
$$2\,\mathrm{i}\sqrt{\frac{b}{c}}\,e\left(-128 c^4 d^4+256 b c^3 d^3 e-135 b^2 c^2 d^2 e^2+7 b^3 c d e^3+b^4 e^4\right)\sqrt{1+\frac{b}{c x}}\sqrt{1+\frac{d}{e x}}\,x\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]+$$
$$\left.\mathrm{i}\sqrt{\frac{b}{c}}\,e\left(-128 c^4 d^4+272 b c^3 d^3 e-159 b^2 c^2 d^2 e^2+13 b^3 c d e^3+2 b^4 e^4\right)\sqrt{1+\frac{b}{c x}}\sqrt{1+\frac{d}{e x}}\,x\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]\right)$$

Problem 402: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(b x+c x^2\right)^{5/2}}{(d+e x)^{5/2}}\,dx$$

Optimal (type 4, 401 leaves, 10 steps) :

$$\frac{2\sqrt{d+e\,x}\,\left(128\,c^2\,d^2-176\,b\,c\,d\,e+51\,b^2\,e^2-48\,c\,e\,(2\,c\,d-b\,e)\,x\right)\sqrt{b\,x+c\,x^2}}{21\,e^5}+\frac{10\,(16\,c\,d-7\,b\,e+2\,c\,e\,x)\,\left(b\,x+c\,x^2\right)^{3/2}}{21\,e^3\,\sqrt{d+e\,x}}-$$
$$\frac{2\,\left(b\,x+c\,x^2\right)^{5/2}}{3\,e\,(d+e\,x)^{3/2}}-\left(2\sqrt{-b}\,(2\,c\,d-b\,e)\,\left(128\,c^2\,d^2-128\,b\,c\,d\,e+3\,b^2\,e^2\right)\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]\right)\Big/$$
$$\left(21\sqrt{c}\,e^6\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}\right)+$$
$$\left(4\sqrt{-b}\,d\,(c\,d-b\,e)\,\left(128\,c^2\,d^2-128\,b\,c\,d\,e+27\,b^2\,e^2\right)\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]\right)\Big/$$
$$\left(21\sqrt{c}\,e^6\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}\right)$$

Result (type 4, 442 leaves) :

$$\frac{1}{21\,e^6\,x^{5/2}\,(b+c\,x)^3\,\sqrt{d+e\,x}}\,2\,(x\,(b+c\,x))^{5/2}\left(-\frac{\left(256\,c^3\,d^3-384\,b\,c^2\,d^2\,e+134\,b^2\,c\,d\,e^2-3\,b^3\,e^3\right)(b+c\,x)\,(d+e\,x)}{c\sqrt{x}}+\frac{1}{d+e\,x}e\sqrt{x}\,(b+c\,x)\right.$$
$$\left(b^2\,e^2\left(51\,d^2+67\,d\,e\,x+9\,e^2\,x^2\right)+b\,c\,e\left(-176\,d^3-224\,d^2\,e\,x-25\,d\,e^2\,x^2+9\,e^3\,x^3\right)+c^2\left(128\,d^4+160\,d^3\,e\,x+16\,d^2\,e^2\,x^2-6\,d\,e^3\,x^3+3\,e^4\,x^4\right)\right)+$$
$$i\sqrt{\frac{b}{c}}\,e\left(-256\,c^3\,d^3+384\,b\,c^2\,d^2\,e-134\,b^2\,c\,d\,e^2+3\,b^3\,e^3\right)\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]+$$
$$\left.i\sqrt{\frac{b}{c}}\,e\left(128\,c^3\,d^3-208\,b\,c^2\,d^2\,e+83\,b^2\,c\,d\,e^2-3\,b^3\,e^3\right)\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)$$

- **Problem 403: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(b\,x+c\,x^2\right)^{5/2}}{(d+e\,x)^{7/2}}\,dx$$

Optimal (type 4, 392 leaves, 10 steps) :

$$-\frac{2\left(128 c^2 d^2-112 b c d e+15 b^2 e^2+16 c e (2 c d-b e) x\right) \sqrt{b x+c x^2}}{15 e^5 \sqrt{d+e x}}+\frac{2 (16 c d-5 b e+6 c e x) \left(b x+c x^2\right)^{3/2}}{15 e^3 (d+e x)^{3/2}}-$$

$$\frac{2 \left(b x+c x^2\right)^{5/2}}{5 e (d+e x)^{5/2}}+\frac{4 \sqrt{-b} \sqrt{c} \left(128 c^2 d^2-128 b c d e+23 b^2 e^2\right) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{d+e x}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]}{15 e^6 \sqrt{1+\frac{e x}{d}} \sqrt{b x+c x^2}}-$$

$$\left(2 \sqrt{-b} (2 c d-b e) \left(128 c^2 d^2-128 b c d e+15 b^2 e^2\right) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{1+\frac{e x}{d}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/$$

$$\left(15 \sqrt{c}\, e^6 \sqrt{d+e x} \sqrt{b x+c x^2}\right)$$

Result (type 4, 401 leaves) :

$$\frac{1}{15 e^6 x^{5/2} (b+c x)^3 \sqrt{d+e x}} 2 (x (b+c x))^{5/2} \left(\frac{2 \left(128 c^2 d^2-128 b c d e+23 b^2 e^2\right) (b+c x) (d+e x)}{\sqrt{x}}-\frac{1}{(d+e x)^2} e \sqrt{x} (b+c x)\right.$$

$$\left(b^2 e^2 \left(15 d^2+35 d e x+23 e^2 x^2\right)-b c e \left(112 d^3+256 d^2 e x+161 d e^2 x^2+11 e^3 x^3\right)+c^2 \left(128 d^4+288 d^3 e x+176 d^2 e^2 x^2+10 d e^3 x^3-3 e^4 x^4\right)\right)+$$

$$2 i \sqrt{\frac{b}{c}}\, c e \left(128 c^2 d^2-128 b c d e+23 b^2 e^2\right) \sqrt{1+\frac{b}{c x}} \sqrt{1+\frac{d}{e x}}\, x\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right]-$$

$$\left.i \sqrt{\frac{b}{c}}\, c e \left(128 c^2 d^2-144 b c d e+31 b^2 e^2\right) \sqrt{1+\frac{b}{c x}} \sqrt{1+\frac{d}{e x}}\, x\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right]\right)$$

- **Problem 404: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(b x+c x^2\right)^{5/2}}{(d+e x)^{9/2}} \, dx$$

Optimal (type 4, 474 leaves, 10 steps) :

$$\frac{2\,c\,\left(d\,\left(128\,c^2\,d^2-176\,b\,c\,d\,e+51\,b^2\,e^2\right)+e\,\left(32\,c^2\,d^2-32\,b\,c\,d\,e+3\,b^2\,e^2\right)\,x\right)\,\sqrt{b\,x+c\,x^2}}{21\,d\,e^5\,(c\,d-b\,e)\,\sqrt{d+e\,x}}-$$

$$\frac{2\,\left(c\,d^2\,(16\,c\,d-13\,b\,e)+e\,\left(22\,c^2\,d^2-22\,b\,c\,d\,e+3\,b^2\,e^2\right)\,x\right)\,\left(b\,x+c\,x^2\right)^{3/2}}{21\,d\,e^3\,(c\,d-b\,e)\,(d+e\,x)^{5/2}}-\frac{2\,\left(b\,x+c\,x^2\right)^{5/2}}{7\,e\,(d+e\,x)^{7/2}}-$$

$$\left(2\,\sqrt{-b}\,\sqrt{c}\,(2\,c\,d-b\,e)\,\left(128\,c^2\,d^2-128\,b\,c\,d\,e+3\,b^2\,e^2\right)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]\right)\Big/$$

$$\left(21\,d\,e^6\,(c\,d-b\,e)\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}\right)+\frac{4\,\sqrt{-b}\,\sqrt{c}\,\left(128\,c^2\,d^2-128\,b\,c\,d\,e+27\,b^2\,e^2\right)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{21\,e^6\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}$$

Result (type 4, 500 leaves):

$$-\frac{1}{21\,b\,d\,e^6\,(c\,d-b\,e)\,x^3\,(b+c\,x)^3\,(d+e\,x)^{7/2}}\,2\,(x\,(b+c\,x))^{5/2}$$

$$\Bigg(b\,e\,x\,(b+c\,x)\,\left(3\,b^3\,e^6\,x^3-b^2\,c\,d\,e^2\,\left(51\,d^3+169\,d^2\,e\,x+194\,d\,e^2\,x^2+85\,e^3\,x^3\right)-c^3\,d^2\,\left(128\,d^4+416\,d^3\,e\,x+464\,d^2\,e^2\,x^2+186\,d\,e^3\,x^3+7\,e^4\,x^4\right)+\right.$$

$$\left.b\,c^2\,d\,e\,\left(176\,d^4+576\,d^3\,e\,x+649\,d^2\,e^2\,x^2+265\,d\,e^3\,x^3+7\,e^4\,x^4\right)\right)+$$

$$\sqrt{\frac{b}{c}}\,c\,(d+e\,x)^3\,\Bigg(\sqrt{\frac{b}{c}}\,\left(256\,c^3\,d^3-384\,b\,c^2\,d^2\,e+134\,b^2\,c\,d\,e^2-3\,b^3\,e^3\right)\,(b+c\,x)\,(d+e\,x)+$$

$$i\,b\,e\,\left(256\,c^3\,d^3-384\,b\,c^2\,d^2\,e+134\,b^2\,c\,d\,e^2-3\,b^3\,e^3\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]-$$

$$i\,b\,e\,\left(128\,c^3\,d^3-208\,b\,c^2\,d^2\,e+83\,b^2\,c\,d\,e^2-3\,b^3\,e^3\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\Bigg)\Bigg)$$

- **Problem 405: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(b\,x+c\,x^2\right)^{5/2}}{(d+e\,x)^{11/2}}\,dx$$

Optimal (type 4, 570 leaves, 10 steps):

$$-\left(2\left(c\,d^2\left(128\,c^3\,d^3-240\,b\,c^2\,d^2\,e+111\,b^2\,c\,d\,e^2-b^3\,e^3\right)+e\left(160\,c^4\,d^4-320\,b\,c^3\,d^3\,e+171\,b^2\,c^2\,d^2\,e^2-11\,b^3\,c\,d\,e^3-2\,b^4\,e^4\right)x\right)\sqrt{b\,x+c\,x^2}\right)\Big/$$
$$\left(63\,d^2\,e^5\,(c\,d-b\,e)^2\,(d+e\,x)^{3/2}\right)-\frac{2\left(d\left(16\,c^2\,d^2-11\,b\,c\,d\,e-2\,b^2\,e^2\right)+e\left(26\,c^2\,d^2-26\,b\,c\,d\,e+3\,b^2\,e^2\right)x\right)\left(b\,x+c\,x^2\right)^{3/2}}{63\,d\,e^3\,(c\,d-b\,e)\,(d+e\,x)^{7/2}}-\frac{2\left(b\,x+c\,x^2\right)^{5/2}}{9\,e\,(d+e\,x)^{9/2}}+$$
$$\left(4\sqrt{-b}\,\sqrt{c}\left(128\,c^4\,d^4-256\,b\,c^3\,d^3\,e+135\,b^2\,c^2\,d^2\,e^2-7\,b^3\,c\,d\,e^3-b^4\,e^4\right)\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]\right)\Big/$$
$$\left(63\,d^2\,e^6\,(c\,d-b\,e)^2\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}\right)-$$
$$\left(2\sqrt{-b}\,\sqrt{c}\,(2\,c\,d-b\,e)\left(128\,c^2\,d^2-128\,b\,c\,d\,e-b^2\,e^2\right)\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]\right)\Big/$$
$$\left(63\,d\,e^6\,(c\,d-b\,e)\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}\right)$$

Result (type 4, 610 leaves):

$$-\frac{1}{63\, b\, d^2\, e^6\, (c\, d-b\, e)^2\, x^3\, (b+c\, x)^3\, (d+e\, x)^{9/2}}$$

$$2\, (x\, (b+c\, x))^{5/2} \left(b\, e\, x\, (b+c\, x) \left(7\, d^4\, (c\, d-b\, e)^4-19\, d^3\, (c\, d-b\, e)^2 \left(2\, c^2\, d^2-3\, b\, c\, d\, e+b^2\, e^2\right) (d+e\, x)+\right.\right.$$

$$d^2\, (c\, d-b\, e)^2 \left(88\, c^2\, d^2-88\, b\, c\, d\, e+15\, b^2\, e^2\right) (d+e\, x)^2-d\, (c\, d-b\, e) \left(122\, c^3\, d^3-183\, b\, c^2\, d^2\, e+63\, b^2\, c\, d\, e^2-b^3\, e^3\right) (d+e\, x)^3+$$
$$\left.\left(193\, c^4\, d^4-386\, b\, c^3\, d^3\, e+207\, b^2\, c^2\, d^2\, e^2-14\, b^3\, c\, d\, e^3-2\, b^4\, e^4\right) (d+e\, x)^4\right)-$$

$$\sqrt{\frac{b}{c}}\, c\, (d+e\, x)^4 \left(-2 \sqrt{\frac{b}{c}} \left(-128\, c^4\, d^4+256\, b\, c^3\, d^3\, e-135\, b^2\, c^2\, d^2\, e^2+7\, b^3\, c\, d\, e^3+b^4\, e^4\right) (b+c\, x)\, (d+e\, x)+\right.$$

$$2\, i\, b\, e \left(128\, c^4\, d^4-256\, b\, c^3\, d^3\, e+135\, b^2\, c^2\, d^2\, e^2-7\, b^3\, c\, d\, e^3-b^4\, e^4\right) \sqrt{1+\frac{b}{c\, x}} \sqrt{1+\frac{d}{e\, x}}\, x^{3/2}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c\, d}{b\, e}\right]-$$

$$\left.\left. i\, b\, e \left(128\, c^4\, d^4-272\, b\, c^3\, d^3\, e+159\, b^2\, c^2\, d^2\, e^2-13\, b^3\, c\, d\, e^3-2\, b^4\, e^4\right) \sqrt{1+\frac{b}{c\, x}} \sqrt{1+\frac{d}{e\, x}}\, x^{3/2}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c\, d}{b\, e}\right]\right)\right)$$

- **Problem 406: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e\, x)^{7/2}}{\sqrt{b\, x+c\, x^2}}\, dx$$

Optimal (type 4, 379 leaves, 10 steps):

$$\frac{2 e \left(71 c^2 d^2-71 b c d e+24 b^2 e^2\right) \sqrt{d+e x} \sqrt{b x+c x^2}}{105 c^3}+\frac{12 e (2 c d-b e) (d+e x)^{3/2} \sqrt{b x+c x^2}}{35 c^2}+\frac{2 e (d+e x)^{5/2} \sqrt{b x+c x^2}}{7 c}+$$

$$\left(16 \sqrt{-b} (2 c d-b e) \left(11 c^2 d^2-11 b c d e+6 b^2 e^2\right) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{d+e x} \text{ EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/$$

$$\left(105 c^{7/2} \sqrt{1+\frac{e x}{d}} \sqrt{b x+c x^2}\right)-$$

$$\left(2 \sqrt{-b}\, d (c d-b e) \left(71 c^2 d^2-71 b c d e+24 b^2 e^2\right) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{1+\frac{e x}{d}} \text{ EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/$$

$$\left(105 c^{7/2} \sqrt{d+e x} \sqrt{b x+c x^2}\right)$$

Result (type 4, 388 leaves) :

$$\frac{1}{105 c^3 \sqrt{x (b+c x)} \sqrt{d+e x}} 2 \sqrt{x} \left(\frac{8 \left(22 c^3 d^3-33 b c^2 d^2 e+23 b^2 c d e^2-6 b^3 e^3\right) (b+c x) (d+e x)}{c \sqrt{x}}+\right.$$

$$e \sqrt{x} (b+c x) (d+e x) \left(24 b^2 e^2-b c e (89 d+18 e x)+c^2 \left(122 d^2+66 d e x+15 e^2 x^2\right)\right)+$$

$$8 \mathrm{i} \sqrt{\frac{b}{c}}\, e \left(22 c^3 d^3-33 b c^2 d^2 e+23 b^2 c d e^2-6 b^3 e^3\right) \sqrt{1+\frac{b}{c x}} \sqrt{1+\frac{d}{e x}}\, x \text{ EllipticE}\left[\mathrm{i} \text{ ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right]+\frac{1}{b}$$

$$\left.\mathrm{i} \sqrt{\frac{b}{c}} \left(105 c^4 d^4-298 b c^3 d^3 e+353 b^2 c^2 d^2 e^2-208 b^3 c d e^3+48 b^4 e^4\right) \sqrt{1+\frac{b}{c x}} \sqrt{1+\frac{d}{e x}}\, x \text{ EllipticF}\left[\mathrm{i} \text{ ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right]\right)$$

- **Problem 407: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e x)^{5/2}}{\sqrt{b x+c x^2}} \, dx$$

Optimal (type 4, 303 leaves, 9 steps) :

$$\frac{8e(2cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}}{15c^2}+\frac{2e(d+ex)^{3/2}\sqrt{bx+cx^2}}{5c}+$$

$$\frac{2\sqrt{-b}\left(23c^2d^2-23bcde+8b^2e^2\right)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{d+ex}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right],\frac{be}{cd}\right]}{15c^{5/2}\sqrt{1+\frac{ex}{d}}\sqrt{bx+cx^2}}-$$

$$\frac{8\sqrt{-b}\,d(cd-be)(2cd-be)\sqrt{x}\sqrt{1+\frac{cx}{b}}\sqrt{1+\frac{ex}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right],\frac{be}{cd}\right]}{15c^{5/2}\sqrt{d+ex}\sqrt{bx+cx^2}}$$

Result (type 4, 314 leaves):

$$\left(2\sqrt{x}\left(\frac{\left(23c^2d^2-23bcde+8b^2e^2\right)(b+cx)(d+ex)}{c\sqrt{x}}+e\sqrt{x}(b+cx)(d+ex)(11cd-4be+3cex)+\right.\right.$$

$$i\sqrt{\frac{b}{c}}\,e\left(23c^2d^2-23bcde+8b^2e^2\right)\sqrt{1+\frac{b}{cx}}\sqrt{1+\frac{d}{ex}}\,x\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{cd}{be}\right]-1/b\,i\sqrt{\frac{b}{c}}$$

$$\left.\left.\left(-15c^3d^3+34bc^2d^2e-27b^2cde^2+8b^3e^3\right)\sqrt{1+\frac{b}{cx}}\sqrt{1+\frac{d}{ex}}\,x\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{cd}{be}\right]\right)\right)\Big/\left(15c^2\sqrt{x(b+cx)}\sqrt{d+ex}\right)$$

- **Problem 408: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(d+ex)^{3/2}}{\sqrt{bx+cx^2}}\,dx$$

Optimal (type 4, 241 leaves, 8 steps):

$$\frac{2\,e\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}{3\,c}+\frac{4\,\sqrt{-b}\,(2\,c\,d-b\,e)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{3\,c^{3/2}\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}}-$$

$$\frac{2\,\sqrt{-b}\,d\,(c\,d-b\,e)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{3\,c^{3/2}\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}$$

Result (type 4, 246 leaves) :

$$\left(-2\,b\,(b+c\,x)\,(d+e\,x)\,(2\,b\,e-c\,(4\,d+e\,x))-4\,i\,b\,\sqrt{\frac{b}{c}}\,c\,e\,(-2\,c\,d+b\,e)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]+\right.$$

$$\left.2\,i\,\sqrt{\frac{b}{c}}\,c\,\left(3\,c^2\,d^2-5\,b\,c\,d\,e+2\,b^2\,e^2\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\Big/\left(3\,b\,c^2\,\sqrt{x\,(b+c\,x)}\,\sqrt{d+e\,x}\right)$$

- **Problem 412: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{(d+e\,x)^{5/2}\,\sqrt{b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 317 leaves, 9 steps) :

$$-\frac{2\,e\,\sqrt{b\,x+c\,x^2}}{3\,d\,(c\,d-b\,e)\,(d+e\,x)^{3/2}}-\frac{4\,e\,(2\,c\,d-b\,e)\,\sqrt{b\,x+c\,x^2}}{3\,d^2\,(c\,d-b\,e)^2\,\sqrt{d+e\,x}}+\frac{4\,\sqrt{-b}\,\sqrt{c}\,(2\,c\,d-b\,e)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{3\,d^2\,(c\,d-b\,e)^2\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}}-$$

$$\frac{2\,\sqrt{-b}\,\sqrt{c}\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{3\,d\,(c\,d-b\,e)\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}$$

Result (type 4, 290 leaves) :

$$-\left(2\left(-b\,e\,x\,(b+c\,x)\,(b\,e\,(3\,d+2\,e\,x)-c\,d\,(5\,d+4\,e\,x))-\sqrt{\frac{b}{c}}\,c\,(d+e\,x)\left(-2\sqrt{\frac{b}{c}}\,(-2\,c\,d+b\,e)\,(b+c\,x)\,(d+e\,x)+\right.\right.\right.$$

$$2\,i\,b\,e\,(2\,c\,d-b\,e)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]+i\left(3\,c^2\,d^2-5\,b\,c\,d\,e+2\,b^2\,e^2\right)$$

$$\left.\left.\left.\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\right)\right)\Big/\left(3\,b\,d^2\,(c\,d-b\,e)^2\,\sqrt{x\,(b+c\,x)}\,(d+e\,x)^{3/2}\right)$$

Problem 413: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(d+e\,x)^{7/2}\,\sqrt{b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 403 leaves, 10 steps):

$$-\frac{2\,e\,\sqrt{b\,x+c\,x^2}}{5\,d\,(c\,d-b\,e)\,(d+e\,x)^{5/2}}-\frac{8\,e\,(2\,c\,d-b\,e)\,\sqrt{b\,x+c\,x^2}}{15\,d^2\,(c\,d-b\,e)^2\,(d+e\,x)^{3/2}}-\frac{2\,e\,\left(23\,c^2\,d^2-23\,b\,c\,d\,e+8\,b^2\,e^2\right)\,\sqrt{b\,x+c\,x^2}}{15\,d^3\,(c\,d-b\,e)^3\,\sqrt{d+e\,x}}+$$

$$\frac{2\,\sqrt{-b}\,\sqrt{c}\,\left(23\,c^2\,d^2-23\,b\,c\,d\,e+8\,b^2\,e^2\right)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{15\,d^3\,(c\,d-b\,e)^3\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}}-$$

$$\frac{8\,\sqrt{-b}\,\sqrt{c}\,(2\,c\,d-b\,e)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{15\,d^2\,(c\,d-b\,e)^2\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}$$

Result (type 4, 381 leaves):

$$-\frac{1}{15\,b\,d^3\,(c\,d-b\,e)^3\,\sqrt{x\,(b+c\,x)}\,(d+e\,x)^{5/2}}\,2\left(b\,e\,x\,(b+c\,x)\,\left(3\,d^2\,(c\,d-b\,e)^2+4\,d\,(c\,d-b\,e)\,(2\,c\,d-b\,e)\,(d+e\,x)+\left(23\,c^2\,d^2-23\,b\,c\,d\,e+8\,b^2\,e^2\right)\,(d+e\,x)^2\right)-\sqrt{\frac{b}{c}}\,c\,(d+e\,x)^2\left(\sqrt{\frac{b}{c}}\,\left(23\,c^2\,d^2-23\,b\,c\,d\,e+8\,b^2\,e^2\right)\,(b+c\,x)\,(d+e\,x)+i\,b\,e\,\left(23\,c^2\,d^2-23\,b\,c\,d\,e+8\,b^2\,e^2\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]+i\,\left(15\,c^3\,d^3-34\,b\,c^2\,d^2\,e+27\,b^2\,c\,d\,e^2-8\,b^3\,e^3\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\right)$$

- **Problem 414: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(d+e\,x)^{7/2}}{\left(b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 395 leaves, 10 steps):

$$-\frac{2\,(d+e\,x)^{5/2}\,(b\,d+(2\,c\,d-b\,e)\,x)}{b^2\,\sqrt{b\,x+c\,x^2}}+\frac{4\,e\,\left(3\,c^2\,d^2-3\,b\,c\,d\,e+2\,b^2\,e^2\right)\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}{3\,b^2\,c^2}+\frac{2\,e\,(2\,c\,d-b\,e)\,(d+e\,x)^{3/2}\,\sqrt{b\,x+c\,x^2}}{b^2\,c}+\frac{2\,(2\,c\,d-b\,e)\,\left(3\,c^2\,d^2-3\,b\,c\,d\,e+8\,b^2\,e^2\right)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{3\,(-b)^{3/2}\,c^{5/2}\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}}-\frac{4\,d\,(c\,d-b\,e)\,\left(3\,c^2\,d^2-3\,b\,c\,d\,e+2\,b^2\,e^2\right)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{3\,(-b)^{3/2}\,c^{5/2}\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}$$

Result (type 4, 360 leaves):

$$-\frac{1}{3 b^3 c^2 \sqrt{x}\,(b+c x)\,\sqrt{d+e x}}$$

$$2\left(b\,(d+e x)\left(3\,(c d-b e)^3 x+3 c^2 d^3\,(b+c x)-b^2 e^3 x\,(b+c x)\right)-\sqrt{\frac{b}{c}}\left(\sqrt{\frac{b}{c}}\left(6 c^3 d^3-9 b c^2 d^2 e+19 b^2 c d e^2-8 b^3 e^3\right)(b+c x)\,(d+e x)+\right.\right.$$

$$i\, b\, e\left(6 c^3 d^3-9 b c^2 d^2 e+19 b^2 c d e^2-8 b^3 e^3\right)\sqrt{1+\frac{b}{c x}}\,\sqrt{1+\frac{d}{e x}}\;x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]-$$

$$\left.\left.i\, b\, e\left(3 c^3 d^3-18 b c^2 d^2 e+23 b^2 c d e^2-8 b^3 e^3\right)\sqrt{1+\frac{b}{c x}}\,\sqrt{1+\frac{d}{e x}}\;x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]\right)\right)$$

- **Problem 415: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e x)^{5/2}}{\left(b x+c x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 310 leaves, 9 steps) :

$$-\frac{2\,(d+e x)^{3/2}\,(b d+(2 c d-b e)\,x)}{b^2\sqrt{b x+c x^2}}+\frac{2 e\,(2 c d-b e)\,\sqrt{d+e x}\,\sqrt{b x+c x^2}}{b^2 c}+$$

$$\frac{4\left(c^2 d^2-b c d e+b^2 e^2\right)\sqrt{x}\,\sqrt{1+\frac{c x}{b}}\,\sqrt{d+e x}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{(-b)^{3/2} c^{3/2}\sqrt{1+\frac{e x}{d}}\,\sqrt{b x+c x^2}}-$$

$$\frac{2 d\,(c d-b e)\,(2 c d-b e)\,\sqrt{x}\,\sqrt{1+\frac{c x}{b}}\,\sqrt{1+\frac{e x}{d}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{(-b)^{3/2} c^{3/2}\sqrt{d+e x}\,\sqrt{b x+c x^2}}$$

Result (type 4, 262 leaves) :

$$\left(2\,b\,(d+e\,x)\,\left(c^2\,d^2+2\,b^2\,e^2+b\,c\,e\,(-2\,d+e\,x)\right)+4\,i\,\sqrt{\frac{b}{c}}\,c\,e\,\left(c^2\,d^2-b\,c\,d\,e+b^2\,e^2\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]-\right.$$

$$\left.2\,i\,\sqrt{\frac{b}{c}}\,c\,e\,\left(c^2\,d^2-3\,b\,c\,d\,e+2\,b^2\,e^2\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\Big/\left(b^2\,c^2\,\sqrt{x\,(b+c\,x)}\,\sqrt{d+e\,x}\right)$$

- **Problem 416: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(d+e\,x)^{3/2}}{\left(b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 249 leaves, 8 steps) :

$$-\frac{2\,\sqrt{d+e\,x}\,\left(b\,d+(2\,c\,d-b\,e)\,x\right)}{b^2\,\sqrt{b\,x+c\,x^2}}+\frac{2\,(2\,c\,d-b\,e)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{(-b)^{3/2}\,\sqrt{c}\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}}-$$

$$\frac{4\,d\,(c\,d-b\,e)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{(-b)^{3/2}\,\sqrt{c}\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}$$

Result (type 4, 210 leaves) :

$$\left(-2\,i\,\sqrt{\frac{b}{c}}\,c\,e\,(-2\,c\,d+b\,e)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]+\right.$$

$$\left.2\,(c\,d-b\,e)\,\left(b\,(d+e\,x)-i\,\sqrt{\frac{b}{c}}\,c\,e\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\right)\Big/\left(b^2\,c\,\sqrt{x\,(b+c\,x)}\,\sqrt{d+e\,x}\right)$$

■ Problem 417: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{d+e x}}{\left(b x+c x^2\right)^{3/2}} \, dx$$

Optimal (type 4, 231 leaves, 8 steps):

$$-\frac{2 (b+2 c x) \sqrt{d+e x}}{b^2 \sqrt{b x+c x^2}}+\frac{4 \sqrt{c} \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{d+e x} \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{(-b)^{3/2} \sqrt{1+\frac{e x}{d}} \sqrt{b x+c x^2}}-$$
$$\frac{2 (2 c d-b e) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{1+\frac{e x}{d}} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{(-b)^{3/2} \sqrt{c} \sqrt{d+e x} \sqrt{b x+c x^2}}$$

Result (type 4, 186 leaves):

$$\left(2 \sqrt{\frac{b}{c}} (d+e x)+4 i e \sqrt{1+\frac{b}{c x}} \sqrt{1+\frac{d}{e x}} x^{3/2} \, \text{EllipticE}\left[i \, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]-\right.$$
$$\left.2 i e \sqrt{1+\frac{b}{c x}} \sqrt{1+\frac{d}{e x}} x^{3/2} \, \text{EllipticF}\left[i \, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]\right) \Big/ \left(b \sqrt{\frac{b}{c}} \sqrt{x (b+c x)} \sqrt{d+e x}\right)$$

■ Problem 418: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{d+e x} \left(b x+c x^2\right)^{3/2}} \, dx$$

Optimal (type 4, 274 leaves, 8 steps):

$$-\frac{2\sqrt{d+e x}\,(b\,(c d-b e)+c\,(2 c d-b e)\,x)}{b^2 d\,(c d-b e)\sqrt{b x+c x^2}}+\frac{2\sqrt{c}\,(2 c d-b e)\sqrt{x}\sqrt{1+\frac{c x}{b}}\sqrt{d+e x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{(-b)^{3/2} d\,(c d-b e)\sqrt{1+\frac{e x}{d}}\sqrt{b x+c x^2}}-$$

$$\frac{4\sqrt{c}\sqrt{x}\sqrt{1+\frac{c x}{b}}\sqrt{1+\frac{e x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{(-b)^{3/2}\sqrt{d+e x}\sqrt{b x+c x^2}}$$

Result (type 4, 220 leaves) :

$$\left(-2 b c d\,(d+e x)+2 i\sqrt{\frac{b}{c}}\,c e\,(-2 c d+b e)\sqrt{1+\frac{b}{c x}}\sqrt{1+\frac{d}{e x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]-\right.$$

$$\left.2 i\sqrt{\frac{b}{c}}\,c e\,(-c d+b e)\sqrt{1+\frac{b}{c x}}\sqrt{1+\frac{d}{e x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]\right)\Big/\left(b^2 d\,(-c d+b e)\sqrt{x\,(b+c x)}\sqrt{d+e x}\right)$$

- **Problem 419: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{(d+e x)^{3/2}\left(b x+c x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 370 leaves, 9 steps) :

$$-\frac{2\,(b\,(c d-b e)+c\,(2 c d-b e)\,x)}{b^2 d\,(c d-b e)\sqrt{d+e x}\sqrt{b x+c x^2}}-\frac{4 e\left(c^2 d^2-b c d e+b^2 e^2\right)\sqrt{b x+c x^2}}{b^2 d^2\,(c d-b e)^2\sqrt{d+e x}}+$$

$$\frac{4\sqrt{c}\left(c^2 d^2-b c d e+b^2 e^2\right)\sqrt{x}\sqrt{1+\frac{c x}{b}}\sqrt{d+e x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{(-b)^{3/2} d^2\,(c d-b e)^2\sqrt{1+\frac{e x}{d}}\sqrt{b x+c x^2}}-$$

$$\frac{2\sqrt{c}\,(2 c d-b e)\sqrt{x}\sqrt{1+\frac{c x}{b}}\sqrt{1+\frac{e x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{(-b)^{3/2} d\,(c d-b e)\sqrt{d+e x}\sqrt{b x+c x^2}}$$

Result (type 4, 266 leaves) :

$$\left(2\,b\,d\,\left(b^2\,e^2+b\,c\,e^2\,x+c^2\,d\,(d+e\,x)\right)+4\,i\,\sqrt{\frac{b}{c}}\,c\,e\,\left(c^2\,d^2-b\,c\,d\,e+b^2\,e^2\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]-\right.$$

$$\left.2\,i\,\sqrt{\frac{b}{c}}\,c\,e\,\left(c^2\,d^2-3\,b\,c\,d\,e+2\,b^2\,e^2\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\Big/\left(b^2\,d^2\,(c\,d-b\,e)^2\,\sqrt{x\,(b+c\,x)}\,\sqrt{d+e\,x}\right)$$

- **Problem 420: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{(d+e\,x)^{5/2}\,\left(b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 478 leaves, 10 steps) :

$$-\frac{2\,(b\,(c\,d-b\,e)+c\,(2\,c\,d-b\,e)\,x)}{b^2\,d\,(c\,d-b\,e)\,(d+e\,x)^{3/2}\,\sqrt{b\,x+c\,x^2}}-\frac{4\,e\,\left(3\,c^2\,d^2-3\,b\,c\,d\,e+2\,b^2\,e^2\right)\,\sqrt{b\,x+c\,x^2}}{3\,b^2\,d^2\,(c\,d-b\,e)^2\,(d+e\,x)^{3/2}}-\frac{2\,e\,(2\,c\,d-b\,e)\,\left(3\,c^2\,d^2-3\,b\,c\,d\,e+8\,b^2\,e^2\right)\,\sqrt{b\,x+c\,x^2}}{3\,b^2\,d^3\,(c\,d-b\,e)^3\,\sqrt{d+e\,x}}+$$

$$\frac{2\,\sqrt{c}\,(2\,c\,d-b\,e)\,\left(3\,c^2\,d^2-3\,b\,c\,d\,e+8\,b^2\,e^2\right)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{3\,(-b)^{3/2}\,d^3\,(c\,d-b\,e)^3\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}}-$$

$$\frac{4\,\sqrt{c}\,\left(3\,c^2\,d^2-3\,b\,c\,d\,e+2\,b^2\,e^2\right)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{3\,(-b)^{3/2}\,d^2\,(c\,d-b\,e)^2\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}$$

Result (type 4, 420 leaves) :

$$-\frac{1}{3\,b^3\,d^3\,(c\,d-b\,e)^3\,\sqrt{x\,(b+c\,x)}\,\,(d+e\,x)^{3/2}}$$

$$2\left(b\left(b^2\,d\,e^3\,(c\,d-b\,e)\,x\,(b+c\,x)-5\,b^2\,e^3\,(-2\,c\,d+b\,e)\,x\,(b+c\,x)\,(d+e\,x)+3\,c^4\,d^3\,x\,(d+e\,x)^2+3\,(c\,d-b\,e)^3\,(b+c\,x)\,(d+e\,x)^2\right)-\right.$$

$$\sqrt{\frac{b}{c}}\,c\,(d+e\,x)\left(\sqrt{\frac{b}{c}}\,\left(6\,c^3\,d^3-9\,b\,c^2\,d^2\,e+19\,b^2\,c\,d\,e^2-8\,b^3\,e^3\right)\,(b+c\,x)\,(d+e\,x)+\right.$$

$$i\,b\,e\,\left(6\,c^3\,d^3-9\,b\,c^2\,d^2\,e+19\,b^2\,c\,d\,e^2-8\,b^3\,e^3\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]-$$

$$\left.\left.i\,b\,e\,\left(3\,c^3\,d^3-18\,b\,c^2\,d^2\,e+23\,b^2\,c\,d\,e^2-8\,b^3\,e^3\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\right)$$

- **Problem 421: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(d+e\,x)^{9/2}}{\left(b\,x+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 470 leaves, 10 steps) :

$$-\frac{2\,(d+e\,x)^{7/2}\,(b\,d+(2\,c\,d-b\,e)\,x)}{3\,b^2\,\left(b\,x+c\,x^2\right)^{3/2}}+\frac{2\,(d+e\,x)^{3/2}\,\left(b\,c\,d^2\,(8\,c\,d-11\,b\,e)+(2\,c\,d-b\,e)\,\left(8\,c^2\,d^2-8\,b\,c\,d\,e-3\,b^2\,e^2\right)\,x\right)}{3\,b^4\,c\,\sqrt{b\,x+c\,x^2}}-\frac{8\,e\,\left(4\,c^3\,d^3-6\,b\,c^2\,d^2\,e+b^3\,e^3\right)\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}{3\,b^4\,c^2}-$$

$$\left(2\,\left(16\,c^4\,d^4-32\,b\,c^3\,d^3\,e+9\,b^2\,c^2\,d^2\,e^2+7\,b^3\,c\,d\,e^3-8\,b^4\,e^4\right)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\,\frac{b\,e}{c\,d}\right]\right)\Big/$$

$$\left(3\,(-b)^{7/2}\,c^{5/2}\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}\right)+$$

$$\left(8\,d\,(c\,d-b\,e)\,(2\,c\,d-b\,e)\,\left(2\,c^2\,d^2-2\,b\,c\,d\,e-b^2\,e^2\right)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\,\frac{b\,e}{c\,d}\right]\right)\Big/$$

$$\left(3\,(-b)^{7/2}\,c^{5/2}\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}\right)$$

Result (type 4, 451 leaves) :

$$\frac{1}{3\,b^5\,c^2\,(x\,(b+c\,x))^{3/2}\,\sqrt{d+e\,x}}$$

$$\left(2\,\left(b\,(d+e\,x)\,\left(b\,(c\,d-b\,e)^4\,x^2+(c\,d-b\,e)^3\,(8\,c\,d+5\,b\,e)\,x^2\,(b+c\,x)-b\,c^2\,d^4\,(b+c\,x)^2+c^2\,d^3\,(8\,c\,d-13\,b\,e)\,x\,(b+c\,x)^2\right)-\right.\right.$$

$$\sqrt{\frac{b}{c}}\,x\,(b+c\,x)\,\left(\sqrt{\frac{b}{c}}\,\left(16\,c^4\,d^4-32\,b\,c^3\,d^3\,e+9\,b^2\,c^2\,d^2\,e^2+7\,b^3\,c\,d\,e^3-8\,b^4\,e^4\right)\,(b+c\,x)\,(d+e\,x)+\right.$$

$$i\,b\,e\,\left(16\,c^4\,d^4-32\,b\,c^3\,d^3\,e+9\,b^2\,c^2\,d^2\,e^2+7\,b^3\,c\,d\,e^3-8\,b^4\,e^4\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\,\frac{c\,d}{b\,e}\right]-$$

$$\left.\left.\left.i\,b\,e\,\left(8\,c^4\,d^4-17\,b\,c^3\,d^3\,e+6\,b^2\,c^2\,d^2\,e^2+11\,b^3\,c\,d\,e^3-8\,b^4\,e^4\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\,\frac{c\,d}{b\,e}\right]\right)\right)\right)$$

- **Problem 422: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e\,x)^{7/2}}{\left(b\,x+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 383 leaves, 9 steps):

$$-\frac{2\,(d+e\,x)^{5/2}\,(b\,d+(2\,c\,d-b\,e)\,x)}{3\,b^2\,\left(b\,x+c\,x^2\right)^{3/2}}+\frac{2\,\sqrt{d+e\,x}\,\left(b\,c\,d^2\,(8\,c\,d-9\,b\,e)+(2\,c\,d-b\,e)\,\left(8\,c^2\,d^2-8\,b\,c\,d\,e-b^2\,e^2\right)\,x\right)}{3\,b^4\,c\,\sqrt{b\,x+c\,x^2}}-$$

$$\frac{4\,(2\,c\,d-b\,e)\,\left(4\,c^2\,d^2-4\,b\,c\,d\,e-b^2\,e^2\right)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{3\,(-b)^{7/2}\,c^{3/2}\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}}+$$

$$\frac{2\,d\,(c\,d-b\,e)\,\left(16\,c^2\,d^2-16\,b\,c\,d\,e-b^2\,e^2\right)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{3\,(-b)^{7/2}\,c^{3/2}\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}$$

Result (type 4, 405 leaves):

$$\frac{1}{3\,b^5\,c\,(x\,(b+c\,x))^{3/2}\,\sqrt{d+e\,x}}$$

$$2\left(b\,(d+e\,x)\,\left(b\,(c\,d-b\,e)^3\,x^2+2\,(c\,d-b\,e)^2\,(4\,c\,d+b\,e)\,x^2\,(b+c\,x)-b\,c\,d^3\,(b+c\,x)^2+2\,c\,d^2\,(4\,c\,d-5\,b\,e)\,x\,(b+c\,x)^2\right)-\right.$$

$$\sqrt{\frac{b}{c}}\,x\,(b+c\,x)\left(2\,\sqrt{\frac{b}{c}}\,\left(8\,c^3\,d^3-12\,b\,c^2\,d^2\,e+2\,b^2\,c\,d\,e^2+b^3\,e^3\right)\,(b+c\,x)\,(d+e\,x)+\right.$$

$$2\,i\,b\,e\,\left(8\,c^3\,d^3-12\,b\,c^2\,d^2\,e+2\,b^2\,c\,d\,e^2+b^3\,e^3\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\,\frac{c\,d}{b\,e}\right]-$$

$$\left.\left.i\,b\,e\,\left(8\,c^3\,d^3-13\,b\,c^2\,d^2\,e+3\,b^2\,c\,d\,e^2+2\,b^3\,e^3\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\,\frac{c\,d}{b\,e}\right]\right)\right)$$

- **Problem 423: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e\,x)^{5/2}}{\left(b\,x+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 343 leaves, 9 steps) :

$$-\frac{2\,(d+e\,x)^{3/2}\,(b\,d+(2\,c\,d-b\,e)\,x)}{3\,b^2\,\left(b\,x+c\,x^2\right)^{3/2}}+\frac{2\,\sqrt{d+e\,x}\,\left(b\,d\,(8\,c\,d-7\,b\,e)+\left(16\,c^2\,d^2-16\,b\,c\,d\,e+b^2\,e^2\right)\,x\right)}{3\,b^4\,\sqrt{b\,x+c\,x^2}}-$$

$$\frac{2\,\left(16\,c^2\,d^2-16\,b\,c\,d\,e+b^2\,e^2\right)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{3\,(-b)^{7/2}\,\sqrt{c}\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}}+$$

$$\frac{16\,d\,(c\,d-b\,e)\,(2\,c\,d-b\,e)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{3\,(-b)^{7/2}\,\sqrt{c}\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}$$

Result (type 4, 353 leaves) :

$$\frac{1}{3\,b^5\,(x\,(b+c\,x))^{3/2}\,\sqrt{d+e\,x}}$$

$$2\left(b\,(d+e\,x)\,\left(16\,c^3\,d^2\,x^3+8\,b\,c^2\,d\,x^2\,(3\,d-2\,e\,x)-b^3\,\left(d^2+7\,d\,e\,x-2\,e^2\,x^2\right)+b^2\,c\,x\,\left(6\,d^2-25\,d\,e\,x+e^2\,x^2\right)\right)-\sqrt{\frac{b}{c}}\,x\,(b+c\,x)\right.$$

$$\left(\sqrt{\frac{b}{c}}\,\left(16\,c^2\,d^2-16\,b\,c\,d\,e+b^2\,e^2\right)\,(b+c\,x)\,(d+e\,x)+i\,b\,e\,\left(16\,c^2\,d^2-16\,b\,c\,d\,e+b^2\,e^2\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\right.\right.$$

$$\left.\left.\left.\frac{c\,d}{b\,e}\right]-i\,b\,e\,\left(8\,c^2\,d^2-9\,b\,c\,d\,e+b^2\,e^2\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\right)$$

- **Problem 424: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(d+e\,x)^{3/2}}{\left(b\,x+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 344 leaves, 9 steps) :

$$-\frac{2\sqrt{d+e x}\,(b d+(2 c d-b e) x)}{3 b^2 \left(b x+c x^2\right)^{3/2}}+\frac{2\sqrt{d+e x}\,(b (8 c d-5 b e) (c d-b e)+8 c (c d-b e) (2 c d-b e) x)}{3 b^4 (c d-b e) \sqrt{b x+c x^2}}-$$

$$\frac{16\sqrt{c}\,(2 c d-b e)\sqrt{x}\sqrt{1+\frac{c x}{b}}\sqrt{d+e x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{3 (-b)^{7/2}\sqrt{1+\frac{e x}{d}}\sqrt{b x+c x^2}}+$$

$$\frac{2 (4 c d-3 b e) (4 c d-b e)\sqrt{x}\sqrt{1+\frac{c x}{b}}\sqrt{1+\frac{e x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{3 (-b)^{7/2}\sqrt{c}\sqrt{d+e x}\sqrt{b x+c x^2}}$$

Result (type 4, 290 leaves) :

$$\left(2\left(-8 (2 c d-b e) x (b+c x) (d+e x)+\frac{(d+e x)\left(16 c^3 d x^3+b^2 c x (6 d-13 e x)-8 b c^2 x^2 (-3 d+e x)-b^3 (d+4 e x)\right)}{b+c x}+\right.\right.$$

$$8 i\sqrt{\frac{b}{c}}\,c e (-2 c d+b e)\sqrt{1+\frac{b}{c x}}\sqrt{1+\frac{d}{e x}}\,x^{5/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]+$$

$$\left.\left.i\sqrt{\frac{b}{c}}\,c e (8 c d-5 b e)\sqrt{1+\frac{b}{c x}}\sqrt{1+\frac{d}{e x}}\,x^{5/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]\right)\right)\Big/\left(3 b^4 x\sqrt{x (b+c x)}\sqrt{d+e x}\right)$$

- **Problem 425: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{d+e x}}{\left(b x+c x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 359 leaves, 9 steps) :

$$-\frac{2\,(b+2\,c\,x)\,\sqrt{d+e\,x}}{3\,b^2\,\left(b\,x+c\,x^2\right)^{3/2}}+\frac{2\,\sqrt{d+e\,x}\,\left(b\,(c\,d-b\,e)\,(8\,c\,d-b\,e)+c\,\left(16\,c^2\,d^2-16\,b\,c\,d\,e+b^2\,e^2\right)\,x\right)}{3\,b^4\,d\,(c\,d-b\,e)\,\sqrt{b\,x+c\,x^2}}-$$

$$\frac{2\,\sqrt{c}\,\left(16\,c^2\,d^2-16\,b\,c\,d\,e+b^2\,e^2\right)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{3\,(-b)^{7/2}\,d\,(c\,d-b\,e)\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}}+$$

$$\frac{16\,\sqrt{c}\,(2\,c\,d-b\,e)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{3\,(-b)^{7/2}\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}$$

Result (type 4, 375 leaves) :

$$\frac{1}{3\,b^5\,d\,(c\,d-b\,e)\,(x\,(b+c\,x))^{3/2}\,\sqrt{d+e\,x}}$$

$$2\left(b\,(d+e\,x)\,\left(b\,c^2\,d\,(c\,d-b\,e)\,x^2+c^2\,d\,(8\,c\,d-7\,b\,e)\,x^2\,(b+c\,x)+b\,d\,(-c\,d+b\,e)\,(b+c\,x)^2+(c\,d-b\,e)\,(8\,c\,d-b\,e)\,x\,(b+c\,x)^2\right)-\right.$$

$$\sqrt{\frac{b}{c}}\,c\,x\,(b+c\,x)\left(\sqrt{\frac{b}{c}}\,\left(16\,c^2\,d^2-16\,b\,c\,d\,e+b^2\,e^2\right)\,(b+c\,x)\,(d+e\,x)+i\,b\,e\,\left(16\,c^2\,d^2-16\,b\,c\,d\,e+b^2\,e^2\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\right.$$

$$\left.\left.\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]-i\,b\,e\,\left(8\,c^2\,d^2-9\,b\,c\,d\,e+b^2\,e^2\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\right)$$

- **Problem 426: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{d+e\,x}\,\left(b\,x+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 451 leaves, 9 steps) :

$$-\frac{2\sqrt{d+e x}\,(b(c d-b e)+c(2 c d-b e)x)}{3 b^2 d(c d-b e)\left(b x+c x^2\right)^{3/2}}+\frac{2\sqrt{d+e x}\left(b(c d-b e)\left(8 c^2 d^2-5 b c d e-2 b^2 e^2\right)+2 c(2 c d-b e)\left(4 c^2 d^2-4 b c d e-b^2 e^2\right)x\right)}{3 b^4 d^2(c d-b e)^2\sqrt{b x+c x^2}}-$$

$$\frac{4\sqrt{c}\,(2 c d-b e)\left(4 c^2 d^2-4 b c d e-b^2 e^2\right)\sqrt{x}\sqrt{1+\frac{c x}{b}}\sqrt{d+e x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{3(-b)^{7/2}d^2(c d-b e)^2\sqrt{1+\frac{e x}{d}}\sqrt{b x+c x^2}}+$$

$$\frac{2\sqrt{c}\left(16 c^2 d^2-16 b c d e-b^2 e^2\right)\sqrt{x}\sqrt{1+\frac{c x}{b}}\sqrt{1+\frac{e x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{3(-b)^{7/2}d(c d-b e)\sqrt{d+e x}\sqrt{b x+c x^2}}$$

Result (type 4, 429 leaves) :

$$\frac{1}{3 b^5 d^2(c d-b e)^2(x(b+c x))^{3/2}\sqrt{d+e x}}$$

$$2\left(b(d+e x)\left(b c^3 d^2(c d-b e)x^2+2 c^3 d^2(4 c d-5 b e)x^2(b+c x)-b d(c d-b e)^2(b+c x)^2+2(c d-b e)^2(4 c d+b e)x(b+c x)^2\right)-\right.$$

$$\sqrt{\frac{b}{c}}\,c\,x(b+c x)\left(2\sqrt{\frac{b}{c}}\left(8 c^3 d^3-12 b c^2 d^2 e+2 b^2 c d e^2+b^3 e^3\right)(b+c x)(d+e x)+\right.$$

$$2\,\mathbb{i}\,b e\left(8 c^3 d^3-12 b c^2 d^2 e+2 b^2 c d e^2+b^3 e^3\right)\sqrt{1+\frac{b}{c x}}\sqrt{1+\frac{d}{e x}}\,x^{3/2}\,\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]-$$

$$\left.\left.\mathbb{i}\,b e\left(8 c^3 d^3-13 b c^2 d^2 e+3 b^2 c d e^2+2 b^3 e^3\right)\sqrt{1+\frac{b}{c x}}\sqrt{1+\frac{d}{e x}}\,x^{3/2}\,\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]\right)\right)$$

- **Problem 427: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(d+e x)^{3/2} \left(b x+c x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 567 leaves, 10 steps):

$$-\frac{2 (b (c d-b e)+c (2 c d-b e) x)}{3 b^2 d (c d-b e) \sqrt{d+e x} \left(b x+c x^2\right)^{3/2}}+\frac{2 \left(b (c d-b e) \left(8 c^2 d^2-3 b c d e-4 b^2 e^2\right)+4 c \left(4 c^3 d^3-6 b c^2 d^2 e+b^3 e^3\right) x\right)}{3 b^4 d^2 (c d-b e)^2 \sqrt{d+e x} \sqrt{b x+c x^2}}+$$
$$\frac{2 e \left(16 c^4 d^4-32 b c^3 d^3 e+9 b^2 c^2 d^2 e^2+7 b^3 c d e^3-8 b^4 e^4\right) \sqrt{b x+c x^2}}{3 b^4 d^3 (c d-b e)^3 \sqrt{d+e x}}-$$
$$\left(2 \sqrt{c} \left(16 c^4 d^4-32 b c^3 d^3 e+9 b^2 c^2 d^2 e^2+7 b^3 c d e^3-8 b^4 e^4\right) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{d+e x} \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/$$
$$\left(3 (-b)^{7/2} d^3 (c d-b e)^3 \sqrt{1+\frac{e x}{d}} \sqrt{b x+c x^2}\right)+$$
$$\frac{8 \sqrt{c} (2 c d-b e) \left(2 c^2 d^2-2 b c d e-b^2 e^2\right) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{1+\frac{e x}{d}} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]}{3 (-b)^{7/2} d^2 (c d-b e)^2 \sqrt{d+e x} \sqrt{b x+c x^2}}$$

Result (type 4, 504 leaves):

$$-\frac{1}{3\,b^5\,d^3\,(c\,d-b\,e)^3\,(x\,(b+c\,x))^{3/2}\,\sqrt{d+e\,x}}$$

$$2\left(b\left(3\,b^4\,e^5\,x^2\,(b+c\,x)^2+b\,c^4\,d^3\,(-c\,d+b\,e)\,x^2\,(d+e\,x)-c^4\,d^3\,(8\,c\,d-13\,b\,e)\,x^2\,(b+c\,x)\,(d+e\,x)+b\,d\,(c\,d-b\,e)^3\,(b+c\,x)^2\,(d+e\,x)-\right.\right.$$

$$\left.(c\,d-b\,e)^3\,(8\,c\,d+5\,b\,e)\,x\,(b+c\,x)^2\,(d+e\,x)\right)+\sqrt{\frac{b}{c}}\,c\,x\,(b+c\,x)\left(\sqrt{\frac{b}{c}}\left(16\,c^4\,d^4-32\,b\,c^3\,d^3\,e+9\,b^2\,c^2\,d^2\,e^2+7\,b^3\,c\,d\,e^3-8\,b^4\,e^4\right)(b+c\,x)\right.$$

$$(d+e\,x)+\mathrm{i}\,b\,e\left(16\,c^4\,d^4-32\,b\,c^3\,d^3\,e+9\,b^2\,c^2\,d^2\,e^2+7\,b^3\,c\,d\,e^3-8\,b^4\,e^4\right)\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\mathrm{EllipticE}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]-$$

$$\left.\left.\mathrm{i}\,b\,e\left(8\,c^4\,d^4-17\,b\,c^3\,d^3\,e+6\,b^2\,c^2\,d^2\,e^2+11\,b^3\,c\,d\,e^3-8\,b^4\,e^4\right)\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\right)$$

- **Problem 428: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{d+e\,x}}{\sqrt{2\,x-3\,x^2}}\,dx$$

Optimal (type 4, 51 leaves, 4 steps) :

$$\frac{2\,\sqrt{d+e\,x}\,\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\sqrt{\frac{3}{2}}\,\sqrt{x}\right],-\frac{2\,e}{3\,d}\right]}{\sqrt{3}\,\sqrt{1+\frac{e\,x}{d}}}$$

Result (type 4, 117 leaves) :

$$\frac{2\,\sqrt{-\frac{d}{e}}\,(-2+3\,x)\,(d+e\,x)-2\,d\,\sqrt{9-\frac{6}{x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\frac{\sqrt{-\frac{d}{e}}}{\sqrt{x}}\right],-\frac{2\,e}{3\,d}\right]}{3\,\sqrt{-\frac{d}{e}}\,\sqrt{-x\,(-2+3\,x)}\,\sqrt{d+e\,x}}$$

Problem 430: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{d+e x}}{\sqrt{-2 x-3 x^2}} \, dx$$

Optimal (type 4, 53 leaves, 4 steps):

$$-\frac{2 \sqrt{d+e x} \,\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{3}{2}} \sqrt{-x}\right],\frac{2 e}{3 d}\right]}{\sqrt{3} \sqrt{1+\frac{e x}{d}}}$$

Result (type 4, 117 leaves):

$$\frac{2 \sqrt{-\frac{d}{e}} (2+3 x) (d+e x)-2 d \sqrt{9+\frac{6}{x}} \sqrt{1+\frac{d}{e x}} \, x^{3/2} \,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-\frac{d}{e}}}{\sqrt{x}}\right],\frac{2 e}{3 d}\right]}{3 \sqrt{-\frac{d}{e}} \sqrt{-x (2+3 x)} \sqrt{d+e x}}$$

Problem 431: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{d+e x} \sqrt{-2 x-3 x^2}} \, dx$$

Optimal (type 4, 53 leaves, 4 steps):

$$-\frac{2 \sqrt{1+\frac{e x}{d}} \,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{3}{2}} \sqrt{-x}\right],\frac{2 e}{3 d}\right]}{\sqrt{3} \sqrt{d+e x}}$$

Result (type 4, 82 leaves):

$$\frac{i \sqrt{6+\frac{4}{x}} \sqrt{1+\frac{d}{e x}} \, x^{3/2} \,\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3 d}{2 e}\right]}{\sqrt{-x (2+3 x)} \sqrt{d+e x}}$$

Problem 432: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x}}{\sqrt{-x} \sqrt{1+x}} \, dx$$

Optimal (type 4, 12 leaves, 1 step):

$$-2 \,\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{-x}\right],-1\right]$$

Result (type 4, 60 leaves) :

$$-\frac{2\sqrt{2}\sqrt{x}\sqrt{1+x}\left(-\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{1-x}\right],\frac{1}{2}\right]+\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{1-x}\right],\frac{1}{2}\right]\right)}{\sqrt{-x(1+x)}}$$

- **Problem 433: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{1-x}}{\sqrt{-x-x^2}}\,dx$$

Optimal (type 4, 12 leaves, 2 steps) :

$$-2\,\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{-x}\right],-1\right]$$

Result (type 4, 60 leaves) :

$$-\frac{2\sqrt{2}\sqrt{x}\sqrt{1+x}\left(-\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{1-x}\right],\frac{1}{2}\right]+\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{1-x}\right],\frac{1}{2}\right]\right)}{\sqrt{-x(1+x)}}$$

- **Problem 444: Unable to integrate problem.**

$$\int \frac{(d+e\,x)^m}{\left(b\,x+c\,x^2\right)^2}\,dx$$

Optimal (type 5, 180 leaves, 5 steps) :

$$-\frac{(d+e\,x)^{1+m}\,(b\,(c\,d-b\,e)+c\,(2\,c\,d-b\,e)\,x)}{b^2\,d\,(c\,d-b\,e)\,\left(b\,x+c\,x^2\right)}-\frac{c^2\,(2\,c\,d-b\,e\,(2-m))\,(d+e\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{c\,(d+e\,x)}{c\,d-b\,e}\right]}{b^3\,(c\,d-b\,e)^2\,(1+m)}+$$
$$\frac{(2\,c\,d-b\,e\,m)\,(d+e\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,1+\frac{e\,x}{d}\right]}{b^3\,d^2\,(1+m)}$$

Result (type 8, 21 leaves) :

$$\int \frac{(d+e\,x)^m}{\left(b\,x+c\,x^2\right)^2}\,dx$$

- **Problem 445: Unable to integrate problem.**

$$\int \frac{(d+e\,x)^m}{\left(b\,x+c\,x^2\right)^3}\,dx$$

Optimal (type 5, 350 leaves, 6 steps) :

$$-\frac{(d+e x)^{1+m} (b (c d-b e)+c (2 c d-b e) x)}{2 b^2 d (c d-b e) \left(b x+c x^2\right)^2}+$$
$$\left((d+e x)^{1+m} \left(b (c d-b e) \left(6 c^2 d^2-b^2 e^2 (1-m)-b c d e (4+m)\right)+c (2 c d-b e) \left(6 c^2 d^2-6 b c d e-b^2 e^2 (1-m)\right) x\right)\right) /$$
$$\left(2 b^4 d^2 (c d-b e)^2 \left(b x+c x^2\right)\right)+\frac{1}{2 b^5 (c d-b e)^3 (1+m)}$$
$$c^3 \left(12 c^2 d^2-6 b c d e (4-m)+b^2 e^2 \left(12-7 m+m^2\right)\right) (d+e x)^{1+m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{c (d+e x)}{c d-b e}\right]-$$
$$\frac{\left(12 c^2 d^2-6 b c d e m-b^2 e^2 (1-m) m\right) (d+e x)^{1+m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, 1+\frac{e x}{d}\right]}{2 b^5 d^3 (1+m)}$$

Result (type 8, 21 leaves) :

$$\int \frac{(d+e x)^m}{\left(b x+c x^2\right)^3} \, dx$$

- **Problem 446: Result more than twice size of optimal antiderivative.**

$$\int (d+e x)^m \left(b x+c x^2\right)^{3/2} \, dx$$

Optimal (type 6, 105 leaves, 2 steps) :

$$\frac{(d+e x)^{1+m} \left(b x+c x^2\right)^{3/2} \text{AppellF1}\left[1+m, -\frac{3}{2}, -\frac{3}{2}, 2+m, \frac{d+e x}{d}, \frac{c (d+e x)}{c d-b e}\right]}{e (1+m) \left(-\frac{e x}{d}\right)^{3/2} \left(1-\frac{c (d+e x)}{c d-b e}\right)^{3/2}}$$

Result (type 6, 289 leaves) :

$$\frac{2}{35} b d x^2 \sqrt{x (b+c x)} (d+e x)^m \left(\left(49 b \text{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}, -m, \frac{7}{2}, -\frac{c x}{b}, -\frac{e x}{d}\right]\right) / \left(7 b d \text{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}, -m, \frac{7}{2}, -\frac{c x}{b}, -\frac{e x}{d}\right]+\right.\right.$$
$$\left.2 b e m x \text{AppellF1}\left[\frac{7}{2}, -\frac{1}{2}, 1-m, \frac{9}{2}, -\frac{c x}{b}, -\frac{e x}{d}\right]+c d x \text{AppellF1}\left[\frac{7}{2}, \frac{1}{2}, -m, \frac{9}{2}, -\frac{c x}{b}, -\frac{e x}{d}\right]\right)+$$
$$\left(45 c x \text{AppellF1}\left[\frac{7}{2}, -\frac{1}{2}, -m, \frac{9}{2}, -\frac{c x}{b}, -\frac{e x}{d}\right]\right) / \left(9 b d \text{AppellF1}\left[\frac{7}{2}, -\frac{1}{2}, -m, \frac{9}{2}, -\frac{c x}{b}, -\frac{e x}{d}\right]+\right.$$
$$\left.\left.2 b e m x \text{AppellF1}\left[\frac{9}{2}, -\frac{1}{2}, 1-m, \frac{11}{2}, -\frac{c x}{b}, -\frac{e x}{d}\right]+c d x \text{AppellF1}\left[\frac{9}{2}, \frac{1}{2}, -m, \frac{11}{2}, -\frac{c x}{b}, -\frac{e x}{d}\right]\right)\right)$$

- **Problem 449: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d+e x)^m}{\left(b x+c x^2\right)^{3/2}} \, dx$$

Optimal (type 6, 105 leaves, 2 steps) :

$$\frac{\left(-\frac{e x}{d}\right)^{3/2} (d+e x)^{1+m} \left(1-\frac{c (d+e x)}{c d-b e}\right)^{3/2} \text{AppellF1}\left[1+m,\frac{3}{2},\frac{3}{2},2+m,\frac{d+e x}{d},\frac{c (d+e x)}{c d-b e}\right]}{e (1+m) \left(b x+c x^2\right)^{3/2}}$$

Result (type 6, 433 leaves) :

$$\frac{1}{(x (b+c x))^{3/2}} 2 d x (d+e x)^m \left(-\left((b+c x)^2 \text{AppellF1}\left[-\frac{1}{2},-\frac{1}{2},-m,\frac{1}{2},-\frac{c x}{b},-\frac{e x}{d}\right]\right) \Big/ \left(b \left(b d \text{AppellF1}\left[-\frac{1}{2},-\frac{1}{2},-m,\frac{1}{2},-\frac{c x}{b},-\frac{e x}{d}\right]+2 b e m x \text{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1-m,\frac{3}{2},-\frac{c x}{b},-\frac{e x}{d}\right]+c d x \text{AppellF1}\left[\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},-\frac{c x}{b},-\frac{e x}{d}\right]\right)\right) - \left(3 c x (b+c x) \text{AppellF1}\left[\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},-\frac{c x}{b},-\frac{e x}{d}\right]\right) \Big/ \left(b \left(3 b d \text{AppellF1}\left[\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},-\frac{c x}{b},-\frac{e x}{d}\right]+2 b e m x \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1-m,\frac{5}{2},-\frac{c x}{b},-\frac{e x}{d}\right]-c d x \text{AppellF1}\left[\frac{3}{2},\frac{3}{2},-m,\frac{5}{2},-\frac{c x}{b},-\frac{e x}{d}\right]\right)\right) - \left(3 c x \text{AppellF1}\left[\frac{1}{2},\frac{3}{2},-m,\frac{3}{2},-\frac{c x}{b},-\frac{e x}{d}\right]\right) \Big/ \left(3 b d \text{AppellF1}\left[\frac{1}{2},\frac{3}{2},-m,\frac{3}{2},-\frac{c x}{b},-\frac{e x}{d}\right]+2 b e m x \text{AppellF1}\left[\frac{3}{2},\frac{3}{2},1-m,\frac{5}{2},-\frac{c x}{b},-\frac{e x}{d}\right]-3 c d x \text{AppellF1}\left[\frac{3}{2},\frac{5}{2},-m,\frac{5}{2},-\frac{c x}{b},-\frac{e x}{d}\right]\right)\right)$$

- **Problem 618: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e x)^{5/2}}{a+c x^2} \, dx$$

Optimal (type 3, 781 leaves, 12 steps) :

$$\frac{4 d e \sqrt{d+e x}}{c}+\frac{2 e (d+e x)^{3/2}}{3 c}-\frac{e \left(2 c^{3/2} d^3+2 a \sqrt{c} d e^2-\left(3 c d^2-a e^2\right) \sqrt{c d^2+a e^2}\right) \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c} d+\sqrt{c d^2+a e^2}}-\sqrt{2} c^{1/4} \sqrt{d+e x}}{\sqrt{\sqrt{c} d-\sqrt{c d^2+a e^2}}}\right]}{\sqrt{2} c^{7/4} \sqrt{c d^2+a e^2} \sqrt{\sqrt{c} d-\sqrt{c d^2+a e^2}}}+$$

$$\frac{e \left(2 c^{3/2} d^3+2 a \sqrt{c} d e^2-\left(3 c d^2-a e^2\right) \sqrt{c d^2+a e^2}\right) \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c} d+\sqrt{c d^2+a e^2}}+\sqrt{2} c^{1/4} \sqrt{d+e x}}{\sqrt{\sqrt{c} d-\sqrt{c d^2+a e^2}}}\right]}{\sqrt{2} c^{7/4} \sqrt{c d^2+a e^2} \sqrt{\sqrt{c} d-\sqrt{c d^2+a e^2}}}+$$

$$\left(e \left(2 c^{3/2} d^3+2 a \sqrt{c} d e^2+\left(3 c d^2-a e^2\right) \sqrt{c d^2+a e^2}\right) \text{Log}\left[\sqrt{c d^2+a e^2}-\sqrt{2} c^{1/4} \sqrt{\sqrt{c} d+\sqrt{c d^2+a e^2}} \sqrt{d+e x}+\sqrt{c} (d+e x)\right]\right)\Big/$$
$$\left(2 \sqrt{2} c^{7/4} \sqrt{c d^2+a e^2} \sqrt{\sqrt{c} d+\sqrt{c d^2+a e^2}}\right)-$$
$$\left(e \left(2 c^{3/2} d^3+2 a \sqrt{c} d e^2+\left(3 c d^2-a e^2\right) \sqrt{c d^2+a e^2}\right) \text{Log}\left[\sqrt{c d^2+a e^2}+\sqrt{2} c^{1/4} \sqrt{\sqrt{c} d+\sqrt{c d^2+a e^2}} \sqrt{d+e x}+\sqrt{c} (d+e x)\right]\right)\Big/$$
$$\left(2 \sqrt{2} c^{7/4} \sqrt{c d^2+a e^2} \sqrt{\sqrt{c} d+\sqrt{c d^2+a e^2}}\right)$$

Result (type 3, 217 leaves) :

$$\frac{2 e \sqrt{d+e x} (7 d+e x)}{3 c}-\frac{i \left(\sqrt{c} d-i \sqrt{a} e\right)^3 \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{d+e x}}{\sqrt{c d-i \sqrt{a} \sqrt{c} e}}\right]}{\sqrt{a} c^{3/2} \sqrt{c d-i \sqrt{a} \sqrt{c} e}}+\frac{i \left(\sqrt{c} d+i \sqrt{a} e\right)^3 \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{d+e x}}{\sqrt{c d+i \sqrt{a} \sqrt{c} e}}\right]}{\sqrt{a} c^{3/2} \sqrt{c d+i \sqrt{a} \sqrt{c} e}}$$

- **Problem 619: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e x)^{3/2}}{a+c x^2} \, dx$$

Optimal (type 3, 689 leaves, 11 steps) :

$$\frac{2 e \sqrt{d+e x}}{c}-\frac{e \left(c d^2+a e^2-2 \sqrt{c}\, d \sqrt{c d^2+a e^2}\right) \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}-\sqrt{2}\, c^{1/4} \sqrt{d+e x}}{\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}\right]}{\sqrt{2}\, c^{5/4} \sqrt{c d^2+a e^2} \sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}+$$

$$\frac{e \left(c d^2+a e^2-2 \sqrt{c}\, d \sqrt{c d^2+a e^2}\right) \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}+\sqrt{2}\, c^{1/4} \sqrt{d+e x}}{\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}\right]}{\sqrt{2}\, c^{5/4} \sqrt{c d^2+a e^2} \sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}+$$

$$\left(e \left(c d^2+a e^2+2 \sqrt{c}\, d \sqrt{c d^2+a e^2}\right) \text{Log}\left[\sqrt{c d^2+a e^2}-\sqrt{2}\, c^{1/4} \sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}} \sqrt{d+e x}+\sqrt{c}\, (d+e x)\right]\right)\Big/$$

$$\left(2 \sqrt{2}\, c^{5/4} \sqrt{c d^2+a e^2} \sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\right)-$$

$$\left(e \left(c d^2+a e^2+2 \sqrt{c}\, d \sqrt{c d^2+a e^2}\right) \text{Log}\left[\sqrt{c d^2+a e^2}+\sqrt{2}\, c^{1/4} \sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}} \sqrt{d+e x}+\sqrt{c}\, (d+e x)\right]\right)\Big/$$

$$\left(2 \sqrt{2}\, c^{5/4} \sqrt{c d^2+a e^2} \sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\right)$$

Result (type 3, 204 leaves) :

$$\frac{2 e \sqrt{d+e x}}{c}-\frac{i \left(\sqrt{c}\, d-i \sqrt{a}\, e\right)^2 \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{d+e x}}{\sqrt{c d-i \sqrt{a} \sqrt{c}\, e}}\right]}{\sqrt{a}\, c \sqrt{c d-i \sqrt{a} \sqrt{c}\, e}}+\frac{i \left(\sqrt{c}\, d+i \sqrt{a}\, e\right)^2 \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{d+e x}}{\sqrt{c d+i \sqrt{a} \sqrt{c}\, e}}\right]}{\sqrt{a}\, c \sqrt{c d+i \sqrt{a} \sqrt{c}\, e}}$$

- **Problem 620: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{d+e x}}{a+c x^2}\, dx$$

Optimal (type 3, 478 leaves, 10 steps) :

$$\frac{e\,\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}-\sqrt{2}\,c^{1/4}\,\sqrt{d+e\,x}}{\sqrt{\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}}}\right]}{\sqrt{2}\,c^{3/4}\,\sqrt{\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}}}-\frac{e\,\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}+\sqrt{2}\,c^{1/4}\,\sqrt{d+e\,x}}{\sqrt{\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}}}\right]}{\sqrt{2}\,c^{3/4}\,\sqrt{\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}}}+$$

$$\frac{e\,\text{Log}\left[\sqrt{c\,d^2+a\,e^2}-\sqrt{2}\,c^{1/4}\,\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}\,\sqrt{d+e\,x}+\sqrt{c}\,(d+e\,x)\right]}{2\,\sqrt{2}\,c^{3/4}\,\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}}-$$

$$\frac{e\,\text{Log}\left[\sqrt{c\,d^2+a\,e^2}+\sqrt{2}\,c^{1/4}\,\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}\,\sqrt{d+e\,x}+\sqrt{c}\,(d+e\,x)\right]}{2\,\sqrt{2}\,c^{3/4}\,\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}}$$

Result (type 3, 140 leaves) :

$$-\frac{i\left(\sqrt{c\,d-i\,\sqrt{a}\,\sqrt{c}\,e}\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{d+e\,x}}{\sqrt{c\,d-i\,\sqrt{a}\,\sqrt{c}\,e}}\right]-\sqrt{c\,d+i\,\sqrt{a}\,\sqrt{c}\,e}\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{d+e\,x}}{\sqrt{c\,d+i\,\sqrt{a}\,\sqrt{c}\,e}}\right]\right)}{\sqrt{a}\,c}$$

- **Problem 621: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{d+e\,x}\,\left(a+c\,x^2\right)}\,dx$$

Optimal (type 3, 538 leaves, 10 steps) :

$$\frac{e\,\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}-\sqrt{2}\,c^{1/4}\,\sqrt{d+e\,x}}{\sqrt{\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}}}\right]}{\sqrt{2}\,c^{1/4}\,\sqrt{c\,d^2+a\,e^2}\,\sqrt{\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}}}-\frac{e\,\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}+\sqrt{2}\,c^{1/4}\,\sqrt{d+e\,x}}{\sqrt{\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}}}\right]}{\sqrt{2}\,c^{1/4}\,\sqrt{c\,d^2+a\,e^2}\,\sqrt{\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}}}-$$

$$\frac{e\,\text{Log}\left[\sqrt{c\,d^2+a\,e^2}-\sqrt{2}\,c^{1/4}\,\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}\,\sqrt{d+e\,x}+\sqrt{c}\,(d+e\,x)\right]}{2\,\sqrt{2}\,c^{1/4}\,\sqrt{c\,d^2+a\,e^2}\,\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}}+$$

$$\frac{e\,\text{Log}\left[\sqrt{c\,d^2+a\,e^2}+\sqrt{2}\,c^{1/4}\,\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}\,\sqrt{d+e\,x}+\sqrt{c}\,(d+e\,x)\right]}{2\,\sqrt{2}\,c^{1/4}\,\sqrt{c\,d^2+a\,e^2}\,\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}}$$

Result (type 3, 137 leaves) :

$$-\frac{i\left(\frac{\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{d+e\,x}}{\sqrt{c\,d-i\,\sqrt{a}\,\sqrt{c}\,e}}\right]}{\sqrt{c\,d-i\,\sqrt{a}\,\sqrt{c}\,e}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{d+e\,x}}{\sqrt{c\,d+i\,\sqrt{a}\,\sqrt{c}\,e}}\right]}{\sqrt{c\,d+i\,\sqrt{a}\,\sqrt{c}\,e}}\right)}{\sqrt{a}}$$

- **Problem 622: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(d+e\,x)^{3/2}\,\left(a+c\,x^2\right)}\,dx$$

Optimal (type 3, 663 leaves, 11 steps) :

$$-\frac{2\,e}{\left(c\,d^2+a\,e^2\right)\sqrt{d+e\,x}}+\frac{c^{1/4}\,e\,\left(2\,\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}\right)\,\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}-\sqrt{2}\,c^{1/4}\,\sqrt{d+e\,x}}{\sqrt{\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}}}\right]}{\sqrt{2}\,\left(c\,d^2+a\,e^2\right)^{3/2}\sqrt{\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}}}-$$

$$\frac{c^{1/4}\,e\,\left(2\,\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}\right)\,\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}+\sqrt{2}\,c^{1/4}\,\sqrt{d+e\,x}}{\sqrt{\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}}}\right]}{\sqrt{2}\,\left(c\,d^2+a\,e^2\right)^{3/2}\sqrt{\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}}}-$$

$$\left(c^{1/4}\,e\,\left(2\,\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}\right)\,\text{Log}\left[\sqrt{c\,d^2+a\,e^2}-\sqrt{2}\,c^{1/4}\,\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}\,\sqrt{d+e\,x}+\sqrt{c}\,(d+e\,x)\right]\right)\Big/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2+a\,e^2\right)^{3/2}\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}\right)+$$

$$\left(c^{1/4}\,e\,\left(2\,\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}\right)\,\text{Log}\left[\sqrt{c\,d^2+a\,e^2}+\sqrt{2}\,c^{1/4}\,\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}\,\sqrt{d+e\,x}+\sqrt{c}\,(d+e\,x)\right]\right)\Big/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2+a\,e^2\right)^{3/2}\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}\right)$$

Result (type 3, 209 leaves):

$$\frac{-\frac{2\,e}{\sqrt{d+e\,x}}+\frac{\sqrt{c}\,\left(-i\,\sqrt{c}\,d+\sqrt{a}\,e\right)\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{d+e\,x}}{\sqrt{c\,d-i\,\sqrt{a}\,\sqrt{c}\,e}}\right]}{\sqrt{a}\,\sqrt{c\,d-i\,\sqrt{a}\,\sqrt{c}\,e}}+\frac{\sqrt{c}\,\left(i\,\sqrt{c}\,d+\sqrt{a}\,e\right)\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{d+e\,x}}{\sqrt{c\,d+i\,\sqrt{a}\,\sqrt{c}\,e}}\right]}{\sqrt{a}\,\sqrt{c\,d+i\,\sqrt{a}\,\sqrt{c}\,e}}}{c\,d^2+a\,e^2}$$

- **Problem 623: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(d+e\,x)^{5/2}\,\left(a+c\,x^2\right)}\,dx$$

Optimal (type 3, 736 leaves, 12 steps):

$$-\frac{2 e}{3 \left(c d^2+a e^2\right) (d+e x)^{3/2}}-\frac{4 c d e}{\left(c d^2+a e^2\right)^2 \sqrt{d+e x}}+\frac{c^{3/4} e \left(3 c d^2-a e^2-2 \sqrt{c} d \sqrt{c d^2+a e^2}\right) \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c} d+\sqrt{c d^2+a e^2}}-\sqrt{2} c^{1/4} \sqrt{d+e x}}{\sqrt{\sqrt{c} d-\sqrt{c d^2+a e^2}}}\right]}{\sqrt{2} \left(c d^2+a e^2\right)^{5/2} \sqrt{\sqrt{c} d-\sqrt{c d^2+a e^2}}}-$$

$$\frac{c^{3/4} e \left(3 c d^2-a e^2-2 \sqrt{c} d \sqrt{c d^2+a e^2}\right) \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c} d+\sqrt{c d^2+a e^2}}+\sqrt{2} c^{1/4} \sqrt{d+e x}}{\sqrt{\sqrt{c} d-\sqrt{c d^2+a e^2}}}\right]}{\sqrt{2} \left(c d^2+a e^2\right)^{5/2} \sqrt{\sqrt{c} d-\sqrt{c d^2+a e^2}}}-$$

$$\left(c^{3/4} e \left(3 c d^2-a e^2+2 \sqrt{c} d \sqrt{c d^2+a e^2}\right) \text{Log}\left[\sqrt{c d^2+a e^2}-\sqrt{2} c^{1/4} \sqrt{\sqrt{c} d+\sqrt{c d^2+a e^2}} \sqrt{d+e x}+\sqrt{c} (d+e x)\right]\right) \Big/$$

$$\left(2 \sqrt{2} \left(c d^2+a e^2\right)^{5/2} \sqrt{\sqrt{c} d+\sqrt{c d^2+a e^2}}\right)+$$

$$\left(c^{3/4} e \left(3 c d^2-a e^2+2 \sqrt{c} d \sqrt{c d^2+a e^2}\right) \text{Log}\left[\sqrt{c d^2+a e^2}+\sqrt{2} c^{1/4} \sqrt{\sqrt{c} d+\sqrt{c d^2+a e^2}} \sqrt{d+e x}+\sqrt{c} (d+e x)\right]\right) \Big/$$

$$\left(2 \sqrt{2} \left(c d^2+a e^2\right)^{5/2} \sqrt{\sqrt{c} d+\sqrt{c d^2+a e^2}}\right)$$

Result (type 3, 229 leaves) :

$$-\frac{2 e \left(a e^2+c d (7 d+6 e x)\right)}{3 \left(c d^2+a e^2\right)^2 (d+e x)^{3/2}}-\frac{i c \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{d+e x}}{\sqrt{c d-i \sqrt{a} \sqrt{c} e}}\right]}{\sqrt{a} \left(\sqrt{c} d-i \sqrt{a} e\right)^2 \sqrt{c d-i \sqrt{a} \sqrt{c} e}}+\frac{i c \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{d+e x}}{\sqrt{c d+i \sqrt{a} \sqrt{c} e}}\right]}{\sqrt{a} \left(\sqrt{c} d+i \sqrt{a} e\right)^2 \sqrt{c d+i \sqrt{a} \sqrt{c} e}}$$

- **Problem 631: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e x)^{7/2}}{\left(a+c x^2\right)^2} \, dx$$

Optimal (type 3, 887 leaves, 13 steps) :

$$-\frac{e\left(c d^2-5 a e^2\right)\sqrt{d+e x}}{2 a c^2}-\frac{d e (d+e x)^{3/2}}{2 a c}-\frac{(a e-c d x)(d+e x)^{5/2}}{2 a c\left(a+c x^2\right)}+$$

$$\left(e\left(c^2 d^4-4 a c d^2 e^2-5 a^2 e^4+\sqrt{c}\, d\sqrt{c d^2+a e^2}\left(c d^2+13 a e^2\right)\right)\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}-\sqrt{2}\, c^{1/4}\sqrt{d+e x}}{\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}\right]\right)\Big/$$

$$\left(4\sqrt{2}\, a c^{9/4}\sqrt{c d^2+a e^2}\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}\right)-$$

$$\left(e\left(c^2 d^4-4 a c d^2 e^2-5 a^2 e^4+\sqrt{c}\, d\sqrt{c d^2+a e^2}\left(c d^2+13 a e^2\right)\right)\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}+\sqrt{2}\, c^{1/4}\sqrt{d+e x}}{\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}\right]\right)\Big/$$

$$\left(4\sqrt{2}\, a c^{9/4}\sqrt{c d^2+a e^2}\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}\right)-$$

$$\left(e\left(c^2 d^4-4 a c d^2 e^2-5 a^2 e^4-\sqrt{c}\, d\sqrt{c d^2+a e^2}\left(c d^2+13 a e^2\right)\right)\text{Log}\left[\sqrt{c d^2+a e^2}-\sqrt{2}\, c^{1/4}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\sqrt{d+e x}+\sqrt{c}\,(d+e x)\right]\right)\Big/$$

$$\left(8\sqrt{2}\, a c^{9/4}\sqrt{c d^2+a e^2}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\right)+$$

$$\left(e\left(c^2 d^4-4 a c d^2 e^2-5 a^2 e^4-\sqrt{c}\, d\sqrt{c d^2+a e^2}\left(c d^2+13 a e^2\right)\right)\text{Log}\left[\sqrt{c d^2+a e^2}+\sqrt{2}\, c^{1/4}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\sqrt{d+e x}+\sqrt{c}\,(d+e x)\right]\right)\Big/$$

$$\left(8\sqrt{2}\, a c^{9/4}\sqrt{c d^2+a e^2}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\right)$$

Result (type 3, 282 leaves) :

$$\frac{1}{4 a^{3/2} c^2}\left(\frac{2 \sqrt{a} \sqrt{d+e x} \left(5 a^2 e^3+c^2 d^3 x+a c e \left(-3 d^2-3 d e x+4 e^2 x^2\right)\right)}{a+c x^2}+\right.$$

$$\left.\frac{\left(\sqrt{c} d-\mathrm{i} \sqrt{a} e\right)^3 \left(-2 \mathrm{i} \sqrt{c} d+5 \sqrt{a} e\right) \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{d+e x}}{\sqrt{c d-\mathrm{i} \sqrt{a} \sqrt{c} e}}\right]}{\sqrt{c d-\mathrm{i} \sqrt{a} \sqrt{c} e}}+\frac{\left(\sqrt{c} d+\mathrm{i} \sqrt{a} e\right)^3 \left(2 \mathrm{i} \sqrt{c} d+5 \sqrt{a} e\right) \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{d+e x}}{\sqrt{c d+\mathrm{i} \sqrt{a} \sqrt{c} e}}\right]}{\sqrt{c d+\mathrm{i} \sqrt{a} \sqrt{c} e}}\right)$$

- **Problem 632: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e x)^{5/2}}{\left(a+c x^2\right)^2} \, dx$$

Optimal (type 3, 811 leaves, 12 steps) :

$$-\frac{d\,e\,\sqrt{d+e\,x}}{2\,a\,c}-\frac{(a\,e-c\,d\,x)\,(d+e\,x)^{3/2}}{2\,a\,c\,(a+c\,x^2)}+\frac{e\left(c^{3/2}\,d^3+a\,\sqrt{c}\,d\,e^2+\sqrt{c\,d^2+a\,e^2}\,(c\,d^2+3\,a\,e^2)\right)\,\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}-\sqrt{2}\,c^{1/4}\,\sqrt{d+e\,x}}{\sqrt{\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}}}\right]}{4\,\sqrt{2}\,a\,c^{7/4}\,\sqrt{c\,d^2+a\,e^2}\,\sqrt{\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}}}-$$

$$\frac{e\left(c^{3/2}\,d^3+a\,\sqrt{c}\,d\,e^2+\sqrt{c\,d^2+a\,e^2}\,(c\,d^2+3\,a\,e^2)\right)\,\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}+\sqrt{2}\,c^{1/4}\,\sqrt{d+e\,x}}{\sqrt{\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}}}\right]}{4\,\sqrt{2}\,a\,c^{7/4}\,\sqrt{c\,d^2+a\,e^2}\,\sqrt{\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}}}-$$

$$\left(e\left(c^{3/2}\,d^3+a\,\sqrt{c}\,d\,e^2-\sqrt{c\,d^2+a\,e^2}\,(c\,d^2+3\,a\,e^2)\right)\,\text{Log}\left[\sqrt{c\,d^2+a\,e^2}-\sqrt{2}\,c^{1/4}\,\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}\,\sqrt{d+e\,x}+\sqrt{c}\,(d+e\,x)\right]\right)\Big/$$

$$\left(8\,\sqrt{2}\,a\,c^{7/4}\,\sqrt{c\,d^2+a\,e^2}\,\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}\right)+$$

$$\left(e\left(c^{3/2}\,d^3+a\,\sqrt{c}\,d\,e^2-\sqrt{c\,d^2+a\,e^2}\,(c\,d^2+3\,a\,e^2)\right)\,\text{Log}\left[\sqrt{c\,d^2+a\,e^2}+\sqrt{2}\,c^{1/4}\,\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}\,\sqrt{d+e\,x}+\sqrt{c}\,(d+e\,x)\right]\right)\Big/$$

$$\left(8\,\sqrt{2}\,a\,c^{7/4}\,\sqrt{c\,d^2+a\,e^2}\,\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}\right)$$

Result (type 3, 271 leaves) :

$$\frac{1}{4\,a^{3/2}\,c^2}\left(\frac{2\,\sqrt{a}\,c\,\sqrt{d+e\,x}\,\left(c\,d^2\,x-a\,e\,(2\,d+e\,x)\right)}{a+c\,x^2}+\right.$$

$$\left.\frac{\sqrt{c}\,\left(\sqrt{c}\,d-i\,\sqrt{a}\,e\right)^2\,\left(-2\,i\,\sqrt{c}\,d+3\,\sqrt{a}\,e\right)\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{d+e\,x}}{\sqrt{c\,d-i\,\sqrt{a}\,\sqrt{c}\,e}}\right]}{\sqrt{c\,d-i\,\sqrt{a}\,\sqrt{c}\,e}}+\frac{\sqrt{c}\,\left(\sqrt{c}\,d+i\,\sqrt{a}\,e\right)^2\,\left(2\,i\,\sqrt{c}\,d+3\,\sqrt{a}\,e\right)\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{d+e\,x}}{\sqrt{c\,d+i\,\sqrt{a}\,\sqrt{c}\,e}}\right]}{\sqrt{c\,d+i\,\sqrt{a}\,\sqrt{c}\,e}}\right)$$

■ Problem 633: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d+e x)^{3/2}}{\left(a+c x^2\right)^2} \, dx$$

Optimal (type 3, 726 leaves, 11 steps):

$$-\frac{(a e-c d x)\sqrt{d+e x}}{2 a c\left(a+c x^2\right)}+\frac{e\left(c d^2+a e^2+\sqrt{c}\, d\sqrt{c d^2+a e^2}\right)\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}-\sqrt{2}\, c^{1/4}\sqrt{d+e x}}{\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}\right]}{4\sqrt{2}\, a c^{5/4}\sqrt{c d^2+a e^2}\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}-$$

$$\frac{e\left(c d^2+a e^2+\sqrt{c}\, d\sqrt{c d^2+a e^2}\right)\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}+\sqrt{2}\, c^{1/4}\sqrt{d+e x}}{\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}\right]}{4\sqrt{2}\, a c^{5/4}\sqrt{c d^2+a e^2}\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}-$$

$$\left(e\left(c d^2+a e^2-\sqrt{c}\, d\sqrt{c d^2+a e^2}\right)\text{Log}\left[\sqrt{c d^2+a e^2}-\sqrt{2}\, c^{1/4}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\sqrt{d+e x}+\sqrt{c}\,(d+e x)\right]\right)\Big/$$

$$\left(8\sqrt{2}\, a c^{5/4}\sqrt{c d^2+a e^2}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\right)+$$

$$\left(e\left(c d^2+a e^2-\sqrt{c}\, d\sqrt{c d^2+a e^2}\right)\text{Log}\left[\sqrt{c d^2+a e^2}+\sqrt{2}\, c^{1/4}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\sqrt{d+e x}+\sqrt{c}\,(d+e x)\right]\right)\Big/$$

$$\left(8\sqrt{2}\, a c^{5/4}\sqrt{c d^2+a e^2}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\right)$$

Result (type 3, 235 leaves):

$$\frac{1}{4 a^{3/2} c}\left(\frac{2\sqrt{a}\,(-a e+c d x)\sqrt{d+e x}}{a+c x^2}-\frac{\left(2 i c d^2+\sqrt{a}\sqrt{c}\, d e+i a e^2\right)\text{ArcTanh}\left[\frac{\sqrt{c}\sqrt{d+e x}}{\sqrt{c d-i\sqrt{a}\sqrt{c}\, e}}\right]}{\sqrt{c d-i\sqrt{a}\sqrt{c}\, e}}+\frac{i\left(2 c d^2+i\sqrt{a}\sqrt{c}\, d e+a e^2\right)\text{ArcTanh}\left[\frac{\sqrt{c}\sqrt{d+e x}}{\sqrt{c d+i\sqrt{a}\sqrt{c}\, e}}\right]}{\sqrt{c d+i\sqrt{a}\sqrt{c}\, e}}\right)$$

Problem 634: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{d+e x}}{\left(a+c x^2\right)^2}\,dx$$

Optimal (type 3, 675 leaves, 11 steps):

$$\frac{x\sqrt{d+e x}}{2 a\left(a+c x^2\right)}+\frac{e\left(\sqrt{c}\, d+\sqrt{c d^2+a e^2}\right)\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}-\sqrt{2}\, c^{1/4}\sqrt{d+e x}}{\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}\right]}{4\sqrt{2}\, a c^{3/4}\sqrt{c d^2+a e^2}\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}-\frac{e\left(\sqrt{c}\, d+\sqrt{c d^2+a e^2}\right)\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}+\sqrt{2}\, c^{1/4}\sqrt{d+e x}}{\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}\right]}{4\sqrt{2}\, a c^{3/4}\sqrt{c d^2+a e^2}\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}-$$

$$\frac{e\left(d-\frac{\sqrt{c d^2+a e^2}}{\sqrt{c}}\right)\text{Log}\left[\sqrt{c d^2+a e^2}-\sqrt{2}\, c^{1/4}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\sqrt{d+e x}+\sqrt{c}\,(d+e x)\right]}{8\sqrt{2}\, a c^{1/4}\sqrt{c d^2+a e^2}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}}+$$

$$\frac{e\left(d-\frac{\sqrt{c d^2+a e^2}}{\sqrt{c}}\right)\text{Log}\left[\sqrt{c d^2+a e^2}+\sqrt{2}\, c^{1/4}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\sqrt{d+e x}+\sqrt{c}\,(d+e x)\right]}{8\sqrt{2}\, a c^{1/4}\sqrt{c d^2+a e^2}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}}$$

Result (type 3, 208 leaves):

$$\frac{\frac{2\sqrt{a}\, x\sqrt{d+e x}}{a+c x^2}-\frac{\left(2 i c d+\sqrt{a}\sqrt{c}\, e\right)\text{ArcTanh}\left[\frac{\sqrt{c}\sqrt{d+e x}}{\sqrt{c d-i\sqrt{a}\sqrt{c}\, e}}\right]}{c\sqrt{c d-i\sqrt{a}\sqrt{c}\, e}}-\frac{\left(-2 i c d+\sqrt{a}\sqrt{c}\, e\right)\text{ArcTanh}\left[\frac{\sqrt{c}\sqrt{d+e x}}{\sqrt{c d+i\sqrt{a}\sqrt{c}\, e}}\right]}{c\sqrt{c d+i\sqrt{a}\sqrt{c}\, e}}}{4 a^{3/2}}$$

Problem 635: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{d+e x}\left(a+c x^2\right)^2}\,dx$$

Optimal (type 3, 739 leaves, 11 steps):

$$\frac{(a e+c d x)\sqrt{d+e x}}{2 a\left(c d^2+a e^2\right)\left(a+c x^2\right)}+\frac{e\left(c d^2+3 a e^2+\sqrt{c}\, d\sqrt{c d^2+a e^2}\right)\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}-\sqrt{2}\, c^{1/4}\sqrt{d+e x}}{\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}\right]}{4\sqrt{2}\, a c^{1/4}\left(c d^2+a e^2\right)^{3/2}\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}-$$

$$\frac{e\left(c d^2+3 a e^2+\sqrt{c}\, d\sqrt{c d^2+a e^2}\right)\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}+\sqrt{2}\, c^{1/4}\sqrt{d+e x}}{\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}\right]}{4\sqrt{2}\, a c^{1/4}\left(c d^2+a e^2\right)^{3/2}\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}-$$

$$\left(e\left(c d^2+3 a e^2-\sqrt{c}\, d\sqrt{c d^2+a e^2}\right)\text{Log}\left[\sqrt{c d^2+a e^2}-\sqrt{2}\, c^{1/4}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\sqrt{d+e x}+\sqrt{c}\,(d+e x)\right]\right)\Big/$$

$$\left(8\sqrt{2}\, a c^{1/4}\left(c d^2+a e^2\right)^{3/2}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\right)+$$

$$\left(e\left(c d^2+3 a e^2-\sqrt{c}\, d\sqrt{c d^2+a e^2}\right)\text{Log}\left[\sqrt{c d^2+a e^2}+\sqrt{2}\, c^{1/4}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\sqrt{d+e x}+\sqrt{c}\,(d+e x)\right]\right)\Big/$$

$$\left(8\sqrt{2}\, a c^{1/4}\left(c d^2+a e^2\right)^{3/2}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\right)$$

Result (type 3, 245 leaves) :

$$\frac{1}{4 a^{3/2}\left(c d^2+a e^2\right)}\left(\frac{2\sqrt{a}\,(a e+c d x)\sqrt{d+e x}}{a+c x^2}-\right.$$

$$\left.\frac{\left(2 i c d^2+\sqrt{a}\sqrt{c}\, d e+3 i a e^2\right)\text{ArcTanh}\left[\frac{\sqrt{c}\sqrt{d+e x}}{\sqrt{c d-i\sqrt{a}\sqrt{c}\, e}}\right]}{\sqrt{c d-i\sqrt{a}\sqrt{c}\, e}}+\frac{i\left(2 c d^2+i\sqrt{a}\sqrt{c}\, d e+3 a e^2\right)\text{ArcTanh}\left[\frac{\sqrt{c}\sqrt{d+e x}}{\sqrt{c d+i\sqrt{a}\sqrt{c}\, e}}\right]}{\sqrt{c d+i\sqrt{a}\sqrt{c}\, e}}\right)$$

Problem 636: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(d+e x)^{3/2} \left(a+c x^2\right)^2} \, dx$$

Optimal (type 3, 845 leaves, 12 steps):

$$\frac{e \left(c d^2-5 a e^2\right)}{2 a \left(c d^2+a e^2\right)^2 \sqrt{d+e x}}+\frac{a e+c d x}{2 a \left(c d^2+a e^2\right) \sqrt{d+e x} \left(a+c x^2\right)}+$$

$$\frac{c^{1/4} e \left(c^{3/2} d^3+13 a \sqrt{c} d e^2+\left(c d^2-5 a e^2\right) \sqrt{c d^2+a e^2}\right) \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c} d+\sqrt{c d^2+a e^2}}-\sqrt{2} c^{1/4} \sqrt{d+e x}}{\sqrt{\sqrt{c} d-\sqrt{c d^2+a e^2}}}\right]}{4 \sqrt{2} a \left(c d^2+a e^2\right)^{5/2} \sqrt{\sqrt{c} d-\sqrt{c d^2+a e^2}}}-$$

$$\frac{c^{1/4} e \left(c^{3/2} d^3+13 a \sqrt{c} d e^2+\left(c d^2-5 a e^2\right) \sqrt{c d^2+a e^2}\right) \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c} d+\sqrt{c d^2+a e^2}}+\sqrt{2} c^{1/4} \sqrt{d+e x}}{\sqrt{\sqrt{c} d-\sqrt{c d^2+a e^2}}}\right]}{4 \sqrt{2} a \left(c d^2+a e^2\right)^{5/2} \sqrt{\sqrt{c} d-\sqrt{c d^2+a e^2}}}-$$

$$\left(c^{1/4} e \left(c^{3/2} d^3+13 a \sqrt{c} d e^2-\left(c d^2-5 a e^2\right) \sqrt{c d^2+a e^2}\right) \text{Log}\left[\sqrt{c d^2+a e^2}-\sqrt{2} c^{1/4} \sqrt{\sqrt{c} d+\sqrt{c d^2+a e^2}} \sqrt{d+e x}+\sqrt{c} (d+e x)\right]\right) \Big/$$

$$\left(8 \sqrt{2} a \left(c d^2+a e^2\right)^{5/2} \sqrt{\sqrt{c} d+\sqrt{c d^2+a e^2}}\right)+$$

$$\left(c^{1/4} e \left(c^{3/2} d^3+13 a \sqrt{c} d e^2-\left(c d^2-5 a e^2\right) \sqrt{c d^2+a e^2}\right) \text{Log}\left[\sqrt{c d^2+a e^2}+\sqrt{2} c^{1/4} \sqrt{\sqrt{c} d+\sqrt{c d^2+a e^2}} \sqrt{d+e x}+\sqrt{c} (d+e x)\right]\right) \Big/$$

$$\left(8 \sqrt{2} a \left(c d^2+a e^2\right)^{5/2} \sqrt{\sqrt{c} d+\sqrt{c d^2+a e^2}}\right)$$

Result (type 3, 312 leaves):

$$\frac{1}{4\,a^{3/2}}\left(\frac{2\sqrt{a}\,\left(-4\,a^2\,e^3+c^2\,d^2\,x\,(d+e\,x)+a\,c\,e\,\left(2\,d^2+d\,e\,x-5\,e^2\,x^2\right)\right)}{\left(c\,d^2+a\,e^2\right)^2\sqrt{d+e\,x}\,\left(a+c\,x^2\right)}-\right.$$

$$\left.\frac{i\,\sqrt{c}\,\left(2\,\sqrt{c}\,d-5\,i\,\sqrt{a}\,e\right)\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{d+e\,x}}{\sqrt{c\,d-i\,\sqrt{a}\,\sqrt{c}\,e}}\right]}{\left(\sqrt{c}\,d-i\,\sqrt{a}\,e\right)^2\sqrt{c\,d-i\,\sqrt{a}\,\sqrt{c}\,e}}+\frac{i\,\sqrt{c}\,\left(2\,\sqrt{c}\,d+5\,i\,\sqrt{a}\,e\right)\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{d+e\,x}}{\sqrt{c\,d+i\,\sqrt{a}\,\sqrt{c}\,e}}\right]}{\left(\sqrt{c}\,d+i\,\sqrt{a}\,e\right)^2\sqrt{c\,d+i\,\sqrt{a}\,\sqrt{c}\,e}}\right)$$

- **Problem 637: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{(d+e\,x)^{5/2}\,\left(a+c\,x^2\right)^2}\,dx$$

Optimal (type 3, 930 leaves, 13 steps):

$$\frac{e\left(3 c d^2-7 a e^2\right)}{6 a\left(c d^2+a e^2\right)^2 (d+e x)^{3/2}}+\frac{c d e\left(c d^2-19 a e^2\right)}{2 a\left(c d^2+a e^2\right)^3 \sqrt{d+e x}}+\frac{a e+c d x}{2 a\left(c d^2+a e^2\right) (d+e x)^{3/2}\left(a+c x^2\right)}+$$

$$\left(c^{3/4} e\left(c^2 d^4+34 a c d^2 e^2-7 a^2 e^4+\sqrt{c}\, d\left(c d^2-19 a e^2\right) \sqrt{c d^2+a e^2}\right) \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}-\sqrt{2}\, c^{1/4} \sqrt{d+e x}}{\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}\right]\right) \Big/$$

$$\left(4 \sqrt{2}\, a\left(c d^2+a e^2\right)^{7/2} \sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}\right)-$$

$$\left(c^{3/4} e\left(c^2 d^4+34 a c d^2 e^2-7 a^2 e^4+\sqrt{c}\, d\left(c d^2-19 a e^2\right) \sqrt{c d^2+a e^2}\right) \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}+\sqrt{2}\, c^{1/4} \sqrt{d+e x}}{\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}\right]\right) \Big/$$

$$\left(4 \sqrt{2}\, a\left(c d^2+a e^2\right)^{7/2} \sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}\right)-\left(c^{3/4} e\left(c^2 d^4+34 a c d^2 e^2-7 a^2 e^4-\sqrt{c}\, d\left(c d^2-19 a e^2\right) \sqrt{c d^2+a e^2}\right)\right.$$

$$\left.\text{Log}\left[\sqrt{c d^2+a e^2}-\sqrt{2}\, c^{1/4} \sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}} \sqrt{d+e x}+\sqrt{c}\,(d+e x)\right]\right) \Big/$$

$$\left(8 \sqrt{2}\, a\left(c d^2+a e^2\right)^{7/2} \sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\right)+\left(c^{3/4} e\left(c^2 d^4+34 a c d^2 e^2-7 a^2 e^4-\sqrt{c}\, d\left(c d^2-19 a e^2\right) \sqrt{c d^2+a e^2}\right)\right.$$

$$\left.\text{Log}\left[\sqrt{c d^2+a e^2}+\sqrt{2}\, c^{1/4} \sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}} \sqrt{d+e x}+\sqrt{c}\,(d+e x)\right]\right) \Big/ \left(8 \sqrt{2}\, a\left(c d^2+a e^2\right)^{7/2} \sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\right)$$

Result (type 3, 349 leaves) :

$$\frac{1}{12\,a^{3/2}}\left(-\left(2\sqrt{a}\,\left(4\,a^3\,e^5-3\,c^3\,d^3\,x\,(d+e\,x)^2+a^2\,c\,e^3\,\left(55\,d^2+54\,d\,e\,x+7\,e^2\,x^2\right)+a\,c^2\,d\,e\,\left(-9\,d^3-9\,d^2\,e\,x+61\,d\,e^2\,x^2+57\,e^3\,x^3\right)\right)\right)\Big/\right.$$

$$\left(\left(c\,d^2+a\,e^2\right)^3\,(d+e\,x)^{3/2}\,\left(a+c\,x^2\right)\right)-$$

$$\left.\frac{3\,\mathrm{i}\,c\,\left(2\sqrt{c}\,d-7\,\mathrm{i}\sqrt{a}\,e\right)\,\mathrm{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{d+e\,x}}{\sqrt{c\,d-\mathrm{i}\sqrt{a}\,\sqrt{c}\,e}}\right]}{\left(\sqrt{c}\,d-\mathrm{i}\sqrt{a}\,e\right)^3\sqrt{c\,d-\mathrm{i}\sqrt{a}\,\sqrt{c}\,e}}+\frac{3\,\mathrm{i}\,c\,\left(2\sqrt{c}\,d+7\,\mathrm{i}\sqrt{a}\,e\right)\,\mathrm{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{d+e\,x}}{\sqrt{c\,d+\mathrm{i}\sqrt{a}\,\sqrt{c}\,e}}\right]}{\left(\sqrt{c}\,d+\mathrm{i}\sqrt{a}\,e\right)^3\sqrt{c\,d+\mathrm{i}\sqrt{a}\,\sqrt{c}\,e}}\right)$$

- **Problem 643: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(d+e\,x)^{7/2}}{\left(a+c\,x^2\right)^3}\,dx$$

Optimal (type 3, 905 leaves, 12 steps) :

$$-\frac{(a e-c d x)\,(d+e x)^{5/2}}{4 a c\left(a+c x^2\right)^2}-\frac{\sqrt{d+e x}\,\left(a e\left(7 c d^2+5 a e^2\right)-2 c d\left(3 c d^2+2 a e^2\right) x\right)}{16 a^2 c^2\left(a+c x^2\right)}+$$

$$\left(e\left(6 c^2 d^4+11 a c d^2 e^2+5 a^2 e^4+\sqrt{c}\, d \sqrt{c d^2+a e^2}\,\left(6 c d^2+8 a e^2\right)\right) \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}-\sqrt{2}\, c^{1/4} \sqrt{d+e x}}{\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}\right]\right)\Big/$$

$$\left(32 \sqrt{2}\, a^2 c^{9/4} \sqrt{c d^2+a e^2}\, \sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}\right)-$$

$$\left(e\left(6 c^2 d^4+11 a c d^2 e^2+5 a^2 e^4+\sqrt{c}\, d \sqrt{c d^2+a e^2}\,\left(6 c d^2+8 a e^2\right)\right) \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}+\sqrt{2}\, c^{1/4} \sqrt{d+e x}}{\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}\right]\right)\Big/$$

$$\left(32 \sqrt{2}\, a^2 c^{9/4} \sqrt{c d^2+a e^2}\, \sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}\right)-\left(e\left(6 c^2 d^4+11 a c d^2 e^2+5 a^2 e^4-2 \sqrt{c}\, d \sqrt{c d^2+a e^2}\,\left(3 c d^2+4 a e^2\right)\right)\right.$$

$$\left.\text{Log}\left[\sqrt{c d^2+a e^2}-\sqrt{2}\, c^{1/4} \sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\, \sqrt{d+e x}+\sqrt{c}\,(d+e x)\right]\right)\Big/$$

$$\left(64 \sqrt{2}\, a^2 c^{9/4} \sqrt{c d^2+a e^2}\, \sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\right)+\left(e\left(6 c^2 d^4+11 a c d^2 e^2+5 a^2 e^4-2 \sqrt{c}\, d \sqrt{c d^2+a e^2}\,\left(3 c d^2+4 a e^2\right)\right)\right.$$

$$\left.\text{Log}\left[\sqrt{c d^2+a e^2}+\sqrt{2}\, c^{1/4} \sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\, \sqrt{d+e x}+\sqrt{c}\,(d+e x)\right]\right)\Big/\left(64 \sqrt{2}\, a^2 c^{9/4} \sqrt{c d^2+a e^2}\, \sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\right)$$

Result (type 3, 337 leaves) :

$$\frac{1}{32\,a^{5/2}\,c^2}\left(\frac{2\,\sqrt{a}\,\sqrt{d+e\,x}\,\left(-5\,a^3\,e^3+6\,c^3\,d^3\,x^3+a\,c^2\,d\,x\,\left(10\,d^2+d\,e\,x+8\,e^2\,x^2\right)-a^2\,c\,e\,\left(11\,d^2+4\,d\,e\,x+9\,e^2\,x^2\right)\right)}{\left(a+c\,x^2\right)^2}+\right.$$

$$\frac{\left(\sqrt{c}\,d-i\,\sqrt{a}\,e\right)^2\,\left(-12\,i\,c\,d^2+18\,\sqrt{a}\,\sqrt{c}\,d\,e+5\,i\,a\,e^2\right)\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{d+e\,x}}{\sqrt{c\,d-i\,\sqrt{a}\,\sqrt{c}\,e}}\right]}{\sqrt{c\,d-i\,\sqrt{a}\,\sqrt{c}\,e}}+$$

$$\left.\frac{\left(\sqrt{c}\,d+i\,\sqrt{a}\,e\right)^2\,\left(12\,i\,c\,d^2+18\,\sqrt{a}\,\sqrt{c}\,d\,e-5\,i\,a\,e^2\right)\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{d+e\,x}}{\sqrt{c\,d+i\,\sqrt{a}\,\sqrt{c}\,e}}\right]}{\sqrt{c\,d+i\,\sqrt{a}\,\sqrt{c}\,e}}\right)$$

- **Problem 644: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e\,x)^{5/2}}{\left(a+c\,x^2\right)^3}\,dx$$

Optimal (type 3, 846 leaves, 12 steps) :

$$-\frac{(a e-c d x)\,(d+e x)^{3/2}}{4 a c\,\left(a+c x^2\right)^2}-\frac{3 \sqrt{d+e x}\,\left(a d e-\left(2 c d^2+a e^2\right) x\right)}{16 a^2 c\,\left(a+c x^2\right)}+$$

$$\frac{3 e \left(2 c^{3/2} d^3+2 a \sqrt{c}\, d e^2+\sqrt{c d^2+a e^2}\,\left(2 c d^2+a e^2\right)\right) \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}-\sqrt{2}\, c^{1/4} \sqrt{d+e x}}{\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}\right]}{32 \sqrt{2}\, a^2 c^{7/4} \sqrt{c d^2+a e^2}\, \sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}-$$

$$\frac{3 e \left(2 c^{3/2} d^3+2 a \sqrt{c}\, d e^2+\sqrt{c d^2+a e^2}\,\left(2 c d^2+a e^2\right)\right) \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}+\sqrt{2}\, c^{1/4} \sqrt{d+e x}}{\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}\right]}{32 \sqrt{2}\, a^2 c^{7/4} \sqrt{c d^2+a e^2}\, \sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}-$$

$$\left(3 e \left(2 c^{3/2} d^3+2 a \sqrt{c}\, d e^2-\sqrt{c d^2+a e^2}\,\left(2 c d^2+a e^2\right)\right) \text{Log}\left[\sqrt{c d^2+a e^2}-\sqrt{2}\, c^{1/4} \sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\, \sqrt{d+e x}+\sqrt{c}\,(d+e x)\right]\right)\Big/$$

$$\left(64 \sqrt{2}\, a^2 c^{7/4} \sqrt{c d^2+a e^2}\, \sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\right)+$$

$$\left(3 e \left(2 c^{3/2} d^3+2 a \sqrt{c}\, d e^2-\sqrt{c d^2+a e^2}\,\left(2 c d^2+a e^2\right)\right) \text{Log}\left[\sqrt{c d^2+a e^2}+\sqrt{2}\, c^{1/4} \sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\, \sqrt{d+e x}+\sqrt{c}\,(d+e x)\right]\right)\Big/$$

$$\left(64 \sqrt{2}\, a^2 c^{7/4} \sqrt{c d^2+a e^2}\, \sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\right)$$

Result (type 3, 311 leaves) :

$$\frac{1}{32\,a^{5/2}\,c^2}\left(\frac{2\sqrt{a}\;c\sqrt{d+e\,x}\;\left(6\,c^2\,d^2\,x^3-a^2\,e\,(7\,d+e\,x)+a\,c\,x\left(10\,d^2+d\,e\,x+3\,e^2\,x^2\right)\right)}{\left(a+c\,x^2\right)^2}-\right.$$

$$\frac{3\left(4\,\mathrm{i}\,c^2\,d^3+2\sqrt{a}\;c^{3/2}\,d^2\,e+3\,\mathrm{i}\,a\,c\,d\,e^2+a^{3/2}\sqrt{c}\;e^3\right)\text{ArcTanh}\left[\frac{\sqrt{c}\;\sqrt{d+e\,x}}{\sqrt{c\,d-\mathrm{i}\sqrt{a}\;\sqrt{c}\;e}}\right]}{\sqrt{c\,d-\mathrm{i}\sqrt{a}\;\sqrt{c}\;e}}+$$

$$\left.3\sqrt{c\,d+\mathrm{i}\sqrt{a}\;\sqrt{c}\;e}\;\left(4\,\mathrm{i}\,c\,d^2+2\sqrt{a}\;\sqrt{c}\;d\,e+\mathrm{i}\,a\,e^2\right)\text{ArcTanh}\left[\frac{\sqrt{c\,d+\mathrm{i}\sqrt{a}\;\sqrt{c}\;e}\;\sqrt{d+e\,x}}{\sqrt{c}\;d+\mathrm{i}\sqrt{a}\;e}\right]\right)$$

- **Problem 645: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(d+e\,x)^{3/2}}{\left(a+c\,x^2\right)^3}\,dx$$

Optimal (type 3, 769 leaves, 12 steps) :

$$-\frac{(a e-c d x)\sqrt{d+e x}}{4 a c\left(a+c x^2\right)^2}+\frac{(a e+6 c d x)\sqrt{d+e x}}{16 a^2 c\left(a+c x^2\right)}+\frac{3 e\left(2 c d^2+a e^2+2\sqrt{c}\, d\sqrt{c d^2+a e^2}\right)\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}-\sqrt{2}\, c^{1/4}\sqrt{d+e x}}{\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}\right]}{32\sqrt{2}\, a^2 c^{5/4}\sqrt{c d^2+a e^2}\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}-$$

$$\frac{3 e\left(2 c d^2+a e^2+2\sqrt{c}\, d\sqrt{c d^2+a e^2}\right)\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}+\sqrt{2}\, c^{1/4}\sqrt{d+e x}}{\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}\right]}{32\sqrt{2}\, a^2 c^{5/4}\sqrt{c d^2+a e^2}\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}-$$

$$\left(3 e\left(2 c d^2+a e^2-2\sqrt{c}\, d\sqrt{c d^2+a e^2}\right)\text{Log}\left[\sqrt{c d^2+a e^2}-\sqrt{2}\, c^{1/4}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\sqrt{d+e x}+\sqrt{c}\,(d+e x)\right]\right)\Big/$$

$$\left(64\sqrt{2}\, a^2 c^{5/4}\sqrt{c d^2+a e^2}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\right)+$$

$$\left(3 e\left(2 c d^2+a e^2-2\sqrt{c}\, d\sqrt{c d^2+a e^2}\right)\text{Log}\left[\sqrt{c d^2+a e^2}+\sqrt{2}\, c^{1/4}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\sqrt{d+e x}+\sqrt{c}\,(d+e x)\right]\right)\Big/$$

$$\left(64\sqrt{2}\, a^2 c^{5/4}\sqrt{c d^2+a e^2}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\right)$$

Result (type 3, 253 leaves):

$$\frac{1}{32 a^{5/2} c}\left(\frac{2\sqrt{a}\sqrt{d+e x}\left(-3 a^2 e+6 c^2 d x^3+a c x(10 d+e x)\right)}{\left(a+c x^2\right)^2}-\right.$$

$$\left.\frac{3 i\left(4 c d^2-2 i\sqrt{a}\sqrt{c}\, d e+a e^2\right)\text{ArcTanh}\left[\frac{\sqrt{c}\sqrt{d+e x}}{\sqrt{c d-i\sqrt{a}\sqrt{c}\, e}}\right]}{\sqrt{c d-i\sqrt{a}\sqrt{c}\, e}}+\frac{3 i\left(4 c d^2+2 i\sqrt{a}\sqrt{c}\, d e+a e^2\right)\text{ArcTanh}\left[\frac{\sqrt{c}\sqrt{d+e x}}{\sqrt{c d+i\sqrt{a}\sqrt{c}\, e}}\right]}{\sqrt{c d+i\sqrt{a}\sqrt{c}\, e}}\right)$$

■ Problem 646: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{d+e\,x}}{\left(a+c\,x^2\right)^3}\,dx$$

Optimal (type 3, 849 leaves, 12 steps):

$$\frac{x\sqrt{d+e\,x}}{4\,a\left(a+c\,x^2\right)^2}+\frac{\sqrt{d+e\,x}\,\left(a\,d\,e+\left(6\,c\,d^2+5\,a\,e^2\right)x\right)}{16\,a^2\left(c\,d^2+a\,e^2\right)\left(a+c\,x^2\right)}+$$

$$\frac{e\left(6\,c^{3/2}\,d^3+8\,a\sqrt{c}\,d\,e^2+\sqrt{c\,d^2+a\,e^2}\,\left(6\,c\,d^2+5\,a\,e^2\right)\right)\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}-\sqrt{2}\,c^{1/4}\sqrt{d+e\,x}}{\sqrt{\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}}}\right]}{32\sqrt{2}\,a^2\,c^{3/4}\left(c\,d^2+a\,e^2\right)^{3/2}\sqrt{\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}}}-$$

$$\frac{e\left(6\,c^{3/2}\,d^3+8\,a\sqrt{c}\,d\,e^2+\sqrt{c\,d^2+a\,e^2}\,\left(6\,c\,d^2+5\,a\,e^2\right)\right)\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}+\sqrt{2}\,c^{1/4}\sqrt{d+e\,x}}{\sqrt{\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}}}\right]}{32\sqrt{2}\,a^2\,c^{3/4}\left(c\,d^2+a\,e^2\right)^{3/2}\sqrt{\sqrt{c}\,d-\sqrt{c\,d^2+a\,e^2}}}-$$

$$\left(e\left(6\,c^{3/2}\,d^3+8\,a\sqrt{c}\,d\,e^2-\sqrt{c\,d^2+a\,e^2}\,\left(6\,c\,d^2+5\,a\,e^2\right)\right)\text{Log}\left[\sqrt{c\,d^2+a\,e^2}-\sqrt{2}\,c^{1/4}\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}\,\sqrt{d+e\,x}+\sqrt{c}\,(d+e\,x)\right]\right)\Big/$$

$$\left(64\sqrt{2}\,a^2\,c^{3/4}\left(c\,d^2+a\,e^2\right)^{3/2}\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}\right)+$$

$$\left(e\left(6\,c^{3/2}\,d^3+8\,a\sqrt{c}\,d\,e^2-\sqrt{c\,d^2+a\,e^2}\,\left(6\,c\,d^2+5\,a\,e^2\right)\right)\text{Log}\left[\sqrt{c\,d^2+a\,e^2}+\sqrt{2}\,c^{1/4}\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}\,\sqrt{d+e\,x}+\sqrt{c}\,(d+e\,x)\right]\right)\Big/$$

$$\left(64\sqrt{2}\,a^2\,c^{3/4}\left(c\,d^2+a\,e^2\right)^{3/2}\sqrt{\sqrt{c}\,d+\sqrt{c\,d^2+a\,e^2}}\right)$$

Result (type 3, 318 leaves):

$$\frac{1}{32\,a^{5/2}}\left(\frac{2\sqrt{a}\,\sqrt{d+e\,x}\,\left(4\,a\,x+\frac{\left(a+c\,x^2\right)\,\left(6\,c\,d^2\,x+a\,e\,\left(d+5\,e\,x\right)\right)}{c\,d^2+a\,e^2}\right)}{\left(a+c\,x^2\right)^2}-\right.$$

$$\left.\frac{i\,\left(12\,c\,d^2-18\,i\,\sqrt{a}\,\sqrt{c}\,d\,e-5\,a\,e^2\right)\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{d+e\,x}}{\sqrt{c\,d-i\,\sqrt{a}\,\sqrt{c}\,e}}\right]}{\sqrt{c}\,\left(\sqrt{c}\,d-i\,\sqrt{a}\,e\right)\,\sqrt{c\,d-i\,\sqrt{a}\,\sqrt{c}\,e}}+\frac{i\,\left(12\,c\,d^2+18\,i\,\sqrt{a}\,\sqrt{c}\,d\,e-5\,a\,e^2\right)\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{d+e\,x}}{\sqrt{c\,d+i\,\sqrt{a}\,\sqrt{c}\,e}}\right]}{\sqrt{c}\,\left(\sqrt{c}\,d+i\,\sqrt{a}\,e\right)\,\sqrt{c\,d+i\,\sqrt{a}\,\sqrt{c}\,e}}\right)$$

- **Problem 647: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\sqrt{d+e\,x}\,\left(a+c\,x^2\right)^3}\,dx$$

Optimal (type 3, 920 leaves, 12 steps):

$$\frac{(a e+c d x)\sqrt{d+e x}}{4 a\left(c d^2+a e^2\right)\left(a+c x^2\right)^2}+\frac{\sqrt{d+e x}\left(a e\left(c d^2+7 a e^2\right)+6 c d\left(c d^2+2 a e^2\right) x\right)}{16 a^2\left(c d^2+a e^2\right)^2\left(a+c x^2\right)}+$$

$$\left(3 e\left(2 c^2 d^4+5 a c d^2 e^2+7 a^2 e^4+2\sqrt{c}\, d\sqrt{c d^2+a e^2}\left(c d^2+2 a e^2\right)\right)\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}-\sqrt{2}\, c^{1/4}\sqrt{d+e x}}{\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}\right]\right)\Big/$$

$$\left(32\sqrt{2}\, a^2 c^{1/4}\left(c d^2+a e^2\right)^{5/2}\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}\right)-$$

$$\left(3 e\left(2 c^2 d^4+5 a c d^2 e^2+7 a^2 e^4+2\sqrt{c}\, d\sqrt{c d^2+a e^2}\left(c d^2+2 a e^2\right)\right)\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}+\sqrt{2}\, c^{1/4}\sqrt{d+e x}}{\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}}\right]\right)\Big/$$

$$\left(32\sqrt{2}\, a^2 c^{1/4}\left(c d^2+a e^2\right)^{5/2}\sqrt{\sqrt{c}\, d-\sqrt{c d^2+a e^2}}\right)-\left(3 e\left(2 c^2 d^4+5 a c d^2 e^2+7 a^2 e^4-2\sqrt{c}\, d\sqrt{c d^2+a e^2}\left(c d^2+2 a e^2\right)\right)\right.$$

$$\left.\text{Log}\left[\sqrt{c d^2+a e^2}-\sqrt{2}\, c^{1/4}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\sqrt{d+e x}+\sqrt{c}\,(d+e x)\right]\right)\Big/$$

$$\left(64\sqrt{2}\, a^2 c^{1/4}\left(c d^2+a e^2\right)^{5/2}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\right)+\left(3 e\left(2 c^2 d^4+5 a c d^2 e^2+7 a^2 e^4-2\sqrt{c}\, d\sqrt{c d^2+a e^2}\left(c d^2+2 a e^2\right)\right)\right.$$

$$\left.\text{Log}\left[\sqrt{c d^2+a e^2}+\sqrt{2}\, c^{1/4}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\sqrt{d+e x}+\sqrt{c}\,(d+e x)\right]\right)\Big/\left(64\sqrt{2}\, a^2 c^{1/4}\left(c d^2+a e^2\right)^{5/2}\sqrt{\sqrt{c}\, d+\sqrt{c d^2+a e^2}}\right)$$

Result (type 3, 348 leaves) :

$$\frac{1}{32 a^{5/2}}\left(\frac{2\sqrt{a}\sqrt{d+e x}\left(11 a^3 e^3+6 c^3 d^3 x^3+a^2 c e\left(5 d^2+16 d e x+7 e^2 x^2\right)+a c^2 d x\left(10 d^2+d e x+12 e^2 x^2\right)\right)}{\left(c d^2+a e^2\right)^2\left(a+c x^2\right)^2}-\right.$$

$$\left.\frac{3 i\left(4 c d^2-10 i\sqrt{a}\sqrt{c}\, d e-7 a e^2\right)\text{ArcTanh}\left[\frac{\sqrt{c}\sqrt{d+e x}}{\sqrt{c d-i\sqrt{a}\sqrt{c}\, e}}\right]}{\left(\sqrt{c}\, d-i\sqrt{a}\, e\right)^2\sqrt{c d-i\sqrt{a}\sqrt{c}\, e}}+\frac{3 i\left(4 c d^2+10 i\sqrt{a}\sqrt{c}\, d e-7 a e^2\right)\text{ArcTanh}\left[\frac{\sqrt{c}\sqrt{d+e x}}{\sqrt{c d+i\sqrt{a}\sqrt{c}\, e}}\right]}{\left(\sqrt{c}\, d+i\sqrt{a}\, e\right)^2\sqrt{c d+i\sqrt{a}\sqrt{c}\, e}}\right)$$

Problem 648: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{2+3x}}{1+x^2}\,dx$$

Optimal (type 3, 214 leaves, 10 steps):

$$-\frac{3\,\text{ArcTan}\left[\frac{\sqrt{2\left(2+\sqrt{13}\right)}-2\sqrt{2+3x}}{\sqrt{2\left(-2+\sqrt{13}\right)}}\right]}{\sqrt{2\left(-2+\sqrt{13}\right)}}+\frac{3\,\text{ArcTan}\left[\frac{\sqrt{2\left(2+\sqrt{13}\right)}+2\sqrt{2+3x}}{\sqrt{2\left(-2+\sqrt{13}\right)}}\right]}{\sqrt{2\left(-2+\sqrt{13}\right)}}+$$

$$\frac{3\,\text{Log}\left[2+\sqrt{13}+3x-\sqrt{2\left(2+\sqrt{13}\right)}\sqrt{2+3x}\right]}{2\sqrt{2\left(2+\sqrt{13}\right)}}-\frac{3\,\text{Log}\left[2+\sqrt{13}+3x+\sqrt{2\left(2+\sqrt{13}\right)}\sqrt{2+3x}\right]}{2\sqrt{2\left(2+\sqrt{13}\right)}}$$

Result (type 3, 59 leaves):

$$\frac{(3-2\,i)\,\text{ArcTan}\left[\frac{\sqrt{2+3x}}{\sqrt{-2-3i}}\right]}{\sqrt{-2-3\,i}}+\frac{(3+2\,i)\,\text{ArcTan}\left[\frac{\sqrt{2+3x}}{\sqrt{-2+3i}}\right]}{\sqrt{-2+3\,i}}$$

Problem 649: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{c+dx}}{1+x^2}\,dx$$

Optimal (type 3, 316 leaves, 10 steps):

$$\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{c+\sqrt{c^2+d^2}}-\sqrt{2}\sqrt{c+dx}}{\sqrt{c-\sqrt{c^2+d^2}}}\right]}{\sqrt{2}\sqrt{c-\sqrt{c^2+d^2}}}-\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{c+\sqrt{c^2+d^2}}+\sqrt{2}\sqrt{c+dx}}{\sqrt{c-\sqrt{c^2+d^2}}}\right]}{\sqrt{2}\sqrt{c-\sqrt{c^2+d^2}}}+$$

$$\frac{d\,\text{Log}\left[c+\sqrt{c^2+d^2}+dx-\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}\sqrt{c+dx}\right]}{2\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}}-\frac{d\,\text{Log}\left[c+\sqrt{c^2+d^2}+dx+\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}\sqrt{c+dx}\right]}{2\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}}$$

Result (type 3, 75 leaves):

$$-i\sqrt{c-i\,d}\;\text{ArcTanh}\left[\frac{\sqrt{c+d\,x}}{\sqrt{c-i\,d}}\right]+i\sqrt{c+i\,d}\;\text{ArcTanh}\left[\frac{\sqrt{c+d\,x}}{\sqrt{c+i\,d}}\right]$$

- **Problem 652: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{2+3\,x}}{a+b\,x^2}\,dx$$

Optimal (type 3, 427 leaves, 10 steps):

$$\frac{3\,\text{ArcTanh}\left[\frac{\sqrt{2\sqrt{b}+\sqrt{9\,a+4\,b}}-\sqrt{2}\,b^{1/4}\sqrt{2+3\,x}}{\sqrt{2\sqrt{b}-\sqrt{9\,a+4\,b}}}\right]}{\sqrt{2}\,b^{3/4}\sqrt{2\sqrt{b}-\sqrt{9\,a+4\,b}}}-\frac{3\,\text{ArcTanh}\left[\frac{\sqrt{2\sqrt{b}+\sqrt{9\,a+4\,b}}+\sqrt{2}\,b^{1/4}\sqrt{2+3\,x}}{\sqrt{2\sqrt{b}-\sqrt{9\,a+4\,b}}}\right]}{\sqrt{2}\,b^{3/4}\sqrt{2\sqrt{b}-\sqrt{9\,a+4\,b}}}+$$

$$\frac{3\,\text{Log}\left[\sqrt{9\,a+4\,b}-\sqrt{2}\,b^{1/4}\sqrt{2\sqrt{b}+\sqrt{9\,a+4\,b}}\,\sqrt{2+3\,x}+\sqrt{b}\,(2+3\,x)\right]}{2\sqrt{2}\,b^{3/4}\sqrt{2\sqrt{b}+\sqrt{9\,a+4\,b}}}-$$

$$\frac{3\,\text{Log}\left[\sqrt{9\,a+4\,b}+\sqrt{2}\,b^{1/4}\sqrt{2\sqrt{b}+\sqrt{9\,a+4\,b}}\,\sqrt{2+3\,x}+\sqrt{b}\,(2+3\,x)\right]}{2\sqrt{2}\,b^{3/4}\sqrt{2\sqrt{b}+\sqrt{9\,a+4\,b}}}$$

Result (type 3, 186 leaves):

$$\frac{(-1)^{1/4}\left(\frac{\left(3\sqrt{a}-2\,i\sqrt{b}\right)\text{ArcTan}\left[\frac{(-1)^{1/4}\sqrt{b}\,\sqrt{2+3\,x}}{\sqrt{3\sqrt{a}\,\sqrt{b}-2\,i\,b}}\right]}{\sqrt{3\sqrt{a}\,\sqrt{b}-2\,i\,b}}+\frac{i\left(3\sqrt{a}+2\,i\sqrt{b}\right)\text{ArcTan}\left[\frac{(-1)^{3/4}\sqrt{b}\,\sqrt{2+3\,x}}{\sqrt{3\sqrt{a}\,\sqrt{b}+2\,i\,b}}\right]}{\sqrt{3\sqrt{a}\,\sqrt{b}+2\,i\,b}}\right)}{\sqrt{a}\,\sqrt{b}}$$

- **Problem 654: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{1+x}}{1+x^2}\,dx$$

Optimal (type 3, 205 leaves, 10 steps):

$$-\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\ \text{ArcTan}\left[\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right]+\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\ \text{ArcTan}\left[\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right]+$$

$$\frac{\text{Log}\left[1+\sqrt{2}+x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right]}{2\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\text{Log}\left[1+\sqrt{2}+x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right]}{2\sqrt{2\left(1+\sqrt{2}\right)}}$$

Result (type 3, 51 leaves) :

$$-\frac{2\,\text{ArcTan}\left[\frac{\sqrt{1+x}}{\sqrt{-1-i}}\right]}{(-1-i)^{3/2}}-\frac{2\,\text{ArcTan}\left[\frac{\sqrt{1+x}}{\sqrt{-1+i}}\right]}{(-1+i)^{3/2}}$$

- **Problem 655: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\sqrt{1+x}\,\left(1+x^2\right)}\,dx$$

Optimal (type 3, 198 leaves, 10 steps) :

$$-\frac{1}{2}\sqrt{1+\sqrt{2}}\ \text{ArcTan}\left[\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right]+\frac{1}{2}\sqrt{1+\sqrt{2}}\ \text{ArcTan}\left[\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right]-$$

$$\frac{\text{Log}\left[1+\sqrt{2}+x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right]}{4\sqrt{1+\sqrt{2}}}+\frac{\text{Log}\left[1+\sqrt{2}+x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right]}{4\sqrt{1+\sqrt{2}}}$$

Result (type 3, 55 leaves) :

$$-\frac{i\,\text{ArcTan}\left[\frac{\sqrt{1+x}}{\sqrt{-1-i}}\right]}{\sqrt{-1-i}}+\frac{i\,\text{ArcTan}\left[\frac{\sqrt{1+x}}{\sqrt{-1+i}}\right]}{\sqrt{-1+i}}$$

- **Problem 656: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{-1+x}}{\left(1+x^2\right)^3}\,dx$$

Optimal (type 3, 272 leaves, 12 steps) :

$$\frac{\sqrt{-1+x}\,x}{4\left(1+x^2\right)^2}-\frac{(1-11x)\sqrt{-1+x}}{32\left(1+x^2\right)}-\frac{1}{64}\sqrt{\frac{1}{2}\left(-527+373\sqrt{2}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{2}\right)}-2\sqrt{-1+x}}{\sqrt{2\left(1+\sqrt{2}\right)}}\right]+$$

$$\frac{1}{64}\sqrt{\frac{1}{2}\left(-527+373\sqrt{2}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{2}\right)}+2\sqrt{-1+x}}{\sqrt{2\left(1+\sqrt{2}\right)}}\right]-\frac{1}{128}\sqrt{\frac{1}{2}\left(527+373\sqrt{2}\right)}\,\text{Log}\left[1-\sqrt{2}-\sqrt{2\left(-1+\sqrt{2}\right)}\,\sqrt{-1+x}-x\right]+$$

$$\frac{1}{128}\sqrt{\frac{1}{2}\left(527+373\sqrt{2}\right)}\,\text{Log}\left[1-\sqrt{2}+\sqrt{2\left(-1+\sqrt{2}\right)}\,\sqrt{-1+x}-x\right]$$

Result (type 3, 90 leaves) :

$$\frac{1}{64}\left(\frac{2\sqrt{-1+x}\left(-1+19x-x^2+11x^3\right)}{\left(1+x^2\right)^2}-(7-18\,i)\sqrt{1-i}\,\text{ArcTan}\left[\frac{\sqrt{-1+x}}{\sqrt{1-i}}\right]-(7+18\,i)\sqrt{1+i}\,\text{ArcTan}\left[\frac{\sqrt{-1+x}}{\sqrt{1+i}}\right]\right)$$

- **Problem 657: Result unnecessarily involves imaginary or complex numbers.**

$$\int (d+e\,x)^{3/2}\sqrt{a+c\,x^2}\,dx$$

Optimal (type 4, 398 leaves, 7 steps) :

$$\frac{2\sqrt{d+e\,x}\left(3\,c\,d^2-5\,a\,e^2+24\,c\,d\,e\,x\right)\sqrt{a+c\,x^2}}{105\,c\,e}+\frac{2\,e\sqrt{d+e\,x}\left(a+c\,x^2\right)^{3/2}}{7\,c}+$$

$$\frac{4\sqrt{-a}\,d\left(3\,c\,d^2-29\,a\,e^2\right)\sqrt{d+e\,x}\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]}{105\sqrt{c}\,e^2\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{a+c\,x^2}}-$$

$$\left(4\sqrt{-a}\left(3\,c\,d^2-5\,a\,e^2\right)\left(c\,d^2+a\,e^2\right)\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]\right)\Big/$$

$$\left(105\,c^{3/2}\,e^2\sqrt{d+e\,x}\sqrt{a+c\,x^2}\right)$$

Result (type 4, 582 leaves) :

$$\frac{1}{105\sqrt{a+c x^2}}\sqrt{d+e x}\left(\frac{2\left(a+c x^2\right)\left(10 a e^2+3 c\left(d^2+8 d e x+5 e^2 x^2\right)\right)}{c e}+\right.$$

$$\frac{1}{c e^3\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\,(d+e x)}4\left(-d e^2\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\left(-29 a^2 e^2+3 c^2 d^2 x^2+a c\left(3 d^2-29 e^2 x^2\right)\right)+\right.$$

$$\sqrt{c}\,d\left(3 i c^{3/2} d^3-3\sqrt{a}\,c d^2 e-29 i a\sqrt{c}\,d e^2+29 a^{3/2} e^3\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}\,(d+e x)^{3/2}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]+\sqrt{a}\,e\left(3 c^{3/2} d^3+27 i\sqrt{a}\,c d^2 e-29 a\sqrt{c}\,d e^2-5 i a^{3/2} e^3\right)$$

$$\left.\left.\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}\,(d+e x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]\right)\right)$$

- **Problem 658: Result unnecessarily involves imaginary or complex numbers.**

$$\int\sqrt{d+e x}\,\sqrt{a+c x^2}\,dx$$

Optimal (type 4, 362 leaves, 7 steps) :

$$-\frac{4 d \sqrt{d+e x} \sqrt{a+c x^2}}{15 e}+\frac{2 (d+e x)^{3/2} \sqrt{a+c x^2}}{5 e}+\frac{4 \sqrt{-a} \left(c d^2-3 a e^2\right) \sqrt{d+e x} \sqrt{1+\frac{c x^2}{a}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]}{15 \sqrt{c} e^2 \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{a+c x^2}}-$$

$$\frac{4 \sqrt{-a} d \left(c d^2+a e^2\right) \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{1+\frac{c x^2}{a}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]}{15 \sqrt{c} e^2 \sqrt{d+e x} \sqrt{a+c x^2}}$$

Result (type 4, 536 leaves) :

$$\frac{1}{15 \sqrt{a+c x^2}}\sqrt{d+e x}$$

$$\left(\frac{2 (d+3 e x) \left(a+c x^2\right)}{e}-\left(4 \left(e^2 \sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}} \left(-3 a^2 e^2+c^2 d^2 x^2+a c \left(d^2-3 e^2 x^2\right)\right)+\sqrt{c} \left(-i c^{3/2} d^3+\sqrt{a} c d^2 e+3 i a \sqrt{c} d e^2-3 a^{3/2} e^3\right)\right.\right.\right.$$

$$\sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a} e}{\sqrt{c}}-e x}{d+e x}} (d+e x)^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c} d-i \sqrt{a} e}{\sqrt{c} d+i \sqrt{a} e}\right]-$$

$$\sqrt{a} \sqrt{c} e \left(c d^2+4 i \sqrt{a} \sqrt{c} d e-3 a e^2\right) \sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a} e}{\sqrt{c}}-e x}{d+e x}} (d+e x)^{3/2}$$

$$\left.\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c} d-i \sqrt{a} e}{\sqrt{c} d+i \sqrt{a} e}\right]\right)\right)\Big/\left(c e^3 \sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}} (d+e x)\right)\right)$$

- **Problem 659: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{a+c x^2}}{\sqrt{d+e x}} \, dx$$

Optimal (type 4, 322 leaves, 6 steps) :

$$\frac{2\sqrt{d+e x}\sqrt{a+c x^2}}{3 e}+\frac{4\sqrt{-a}\sqrt{c}\, d\sqrt{d+e x}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\, d-a e}\right]}{3 e^2\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}}\sqrt{a+c x^2}}-$$

$$\frac{4\sqrt{-a}\left(c d^2+a e^2\right)\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\, d-a e}\right]}{3\sqrt{c}\, e^2\sqrt{d+e x}\sqrt{a+c x^2}}$$

Result (type 4, 456 leaves) :

$$\frac{1}{3 e^3\sqrt{a+c x^2}}$$

$$2\sqrt{d+e x}\left(e^2\left(a+c x^2\right)-1\Big/\left(\sqrt{-d-\frac{i\sqrt{a}\, e}{\sqrt{c}}}\,(d+e x)\right)2\left(d e^2\sqrt{-d-\frac{i\sqrt{a}\, e}{\sqrt{c}}}\left(a+c x^2\right)+\sqrt{c}\, d\left(-i\sqrt{c}\, d+\sqrt{a}\, e\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\, e}{\sqrt{c}}-e x}{d+e x}}\right.\right.$$

$$(d+e x)^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\, d-i\sqrt{a}\, e}{\sqrt{c}\, d+i\sqrt{a}\, e}\right]-$$

$$\left.\left.\sqrt{a}\, e\left(\sqrt{c}\, d+i\sqrt{a}\, e\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\, e}{\sqrt{c}}-e x}{d+e x}}\,(d+e x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\, d-i\sqrt{a}\, e}{\sqrt{c}\, d+i\sqrt{a}\, e}\right]\right)\right)$$

- **Problem 660: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{a+c x^2}}{(d+e x)^{3/2}}\,dx$$

Optimal (type 4, 305 leaves, 6 steps) :

$$-\frac{2\sqrt{a+c\,x^2}}{e\sqrt{d+e\,x}}-\frac{4\sqrt{-a}\,\sqrt{c}\,\sqrt{d+e\,x}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\,-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]}{e^2\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{a+c\,x^2}}+$$

$$\frac{4\sqrt{-a}\,\sqrt{c}\,d\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\,-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]}{e^2\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 419 leaves) :

$$\frac{1}{e^3\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}}$$

$$2\left(e^2\left(a+c\,x^2\right)+1\Big/\left(\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\right)2\sqrt{c}\,\left(-i\sqrt{c}\,d+\sqrt{a}\,e\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\,\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\right.\right.$$

$$\left.\left.\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]-\frac{2\sqrt{a}\,\sqrt{c}\,e\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\,\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\,\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]}{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}\right)$$

- **Problem 661: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{a+c\,x^2}}{(d+e\,x)^{5/2}}\,dx$$

Optimal (type 4, 366 leaves, 7 steps) :

$$-\frac{2\sqrt{a+c x^2}}{3 e (d+e x)^{3/2}}+\frac{4 c d \sqrt{a+c x^2}}{3 e \left(c d^2+a e^2\right) \sqrt{d+e x}}+\frac{4 \sqrt{-a}\, c^{3/2} d \sqrt{d+e x} \sqrt{1+\frac{c x^2}{a}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c}\, d-a e}\right]}{3 e^2 \left(c d^2+a e^2\right) \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}} \sqrt{a+c x^2}}-$$

$$\frac{4 \sqrt{-a} \sqrt{c} \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}} \sqrt{1+\frac{c x^2}{a}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c}\, d-a e}\right]}{3 e^2 \sqrt{d+e x} \sqrt{a+c x^2}}$$

Result (type 4, 504 leaves) :

$$\frac{2 \sqrt{a+c x^2} \left(-a e^2+c d (d+2 e x)\right)}{3 \left(c d^2 e+a e^3\right) (d+e x)^{3/2}}-$$

$$\left(4 c \left(d e^2 \sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}} \left(a+c x^2\right)+\sqrt{c}\, d \left(-i \sqrt{c}\, d+\sqrt{a}\, e\right) \sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a}\, e}{\sqrt{c}}-e x}{d+e x}} (d+e x)^{3/2} \text{EllipticE}\left[\right.\right.\right.$$

$$\left. i \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\, d-i \sqrt{a}\, e}{\sqrt{c}\, d+i \sqrt{a}\, e}\right]-\sqrt{a}\, e \left(\sqrt{c}\, d+i \sqrt{a}\, e\right) \sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a}\, e}{\sqrt{c}}-e x}{d+e x}} (d+e x)^{3/2}$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\, d-i \sqrt{a}\, e}{\sqrt{c}\, d+i \sqrt{a}\, e}\right]\right)\right) \Big/ \left(3 e^3 \sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}} \left(c d^2+a e^2\right) \sqrt{d+e x} \sqrt{a+c x^2}\right)$$

- **Problem 662: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{a+c x^2}}{(d+e x)^{7/2}}\, dx$$

Optimal (type 4, 444 leaves, 8 steps) :

$$-\frac{2\sqrt{a+c x^2}}{5 e (d+e x)^{5/2}}+\frac{4 c d \sqrt{a+c x^2}}{15 e \left(c d^2+a e^2\right) (d+e x)^{3/2}}+\frac{4 c \left(c d^2-3 a e^2\right) \sqrt{a+c x^2}}{15 e \left(c d^2+a e^2\right)^2 \sqrt{d+e x}}+$$

$$\frac{4 \sqrt{-a}\, c^{3/2} \left(c d^2-3 a e^2\right) \sqrt{d+e x} \sqrt{1+\frac{c x^2}{a}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c}\, d-a e}\right]}{15 e^2 \left(c d^2+a e^2\right)^2 \sqrt{\frac{\sqrt{c}\, (d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}} \sqrt{a+c x^2}}-$$

$$\frac{4 \sqrt{-a}\, c^{3/2} d \sqrt{\frac{\sqrt{c}\, (d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}} \sqrt{1+\frac{c x^2}{a}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c}\, d-a e}\right]}{15 e^2 \left(c d^2+a e^2\right) \sqrt{d+e x} \sqrt{a+c x^2}}$$

Result (type 4, 602 leaves) :

$$\frac{1}{15 e^3 \left(c d^2+a e^2\right)^2 (d+e x)^{5/2} \sqrt{a+c x^2}}$$

$$2 \left(-e^2 \left(a+c x^2\right) \left(3 a^2 e^4-c^2 d^2 \left(d^2+6 d e x+2 e^2 x^2\right)+2 a c e^2 \left(5 d^2+5 d e x+3 e^2 x^2\right)\right)+\frac{1}{\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}} 2 c (d+e x)^2\right.$$

$$\left(-e^2 \sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}} \left(-3 a^2 e^2+c^2 d^2 x^2+a c \left(d^2-3 e^2 x^2\right)\right)+\sqrt{c} \left(i c^{3/2} d^3-\sqrt{a}\, c d^2 e-3 i a \sqrt{c}\, d e^2+3 a^{3/2} e^3\right) \sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\right.$$

$$\sqrt{-\frac{\frac{i \sqrt{a}\, e}{\sqrt{c}}-e x}{d+e x}}\, (d+e x)^{3/2}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\, d-i \sqrt{a}\, e}{\sqrt{c}\, d+i \sqrt{a}\, e}\right]+\sqrt{a} \sqrt{c}\, e \left(c d^2+4 i \sqrt{a} \sqrt{c}\, d e-3 a e^2\right)$$

$$\left.\left.\sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a}\, e}{\sqrt{c}}-e x}{d+e x}}\, (d+e x)^{3/2}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\, d-i \sqrt{a}\, e}{\sqrt{c}\, d+i \sqrt{a}\, e}\right]\right)\right)$$

- **Problem 663: Result unnecessarily involves imaginary or complex numbers.**

$$\int (d+e x)^{3/2} \left(a+c x^2\right)^{3/2} dx$$

Optimal (type 4, 497 leaves, 8 steps):

$$\frac{4 \sqrt{d+e x} \left(4 c^2 d^4+21 a c d^2 e^2-15 a^2 e^4-3 c d e \left(c d^2-31 a e^2\right) x\right) \sqrt{a+c x^2}}{1155 c e^3}+\frac{2 \sqrt{d+e x} \left(c d^2-3 a e^2+28 c d e x\right) \left(a+c x^2\right)^{3/2}}{231 c e}+$$

$$\frac{2 e \sqrt{d+e x} \left(a+c x^2\right)^{5/2}}{11 c}+\left(32 \sqrt{-a} d \left(c d^2-3 a e^2\right) \left(c d^2+9 a e^2\right) \sqrt{d+e x} \sqrt{1+\frac{c x^2}{a}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]\right)\Big/$$

$$\left(1155 \sqrt{c} e^4 \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{a+c x^2}\right)-$$

$$\left(8 \sqrt{-a} \left(c d^2+a e^2\right) \left(4 c^2 d^4+21 a c d^2 e^2-15 a^2 e^4\right) \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{1+\frac{c x^2}{a}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]\right)\Big/$$

$$\left(1155 c^{3/2} e^4 \sqrt{d+e x} \sqrt{a+c x^2}\right)$$

Result (type 4, 695 leaves):

$$\frac{1}{1155\, c\, e^5 \sqrt{a+c\,x^2}} 2\sqrt{d+e\,x}$$

$$\left(e^2\left(a+c\,x^2\right)\left(60\,a^2\,e^4+a\,c\,e^2\left(47\,d^2+326\,d\,e\,x+195\,e^2\,x^2\right)+c^2\left(8\,d^4-6\,d^3\,e\,x+5\,d^2\,e^2\,x^2+140\,d\,e^3\,x^3+105\,e^4\,x^4\right)\right)+\frac{1}{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\,(d+e\,x)}\right.$$

$$4\left(-4\,d\,e^2\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\left(c^2\,d^4+6\,a\,c\,d^2\,e^2-27\,a^2\,e^4\right)\left(a+c\,x^2\right)+4\sqrt{c}\,d\left(i\,c^{5/2}\,d^5-\sqrt{a}\,c^2\,d^4\,e+6\,i\,a\,c^{3/2}\,d^3\,e^2-6\,a^{3/2}\,c\,d^2\,e^3-\right.\right.$$

$$\left.27\,i\,a^2\sqrt{c}\,d\,e^4+27\,a^{5/2}\,e^5\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]+$$

$$\sqrt{a}\,e\left(4\,c^{5/2}\,d^5+i\sqrt{a}\,c^2\,d^4\,e+24\,a\,c^{3/2}\,d^3\,e^2+114\,i\,a^{3/2}\,c\,d^2\,e^3-108\,a^2\sqrt{c}\,d\,e^4-15\,i\,a^{5/2}\,e^5\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}$$

$$\left.\left.\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]\right)\right)$$

- **Problem 664: Result unnecessarily involves imaginary or complex numbers.**

$$\int\sqrt{d+e\,x}\,\left(a+c\,x^2\right)^{3/2}\,dx$$

Optimal (type 4, 448 leaves, 8 steps) :

$$\frac{4\sqrt{d+e\,x}\,\left(4\,d\,\left(c\,d^2+3\,a\,e^2\right)-3\,e\,\left(c\,d^2-7\,a\,e^2\right)\,x\right)\,\sqrt{a+c\,x^2}}{315\,e^3}-\frac{4\,d\,\sqrt{d+e\,x}\,\left(a+c\,x^2\right)^{3/2}}{21\,e}+\frac{2\,(d+e\,x)^{3/2}\,\left(a+c\,x^2\right)^{3/2}}{9\,e}+$$

$$\left(8\sqrt{-a}\,\left(4\,c^2\,d^4+15\,a\,c\,d^2\,e^2-21\,a^2\,e^4\right)\,\sqrt{d+e\,x}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]\right)\Big/$$

$$\left(315\sqrt{c}\,e^4\,\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{a+c\,x^2}\right)-$$

$$\left(32\sqrt{-a}\,d\,\left(c\,d^2+a\,e^2\right)\,\left(c\,d^2+3\,a\,e^2\right)\,\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]\right)\Big/$$

$$\left(315\sqrt{c}\,e^4\,\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}\right)$$

Result (type 4, 646 leaves) :

$$\frac{1}{315\sqrt{a+c\,x^2}}\sqrt{d+e\,x}\left(\frac{2\left(a+c\,x^2\right)\left(a\,e^2\,(29\,d+77\,e\,x)+c\left(8\,d^3-6\,d^2\,e\,x+5\,d\,e^2\,x^2+35\,e^3\,x^3\right)\right)}{e^3}+\right.$$

$$\frac{1}{c\,e^5\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\,(d+e\,x)}8\left(-e^2\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\left(4\,c^2\,d^4+15\,a\,c\,d^2\,e^2-21\,a^2\,e^4\right)\left(a+c\,x^2\right)+\right.$$

$$\sqrt{c}\left(4\,i\,c^{5/2}\,d^5-4\sqrt{a}\,c^2\,d^4\,e+15\,i\,a\,c^{3/2}\,d^3\,e^2-15\,a^{3/2}\,c\,d^2\,e^3-21\,i\,a^2\sqrt{c}\,d\,e^4+21\,a^{5/2}\,e^5\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]+\sqrt{a}\sqrt{c}\,e\left(4\,c^2\,d^4+i\sqrt{a}\,c^{3/2}\,d^3\,e+15\,a\,c\,d^2\,e^2+33\,i\,a^{3/2}\sqrt{c}\,d\,e^3-21\,a^2\,e^4\right)$$

$$\left.\left.\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]\right)\right)$$

- **Problem 665: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(a+c\,x^2\right)^{3/2}}{\sqrt{d+e\,x}}\,dx$$

Optimal (type 4, 393 leaves, 7 steps) :

$$\frac{4\sqrt{d+e x}\left(4 c d^2+5 a e^2-3 c d e x\right)\sqrt{a+c x^2}}{35 e^3}+\frac{2\sqrt{d+e x}\left(a+c x^2\right)^{3/2}}{7 e}+$$

$$\frac{32\sqrt{-a}\sqrt{c}\, d\left(c d^2+2 a e^2\right)\sqrt{d+e x}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\,d-a e}\right]}{35 e^4\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{a+c x^2}}-$$

$$\left(8\sqrt{-a}\left(c d^2+a e^2\right)\left(4 c d^2+5 a e^2\right)\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\,d-a e}\right]\right)\Big/$$

$$\left(35\sqrt{c}\,e^4\sqrt{d+e x}\sqrt{a+c x^2}\right)$$

Result (type 4, 575 leaves):

$$\frac{1}{35\sqrt{a+c x^2}}\sqrt{d+e x}$$

$$\left(\frac{2\left(a+c x^2\right)\left(15 a e^2+c\left(8 d^2-6 d e x+5 e^2 x^2\right)\right)}{e^3}-\frac{1}{e^5\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\,(d+e x)}8\left(4 d e^2\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\left(2 a^2 e^2+c^2 d^2 x^2+a c\left(d^2+2 e^2 x^2\right)\right)+\right.\right.$$

$$4\sqrt{c}\,d\left(-i c^{3/2} d^3+\sqrt{a}\,c d^2 e-2 i a\sqrt{c}\,d e^2+2 a^{3/2} e^3\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}\,(d+e x)^{3/2}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]-\sqrt{a}\,e\left(4 c^{3/2} d^3+i\sqrt{a}\,c d^2 e+8 a\sqrt{c}\,d e^2+5 i a^{3/2} e^3\right)$$

$$\left.\left.\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}\,(d+e x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]\right)\right)$$

Problem 666: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+c x^2\right)^{3/2}}{(d+e x)^{3/2}} \, dx$$

Optimal (type 4, 369 leaves, 7 steps):

$$-\frac{4 c (4 d-3 e x) \sqrt{d+e x} \sqrt{a+c x^2}}{5 e^3}-\frac{2 \left(a+c x^2\right)^{3/2}}{e \sqrt{d+e x}}-$$

$$\frac{8 \sqrt{-a} \sqrt{c} \left(4 c d^2+3 a e^2\right) \sqrt{d+e x} \sqrt{1+\frac{c x^2}{a}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]}{5 e^4 \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{a+c x^2}}+$$

$$\frac{32 \sqrt{-a} \sqrt{c} d \left(c d^2+a e^2\right) \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{1+\frac{c x^2}{a}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]}{5 e^4 \sqrt{d+e x} \sqrt{a+c x^2}}$$

Result (type 4, 565 leaves):

$$\frac{2\sqrt{a+c x^2}\left(-5 a e^2+c\left(-8 d^2-2 d e x+e^2 x^2\right)\right)}{5 e^3\sqrt{d+e x}}+\frac{1}{5 e^5\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\sqrt{d+e x}\sqrt{a+c x^2}}$$

$$8\left(e^2\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\left(3 a^2 e^2+4 c^2 d^2 x^2+a c\left(4 d^2+3 e^2 x^2\right)\right)+\sqrt{c}\left(-4 i c^{3/2} d^3+4\sqrt{a}\,c d^2 e-3 i a\sqrt{c}\,d e^2+3 a^{3/2} e^3\right)\right.$$

$$\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}(d+e x)^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]-\sqrt{a}\sqrt{c}\,e$$

$$\left.\left(4 c d^2+i\sqrt{a}\sqrt{c}\,d e+3 a e^2\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}(d+e x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]\right)$$

- **Problem 667: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(a+c x^2\right)^{3/2}}{(d+e x)^{5/2}}\,dx$$

Optimal (type 4, 358 leaves, 7 steps) :

$$\frac{4 c (4 d+e x)\sqrt{a+c x^2}}{3 e^3\sqrt{d+e x}}-\frac{2\left(a+c x^2\right)^{3/2}}{3 e (d+e x)^{3/2}}+\frac{32\sqrt{-a}\,c^{3/2} d\sqrt{d+e x}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\,d-a e}\right]}{3 e^4\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{a+c x^2}}-$$

$$\frac{8\sqrt{-a}\sqrt{c}\left(4 c d^2+a e^2\right)\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\,d-a e}\right]}{3 e^4\sqrt{d+e x}\sqrt{a+c x^2}}$$

Result (type 4, 494 leaves) :

$$\frac{2\sqrt{a+c x^2}\left(-a e^2+c\left(8 d^2+10 d e x+e^2 x^2\right)\right)}{3 e^3 (d+e x)^{3/2}}-$$

$$\left(8 c \left(4 d e^2 \sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}} \left(a+c x^2\right)+4 \sqrt{c} d \left(-i \sqrt{c} d+\sqrt{a} e\right) \sqrt{\frac{e\left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a} e}{\sqrt{c}}-e x}{d+e x}} (d+e x)^{3/2}\right.\right.$$

$$\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right], \frac{\sqrt{c} d-i \sqrt{a} e}{\sqrt{c} d+i \sqrt{a} e}\right]-\sqrt{a} e \left(4 \sqrt{c} d+i \sqrt{a} e\right) \sqrt{\frac{e\left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a} e}{\sqrt{c}}-e x}{d+e x}}$$

$$\left.\left.(d+e x)^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right], \frac{\sqrt{c} d-i \sqrt{a} e}{\sqrt{c} d+i \sqrt{a} e}\right]\right)\right) \Big/ \left(3 e^5 \sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}} \sqrt{d+e x} \sqrt{a+c x^2}\right)$$

- **Problem 668: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(a+c x^2\right)^{3/2}}{(d+e x)^{7/2}} \, dx$$

Optimal (type 4, 410 leaves, 7 steps):

$$-\frac{4 c \left(2 d \left(2 c d^2+a e^2\right)+e \left(5 c d^2+3 a e^2\right) x\right) \sqrt{a+c x^2}}{5 e^3 \left(c d^2+a e^2\right) (d+e x)^{3/2}}-\frac{2 \left(a+c x^2\right)^{3/2}}{5 e (d+e x)^{5/2}}-$$

$$\frac{8 \sqrt{-a}\, c^{3/2} \left(4 c d^2+3 a e^2\right) \sqrt{d+e x} \sqrt{1+\frac{c x^2}{a}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right], -\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]}{5 e^4 \left(c d^2+a e^2\right) \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{a+c x^2}}+$$

$$\frac{32 \sqrt{-a}\, c^{3/2} d \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{1+\frac{c x^2}{a}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right], -\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]}{5 e^4 \sqrt{d+e x} \sqrt{a+c x^2}}$$

Result (type 4, 602 leaves):

$$\frac{1}{5\,e^5\left(c\,d^2+a\,e^2\right)(d+e\,x)^{5/2}\sqrt{a+c\,x^2}}$$

$$2\left(-e^2\left(a+c\,x^2\right)\left(\left(c\,d^2+a\,e^2\right)^2-4\,c\,d\left(c\,d^2+a\,e^2\right)(d+e\,x)+c\left(11\,c\,d^2+7\,a\,e^2\right)(d+e\,x)^2\right)+\frac{1}{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}\,4\,c\,(d+e\,x)^2\right.$$

$$\left(e^2\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\left(3\,a^2\,e^2+4\,c^2\,d^2\,x^2+a\,c\left(4\,d^2+3\,e^2\,x^2\right)\right)+\sqrt{c}\left(-4\,i\,c^{3/2}\,d^3+4\,\sqrt{a}\,c\,d^2\,e-3\,i\,a\,\sqrt{c}\,d\,e^2+3\,a^{3/2}\,e^3\right)\right.$$

$$\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]-\sqrt{a}\,\sqrt{c}\,e$$

$$\left.\left.\left(4\,c\,d^2+i\sqrt{a}\,\sqrt{c}\,d\,e+3\,a\,e^2\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]\right)\right)$$

- **Problem 669: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(a+c\,x^2\right)^{3/2}}{(d+e\,x)^{9/2}}\,dx$$

Optimal (type 4, 491 leaves, 8 steps) :

$$\frac{32\,c^2\,d\,\left(c\,d^2+2\,a\,e^2\right)\sqrt{a+c\,x^2}}{35\,e^3\,\left(c\,d^2+a\,e^2\right)^2\sqrt{d+e\,x}}-\frac{4\,c\,\left(2\,d\,\left(2\,c\,d^2+a\,e^2\right)+e\,\left(7\,c\,d^2+5\,a\,e^2\right)x\right)\sqrt{a+c\,x^2}}{35\,e^3\,\left(c\,d^2+a\,e^2\right)\,(d+e\,x)^{5/2}}-$$

$$\frac{2\,\left(a+c\,x^2\right)^{3/2}}{7\,e\,(d+e\,x)^{7/2}}+\frac{32\,\sqrt{-a}\,c^{5/2}\,d\,\left(c\,d^2+2\,a\,e^2\right)\sqrt{d+e\,x}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]}{35\,e^4\,\left(c\,d^2+a\,e^2\right)^2\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{a+c\,x^2}}-$$

$$\frac{8\,\sqrt{-a}\,c^{3/2}\,\left(4\,c\,d^2+5\,a\,e^2\right)\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]}{35\,e^4\,\left(c\,d^2+a\,e^2\right)\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 659 leaves) :

$$\frac{1}{35\,e^5\left(c\,d^2+a\,e^2\right)^2(d+e\,x)^{7/2}\sqrt{a+c\,x^2}}$$

$$2\left(-e^2\left(a+c\,x^2\right)\left(5\left(c\,d^2+a\,e^2\right)^3-16\,c\,d\left(c\,d^2+a\,e^2\right)^2(d+e\,x)+c\left(c\,d^2+a\,e^2\right)\left(19\,c\,d^2+15\,a\,e^2\right)(d+e\,x)^2-16\,c^2\,d\left(c\,d^2+2\,a\,e^2\right)(d+e\,x)^3\right)-\right.$$

$$\frac{1}{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}\,4\,c^2\,(d+e\,x)^3\left(4\,d\,e^2\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\left(2\,a^2\,e^2+c^2\,d^2\,x^2+a\,c\left(d^2+2\,e^2\,x^2\right)\right)+\right.$$

$$4\sqrt{c}\,d\left(-i\,c^{3/2}\,d^3+\sqrt{a}\,c\,d^2\,e-2\,i\,a\sqrt{c}\,d\,e^2+2\,a^{3/2}\,e^3\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]-\sqrt{a}\,e\left(4\,c^{3/2}\,d^3+i\sqrt{a}\,c\,d^2\,e+8\,a\sqrt{c}\,d\,e^2+5\,i\,a^{3/2}\,e^3\right)$$

$$\left.\left.\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]\right)\right)$$

- **Problem 670: Result unnecessarily involves imaginary or complex numbers.**

$$\int\sqrt{d+e\,x}\,\left(a+c\,x^2\right)^{5/2}dx$$

Optimal (type 4, 566 leaves, 9 steps) :

$$\frac{8\sqrt{d+e x}\left(d\left(32 c^2 d^4+113 a c d^2 e^2+177 a^2 e^4\right)-3 e\left(8 c^2 d^4+27 a c d^2 e^2-77 a^2 e^4\right) x\right)\sqrt{a+c x^2}}{9009 e^5}+$$

$$\frac{20\sqrt{d+e x}\left(4 d\left(2 c d^2+5 a e^2\right)-7 e\left(c d^2-11 a e^2\right) x\right)\left(a+c x^2\right)^{3/2}}{9009 e^3}-\frac{20 d\sqrt{d+e x}\left(a+c x^2\right)^{5/2}}{143 e}+\frac{2(d+e x)^{3/2}\left(a+c x^2\right)^{5/2}}{13 e}+$$

$$\left(16\sqrt{-a}\left(32 c^3 d^6+137 a c^2 d^4 e^2+258 a^2 c d^2 e^4-231 a^3 e^6\right)\sqrt{d+e x}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\,d-a e}\right]\right)\bigg/$$

$$\left(9009\sqrt{c}\,e^6\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{a+c x^2}\right)-$$

$$\left(16\sqrt{-a}\,d\left(c d^2+a e^2\right)\left(32 c^2 d^4+113 a c d^2 e^2+177 a^2 e^4\right)\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\,d-a e}\right]\right)\bigg/$$

$$\left(9009\sqrt{c}\,e^6\sqrt{d+e x}\sqrt{a+c x^2}\right)$$

Result (type 4, 967 leaves) :

$$\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}\,\left(\frac{2\,d\,\left(128\,c^2\,d^4+532\,a\,c\,d^2\,e^2+971\,a^2\,e^4\right)}{9009\,e^5}+\frac{2\,\left(-96\,c^2\,d^4-394\,a\,c\,d^2\,e^2+2387\,a^2\,e^4\right)\,x}{9009\,e^4}+\frac{4\,c\,d\,\left(40\,c\,d^2+163\,a\,e^2\right)\,x^2}{9009\,e^3}+\frac{4\,c\,\left(-5\,c\,d^2+154\,a\,e^2\right)\,x^3}{1287\,e^2}+\frac{2\,c^2\,d\,x^4}{143\,e}+\frac{2\,c^2\,x^5}{13}\right)+$$

$$\frac{1}{9009\,c\,e^7\,\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}\,\sqrt{a+\frac{c\,(d+e\,x)^2\,\left(-1+\frac{d}{d+e\,x}\right)^2}{e^2}}}\,8\,(d+e\,x)^{3/2}\left(-2\,\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}\,\left(-\frac{231\,a^4\,e^8}{(d+e\,x)^2}+a^2\,c^2\,d^2\,e^4\,\left(258+\frac{395\,d^2}{(d+e\,x)^2}-\frac{516\,d}{d+e\,x}\right)+\right.\right.$$

$$\left.a\,c^3\,d^4\,e^2\,\left(137+\frac{169\,d^2}{(d+e\,x)^2}-\frac{274\,d}{d+e\,x}\right)+32\,c^4\,d^6\,\left(-1+\frac{d}{d+e\,x}\right)^2+3\,a^3\,c\,e^6\,\left(-77+\frac{9\,d^2}{(d+e\,x)^2}+\frac{154\,d}{d+e\,x}\right)\right)+\frac{1}{\sqrt{d+e\,x}}$$

$$2\,\sqrt{c}\,\left(32\,i\,c^{7/2}\,d^7-32\,\sqrt{a}\,c^3\,d^6\,e+137\,i\,a\,c^{5/2}\,d^5\,e^2-137\,a^{3/2}\,c^2\,d^4\,e^3+258\,i\,a^2\,c^{3/2}\,d^3\,e^4-258\,a^{5/2}\,c\,d^2\,e^5-231\,i\,a^3\,\sqrt{c}\,d\,e^6+231\,a^{7/2}\,e^7\right)$$

$$\sqrt{1-\frac{d}{d+e\,x}-\frac{i\,\sqrt{a}\,e}{\sqrt{c}\,(d+e\,x)}}\,\sqrt{1-\frac{d}{d+e\,x}+\frac{i\,\sqrt{a}\,e}{\sqrt{c}\,(d+e\,x)}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\,\frac{\sqrt{c}\,d-i\,\sqrt{a}\,e}{\sqrt{c}\,d+i\,\sqrt{a}\,e}\right]+\frac{1}{\sqrt{d+e\,x}}$$

$$2\,\sqrt{a}\,\sqrt{c}\,e\,\left(32\,c^3\,d^6+8\,i\,\sqrt{a}\,c^{5/2}\,d^5\,e+137\,a\,c^2\,d^4\,e^2+32\,i\,a^{3/2}\,c^{3/2}\,d^3\,e^3+258\,a^2\,c\,d^2\,e^4+408\,i\,a^{5/2}\,\sqrt{c}\,d\,e^5-231\,a^3\,e^6\right)$$

$$\left.\sqrt{1-\frac{d}{d+e\,x}-\frac{i\,\sqrt{a}\,e}{\sqrt{c}\,(d+e\,x)}}\,\sqrt{1-\frac{d}{d+e\,x}+\frac{i\,\sqrt{a}\,e}{\sqrt{c}\,(d+e\,x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\,\frac{\sqrt{c}\,d-i\,\sqrt{a}\,e}{\sqrt{c}\,d+i\,\sqrt{a}\,e}\right]\right)$$

- **Problem 671: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(a+c\,x^2\right)^{5/2}}{\sqrt{d+e\,x}}\,dx$$

Optimal (type 4, 494 leaves, 8 steps) :

$$\frac{8\sqrt{d+e\,x}\,\left(32\,c^2\,d^4+69\,a\,c\,d^2\,e^2+45\,a^2\,e^4-24\,c\,d\,e\,\left(c\,d^2+2\,a\,e^2\right)\,x\right)\,\sqrt{a+c\,x^2}}{693\,e^5}+$$

$$\frac{20\sqrt{d+e\,x}\,\left(8\,c\,d^2+9\,a\,e^2-7\,c\,d\,e\,x\right)\,\left(a+c\,x^2\right)^{3/2}}{693\,e^3}+\frac{2\sqrt{d+e\,x}\,\left(a+c\,x^2\right)^{5/2}}{11\,e}+$$

$$\left(16\sqrt{-a}\,\sqrt{c}\,d\,\left(32\,c^2\,d^4+93\,a\,c\,d^2\,e^2+93\,a^2\,e^4\right)\,\sqrt{d+e\,x}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\,-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]\right)\Big/$$

$$\left(693\,e^6\,\sqrt{\frac{\sqrt{c}\,\left(d+e\,x\right)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{a+c\,x^2}\right)-$$

$$\left(16\sqrt{-a}\,\left(c\,d^2+a\,e^2\right)\,\left(32\,c^2\,d^4+69\,a\,c\,d^2\,e^2+45\,a^2\,e^4\right)\,\sqrt{\frac{\sqrt{c}\,\left(d+e\,x\right)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\,-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]\right)\Big/$$

$$\left(693\sqrt{c}\,e^6\,\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}\right)$$

Result (type 4, 634 leaves) :

$$\frac{1}{693\, e^7 \sqrt{a+c\,x^2}}\, 2\sqrt{d+e\,x}\, \Bigg(-\frac{8\,d\,e^2 \left(32\,c^2 d^4+93\,a\,c\,d^2 e^2+93\,a^2 e^4\right) \left(a+c\,x^2\right)}{d+e\,x}+$$

$$e^2 \left(a+c\,x^2\right) \left(333\,a^2 e^4+2\,a\,c\,e^2 \left(178\,d^2-131\,d\,e\,x+108\,e^2 x^2\right)+c^2 \left(128\,d^4-96\,d^3 e\,x+80\,d^2 e^2 x^2-70\,d\,e^3 x^3+63\,e^4 x^4\right)\right)-8\,i\,c\,d\,\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}$$

$$\left(32\,c^2 d^4+93\,a\,c\,d^2 e^2+93\,a^2 e^4\right) \sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\, \sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\, \sqrt{d+e\,x}\, \text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right], \frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]+$$

$$\frac{1}{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}} 8\sqrt{a}\,e \left(32\,c^{5/2} d^5+8\,i\sqrt{a}\,c^2 d^4 e+93\,a\,c^{3/2} d^3 e^2+21\,i\,a^{3/2} c\,d^2 e^3+93\,a^2 \sqrt{c}\,d\,e^4+45\,i\,a^{5/2} e^5\right)$$

$$\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\, \sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\, \sqrt{d+e\,x}\, \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right], \frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]\Bigg)$$

- **Problem 672: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(a+c\,x^2\right)^{5/2}}{(d+e\,x)^{3/2}}\, dx$$

Optimal (type 4, 457 leaves, 8 steps) :

$$-\frac{8\,c\,\sqrt{d+e\,x}\,\left(d\,\left(32\,c\,d^2+33\,a\,e^2\right)-3\,e\,\left(8\,c\,d^2+7\,a\,e^2\right)\,x\right)\,\sqrt{a+c\,x^2}}{63\,e^5}-\frac{20\,c\,(8\,d-7\,e\,x)\,\sqrt{d+e\,x}\,\left(a+c\,x^2\right)^{3/2}}{63\,e^3}-\frac{2\,\left(a+c\,x^2\right)^{5/2}}{e\,\sqrt{d+e\,x}}-$$

$$\left(16\,\sqrt{-a}\,\sqrt{c}\,\left(32\,c^2\,d^4+57\,a\,c\,d^2\,e^2+21\,a^2\,e^4\right)\,\sqrt{d+e\,x}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]\right)\Big/$$

$$\left(63\,e^6\,\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{a+c\,x^2}\right)+\frac{1}{63\,e^6\,\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}}$$

$$16\,\sqrt{-a}\,\sqrt{c}\,d\,\left(c\,d^2+a\,e^2\right)\,\left(32\,c\,d^2+33\,a\,e^2\right)\,\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]$$

Result (type 4, 684 leaves) :

$$\frac{1}{63\sqrt{a+c x^2}}\sqrt{d+e x}\left(-\frac{1}{e^5 (d+e x)}2 \left(a+c x^2\right) \left(63 a^2 e^4+2 a c e^2 \left(106 d^2+29 d e x-14 e^2 x^2\right)+c^2 \left(128 d^4+32 d^3 e x-16 d^2 e^2 x^2+10 d e^3 x^3-7 e^4 x^4\right)\right)+\right.$$

$$\frac{1}{e^7 \sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}} (d+e x)}16 \left(e^2 \sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}} \left(32 c^2 d^4+57 a c d^2 e^2+21 a^2 e^4\right) \left(a+c x^2\right)+\right.$$

$$\sqrt{c} \left(-32 i c^{5/2} d^5+32 \sqrt{a} c^2 d^4 e-57 i a c^{3/2} d^3 e^2+57 a^{3/2} c d^2 e^3-21 i a^2 \sqrt{c} d e^4+21 a^{5/2} e^5\right) \sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a} e}{\sqrt{c}}-e x}{d+e x}}$$

$$(d+e x)^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right], \frac{\sqrt{c} d-i \sqrt{a} e}{\sqrt{c} d+i \sqrt{a} e}\right]-\sqrt{a} \sqrt{c} e \left(32 c^2 d^4+8 i \sqrt{a} c^{3/2} d^3 e+57 a c d^2 e^2+\right.$$

$$\left.\left.\left.12 i a^{3/2} \sqrt{c} d e^3+21 a^2 e^4\right) \sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a} e}{\sqrt{c}}-e x}{d+e x}} (d+e x)^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right], \frac{\sqrt{c} d-i \sqrt{a} e}{\sqrt{c} d+i \sqrt{a} e}\right]\right)\right)$$

- **Problem 673: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(a+c x^2\right)^{5/2}}{(d+e x)^{5/2}} \, dx$$

Optimal (type 4, 430 leaves, 8 steps) :

$$\frac{8\,c\,\sqrt{d+e\,x}\,\left(32\,c\,d^2+5\,a\,e^2-24\,c\,d\,e\,x\right)\,\sqrt{a+c\,x^2}}{21\,e^5}+\frac{20\,c\,(8\,d+e\,x)\,\left(a+c\,x^2\right)^{3/2}}{21\,e^3\,\sqrt{d+e\,x}}-\frac{2\,\left(a+c\,x^2\right)^{5/2}}{3\,e\,(d+e\,x)^{3/2}}+$$

$$\frac{16\,\sqrt{-a}\,c^{3/2}\,d\,\left(32\,c\,d^2+29\,a\,e^2\right)\,\sqrt{d+e\,x}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]}{21\,e^6\,\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{a+c\,x^2}}-\frac{1}{21\,e^6\,\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}}$$

$$16\,\sqrt{-a}\,\sqrt{c}\,\left(c\,d^2+a\,e^2\right)\,\left(32\,c\,d^2+5\,a\,e^2\right)\,\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]$$

Result (type 4, 637 leaves) :

$$\frac{1}{21\sqrt{a+c x^2}}\sqrt{d+e x}\left(\frac{1}{e^5 (d+e x)^2}2\left(a+c x^2\right)\left(-7 a^2 e^4+2 a c e^2\left(50 d^2+65 d e x+8 e^2 x^2\right)+c^2\left(128 d^4+160 d^3 e x+16 d^2 e^2 x^2-6 d e^3 x^3+3 e^4 x^4\right)\right)-\right.$$

$$\frac{1}{e^7\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\,(d+e x)}16 c\left(d e^2\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\left(29 a^2 e^2+32 c^2 d^2 x^2+a c\left(32 d^2+29 e^2 x^2\right)\right)+\right.$$

$$\sqrt{c}\,d\left(-32 i c^{3/2} d^3+32\sqrt{a}\,c d^2 e-29 i a\sqrt{c}\,d e^2+29 a^{3/2} e^3\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}\,(d+e x)^{3/2}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]-\sqrt{a}\,e\left(32 c^{3/2} d^3+8 i\sqrt{a}\,c d^2 e+29 a\sqrt{c}\,d e^2+5 i a^{3/2} e^3\right)$$

$$\left.\left.\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}\,(d+e x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]\right)\right)$$

- **Problem 674: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(a+c x^2\right)^{5/2}}{(d+e x)^{7/2}}\,dx$$

Optimal (type 4, 420 leaves, 8 steps) :

$$-\frac{8\,c\,\left(32\,c\,d^2+9\,a\,e^2+8\,c\,d\,e\,x\right)\sqrt{a+c\,x^2}}{15\,e^5\,\sqrt{d+e\,x}}+\frac{4\,c\,(8\,d+3\,e\,x)\,\left(a+c\,x^2\right)^{3/2}}{15\,e^3\,(d+e\,x)^{3/2}}-\frac{2\,\left(a+c\,x^2\right)^{5/2}}{5\,e\,(d+e\,x)^{5/2}}-$$

$$\frac{16\,\sqrt{-a}\,c^{3/2}\,\left(32\,c\,d^2+9\,a\,e^2\right)\sqrt{d+e\,x}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]}{15\,e^6\,\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{a+c\,x^2}}+$$

$$\frac{16\,\sqrt{-a}\,c^{3/2}\,d\,\left(32\,c\,d^2+17\,a\,e^2\right)\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]}{15\,e^6\,\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 613 leaves) :

$$\frac{1}{15\,e^7\,(d+e\,x)^{5/2}\,\sqrt{a+c\,x^2}}$$

$$2\left(-e^2\,\left(a+c\,x^2\right)\,\left(3\,a^2\,e^4+2\,a\,c\,e^2\,\left(10\,d^2+25\,d\,e\,x+18\,e^2\,x^2\right)+c^2\,\left(128\,d^4+288\,d^3\,e\,x+176\,d^2\,e^2\,x^2+10\,d\,e^3\,x^3-3\,e^4\,x^4\right)\right)+\frac{1}{\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}}\right.$$

$$8\,c\,(d+e\,x)^2\left(e^2\,\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}\,\left(9\,a^2\,e^2+32\,c^2\,d^2\,x^2+a\,c\,\left(32\,d^2+9\,e^2\,x^2\right)\right)+\sqrt{c}\,\left(-32\,i\,c^{3/2}\,d^3+32\,\sqrt{a}\,c\,d^2\,e-9\,i\,a\,\sqrt{c}\,d\,e^2+9\,a^{3/2}\,e^3\right)\right.$$

$$\sqrt{\frac{e\left(\frac{i\,\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\,\sqrt{-\frac{\frac{i\,\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\,\sqrt{a}\,e}{\sqrt{c}\,d+i\,\sqrt{a}\,e}\right]-\sqrt{a}\,\sqrt{c}\,e$$

$$\left.\left.\left(32\,c\,d^2+8\,i\,\sqrt{a}\,\sqrt{c}\,d\,e+9\,a\,e^2\right)\sqrt{\frac{e\left(\frac{i\,\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\,\sqrt{-\frac{\frac{i\,\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\,\sqrt{a}\,e}{\sqrt{c}\,d+i\,\sqrt{a}\,e}\right]\right)\right)$$

Problem 675: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+c x^2\right)^{5/2}}{(d+e x)^{9/2}} \, dx$$

Optimal (type 4, 498 leaves, 8 steps):

$$\frac{8 c^2 \left(d \left(32 c d^2+29 a e^2\right)+e \left(8 c d^2+5 a e^2\right) x\right) \sqrt{a+c x^2}}{21 e^5 \left(c d^2+a e^2\right) \sqrt{d+e x}}-\frac{4 c \left(2 d \left(4 c d^2+a e^2\right)+e \left(11 c d^2+5 a e^2\right) x\right) \left(a+c x^2\right)^{3/2}}{21 e^3 \left(c d^2+a e^2\right) (d+e x)^{5/2}}-$$

$$\frac{2 \left(a+c x^2\right)^{5/2}}{7 e (d+e x)^{7/2}}+\frac{16 \sqrt{-a}\, c^{5/2} d \left(32 c d^2+29 a e^2\right) \sqrt{d+e x} \sqrt{1+\frac{c x^2}{a}} \,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]}{21 e^6 \left(c d^2+a e^2\right) \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{a+c x^2}}-$$

$$\frac{16 \sqrt{-a}\, c^{3/2} \left(32 c d^2+5 a e^2\right) \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{1+\frac{c x^2}{a}} \,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]}{21 e^6 \sqrt{d+e x} \sqrt{a+c x^2}}$$

Result (type 4, 677 leaves):

$$\frac{1}{21\sqrt{a+c x^2}}\sqrt{d+e x}\left(\frac{2\left(a+c x^2\right)\left(7 c^2-\frac{3\left(c d^2+a e^2\right)^2}{(d+e x)^4}+\frac{18 c d\left(c d^2+a e^2\right)}{(d+e x)^3}-\frac{4 c\left(13 c d^2+4 a e^2\right)}{(d+e x)^2}+\frac{2 c^2 d\left(79 c d^2+67 a e^2\right)}{\left(c d^2+a e^2\right)(d+e x)}\right)}{e^5}-\right.$$

$$\frac{1}{e^7\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\left(c d^2+a e^2\right)(d+e x)}16 c^2\left(d e^2\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\left(29 a^2 e^2+32 c^2 d^2 x^2+a c\left(32 d^2+29 e^2 x^2\right)\right)+\right.$$

$$\sqrt{c}\,d\left(-32 i c^{3/2} d^3+32\sqrt{a}\,c d^2 e-29 i a\sqrt{c}\,d e^2+29 a^{3/2} e^3\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}(d+e x)^{3/2}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]-\sqrt{a}\,e\left(32 c^{3/2} d^3+8 i\sqrt{a}\,c d^2 e+29 a\sqrt{c}\,d e^2+5 i a^{3/2} e^3\right)$$

$$\left.\left.\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}(d+e x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]\right)\right)$$

- **Problem 676: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(a+c x^2\right)^{5/2}}{(d+e x)^{11/2}}\,dx$$

Optimal (type 4, 553 leaves, 8 steps) :

$$-\frac{8 c^2 \left(d \left(32 c^2 d^4+49 a c d^2 e^2+9 a^2 e^4\right)+e \left(40 c^2 d^4+69 a c d^2 e^2+21 a^2 e^4\right) x\right) \sqrt{a+c x^2}}{63 e^5 \left(c d^2+a e^2\right)^2 (d+e x)^{3/2}}-$$

$$\frac{4 c \left(2 d \left(4 c d^2+a e^2\right)+e \left(13 c d^2+7 a e^2\right) x\right) \left(a+c x^2\right)^{3/2}}{63 e^3 \left(c d^2+a e^2\right) (d+e x)^{7/2}}-\frac{2 \left(a+c x^2\right)^{5/2}}{9 e (d+e x)^{9/2}}-$$

$$\left(16 \sqrt{-a}\, c^{5/2} \left(32 c^2 d^4+57 a c d^2 e^2+21 a^2 e^4\right) \sqrt{d+e x} \sqrt{1+\frac{c x^2}{a}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right], -\frac{2 a e}{\sqrt{-a} \sqrt{c}\, d-a e}\right]\right) \Big/$$

$$\left(63 e^6 \left(c d^2+a e^2\right)^2 \sqrt{\frac{\sqrt{c}\, (d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}} \sqrt{a+c x^2}\right)+$$

$$\frac{16 \sqrt{-a}\, c^{5/2} d \left(32 c d^2+33 a e^2\right) \sqrt{\frac{\sqrt{c}\, (d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}} \sqrt{1+\frac{c x^2}{a}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right], -\frac{2 a e}{\sqrt{-a} \sqrt{c}\, d-a e}\right]}{63 e^6 \left(c d^2+a e^2\right) \sqrt{d+e x} \sqrt{a+c x^2}}$$

Result (type 4, 913 leaves) :

$$\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}\left(-\frac{2\left(c\,d^2+a\,e^2\right)^2}{9\,e^5\,(d+e\,x)^5}+\frac{76\,c\,d\left(c\,d^2+a\,e^2\right)}{63\,e^5\,(d+e\,x)^4}-\frac{8\,c\left(22\,c\,d^2+7\,a\,e^2\right)}{63\,e^5\,(d+e\,x)^3}+\frac{4\,c^2\,d\left(61\,c\,d^2+57\,a\,e^2\right)}{63\,e^5\left(c\,d^2+a\,e^2\right)(d+e\,x)^2}-\frac{2\,c^2\left(193\,c^2\,d^4+330\,a\,c\,d^2\,e^2+105\,a^2\,e^4\right)}{63\,e^5\left(c\,d^2+a\,e^2\right)^2(d+e\,x)}\right)+$$

$$\frac{1}{63\,e^7\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\left(c\,d^2+a\,e^2\right)^2\sqrt{a+\frac{c\,(d+e\,x)^2\left(-1+\frac{d}{d+e\,x}\right)^2}{e^2}}}$$

$$16\,c^2\,(d+e\,x)^{3/2}\left(\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\left(\frac{21\,a^3\,e^6}{(d+e\,x)^2}+a\,c^2\,d^2\,e^2\left(57+\frac{89\,d^2}{(d+e\,x)^2}-\frac{114\,d}{d+e\,x}\right)+3\,a^2\,c\,e^4\left(7+\frac{26\,d^2}{(d+e\,x)^2}-\frac{14\,d}{d+e\,x}\right)+32\,c^3\,d^4\left(-1+\frac{d}{d+e\,x}\right)^2\right)+\right.$$

$$\frac{1}{\sqrt{d+e\,x}}\sqrt{c}\left(-32\,i\,c^{5/2}\,d^5+32\sqrt{a}\,c^2\,d^4\,e-57\,i\,a\,c^{3/2}\,d^3\,e^2+57\,a^{3/2}\,c\,d^2\,e^3-21\,i\,a^2\sqrt{c}\,d\,e^4+21\,a^{5/2}\,e^5\right)$$

$$\sqrt{1-\frac{d}{d+e\,x}-\frac{i\sqrt{a}\,e}{\sqrt{c}\,(d+e\,x)}}\sqrt{1-\frac{d}{d+e\,x}+\frac{i\sqrt{a}\,e}{\sqrt{c}\,(d+e\,x)}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]-$$

$$\frac{1}{\sqrt{d+e\,x}}\sqrt{a}\,\sqrt{c}\,e\left(32\,c^2\,d^4+8\,i\sqrt{a}\,c^{3/2}\,d^3\,e+57\,a\,c\,d^2\,e^2+12\,i\,a^{3/2}\sqrt{c}\,d\,e^3+21\,a^2\,e^4\right)\sqrt{1-\frac{d}{d+e\,x}-\frac{i\sqrt{a}\,e}{\sqrt{c}\,(d+e\,x)}}$$

$$\left.\sqrt{1-\frac{d}{d+e\,x}+\frac{i\sqrt{a}\,e}{\sqrt{c}\,(d+e\,x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]\right)$$

- **Problem 677: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(d+e\,x)^{7/2}}{\sqrt{a+c\,x^2}}\,dx$$

Optimal (type 4, 413 leaves, 8 steps) :

$$\frac{2 e \left(71 c d^2-25 a e^2\right) \sqrt{d+e x} \sqrt{a+c x^2}}{105 c^2}+\frac{24 d e (d+e x)^{3/2} \sqrt{a+c x^2}}{35 c}+\frac{2 e (d+e x)^{5/2} \sqrt{a+c x^2}}{7 c}-$$

$$\frac{32 \sqrt{-a}\, d \left(11 c d^2-13 a e^2\right) \sqrt{d+e x} \sqrt{1+\frac{c x^2}{a}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c}\, d-a e}\right]}{105 c^{3/2} \sqrt{\frac{\sqrt{c}\, (d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}} \sqrt{a+c x^2}}+$$

$$\left(2 \sqrt{-a} \left(71 c d^2-25 a e^2\right) \left(c d^2+a e^2\right) \sqrt{\frac{\sqrt{c}\, (d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}} \sqrt{1+\frac{c x^2}{a}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c}\, d-a e}\right]\right)\Big/$$

$$\left(105 c^{5/2} \sqrt{d+e x} \sqrt{a+c x^2}\right)$$

Result (type 4, 548 leaves) :

$$\frac{1}{105 c^2 \sqrt{a+c x^2}} 2 \sqrt{d+e x} \left(\frac{16 d e \left(-13 a^2 e^2+11 c^2 d^2 x^2+a c \left(11 d^2-13 e^2 x^2\right)\right)}{d+e x}+\left(a+c x^2\right) \left(-25 a e^3+c e \left(122 d^2+66 d e x+15 e^2 x^2\right)\right)+\frac{1}{e}\right.$$

$$16 i c d \sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}} \left(11 c d^2-13 a e^2\right) \sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a}\, e}{\sqrt{c}}-e x}{d+e x}} \sqrt{d+e x}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\, d-i \sqrt{a}\, e}{\sqrt{c}\, d+i \sqrt{a}\, e}\right]+$$

$$\frac{1}{e \sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}} \left(105 i c^2 d^4-176 \sqrt{a}\, c^{3/2} d^3 e-254 i a c d^2 e^2+208 a^{3/2} \sqrt{c}\, d e^3+25 i a^2 e^4\right)$$

$$\left.\sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a}\, e}{\sqrt{c}}-e x}{d+e x}} \sqrt{d+e x}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\, d-i \sqrt{a}\, e}{\sqrt{c}\, d+i \sqrt{a}\, e}\right]\right)$$

- **Problem 678: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e\,x)^{5/2}}{\sqrt{a+c\,x^2}}\,dx$$

Optimal (type 4, 359 leaves, 7 steps) :

$$\frac{16\,d\,e\,\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}}{15\,c}+\frac{2\,e\,(d+e\,x)^{3/2}\,\sqrt{a+c\,x^2}}{5\,c}-\frac{2\,\sqrt{-a}\,\left(23\,c\,d^2-9\,a\,e^2\right)\sqrt{d+e\,x}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]}{15\,c^{3/2}\,\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{a+c\,x^2}}+$$

$$\frac{16\,\sqrt{-a}\,d\,\left(c\,d^2+a\,e^2\right)\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]}{15\,c^{3/2}\,\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 557 leaves) :

$$\frac{1}{15\,\sqrt{a+c\,x^2}}$$

$$\sqrt{d+e\,x}\,\left(\frac{2\,e\,(11\,d+3\,e\,x)\,\left(a+c\,x^2\right)}{c}+\frac{1}{c^2\,e\,\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}\,(d+e\,x)}\,2\,\left(e^2\,\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}\,\left(-9\,a^2\,e^2+23\,c^2\,d^2\,x^2+a\,c\,\left(23\,d^2-9\,e^2\,x^2\right)\right)+\sqrt{c}\right.\right.$$

$$\left(-23\,i\,c^{3/2}\,d^3+23\,\sqrt{a}\,c\,d^2\,e+9\,i\,a\,\sqrt{c}\,d\,e^2-9\,a^{3/2}\,e^3\right)\sqrt{\frac{e\,\left(\frac{i\,\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\,\sqrt{-\frac{\frac{i\,\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\,\sqrt{a}\,e}{\sqrt{c}\,d+i\,\sqrt{a}\,e}\right]+\sqrt{c}\,\left(15\,i\,c^{3/2}\,d^3-23\,\sqrt{a}\,c\,d^2\,e-17\,i\,a\,\sqrt{c}\,d\,e^2+9\,a^{3/2}\,e^3\right)$$

$$\left.\left.\sqrt{\frac{e\,\left(\frac{i\,\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\,\sqrt{-\frac{\frac{i\,\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\,\sqrt{a}\,e}{\sqrt{c}\,d+i\,\sqrt{a}\,e}\right]\right)\right)$$

- **Problem 679: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e x)^{3/2}}{\sqrt{a+c x^2}} \, dx$$

Optimal (type 4, 317 leaves, 6 steps):

$$\frac{2 e \sqrt{d+e x} \sqrt{a+c x^2}}{3 c} - \frac{8 \sqrt{-a} \, d \sqrt{d+e x} \sqrt{1+\frac{c x^2}{a}} \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right], -\frac{2 a e}{\sqrt{-a} \sqrt{c} d - a e}\right]}{3 \sqrt{c} \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{a+c x^2}} + \frac{2 \sqrt{-a} \left(c d^2+a e^2\right) \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{1+\frac{c x^2}{a}} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right], -\frac{2 a e}{\sqrt{-a} \sqrt{c} d - a e}\right]}{3 c^{3/2} \sqrt{d+e x} \sqrt{a+c x^2}}$$

Result (type 4, 445 leaves):

$$\frac{1}{3 c e \sqrt{a+c x^2}} 2 \sqrt{d+e x} \left(e^2 \left(a+c x^2\right) + \frac{4 d e^2 \left(a+c x^2\right)}{d+e x} + 4 i c d \sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}} \sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a} e}{\sqrt{c}} - e x}{d+e x}} \sqrt{d+e x} \, \text{EllipticE}\left[i \, \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right], \frac{\sqrt{c} d - i \sqrt{a} e}{\sqrt{c} d + i \sqrt{a} e}\right] + 1 \Big/ \left(\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}\right) i \left(3 c d^2 + 4 i \sqrt{a} \sqrt{c} d e - a e^2\right) \sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a} e}{\sqrt{c}} - e x}{d+e x}} \sqrt{d+e x} \, \text{EllipticF}\left[i \, \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right], \frac{\sqrt{c} d - i \sqrt{a} e}{\sqrt{c} d + i \sqrt{a} e}\right]\right)$$

- **Problem 680: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{d+e x}}{\sqrt{a+c x^2}} \, dx$$

Optimal (type 4, 136 leaves, 2 steps):

$$-\frac{2\sqrt{-a}\sqrt{d+e x}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\,d-a e}\right]}{\sqrt{c}\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{a+c x^2}}$$

Result (type 4, 294 leaves):

$$\left(2\,i\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\sqrt{\frac{e\left(\sqrt{a}+i\sqrt{c}\,x\right)}{-i\sqrt{c}\,d+\sqrt{a}\,e}}\sqrt{d+e x}\right.$$
$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d-i\sqrt{a}\,e}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d-i\sqrt{a}\,e}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]\right)\right)\Big/$$
$$\left(\sqrt{c}\,e\sqrt{\frac{\sqrt{c}\,(d+e x)}{e\left(i\sqrt{a}+\sqrt{c}\,x\right)}}\sqrt{a+c x^2}\right)$$

Problem 681: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{d+e x}\sqrt{a+c x^2}}\,dx$$

Optimal (type 4, 136 leaves, 2 steps):

$$-\frac{2\sqrt{-a}\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\,d-a e}\right]}{\sqrt{c}\sqrt{d+e x}\sqrt{a+c x^2}}$$

Result (type 4, 186 leaves):

$$\frac{2\,i\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}\,(d+e x)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]}{e\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\sqrt{a+c x^2}}$$

- **Problem 682: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(d+e x)^{3/2}\sqrt{a+c x^2}}\,dx$$

Optimal (type 4, 186 leaves, 4 steps):

$$-\frac{2 e \sqrt{a+c x^2}}{\left(c d^2+a e^2\right) \sqrt{d+e x}}-\frac{2 \sqrt{-a}\, \sqrt{c}\, \sqrt{d+e x}\, \sqrt{1+\frac{c x^2}{a}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\, \sqrt{c}\, d-a e}\right]}{\left(c d^2+a e^2\right) \sqrt{\frac{\sqrt{c}\, (d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}}\, \sqrt{a+c x^2}}$$

Result (type 4, 331 leaves):

$$-\frac{2 e \sqrt{a+c x^2}}{\left(c d^2+a e^2\right) \sqrt{d+e x}}-\left(2 \sqrt{c}\, \sqrt{\frac{e \left(\sqrt{a}+i \sqrt{c}\, x\right)}{-i \sqrt{c}\, d+\sqrt{a}\, e}}\, \sqrt{d+e x}\right.$$

$$\left.\left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}\, (d+e x)}{\sqrt{c}\, d-i \sqrt{a}\, e}}\right],\frac{\sqrt{c}\, d-i \sqrt{a}\, e}{\sqrt{c}\, d+i \sqrt{a}\, e}\right]-\text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}\, (d+e x)}{\sqrt{c}\, d-i \sqrt{a}\, e}}\right],\frac{\sqrt{c}\, d-i \sqrt{a}\, e}{\sqrt{c}\, d+i \sqrt{a}\, e}\right]\right)\right)\Bigg/$$

$$\left(e \left(i \sqrt{c}\, d+\sqrt{a}\, e\right) \sqrt{\frac{\sqrt{c}\, (d+e x)}{e \left(i \sqrt{a}+\sqrt{c}\, x\right)}}\, \sqrt{a+c x^2}\right)$$

- **Problem 683: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(d+e x)^{5/2}\sqrt{a+c x^2}}\,dx$$

Optimal (type 4, 382 leaves, 7 steps):

$$-\frac{2 e \sqrt{a+c x^2}}{3 \left(c d^2+a e^2\right) (d+e x)^{3/2}}-\frac{8 c d e \sqrt{a+c x^2}}{3 \left(c d^2+a e^2\right)^2 \sqrt{d+e x}}-\frac{8 \sqrt{-a}\, c^{3/2} d \sqrt{d+e x} \sqrt{1+\frac{c x^2}{a}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c}\, d-a e}\right]}{3 \left(c d^2+a e^2\right)^2 \sqrt{\frac{\sqrt{c}\, (d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}} \sqrt{a+c x^2}}+$$

$$\frac{2 \sqrt{-a} \sqrt{c} \sqrt{\frac{\sqrt{c}\, (d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}} \sqrt{1+\frac{c x^2}{a}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c}\, d-a e}\right]}{3 \left(c d^2+a e^2\right) \sqrt{d+e x} \sqrt{a+c x^2}}$$

Result (type 4, 494 leaves) :

$$\left(2 \left(-e^2 \left(a+c x^2\right) \left(a e^2+c d (5 d+4 e x)\right)+\right.\right.$$

$$1\Big/\left(\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}\ c\, (d+e x)\right) \left(4 d e^2 \sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}} \left(a+c x^2\right)+4 \sqrt{c}\, d \left(-i \sqrt{c}\, d+\sqrt{a}\, e\right) \sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a}\, e}{\sqrt{c}}-e x}{d+e x}}\ (d+e x)^{3/2}\right.$$

$$\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\, d-i \sqrt{a}\, e}{\sqrt{c}\, d+i \sqrt{a}\, e}\right]+i \left(3 c d^2+4 i \sqrt{a} \sqrt{c}\, d e-a e^2\right) \sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a}\, e}{\sqrt{c}}-e x}{d+e x}}$$

$$\left.\left.\left.(d+e x)^{3/2}\ \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\, d-i \sqrt{a}\, e}{\sqrt{c}\, d+i \sqrt{a}\, e}\right]\right)\right)\right)\Big/\left(3 e \left(c d^2+a e^2\right)^2 (d+e x)^{3/2} \sqrt{a+c x^2}\right)$$

- **Problem 684: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(d+e x)^{7/2} \sqrt{a+c x^2}}\, dx$$

Optimal (type 4, 447 leaves, 8 steps) :

$$-\frac{2 e \sqrt{a+c x^2}}{5 \left(c d^2+a e^2\right) (d+e x)^{5/2}}-\frac{16 c d e \sqrt{a+c x^2}}{15 \left(c d^2+a e^2\right)^2 (d+e x)^{3/2}}-\frac{2 c e \left(23 c d^2-9 a e^2\right) \sqrt{a+c x^2}}{15 \left(c d^2+a e^2\right)^3 \sqrt{d+e x}}-$$

$$\frac{2 \sqrt{-a}\, c^{3/2} \left(23 c d^2-9 a e^2\right) \sqrt{d+e x} \sqrt{1+\frac{c x^2}{a}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right], -\frac{2 a e}{\sqrt{-a} \sqrt{c}\, d-a e}\right]}{15 \left(c d^2+a e^2\right)^3 \sqrt{\frac{\sqrt{c}\, (d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}} \sqrt{a+c x^2}}+$$

$$\frac{16 \sqrt{-a}\, c^{3/2} d \sqrt{\frac{\sqrt{c}\, (d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}} \sqrt{1+\frac{c x^2}{a}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right], -\frac{2 a e}{\sqrt{-a} \sqrt{c}\, d-a e}\right]}{15 \left(c d^2+a e^2\right)^2 \sqrt{d+e x} \sqrt{a+c x^2}}$$

Result (type 4, 618 leaves) :

$$\frac{1}{15 e \left(c d^2+a e^2\right)^3 (d+e x)^{5/2} \sqrt{a+c x^2}}$$

$$2 \left(-e^2 \left(a+c x^2\right) \left(3 \left(c d^2+a e^2\right)^2+8 c d \left(c d^2+a e^2\right) (d+e x)+c \left(23 c d^2-9 a e^2\right) (d+e x)^2\right)-\frac{1}{\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}} c (d+e x)^2\right.$$

$$\left(-e^2 \sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}} \left(-9 a^2 e^2+23 c^2 d^2 x^2+a c \left(23 d^2-9 e^2 x^2\right)\right)+\sqrt{c} \left(23 i c^{3/2} d^3-23 \sqrt{a}\, c d^2 e-9 i a \sqrt{c}\, d e^2+9 a^{3/2} e^3\right) \sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\right.$$

$$\sqrt{-\frac{\frac{i \sqrt{a}\, e}{\sqrt{c}}-e x}{d+e x}} (d+e x)^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right], \frac{\sqrt{c}\, d-i \sqrt{a}\, e}{\sqrt{c}\, d+i \sqrt{a}\, e}\right]+\sqrt{c} \left(-15 i c^{3/2} d^3+23 \sqrt{a}\, c d^2 e+\right.$$

$$\left.\left.17 i a \sqrt{c}\, d e^2-9 a^{3/2} e^3\right) \sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a}\, e}{\sqrt{c}}-e x}{d+e x}} (d+e x)^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right], \frac{\sqrt{c}\, d-i \sqrt{a}\, e}{\sqrt{c}\, d+i \sqrt{a}\, e}\right]\right)\right)$$

■ Problem 685: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d+e x)^{7/2}}{\left(a+c x^2\right)^{3/2}} \, dx$$

Optimal (type 4, 426 leaves, 8 steps):

$$-\frac{(a e-c d x)\,(d+e x)^{5/2}}{a c \sqrt{a+c x^2}}-\frac{e\left(3 c d^2-5 a e^2\right)\sqrt{d+e x}\,\sqrt{a+c x^2}}{3 a c^2}-\frac{d e\,(d+e x)^{3/2}\sqrt{a+c x^2}}{a c}-$$

$$\frac{d\left(3 c d^2-29 a e^2\right)\sqrt{d+e x}\,\sqrt{1+\frac{c x^2}{a}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\,\sqrt{c}\,d-a e}\right]}{3\sqrt{-a}\;c^{3/2}\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\;\sqrt{a+c x^2}}+$$

$$\frac{\left(3 c d^2-5 a e^2\right)\left(c d^2+a e^2\right)\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\;\sqrt{1+\frac{c x^2}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\,\sqrt{c}\,d-a e}\right]}{3\sqrt{-a}\;c^{5/2}\sqrt{d+e x}\;\sqrt{a+c x^2}}$$

Result (type 4, 586 leaves):

$$\frac{1}{6\sqrt{a+c x^2}}\sqrt{d+e x}\left(\frac{10 a e^3}{c^2}+\frac{6 d^3 x}{a}+\frac{2 e\left(-9 d^2-9 d e x+2 e^2 x^2\right)}{c}+\right.$$

$$\frac{1}{a c^2 e\sqrt{-d-\frac{i\sqrt{a} e}{\sqrt{c}}}\,(d+e x)}2\left(-d e^2\sqrt{-d-\frac{i\sqrt{a} e}{\sqrt{c}}}\left(-29 a^2 e^2+3 c^2 d^2 x^2+a c\left(3 d^2-29 e^2 x^2\right)\right)+\right.$$

$$\sqrt{c}\, d\left(3 i c^{3/2} d^3-3\sqrt{a}\, c d^2 e-29 i a\sqrt{c}\, d e^2+29 a^{3/2} e^3\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a} e}{\sqrt{c}}-e x}{d+e x}}\,(d+e x)^{3/2}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\, d-i\sqrt{a}\, e}{\sqrt{c}\, d+i\sqrt{a}\, e}\right]+\sqrt{a}\, e\left(3 c^{3/2} d^3+27 i\sqrt{a}\, c d^2 e-29 a\sqrt{c}\, d e^2-5 i a^{3/2} e^3\right)$$

$$\left.\left.\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a} e}{\sqrt{c}}-e x}{d+e x}}\,(d+e x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\, d-i\sqrt{a}\, e}{\sqrt{c}\, d+i\sqrt{a}\, e}\right]\right)\right)$$

- **Problem 686: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(d+e x)^{5/2}}{\left(a+c x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 363 leaves, 7 steps) :

$$-\frac{(a e-c d x)(d+e x)^{3/2}}{a c \sqrt{a+c x^2}}-\frac{d e \sqrt{d+e x} \sqrt{a+c x^2}}{a c}-\frac{\left(c d^2-3 a e^2\right) \sqrt{d+e x} \sqrt{1+\frac{c x^2}{a}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]}{\sqrt{-a} c^{3/2} \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{a+c x^2}}+$$

$$\frac{d\left(c d^2+a e^2\right) \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{1+\frac{c x^2}{a}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]}{\sqrt{-a} c^{3/2} \sqrt{d+e x} \sqrt{a+c x^2}}$$

Result (type 4, 495 leaves) :

$$\frac{1}{a c^2 \sqrt{a+c x^2}} \sqrt{d+e x}$$

$$\left(c\left(c d^2 x-a e(2 d+e x)\right)-\frac{e\left(-3 a^2 e^2+c^2 d^2 x^2+a c\left(d^2-3 e^2 x^2\right)\right)}{d+e x}-1/ e i c \sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}\left(c d^2-3 a e^2\right) \sqrt{\frac{e\left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a} e}{\sqrt{c}}-e x}{d+e x}}\right.$$

$$\sqrt{d+e x} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right], \frac{\sqrt{c} d-i \sqrt{a} e}{\sqrt{c} d+i \sqrt{a} e}\right]+1 \Big/ \left(\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}\right) \sqrt{a} \sqrt{c}\left(c d^2+4 i \sqrt{a} \sqrt{c} d e-3 a e^2\right)$$

$$\left.\sqrt{\frac{e\left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a} e}{\sqrt{c}}-e x}{d+e x}} \sqrt{d+e x} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right], \frac{\sqrt{c} d-i \sqrt{a} e}{\sqrt{c} d+i \sqrt{a} e}\right]\right)$$

- **Problem 687: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e x)^{3/2}}{\left(a+c x^2\right)^{3/2}} \, dx$$

Optimal (type 4, 321 leaves, 6 steps) :

$$-\frac{(a e-c d x)\sqrt{d+e x}}{a c\sqrt{a+c x^2}}-\frac{d\sqrt{d+e x}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\,d-a e}\right]}{\sqrt{-a}\sqrt{c}\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{a+c x^2}}+$$

$$\frac{\left(c d^2+a e^2\right)\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\,d-a e}\right]}{\sqrt{-a}\,c^{3/2}\sqrt{d+e x}\sqrt{a+c x^2}}$$

Result (type 4, 542 leaves) :

$$\frac{(-a e+c d x)\sqrt{d+e x}}{a c\sqrt{a+c x^2}}-$$

$$\left((d+e x)^{3/2}\left(d\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\left(\frac{a e^2}{(d+e x)^2}+c\left(-1+\frac{d}{d+e x}\right)^2\right)+1\Big/\left(\sqrt{d+e x}\right)\sqrt{c}\,d\left(-i\sqrt{c}\,d+\sqrt{a}\,e\right)\sqrt{1-\frac{d}{d+e x}-\frac{i\sqrt{a}\,e}{\sqrt{c}\,(d+e x)}}\right.\right.$$

$$\sqrt{1-\frac{d}{d+e x}+\frac{i\sqrt{a}\,e}{\sqrt{c}\,(d+e x)}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]-$$

$$1\Big/\left(\sqrt{d+e x}\right)\sqrt{a}\,e\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\sqrt{1-\frac{d}{d+e x}-\frac{i\sqrt{a}\,e}{\sqrt{c}\,(d+e x)}}\sqrt{1-\frac{d}{d+e x}+\frac{i\sqrt{a}\,e}{\sqrt{c}\,(d+e x)}}$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]\right)\right)\Big/\left(a c e\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\sqrt{a+\frac{c\,(d+e x)^2\left(-1+\frac{d}{d+e x}\right)^2}{e^2}}\right)$$

- **Problem 688: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{d+e x}}{\left(a+c x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 298 leaves, 7 steps) :

$$\frac{x\sqrt{d+e x}}{a\sqrt{a+c x^2}}-\frac{\sqrt{d+e x}\sqrt{1+\frac{c x^2}{a}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\,d-a e}\right]}{\sqrt{-a}\sqrt{c}\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{a+c x^2}}+$$

$$\frac{d\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{1+\frac{c x^2}{a}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\,d-a e}\right]}{\sqrt{-a}\sqrt{c}\sqrt{d+e x}\sqrt{a+c x^2}}$$

Result (type 4, 408 leaves) :

$$\frac{1}{a\sqrt{a+c x^2}}\sqrt{d+e x}\left(x-\frac{e\left(a+c x^2\right)}{c\,(d+e x)}-\frac{i\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}\sqrt{d+e x}\ \text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]}{e}+\right.$$

$$\left.\frac{\sqrt{a}\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}\sqrt{d+e x}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]}{\sqrt{c}\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}\right)$$

- **Problem 689: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\sqrt{d+e x}\left(a+c x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 331 leaves, 6 steps) :

$$\frac{(a e+c d x)\sqrt{d+e x}}{a\left(c d^2+a e^2\right)\sqrt{a+c x^2}}-\frac{\sqrt{c}\, d\sqrt{d+e x}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\, d-a e}\right]}{\sqrt{-a}\left(c d^2+a e^2\right)\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}}\sqrt{a+c x^2}}+$$

$$\frac{\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\, d-a e}\right]}{\sqrt{-a}\sqrt{c}\sqrt{d+e x}\sqrt{a+c x^2}}$$

Result (type 4, 430 leaves) :

$$\left(e\left(\sqrt{c}\, d-i\sqrt{a}\, e\right)\sqrt{-d-\frac{i\sqrt{a}\, e}{\sqrt{c}}}\, x+\right.$$

$$i\sqrt{c}\, d\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\, e}{\sqrt{c}}-e x}{d+e x}}\,(d+e x)^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\, d-i\sqrt{a}\, e}{\sqrt{c}\, d+i\sqrt{a}\, e}\right]+$$

$$\left.\sqrt{a}\, e\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\, e}{\sqrt{c}}-e x}{d+e x}}\,(d+e x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\, d-i\sqrt{a}\, e}{\sqrt{c}\, d+i\sqrt{a}\, e}\right]\right)\Big/$$

$$\left(a e\left(\sqrt{c}\, d-i\sqrt{a}\, e\right)\sqrt{-d-\frac{i\sqrt{a}\, e}{\sqrt{c}}}\sqrt{d+e x}\sqrt{a+c x^2}\right)$$

- **Problem 690: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(d+e x)^{3/2}\left(a+c x^2\right)^{3/2}}\, dx$$

Optimal (type 4, 406 leaves, 7 steps) :

$$\frac{a e+c d x}{a\left(c d^2+a e^2\right) \sqrt{d+e x} \sqrt{a+c x^2}}+\frac{e\left(c d^2-3 a e^2\right) \sqrt{a+c x^2}}{a\left(c d^2+a e^2\right)^2 \sqrt{d+e x}}-$$

$$\frac{\sqrt{c}\left(c d^2-3 a e^2\right) \sqrt{d+e x} \sqrt{1+\frac{c x^2}{a}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]}{\sqrt{-a}\left(c d^2+a e^2\right)^2 \sqrt{\frac{\sqrt{c}(d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{a+c x^2}}+$$

$$\frac{\sqrt{c} d \sqrt{\frac{\sqrt{c}(d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{1+\frac{c x^2}{a}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]}{\sqrt{-a}\left(c d^2+a e^2\right) \sqrt{d+e x} \sqrt{a+c x^2}}$$

Result (type 4, 583 leaves) :

$$\frac{1}{a e \sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}\left(c d^2+a e^2\right)^2 \sqrt{d+e x} \sqrt{a+c x^2}}$$

$$\left(-e^2 \sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}\left(-3 a^2 e^2+c^2 d^2 x^2+a c\left(d^2-3 e^2 x^2\right)\right)+e \sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}\left(-2 a e^3\left(a+c x^2\right)+c(d+e x)\left(c d^2 x+a e(2 d-e x)\right)\right)+\right.$$

$$\sqrt{c}\left(i c^{3/2} d^3-\sqrt{a} c d^2 e-3 i a \sqrt{c} d e^2+3 a^{3/2} e^3\right) \sqrt{\frac{e\left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a} e}{\sqrt{c}}-e x}{d+e x}}$$

$$(d+e x)^{3/2} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c} d-i \sqrt{a} e}{\sqrt{c} d+i \sqrt{a} e}\right]+$$

$$\left.\sqrt{a} \sqrt{c} e\left(c d^2+4 i \sqrt{a} \sqrt{c} d e-3 a e^2\right) \sqrt{\frac{e\left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a} e}{\sqrt{c}}-e x}{d+e x}}(d+e x)^{3/2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c} d-i \sqrt{a} e}{\sqrt{c} d+i \sqrt{a} e}\right]\right)$$

▪ Problem 691: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(d+e x)^{5/2} \left(a+c x^2\right)^{3/2}} \, dx$$

Optimal (type 4, 485 leaves, 8 steps) :

$$\frac{a e+c d x}{a \left(c d^2+a e^2\right) (d+e x)^{3/2} \sqrt{a+c x^2}}+\frac{e \left(3 c d^2-5 a e^2\right) \sqrt{a+c x^2}}{3 a \left(c d^2+a e^2\right)^2 (d+e x)^{3/2}}+\frac{c d e \left(3 c d^2-29 a e^2\right) \sqrt{a+c x^2}}{3 a \left(c d^2+a e^2\right)^3 \sqrt{d+e x}}-$$

$$\frac{c^{3/2} d \left(3 c d^2-29 a e^2\right) \sqrt{d+e x} \sqrt{1+\frac{c x^2}{a}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]}{3 \sqrt{-a} \left(c d^2+a e^2\right)^3 \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{a+c x^2}}+$$

$$\frac{\sqrt{c} \left(3 c d^2-5 a e^2\right) \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{1+\frac{c x^2}{a}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]}{3 \sqrt{-a} \left(c d^2+a e^2\right)^2 \sqrt{d+e x} \sqrt{a+c x^2}}$$

Result (type 4, 634 leaves) :

$$\frac{1}{3\,a\,\left(c\,d^2+a\,e^2\right)^3\,(d+e\,x)^{3/2}\,\sqrt{a+c\,x^2}}$$

$$\left(-2\,a\,e^3\,\left(c\,d^2+a\,e^2\right)\,\left(a+c\,x^2\right)-20\,a\,c\,d\,e^3\,(d+e\,x)\,\left(a+c\,x^2\right)+3\,c\,(d+e\,x)^2\,\left(-a^2\,e^3+c^2\,d^3\,x+3\,a\,c\,d\,e\,(d-e\,x)\right)+\right.$$

$$\frac{1}{e\,\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}}\,c\,(d+e\,x)\,\left(-d\,e^2\,\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}\,\left(-29\,a^2\,e^2+3\,c^2\,d^2\,x^2+a\,c\,\left(3\,d^2-29\,e^2\,x^2\right)\right)+\right.$$

$$\sqrt{c}\,d\,\left(3\,i\,c^{3/2}\,d^3-3\,\sqrt{a}\,c\,d^2\,e-29\,i\,a\,\sqrt{c}\,d\,e^2+29\,a^{3/2}\,e^3\right)\,\sqrt{\frac{e\,\left(\frac{i\,\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\,\sqrt{-\frac{\frac{i\,\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\,\frac{\sqrt{c}\,d-i\,\sqrt{a}\,e}{\sqrt{c}\,d+i\,\sqrt{a}\,e}\right]+\sqrt{a}\,e\,\left(3\,c^{3/2}\,d^3+27\,i\,\sqrt{a}\,c\,d^2\,e-29\,a\,\sqrt{c}\,d\,e^2-5\,i\,a^{3/2}\,e^3\right)$$

$$\left.\left.\sqrt{\frac{e\,\left(\frac{i\,\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\,\sqrt{-\frac{\frac{i\,\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\,\frac{\sqrt{c}\,d-i\,\sqrt{a}\,e}{\sqrt{c}\,d+i\,\sqrt{a}\,e}\right]\right)\right)$$

- **Problem 692: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(d+e\,x)^{9/2}}{\left(a+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 475 leaves, 8 steps) :

$$-\frac{(a e-c d x)(d+e x)^{7/2}}{3 a c\left(a+c x^2\right)^{3/2}}-\frac{(d+e x)^{3/2}\left(a e\left(c d^2+7 a e^2\right)-2 c d\left(2 c d^2+5 a e^2\right) x\right)}{6 a^2 c^2 \sqrt{a+c x^2}}-\frac{2 d e\left(c d^2+3 a e^2\right) \sqrt{d+e x} \sqrt{a+c x^2}}{3 a^2 c^2}+$$

$$\frac{\left(4 c^2 d^4+15 a c d^2 e^2-21 a^2 e^4\right) \sqrt{d+e x} \sqrt{1+\frac{c x^2}{a}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c}\, d-a e}\right]}{6 (-a)^{3/2} c^{5/2} \sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}} \sqrt{a+c x^2}}-$$

$$\frac{2 d\left(c d^2+a e^2\right)\left(c d^2+3 a e^2\right) \sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}} \sqrt{1+\frac{c x^2}{a}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c}\, d-a e}\right]}{3 (-a)^{3/2} c^{5/2} \sqrt{d+e x} \sqrt{a+c x^2}}$$

Result (type 4, 851 leaves) :

$$\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}\left(\frac{-4\,a\,c\,d^3\,e+4\,a^2\,d\,e^3+c^2\,d^4\,x-6\,a\,c\,d^2\,e^2\,x+a^2\,e^4\,x}{3\,a\,c^2\,\left(a+c\,x^2\right)^2}+\frac{a\,c\,d^3\,e-27\,a^2\,d\,e^3+4\,c^2\,d^4\,x+15\,a\,c\,d^2\,e^2\,x-9\,a^2\,e^4\,x}{6\,a^2\,c^2\,\left(a+c\,x^2\right)}\right)+$$

$$\frac{1}{12\,a^2\,c^3\,e\,\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\,\sqrt{a+\frac{c\,(d+e\,x)^2\left(-1+\frac{d}{d+e\,x}\right)^2}{e^2}}}$$

$$(d+e\,x)^{3/2}\left(2\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\left(\frac{21\,a^3\,e^6}{(d+e\,x)^2}+3\,a^2\,c\,e^4\left(7+\frac{2\,d^2}{(d+e\,x)^2}-\frac{14\,d}{d+e\,x}\right)-4\,c^3\,d^4\left(-1+\frac{d}{d+e\,x}\right)^2+a\,c^2\,d^2\,e^2\left(-15-\frac{19\,d^2}{(d+e\,x)^2}+\frac{30\,d}{d+e\,x}\right)\right)+\right.$$

$$\frac{1}{\sqrt{d+e\,x}}2\sqrt{c}\,\left(4\,i\,c^{5/2}\,d^5-4\sqrt{a}\,c^2\,d^4\,e+15\,i\,a\,c^{3/2}\,d^3\,e^2-15\,a^{3/2}\,c\,d^2\,e^3-21\,i\,a^2\sqrt{c}\,d\,e^4+21\,a^{5/2}\,e^5\right)$$

$$\sqrt{1-\frac{d}{d+e\,x}-\frac{i\sqrt{a}\,e}{\sqrt{c}\,(d+e\,x)}}\,\sqrt{1-\frac{d}{d+e\,x}+\frac{i\sqrt{a}\,e}{\sqrt{c}\,(d+e\,x)}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]+$$

$$\frac{1}{\sqrt{d+e\,x}}2\sqrt{a}\,\sqrt{c}\,e\left(4\,c^2\,d^4+i\sqrt{a}\,c^{3/2}\,d^3\,e+15\,a\,c\,d^2\,e^2+33\,i\,a^{3/2}\sqrt{c}\,d\,e^3-21\,a^2\,e^4\right)\sqrt{1-\frac{d}{d+e\,x}-\frac{i\sqrt{a}\,e}{\sqrt{c}\,(d+e\,x)}}$$

$$\left.\sqrt{1-\frac{d}{d+e\,x}+\frac{i\sqrt{a}\,e}{\sqrt{c}\,(d+e\,x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]\right)$$

- **Problem 693: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(d+e\,x)^{7/2}}{\left(a+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 418 leaves, 7 steps) :

$$-\frac{(a e-c d x)(d+e x)^{5/2}}{3 a c\left(a+c x^2\right)^{3/2}}-\frac{\sqrt{d+e x}\left(a e\left(3 c d^2+5 a e^2\right)-2 c d\left(2 c d^2+3 a e^2\right) x\right)}{6 a^2 c^2 \sqrt{a+c x^2}}+$$

$$\frac{2 d\left(c d^2+2 a e^2\right)\sqrt{d+e x}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\,d-a e}\right]}{3(-a)^{3/2} c^{3/2}\sqrt{\frac{\sqrt{c}(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{a+c x^2}}-$$

$$\frac{\left(c d^2+a e^2\right)\left(4 c d^2+5 a e^2\right)\sqrt{\frac{\sqrt{c}(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\,d-a e}\right]}{6(-a)^{3/2} c^{5/2}\sqrt{d+e x}\sqrt{a+c x^2}}$$

Result (type 4, 627 leaves) :

$$\frac{1}{6\sqrt{a+c x^2}}\sqrt{d+e x}\left(\frac{-5 a^3 e^3+4 c^3 d^3 x^3+a^2 c e\left(-5 d^2+2 d e x-7 e^2 x^2\right)+a c^2 d x\left(6 d^2+d e x+8 e^2 x^2\right)}{a^2 c^2\left(a+c x^2\right)}-\frac{1}{a^2 c^2 e\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\,(d+e x)}\left(4 d e^2\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\right.\right.$$

$$\left(2 a^2 e^2+c^2 d^2 x^2+a c\left(d^2+2 e^2 x^2\right)\right)+4\sqrt{c}\,d\left(-i c^{3/2} d^3+\sqrt{a}\,c d^2 e-2 i a\sqrt{c}\,d e^2+2 a^{3/2} e^3\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}$$

$$(d+e x)^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]-\sqrt{a}\,e\left(4 c^{3/2} d^3+i\sqrt{a}\,c d^2 e+8 a\sqrt{c}\,d e^2+5 i a^{3/2} e^3\right)$$

$$\left.\left.\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}\,(d+e x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]\right)\right)$$

- **Problem 694: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e x)^{5/2}}{\left(a+c x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 392 leaves, 7 steps):

$$-\frac{(a e-c d x)\,(d+e x)^{3/2}}{3 a c \left(a+c x^2\right)^{3/2}}-\frac{\sqrt{d+e x}\,\left(a d e-\left(4 c d^2+3 a e^2\right) x\right)}{6 a^2 c \sqrt{a+c x^2}}+\frac{\left(4 c d^2+3 a e^2\right) \sqrt{d+e x}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\,\sqrt{c}\,d-a e}\right]}{6 (-a)^{3/2} c^{3/2} \sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{a+c x^2}}-$$

$$\frac{2 d \left(c d^2+a e^2\right) \sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\,\sqrt{c}\,d-a e}\right]}{3 (-a)^{3/2} c^{3/2} \sqrt{d+e x}\,\sqrt{a+c x^2}}$$

Result (type 4, 597 leaves):

$$\frac{1}{12 \sqrt{a+c x^2}} \sqrt{d+e x} \left(\frac{2 \left(4 c^2 d^2 x^3+a^2 e (-3 d+e x)+a c x \left(6 d^2+d e x+3 e^2 x^2\right)\right)}{a^2 c \left(a+c x^2\right)}+\frac{1}{a^2 c^2 e \sqrt{-d-\frac{i \sqrt{a}\,e}{\sqrt{c}}}} (d+e x)\right.$$

$$\left(-\frac{2 e^2 \sqrt{-d-\frac{i \sqrt{a}\,e}{\sqrt{c}}}\,\left(3 a^2 e^2+4 c^2 d^2 x^2+a c \left(4 d^2+3 e^2 x^2\right)\right)}{(d+e x)^2}+1\Big/\left(\sqrt{d+e x}\right) 2 i \sqrt{c}\,\left(4 c^{3/2} d^3+4 i \sqrt{a}\,c d^2 e+3 a \sqrt{c}\,d e^2+3 i a^{3/2} e^3\right)\right.$$

$$\sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\,\sqrt{-\frac{\frac{i \sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i \sqrt{a}\,e}{\sqrt{c}\,d+i \sqrt{a}\,e}\right]+1\Big/\left(\sqrt{d+e x}\right) 2 \sqrt{a}\,\sqrt{c}\,e$$

$$\left.\left.\left(4 c d^2+i \sqrt{a}\,\sqrt{c}\,d e+3 a e^2\right) \sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\,\sqrt{-\frac{\frac{i \sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i \sqrt{a}\,e}{\sqrt{c}\,d+i \sqrt{a}\,e}\right]\right)\right)$$

Problem 695: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d+e x)^{3/2}}{\left(a+c x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 368 leaves, 7 steps):

$$-\frac{(a e-c d x) \sqrt{d+e x}}{3 a c \left(a+c x^2\right)^{3/2}}+\frac{(a e+4 c d x) \sqrt{d+e x}}{6 a^2 c \sqrt{a+c x^2}}+\frac{2 d \sqrt{d+e x} \sqrt{1+\frac{c x^2}{a}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]}{3 (-a)^{3/2} \sqrt{c} \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{a+c x^2}}-$$

$$\frac{\left(4 c d^2+a e^2\right) \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{1+\frac{c x^2}{a}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]}{6 (-a)^{3/2} c^{3/2} \sqrt{d+e x} \sqrt{a+c x^2}}$$

Result (type 4, 504 leaves):

$$\frac{1}{12 \sqrt{a+c x^2}}$$

$$\sqrt{d+e x} \left(\frac{-2 a^2 e+8 c^2 d x^3+2 a c x (6 d+e x)}{a^2 c \left(a+c x^2\right)}+1 \Bigg/ \left(a^2 c e \sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}\right) (d+e x) \left(-\frac{8 d e^2 \sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}} \left(a+c x^2\right)}{(d+e x)^2}+1 \Big/ \left(\sqrt{d+e x}\right) 8\right.\right.$$

$$i \sqrt{c} d \left(\sqrt{c} d+i \sqrt{a} e\right) \sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a} e}{\sqrt{c}}-e x}{d+e x}} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c} d-i \sqrt{a} e}{\sqrt{c} d+i \sqrt{a} e}\right]+$$

$$\left.\left.1 \Big/ \left(\sqrt{d+e x}\right) 2 \sqrt{a} e \left(4 \sqrt{c} d+i \sqrt{a} e\right) \sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a} e}{\sqrt{c}}-e x}{d+e x}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c} d-i \sqrt{a} e}{\sqrt{c} d+i \sqrt{a} e}\right]\right)\right)$$

Problem 696: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{d+e x}}{\left(a+c x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 392 leaves, 7 steps):

$$\frac{x\sqrt{d+e x}}{3 a\left(a+c x^2\right)^{3/2}}+\frac{\sqrt{d+e x}\left(a d e+\left(4 c d^2+3 a e^2\right) x\right)}{6 a^2\left(c d^2+a e^2\right)\sqrt{a+c x^2}}+\frac{\left(4 c d^2+3 a e^2\right)\sqrt{d+e x}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\,d-a e}\right]}{6(-a)^{3/2}\sqrt{c}\left(c d^2+a e^2\right)\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{a+c x^2}}-$$

$$\frac{2 d\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\,d-a e}\right]}{3(-a)^{3/2}\sqrt{c}\sqrt{d+e x}\sqrt{a+c x^2}}$$

Result (type 4, 619 leaves):

$$\frac{1}{12\sqrt{a+c x^2}}$$

$$\sqrt{d+e x}\left(\frac{2\left(4 c^2 d^2 x^3+a^2 e\,(d+5 e x)+a c x\left(6 d^2+d e x+3 e^2 x^2\right)\right)}{a^2\left(c d^2+a e^2\right)\left(a+c x^2\right)}+(d+e x)\left(-\frac{2 e^2\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\left(3 a^2 e^2+4 c^2 d^2 x^2+a c\left(4 d^2+3 e^2 x^2\right)\right)}{(d+e x)^2}+\right.\right.$$

$$1\Big/\left(\sqrt{d+e x}\right) 2 i\sqrt{c}\left(4 c^{3/2} d^3+4 i\sqrt{a}\,c d^2 e+3 a\sqrt{c}\,d e^2+3 i a^{3/2} e^3\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]+1\Big/\left(\sqrt{d+e x}\right) 2\sqrt{a}\sqrt{c}\,e\left(4 c d^2+i\sqrt{a}\sqrt{c}\,d e+3 a e^2\right)$$

$$\left.\left.\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]\right)\right)\Big/\left(a^2 c e\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\left(c d^2+a e^2\right)\right)$$

- **Problem 697: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\sqrt{d+e x}\left(a+c x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 450 leaves, 7 steps):

$$\frac{(a e+c d x)\sqrt{d+e x}}{3 a\left(c d^2+a e^2\right)\left(a+c x^2\right)^{3/2}}+\frac{\sqrt{d+e x}\left(a e\left(c d^2+5 a e^2\right)+4 c d\left(c d^2+2 a e^2\right) x\right)}{6 a^2\left(c d^2+a e^2\right)^2\sqrt{a+c x^2}}+$$

$$\frac{2\sqrt{c}\, d\left(c d^2+2 a e^2\right)\sqrt{d+e x}\sqrt{1+\frac{c x^2}{a}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\, d-a e}\right]}{3(-a)^{3/2}\left(c d^2+a e^2\right)^2\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}}\sqrt{a+c x^2}}-$$

$$\frac{\left(4 c d^2+5 a e^2\right)\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}}\sqrt{1+\frac{c x^2}{a}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\, d-a e}\right]}{6(-a)^{3/2}\sqrt{c}\left(c d^2+a e^2\right)\sqrt{d+e x}\sqrt{a+c x^2}}$$

Result (type 4, 570 leaves):

$$\frac{1}{6 a^2 \left(c d^2+a e^2\right)^2 \sqrt{a+c x^2}}$$

$$\sqrt{d+e x} \left(a c d^2 e+5 a^2 e^3+4 c^2 d^3 x+8 a c d e^2 x+\frac{2 a \left(c d^2+a e^2\right) (a e+c d x)}{a+c x^2}-\frac{4 d e \left(2 a^2 e^2+c^2 d^2 x^2+a c \left(d^2+2 e^2 x^2\right)\right)}{d+e x}-\frac{1}{e}\right.$$

$$4 i c d \sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}} \left(c d^2+2 a e^2\right) \sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a}\, e}{\sqrt{c}}-e x}{d+e x}} \sqrt{d+e x}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right], \frac{\sqrt{c}\, d-i \sqrt{a}\, e}{\sqrt{c}\, d+i \sqrt{a}\, e}\right]+$$

$$\frac{1}{\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}} \sqrt{a} \left(4 c^{3/2} d^3+i \sqrt{a}\, c d^2 e+8 a \sqrt{c}\, d e^2+5 i a^{3/2} e^3\right) \sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}$$

$$\left.\sqrt{-\frac{\frac{i \sqrt{a}\, e}{\sqrt{c}}-e x}{d+e x}} \sqrt{d+e x}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right], \frac{\sqrt{c}\, d-i \sqrt{a}\, e}{\sqrt{c}\, d+i \sqrt{a}\, e}\right]\right)$$

- **Problem 698: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(d+e x)^{3/2} \left(a+c x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 532 leaves, 8 steps) :

$$\frac{a e+c d x}{3 a \left(c d^2+a e^2\right) \sqrt{d+e x} \left(a+c x^2\right)^{3/2}}-\frac{a e \left(c d^2-7 a e^2\right)-4 c d \left(c d^2+3 a e^2\right) x}{6 a^2 \left(c d^2+a e^2\right)^2 \sqrt{d+e x} \sqrt{a+c x^2}}+\frac{e \left(4 c^2 d^4+15 a c d^2 e^2-21 a^2 e^4\right) \sqrt{a+c x^2}}{6 a^2 \left(c d^2+a e^2\right)^3 \sqrt{d+e x}}+$$

$$\frac{\sqrt{c} \left(4 c^2 d^4+15 a c d^2 e^2-21 a^2 e^4\right) \sqrt{d+e x} \sqrt{1+\frac{c x^2}{a}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]}{6 (-a)^{3/2} \left(c d^2+a e^2\right)^3 \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{a+c x^2}}-$$

$$\frac{2 \sqrt{c} d \left(c d^2+3 a e^2\right) \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{1+\frac{c x^2}{a}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]}{3 (-a)^{3/2} \left(c d^2+a e^2\right)^2 \sqrt{d+e x} \sqrt{a+c x^2}}$$

Result (type 4, 669 leaves) :

$$\frac{1}{6 a^2 \left(c d^2+a e^2\right)^3 \sqrt{d+e x} \sqrt{a+c x^2}}\left(21 a^3 e^5-4 c^3 d^4 e x^2-12 a^2 e^5 \left(a+c x^2\right)+3 a^2 c e^3 \left(-5 d^2+7 e^2 x^2\right)-a c^2 d^2 e \left(4 d^2+15 e^2 x^2\right)+\frac{2 a c \left(c d^2+a e^2\right) (d+e x) \left(c d^2 x+a e (2 d-e x)\right)}{a+c x^2}+\right.$$

$$c (d+e x) \left(4 c^2 d^4 x+3 a^2 e^3 (7 d-3 e x)+a c d^2 e (d+15 e x)\right)-\frac{1}{e} i c \sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}} \left(4 c^2 d^4+15 a c d^2 e^2-21 a^2 e^4\right)$$

$$\sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a} e}{\sqrt{c}}-e x}{d+e x}} (d+e x)^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right], \frac{\sqrt{c} d-i \sqrt{a} e}{\sqrt{c} d+i \sqrt{a} e}\right]+$$

$$\frac{1}{\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}} \sqrt{a} \sqrt{c} \left(4 c^2 d^4+i \sqrt{a} c^{3/2} d^3 e+15 a c d^2 e^2+33 i a^{3/2} \sqrt{c} d e^3-21 a^2 e^4\right) \sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}$$

$$\left.\sqrt{-\frac{\frac{i \sqrt{a} e}{\sqrt{c}}-e x}{d+e x}} (d+e x)^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right], \frac{\sqrt{c} d-i \sqrt{a} e}{\sqrt{c} d+i \sqrt{a} e}\right]\right)$$

- **Problem 699: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(d+e x) \left(d^2+3 e^2 x^2\right)^{1/3}} \, dx$$

Optimal (type 3, 151 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{2/3} (d-e x)}{\sqrt{3} d^{1/3} \left(d^2+3 e^2 x^2\right)^{1/3}}\right]}{2^{2/3} \sqrt{3} d^{2/3} e}-\frac{\text{Log}[d+e x]}{2\times 2^{2/3} d^{2/3} e}+\frac{\text{Log}\left[3 d e^2-3 e^3 x-3\times 2^{1/3} d^{1/3} e^2 \left(d^2+3 e^2 x^2\right)^{1/3}\right]}{2\times 2^{2/3} d^{2/3} e}$$

Result (type 6, 176 leaves):

$$-\frac{\left(\frac{e\left(\sqrt{3}\sqrt{-\frac{d^2}{e^2}}+3x\right)}{d+e x}\right)^{1/3}\left(\frac{e\left(-3\sqrt{3}\sqrt{-\frac{d^2}{e^2}}+9x\right)}{d+e x}\right)^{1/3}\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{3d-\sqrt{3}\sqrt{-\frac{d^2}{e^2}}\,e}{3d+3ex},\frac{3d+\sqrt{3}\sqrt{-\frac{d^2}{e^2}}\,e}{3d+3ex}\right]}{2e\left(d^2+3e^2x^2\right)^{1/3}}$$

Problem 700: Result unnecessarily involves higher level functions.

$$\int\frac{(2+3x)^3}{\left(4+27x^2\right)^{1/3}}\,dx$$

Optimal (type 4, 558 leaves, 6 steps):

$$\frac{1}{30}(2+3x)^2\left(4+27x^2\right)^{2/3}+\frac{4}{35}(7+4x)\left(4+27x^2\right)^{2/3}-\frac{96x}{7\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)}+$$

$$\left(16\times2^{1/3}\sqrt{2+\sqrt{3}}\left(2^{2/3}-\left(4+27x^2\right)^{1/3}\right)\sqrt{\frac{2\times2^{1/3}+2^{2/3}\left(4+27x^2\right)^{1/3}+\left(4+27x^2\right)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{2^{2/3}\left(1+\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}}{2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Bigg/\left(21\times3^{3/4}x\sqrt{-\frac{2^{2/3}-\left(4+27x^2\right)^{1/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)^2}}\right)-$$

$$\left(32\times2^{5/6}\left(2^{2/3}-\left(4+27x^2\right)^{1/3}\right)\sqrt{\frac{2\times2^{1/3}+2^{2/3}\left(4+27x^2\right)^{1/3}+\left(4+27x^2\right)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{2/3}\left(1+\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}}{2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Bigg/$$

$$\left(63\times3^{1/4}x\sqrt{-\frac{2^{2/3}-\left(4+27x^2\right)^{1/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 97 leaves):

$$\frac{1}{210\left(4+27x^2\right)^{1/3}}$$

$$\left(784+720x+5544x^2+4860x^3+1701x^4+80\times6^{1/3}\left(2\sqrt{3}-9ix\right)^{1/3}\left(-2i+3\sqrt{3}x\right)\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{1}{2}+\frac{3}{4}i\sqrt{3}x\right]\right)$$

Problem 701: Result unnecessarily involves higher level functions.

$$\int\frac{(2+3x)^2}{\left(4+27x^2\right)^{1/3}}\,dx$$

Optimal (type 4, 551 leaves, 6 steps) :

$$\frac{5}{21}\left(4+27x^2\right)^{2/3}+\frac{1}{21}(2+3x)\left(4+27x^2\right)^{2/3}-\frac{72x}{7\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)}+$$

$$\left(4\times 2^{1/3}\sqrt{2+\sqrt{3}}\left(2^{2/3}-\left(4+27x^2\right)^{1/3}\right)\sqrt{\frac{2\times 2^{1/3}+2^{2/3}\left(4+27x^2\right)^{1/3}+\left(4+27x^2\right)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{2^{2/3}\left(1+\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}}{2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/\left(7\times 3^{3/4}x\sqrt{-\frac{2^{2/3}-\left(4+27x^2\right)^{1/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)^2}}\right)-$$

$$\left(8\times 2^{5/6}\left(2^{2/3}-\left(4+27x^2\right)^{1/3}\right)\sqrt{\frac{2\times 2^{1/3}+2^{2/3}\left(4+27x^2\right)^{1/3}+\left(4+27x^2\right)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{2/3}\left(1+\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}}{2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(21\times 3^{1/4}x\sqrt{-\frac{2^{2/3}-\left(4+27x^2\right)^{1/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 92 leaves) :

$$\frac{1}{21\left(4+27x^2\right)^{1/3}}\left(28+12x+189x^2+81x^3+6\times 6^{1/3}\left(2\sqrt{3}-9\,i\,x\right)^{1/3}\left(-2\,i+3\sqrt{3}\,x\right)\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{1}{2}+\frac{3}{4}\,i\sqrt{3}\,x\right]\right)$$

- **Problem 702: Result unnecessarily involves higher level functions.**

$$\int\frac{2+3x}{\left(4+27x^2\right)^{1/3}}\,dx$$

Optimal (type 4, 529 leaves, 5 steps) :

$$\frac{1}{12}\left(4+27x^2\right)^{2/3}-\frac{6x}{2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}}+\left(2^{1/3}\sqrt{2+\sqrt{3}}\left(2^{2/3}-\left(4+27x^2\right)^{1/3}\right)\sqrt{\frac{2\times 2^{1/3}+2^{2/3}\left(4+27x^2\right)^{1/3}+\left(4+27x^2\right)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{2^{2/3}\left(1+\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}}{2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Bigg/\left(3\times 3^{3/4}x\sqrt{-\frac{2^{2/3}-\left(4+27x^2\right)^{1/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)^2}}\right)-$$

$$\left(2\times 2^{5/6}\left(2^{2/3}-\left(4+27x^2\right)^{1/3}\right)\sqrt{\frac{2\times 2^{1/3}+2^{2/3}\left(4+27x^2\right)^{1/3}+\left(4+27x^2\right)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{2/3}\left(1+\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}}{2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Bigg/$$

$$\left(9\times 3^{1/4}x\sqrt{-\frac{2^{2/3}-\left(4+27x^2\right)^{1/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 40 leaves):

$$\frac{1}{12}\left(4+27x^2\right)^{2/3}+2^{1/3}x\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{27x^2}{4}\right]$$

- **Problem 703: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\frac{1}{(2+3x)\left(4+27x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 97 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{1/3}(2-3x)}{\sqrt{3}\left(4+27x^2\right)^{1/3}}\right]}{6\times 2^{1/3}\sqrt{3}}-\frac{\text{Log}[2+3x]}{12\times 2^{1/3}}+\frac{\text{Log}\left[54-81x-27\times 2^{2/3}\left(4+27x^2\right)^{1/3}\right]}{12\times 2^{1/3}}$$

Result (type 6, 285 leaves):

$$-\left(5(2+3x)\left(-2i\sqrt{3}+9x\right)\left(2i\sqrt{3}+9x\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{6-2i\sqrt{3}}{6+9x},\frac{6+2i\sqrt{3}}{6+9x}\right]\right)\Big/$$

$$\left(2\left(4+27x^2\right)^{4/3}\left(15(2+3x)\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{6-2i\sqrt{3}}{6+9x},\frac{6+2i\sqrt{3}}{6+9x}\right]+\right.\right.$$

$$\left.\left.\left(6+2i\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{4}{3},\frac{8}{3},\frac{6-2i\sqrt{3}}{6+9x},\frac{6+2i\sqrt{3}}{6+9x}\right]+2\left(3-i\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},\frac{1}{3},\frac{8}{3},\frac{6-2i\sqrt{3}}{6+9x},\frac{6+2i\sqrt{3}}{6+9x}\right]\right)\right)$$

- **Problem 704: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(2+3x)^2 \left(4+27x^2\right)^{1/3}} \, dx$$

Optimal (type 4, 634 leaves, 7 steps):

$$-\frac{\left(4+27x^2\right)^{2/3}}{48\,(2+3x)} - \frac{3x}{16\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)} - \frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{1/3}(2-3x)}{\sqrt{3}\left(4+27x^2\right)^{1/3}}\right]}{24\times 2^{1/3}\sqrt{3}} +$$

$$\left(\sqrt{2+\sqrt{3}}\left(2^{2/3}-\left(4+27x^2\right)^{1/3}\right)\sqrt{\frac{2\times 2^{1/3}+2^{2/3}\left(4+27x^2\right)^{1/3}+\left(4+27x^2\right)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{2^{2/3}\left(1+\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}}{2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(48\times 2^{2/3}\,3^{3/4}\,x\sqrt{-\frac{2^{2/3}-\left(4+27x^2\right)^{1/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)^2}}\right) -$$

$$\left(\left(2^{2/3}-\left(4+27x^2\right)^{1/3}\right)\sqrt{\frac{2\times 2^{1/3}+2^{2/3}\left(4+27x^2\right)^{1/3}+\left(4+27x^2\right)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{2/3}\left(1+\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}}{2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(72\times 2^{1/6}\,3^{1/4}\,x\sqrt{-\frac{2^{2/3}-\left(4+27x^2\right)^{1/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)^2}}\right) - \frac{\text{Log}[2+3x]}{48\times 2^{1/3}} + \frac{\text{Log}\left[54-81x-27\times 2^{2/3}\left(4+27x^2\right)^{1/3}\right]}{48\times 2^{1/3}}$$

Result (type 6, 376 leaves):

$$\frac{1}{1728\left(4+27x^2\right)^{4/3}}\left(-\frac{36\left(4+27x^2\right)^2}{2+3x} - \left(1080\,(2+3x)\left(-2\,i\sqrt{3}+9x\right)\left(2\,i\sqrt{3}+9x\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{6-2\,i\sqrt{3}}{6+9x},\frac{6+2\,i\sqrt{3}}{6+9x}\right]\right)\Big/\right.$$

$$\left(15\,(2+3x)\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{6-2\,i\sqrt{3}}{6+9x},\frac{6+2\,i\sqrt{3}}{6+9x}\right]+\right.$$

$$\left.\left(6+2\,i\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{4}{3},\frac{8}{3},\frac{6-2\,i\sqrt{3}}{6+9x},\frac{6+2\,i\sqrt{3}}{6+9x}\right]+2\left(3-i\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},\frac{1}{3},\frac{8}{3},\frac{6-2\,i\sqrt{3}}{6+9x},\frac{6+2\,i\sqrt{3}}{6+9x}\right]\right)+$$

$$\left.3\times 3^{5/6}\left(4\sqrt{3}-18\,i\,x\right)^{1/3}\left(-2\,i\sqrt{3}+9x\right)\left(4+27x^2\right)\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{1}{2}+\frac{3}{4}\,i\sqrt{3}\,x\right]\right)$$

- **Problem 705: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(2+3x)^3 \left(4+27x^2\right)^{1/3}} \, dx$$

Optimal (type 4, 656 leaves, 8 steps):

$$-\frac{\left(4+27x^2\right)^{2/3}}{96\,(2+3x)^2}-\frac{\left(4+27x^2\right)^{2/3}}{96\,(2+3x)}-\frac{3x}{32\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)}-\frac{\mathrm{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{1/3}(2-3x)}{\sqrt{3}\left(4+27x^2\right)^{1/3}}\right]}{96\times 2^{1/3}\sqrt{3}}+$$

$$\left(\sqrt{2+\sqrt{3}}\left(2^{2/3}-\left(4+27x^2\right)^{1/3}\right)\sqrt{\frac{2\times 2^{1/3}+2^{2/3}\left(4+27x^2\right)^{1/3}+\left(4+27x^2\right)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)^2}}\ \mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\frac{2^{2/3}\left(1+\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}}{2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Bigg/$$

$$\left(96\times 2^{2/3}\,3^{3/4}\,x\sqrt{-\frac{2^{2/3}-\left(4+27x^2\right)^{1/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)^2}}\right)-$$

$$\left(\left(2^{2/3}-\left(4+27x^2\right)^{1/3}\right)\sqrt{\frac{2\times 2^{1/3}+2^{2/3}\left(4+27x^2\right)^{1/3}+\left(4+27x^2\right)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)^2}}\ \mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{2^{2/3}\left(1+\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}}{2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Bigg/$$

$$\left(144\times 2^{1/6}\,3^{1/4}\,x\sqrt{-\frac{2^{2/3}-\left(4+27x^2\right)^{1/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4+27x^2\right)^{1/3}\right)^2}}\right)-\frac{\mathrm{Log}[2+3x]}{192\times 2^{1/3}}+\frac{\mathrm{Log}\left[54-81x-27\times 2^{2/3}\left(4+27x^2\right)^{1/3}\right]}{192\times 2^{1/3}}$$

Result (type 6, 379 leaves):

$$\frac{1}{3456\left(4+27x^2\right)^{4/3}}\left(-\frac{108\,(1+x)\left(4+27x^2\right)^2}{(2+3x)^2}-\left(540\,(2+3x)\left(-2i\sqrt{3}+9x\right)\left(2i\sqrt{3}+9x\right)\mathrm{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{6-2i\sqrt{3}}{6+9x},\frac{6+2i\sqrt{3}}{6+9x}\right]\right)\Bigg/\right.$$

$$\left(15\,(2+3x)\,\mathrm{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{6-2i\sqrt{3}}{6+9x},\frac{6+2i\sqrt{3}}{6+9x}\right]+\right.$$

$$\left.\left(6+2i\sqrt{3}\right)\mathrm{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{4}{3},\frac{8}{3},\frac{6-2i\sqrt{3}}{6+9x},\frac{6+2i\sqrt{3}}{6+9x}\right]+2\left(3-i\sqrt{3}\right)\mathrm{AppellF1}\left[\frac{5}{3},\frac{4}{3},\frac{1}{3},\frac{8}{3},\frac{6-2i\sqrt{3}}{6+9x},\frac{6+2i\sqrt{3}}{6+9x}\right]\right)+$$

$$\left.3\times 3^{5/6}\left(4\sqrt{3}-18ix\right)^{1/3}\left(-2i\sqrt{3}+9x\right)\left(4+27x^2\right)\mathrm{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{1}{2}+\frac{3}{4}i\sqrt{3}\,x\right]\right)$$

Problem 706: Result unnecessarily involves higher level functions.

$$\int \frac{(2+3\,i\,x)^3}{\left(4-27\,x^2\right)^{1/3}}\,dx$$

Optimal (type 4, 564 leaves, 6 steps):

$$-\frac{4}{35}\,(7\,i-4\,x)\,\left(4-27\,x^2\right)^{2/3}-\frac{1}{30}\,i\,(2+3\,i\,x)^2\,\left(4-27\,x^2\right)^{2/3}-\frac{96\,x}{7\,\left(2^{2/3}\,\left(1-\sqrt{3}\right)-\left(4-27\,x^2\right)^{1/3}\right)}-$$
$$\left(16\times 2^{1/3}\,\sqrt{2+\sqrt{3}}\,\left(2^{2/3}-\left(4-27\,x^2\right)^{1/3}\right)\,\sqrt{\frac{2\times 2^{1/3}+2^{2/3}\,\left(4-27\,x^2\right)^{1/3}+\left(4-27\,x^2\right)^{2/3}}{\left(2^{2/3}\,\left(1-\sqrt{3}\right)-\left(4-27\,x^2\right)^{1/3}\right)^2}}\right.$$
$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{2^{2/3}\,\left(1+\sqrt{3}\right)-\left(4-27\,x^2\right)^{1/3}}{2^{2/3}\,\left(1-\sqrt{3}\right)-\left(4-27\,x^2\right)^{1/3}}\right],\,-7+4\,\sqrt{3}\right]\right)\Bigg/\left(21\times 3^{3/4}\,x\,\sqrt{-\frac{2^{2/3}-\left(4-27\,x^2\right)^{1/3}}{\left(2^{2/3}\,\left(1-\sqrt{3}\right)-\left(4-27\,x^2\right)^{1/3}\right)^2}}\right)+$$
$$\left(32\times 2^{5/6}\,\left(2^{2/3}-\left(4-27\,x^2\right)^{1/3}\right)\,\sqrt{\frac{2\times 2^{1/3}+2^{2/3}\,\left(4-27\,x^2\right)^{1/3}+\left(4-27\,x^2\right)^{2/3}}{\left(2^{2/3}\,\left(1-\sqrt{3}\right)-\left(4-27\,x^2\right)^{1/3}\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{2/3}\,\left(1+\sqrt{3}\right)-\left(4-27\,x^2\right)^{1/3}}{2^{2/3}\,\left(1-\sqrt{3}\right)-\left(4-27\,x^2\right)^{1/3}}\right],\,-7+4\,\sqrt{3}\right]\right)\Bigg/$$
$$\left(63\times 3^{1/4}\,x\,\sqrt{-\frac{2^{2/3}-\left(4-27\,x^2\right)^{1/3}}{\left(2^{2/3}\,\left(1-\sqrt{3}\right)-\left(4-27\,x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 60 leaves):

$$\left(4-27\,x^2\right)^{2/3}\,\left(-\frac{14\,i}{15}+\frac{6\,x}{7}+\frac{3\,i\,x^2}{10}\right)+\frac{16}{7}\,2^{1/3}\,x\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,\frac{1}{2},\,\frac{3}{2},\,\frac{27\,x^2}{4}\right]$$

Problem 707: Result unnecessarily involves higher level functions.

$$\int \frac{(2+3\,i\,x)^2}{\left(4-27\,x^2\right)^{1/3}}\,dx$$

Optimal (type 4, 557 leaves, 6 steps):

$$-\frac{5}{21}\,i\,\left(4-27x^2\right)^{2/3}-\frac{1}{21}\,i\,(2+3\,i\,x)\,\left(4-27x^2\right)^{2/3}-\frac{72x}{7\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}\right)}-$$

$$\left(4\times 2^{1/3}\sqrt{2+\sqrt{3}}\,\left(2^{2/3}-\left(4-27x^2\right)^{1/3}\right)\sqrt{\frac{2\times 2^{1/3}+2^{2/3}\left(4-27x^2\right)^{1/3}+\left(4-27x^2\right)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{2^{2/3}\left(1+\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}}{2^{2/3}\left(1-\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}}\right],\,-7+4\sqrt{3}\right]\right)\Bigg/\left(7\times 3^{3/4}\,x\sqrt{-\frac{2^{2/3}-\left(4-27x^2\right)^{1/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(8\times 2^{5/6}\left(2^{2/3}-\left(4-27x^2\right)^{1/3}\right)\sqrt{\frac{2\times 2^{1/3}+2^{2/3}\left(4-27x^2\right)^{1/3}+\left(4-27x^2\right)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{2/3}\left(1+\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}}{2^{2/3}\left(1-\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}}\right],\,-7+4\sqrt{3}\right]\right)\Bigg/$$

$$\left(21\times 3^{1/4}\,x\sqrt{-\frac{2^{2/3}-\left(4-27x^2\right)^{1/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 51 leaves) :

$$\left(-\frac{i}{3}+\frac{x}{7}\right)\left(4-27x^2\right)^{2/3}+\frac{12}{7}\,2^{1/3}\,x\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,\frac{1}{2},\,\frac{3}{2},\,\frac{27x^2}{4}\right]$$

- **Problem 708: Result unnecessarily involves higher level functions.**

$$\int\frac{2+3\,i\,x}{\left(4-27x^2\right)^{1/3}}\,dx$$

Optimal (type 4, 531 leaves, 5 steps) :

$$-\frac{1}{12}\, i \left(4-27 x^2\right)^{2/3}-\frac{6 x}{2^{2/3} \left(1-\sqrt{3}\right)-\left(4-27 x^2\right)^{1/3}}-\left(2^{1/3} \sqrt{2+\sqrt{3}} \left(2^{2/3}-\left(4-27 x^2\right)^{1/3}\right) \sqrt{\frac{2\times 2^{1/3}+2^{2/3} \left(4-27 x^2\right)^{1/3}+\left(4-27 x^2\right)^{2/3}}{\left(2^{2/3} \left(1-\sqrt{3}\right)-\left(4-27 x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{2^{2/3} \left(1+\sqrt{3}\right)-\left(4-27 x^2\right)^{1/3}}{2^{2/3} \left(1-\sqrt{3}\right)-\left(4-27 x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right) \Big/ \left(3\times 3^{3/4}\, x \sqrt{-\frac{2^{2/3}-\left(4-27 x^2\right)^{1/3}}{\left(2^{2/3} \left(1-\sqrt{3}\right)-\left(4-27 x^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(2\times 2^{5/6} \left(2^{2/3}-\left(4-27 x^2\right)^{1/3}\right) \sqrt{\frac{2\times 2^{1/3}+2^{2/3} \left(4-27 x^2\right)^{1/3}+\left(4-27 x^2\right)^{2/3}}{\left(2^{2/3} \left(1-\sqrt{3}\right)-\left(4-27 x^2\right)^{1/3}\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{2/3} \left(1+\sqrt{3}\right)-\left(4-27 x^2\right)^{1/3}}{2^{2/3} \left(1-\sqrt{3}\right)-\left(4-27 x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right) \Big/$$

$$\left(9\times 3^{1/4}\, x \sqrt{-\frac{2^{2/3}-\left(4-27 x^2\right)^{1/3}}{\left(2^{2/3} \left(1-\sqrt{3}\right)-\left(4-27 x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 42 leaves):

$$-\frac{1}{12}\, i \left(4-27 x^2\right)^{2/3}+2^{1/3}\, x\, \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{27 x^2}{4}\right]$$

- **Problem 709: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(2+3 i x) \left(4-27 x^2\right)^{1/3}}\, dx$$

Optimal (type 3, 109 leaves, 1 step):

$$\frac{i\, \text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{1/3} (2-3 i x)}{\sqrt{3} \left(4-27 x^2\right)^{1/3}}\right]}{6\times 2^{1/3} \sqrt{3}}+\frac{i\, \text{Log}[2+3 i x]}{12\times 2^{1/3}}-\frac{i\, \text{Log}\left[-54+81 i x+27\times 2^{2/3} \left(4-27 x^2\right)^{1/3}\right]}{12\times 2^{1/3}}$$

Result (type 6, 125 leaves):

$$\frac{i \left(\frac{2 \sqrt{3}-9 x}{2 i-3 x}\right)^{1/3} \left(\frac{2 \sqrt{3}+9 x}{-2 i+3 x}\right)^{1/3} \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{2 \left(3 i+\sqrt{3}\right)}{6 i-9 x},\frac{2 \left(-3 i+\sqrt{3}\right)}{-6 i+9 x}\right]}{2\times 3^{2/3} \left(4-27 x^2\right)^{1/3}}$$

- **Problem 710: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(2+3 i x)^2 \left(4-27 x^2\right)^{1/3}}\, dx$$

Optimal (type 4, 650 leaves, 7 steps):

$$\frac{i\left(4-27x^2\right)^{2/3}}{48\,(2+3\,i\,x)}-\frac{3x}{16\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}\right)}+\frac{i\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{1/3}\,(2-3\,i\,x)}{\sqrt{3}\,\left(4-27x^2\right)^{1/3}}\right]}{24\times 2^{1/3}\sqrt{3}}-$$

$$\left(\sqrt{2+\sqrt{3}}\,\left(2^{2/3}-\left(4-27x^2\right)^{1/3}\right)\sqrt{\frac{2\times 2^{1/3}+2^{2/3}\left(4-27x^2\right)^{1/3}+\left(4-27x^2\right)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{2^{2/3}\left(1+\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}}{2^{2/3}\left(1-\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(48\times 2^{2/3}\,3^{3/4}\,x\,\sqrt{-\frac{2^{2/3}-\left(4-27x^2\right)^{1/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(\left(2^{2/3}-\left(4-27x^2\right)^{1/3}\right)\sqrt{\frac{2\times 2^{1/3}+2^{2/3}\left(4-27x^2\right)^{1/3}+\left(4-27x^2\right)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{2/3}\left(1+\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}}{2^{2/3}\left(1-\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(72\times 2^{1/6}\,3^{1/4}\,x\,\sqrt{-\frac{2^{2/3}-\left(4-27x^2\right)^{1/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}\right)^2}}\right)+\frac{i\,\text{Log}[2+3\,i\,x]}{48\times 2^{1/3}}-\frac{i\,\text{Log}\left[-54+81\,i\,x+27\times 2^{2/3}\left(4-27x^2\right)^{1/3}\right]}{48\times 2^{1/3}}$$

Result (type 6, 132 leaves) :

$$\frac{\left(\frac{2\sqrt{3}-9x}{2\,i-3x}\right)^{1/3}\left(\frac{2\sqrt{3}+9x}{-2\,i+3x}\right)^{1/3}\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{1}{3},\frac{8}{3},\frac{2\left(3\,i+\sqrt{3}\right)}{6\,i-9x},\frac{2\left(-3\,i+\sqrt{3}\right)}{-6\,i+9x}\right]}{5\times 3^{2/3}\,(-2\,i+3x)\left(4-27x^2\right)^{1/3}}$$

- **Problem 711: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{(2+3\,i\,x)^3\left(4-27x^2\right)^{1/3}}\,dx$$

Optimal (type 4, 676 leaves, 8 steps) :

$$\frac{i\left(4-27x^2\right)^{2/3}}{96\,(2+3\,i\,x)^2}+\frac{i\left(4-27x^2\right)^{2/3}}{96\,(2+3\,i\,x)}-\frac{3\,x}{32\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}\right)}+\frac{i\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{1/3}\,(2-3\,i\,x)}{\sqrt{3}\,\left(4-27x^2\right)^{1/3}}\right]}{96\times 2^{1/3}\sqrt{3}}-$$

$$\left(\sqrt{2+\sqrt{3}}\,\left(2^{2/3}-\left(4-27x^2\right)^{1/3}\right)\sqrt{\frac{2\times 2^{1/3}+2^{2/3}\left(4-27x^2\right)^{1/3}+\left(4-27x^2\right)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{2^{2/3}\left(1+\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}}{2^{2/3}\left(1-\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(96\times 2^{2/3}\,3^{3/4}\,x\sqrt{-\frac{2^{2/3}-\left(4-27x^2\right)^{1/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(\left(2^{2/3}-\left(4-27x^2\right)^{1/3}\right)\sqrt{\frac{2\times 2^{1/3}+2^{2/3}\left(4-27x^2\right)^{1/3}+\left(4-27x^2\right)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{2/3}\left(1+\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}}{2^{2/3}\left(1-\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(144\times 2^{1/6}\,3^{1/4}\,x\sqrt{-\frac{2^{2/3}-\left(4-27x^2\right)^{1/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\left(4-27x^2\right)^{1/3}\right)^2}}\right)+\frac{i\,\text{Log}[2+3\,i\,x]}{192\times 2^{1/3}}-\frac{i\,\text{Log}\left[-54+81\,i\,x+27\times 2^{2/3}\left(4-27x^2\right)^{1/3}\right]}{192\times 2^{1/3}}$$

Result (type 6, 134 leaves) :

$$-\frac{i\left(\frac{2\sqrt{3}-9x}{2\,i-3x}\right)^{1/3}\left(\frac{2\sqrt{3}+9x}{-2\,i+3x}\right)^{1/3}\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},\frac{1}{3},\frac{11}{3},\frac{2\left(3\,i+\sqrt{3}\right)}{6\,i-9x},\frac{2\left(-3\,i+\sqrt{3}\right)}{-6\,i+9x}\right]}{8\times 3^{2/3}\,(2\,i-3x)^2\left(4-27x^2\right)^{1/3}}$$

- **Problem 712: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\frac{1}{\left(\sqrt{3}+x\right)\left(1+x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 104 leaves, 1 step) :

$$-\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{2/3}\left(\sqrt{3}-x\right)}{3\left(1+x^2\right)^{1/3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{Log}\left[\sqrt{3}+x\right]}{2\times 2^{2/3}}+\frac{\text{Log}\left[\sqrt{3}-x-2^{1/3}\sqrt{3}\left(1+x^2\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 6, 256 leaves) :

$$-\left(15\left(\sqrt{3}+x\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{-i+\sqrt{3}}{\sqrt{3}+x},\frac{i+\sqrt{3}}{\sqrt{3}+x}\right]\right)\Big/$$
$$\left(2\left(1+x^2\right)^{1/3}\left(5\left(\sqrt{3}+x\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{-i+\sqrt{3}}{\sqrt{3}+x},\frac{i+\sqrt{3}}{\sqrt{3}+x}\right]+\left(i+\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{4}{3},\frac{8}{3},\frac{-i+\sqrt{3}}{\sqrt{3}+x},\frac{i+\sqrt{3}}{\sqrt{3}+x}\right]+\right.\right.$$
$$\left.\left.\left(-i+\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},\frac{1}{3},\frac{8}{3},\frac{-i+\sqrt{3}}{\sqrt{3}+x},\frac{i+\sqrt{3}}{\sqrt{3}+x}\right]\right)\right)$$

- **Problem 713: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left(\sqrt{3}-x\right)\left(1+x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 101 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{2/3}\left(\sqrt{3}+x\right)}{3\left(1+x^2\right)^{1/3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{Log}\left[\sqrt{3}-x\right]}{2\times 2^{2/3}}-\frac{\text{Log}\left[\sqrt{3}+x-2^{1/3}\sqrt{3}\left(1+x^2\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 6, 276 leaves):

$$-\left(15\left(-\sqrt{3}+x\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{-i+\sqrt{3}}{\sqrt{3}-x},\frac{i+\sqrt{3}}{\sqrt{3}-x}\right]\right)\Big/$$
$$\left(2\left(1+x^2\right)^{1/3}\left(5\left(\sqrt{3}-x\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{-i+\sqrt{3}}{\sqrt{3}-x},\frac{i+\sqrt{3}}{\sqrt{3}-x}\right]+\left(i+\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{4}{3},\frac{8}{3},\frac{-i+\sqrt{3}}{\sqrt{3}-x},\frac{i+\sqrt{3}}{\sqrt{3}-x}\right]+\right.\right.$$
$$\left.\left.\left(-i+\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},\frac{1}{3},\frac{8}{3},\frac{-i+\sqrt{3}}{\sqrt{3}-x},\frac{i+\sqrt{3}}{\sqrt{3}-x}\right]\right)\right)$$

- **Problem 714: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(3-x)\left(1-x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 78 leaves, 2 steps):

$$-\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{(1+x)^{2/3}}{\sqrt{3}\,(1-x)^{1/3}}\right]-\frac{1}{4}\text{Log}[3-x]+\frac{3}{8}\text{Log}\left[-(1-x)^{1/3}-\frac{1}{2}(1+x)^{2/3}\right]$$

Result (type 6, 139 leaves):

$$\left(15\,(-3+x)\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},-\frac{4}{-3+x},-\frac{2}{-3+x}\right]\right)\Big/\left(2\,\left(1-x^2\right)^{1/3}\left(5\,(-3+x)\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},-\frac{4}{-3+x},-\frac{2}{-3+x}\right]-2\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{4}{3},\frac{8}{3},-\frac{4}{-3+x},-\frac{2}{-3+x}\right]+2\,\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},\frac{1}{3},\frac{8}{3},-\frac{4}{-3+x},-\frac{2}{-3+x}\right]\right)\right)\right)$$

Problem 715: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(3+x)\,\left(1-x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 76 leaves, 2 steps):

$$\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{(1-x)^{2/3}}{\sqrt{3}\,(1+x)^{1/3}}\right]+\frac{1}{4}\,\text{Log}[3+x]-\frac{3}{8}\,\text{Log}\left[-\frac{1}{2}\,(1-x)^{2/3}-(1+x)^{1/3}\right]$$

Result (type 6, 139 leaves):

$$-\left(15\,(3+x)\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{4}{3+x},\frac{2}{3+x}\right]\right)\Big/\left(2\,\left(1-x^2\right)^{1/3}\left(5\,(3+x)\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{4}{3+x},\frac{2}{3+x}\right]+2\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{4}{3},\frac{8}{3},\frac{4}{3+x},\frac{2}{3+x}\right]+2\,\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},\frac{1}{3},\frac{8}{3},\frac{4}{3+x},\frac{2}{3+x}\right]\right)\right)\right)$$

Problem 716: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(d+e\,x)\,\left(d^2-9\,e^2\,x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 206 leaves, 3 steps):

$$\frac{\sqrt{3}\,\left(1-\frac{9\,e^2\,x^2}{d^2}\right)^{1/3}\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{\left(1-\frac{3\,e\,x}{d}\right)^{2/3}}{\sqrt{3}\,\left(1+\frac{3\,e\,x}{d}\right)^{1/3}}\right]}{4\,e\,\left(d^2-9\,e^2\,x^2\right)^{1/3}}+\frac{\left(1-\frac{9\,e^2\,x^2}{d^2}\right)^{1/3}\text{Log}[d+e\,x]}{4\,e\,\left(d^2-9\,e^2\,x^2\right)^{1/3}}-\frac{3\,\left(1-\frac{9\,e^2\,x^2}{d^2}\right)^{1/3}\text{Log}\left[-\frac{1}{2}\,\left(1-\frac{3\,e\,x}{d}\right)^{2/3}-\left(1+\frac{3\,e\,x}{d}\right)^{1/3}\right]}{8\,e\,\left(d^2-9\,e^2\,x^2\right)^{1/3}}$$

Result (type 6, 155 leaves):

$$-\frac{3^{1/3}\left(-\frac{e\left(\sqrt{\frac{d^2}{e^2}}-3\,x\right)}{d+e\,x}\right)^{1/3}\left(\frac{e\left(\sqrt{\frac{d^2}{e^2}}+3\,x\right)}{d+e\,x}\right)^{1/3}\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{3\,d-\sqrt{\frac{d^2}{e^2}}\,e}{3\,d+3\,e\,x},\frac{3\,d+\sqrt{\frac{d^2}{e^2}}\,e}{3\,d+3\,e\,x}\right]}{2\,e\,\left(d^2-9\,e^2\,x^2\right)^{1/3}}$$

Problem 717: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+b\,x)\,\left(c+d\,x^2\right)^{1/4}}\,dx$$

Optimal (type 4, 278 leaves, 10 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{b}\,\left(c+d\,x^2\right)^{1/4}}{\left(b^2\,c+a^2\,d\right)^{1/4}}\right]}{\sqrt{b}\,\left(b^2\,c+a^2\,d\right)^{1/4}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\,\left(c+d\,x^2\right)^{1/4}}{\left(b^2\,c+a^2\,d\right)^{1/4}}\right]}{\sqrt{b}\,\left(b^2\,c+a^2\,d\right)^{1/4}}-$$

$$\frac{a\,c^{1/4}\,\sqrt{-\frac{d\,x^2}{c}}\,\text{EllipticPi}\left[-\frac{b\,\sqrt{c}}{\sqrt{b^2\,c+a^2\,d}},\,\text{ArcSin}\left[\frac{\left(c+d\,x^2\right)^{1/4}}{c^{1/4}}\right],\,-1\right]}{b\,\sqrt{b^2\,c+a^2\,d}\,\,x}+\frac{a\,c^{1/4}\,\sqrt{-\frac{d\,x^2}{c}}\,\text{EllipticPi}\left[\frac{b\,\sqrt{c}}{\sqrt{b^2\,c+a^2\,d}},\,\text{ArcSin}\left[\frac{\left(c+d\,x^2\right)^{1/4}}{c^{1/4}}\right],\,-1\right]}{b\,\sqrt{b^2\,c+a^2\,d}\,\,x}$$

Result (type 6, 126 leaves):

$$-\frac{2\left(\frac{b\left(-\sqrt{-\frac{c}{d}}+x\right)}{a+b\,x}\right)^{1/4}\left(\frac{b\left(\sqrt{-\frac{c}{d}}+x\right)}{a+b\,x}\right)^{1/4}\text{AppellF1}\left[\frac{1}{2},\,\frac{1}{4},\,\frac{1}{4},\,\frac{3}{2},\,\frac{a-b\,\sqrt{-\frac{c}{d}}}{a+b\,x},\,\frac{a+b\,\sqrt{-\frac{c}{d}}}{a+b\,x}\right]}{b\,\left(c+d\,x^2\right)^{1/4}}$$

Problem 718: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a+b\,x\right)\,\left(c+d\,x^2\right)^{3/4}}\,dx$$

Optimal (type 4, 268 leaves, 11 steps):

$$-\frac{\sqrt{b}\,\text{ArcTan}\left[\frac{\sqrt{b}\,\left(c+d\,x^2\right)^{1/4}}{\left(b^2\,c+a^2\,d\right)^{1/4}}\right]}{\left(b^2\,c+a^2\,d\right)^{3/4}}-\frac{\sqrt{b}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\left(c+d\,x^2\right)^{1/4}}{\left(b^2\,c+a^2\,d\right)^{1/4}}\right]}{\left(b^2\,c+a^2\,d\right)^{3/4}}+$$

$$\frac{a\,c^{1/4}\,\sqrt{-\frac{d\,x^2}{c}}\,\text{EllipticPi}\left[-\frac{b\,\sqrt{c}}{\sqrt{b^2\,c+a^2\,d}},\,\text{ArcSin}\left[\frac{\left(c+d\,x^2\right)^{1/4}}{c^{1/4}}\right],\,-1\right]}{\left(b^2\,c+a^2\,d\right)\,x}+\frac{a\,c^{1/4}\,\sqrt{-\frac{d\,x^2}{c}}\,\text{EllipticPi}\left[\frac{b\,\sqrt{c}}{\sqrt{b^2\,c+a^2\,d}},\,\text{ArcSin}\left[\frac{\left(c+d\,x^2\right)^{1/4}}{c^{1/4}}\right],\,-1\right]}{\left(b^2\,c+a^2\,d\right)\,x}$$

Result (type 6, 128 leaves):

$$-\frac{2\left(\frac{b\left(-\sqrt{-\frac{c}{d}}+x\right)}{a+b\,x}\right)^{3/4}\left(\frac{b\left(\sqrt{-\frac{c}{d}}+x\right)}{a+b\,x}\right)^{3/4}\text{AppellF1}\left[\frac{3}{2},\,\frac{3}{4},\,\frac{3}{4},\,\frac{5}{2},\,\frac{a-b\,\sqrt{-\frac{c}{d}}}{a+b\,x},\,\frac{a+b\,\sqrt{-\frac{c}{d}}}{a+b\,x}\right]}{3\,b\,\left(c+d\,x^2\right)^{3/4}}$$

Problem 719: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(d+e x)^{3/2} \left(a+c x^2\right)^{1/4}} \, dx$$

Optimal (type 5, 200 leaves, 1 step):

$$-\frac{2 \left(\sqrt{-a}-\sqrt{c} x\right) \left(-\frac{\left(\sqrt{c} d+\sqrt{-a} e\right) \left(\sqrt{-a}+\sqrt{c} x\right)}{\left(\sqrt{c} d-\sqrt{-a} e\right) \left(\sqrt{-a}-\sqrt{c} x\right)}\right)^{1/4} \text{Hypergeometric2F1}\left[-\frac{1}{2},\frac{1}{4},\frac{1}{2},\frac{2 \sqrt{-a} \sqrt{c} (d+e x)}{\left(\sqrt{c} d-\sqrt{-a} e\right) \left(\sqrt{-a}-\sqrt{c} x\right)}\right]}{\left(\sqrt{c} d+\sqrt{-a} e\right) \sqrt{d+e x} \left(a+c x^2\right)^{1/4}}$$

Result (type 5, 171 leaves):

$$\frac{2\times 2^{3/4} \left(i \sqrt{a}+\sqrt{c} x\right) \left(\frac{d-\frac{i \sqrt{a} e}{\sqrt{c}}+\frac{i \sqrt{c} d x}{\sqrt{a}}+e x}{d+e x}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{d+\frac{i \sqrt{a} e}{\sqrt{c}}-\frac{i \sqrt{c} d x}{\sqrt{a}}+e x}{2 d+2 e x}\right]}{3 \left(\sqrt{c} d-i \sqrt{a} e\right) \sqrt{d+e x} \left(a+c x^2\right)^{1/4}}$$

Problem 720: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(1+x) \left(1+x^2\right)^{1/6}} \, dx$$

Optimal (type 6, 203 leaves, 15 steps):

$$x \,\text{AppellF1}\left[\frac{1}{2},1,\frac{1}{6},\frac{3}{2},x^2,-x^2\right]-\frac{\sqrt{3} \,\text{ArcTan}\left[\frac{1-2^{5/6} \left(1+x^2\right)^{1/6}}{\sqrt{3}}\right]}{2\times 2^{1/6}}+\frac{\sqrt{3} \,\text{ArcTan}\left[\frac{1+2^{5/6} \left(1+x^2\right)^{1/6}}{\sqrt{3}}\right]}{2\times 2^{1/6}}-$$
$$\frac{\text{ArcTanh}\left[\frac{\left(1+x^2\right)^{1/6}}{2^{1/6}}\right]}{2^{1/6}}+\frac{\text{Log}\left[2^{1/3}-2^{1/6} \left(1+x^2\right)^{1/6}+\left(1+x^2\right)^{1/3}\right]}{4\times 2^{1/6}}-\frac{\text{Log}\left[2^{1/3}+2^{1/6} \left(1+x^2\right)^{1/6}+\left(1+x^2\right)^{1/3}\right]}{4\times 2^{1/6}}$$

Result (type 6, 154 leaves):

$$-\left((12+12 i) (1+x) \,\text{AppellF1}\left[\frac{1}{3},\frac{1}{6},\frac{1}{6},\frac{4}{3},\frac{1-i}{1+x},\frac{1+i}{1+x}\right]\right) \Big/ \left(\left(1+x^2\right)^{1/6}\right.$$
$$\left.\left((4+4 i) (1+x) \,\text{AppellF1}\left[\frac{1}{3},\frac{1}{6},\frac{1}{6},\frac{4}{3},\frac{1-i}{1+x},\frac{1+i}{1+x}\right]+i \,\text{AppellF1}\left[\frac{4}{3},\frac{1}{6},\frac{7}{6},\frac{7}{3},\frac{1-i}{1+x},\frac{1+i}{1+x}\right]+\text{AppellF1}\left[\frac{4}{3},\frac{7}{6},\frac{1}{6},\frac{7}{3},\frac{1-i}{1+x},\frac{1+i}{1+x}\right]\right)\right)$$

Problem 724: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d+e x)^m}{a+c x^2} \, dx$$

Optimal (type 5, 167 leaves, 4 steps):

$$\frac{(d+e x)^{1+m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right]}{2\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+m)}-\frac{(d+e x)^{1+m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right]}{2\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(1+m)}$$

Result (type 5, 210 leaves):

$$-\frac{1}{2\sqrt{a}\sqrt{c}\,m}\,i\,(d+e x)^m\left(\left(\frac{\sqrt{c}\,(d+e x)}{e\left(-i\sqrt{a}+\sqrt{c}\,x\right)}\right)^{-m}\text{Hypergeometric2F1}\left[-m,-m,1-m,\frac{\sqrt{c}\,d+i\sqrt{a}\,e}{i\sqrt{a}\,e-\sqrt{c}\,e x}\right]-\left(\frac{\sqrt{c}\,(d+e x)}{e\left(i\sqrt{a}+\sqrt{c}\,x\right)}\right)^{-m}\text{Hypergeometric2F1}\left[-m,-m,1-m,-\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{i\sqrt{a}\,e+\sqrt{c}\,e x}\right]\right)$$

Problem 725: Unable to integrate problem.

$$\int\frac{(d+e x)^m}{\left(a+c x^2\right)^2}\,dx$$

Optimal (type 5, 304 leaves, 5 steps):

$$\frac{(a e+c d x)(d+e x)^{1+m}}{2a\left(c d^2+a e^2\right)\left(a+c x^2\right)}-\frac{\left(c d^2+a e^2(1-m)+\sqrt{-a}\sqrt{c}\,d e m\right)(d+e x)^{1+m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right]}{4(-a)^{3/2}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(c d^2+a e^2\right)(1+m)}+\frac{\left(c d^2+a e^2(1-m)-\sqrt{-a}\sqrt{c}\,d e m\right)(d+e x)^{1+m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right]}{4(-a)^{3/2}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)\left(c d^2+a e^2\right)(1+m)}$$

Result (type 8, 19 leaves):

$$\int\frac{(d+e x)^m}{\left(a+c x^2\right)^2}\,dx$$

Problem 726: Unable to integrate problem.

$$\int\frac{(d+e x)^m}{\left(a+c x^2\right)^3}\,dx$$

Optimal (type 5, 472 leaves, 6 steps):

$$\frac{(a e+c d x)(d+e x)^{1+m}}{4 a\left(c d^2+a e^2\right)\left(a+c x^2\right)^2}+\frac{(d+e x)^{1+m}\left(a e\left(a e^2(3-m)+c d^2(1+m)\right)+c d\left(3 c d^2+a e^2(5-2 m)\right) x\right)}{8 a^2\left(c d^2+a e^2\right)^2\left(a+c x^2\right)}+$$
$$\left(\left(a \sqrt{c}\, d e\left(3 c d^2+a e^2(5-2 m)\right) m-\sqrt{-a}\left(3 c^2 d^4+a c d^2 e^2\left(6-2 m-m^2\right)+a^2 e^4\left(3-4 m+m^2\right)\right)\right)\right.$$
$$\left.(d+e x)^{1+m} \text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{\sqrt{c}(d+e x)}{\sqrt{c}\, d-\sqrt{-a}\, e}\right]\right) \Big/ \left(16 a^3\left(\sqrt{c}\, d-\sqrt{-a}\, e\right)\left(c d^2+a e^2\right)^2(1+m)\right)+$$
$$\left(\left(a \sqrt{c}\, d e\left(3 c d^2+a e^2(5-2 m)\right) m+\sqrt{-a}\left(3 c^2 d^4+a c d^2 e^2\left(6-2 m-m^2\right)+a^2 e^4\left(3-4 m+m^2\right)\right)\right)(d+e x)^{1+m}\right.$$
$$\left.\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{\sqrt{c}(d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}\right]\right) \Big/ \left(16 a^3\left(\sqrt{c}\, d+\sqrt{-a}\, e\right)\left(c d^2+a e^2\right)^2(1+m)\right)$$

Result (type 8, 19 leaves):

$$\int \frac{(d+e x)^m}{\left(a+c x^2\right)^3}\, dx$$

- **Problem 727: Unable to integrate problem.**

$$\int (d+e x)^m\left(a+c x^2\right)^{3/2} dx$$

Optimal (type 6, 154 leaves, 2 steps):

$$\frac{(d+e x)^{1+m}\left(a+c x^2\right)^{3/2} \text{AppellF1}\left[1+m,-\frac{3}{2},-\frac{3}{2},2+m,\frac{d+e x}{d-\frac{\sqrt{-a}\, e}{\sqrt{c}}},\frac{d+e x}{d+\frac{\sqrt{-a}\, e}{\sqrt{c}}}\right]}{e(1+m)\left(1-\frac{d+e x}{d-\frac{\sqrt{-a}\, e}{\sqrt{c}}}\right)^{3/2}\left(1-\frac{d+e x}{d+\frac{\sqrt{-a}\, e}{\sqrt{c}}}\right)^{3/2}}$$

Result (type 8, 21 leaves):

$$\int (d+e x)^m\left(a+c x^2\right)^{3/2} dx$$

- **Problem 743: Attempted integration timed out after 120 seconds.**

$$\int (d+e x)^{-4-2 p}\left(a+c x^2\right)^p dx$$

Optimal (type 5, 347 leaves, 3 steps):

$$-\frac{e\,(d+e\,x)^{-3-2\,p}\,\left(a+c\,x^2\right)^{1+p}}{\left(c\,d^2+a\,e^2\right)\,(3+2\,p)}-\frac{c\,d\,e\,(2+p)\,(d+e\,x)^{-2\,(1+p)}\,\left(a+c\,x^2\right)^{1+p}}{\left(c\,d^2+a\,e^2\right)^2\,(1+p)\,(3+2\,p)}+\left(c\,\left(a\,e^2-c\,d^2\,(3+2\,p)\right)\,\left(\sqrt{-a}-\sqrt{c}\,x\right)\,\left(-\frac{\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)\,\left(\sqrt{-a}+\sqrt{c}\,x\right)}{\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\,\left(\sqrt{-a}-\sqrt{c}\,x\right)}\right)^{-p}\,(d+e\,x)^{-1-2\,p}\,\left(a+c\,x^2\right)^p\,\text{Hypergeometric2F1}\left[-1-2\,p,\,-p,\,-2\,p,\,\frac{2\,\sqrt{-a}\,\sqrt{c}\,(d+e\,x)}{\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\,\left(\sqrt{-a}-\sqrt{c}\,x\right)}\right]\right)\Big/\left(\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)\,\left(c\,d^2+a\,e^2\right)^2\,(1+2\,p)\,(3+2\,p)\right)$$

Result (type 1, 1 leaves):

???

■ Problem 744: Attempted integration timed out after 120 seconds.

$$\int (d+e\,x)^{-5-2\,p}\,\left(a+c\,x^2\right)^p\,dx$$

Optimal (type 5, 436 leaves, 4 steps):

$$-\frac{c\,d\,e\,(3+p)\,(d+e\,x)^{-3-2\,p}\,\left(a+c\,x^2\right)^{1+p}}{\left(c\,d^2+a\,e^2\right)^2\,(2+p)\,(3+2\,p)}+\frac{c\,e\,\left(a\,e^2\,(3+2\,p)-c\,d^2\,\left(9+8\,p+2\,p^2\right)\right)\,(d+e\,x)^{-2\,(1+p)}\,\left(a+c\,x^2\right)^{1+p}}{2\,\left(c\,d^2+a\,e^2\right)^3\,(1+p)\,(2+p)\,(3+2\,p)}-\frac{e\,(d+e\,x)^{-2\,(2+p)}\,\left(a+c\,x^2\right)^{1+p}}{2\,\left(c\,d^2+a\,e^2\right)\,(2+p)}+\left(c^2\,d\,\left(3\,a\,e^2-c\,d^2\,(3+2\,p)\right)\,\left(\sqrt{-a}-\sqrt{c}\,x\right)\,\left(-\frac{\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)\,\left(\sqrt{-a}+\sqrt{c}\,x\right)}{\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\,\left(\sqrt{-a}-\sqrt{c}\,x\right)}\right)^{-p}\,(d+e\,x)^{-1-2\,p}\,\left(a+c\,x^2\right)^p\,\text{Hypergeometric2F1}\left[-1-2\,p,\,-p,\,-2\,p,\,\frac{2\,\sqrt{-a}\,\sqrt{c}\,(d+e\,x)}{\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\,\left(\sqrt{-a}-\sqrt{c}\,x\right)}\right]\right)\Big/\left(\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)\,\left(c\,d^2+a\,e^2\right)^3\,(1+2\,p)\,(3+2\,p)\right)$$

Result (type 1, 1 leaves):

???

■ Problem 745: Attempted integration timed out after 120 seconds.

$$\int (d+e\,x)^{-6-2\,p}\,\left(a+c\,x^2\right)^p\,dx$$

Optimal (type 5, 559 leaves, 5 steps):

$$-\frac{e\,(d+e\,x)^{-5-2\,p}\,(a+c\,x^2)^{1+p}}{(c\,d^2+a\,e^2)\,(5+2\,p)}+\frac{c\,e\,(3\,a\,e^2\,(2+p)-c\,d^2\,(18+11\,p+2\,p^2))\,(d+e\,x)^{-3-2\,p}\,(a+c\,x^2)^{1+p}}{(c\,d^2+a\,e^2)^3\,(2+p)\,(3+2\,p)\,(5+2\,p)}+$$

$$\frac{c^2\,d\,e\,(3+p)\,(a\,e^2\,(8+5\,p)-c\,d^2\,(8+7\,p+2\,p^2))\,(d+e\,x)^{-2\,(1+p)}\,(a+c\,x^2)^{1+p}}{(c\,d^2+a\,e^2)^4\,(1+p)\,(2+p)\,(3+2\,p)\,(5+2\,p)}-\frac{c\,d\,e\,(4+p)\,(d+e\,x)^{-2\,(2+p)}\,(a+c\,x^2)^{1+p}}{(c\,d^2+a\,e^2)^2\,(2+p)\,(5+2\,p)}-$$

$$\left(c^2\,(3\,a^2\,e^4-6\,a\,c\,d^2\,e^2\,(5+2\,p)+c^2\,d^4\,(15+16\,p+4\,p^2))\,(\sqrt{-a}-\sqrt{c}\,x)\left(-\frac{(\sqrt{c}\,d+\sqrt{-a}\,e)\,(\sqrt{-a}+\sqrt{c}\,x)}{(\sqrt{c}\,d-\sqrt{-a}\,e)\,(\sqrt{-a}-\sqrt{c}\,x)}\right)^{-p}(d+e\,x)^{-1-2\,p}\,(a+c\,x^2)^p\right.$$

$$\left.\text{Hypergeometric2F1}\left[-1-2\,p,\,-p,\,-2\,p,\,\frac{2\,\sqrt{-a}\,\sqrt{c}\,(d+e\,x)}{(\sqrt{c}\,d-\sqrt{-a}\,e)\,(\sqrt{-a}-\sqrt{c}\,x)}\right]\right)\Big/\left((\sqrt{c}\,d+\sqrt{-a}\,e)\,(c\,d^2+a\,e^2)^4\,(1+2\,p)\,(3+2\,p)\,(5+2\,p)\right)$$

Result (type 1, 1 leaves):

???

Problem 821: Result more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{1-x^2}}{1+x}\,dx$$

Optimal (type 3, 14 leaves, 2 steps):

$$\sqrt{1-x^2}+\text{ArcSin}[x]$$

Result (type 3, 46 leaves):

$$\sqrt{1-x^2}\left(1-\frac{2\,\text{Log}\left[\sqrt{-1+x}+\sqrt{1+x}\right]}{\sqrt{-1+x}\,\sqrt{1+x}}\right)$$

Problem 823: Result more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{1-x^2}}{1-x}\,dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$-\sqrt{1-x^2}+\text{ArcSin}[x]$$

Result (type 3, 54 leaves):

$$\sqrt{1-x^2}\left(-1+\frac{2\,\text{Log}\left[\sqrt{-1-x}+\sqrt{1-x}\right]}{\sqrt{-1-x}\,\sqrt{1-x}}\right)$$

Problem 930: Result unnecessarily involves higher level functions.

$$\int \sqrt{2+e x}\,\left(12-3 e^2 x^2\right)^{1/4}\,dx$$

Optimal (type 3, 309 leaves, 14 steps):

$$\frac{3\times 3^{1/4}\,(2-e x)^{1/4}\,(2+e x)^{3/4}}{2 e}-\frac{3^{1/4}\,(2-e x)^{5/4}\,(2+e x)^{3/4}}{2 e}+\frac{3\times 3^{1/4}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,(2-e x)^{1/4}}{(2+e x)^{1/4}}\right]}{\sqrt{2}\,e}-\frac{3\times 3^{1/4}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,(2-e x)^{1/4}}{(2+e x)^{1/4}}\right]}{\sqrt{2}\,e}+\frac{3\times 3^{1/4}\,\text{Log}\left[\frac{\sqrt{6-3 e x}-\sqrt{6}\,(2-e x)^{1/4}\,(2+e x)^{1/4}+\sqrt{3}\,\sqrt{2+e x}}{\sqrt{2+e x}}\right]}{2\sqrt{2}\,e}-\frac{3\times 3^{1/4}\,\text{Log}\left[\frac{\sqrt{6-3 e x}+\sqrt{6}\,(2-e x)^{1/4}\,(2+e x)^{1/4}+\sqrt{3}\,\sqrt{2+e x}}{\sqrt{2+e x}}\right]}{2\sqrt{2}\,e}$$

Result (type 5, 86 leaves):

$$\frac{\sqrt{2+e x}\,\left(12-3 e^2 x^2\right)^{1/4}\left(-2-e x+e^2 x^2-\sqrt{2}\,(2-e x)^{3/4}\,\text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{1}{4}(2+e x)\right]\right)}{2 e\,(-2+e x)}$$

Problem 931: Result unnecessarily involves higher level functions.

$$\int \frac{\left(12-3 e^2 x^2\right)^{1/4}}{\sqrt{2+e x}}\,dx$$

Optimal (type 3, 269 leaves, 13 steps):

$$\frac{3^{1/4}\,(2-e x)^{1/4}\,(2+e x)^{3/4}}{e}+\frac{\sqrt{2}\,3^{1/4}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,(2-e x)^{1/4}}{(2+e x)^{1/4}}\right]}{e}-\frac{\sqrt{2}\,3^{1/4}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,(2-e x)^{1/4}}{(2+e x)^{1/4}}\right]}{e}+\frac{3^{1/4}\,\text{Log}\left[\frac{\sqrt{6-3 e x}-\sqrt{6}\,(2-e x)^{1/4}\,(2+e x)^{1/4}+\sqrt{3}\,\sqrt{2+e x}}{\sqrt{2+e x}}\right]}{\sqrt{2}\,e}-\frac{3^{1/4}\,\text{Log}\left[\frac{\sqrt{6-3 e x}+\sqrt{6}\,(2-e x)^{1/4}\,(2+e x)^{1/4}+\sqrt{3}\,\sqrt{2+e x}}{\sqrt{2+e x}}\right]}{\sqrt{2}\,e}$$

Result (type 5, 81 leaves):

$$\frac{\sqrt{2+e x}\,\left(4-e^2 x^2\right)^{1/4}\left(-6+3 e x-\sqrt{2}\,(2-e x)^{3/4}\,\text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{1}{4}(2+e x)\right]\right)}{3^{3/4}\,e\,(-2+e x)}$$

Problem 932: Result unnecessarily involves higher level functions.

$$\int \frac{\left(12-3 e^2 x^2\right)^{1/4}}{(2+e x)^{3/2}}\,dx$$

Optimal (type 3, 270 leaves, 13 steps):

$$-\frac{4\times 3^{1/4}(2-e\,x)^{1/4}}{e\,(2+e\,x)^{1/4}}-\frac{\sqrt{2}\;3^{1/4}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,(2-e\,x)^{1/4}}{(2+e\,x)^{1/4}}\right]}{e}+\frac{\sqrt{2}\;3^{1/4}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,(2-e\,x)^{1/4}}{(2+e\,x)^{1/4}}\right]}{e}-$$
$$\frac{3^{1/4}\,\text{Log}\left[\frac{\sqrt{6-3\,e\,x}-\sqrt{6}\,(2-e\,x)^{1/4}\,(2+e\,x)^{1/4}+\sqrt{3}\,\sqrt{2+e\,x}}{\sqrt{2+e\,x}}\right]}{\sqrt{2}\,e}+\frac{3^{1/4}\,\text{Log}\left[\frac{\sqrt{6-3\,e\,x}+\sqrt{6}\,(2-e\,x)^{1/4}\,(2+e\,x)^{1/4}+\sqrt{3}\,\sqrt{2+e\,x}}{\sqrt{2+e\,x}}\right]}{\sqrt{2}\,e}$$

Result (type 5, 85 leaves):

$$\frac{\left(4-e^2\,x^2\right)^{1/4}\left(24-12\,e\,x+\sqrt{2}\,(2-e\,x)^{3/4}\,(2+e\,x)\,\text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{1}{4}\,(2+e\,x)\right]\right)}{3^{3/4}\,e\,(-2+e\,x)\,\sqrt{2+e\,x}}$$

■ Problem 937: Result unnecessarily involves higher level functions.

$$\int\frac{(2+e\,x)^{5/2}}{\left(12-3\,e^2\,x^2\right)^{1/4}}\,dx$$

Optimal (type 3, 340 leaves, 15 steps):

$$-\frac{5\times 3^{3/4}\,(2-e\,x)^{3/4}\,(2+e\,x)^{1/4}}{2\,e}-\frac{3^{3/4}\,(2-e\,x)^{3/4}\,(2+e\,x)^{5/4}}{2\,e}-\frac{(2-e\,x)^{3/4}\,(2+e\,x)^{9/4}}{3\times 3^{1/4}\,e}+\frac{5\times 3^{3/4}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,(2-e\,x)^{1/4}}{(2+e\,x)^{1/4}}\right]}{\sqrt{2}\,e}-$$
$$\frac{5\times 3^{3/4}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,(2-e\,x)^{1/4}}{(2+e\,x)^{1/4}}\right]}{\sqrt{2}\,e}-\frac{5\times 3^{3/4}\,\text{Log}\left[\frac{\sqrt{6-3\,e\,x}-\sqrt{6}\,(2-e\,x)^{1/4}\,(2+e\,x)^{1/4}+\sqrt{3}\,\sqrt{2+e\,x}}{\sqrt{2+e\,x}}\right]}{2\,\sqrt{2}\,e}+\frac{5\times 3^{3/4}\,\text{Log}\left[\frac{\sqrt{6-3\,e\,x}+\sqrt{6}\,(2-e\,x)^{1/4}\,(2+e\,x)^{1/4}+\sqrt{3}\,\sqrt{2+e\,x}}{\sqrt{2+e\,x}}\right]}{2\,\sqrt{2}\,e}$$

Result (type 5, 88 leaves):

$$\frac{\sqrt{2+e\,x}\,\left(-142+37\,e\,x+13\,e^2\,x^2+2\,e^3\,x^3+90\,\sqrt{2}\,(2-e\,x)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{1}{4}\,(2+e\,x)\right]\right)}{6\,e\,\left(12-3\,e^2\,x^2\right)^{1/4}}$$

■ Problem 938: Result unnecessarily involves higher level functions.

$$\int\frac{(2+e\,x)^{3/2}}{\left(12-3\,e^2\,x^2\right)^{1/4}}\,dx$$

Optimal (type 3, 309 leaves, 14 steps):

$$-\frac{5\,(2-e\,x)^{3/4}\,(2+e\,x)^{1/4}}{2\times 3^{1/4}\,e}-\frac{(2-e\,x)^{3/4}\,(2+e\,x)^{5/4}}{2\times 3^{1/4}\,e}+\frac{5\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,(2-e\,x)^{1/4}}{(2+e\,x)^{1/4}}\right]}{\sqrt{2}\;3^{1/4}\,e}-$$
$$\frac{5\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,(2-e\,x)^{1/4}}{(2+e\,x)^{1/4}}\right]}{\sqrt{2}\;3^{1/4}\,e}-\frac{5\,\text{Log}\left[\frac{\sqrt{6-3\,e\,x}-\sqrt{6}\,(2-e\,x)^{1/4}\,(2+e\,x)^{1/4}+\sqrt{3}\,\sqrt{2+e\,x}}{\sqrt{2+e\,x}}\right]}{2\,\sqrt{2}\;3^{1/4}\,e}+\frac{5\,\text{Log}\left[\frac{\sqrt{6-3\,e\,x}+\sqrt{6}\,(2-e\,x)^{1/4}\,(2+e\,x)^{1/4}+\sqrt{3}\,\sqrt{2+e\,x}}{\sqrt{2+e\,x}}\right]}{2\,\sqrt{2}\;3^{1/4}\,e}$$

Result (type 5, 79 leaves):

$$\frac{\sqrt{2+e\,x}\,\left(-14+5\,e\,x+e^2\,x^2+10\,\sqrt{2}\,(2-e\,x)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{1}{4}\,(2+e\,x)\right]\right)}{2\,e\,\left(12-3\,e^2\,x^2\right)^{1/4}}$$

Problem 939: Result unnecessarily involves higher level functions.

$$\int\frac{\sqrt{2+e\,x}}{\left(12-3\,e^2\,x^2\right)^{1/4}}\,dx$$

Optimal (type 3, 270 leaves, 13 steps):

$$-\frac{(2-e\,x)^{3/4}\,(2+e\,x)^{1/4}}{3^{1/4}\,e}+\frac{\sqrt{2}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,(2-e\,x)^{1/4}}{(2+e\,x)^{1/4}}\right]}{3^{1/4}\,e}-\frac{\sqrt{2}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,(2-e\,x)^{1/4}}{(2+e\,x)^{1/4}}\right]}{3^{1/4}\,e}-\frac{\text{Log}\left[\frac{\sqrt{6-3\,e\,x}-\sqrt{6}\,(2-e\,x)^{1/4}\,(2+e\,x)^{1/4}+\sqrt{3}\,\sqrt{2+e\,x}}{\sqrt{2+e\,x}}\right]}{\sqrt{2}\,3^{1/4}\,e}+\frac{\text{Log}\left[\frac{\sqrt{6-3\,e\,x}+\sqrt{6}\,(2-e\,x)^{1/4}\,(2+e\,x)^{1/4}+\sqrt{3}\,\sqrt{2+e\,x}}{\sqrt{2+e\,x}}\right]}{\sqrt{2}\,3^{1/4}\,e}$$

Result (type 5, 68 leaves):

$$\frac{\sqrt{2+e\,x}\,\left(-2+e\,x+2\,\sqrt{2}\,(2-e\,x)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{1}{4}\,(2+e\,x)\right]\right)}{e\,\left(12-3\,e^2\,x^2\right)^{1/4}}$$

Problem 940: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\sqrt{2+e\,x}\,\left(12-3\,e^2\,x^2\right)^{1/4}}\,dx$$

Optimal (type 3, 241 leaves, 12 steps):

$$\frac{\sqrt{2}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,(2-e\,x)^{1/4}}{(2+e\,x)^{1/4}}\right]}{3^{1/4}\,e}-\frac{\sqrt{2}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,(2-e\,x)^{1/4}}{(2+e\,x)^{1/4}}\right]}{3^{1/4}\,e}-\frac{\text{Log}\left[\frac{\sqrt{6-3\,e\,x}-\sqrt{6}\,(2-e\,x)^{1/4}\,(2+e\,x)^{1/4}+\sqrt{3}\,\sqrt{2+e\,x}}{\sqrt{2+e\,x}}\right]}{\sqrt{2}\,3^{1/4}\,e}+\frac{\text{Log}\left[\frac{\sqrt{6-3\,e\,x}+\sqrt{6}\,(2-e\,x)^{1/4}\,(2+e\,x)^{1/4}+\sqrt{3}\,\sqrt{2+e\,x}}{\sqrt{2+e\,x}}\right]}{\sqrt{2}\,3^{1/4}\,e}$$

Result (type 5, 58 leaves):

$$\frac{2\,(2-e\,x)^{1/4}\,\sqrt{4+2\,e\,x}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{1}{4}\,(2+e\,x)\right]}{e\,\left(12-3\,e^2\,x^2\right)^{1/4}}$$

Problem 949: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b x)^m}{\left(a^2-b^2 x^2\right)^2} \, dx$$

Optimal (type 5, 44 leaves, 2 steps):

$$-\frac{(a+b x)^{-1+m} \text{Hypergeometric2F1}\left[2,-1+m,m,\frac{a+b x}{2 a}\right]}{4 a^2 b (1-m)}$$

Result (type 5, 102 leaves):

$$\frac{1}{16 a^4 b}(a+b x)^m \left(4 a \left(\frac{1}{m}+\frac{a}{(-1+m)(a+b x)}\right)+\frac{2 (a+b x) \text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{a+b x}{2 a}\right]}{1+m}+\frac{(a+b x) \text{Hypergeometric2F1}\left[2,1+m,2+m,\frac{a+b x}{2 a}\right]}{1+m}\right)$$

Problem 951: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (d+e x)^m \left(d^2-e^2 x^2\right)^{7/2} \, dx$$

Optimal (type 5, 59 leaves, 3 steps):

$$\frac{(d+e x)^m \left(d^2-e^2 x^2\right)^{9/2} \text{Hypergeometric2F1}\left[1,9+m,\frac{11}{2}+m,\frac{d+e x}{2 d}\right]}{d e (9+2 m)}$$

Result (type 6, 531 leaves):

$$\frac{1}{21 e} 2 d \sqrt{d-e x} (d+e x)^m \left(-\left(84 d^4 e^3 x^3 \sqrt{d+e x} \text{AppellF1}\left[3,-\frac{1}{2},-\frac{1}{2}-m,4,\frac{e x}{d},-\frac{e x}{d}\right]\right)\Big/\left(8 d \text{AppellF1}\left[3,-\frac{1}{2},-\frac{1}{2}-m,4,\frac{e x}{d},-\frac{e x}{d}\right]+e x \left((1+2 m) \text{AppellF1}\left[4,-\frac{1}{2},\frac{1}{2}-m,5,\frac{e x}{d},-\frac{e x}{d}\right]-\text{AppellF1}\left[4,\frac{1}{2},-\frac{1}{2}-m,5,\frac{e x}{d},-\frac{e x}{d}\right]\right)\right)+\left(378 d^2 e^5 x^5 \sqrt{d+e x} \text{AppellF1}\left[5,-\frac{1}{2},-\frac{1}{2}-m,6,\frac{e x}{d},-\frac{e x}{d}\right]\right)\Big/\left(60 d \text{AppellF1}\left[5,-\frac{1}{2},-\frac{1}{2}-m,6,\frac{e x}{d},-\frac{e x}{d}\right]+5 e x \left((1+2 m) \text{AppellF1}\left[6,-\frac{1}{2},\frac{1}{2}-m,7,\frac{e x}{d},-\frac{e x}{d}\right]-\text{AppellF1}\left[6,\frac{1}{2},-\frac{1}{2}-m,7,\frac{e x}{d},-\frac{e x}{d}\right]\right)\right)-\left(24 e^7 x^7 \sqrt{d+e x} \text{AppellF1}\left[7,-\frac{1}{2},-\frac{1}{2}-m,8,\frac{e x}{d},-\frac{e x}{d}\right]\right)\Big/\left(16 d \text{AppellF1}\left[7,-\frac{1}{2},-\frac{1}{2}-m,8,\frac{e x}{d},-\frac{e x}{d}\right]+e x \left((1+2 m) \text{AppellF1}\left[8,-\frac{1}{2},\frac{1}{2}-m,9,\frac{e x}{d},-\frac{e x}{d}\right]-\text{AppellF1}\left[8,\frac{1}{2},-\frac{1}{2}-m,9,\frac{e x}{d},-\frac{e x}{d}\right]\right)\right)-7\times 2^{\frac{1}{2}+m} d^5 \sqrt{d-e x} \left(1+\frac{e x}{d}\right)^{-\frac{1}{2}-m} \sqrt{d^2-e^2 x^2} \text{Hypergeometric2F1}\left[\frac{3}{2},-\frac{1}{2}-m,\frac{5}{2},\frac{d-e x}{2 d}\right]\right)$$

- **Problem 952: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (d+e x)^m \left(d^2-e^2 x^2\right)^{5/2} \, dx$$

Optimal (type 5, 83 leaves, 3 steps):

$$-\frac{2^{\frac{7}{2}+m} (d+e x)^m \left(1+\frac{e x}{d}\right)^{-\frac{7}{2}-m} \left(d^2-e^2 x^2\right)^{7/2} \text{Hypergeometric2F1}\left[\frac{7}{2},-\frac{5}{2}-m,\frac{9}{2},\frac{d-e x}{2 d}\right]}{7 d e}$$

Result (type 6, 389 leaves):

$$\frac{1}{3 e} 2 d \sqrt{d-e x} (d+e x)^m \left(-\left(8 d^2 e^3 x^3 \sqrt{d+e x} \text{AppellF1}\left[3,-\frac{1}{2},-\frac{1}{2}-m,4,\frac{e x}{d},-\frac{e x}{d}\right]\right) \Big/ \left(8 d \text{AppellF1}\left[3,-\frac{1}{2},-\frac{1}{2}-m,4,\frac{e x}{d},-\frac{e x}{d}\right]+ e x \left((1+2 m) \text{AppellF1}\left[4,-\frac{1}{2},\frac{1}{2}-m,5,\frac{e x}{d},-\frac{e x}{d}\right]-\text{AppellF1}\left[4,\frac{1}{2},-\frac{1}{2}-m,5,\frac{e x}{d},-\frac{e x}{d}\right]\right)\right)+ \left(18 e^5 x^5 \sqrt{d+e x} \text{AppellF1}\left[5,-\frac{1}{2},-\frac{1}{2}-m,6,\frac{e x}{d},-\frac{e x}{d}\right]\right) \Big/ \left(60 d \text{AppellF1}\left[5,-\frac{1}{2},-\frac{1}{2}-m,6,\frac{e x}{d},-\frac{e x}{d}\right]+ 5 e x \left((1+2 m) \text{AppellF1}\left[6,-\frac{1}{2},\frac{1}{2}-m,7,\frac{e x}{d},-\frac{e x}{d}\right]-\text{AppellF1}\left[6,\frac{1}{2},-\frac{1}{2}-m,7,\frac{e x}{d},-\frac{e x}{d}\right]\right)\right)- 2^{\frac{1}{2}+m} d^3 \sqrt{d-e x} \left(1+\frac{e x}{d}\right)^{-\frac{1}{2}-m} \sqrt{d^2-e^2 x^2} \text{Hypergeometric2F1}\left[\frac{3}{2},-\frac{1}{2}-m,\frac{5}{2},\frac{d-e x}{2 d}\right]\right)$$

- **Problem 953: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (d+e x)^m \left(d^2-e^2 x^2\right)^{3/2} \, dx$$

Optimal (type 5, 59 leaves, 3 steps):

$$\frac{(d+e x)^m \left(d^2-e^2 x^2\right)^{5/2} \text{Hypergeometric2F1}\left[1,5+m,\frac{7}{2}+m,\frac{d+e x}{2 d}\right]}{d e (5+2 m)}$$

Result (type 6, 244 leaves):

$$\frac{1}{3 e} 2 d \sqrt{d-e x} (d+e x)^m \left(-\left(4 e^3 x^3 \sqrt{d+e x} \text{AppellF1}\left[3,-\frac{1}{2},-\frac{1}{2}-m,4,\frac{e x}{d},-\frac{e x}{d}\right]\right) \Big/ \left(8 d \text{AppellF1}\left[3,-\frac{1}{2},-\frac{1}{2}-m,4,\frac{e x}{d},-\frac{e x}{d}\right]+ e x \left((1+2 m) \text{AppellF1}\left[4,-\frac{1}{2},\frac{1}{2}-m,5,\frac{e x}{d},-\frac{e x}{d}\right]-\text{AppellF1}\left[4,\frac{1}{2},-\frac{1}{2}-m,5,\frac{e x}{d},-\frac{e x}{d}\right]\right)\right)- 2^{\frac{1}{2}+m} d \sqrt{d-e x} \left(1+\frac{e x}{d}\right)^{-\frac{1}{2}-m} \sqrt{d^2-e^2 x^2} \text{Hypergeometric2F1}\left[\frac{3}{2},-\frac{1}{2}-m,\frac{5}{2},\frac{d-e x}{2 d}\right]\right)$$

Problem 960: Result more than twice size of optimal antiderivative.

$$\int (d+e x)^3 \left(1-\frac{e^2 x^2}{d^2}\right)^p dx$$

Optimal (type 5, 57 leaves, 2 steps):

$$-\frac{2^{3+p} d^4 \left(\frac{d-e x}{d}\right)^{1+p} \text{Hypergeometric2F1}\left[-3-p, 1+p, 2+p, \frac{d-e x}{2 d}\right]}{e (1+p)}$$

Result (type 5, 240 leaves):

$$\frac{1}{2 e (1+p) (2+p)} \left(7 d^4+3 d^4 p-7 d^4 \left(1-\frac{e^2 x^2}{d^2}\right)^p-3 d^4 p \left(1-\frac{e^2 x^2}{d^2}\right)^p+6 d^2 e^2 x^2 \left(1-\frac{e^2 x^2}{d^2}\right)^p+2 d^2 e^2 p x^2 \left(1-\frac{e^2 x^2}{d^2}\right)^p+e^4 x^4 \left(1-\frac{e^2 x^2}{d^2}\right)^p+e^4 p x^4 \left(1-\frac{e^2 x^2}{d^2}\right)^p+ 2 d^3 e \left(2+3 p+p^2\right) x \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, \frac{e^2 x^2}{d^2}\right]+2 d e^3 \left(2+3 p+p^2\right) x^3 \text{Hypergeometric2F1}\left[\frac{3}{2}, -p, \frac{5}{2}, \frac{e^2 x^2}{d^2}\right]\right)$$

Problem 967: Result more than twice size of optimal antiderivative.

$$\int (a+b x)^3 \left(a^2-b^2 x^2\right)^p dx$$

Optimal (type 5, 60 leaves, 2 steps):

$$\frac{(a+b x)^3 \left(a^2-b^2 x^2\right)^{1+p} \text{Hypergeometric2F1}\left[1, 5+2 p, 5+p, \frac{a+b x}{2 a}\right]}{2 a b (4+p)}$$

Result (type 5, 271 leaves):

$$\frac{1}{2 b (1+p) (2+p)} \left(a^2-b^2 x^2\right)^p \left(1-\frac{b^2 x^2}{a^2}\right)^{-p} \left(7 a^4+3 a^4 p-7 a^4 \left(1-\frac{b^2 x^2}{a^2}\right)^p-3 a^4 p \left(1-\frac{b^2 x^2}{a^2}\right)^p+6 a^2 b^2 x^2 \left(1-\frac{b^2 x^2}{a^2}\right)^p+2 a^2 b^2 p x^2 \left(1-\frac{b^2 x^2}{a^2}\right)^p+b^4 x^4 \left(1-\frac{b^2 x^2}{a^2}\right)^p+b^4 p x^4 \left(1-\frac{b^2 x^2}{a^2}\right)^p+ 2 a^3 b \left(2+3 p+p^2\right) x \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, \frac{b^2 x^2}{a^2}\right]+2 a b^3 \left(2+3 p+p^2\right) x^3 \text{Hypergeometric2F1}\left[\frac{3}{2}, -p, \frac{5}{2}, \frac{b^2 x^2}{a^2}\right]\right)$$

Problem 968: Result more than twice size of optimal antiderivative.

$$\int (a+b x)^2 \left(a^2-b^2 x^2\right)^p dx$$

Optimal (type 5, 60 leaves, 2 steps):

$$\frac{(a+b x)^2 \left(a^2-b^2 x^2\right)^{1+p} \text{Hypergeometric2F1}\left[1, 2 (2+p), 4+p, \frac{a+b x}{2 a}\right]}{2 a b (3+p)}$$

Result (type 5, 150 leaves):

$$\frac{1}{3\,b\,(1+p)}\left(a^2-b^2x^2\right)^p\left(1-\frac{b^2x^2}{a^2}\right)^{-p}\left(3\,a\,b^2x^2\left(1-\frac{b^2x^2}{a^2}\right)^p-3\,a^3\left(-1+\left(1-\frac{b^2x^2}{a^2}\right)^p\right)+3\,a^2\,b\,(1+p)\,x\,\text{Hypergeometric2F1}\left[\frac{1}{2},-p,\frac{3}{2},\frac{b^2x^2}{a^2}\right]+b^3\,(1+p)\,x^3\,\text{Hypergeometric2F1}\left[\frac{3}{2},-p,\frac{5}{2},\frac{b^2x^2}{a^2}\right]\right)$$

- **Problem 973: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+b\,x)^{3/2}\left(a^2-b^2x^2\right)^p\,dx$$

Optimal (type 5, 85 leaves, 3 steps):

$$-\frac{2^{\frac{3}{2}+p}\sqrt{a+b\,x}\,\left(1+\frac{b\,x}{a}\right)^{-\frac{3}{2}-p}\left(a^2-b^2x^2\right)^{1+p}\,\text{Hypergeometric2F1}\left[-\frac{3}{2}-p,1+p,2+p,\frac{a-b\,x}{2\,a}\right]}{b\,(1+p)}$$

Result (type 6, 246 leaves):

$$\frac{1}{b}a\,(a-b\,x)^p\sqrt{a+b\,x}\left(\left(3\,b^2x^2\,(a+b\,x)^p\,\text{AppellF1}\left[2,-p,-\frac{1}{2}-p,3,\frac{b\,x}{a},-\frac{b\,x}{a}\right]\right)\Big/\left(6\,a\,\text{AppellF1}\left[2,-p,-\frac{1}{2}-p,3,\frac{b\,x}{a},-\frac{b\,x}{a}\right]+b\,x\left(-2\,p\,\text{AppellF1}\left[3,1-p,-\frac{1}{2}-p,4,\frac{b\,x}{a},-\frac{b\,x}{a}\right]+(1+2\,p)\,\text{AppellF1}\left[3,-p,\frac{1}{2}-p,4,\frac{b\,x}{a},-\frac{b\,x}{a}\right]\right)\right)-\frac{2^{\frac{1}{2}+p}\,(a-b\,x)^{1-p}\left(1+\frac{b\,x}{a}\right)^{-\frac{1}{2}-p}\left(a^2-b^2x^2\right)^p\,\text{Hypergeometric2F1}\left[-\frac{1}{2}-p,1+p,2+p,\frac{a-b\,x}{2\,a}\right]}{1+p}\right)$$

- **Problem 1109: Result more than twice size of optimal antiderivative.**

$$\int (b\,d+2\,c\,d\,x)^4\left(a+b\,x+c\,x^2\right)\,dx$$

Optimal (type 1, 45 leaves, 2 steps):

$$-\frac{\left(b^2-4\,a\,c\right)d^4\,(b+2\,c\,x)^5}{40\,c^2}+\frac{d^4\,(b+2\,c\,x)^7}{56\,c^2}$$

Result (type 1, 102 leaves):

$$d^4\left(a\,b^4x+\frac{1}{2}b^3\left(b^2+8\,a\,c\right)x^2+b^2c\left(3\,b^2+8\,a\,c\right)x^3+8\,b\,c^2\left(b^2+a\,c\right)x^4+\frac{8}{5}c^3\left(7\,b^2+2\,a\,c\right)x^5+8\,b\,c^4x^6+\frac{16\,c^5x^7}{7}\right)$$

- **Problem 1121: Result more than twice size of optimal antiderivative.**

$$\int (b\,d+2\,c\,d\,x)^5\left(a+b\,x+c\,x^2\right)^2\,dx$$

Optimal (type 1, 73 leaves, 2 steps):

$$\frac{\left(b^2-4ac\right)^2 d^5 (b+2cx)^6}{192c^3}-\frac{\left(b^2-4ac\right) d^5 (b+2cx)^8}{128c^3}+\frac{d^5 (b+2cx)^{10}}{320c^3}$$

Result (type 1, 168 leaves):

$$\frac{1}{15} d^5 x (b+cx) \left(5a^2 \left(3b^4+12b^3cx+28b^2c^2x^2+32bc^3x^3+16c^4x^4\right)+x^2 (b+cx)^2 \left(5b^4+30b^3cx+78b^2c^2x^2+96bc^3x^3+48c^4x^4\right)+5ax \left(3b^5+19b^4cx+56b^3c^2x^2+88b^2c^3x^3+72bc^4x^4+24c^5x^5\right)\right)$$

- **Problem 1122: Result more than twice size of optimal antiderivative.**

$$\int (bd+2cdx)^4 \left(a+bx+cx^2\right)^2 \, dx$$

Optimal (type 1, 73 leaves, 2 steps):

$$\frac{\left(b^2-4ac\right)^2 d^4 (b+2cx)^5}{160c^3}-\frac{\left(b^2-4ac\right) d^4 (b+2cx)^7}{112c^3}+\frac{d^4 (b+2cx)^9}{288c^3}$$

Result (type 1, 179 leaves):

$$d^4 \left(a^2b^4x+ab^3\left(b^2+4ac\right)x^2+\frac{1}{3}b^2\left(b^4+18ab^2c+24a^2c^2\right)x^3+\frac{1}{2}bc\left(5b^4+32ab^2c+16a^2c^2\right)x^4+\frac{1}{5}c^2\left(41b^4+112ab^2c+16a^2c^2\right)x^5+\frac{4}{3}bc^3\left(11b^2+12ac\right)x^6+\frac{8}{7}c^4\left(13b^2+4ac\right)x^7+8bc^5x^8+\frac{16c^6x^9}{9}\right)$$

- **Problem 1125: Result more than twice size of optimal antiderivative.**

$$\int (bd+2cdx) \left(a+bx+cx^2\right)^2 \, dx$$

Optimal (type 1, 17 leaves, 1 step):

$$\frac{1}{3} d \left(a+bx+cx^2\right)^3$$

Result (type 1, 37 leaves):

$$\frac{1}{3} dx (b+cx) \left(3a^2+3ax(b+cx)+x^2(b+cx)^2\right)$$

- **Problem 1137: Result more than twice size of optimal antiderivative.**

$$\int (bd+2cdx)^5 \left(a+bx+cx^2\right)^3 \, dx$$

Optimal (type 1, 101 leaves, 2 steps):

$$-\frac{\left(b^2-4ac\right)^3 d^5 (b+2cx)^6}{768c^4}+\frac{3\left(b^2-4ac\right)^2 d^5 (b+2cx)^8}{1024c^4}-\frac{3\left(b^2-4ac\right) d^5 (b+2cx)^{10}}{1280c^4}+\frac{d^5 (b+2cx)^{12}}{1536c^4}$$

Result (type 1, 224 leaves):

$$\frac{1}{60} d^5 x (b + c x) \left(20 a^3 \left(3 b^4 + 12 b^3 c x + 28 b^2 c^2 x^2 + 32 b c^3 x^3 + 16 c^4 x^4\right) + 12 a x^2 (b + c x)^2 \left(5 b^4 + 30 b^3 c x + 78 b^2 c^2 x^2 + 96 b c^3 x^3 + 48 c^4 x^4\right) + x^3 (b + c x)^3 \left(15 b^4 + 96 b^3 c x + 256 b^2 c^2 x^2 + 320 b c^3 x^3 + 160 c^4 x^4\right) + 30 a^2 x \left(3 b^5 + 19 b^4 c x + 56 b^3 c^2 x^2 + 88 b^2 c^3 x^3 + 72 b c^4 x^4 + 24 c^5 x^5\right)\right)$$

- **Problem 1138: Result more than twice size of optimal antiderivative.**

$$\int (b d + 2 c d x)^4 \left(a + b x + c x^2\right)^3 \, dx$$

Optimal (type 1, 101 leaves, 2 steps):

$$-\frac{\left(b^2 - 4 a c\right)^3 d^4 (b + 2 c x)^5}{640 c^4} + \frac{3 \left(b^2 - 4 a c\right)^2 d^4 (b + 2 c x)^7}{896 c^4} - \frac{\left(b^2 - 4 a c\right) d^4 (b + 2 c x)^9}{384 c^4} + \frac{d^4 (b + 2 c x)^{11}}{1408 c^4}$$

Result (type 1, 259 leaves):

$$d^4 \left(a^3 b^4 x + \frac{1}{2} a^2 b^3 \left(3 b^2 + 8 a c\right) x^2 + a b^2 \left(b^4 + 9 a b^2 c + 8 a^2 c^2\right) x^3 + \frac{1}{4} b \left(b^6 + 30 a b^4 c + 96 a^2 b^2 c^2 + 32 a^3 c^3\right) x^4 + \frac{1}{5} c \left(11 b^6 + 123 a b^4 c + 168 a^2 b^2 c^2 + 16 a^3 c^3\right) x^5 + \frac{1}{2} b c^2 \left(17 b^4 + 88 a b^2 c + 48 a^2 c^2\right) x^6 + \frac{3}{7} c^3 \left(43 b^4 + 104 a b^2 c + 16 a^2 c^2\right) x^7 + 24 b c^4 \left(b^2 + a c\right) x^8 + \frac{8}{3} c^5 \left(7 b^2 + 2 a c\right) x^9 + 8 b c^6 x^{10} + \frac{16 c^7 x^{11}}{11}\right)$$

- **Problem 1139: Result more than twice size of optimal antiderivative.**

$$\int (b d + 2 c d x)^3 \left(a + b x + c x^2\right)^3 \, dx$$

Optimal (type 1, 55 leaves, 2 steps):

$$\frac{1}{20} \left(b^2 - 4 a c\right) d^3 \left(a + b x + c x^2\right)^4 + \frac{1}{5} d^3 (b + 2 c x)^2 \left(a + b x + c x^2\right)^4$$

Result (type 1, 132 leaves):

$$\frac{1}{20} d^3 x (b + c x) \left(20 a^3 \left(b^2 + 2 b c x + 2 c^2 x^2\right) + 20 a x^2 (b + c x)^2 \left(b^2 + 3 b c x + 3 c^2 x^2\right) + x^3 (b + c x)^3 \left(5 b^2 + 16 b c x + 16 c^2 x^2\right) + 10 a^2 x \left(3 b^3 + 11 b^2 c x + 16 b c^2 x^2 + 8 c^3 x^3\right)\right)$$

- **Problem 1141: Result more than twice size of optimal antiderivative.**

$$\int (b d + 2 c d x) \left(a + b x + c x^2\right)^3 \, dx$$

Optimal (type 1, 17 leaves, 1 step):

$$\frac{1}{4} d \left(a + b x + c x^2\right)^4$$

Result (type 1, 52 leaves):

$$\frac{1}{4}\,d\,x\,(b+c\,x)\,\left(4\,a^3+6\,a^2\,x\,(b+c\,x)+4\,a\,x^2\,(b+c\,x)^2+x^3\,(b+c\,x)^3\right)$$

- **Problem 1150: Result more than twice size of optimal antiderivative.**

$$\int\frac{\left(a+b\,x+c\,x^2\right)^3}{(b\,d+2\,c\,d\,x)^9}\,dx$$

Optimal (type 1, 37 leaves, 1 step) :

$$\frac{\left(a+b\,x+c\,x^2\right)^4}{4\,\left(b^2-4\,a\,c\right)\,d^9\,(b+2\,c\,x)^8}$$

Result (type 1, 96 leaves) :

$$\frac{b^6-12\,a\,b^4\,c+48\,a^2\,b^2\,c^2-64\,a^3\,c^3-4\,\left(b^2-4\,a\,c\right)^2\,(b+2\,c\,x)^2+6\,\left(b^2-4\,a\,c\right)\,(b+2\,c\,x)^4-4\,(b+2\,c\,x)^6}{1024\,c^4\,d^9\,(b+2\,c\,x)^8}$$

- **Problem 1327: Result unnecessarily involves imaginary or complex numbers.**

$$\int(b\,d+2\,c\,d\,x)^{7/2}\,\sqrt{a+b\,x+c\,x^2}\,dx$$

Optimal (type 4, 227 leaves, 6 steps) :

$$-\frac{10\,\left(b^2-4\,a\,c\right)^2\,d^3\,\sqrt{b\,d+2\,c\,d\,x}\,\sqrt{a+b\,x+c\,x^2}}{231\,c}-\frac{2\,\left(b^2-4\,a\,c\right)\,d\,(b\,d+2\,c\,d\,x)^{5/2}\,\sqrt{a+b\,x+c\,x^2}}{77\,c}+$$
$$\frac{(b\,d+2\,c\,d\,x)^{9/2}\,\sqrt{a+b\,x+c\,x^2}}{11\,c\,d}-\frac{5\,\left(b^2-4\,a\,c\right)^{13/4}\,d^{7/2}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\,\sqrt{d}}\right],\,-1\right]}{231\,c^2\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 223 leaves) :

$$\frac{1}{231\,c^2\,\sqrt{a+x\,(b+c\,x)}}\,(d\,(b+2\,c\,x))^{7/2}\left(1\Big/(b+2\,c\,x)^3 c\,(a+x\,(b+c\,x))\,\left(5\,b^4+144\,b^3\,c\,x+96\,b\,c^2\,x\,\left(a+7\,c\,x^2\right)+8\,b^2\,c\,\left(13\,a+60\,c\,x^2\right)+16\,c^2\,\left(-10\,a^2+6\,a\,c\,x^2+21\,c^2\,x^4\right)\right)-\frac{5\,\mathrm{i}\,\left(b^2-4\,a\,c\right)^3\sqrt{\frac{c\,(a+x\,(b+c\,x))}{(b+2\,c\,x)^2}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4\,a\,c}}}{\sqrt{b+2\,c\,x}}\right],-1\right]}{\sqrt{-\sqrt{b^2-4\,a\,c}}\,(b+2\,c\,x)^{5/2}}\right)$$

- **Problem 1328: Result unnecessarily involves imaginary or complex numbers.**

$$\int (b\,d+2\,c\,d\,x)^{3/2}\,\sqrt{a+b\,x+c\,x^2}\,dx$$

Optimal (type 4, 180 leaves, 5 steps):

$$-\frac{2\,\left(b^2-4\,a\,c\right)\,d\,\sqrt{b\,d+2\,c\,d\,x}\,\sqrt{a+b\,x+c\,x^2}}{21\,c}+\frac{(b\,d+2\,c\,d\,x)^{5/2}\,\sqrt{a+b\,x+c\,x^2}}{7\,c\,d}-\frac{\left(b^2-4\,a\,c\right)^{9/4}\,d^{3/2}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\sqrt{d}}\right],-1\right]}{21\,c^2\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 175 leaves):

$$\frac{1}{21\,c^2\,\sqrt{a+x\,(b+c\,x)}}\,(d\,(b+2\,c\,x))^{3/2}\left(\frac{c\,(a+x\,(b+c\,x))\,\left(b^2+12\,b\,c\,x+4\,c\,\left(2\,a+3\,c\,x^2\right)\right)}{b+2\,c\,x}-\frac{\mathrm{i}\,\left(b^2-4\,a\,c\right)^2\sqrt{\frac{c\,(a+x\,(b+c\,x))}{(b+2\,c\,x)^2}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4\,a\,c}}}{\sqrt{b+2\,c\,x}}\right],-1\right]}{\sqrt{-\sqrt{b^2-4\,a\,c}}\,\sqrt{b+2\,c\,x}}\right)$$

Problem 1329: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b x+c x^2}}{\sqrt{b d+2 c d x}}\,dx$$

Optimal (type 4, 137 leaves, 4 steps):

$$\frac{\sqrt{b d+2 c d x}\,\sqrt{a+b x+c x^2}}{3 c d}-\frac{\left(b^2-4 a c\right)^{5/4}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4}\sqrt{d}}\right],-1\right]}{3 c^2\sqrt{d}\,\sqrt{a+b x+c x^2}}$$

Result (type 4, 149 leaves):

$$\frac{c\,(b+2 c x)\,(a+x\,(b+c x))-\frac{i\left(b^2-4 a c\right)(b+2 c x)^{3/2}\sqrt{\frac{c\,(a+x\,(b+c x))}{(b+2 c x)^2}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4 a c}}}{\sqrt{b+2 c x}}\right],-1\right]}{\sqrt{-\sqrt{b^2-4 a c}}}}{3 c^2\sqrt{d\,(b+2 c x)}\,\sqrt{a+x\,(b+c x)}}$$

Problem 1330: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b x+c x^2}}{(b d+2 c d x)^{5/2}}\,dx$$

Optimal (type 4, 137 leaves, 4 steps):

$$-\frac{\sqrt{a+b x+c x^2}}{3 c d\,(b d+2 c d x)^{3/2}}+\frac{\left(b^2-4 a c\right)^{1/4}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4}\sqrt{d}}\right],-1\right]}{3 c^2 d^{5/2}\sqrt{a+b x+c x^2}}$$

Result (type 4, 157 leaves):

$$\left(-c\sqrt{-\sqrt{b^2-4 a c}}\,(a+x\,(b+c x))+i\,(b+2 c x)^{5/2}\sqrt{\frac{c\,(a+x\,(b+c x))}{(b+2 c x)^2}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4 a c}}}{\sqrt{b+2 c x}}\right],-1\right]\right)\Big/$$

$$\left(3 c^2\sqrt{-\sqrt{b^2-4 a c}}\,d\,(d\,(b+2 c x))^{3/2}\sqrt{a+x\,(b+c x)}\right)$$

Problem 1331: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b x+c x^2}}{(b d+2 c d x)^{9/2}} \, dx$$

Optimal (type 4, 184 leaves, 5 steps):

$$-\frac{\sqrt{a+b x+c x^2}}{7 c d (b d+2 c d x)^{7/2}}+\frac{2 \sqrt{a+b x+c x^2}}{21 c \left(b^2-4 a c\right) d^3 (b d+2 c d x)^{3/2}}+\frac{\sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4} \sqrt{d}}\right],-1\right]}{21 c^2 \left(b^2-4 a c\right)^{3/4} d^{9/2} \sqrt{a+b x+c x^2}}$$

Result (type 4, 175 leaves):

$$\left(c (b+2 c x) (a+x (b+c x)) \left(-b^2+8 b c x+4 c \left(3 a+2 c x^2\right)\right)+\frac{i (b+2 c x)^{11/2} \sqrt{\frac{c (a+x (b+c x))}{(b+2 c x)^2}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4 a c}}}{\sqrt{b+2 c x}}\right],-1\right]}{\sqrt{-\sqrt{b^2-4 a c}}}\right) \Big/$$
$$\left(21 c^2 \left(b^2-4 a c\right) (d (b+2 c x))^{9/2} \sqrt{a+x (b+c x)}\right)$$

Problem 1332: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b x+c x^2}}{(b d+2 c d x)^{13/2}} \, dx$$

Optimal (type 4, 231 leaves, 6 steps):

$$-\frac{\sqrt{a+b x+c x^2}}{11 c d (b d+2 c d x)^{11/2}}+\frac{2 \sqrt{a+b x+c x^2}}{77 c \left(b^2-4 a c\right) d^3 (b d+2 c d x)^{7/2}}+$$
$$\frac{10 \sqrt{a+b x+c x^2}}{231 c \left(b^2-4 a c\right)^2 d^5 (b d+2 c d x)^{3/2}}+\frac{5 \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4} \sqrt{d}}\right],-1\right]}{231 c^2 \left(b^2-4 a c\right)^{7/4} d^{13/2} \sqrt{a+b x+c x^2}}$$

Result (type 4, 193 leaves):

$$\left(-c\,(b+2\,c\,x)\,(a+x\,(b+c\,x))\,\left(21\,\left(b^2-4\,a\,c\right)^2-6\,\left(b^2-4\,a\,c\right)\,(b+2\,c\,x)^2-10\,(b+2\,c\,x)^4\right)+\right.$$

$$\left.\frac{5\,i\,(b+2\,c\,x)^{15/2}\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{(b+2\,c\,x)^2}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4\,a\,c}}}{\sqrt{b+2\,c\,x}}\right],-1\right]}{\sqrt{-\sqrt{b^2-4\,a\,c}}}\right)\Big/\left(231\,c^2\,\left(b^2-4\,a\,c\right)^2\,(d\,(b+2\,c\,x))^{13/2}\,\sqrt{a+x\,(b+c\,x)}\right)$$

- **Problem 1333: Result unnecessarily involves imaginary or complex numbers.**

$$\int(b\,d+2\,c\,d\,x)^{5/2}\,\sqrt{a+b\,x+c\,x^2}\,dx$$

Optimal (type 4, 279 leaves, 8 steps):

$$-\frac{2\,\left(b^2-4\,a\,c\right)\,d\,(b\,d+2\,c\,d\,x)^{3/2}\,\sqrt{a+b\,x+c\,x^2}}{45\,c}+\frac{(b\,d+2\,c\,d\,x)^{7/2}\,\sqrt{a+b\,x+c\,x^2}}{9\,c\,d}-$$

$$\frac{\left(b^2-4\,a\,c\right)^{11/4}\,d^{5/2}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\,\sqrt{d}}\right],-1\right]}{15\,c^2\,\sqrt{a+b\,x+c\,x^2}}+$$

$$\frac{\left(b^2-4\,a\,c\right)^{11/4}\,d^{5/2}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\,\sqrt{d}}\right],-1\right]}{15\,c^2\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 208 leaves):

$$\frac{1}{45\,c^2\,\sqrt{a+x\,(b+c\,x)}}$$

$$(d\,(b+2\,c\,x))^{5/2}\left(\frac{c\,(a+x\,(b+c\,x))\,\left(3\,b^2+20\,b\,c\,x+4\,c\,\left(2\,a+5\,c\,x^2\right)\right)}{b+2\,c\,x}+1\Big/\left(-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{5/2}3\,i\,\left(b^2-4\,a\,c\right)^{3/2}\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]\right)\right)$$

Problem 1334: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{b d + 2 c d x}\ \sqrt{a + b x + c x^2}\, dx$$

Optimal (type 4, 236 leaves, 7 steps):

$$\frac{(b d + 2 c d x)^{3/2} \sqrt{a + b x + c x^2}}{5 c d} - \frac{(b^2 - 4 a c)^{7/4} \sqrt{d} \sqrt{-\frac{c (a+b x+c x^2)}{b^2-4 a c}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{(b^2-4 a c)^{1/4} \sqrt{d}}\right], -1\right]}{5 c^2 \sqrt{a + b x + c x^2}} + \frac{(b^2 - 4 a c)^{7/4} \sqrt{d} \sqrt{-\frac{c (a+b x+c x^2)}{b^2-4 a c}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{(b^2-4 a c)^{1/4} \sqrt{d}}\right], -1\right]}{5 c^2 \sqrt{a + b x + c x^2}}$$

Result (type 4, 246 leaves):

$$-\left(d \sqrt{-\frac{b + 2 c x}{\sqrt{b^2 - 4 a c}}} \left(-\frac{c (b + 2 c x)^2 (a + x (b + c x))}{\sqrt{-\frac{b+2 c x}{\sqrt{b^2-4 a c}}}} + i (b^2 - 4 a c)^2 \sqrt{\frac{c (a + x (b + c x))}{-b^2 + 4 a c}}\ \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{-\frac{b + 2 c x}{\sqrt{b^2 - 4 a c}}}\right], -1\right] - i (b^2 - 4 a c)^2 \sqrt{\frac{c (a + x (b + c x))}{-b^2 + 4 a c}}\ \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{-\frac{b + 2 c x}{\sqrt{b^2 - 4 a c}}}\right], -1\right]\right)\right) \Big/ \left(5 c^2 \sqrt{d (b + 2 c x)}\ \sqrt{a + x (b + c x)}\right)$$

Problem 1335: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + b x + c x^2}}{(b d + 2 c d x)^{3/2}}\, dx$$

Optimal (type 4, 229 leaves, 7 steps):

$$-\frac{\sqrt{a + b x + c x^2}}{c d \sqrt{b d + 2 c d x}} + \frac{(b^2 - 4 a c)^{3/4} \sqrt{-\frac{c (a+b x+c x^2)}{b^2-4 a c}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{(b^2-4 a c)^{1/4} \sqrt{d}}\right], -1\right]}{c^2 d^{3/2} \sqrt{a + b x + c x^2}} - \frac{(b^2 - 4 a c)^{3/4} \sqrt{-\frac{c (a+b x+c x^2)}{b^2-4 a c}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{(b^2-4 a c)^{1/4} \sqrt{d}}\right], -1\right]}{c^2 d^{3/2} \sqrt{a + b x + c x^2}}$$

Result (type 4, 176 leaves):

$$\left(-c\,(b+2\,c\,x)\,(a+x\,(b+c\,x))+1\Big/\left(-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{3/2}\,i\,(b+2\,c\,x)^3\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\right.$$
$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]\right)\right)\Big/\left(c^2\,(d\,(b+2\,c\,x))^{3/2}\,\sqrt{a+x\,(b+c\,x)}\right)$$

- **Problem 1336: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{a+b\,x+c\,x^2}}{(b\,d+2\,c\,d\,x)^{7/2}}\,dx$$

Optimal (type 4, 283 leaves, 8 steps):

$$-\frac{\sqrt{a+b\,x+c\,x^2}}{5\,c\,d\,(b\,d+2\,c\,d\,x)^{5/2}}+\frac{2\,\sqrt{a+b\,x+c\,x^2}}{5\,c\,\left(b^2-4\,a\,c\right)\,d^3\,\sqrt{b\,d+2\,c\,d\,x}}-$$
$$\frac{\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{(b^2-4\,a\,c)^{1/4}\,\sqrt{d}}\right],-1\right]}{5\,c^2\,\left(b^2-4\,a\,c\right)^{1/4}\,d^{7/2}\,\sqrt{a+b\,x+c\,x^2}}+\frac{\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{(b^2-4\,a\,c)^{1/4}\,\sqrt{d}}\right],-1\right]}{5\,c^2\,\left(b^2-4\,a\,c\right)^{1/4}\,d^{7/2}\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 208 leaves):

$$\left(-\frac{c\,(a+x\,(b+c\,x))\,\left(b^2+8\,b\,c\,x+4\,c\,\left(a+2\,c\,x^2\right)\right)}{-b^2+4\,a\,c}-i\,(b+2\,c\,x)^2\,\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\right.$$
$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]\right)\right)\Big/\left(5\,c^2\,d\,(d\,(b+2\,c\,x))^{5/2}\,\sqrt{a+x\,(b+c\,x)}\right)$$

- **Problem 1337: Result unnecessarily involves imaginary or complex numbers.**

$$\int(b\,d+2\,c\,d\,x)^{7/2}\,\left(a+b\,x+c\,x^2\right)^{3/2}\,dx$$

Optimal (type 4, 274 leaves, 7 steps):

$$\frac{\left(b^2-4ac\right)^3 d^3 \sqrt{bd+2cdx}\,\sqrt{a+bx+cx^2}}{231c^2}+\frac{\left(b^2-4ac\right)^2 d\,(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}}{385c^2}-\frac{\left(b^2-4ac\right)(bd+2cdx)^{9/2}\sqrt{a+bx+cx^2}}{110c^2 d}+$$

$$\frac{(bd+2cdx)^{9/2}\left(a+bx+cx^2\right)^{3/2}}{15cd}+\frac{\left(b^2-4ac\right)^{17/4} d^{7/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)^{1/4}\sqrt{d}}\right],-1\right]}{462c^3\sqrt{a+bx+cx^2}}$$

Result (type 4, 295 leaves):

$$\frac{1}{2310c^3\sqrt{a+x(b+cx)}}(d(b+2cx))^{7/2}$$

$$\left(1/(b+2cx)^3 c\,(a+x(b+cx))\left(-5b^6+10b^5cx+16b^3c^2x\left(107a+266cx^2\right)+2b^4c\left(35a+453cx^2\right)+32bc^3x\left(12a^2+238acx^2+231c^2x^4\right)+\right.\right.$$

$$\left.16b^2c^2\left(36a^2+345acx^2+518c^2x^4\right)+32c^3\left(-20a^3+12a^2cx^2+119ac^2x^4+77c^3x^6\right)\right)+$$

$$\left.\frac{5i\left(b^2-4ac\right)^4\sqrt{\frac{c(a+x(b+cx))}{(b+2cx)^2}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4ac}}}{\sqrt{b+2cx}}\right],-1\right]}{\sqrt{-\sqrt{b^2-4ac}}\,(b+2cx)^{5/2}}\right)$$

- **Problem 1338: Result unnecessarily involves imaginary or complex numbers.**

$$\int (bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}\,dx$$

Optimal (type 4, 227 leaves, 6 steps):

$$\frac{\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}\,\sqrt{a+bx+cx^2}}{77c^2}-\frac{3\left(b^2-4ac\right)(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}}{154c^2 d}+$$

$$\frac{(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^{3/2}}{11cd}+\frac{\left(b^2-4ac\right)^{13/4} d^{3/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)^{1/4}\sqrt{d}}\right],-1\right]}{154c^3\sqrt{a+bx+cx^2}}$$

Result (type 4, 225 leaves):

$$\frac{1}{154\, c^3 \sqrt{a + x\, (b + c\, x)}} (d\, (b + 2\, c\, x))^{3/2} \left(1 / (b + 2\, c\, x) c\, (a + x\, (b + c\, x))\, \left(-b^4 + 2\, b^3\, c\, x + 8\, b\, c^2\, x\, \left(13\, a + 14\, c\, x^2\right) + 2\, b^2\, c\, \left(5\, a + 29\, c\, x^2\right) + 8\, c^2\, \left(4\, a^2 + 13\, a\, c\, x^2 + 7\, c^2\, x^4\right)\right) + \frac{i\, \left(b^2 - 4\, a\, c\right)^3 \sqrt{\frac{c\, (a+x\, (b+c\, x))}{(b+2\, c\, x)^2}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4\, a\, c}}}{\sqrt{b+2\, c\, x}}\right], -1\right]}{\sqrt{-\sqrt{b^2 - 4\, a\, c}}\, \sqrt{b + 2\, c\, x}}\right)$$

- **Problem 1339: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(a + b\, x + c\, x^2\right)^{3/2}}{\sqrt{b\, d + 2\, c\, d\, x}}\, dx$$

Optimal (type 4, 182 leaves, 5 steps):

$$-\frac{\left(b^2 - 4\, a\, c\right) \sqrt{b\, d + 2\, c\, d\, x}\, \sqrt{a + b\, x + c\, x^2}}{14\, c^2\, d} + \frac{\sqrt{b\, d + 2\, c\, d\, x}\, \left(a + b\, x + c\, x^2\right)^{3/2}}{7\, c\, d} + \frac{\left(b^2 - 4\, a\, c\right)^{9/4} \sqrt{-\frac{c\, (a+b\, x+c\, x^2)}{b^2-4\, a\, c}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\, d+2\, c\, d\, x}}{(b^2-4\, a\, c)^{1/4}\sqrt{d}}\right], -1\right]}{14\, c^3\, \sqrt{d}\, \sqrt{a + b\, x + c\, x^2}}$$

Result (type 4, 174 leaves):

$$\left(c\, (b + 2\, c\, x)\, (a + x\, (b + c\, x))\, \left(-b^2 + 2\, b\, c\, x + 2\, c\, \left(3\, a + c\, x^2\right)\right) + \frac{i\, \left(b^2 - 4\, a\, c\right)^2\, (b + 2\, c\, x)^{3/2} \sqrt{\frac{c\, (a+x\, (b+c\, x))}{(b+2\, c\, x)^2}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4\, a\, c}}}{\sqrt{b+2\, c\, x}}\right], -1\right]}{\sqrt{-\sqrt{b^2 - 4\, a\, c}}}\right) \Big/ \left(14\, c^3\, \sqrt{d\, (b + 2\, c\, x)}\, \sqrt{a + x\, (b + c\, x)}\right)$$

Problem 1340: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x+c x^2\right)^{3/2}}{(b d+2 c d x)^{5/2}}\,dx$$

Optimal (type 4, 174 leaves, 5 steps):

$$\frac{\sqrt{b d+2 c d x}\,\sqrt{a+b x+c x^2}}{6 c^2 d^3}-\frac{\left(a+b x+c x^2\right)^{3/2}}{3 c d\,(b d+2 c d x)^{3/2}}-\frac{\left(b^2-4 a c\right)^{5/4}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4}\sqrt{d}}\right],-1\right]}{6 c^3 d^{5/2}\sqrt{a+b x+c x^2}}$$

Result (type 4, 170 leaves):

$$\left(c\,(b+2 c x)\,(a+x\,(b+c x))\,\left(b^2+2 b c x+2 c\left(-a+c x^2\right)\right)-\frac{i\left(b^2-4 a c\right)(b+2 c x)^{7/2}\sqrt{\frac{c\,(a+x\,(b+c x))}{(b+2 c x)^2}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4 a c}}}{\sqrt{b+2 c x}}\right],-1\right]}{\sqrt{-\sqrt{b^2-4 a c}}}\right)\Big/$$

$$\left(6 c^3\,(d\,(b+2 c x))^{5/2}\sqrt{a+x\,(b+c x)}\right)$$

Problem 1341: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x+c x^2\right)^{3/2}}{(b d+2 c d x)^{9/2}}\,dx$$

Optimal (type 4, 174 leaves, 5 steps):

$$-\frac{\sqrt{a+b x+c x^2}}{14 c^2 d^3\,(b d+2 c d x)^{3/2}}-\frac{\left(a+b x+c x^2\right)^{3/2}}{7 c d\,(b d+2 c d x)^{7/2}}+\frac{\left(b^2-4 a c\right)^{1/4}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4}\sqrt{d}}\right],-1\right]}{14 c^3 d^{9/2}\sqrt{a+b x+c x^2}}$$

Result (type 4, 159 leaves):

$$\left(c\,(b+2 c x)\,(a+x\,(b+c x))\,\left(b^2-4 a c-3\,(b+2 c x)^2\right)+\frac{2 i\,(b+2 c x)^{11/2}\sqrt{\frac{c\,(a+x\,(b+c x))}{(b+2 c x)^2}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4 a c}}}{\sqrt{b+2 c x}}\right],-1\right]}{\sqrt{-\sqrt{b^2-4 a c}}}\right)\Big/$$

$$\left(28 c^3\,(d\,(b+2 c x))^{9/2}\sqrt{a+x\,(b+c x)}\right)$$

Problem 1342: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x+c x^2\right)^{3/2}}{(b d+2 c d x)^{13/2}} \, dx$$

Optimal (type 4, 221 leaves, 6 steps):

$$-\frac{3 \sqrt{a+b x+c x^2}}{154 c^2 d^3 (b d+2 c d x)^{7/2}}+\frac{\sqrt{a+b x+c x^2}}{77 c^2 \left(b^2-4 a c\right) d^5 (b d+2 c d x)^{3/2}}-\frac{\left(a+b x+c x^2\right)^{3/2}}{11 c d (b d+2 c d x)^{11/2}}+\frac{\sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4} \sqrt{d}}\right],-1\right]}{154 c^3 \left(b^2-4 a c\right)^{3/4} d^{13/2} \sqrt{a+b x+c x^2}}$$

Result (type 4, 192 leaves):

$$\left(c (b+2 c x) (a+x (b+c x)) \left(7 \left(b^2-4 a c\right)^2-13 \left(b^2-4 a c\right) (b+2 c x)^2+4 (b+2 c x)^4\right)+\frac{2 i (b+2 c x)^{15/2} \sqrt{\frac{c (a+x (b+c x))}{(b+2 c x)^2}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4 a c}}}{\sqrt{b+2 c x}}\right],-1\right]}{\sqrt{-\sqrt{b^2-4 a c}}}\right) \Big/ \left(308 c^3 \left(b^2-4 a c\right) (d (b+2 c x))^{13/2} \sqrt{a+x (b+c x)}\right)$$

Problem 1343: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x+c x^2\right)^{3/2}}{(b d+2 c d x)^{17/2}} \, dx$$

Optimal (type 4, 268 leaves, 7 steps):

$$-\frac{\sqrt{a+b x+c x^2}}{110 c^2 d^3 (b d+2 c d x)^{11/2}}+\frac{\sqrt{a+b x+c x^2}}{385 c^2 \left(b^2-4 a c\right) d^5 (b d+2 c d x)^{7/2}}+\frac{\sqrt{a+b x+c x^2}}{231 c^2 \left(b^2-4 a c\right)^2 d^7 (b d+2 c d x)^{3/2}}-\frac{\left(a+b x+c x^2\right)^{3/2}}{15 c d (b d+2 c d x)^{15/2}}+\frac{\sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4} \sqrt{d}}\right],-1\right]}{462 c^3 \left(b^2-4 a c\right)^{7/4} d^{17/2} \sqrt{a+b x+c x^2}}$$

Result (type 4, 212 leaves) :

$$\left(c\,(b+2\,c\,x)\,(a+x\,(b+c\,x))\,\left(77\,\left(b^2-4\,a\,c\right)^3-119\,\left(b^2-4\,a\,c\right)^2\,(b+2\,c\,x)^2+12\,\left(b^2-4\,a\,c\right)\,(b+2\,c\,x)^4+20\,(b+2\,c\,x)^6\right)+\frac{10\,\mathrm{i}\,(b+2\,c\,x)^{19/2}\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{(b+2\,c\,x)^2}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4\,a\,c}}}{\sqrt{b+2\,c\,x}}\right],-1\right]}{\sqrt{-\sqrt{b^2-4\,a\,c}}}\right)\Big/\left(4620\,c^3\,\left(b^2-4\,a\,c\right)^2\,(d\,(b+2\,c\,x))^{17/2}\,\sqrt{a+x\,(b+c\,x)}\right)$$

- **Problem 1344: Result unnecessarily involves imaginary or complex numbers.**

$$\int(b\,d+2\,c\,d\,x)^{5/2}\,\left(a+b\,x+c\,x^2\right)^{3/2}\,dx$$

Optimal (type 4, 326 leaves, 9 steps) :

$$\frac{\left(b^2-4\,a\,c\right)^2\,d\,(b\,d+2\,c\,d\,x)^{3/2}\,\sqrt{a+b\,x+c\,x^2}}{195\,c^2}-\frac{\left(b^2-4\,a\,c\right)\,(b\,d+2\,c\,d\,x)^{7/2}\,\sqrt{a+b\,x+c\,x^2}}{78\,c^2\,d}+$$

$$\frac{(b\,d+2\,c\,d\,x)^{7/2}\,\left(a+b\,x+c\,x^2\right)^{3/2}}{13\,c\,d}+\frac{\left(b^2-4\,a\,c\right)^{15/4}\,d^{5/2}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\,\sqrt{d}}\right],-1\right]}{130\,c^3\,\sqrt{a+b\,x+c\,x^2}}-$$

$$\frac{\left(b^2-4\,a\,c\right)^{15/4}\,d^{5/2}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\,\sqrt{d}}\right],-1\right]}{130\,c^3\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 256 leaves) :

$$\frac{1}{390\, c^3 \sqrt{a + x\,(b + c\,x)}}\,(d\,(b + 2\,c\,x))^{5/2} \Bigg(1 \big/ (b + 2\,c\,x)\,c\,(a + x\,(b + c\,x))\,\left(-3\,b^4 + 10\,b^3\,c\,x + 40\,b\,c^2\,x\,\left(5\,a + 6\,c\,x^2\right) + 2\,b^2\,c\,\left(17\,a + 65\,c\,x^2\right) + 8\,c^2\,\left(4\,a^2 + 25\,a\,c\,x^2 + 15\,c^2\,x^4\right)\right) - 1 \Bigg/ \left(-\frac{b + 2\,c\,x}{\sqrt{b^2 - 4\,a\,c}}\right)^{5/2} 3\,\mathrm{i}\,\left(b^2 - 4\,a\,c\right)^{5/2} \sqrt{\frac{c\,(a + x\,(b + c\,x))}{-b^2 + 4\,a\,c}} \left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{b + 2\,c\,x}{\sqrt{b^2 - 4\,a\,c}}}\right], -1\right] - \text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{b + 2\,c\,x}{\sqrt{b^2 - 4\,a\,c}}}\right], -1\right]\right)\Bigg)$$

- **Problem 1345: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{b\,d + 2\,c\,d\,x}\,\left(a + b\,x + c\,x^2\right)^{3/2} \, dx$$

Optimal (type 4, 281 leaves, 8 steps):

$$-\frac{\left(b^2 - 4\,a\,c\right)\,(b\,d + 2\,c\,d\,x)^{3/2} \sqrt{a + b\,x + c\,x^2}}{30\,c^2\,d} + \frac{(b\,d + 2\,c\,d\,x)^{3/2} \left(a + b\,x + c\,x^2\right)^{3/2}}{9\,c\,d} + \frac{\left(b^2 - 4\,a\,c\right)^{11/4} \sqrt{d}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\sqrt{d}}\right], -1\right]}{30\,c^3 \sqrt{a + b\,x + c\,x^2}} - \frac{\left(b^2 - 4\,a\,c\right)^{11/4} \sqrt{d}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\sqrt{d}}\right], -1\right]}{30\,c^3 \sqrt{a + b\,x + c\,x^2}}$$

Result (type 4, 206 leaves):

$$\frac{1}{90\,c^3\sqrt{a+x\,(b+c\,x)}}\sqrt{d\,(b+2\,c\,x)}\left(c\,(b+2\,c\,x)\,(a+x\,(b+c\,x))\,\left(-3\,b^2+10\,b\,c\,x+2\,c\,\left(11\,a+5\,c\,x^2\right)\right)-1\Big/\left(\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right)3\,i\,\left(b^2-4\,a\,c\right)^{5/2}\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]\right)\right)$$

Problem 1346: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,x+c\,x^2\right)^{3/2}}{(b\,d+2\,c\,d\,x)^{3/2}}\,dx$$

Optimal (type 4, 271 leaves, 8 steps):

$$\frac{3\,(b\,d+2\,c\,d\,x)^{3/2}\sqrt{a+b\,x+c\,x^2}}{10\,c^2\,d^3}-\frac{\left(a+b\,x+c\,x^2\right)^{3/2}}{c\,d\,\sqrt{b\,d+2\,c\,d\,x}}-\frac{3\,\left(b^2-4\,a\,c\right)^{7/4}\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\sqrt{d}}\right],-1\right]}{10\,c^3\,d^{3/2}\sqrt{a+b\,x+c\,x^2}}+\frac{3\,\left(b^2-4\,a\,c\right)^{7/4}\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\sqrt{d}}\right],-1\right]}{10\,c^3\,d^{3/2}\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 200 leaves):

$$\left(c\,(a+x\,(b+c\,x))\,\left(3\,b^2+2\,b\,c\,x+2\,c\,\left(-5\,a+c\,x^2\right)\right)-3\,i\,\left(b^2-4\,a\,c\right)^2\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]\right)\right)\Big/\left(10\,c^3\,d\,\sqrt{d\,(b+2\,c\,x)}\,\sqrt{a+x\,(b+c\,x)}\right)$$

Problem 1347: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,x+c\,x^2\right)^{3/2}}{(b\,d+2\,c\,d\,x)^{7/2}}\,dx$$

Optimal (type 4, 273 leaves, 8 steps):

$$-\frac{3\sqrt{a+b\,x+c\,x^2}}{10\,c^2\,d^3\,\sqrt{b\,d+2\,c\,d\,x}}-\frac{\left(a+b\,x+c\,x^2\right)^{3/2}}{5\,c\,d\,(b\,d+2\,c\,d\,x)^{5/2}}+\frac{3\left(b^2-4\,a\,c\right)^{3/4}\sqrt{-\frac{c\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\sqrt{d}}\right],-1\right]}{10\,c^3\,d^{7/2}\,\sqrt{a+b\,x+c\,x^2}}-\frac{3\left(b^2-4\,a\,c\right)^{3/4}\sqrt{-\frac{c\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\sqrt{d}}\right],-1\right]}{10\,c^3\,d^{7/2}\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 198 leaves) :

$$\left(-c\,(a+x\,(b+c\,x))\left(3\,b^2+14\,b\,c\,x+2\,c\left(a+7\,c\,x^2\right)\right)+1\Big/\left(-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{3/2} 3\,i\,(b+2\,c\,x)^4\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]\right)\right)\Big/\left(10\,c^3\,d\,(d\,(b+2\,c\,x))^{5/2}\sqrt{a+x\,(b+c\,x)}\right)$$

- **Problem 1348: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(a+b\,x+c\,x^2\right)^{3/2}}{(b\,d+2\,c\,d\,x)^{11/2}}\,dx$$

Optimal (type 4, 320 leaves, 9 steps) :

$$-\frac{\sqrt{a+b\,x+c\,x^2}}{30\,c^2\,d^3\,(b\,d+2\,c\,d\,x)^{5/2}}+\frac{\sqrt{a+b\,x+c\,x^2}}{15\,c^2\left(b^2-4\,a\,c\right)d^5\,\sqrt{b\,d+2\,c\,d\,x}}-\frac{\left(a+b\,x+c\,x^2\right)^{3/2}}{9\,c\,d\,(b\,d+2\,c\,d\,x)^{9/2}}-\frac{\sqrt{-\frac{c\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\sqrt{d}}\right],-1\right]}{30\,c^3\left(b^2-4\,a\,c\right)^{1/4}d^{11/2}\sqrt{a+b\,x+c\,x^2}}+\frac{\sqrt{-\frac{c\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\sqrt{d}}\right],-1\right]}{30\,c^3\left(b^2-4\,a\,c\right)^{1/4}d^{11/2}\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 229 leaves) :

$$\left(\frac{c\,(b+2\,c\,x)\,(a+x\,(b+c\,x))\left(5\left(b^2-4\,a\,c\right)^2-11\left(b^2-4\,a\,c\right)(b+2\,c\,x)^2+12\,(b+2\,c\,x)^4\right)}{b^2-4\,a\,c}-6\,i\,(b+2\,c\,x)^5\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]\right)\right)\Big/\left(180\,c^3\,(d\,(b+2\,c\,x))^{11/2}\sqrt{a+x\,(b+c\,x)}\right)$$

Problem 1349: Result unnecessarily involves imaginary or complex numbers.

$$\int (b d + 2 c d x)^{7/2} \left(a + b x + c x^2\right)^{5/2} dx$$

Optimal (type 4, 321 leaves, 8 steps):

$$-\frac{5 \left(b^2 - 4 a c\right)^4 d^3 \sqrt{b d + 2 c d x} \sqrt{a + b x + c x^2}}{8778 c^3} - \frac{\left(b^2 - 4 a c\right)^3 d (b d + 2 c d x)^{5/2} \sqrt{a + b x + c x^2}}{2926 c^3} + \frac{\left(b^2 - 4 a c\right)^2 (b d + 2 c d x)^{9/2} \sqrt{a + b x + c x^2}}{836 c^3 d} - \frac{\left(b^2 - 4 a c\right) (b d + 2 c d x)^{9/2} \left(a + b x + c x^2\right)^{3/2}}{114 c^2 d} + \frac{(b d + 2 c d x)^{9/2} \left(a + b x + c x^2\right)^{5/2}}{19 c d} - \frac{5 \left(b^2 - 4 a c\right)^{21/4} d^{7/2} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4} \sqrt{d}}\right], -1\right]}{17556 c^4 \sqrt{a + b x + c x^2}}$$

Result (type 4, 386 leaves):

$$\frac{1}{17556 c^4 \sqrt{a + x (b + c x)}} (d (b + 2 c x))^{7/2} \left(\frac{1}{(b + 2 c x)^3} c (a + x (b + c x)) \left(5 b^8 - 10 b^7 c x + 18 b^6 c \left(-5 a + c x^2\right) + 8 b^5 c^2 x \left(22 a + 623 c x^2\right) + 32 b^3 c^3 x \left(433 a^2 + 2142 a c x^2 + 2310 c^2 x^4\right) + 4 b^4 c^2 \left(157 a^2 + 3684 a c x^2 + 7399 c^2 x^4\right) + 128 b c^4 x \left(12 a^3 + 469 a^2 c x^2 + 924 a c^2 x^4 + 462 c^3 x^6\right) + 32 b^2 c^3 \left(92 a^3 + 1371 a^2 c x^2 + 4151 a c^2 x^4 + 2926 c^3 x^6\right) + 64 c^4 \left(-40 a^4 + 24 a^3 c x^2 + 469 a^2 c^2 x^4 + 616 a c^3 x^6 + 231 c^4 x^8\right)\right) - \frac{5 i \left(b^2 - 4 a c\right)^5 \sqrt{\frac{c (a+x (b+c x))}{(b+2 c x)^2}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4 a c}}}{\sqrt{b+2 c x}}\right], -1\right]}{\sqrt{-\sqrt{b^2 - 4 a c}} (b + 2 c x)^{5/2}}\right)$$

Problem 1350: Result unnecessarily involves imaginary or complex numbers.

$$\int (b d + 2 c d x)^{3/2} \left(a + b x + c x^2\right)^{5/2} dx$$

Optimal (type 4, 274 leaves, 7 steps):

$$-\frac{\left(b^2-4ac\right)^3 d\sqrt{bd+2cdx}\,\sqrt{a+bx+cx^2}}{462c^3}+\frac{\left(b^2-4ac\right)^2 (bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}}{308c^3d}-\frac{\left(b^2-4ac\right)(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^{3/2}}{66c^2d}+$$

$$\frac{(bd+2cdx)^{5/2}\left(a+bx+cx^2\right)^{5/2}}{15cd}-\frac{\left(b^2-4ac\right)^{17/4}d^{3/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)^{1/4}\sqrt{d}}\right],-1\right]}{924c^4\sqrt{a+bx+cx^2}}$$

Result (type 4, 295 leaves) :

$$\frac{1}{4620c^4\sqrt{a+x(b+cx)}}(d(b+2cx))^{3/2}\left(1/(b+2cx)\,c\,(a+x(b+cx))\left(5b^6-10b^5cx+2b^4c\left(-35a+9cx^2\right)+8b^3c^2x\left(17a+161cx^2\right)+16bc^3x\left(207a^2+448acx^2+231c^2x^4\right)+\right.\right.$$

$$\left.4b^2c^2\left(87a^2+930acx^2+931c^2x^4\right)+16c^3\left(40a^3+207a^2cx^2+224ac^2x^4+77c^3x^6\right)\right)-$$

$$\left.\frac{5i\left(b^2-4ac\right)^4\sqrt{\frac{c(a+x(b+cx))}{(b+2cx)^2}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4ac}}}{\sqrt{b+2cx}}\right],-1\right]}{\sqrt{-\sqrt{b^2-4ac}}\,\sqrt{b+2cx}}\right)$$

- **Problem 1351: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(a+bx+cx^2\right)^{5/2}}{\sqrt{bd+2cdx}}\,dx$$

Optimal (type 4, 229 leaves, 6 steps) :

$$\frac{5\left(b^2-4ac\right)^2\sqrt{bd+2cdx}\,\sqrt{a+bx+cx^2}}{308c^3d}-\frac{5\left(b^2-4ac\right)\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}}{154c^2d}+$$

$$\frac{\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{5/2}}{11cd}-\frac{5\left(b^2-4ac\right)^{13/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)^{1/4}\sqrt{d}}\right],-1\right]}{308c^4\sqrt{d}\,\sqrt{a+bx+cx^2}}$$

Result (type 4, 223 leaves) :

$$\left(c\,(b+2\,c\,x)\,(a+x\,(b+c\,x))\,\left(5\,b^4-10\,b^3\,c\,x+8\,b\,c^2\,x\,\left(12\,a+7\,c\,x^2\right)+2\,b^2\,c\,\left(-25\,a+9\,c\,x^2\right)+4\,c^2\,\left(37\,a^2+24\,a\,c\,x^2+7\,c^2\,x^4\right)\right)-\right.$$

$$\left.\frac{5\,\mathrm{i}\,\left(b^2-4\,a\,c\right)^3\,(b+2\,c\,x)^{3/2}\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{(b+2\,c\,x)^2}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4\,a\,c}}}{\sqrt{b+2\,c\,x}}\right],-1\right]}{\sqrt{-\sqrt{b^2-4\,a\,c}}}\right)\Big/\left(308\,c^4\,\sqrt{d\,(b+2\,c\,x)}\,\sqrt{a+x\,(b+c\,x)}\right)$$

- **Problem 1352: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(a+b\,x+c\,x^2\right)^{5/2}}{(b\,d+2\,c\,d\,x)^{5/2}}\,dx$$

Optimal (type 4, 219 leaves, 6 steps):

$$-\frac{5\,\left(b^2-4\,a\,c\right)\,\sqrt{b\,d+2\,c\,d\,x}\,\sqrt{a+b\,x+c\,x^2}}{84\,c^3\,d^3}+\frac{5\,\sqrt{b\,d+2\,c\,d\,x}\,\left(a+b\,x+c\,x^2\right)^{3/2}}{42\,c^2\,d^3}-$$

$$\frac{\left(a+b\,x+c\,x^2\right)^{5/2}}{3\,c\,d\,(b\,d+2\,c\,d\,x)^{3/2}}+\frac{5\,\left(b^2-4\,a\,c\right)^{9/4}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\,\sqrt{d}}\right],-1\right]}{84\,c^4\,d^{5/2}\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 201 leaves):

$$\left(\frac{(b+2\,c\,x)^3\,(a+x\,(b+c\,x))\,\left(-13\,b^2+64\,a\,c+12\,b\,c\,x+12\,c^2\,x^2-\frac{7\,\left(b^2-4\,a\,c\right)^2}{(b+2\,c\,x)^2}\right)}{4\,c^3}+\right.$$

$$\left.\frac{5\,\mathrm{i}\,\left(b^2-4\,a\,c\right)^2\,(b+2\,c\,x)^{7/2}\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{(b+2\,c\,x)^2}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4\,a\,c}}}{\sqrt{b+2\,c\,x}}\right],-1\right]}{c^4\,\sqrt{-\sqrt{b^2-4\,a\,c}}}\right)\Big/\left(84\,(d\,(b+2\,c\,x))^{5/2}\,\sqrt{a+x\,(b+c\,x)}\right)$$

Problem 1353: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x+c x^2\right)^{5/2}}{(b d+2 c d x)^{9/2}} \, dx$$

Optimal (type 4, 211 leaves, 6 steps):

$$\frac{5 \sqrt{b d+2 c d x} \sqrt{a+b x+c x^2}}{84 c^3 d^5}-\frac{5 \left(a+b x+c x^2\right)^{3/2}}{42 c^2 d^3 (b d+2 c d x)^{3/2}}-\frac{\left(a+b x+c x^2\right)^{5/2}}{7 c d (b d+2 c d x)^{7/2}}-\frac{5 \left(b^2-4 a c\right)^{5/4} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4} \sqrt{d}}\right],-1\right]}{84 c^4 d^{9/2} \sqrt{a+b x+c x^2}}$$

Result (type 4, 195 leaves):

$$\left(-\frac{(b+2 c x) (a+x (b+c x)) \left(3 \left(b^2-4 a c\right)^2-16 \left(b^2-4 a c\right) (b+2 c x)^2-7 (b+2 c x)^4\right)}{4 c^3}-\frac{5 i \left(b^2-4 a c\right) (b+2 c x)^{11/2} \sqrt{\frac{c (a+x (b+c x))}{(b+2 c x)^2}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4 a c}}}{\sqrt{b+2 c x}}\right],-1\right]}{c^4 \sqrt{-\sqrt{b^2-4 a c}}}\right) \Bigg/ \left(84 (d (b+2 c x))^{9/2} \sqrt{a+x (b+c x)}\right)$$

Problem 1354: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x+c x^2\right)^{5/2}}{(b d+2 c d x)^{13/2}} \, dx$$

Optimal (type 4, 211 leaves, 6 steps):

$$-\frac{5 \sqrt{a+b x+c x^2}}{308 c^3 d^5 (b d+2 c d x)^{3/2}}-\frac{5 \left(a+b x+c x^2\right)^{3/2}}{154 c^2 d^3 (b d+2 c d x)^{7/2}}-\frac{\left(a+b x+c x^2\right)^{5/2}}{11 c d (b d+2 c d x)^{11/2}}+\frac{5 \left(b^2-4 a c\right)^{1/4} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4} \sqrt{d}}\right],-1\right]}{308 c^4 d^{13/2} \sqrt{a+b x+c x^2}}$$

Result (type 4, 187 leaves):

$$\left(-\frac{(b+2cx)(a+x(b+cx))\left(7\left(b^2-4ac\right)^2-24\left(b^2-4ac\right)(b+2cx)^2+37(b+2cx)^4\right)}{4c^3}+\frac{5\,i\,(b+2cx)^{15/2}\sqrt{\frac{c(a+x(b+cx))}{(b+2cx)^2}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4ac}}}{\sqrt{b+2cx}}\right],-1\right]}{c^4\sqrt{-\sqrt{b^2-4ac}}}\right)\Big/\left(308\,(d(b+2cx))^{13/2}\sqrt{a+x(b+cx)}\right)$$

- **Problem 1355: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^{17/2}}\,dx$$

Optimal (type 4, 258 leaves, 7 steps):

$$-\frac{\sqrt{a+bx+cx^2}}{308c^3d^5(bd+2cdx)^{7/2}}+\frac{\sqrt{a+bx+cx^2}}{462c^3\left(b^2-4ac\right)d^7(bd+2cdx)^{3/2}}-\frac{\left(a+bx+cx^2\right)^{3/2}}{66c^2d^3(bd+2cdx)^{11/2}}-\frac{\left(a+bx+cx^2\right)^{5/2}}{15cd(bd+2cdx)^{15/2}}+\frac{\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)^{1/4}\sqrt{d}}\right],-1\right]}{924c^4\left(b^2-4ac\right)^{3/4}d^{17/2}\sqrt{a+bx+cx^2}}$$

Result (type 4, 213 leaves):

$$\left(-c(b+2cx)(a+x(b+cx))\left(77\left(b^2-4ac\right)^3-224\left(b^2-4ac\right)^2(b+2cx)^2+207\left(b^2-4ac\right)(b+2cx)^4-40(b+2cx)^6\right)+\frac{20\,i\,(b+2cx)^{19/2}\sqrt{\frac{c(a+x(b+cx))}{(b+2cx)^2}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4ac}}}{\sqrt{b+2cx}}\right],-1\right]}{\sqrt{-\sqrt{b^2-4ac}}}\right)\Big/\left(18480\,c^4\left(b^2-4ac\right)(d(b+2cx))^{17/2}\sqrt{a+x(b+cx)}\right)$$

Problem 1356: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x+c x^2\right)^{5/2}}{(b d+2 c d x)^{21/2}} \, dx$$

Optimal (type 4, 305 leaves, 8 steps):

$$-\frac{\sqrt{a+b x+c x^2}}{836 c^3 d^5 (b d+2 c d x)^{11/2}}+\frac{\sqrt{a+b x+c x^2}}{2926 c^3 \left(b^2-4 a c\right) d^7 (b d+2 c d x)^{7/2}}+\frac{5 \sqrt{a+b x+c x^2}}{8778 c^3 \left(b^2-4 a c\right)^2 d^9 (b d+2 c d x)^{3/2}}-\frac{\left(a+b x+c x^2\right)^{3/2}}{114 c^2 d^3 (b d+2 c d x)^{15/2}}-\frac{\left(a+b x+c x^2\right)^{5/2}}{19 c d (b d+2 c d x)^{19/2}}+\frac{5 \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4} \sqrt{d}}\right],-1\right]}{17556 c^4 \left(b^2-4 a c\right)^{7/4} d^{21/2} \sqrt{a+b x+c x^2}}$$

Result (type 4, 233 leaves):

$$\left(-c (b+2 c x) (a+x (b+c x)) \left(231 \left(b^2-4 a c\right)^4-616 \left(b^2-4 a c\right)^3 (b+2 c x)^2+469 \left(b^2-4 a c\right)^2 (b+2 c x)^4-24 \left(b^2-4 a c\right) (b+2 c x)^6-40 (b+2 c x)^8\right)+\frac{20 i (b+2 c x)^{23/2} \sqrt{\frac{c (a+x (b+c x))}{(b+2 c x)^2}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4 a c}}}{\sqrt{b+2 c x}}\right],-1\right]}{\sqrt{-\sqrt{b^2-4 a c}}}\right) \Bigg/ \left(70224 c^4 \left(b^2-4 a c\right)^2 (d (b+2 c x))^{21/2} \sqrt{a+x (b+c x)}\right)$$

Problem 1357: Result unnecessarily involves imaginary or complex numbers.

$$\int (b d+2 c d x)^{5/2} \left(a+b x+c x^2\right)^{5/2} \, dx$$

Optimal (type 4, 373 leaves, 10 steps):

$$-\frac{\left(b^2-4ac\right)^3 d\,(b d+2 c d x)^{3/2}\sqrt{a+bx+cx^2}}{1326\,c^3}+\frac{5\left(b^2-4ac\right)^2 (b d+2 c d x)^{7/2}\sqrt{a+bx+cx^2}}{2652\,c^3\,d}-\frac{5\left(b^2-4ac\right)(b d+2 c d x)^{7/2}\left(a+bx+cx^2\right)^{3/2}}{442\,c^2\,d}+$$

$$\frac{(b d+2 c d x)^{7/2}\left(a+bx+cx^2\right)^{5/2}}{17\,c\,d}-\frac{\left(b^2-4ac\right)^{19/4} d^{5/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4ac\right)^{1/4}\sqrt{d}}\right],-1\right]}{884\,c^4\sqrt{a+bx+cx^2}}+$$

$$\frac{\left(b^2-4ac\right)^{19/4} d^{5/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4ac\right)^{1/4}\sqrt{d}}\right],-1\right]}{884\,c^4\sqrt{a+bx+cx^2}}$$

Result (type 4, 327 leaves) :

$$\frac{1}{2652\,c^4\sqrt{a+x\,(b+c\,x)}}\,(d\,(b+2\,c\,x))^{5/2}$$

$$\Bigg(1/(b+2\,c\,x)\,c\,(a+x\,(b+c\,x))\,\left(3\,b^6-10\,b^5\,c\,x+8\,b^3\,c^2\,x\left(19\,a+87\,c\,x^2\right)+b^4\left(-46\,a\,c+26\,c^2\,x^2\right)+16\,b\,c^3\,x\left(89\,a^2+216\,a\,c\,x^2+117\,c^2\,x^4\right)+\right.$$

$$\left.4\,b^2\,c^2\left(65\,a^2+470\,a\,c\,x^2+477\,c^2\,x^4\right)+16\,c^3\left(8\,a^3+89\,a^2\,c\,x^2+108\,a\,c^2\,x^4+39\,c^3\,x^6\right)\right)+1\Bigg/\left(-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{5/2}3\,i\,\left(b^2-4\,a\,c\right)^{7/2}$$

$$\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]\right)\Bigg)$$

- **Problem 1358: Result unnecessarily involves imaginary or complex numbers.**

$$\int\sqrt{b\,d+2\,c\,d\,x}\,\left(a+b\,x+c\,x^2\right)^{5/2}\,dx$$

Optimal (type 4, 328 leaves, 9 steps) :

$$\frac{\left(b^2-4ac\right)^2 (bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{156c^3 d}-\frac{5\left(b^2-4ac\right)(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{234c^2 d}+$$

$$\frac{(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{5/2}}{13cd}-\frac{\left(b^2-4ac\right)^{15/4}\sqrt{d}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)^{1/4}\sqrt{d}}\right],-1\right]}{156c^4\sqrt{a+bx+cx^2}}+$$

$$\frac{\left(b^2-4ac\right)^{15/4}\sqrt{d}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)^{1/4}\sqrt{d}}\right],-1\right]}{156c^4\sqrt{a+bx+cx^2}}$$

Result (type 4, 255 leaves) :

$$\frac{1}{468c^4\sqrt{a+x(b+cx)}}\sqrt{d(b+2cx)}\left(c(b+2cx)(a+x(b+cx))\left(3b^4-10b^3cx+8bc^2x\left(14a+9cx^2\right)+b^2\left(-34ac+26c^2x^2\right)+4c^2\left(31a^2+28acx^2+9c^2x^4\right)\right)+\right.$$

$$1\Big/\left(\sqrt{-\frac{b+2cx}{\sqrt{b^2-4ac}}}\right)3\,i\left(b^2-4ac\right)^{7/2}\sqrt{\frac{c(a+x(b+cx))}{-b^2+4ac}}$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2cx}{\sqrt{b^2-4ac}}}\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2cx}{\sqrt{b^2-4ac}}}\right],-1\right]\right)\right)$$

- **Problem 1359: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(a+bx+cx^2\right)^{5/2}}{(bd+2cdx)^{3/2}}\,dx$$

Optimal (type 4, 316 leaves, 9 steps) :

$$-\frac{\left(b^2-4ac\right)(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}{12c^3d^3}+\frac{5(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{18c^2d^3}-\frac{\left(a+bx+cx^2\right)^{5/2}}{cd\sqrt{bd+2cdx}}+$$

$$\frac{\left(b^2-4ac\right)^{11/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)^{1/4}\sqrt{d}}\right],-1\right]}{12c^4d^{3/2}\sqrt{a+bx+cx^2}}-\frac{\left(b^2-4ac\right)^{11/4}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)^{1/4}\sqrt{d}}\right],-1\right]}{12c^4d^{3/2}\sqrt{a+bx+cx^2}}$$

Result (type 4, 246 leaves):

$$\left(c\,(a+x\,(b+c\,x))\,\left(-3\,b^4-2\,b^3\,c\,x+8\,b\,c^2\,x\,\left(2\,a+c\,x^2\right)+2\,b^2\,c\,\left(11\,a+c\,x^2\right)+4\,c^2\,\left(-9\,a^2+4\,a\,c\,x^2+c^2\,x^4\right)\right)+3\,\mathrm{i}\,\left(b^2-4\,a\,c\right)^3\,\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]-\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]\right)\right)\Big/\left(36\,c^4\,d\,\sqrt{d\,(b+2\,c\,x)}\,\sqrt{a+x\,(b+c\,x)}\right)$$

Problem 1360: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,x+c\,x^2\right)^{5/2}}{(b\,d+2\,c\,d\,x)^{7/2}}\,dx$$

Optimal (type 4, 310 leaves, 9 steps):

$$\frac{3\,(b\,d+2\,c\,d\,x)^{3/2}\,\sqrt{a+b\,x+c\,x^2}}{20\,c^3\,d^5}-\frac{\left(a+b\,x+c\,x^2\right)^{3/2}}{2\,c^2\,d^3\,\sqrt{b\,d+2\,c\,d\,x}}-\frac{\left(a+b\,x+c\,x^2\right)^{5/2}}{5\,c\,d\,(b\,d+2\,c\,d\,x)^{5/2}}-\frac{3\,\left(b^2-4\,a\,c\right)^{7/4}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\,\sqrt{d}}\right],-1\right]}{20\,c^4\,d^{7/2}\,\sqrt{a+b\,x+c\,x^2}}+\frac{3\,\left(b^2-4\,a\,c\right)^{7/4}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\,\sqrt{d}}\right],-1\right]}{20\,c^4\,d^{7/2}\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 253 leaves):

$$\left(c\,(a+x\,(b+c\,x))\,\left(3\,b^4+14\,b^3\,c\,x+8\,b\,c^2\,x\,\left(-6\,a+c\,x^2\right)+2\,b^2\,c\,\left(-5\,a+9\,c\,x^2\right)+4\,c^2\,\left(-a^2-12\,a\,c\,x^2+c^2\,x^4\right)\right)-1\Big/\left(-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{3/2}\,3\,\mathrm{i}\,\left(b^2-4\,a\,c\right)\,(b+2\,c\,x)^4\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]-\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]\right)\right)\Big/\left(20\,c^4\,d\,(d\,(b+2\,c\,x))^{5/2}\,\sqrt{a+x\,(b+c\,x)}\right)$$

Problem 1361: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x+c x^2\right)^{5/2}}{(b d+2 c d x)^{11/2}} \, dx$$

Optimal (type 4, 310 leaves, 9 steps):

$$-\frac{\sqrt{a+b x+c x^2}}{12 c^3 d^5 \sqrt{b d+2 c d x}}-\frac{\left(a+b x+c x^2\right)^{3/2}}{18 c^2 d^3 (b d+2 c d x)^{5/2}}-\frac{\left(a+b x+c x^2\right)^{5/2}}{9 c d (b d+2 c d x)^{9/2}}+$$
$$\frac{\left(b^2-4 a c\right)^{3/4} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4} \sqrt{d}}\right],-1\right]}{12 c^4 d^{11/2} \sqrt{a+b x+c x^2}}-\frac{\left(b^2-4 a c\right)^{3/4} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4} \sqrt{d}}\right],-1\right]}{12 c^4 d^{11/2} \sqrt{a+b x+c x^2}}$$

Result (type 4, 251 leaves):

$$\left(-1\Big/\left(3 c^3\right) (b+2 c x) (a+x (b+c x)) \left(3 b^4+26 b^3 c x+8 b c^2 x \left(2 a+15 c x^2\right)+2 b^2 c \left(a+43 c x^2\right)+4 c^2 \left(a^2+4 a c x^2+15 c^2 x^4\right)\right)+\right.$$
$$1\Big/\left(c^4 \left(-\frac{b+2 c x}{\sqrt{b^2-4 a c}}\right)^{3/2}\right) i (b+2 c x)^7 \sqrt{\frac{c (a+x (b+c x))}{-b^2+4 a c}}$$
$$\left.\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{b+2 c x}{\sqrt{b^2-4 a c}}}\right],-1\right]-\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{b+2 c x}{\sqrt{b^2-4 a c}}}\right],-1\right]\right)\right)\Big/\left(12 (d (b+2 c x))^{11/2} \sqrt{a+x (b+c x)}\right)$$

Problem 1362: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x+c x^2\right)^{5/2}}{(b d+2 c d x)^{15/2}} \, dx$$

Optimal (type 4, 357 leaves, 10 steps):

$$-\frac{\sqrt{a+b x+c x^2}}{156 c^3 d^5 (b d+2 c d x)^{5/2}}+\frac{\sqrt{a+b x+c x^2}}{78 c^3 \left(b^2-4 a c\right) d^7 \sqrt{b d+2 c d x}}-\frac{5 \left(a+b x+c x^2\right)^{3/2}}{234 c^2 d^3 (b d+2 c d x)^{9/2}}-\frac{\left(a+b x+c x^2\right)^{5/2}}{13 c d (b d+2 c d x)^{13/2}}-$$
$$\frac{\sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4} \sqrt{d}}\right],-1\right]}{156 c^4 \left(b^2-4 a c\right)^{1/4} d^{15/2} \sqrt{a+b x+c x^2}}+\frac{\sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4} \sqrt{d}}\right],-1\right]}{156 c^4 \left(b^2-4 a c\right)^{1/4} d^{15/2} \sqrt{a+b x+c x^2}}$$

Result (type 4, 254 leaves):

$$\left(-1\Big/\left(12\,c^3\,\left(b^2-4\,a\,c\right)\right)\,(b+2\,c\,x)\,\,(a+x\,(b+c\,x))\,\left(9\,\left(b^2-4\,a\,c\right)^3-28\,\left(b^2-4\,a\,c\right)^2\,(b+2\,c\,x)^2+31\,\left(b^2-4\,a\,c\right)\,(b+2\,c\,x)^4-24\,(b+2\,c\,x)^6\right)-\right.$$

$$1\Big/c^4\,i\,(b+2\,c\,x)^7\,\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]\right)\right)\Big/\left(156\,(d\,(b+2\,c\,x))^{15/2}\,\sqrt{a+x\,(b+c\,x)}\right)$$

- **Problem 1363: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(b\,d+2\,c\,d\,x)^{7/2}}{\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 174 leaves, 5 steps):

$$\frac{20}{21}\left(b^2-4\,a\,c\right)\,d^3\,\sqrt{b\,d+2\,c\,d\,x}\,\sqrt{a+b\,x+c\,x^2}+\frac{4}{7}\,d\,(b\,d+2\,c\,d\,x)^{5/2}\,\sqrt{a+b\,x+c\,x^2}+$$

$$\frac{10\,\left(b^2-4\,a\,c\right)^{9/4}\,d^{7/2}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\,\sqrt{d}}\right],-1\right]}{21\,c\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 176 leaves):

$$\frac{1}{21\,\sqrt{a+x\,(b+c\,x)}}$$

$$(d\,(b+2\,c\,x))^{7/2}\left(\frac{16\,(a+x\,(b+c\,x))\,\left(2\,b^2+3\,b\,c\,x+c\,\left(-5\,a+3\,c\,x^2\right)\right)}{(b+2\,c\,x)^3}+\frac{10\,i\,\left(b^2-4\,a\,c\right)^2\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{(b+2\,c\,x)^2}}\,\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4\,a\,c}}}{\sqrt{b+2\,c\,x}}\right],-1\right]}{c\,\sqrt{-\sqrt{b^2-4\,a\,c}}\,\,(b+2\,c\,x)^{5/2}}\right)$$

- **Problem 1364: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(b\,d+2\,c\,d\,x)^{3/2}}{\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 132 leaves, 4 steps):

$$\frac{4}{3} d \sqrt{b d+2 c d x} \sqrt{a+b x+c x^2}+\frac{2 \left(b^2-4 a c\right)^{5/4} d^{3/2} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4} \sqrt{d}}\right],-1\right]}{3 c \sqrt{a+b x+c x^2}}$$

Result (type 4, 144 leaves):

$$\frac{2 d \sqrt{d (b+2 c x)} \left(2 (a+x (b+c x))+\frac{i \left(b^2-4 a c\right) \sqrt{b+2 c x} \sqrt{\frac{c (a+x (b+c x))}{(b+2 c x)^2}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4 a c}}}{\sqrt{b+2 c x}}\right],-1\right]}{c \sqrt{-\sqrt{b^2-4 a c}}}\right)}{3 \sqrt{a+x (b+c x)}}$$

■ **Problem 1365: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{b d+2 c d x} \sqrt{a+b x+c x^2}} \, dx$$

Optimal (type 4, 97 leaves, 3 steps):

$$\frac{2 \left(b^2-4 a c\right)^{1/4} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4} \sqrt{d}}\right],-1\right]}{c \sqrt{d} \sqrt{a+b x+c x^2}}$$

Result (type 4, 116 leaves):

$$\frac{2 i \sqrt{a+x (b+c x)} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4 a c}}}{\sqrt{b+2 c x}}\right],-1\right]}{\sqrt{-\sqrt{b^2-4 a c}} \sqrt{b+2 c x} \sqrt{d (b+2 c x)} \sqrt{\frac{c (a+x (b+c x))}{(b+2 c x)^2}}}$$

■ **Problem 1366: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(b d+2 c d x)^{5/2} \sqrt{a+b x+c x^2}} \, dx$$

Optimal (type 4, 144 leaves, 4 steps):

$$\frac{4 \sqrt{a+b x+c x^2}}{3 \left(b^2-4 a c\right) d (b d+2 c d x)^{3/2}}+\frac{2 \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4} \sqrt{d}}\right],-1\right]}{3 c \left(b^2-4 a c\right)^{3/4} d^{5/2} \sqrt{a+b x+c x^2}}$$

Result (type 4, 167 leaves):

$$\left(4\,c\,\sqrt{-\sqrt{b^2-4\,a\,c}}\;(a+x\,(b+c\,x))+2\,\mathrm{i}\,(b+2\,c\,x)^{5/2}\sqrt{\frac{c\,(a+x\,(b+c\,x))}{(b+2\,c\,x)^2}}\;\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4\,a\,c}}}{\sqrt{b+2\,c\,x}}\right],\,-1\right]\right)\Big/$$

$$\left(3\,c\,\sqrt{-\sqrt{b^2-4\,a\,c}}\;\left(b^2-4\,a\,c\right)\,d\,(d\,(b+2\,c\,x))^{3/2}\,\sqrt{a+x\,(b+c\,x)}\right)$$

- **Problem 1367: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{(b\,d+2\,c\,d\,x)^{9/2}\,\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 188 leaves, 5 steps):

$$\frac{4\,\sqrt{a+b\,x+c\,x^2}}{7\,\left(b^2-4\,a\,c\right)\,d\,(b\,d+2\,c\,d\,x)^{7/2}}+\frac{20\,\sqrt{a+b\,x+c\,x^2}}{21\,\left(b^2-4\,a\,c\right)^2\,d^3\,(b\,d+2\,c\,d\,x)^{3/2}}+\frac{10\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\,\sqrt{d}}\right],\,-1\right]}{21\,c\,\left(b^2-4\,a\,c\right)^{7/4}\,d^{9/2}\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 172 leaves):

$$\left(2\left(2\,(b+2\,c\,x)\,(a+x\,(b+c\,x))\,\left(3\,\left(b^2-4\,a\,c\right)+5\,(b+2\,c\,x)^2\right)+\frac{5\,\mathrm{i}\,(b+2\,c\,x)^{11/2}\sqrt{\frac{c\,(a+x\,(b+c\,x))}{(b+2\,c\,x)^2}}\;\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4\,a\,c}}}{\sqrt{b+2\,c\,x}}\right],\,-1\right]}{c\,\sqrt{-\sqrt{b^2-4\,a\,c}}}\right)\right)\Big/$$

$$\left(21\,\left(b^2-4\,a\,c\right)^2\,(d\,(b+2\,c\,x))^{9/2}\,\sqrt{a+x\,(b+c\,x)}\right)$$

- **Problem 1368: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(b\,d+2\,c\,d\,x)^{9/2}}{\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 273 leaves, 8 steps):

$$\frac{28}{45}\left(b^2-4ac\right)d^3\,(b d+2 c d x)^{3/2}\sqrt{a+b x+c x^2}+\frac{4}{9}d\,(b d+2 c d x)^{7/2}\sqrt{a+b x+c x^2}+$$

$$\frac{14\left(b^2-4ac\right)^{11/4}d^{9/2}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4}\sqrt{d}}\right],-1\right]}{15 c\sqrt{a+b x+c x^2}}-$$

$$\frac{14\left(b^2-4ac\right)^{11/4}d^{9/2}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4}\sqrt{d}}\right],-1\right]}{15 c\sqrt{a+b x+c x^2}}$$

Result (type 4, 205 leaves) :

$$\left(2\,(d\,(b+2 c x))^{9/2}\left(8\,(a+x\,(b+c x))\left(3 b^2+5 b c x+c\left(-7 a+5 c x^2\right)\right)+1\Big/\left(c\left(-\frac{b+2 c x}{\sqrt{b^2-4 a c}}\right)^{3/2}\right)21\,\text{i}\left(b^2-4 a c\right)^2\sqrt{\frac{c\,(a+x\,(b+c x))}{-b^2+4 a c}}\right.\right.$$

$$\left.\left.\left(\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{-\frac{b+2 c x}{\sqrt{b^2-4 a c}}}\right],-1\right]-\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{-\frac{b+2 c x}{\sqrt{b^2-4 a c}}}\right],-1\right]\right)\right)\right)\Big/\left(45\,(b+2 c x)^3\sqrt{a+x\,(b+c x)}\right)$$

- **Problem 1369: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(b d+2 c d x)^{5/2}}{\sqrt{a+b x+c x^2}}\,dx$$

Optimal (type 4, 231 leaves, 7 steps) :

$$\frac{4}{5}d\,(b d+2 c d x)^{3/2}\sqrt{a+b x+c x^2}+\frac{6\left(b^2-4ac\right)^{7/4}d^{5/2}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4}\sqrt{d}}\right],-1\right]}{5 c\sqrt{a+b x+c x^2}}-$$

$$\frac{6\left(b^2-4ac\right)^{7/4}d^{5/2}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4}\sqrt{d}}\right],-1\right]}{5 c\sqrt{a+b x+c x^2}}$$

Result (type 4, 248 leaves) :

$$-\left(2\, d^3 \sqrt{-\frac{b+2\, c\, x}{\sqrt{b^2-4\, a\, c}}} \left(-\frac{2\, c\, (b+2\, c\, x)^2\, (a+x\, (b+c\, x))}{\sqrt{-\frac{b+2\, c\, x}{\sqrt{b^2-4\, a\, c}}}} - 3\, i\, \left(b^2-4\, a\, c\right)^2 \sqrt{\frac{c\, (a+x\, (b+c\, x))}{-b^2+4\, a\, c}}\; \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{-\frac{b+2\, c\, x}{\sqrt{b^2-4\, a\, c}}}\right], -1\right] + 3\, i\, \left(b^2-4\, a\, c\right)^2 \sqrt{\frac{c\, (a+x\, (b+c\, x))}{-b^2+4\, a\, c}}\; \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{-\frac{b+2\, c\, x}{\sqrt{b^2-4\, a\, c}}}\right], -1\right]\right)\right) \Big/ \left(5\, c\, \sqrt{d\, (b+2\, c\, x)}\, \sqrt{a+x\, (b+c\, x)}\right)$$

Problem 1370: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{b\, d+2\, c\, d\, x}}{\sqrt{a+b\, x+c\, x^2}}\, dx$$

Optimal (type 4, 195 leaves, 6 steps):

$$\frac{2\, \left(b^2-4\, a\, c\right)^{3/4} \sqrt{d}\, \sqrt{-\frac{c\, (a+b\, x+c\, x^2)}{b^2-4\, a\, c}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b\, d+2\, c\, d\, x}}{\left(b^2-4\, a\, c\right)^{1/4} \sqrt{d}}\right], -1\right]}{c\, \sqrt{a+b\, x+c\, x^2}} - \frac{2\, \left(b^2-4\, a\, c\right)^{3/4} \sqrt{d}\, \sqrt{-\frac{c\, (a+b\, x+c\, x^2)}{b^2-4\, a\, c}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\, d+2\, c\, d\, x}}{\left(b^2-4\, a\, c\right)^{1/4} \sqrt{d}}\right], -1\right]}{c\, \sqrt{a+b\, x+c\, x^2}}$$

Result (type 4, 151 leaves):

$$\left(2\, i\, (d\, (b+2\, c\, x))^{3/2} \sqrt{\frac{c\, (a+x\, (b+c\, x))}{-b^2+4\, a\, c}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{-\frac{b+2\, c\, x}{\sqrt{b^2-4\, a\, c}}}\right], -1\right] - \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{-\frac{b+2\, c\, x}{\sqrt{b^2-4\, a\, c}}}\right], -1\right]\right)\right) \Big/ \left(c\, d\, \left(-\frac{b+2\, c\, x}{\sqrt{b^2-4\, a\, c}}\right)^{3/2} \sqrt{a+x\, (b+c\, x)}\right)$$

Problem 1371: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(b\, d+2\, c\, d\, x)^{3/2} \sqrt{a+b\, x+c\, x^2}}\, dx$$

Optimal (type 4, 237 leaves, 7 steps):

$$\frac{4\sqrt{a+b\,x+c\,x^2}}{\left(b^2-4\,a\,c\right)\,d\,\sqrt{b\,d+2\,c\,d\,x}}-\frac{2\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\sqrt{d}}\right],\,-1\right]}{c\,\left(b^2-4\,a\,c\right)^{1/4}\,d^{3/2}\,\sqrt{a+b\,x+c\,x^2}}+\frac{2\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\sqrt{d}}\right],\,-1\right]}{c\,\left(b^2-4\,a\,c\right)^{1/4}\,d^{3/2}\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 243 leaves) :

$$-\left(2\,i\left(2\,i\,c\,(a+x\,(b+c\,x))+\left(b^2-4\,a\,c\right)\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],\,-1\right]-\left(b^2-4\,a\,c\right)\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],\,-1\right]\right)\right)\Big/\left(c\,\left(b^2-4\,a\,c\right)\,d\,\sqrt{d\,(b+2\,c\,x)}\;\sqrt{a+x\,(b+c\,x)}\right)$$

- **Problem 1372: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{(b\,d+2\,c\,d\,x)^{7/2}\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 287 leaves, 8 steps) :

$$\frac{4\sqrt{a+b\,x+c\,x^2}}{5\,\left(b^2-4\,a\,c\right)\,d\,(b\,d+2\,c\,d\,x)^{5/2}}+\frac{12\sqrt{a+b\,x+c\,x^2}}{5\,\left(b^2-4\,a\,c\right)^2\,d^3\,\sqrt{b\,d+2\,c\,d\,x}}-$$

$$\frac{6\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\sqrt{d}}\right],\,-1\right]}{5\,c\,\left(b^2-4\,a\,c\right)^{5/4}\,d^{7/2}\,\sqrt{a+b\,x+c\,x^2}}+\frac{6\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\sqrt{d}}\right],\,-1\right]}{5\,c\,\left(b^2-4\,a\,c\right)^{5/4}\,d^{7/2}\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 206 leaves) :

$$\left(2\left(8\,(a+x\,(b+c\,x))\,\left(b^2+3\,b\,c\,x+c\,\left(-a+3\,c\,x^2\right)\right)-1\Big/\left(c\left(-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{3/2}\right)3\,i\,(b+2\,c\,x)^4\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],\,-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],\,-1\right]\right)\right)\right)\Big/\left(5\,\left(b^2-4\,a\,c\right)^2\,d\,(d\,(b+2\,c\,x))^{5/2}\,\sqrt{a+x\,(b+c\,x)}\right)$$

Problem 1379: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(b\,d+2\,c\,d\,x)^{11/2}}{\left(a+b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 205 leaves, 6 steps):

$$-\frac{2\,d\,(b\,d+2\,c\,d\,x)^{9/2}}{\sqrt{a+b\,x+c\,x^2}}+\frac{120}{7}\,c\,\left(b^2-4\,a\,c\right)\,d^5\,\sqrt{b\,d+2\,c\,d\,x}\,\sqrt{a+b\,x+c\,x^2}+$$
$$\frac{72}{7}\,c\,d^3\,(b\,d+2\,c\,d\,x)^{5/2}\,\sqrt{a+b\,x+c\,x^2}+\frac{60\,\left(b^2-4\,a\,c\right)^{9/4}\,d^{11/2}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\,\sqrt{d}}\right],-1\right]}{7\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 202 leaves):

$$\frac{1}{7\,\sqrt{a+x\,(b+c\,x)}}(d\,(b+2\,c\,x))^{11/2}\left(\frac{2\,(a+x\,(b+c\,x))\,\left(-8\,c\,\left(-5\,b^2+16\,a\,c\right)+32\,b\,c^2\,x+32\,c^3\,x^2-\frac{7\,\left(b^2-4\,a\,c\right)^2}{a+x\,(b+c\,x)}\right)}{(b+2\,c\,x)^5}+\right.$$
$$\left.\frac{60\,i\,\left(b^2-4\,a\,c\right)^2\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{(b+2\,c\,x)^2}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4\,a\,c}}}{\sqrt{b+2\,c\,x}}\right],-1\right]}{\sqrt{-\sqrt{b^2-4\,a\,c}}\,\,(b+2\,c\,x)^{9/2}}\right)$$

Problem 1380: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(b\,d+2\,c\,d\,x)^{7/2}}{\left(a+b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 162 leaves, 5 steps):

$$-\frac{2\,d\,(b\,d+2\,c\,d\,x)^{5/2}}{\sqrt{a+b\,x+c\,x^2}}+\frac{40}{3}\,c\,d^3\,\sqrt{b\,d+2\,c\,d\,x}\,\sqrt{a+b\,x+c\,x^2}+\frac{20\,\left(b^2-4\,a\,c\right)^{5/4}\,d^{7/2}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\,\sqrt{d}}\right],-1\right]}{3\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 175 leaves):

$$-\left(2\,d^3\sqrt{d\,(b+2\,c\,x)}\left(\sqrt{-\sqrt{b^2-4\,a\,c}}\,\left(3\,b^2-8\,b\,c\,x-4\,c\,\left(5\,a+2\,c\,x^2\right)\right)-\right.\right.$$

$$\left.\left.10\,i\,\left(b^2-4\,a\,c\right)\sqrt{b+2\,c\,x}\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{(b+2\,c\,x)^2}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4\,a\,c}}}{\sqrt{b+2\,c\,x}}\right],-1\right]\right)\right)\Big/\left(3\,\sqrt{-\sqrt{b^2-4\,a\,c}}\,\sqrt{a+x\,(b+c\,x)}\right)$$

- **Problem 1381: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(b\,d+2\,c\,d\,x)^{3/2}}{\left(a+b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 125 leaves, 4 steps):

$$-\frac{2\,d\,\sqrt{b\,d+2\,c\,d\,x}}{\sqrt{a+b\,x+c\,x^2}}+\frac{4\,\left(b^2-4\,a\,c\right)^{1/4}\,d^{3/2}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\sqrt{d}}\right],-1\right]}{\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 138 leaves):

$$-\left(2\,d\,\sqrt{d\,(b+2\,c\,x)}\left(\sqrt{-\sqrt{b^2-4\,a\,c}}\,-2\,i\,\sqrt{b+2\,c\,x}\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{(b+2\,c\,x)^2}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4\,a\,c}}}{\sqrt{b+2\,c\,x}}\right],-1\right]\right)\right)\Big/$$

$$\left(\sqrt{-\sqrt{b^2-4\,a\,c}}\,\sqrt{a+x\,(b+c\,x)}\right)$$

- **Problem 1382: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\sqrt{b\,d+2\,c\,d\,x}\,\left(a+b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 137 leaves, 4 steps):

$$-\frac{2\,\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)\,d\,\sqrt{a+b\,x+c\,x^2}}-\frac{4\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\sqrt{d}}\right],-1\right]}{\left(b^2-4\,a\,c\right)^{3/4}\sqrt{d}\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 150 leaves) :

$$-\left(2\sqrt{d\,(b+2\,c\,x)}\,\left(\sqrt{-\sqrt{b^2-4\,a\,c}}+2\,i\,\sqrt{b+2\,c\,x}\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{(b+2\,c\,x)^2}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4\,a\,c}}}{\sqrt{b+2\,c\,x}}\right],-1\right]\right)\right)\Big/$$
$$\left(\sqrt{-\sqrt{b^2-4\,a\,c}}\,\left(b^2-4\,a\,c\right)\,d\,\sqrt{a+x\,(b+c\,x)}\right)$$

■ **Problem 1383: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{(b\,d+2\,c\,d\,x)^{5/2}\,\left(a+b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 184 leaves, 5 steps) :

$$-\frac{2}{\left(b^2-4\,a\,c\right)\,d\,(b\,d+2\,c\,d\,x)^{3/2}\,\sqrt{a+b\,x+c\,x^2}}-\frac{40\,c\,\sqrt{a+b\,x+c\,x^2}}{3\,\left(b^2-4\,a\,c\right)^2\,d\,(b\,d+2\,c\,d\,x)^{3/2}}-\frac{20\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\,\sqrt{d}}\right],-1\right]}{3\,\left(b^2-4\,a\,c\right)^{7/4}\,d^{5/2}\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 177 leaves) :

$$-\left(2\left(\sqrt{-\sqrt{b^2-4\,a\,c}}\,\left(3\,b^2+20\,b\,c\,x+4\,c\,\left(2\,a+5\,c\,x^2\right)\right)+10\,i\,(b+2\,c\,x)^{5/2}\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{(b+2\,c\,x)^2}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4\,a\,c}}}{\sqrt{b+2\,c\,x}}\right],-1\right]\right)\right)\Big/$$
$$\left(3\sqrt{-\sqrt{b^2-4\,a\,c}}\,\left(b^2-4\,a\,c\right)^2\,d\,(d\,(b+2\,c\,x))^{3/2}\,\sqrt{a+x\,(b+c\,x)}\right)$$

■ **Problem 1384: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(b\,d+2\,c\,d\,x)^{9/2}}{\left(a+b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 258 leaves, 8 steps) :

$$-\frac{2\,d\,(b\,d+2\,c\,d\,x)^{7/2}}{\sqrt{a+b\,x+c\,x^2}}+\frac{56}{5}\,c\,d^3\,(b\,d+2\,c\,d\,x)^{3/2}\,\sqrt{a+b\,x+c\,x^2}+\frac{84\,\left(b^2-4\,a\,c\right)^{7/4}\,d^{9/2}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\,\sqrt{d}}\right],-1\right]}{5\,\sqrt{a+b\,x+c\,x^2}}-$$

$$\frac{84\,\left(b^2-4\,a\,c\right)^{7/4}\,d^{9/2}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\,\sqrt{d}}\right],-1\right]}{5\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 258 leaves) :

$$-\left(2\,d^5\,\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\frac{(b+2\,c\,x)^2\,\left(5\,b^2-8\,b\,c\,x-4\,c\,\left(7\,a+2\,c\,x^2\right)\right)}{\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}-42\,i\,\left(b^2-4\,a\,c\right)^2\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]+\right.$$

$$\left.\left.42\,i\,\left(b^2-4\,a\,c\right)^2\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]\right)\right)\Big/\left(5\,\sqrt{d\,(b+2\,c\,x)}\,\sqrt{a+x\,(b+c\,x)}\right)$$

- **Problem 1385: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(b\,d+2\,c\,d\,x)^{5/2}}{\left(a+b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 219 leaves, 7 steps) :

$$-\frac{2\,d\,(b\,d+2\,c\,d\,x)^{3/2}}{\sqrt{a+b\,x+c\,x^2}}+\frac{12\,\left(b^2-4\,a\,c\right)^{3/4}\,d^{5/2}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\,\sqrt{d}}\right],-1\right]}{\sqrt{a+b\,x+c\,x^2}}-$$

$$\frac{12\,\left(b^2-4\,a\,c\right)^{3/4}\,d^{5/2}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\,\sqrt{d}}\right],-1\right]}{\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 229 leaves) :

$$\left(2\,\mathrm{i}\,d^3\left(\mathrm{i}\,(b+2\,c\,x)^2+6\,(b^2-4\,a\,c)\,\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]-6\,(b^2-4\,a\,c)\,\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]\right)\right)\Big/\left(\sqrt{d\,(b+2\,c\,x)}\,\sqrt{a+x\,(b+c\,x)}\right)$$

Problem 1386: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{b\,d+2\,c\,d\,x}}{(a+b\,x+c\,x^2)^{3/2}}\,dx$$

Optimal (type 4, 231 leaves, 7 steps) :

$$-\frac{2\,(b\,d+2\,c\,d\,x)^{3/2}}{(b^2-4\,a\,c)\,d\,\sqrt{a+b\,x+c\,x^2}}+\frac{4\,\sqrt{d}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{(b^2-4\,a\,c)^{1/4}\,\sqrt{d}}\right],-1\right]}{(b^2-4\,a\,c)^{1/4}\,\sqrt{a+b\,x+c\,x^2}}-\frac{4\,\sqrt{d}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{(b^2-4\,a\,c)^{1/4}\,\sqrt{d}}\right],-1\right]}{(b^2-4\,a\,c)^{1/4}\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 238 leaves) :

$$-\left(2\,\mathrm{i}\,\sqrt{d\,(b+2\,c\,x)}\,\left(\frac{\mathrm{i}\,(b+2\,c\,x)^2}{\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}+2\,(b^2-4\,a\,c)\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]-2\,(b^2-4\,a\,c)\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]\right)\right)\Big/\left((b^2-4\,a\,c)^{3/2}\,\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+x\,(b+c\,x)}\right)$$

Problem 1387: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{(b\,d+2\,c\,d\,x)^{3/2}\,(a+b\,x+c\,x^2)^{3/2}}\,dx$$

Optimal (type 4, 274 leaves, 8 steps):

$$-\frac{2}{\left(b^2-4ac\right) d \sqrt{bd+2cdx}\ \sqrt{a+bx+cx^2}}-\frac{24c\sqrt{a+bx+cx^2}}{\left(b^2-4ac\right)^2 d\sqrt{bd+2cdx}}+$$
$$\frac{12\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)^{1/4}\sqrt{d}}\right],-1\right]}{\left(b^2-4ac\right)^{5/4} d^{3/2}\sqrt{a+bx+cx^2}}-\frac{12\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)^{1/4}\sqrt{d}}\right],-1\right]}{\left(b^2-4ac\right)^{5/4} d^{3/2}\sqrt{a+bx+cx^2}}$$

Result (type 4, 253 leaves):

$$\left(2\,i\left(i\left(b^2+12bcx+4c\left(2a+3cx^2\right)\right)+6\left(b^2-4ac\right)\sqrt{-\frac{b+2cx}{\sqrt{b^2-4ac}}}\ \sqrt{\frac{c\left(a+x\left(b+cx\right)\right)}{-b^2+4ac}}\ \text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2cx}{\sqrt{b^2-4ac}}}\right],-1\right]-\right.\right.$$
$$\left.\left.6\left(b^2-4ac\right)\sqrt{-\frac{b+2cx}{\sqrt{b^2-4ac}}}\ \sqrt{\frac{c\left(a+x\left(b+cx\right)\right)}{-b^2+4ac}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2cx}{\sqrt{b^2-4ac}}}\right],-1\right]\right)\right)\Big/$$
$$\left(\left(b^2-4ac\right)^2 d\sqrt{d\left(b+2cx\right)}\ \sqrt{a+x\left(b+cx\right)}\right)$$

- **Problem 1388: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\left(bd+2cdx\right)^{7/2}\left(a+bx+cx^2\right)^{3/2}}\,dx$$

Optimal (type 4, 325 leaves, 9 steps):

$$-\frac{2}{\left(b^2-4ac\right) d\left(bd+2cdx\right)^{5/2}\sqrt{a+bx+cx^2}}-\frac{56c\sqrt{a+bx+cx^2}}{5\left(b^2-4ac\right)^2 d\left(bd+2cdx\right)^{5/2}}-\frac{168c\sqrt{a+bx+cx^2}}{5\left(b^2-4ac\right)^3 d^3\sqrt{bd+2cdx}}+$$
$$\frac{84\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)^{1/4}\sqrt{d}}\right],-1\right]}{5\left(b^2-4ac\right)^{9/4} d^{7/2}\sqrt{a+bx+cx^2}}-\frac{84\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)^{1/4}\sqrt{d}}\right],-1\right]}{5\left(b^2-4ac\right)^{9/4} d^{7/2}\sqrt{a+bx+cx^2}}$$

Result (type 4, 221 leaves):

$$\left(2\left(-5(b+2cx)^4-8c\left(b^2-4ac\right)(a+x(b+cx))-64c(b+2cx)^2(a+x(b+cx))+1\Big/\left(-\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^{3/2}42\,i\,(b+2cx)^4\right.\right.$$
$$\left.\left.\sqrt{\frac{c(a+x(b+cx))}{-b^2+4ac}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2cx}{\sqrt{b^2-4ac}}}\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2cx}{\sqrt{b^2-4ac}}}\right],-1\right]\right)\right)\right)\Big/$$
$$\left(5\left(b^2-4ac\right)^3d\,(d(b+2cx))^{5/2}\sqrt{a+x(b+cx)}\right)$$

Problem 1389: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(b\,d+2\,c\,d\,x)^{15/2}}{\left(a+b\,x+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 247 leaves, 7 steps):

$$-\frac{2d(bd+2cdx)^{13/2}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{52cd^3(bd+2cdx)^{9/2}}{3\sqrt{a+bx+cx^2}}+\frac{1040}{7}c^2\left(b^2-4ac\right)d^7\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}+$$
$$\frac{624}{7}c^2d^5(bd+2cdx)^{5/2}\sqrt{a+bx+cx^2}+\frac{520c\left(b^2-4ac\right)^{9/4}d^{15/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)^{1/4}\sqrt{d}}\right],-1\right]}{7\sqrt{a+bx+cx^2}}$$

Result (type 4, 231 leaves):

$$\frac{1}{7\sqrt{a+x(b+cx)}}(d(b+2cx))^{15/2}\left(\frac{2(a+x(b+cx))\left(-64c^2\left(-11b^2+38ac\right)+384bc^3x+384c^4x^2-\frac{7\left(b^2-4ac\right)^3}{(a+x(b+cx))^2}-\frac{266c\left(b^2-4ac\right)^2}{a+x(b+cx)}\right)}{3(b+2cx)^7}+\right.$$
$$\left.\frac{520\,i\,c\left(b^2-4ac\right)^2\sqrt{\frac{c(a+x(b+cx))}{(b+2cx)^2}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4ac}}}{\sqrt{b+2cx}}\right],-1\right]}{\sqrt{-\sqrt{b^2-4ac}}\,(b+2cx)^{13/2}}\right)$$

Problem 1390: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(b\,d+2\,c\,d\,x)^{11/2}}{\left(a+b\,x+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 196 leaves, 6 steps):

$$-\frac{2 d (b d+2 c d x)^{9/2}}{3 \left(a+b x+c x^2\right)^{3/2}}-\frac{12 c d^3 (b d+2 c d x)^{5/2}}{\sqrt{a+b x+c x^2}}+80 c^2 d^5 \sqrt{b d+2 c d x} \sqrt{a+b x+c x^2}+\frac{40 c \left(b^2-4 a c\right)^{5/4} d^{11/2} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4} \sqrt{d}}\right],-1\right]}{\sqrt{a+b x+c x^2}}$$

Result (type 4, 201 leaves):

$$\frac{1}{\sqrt{a+x (b+c x)}}(d (b+2 c x))^{11/2} \left(\frac{2 (a+x (b+c x)) \left(32 c^2-\frac{\left(b^2-4 a c\right)^2}{(a+x (b+c x))^2}+\frac{26 c \left(-b^2+4 a c\right)}{a+x (b+c x)}\right)}{3 (b+2 c x)^5}+\frac{40 i c \left(b^2-4 a c\right) \sqrt{\frac{c (a+x (b+c x))}{(b+2 c x)^2}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4 a c}}}{\sqrt{b+2 c x}}\right],-1\right]}{\sqrt{-\sqrt{b^2-4 a c}} (b+2 c x)^{9/2}}\right)$$

Problem 1391: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(b d+2 c d x)^{7/2}}{\left(a+b x+c x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 165 leaves, 5 steps):

$$-\frac{2 d (b d+2 c d x)^{5/2}}{3 \left(a+b x+c x^2\right)^{3/2}}-\frac{20 c d^3 \sqrt{b d+2 c d x}}{3 \sqrt{a+b x+c x^2}}+\frac{40 c \left(b^2-4 a c\right)^{1/4} d^{7/2} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b d+2 c d x}}{\left(b^2-4 a c\right)^{1/4} \sqrt{d}}\right],-1\right]}{3 \sqrt{a+b x+c x^2}}$$

Result (type 4, 165 leaves):

$$\frac{(d (b+2 c x))^{7/2} \left(-\frac{2 \left(b^2+14 b c x+2 c \left(5 a+7 c x^2\right)\right)}{(b+2 c x)^3 (a+x (b+c x))}+\frac{40 i c \sqrt{\frac{c (a+x (b+c x))}{(b+2 c x)^2}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4 a c}}}{\sqrt{b+2 c x}}\right],-1\right]}{\sqrt{-\sqrt{b^2-4 a c}} (b+2 c x)^{5/2}}\right)}{3 \sqrt{a+x (b+c x)}}$$

Problem 1392: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(b d+2 c d x)^{3/2}}{\left(a+b x+c x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 173 leaves, 5 steps):

$$-\frac{2\,d\,\sqrt{b\,d+2\,c\,d\,x}}{3\,\left(a+b\,x+c\,x^2\right)^{3/2}}-\frac{4\,c\,d\,\sqrt{b\,d+2\,c\,d\,x}}{3\,\left(b^2-4\,a\,c\right)\,\sqrt{a+b\,x+c\,x^2}}-\frac{8\,c\,d^{3/2}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\,\sqrt{d}}\right],-1\right]}{3\,\left(b^2-4\,a\,c\right)^{3/4}\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 168 leaves):

$$-\left(2\,d\,\sqrt{d\,\left(b+2\,c\,x\right)}\,\left(\sqrt{b^2-4\,a\,c}\,\left(b^2+2\,b\,c\,x+2\,c\,\left(-a+c\,x^2\right)\right)-4\,i\,\sqrt{-\sqrt{b^2-4\,a\,c}}\,\left(b+2\,c\,x\right)^{5/2}\left(\frac{c\,\left(a+x\,\left(b+c\,x\right)\right)}{\left(b+2\,c\,x\right)^2}\right)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4\,a\,c}}}{\sqrt{b+2\,c\,x}}\right],-1\right]\right)\right)\Big/\left(3\,\left(b^2-4\,a\,c\right)^{3/2}\,\left(a+x\,\left(b+c\,x\right)\right)^{3/2}\right)$$

- **Problem 1393: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\sqrt{b\,d+2\,c\,d\,x}\,\left(a+b\,x+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 187 leaves, 5 steps):

$$-\frac{2\,\sqrt{b\,d+2\,c\,d\,x}}{3\,\left(b^2-4\,a\,c\right)\,d\,\left(a+b\,x+c\,x^2\right)^{3/2}}+\frac{20\,c\,\sqrt{b\,d+2\,c\,d\,x}}{3\,\left(b^2-4\,a\,c\right)^2\,d\,\sqrt{a+b\,x+c\,x^2}}+\frac{40\,c\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{\left(b^2-4\,a\,c\right)^{1/4}\,\sqrt{d}}\right],-1\right]}{3\,\left(b^2-4\,a\,c\right)^{7/4}\,\sqrt{d}\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 168 leaves):

$$\frac{2\left(-\frac{\left(b+2\,c\,x\right)\,\left(b^2-14\,a\,c-10\,c\,x\,\left(b+c\,x\right)\right)}{a+x\,\left(b+c\,x\right)}+\frac{20\,i\,c\,\left(b+2\,c\,x\right)^{3/2}\,\sqrt{\frac{c\,\left(a+x\,\left(b+c\,x\right)\right)}{\left(b+2\,c\,x\right)^2}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4\,a\,c}}}{\sqrt{b+2\,c\,x}}\right],-1\right]}{\sqrt{-\sqrt{b^2-4\,a\,c}}}\right)}{3\,\left(b^2-4\,a\,c\right)^2\,\sqrt{d\,\left(b+2\,c\,x\right)}\,\sqrt{a+x\,\left(b+c\,x\right)}}$$

- **Problem 1394: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\left(b\,d+2\,c\,d\,x\right)^{5/2}\,\left(a+b\,x+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 228 leaves, 6 steps):

$$-\frac{2}{3\left(b^2-4ac\right)d\,(bd+2cdx)^{3/2}\left(a+bx+cx^2\right)^{3/2}}+\frac{12c}{\left(b^2-4ac\right)^2 d\,(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}}+$$

$$\frac{80c^2\sqrt{a+bx+cx^2}}{\left(b^2-4ac\right)^3 d\,(bd+2cdx)^{3/2}}+\frac{40c\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)^{1/4}\sqrt{d}}\right],-1\right]}{\left(b^2-4ac\right)^{11/4}d^{5/2}\sqrt{a+bx+cx^2}}$$

Result (type 4, 200 leaves) :

$$\left(2\left((b+2cx)^3\,(a+x\,(b+cx))\left(\frac{32c^2}{(b+2cx)^2}+\frac{-b^2+4ac}{(a+x\,(b+cx))^2}+\frac{22c}{a+x\,(b+cx)}\right)+\right.\right.$$

$$\left.\left.\frac{60\,\mathrm{i}\,c\,(b+2cx)^{7/2}\sqrt{\frac{c\,(a+x\,(b+cx))}{(b+2cx)^2}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b^2-4ac}}}{\sqrt{b+2cx}}\right],-1\right]}{\sqrt{-\sqrt{b^2-4ac}}}\right)\right)\Big/\left(3\left(b^2-4ac\right)^3\,(d\,(b+2cx))^{5/2}\sqrt{a+x\,(b+cx)}\right)$$

Problem 1395: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(bd+2cdx)^{13/2}}{\left(a+bx+cx^2\right)^{5/2}}\,dx$$

Optimal (type 4, 299 leaves, 9 steps) :

$$-\frac{2d\,(bd+2cdx)^{11/2}}{3\left(a+bx+cx^2\right)^{3/2}}-\frac{44cd^3\,(bd+2cdx)^{7/2}}{3\sqrt{a+bx+cx^2}}+\frac{1232}{15}c^2d^5\,(bd+2cdx)^{3/2}\sqrt{a+bx+cx^2}+$$

$$\frac{616c\left(b^2-4ac\right)^{7/4}d^{13/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)^{1/4}\sqrt{d}}\right],-1\right]}{5\sqrt{a+bx+cx^2}}-$$

$$\frac{616c\left(b^2-4ac\right)^{7/4}d^{13/2}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)^{1/4}\sqrt{d}}\right],-1\right]}{5\sqrt{a+bx+cx^2}}$$

Result (type 4, 251 leaves) :

$$\left(2\,(d\,(b+2\,c\,x))^{13/2}\left(\frac{-5\,b^4-150\,b^3\,c\,x+24\,b\,c^2\,x\,(33\,a+8\,c\,x^2)-2\,b^2\,c\,(55\,a+27\,c\,x^2)+8\,c^2\,(77\,a^2+99\,a\,c\,x^2+12\,c^2\,x^4)}{a+x\,(b+c\,x)}+\right.\right.$$

$$1\Big/\left(\left(-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{3/2}924\,i\,c\,(b^2-4\,a\,c)\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\right.$$

$$\left.\left.\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]\right)\right)\right)\right)\Big/\left(15\,(b+2\,c\,x)^5\sqrt{a+x\,(b+c\,x)}\right)$$

- **Problem 1396: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(b\,d+2\,c\,d\,x)^{9/2}}{(a+b\,x+c\,x^2)^{5/2}}\,dx$$

Optimal (type 4, 258 leaves, 8 steps):

$$-\frac{2\,d\,(b\,d+2\,c\,d\,x)^{7/2}}{3\,(a+b\,x+c\,x^2)^{3/2}}-\frac{28\,c\,d^3\,(b\,d+2\,c\,d\,x)^{3/2}}{3\,\sqrt{a+b\,x+c\,x^2}}+\frac{56\,c\,(b^2-4\,a\,c)^{3/4}\,d^{9/2}\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{(b^2-4\,a\,c)^{1/4}\sqrt{d}}\right],-1\right]}{\sqrt{a+b\,x+c\,x^2}}-$$

$$\frac{56\,c\,(b^2-4\,a\,c)^{3/4}\,d^{9/2}\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{(b^2-4\,a\,c)^{1/4}\sqrt{d}}\right],-1\right]}{\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 194 leaves):

$$\left(2\,(d\,(b+2\,c\,x))^{9/2}\left(-\frac{b^2+18\,b\,c\,x+2\,c\,(7\,a+9\,c\,x^2)}{a+x\,(b+c\,x)}+1\Big/\left(\left(-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{3/2}84\,i\,c\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\right.\right.\right.$$

$$\left.\left.\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],-1\right]\right)\right)\right)\right)\Big/\left(3\,(b+2\,c\,x)^3\sqrt{a+x\,(b+c\,x)}\right)$$

- **Problem 1397: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(b\,d+2\,c\,d\,x)^{5/2}}{(a+b\,x+c\,x^2)^{5/2}}\,dx$$

Optimal (type 4, 264 leaves, 8 steps):

$$-\frac{2\,d\,(b\,d+2\,c\,d\,x)^{3/2}}{3\,(a+b\,x+c\,x^2)^{3/2}}-\frac{4\,c\,d\,(b\,d+2\,c\,d\,x)^{3/2}}{(b^2-4\,a\,c)\,\sqrt{a+b\,x+c\,x^2}}+$$

$$\frac{8\,c\,d^{5/2}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{(b^2-4\,a\,c)^{1/4}\,\sqrt{d}}\right],\,-1\right]}{(b^2-4\,a\,c)^{1/4}\,\sqrt{a+b\,x+c\,x^2}}-\frac{8\,c\,d^{5/2}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{(b^2-4\,a\,c)^{1/4}\,\sqrt{d}}\right],\,-1\right]}{(b^2-4\,a\,c)^{1/4}\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 218 leaves) :

$$\frac{1}{(a+x\,(b+c\,x))^{5/2}}$$

$$(d\,(b+2\,c\,x))^{5/2}\left(-\frac{2\,(a+x\,(b+c\,x))\,\left(b^2+6\,b\,c\,x+2\,c\,\left(a+3\,c\,x^2\right)\right)}{3\,(b^2-4\,a\,c)\,(b+2\,c\,x)}+1\Big/(b+2\,c\,x)^3 8\,i\,c\,\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\,(a+x\,(b+c\,x))^2\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],\,-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}\right],\,-1\right]\right)\right)$$

■ Problem 1398: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{b\,d+2\,c\,d\,x}}{(a+b\,x+c\,x^2)^{5/2}}\,dx$$

Optimal (type 4, 278 leaves, 8 steps) :

$$-\frac{2\,(b\,d+2\,c\,d\,x)^{3/2}}{3\,(b^2-4\,a\,c)\,d\,(a+b\,x+c\,x^2)^{3/2}}+\frac{4\,c\,(b\,d+2\,c\,d\,x)^{3/2}}{(b^2-4\,a\,c)^2\,d\,\sqrt{a+b\,x+c\,x^2}}-$$

$$\frac{8\,c\,\sqrt{d}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{(b^2-4\,a\,c)^{1/4}\,\sqrt{d}}\right],\,-1\right]}{(b^2-4\,a\,c)^{5/4}\,\sqrt{a+b\,x+c\,x^2}}+\frac{8\,c\,\sqrt{d}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,d+2\,c\,d\,x}}{(b^2-4\,a\,c)^{1/4}\,\sqrt{d}}\right],\,-1\right]}{(b^2-4\,a\,c)^{5/4}\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 222 leaves) :

$$\frac{1}{(a+x(b+cx))^{5/2}\sqrt{d(b+2cx)}}\left(-\frac{2(b+2cx)(a+x(b+cx))\left(b^2-6bcx-2c\left(5a+3cx^2\right)\right)}{3\left(b^2-4ac\right)^2}+\right.$$

$$\left(8\,i\,c\,(a+x(b+cx))^2\sqrt{\frac{c(a+x(b+cx))}{-b^2+4ac}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2cx}{\sqrt{b^2-4ac}}}\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2cx}{\sqrt{b^2-4ac}}}\right],-1\right]\right)\right)\Big/$$

$$\left.\left(\left(b^2-4ac\right)^{3/2}\sqrt{-\frac{b+2cx}{\sqrt{b^2-4ac}}}\right)\right)$$

- **Problem 1399: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{(b\,d+2\,c\,d\,x)^{3/2}\left(a+b\,x+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 325 leaves, 9 steps) :

$$-\frac{2}{3\left(b^2-4ac\right)d\sqrt{bd+2cdx}\left(a+bx+cx^2\right)^{3/2}}+\frac{28c}{3\left(b^2-4ac\right)^2d\sqrt{bd+2cdx}\sqrt{a+bx+cx^2}}+\frac{112c^2\sqrt{a+bx+cx^2}}{\left(b^2-4ac\right)^3d\sqrt{bd+2cdx}}-$$

$$\frac{56c\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)^{1/4}\sqrt{d}}\right],-1\right]}{\left(b^2-4ac\right)^{9/4}d^{3/2}\sqrt{a+bx+cx^2}}+\frac{56c\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{bd+2cdx}}{\left(b^2-4ac\right)^{1/4}\sqrt{d}}\right],-1\right]}{\left(b^2-4ac\right)^{9/4}d^{3/2}\sqrt{a+bx+cx^2}}$$

Result (type 4, 270 leaves) :

$$\left(-1\Big/\left(3\left(b^2-4ac\right)^3\right)2(b+2cx)(a+x(b+cx))\left(b^4-14b^3cx-56bc^2x\left(5a+6cx^2\right)-2b^2c\left(11a+91cx^2\right)-8c^2\left(12a^2+35acx^2+21c^2x^4\right)\right)+\right.$$

$$1\Big/\left(b^2-4ac\right)^{3/2}56\,i\,c\left(-\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^{3/2}(a+x(b+cx))^2\sqrt{\frac{c(a+x(b+cx))}{-b^2+4ac}}$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2cx}{\sqrt{b^2-4ac}}}\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b+2cx}{\sqrt{b^2-4ac}}}\right],-1\right]\right)\right)\Big/\left((d(b+2cx))^{3/2}(a+x(b+cx))^{5/2}\right)$$

Problem 1403: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{c\,e+d\,e\,x}\,\sqrt{1-c^2-2\,c\,d\,x-d^2\,x^2}}\,dx$$

Optimal (type 4, 31 leaves, 2 steps):

$$\frac{2\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c\,e+d\,e\,x}}{\sqrt{e}}\right],-1\right]}{d\,\sqrt{e}}$$

Result (type 4, 67 leaves):

$$-\frac{2\,(c+d\,x)^{3/2}\,\sqrt{1-\frac{1}{(c+d\,x)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{\sqrt{c+d\,x}}\right],-1\right]}{d\,\sqrt{e\,(c+d\,x)}\,\sqrt{1-(c+d\,x)^2}}$$

Problem 1412: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b\,x+c\,x^2\right)^{4/3}}{(b\,d+2\,c\,d\,x)^{11/3}}\,dx$$

Optimal (type 3, 320 leaves, 8 steps):

$$-\frac{3\,(d\,(b+2\,c\,x))^{4/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}}{16\,c^2\,\left(b^2-4\,a\,c\right)\,d^5}+\frac{9\,(d\,(b+2\,c\,x))^{4/3}\,\left(a+b\,x+c\,x^2\right)^{4/3}}{16\,c\,\left(b^2-4\,a\,c\right)^2\,d^5}+\frac{3\,\left(a+b\,x+c\,x^2\right)^{7/3}}{4\,\left(b^2-4\,a\,c\right)\,d\,(b\,d+2\,c\,d\,x)^{8/3}}-$$
$$\frac{9\,\left(a+b\,x+c\,x^2\right)^{7/3}}{4\,\left(b^2-4\,a\,c\right)^2\,d^3\,(b\,d+2\,c\,d\,x)^{2/3}}-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,(d\,(b+2\,c\,x))^{2/3}}{c^{1/3}\,d^{2/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}}}{\sqrt{3}}\right]}{16\times 2^{2/3}\,c^{7/3}\,d^{11/3}}-\frac{3\,\text{Log}\left[(d\,(b+2\,c\,x))^{2/3}-2^{2/3}\,c^{1/3}\,d^{2/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right]}{32\times 2^{2/3}\,c^{7/3}\,d^{11/3}}$$

Result (type 5, 167 leaves):

$$\left(-24\,c\,\left(a^2\,c+a\,\left(b^2+6\,b\,c\,x+6\,c^2\,x^2\right)+x\,\left(b^3+6\,b^2\,c\,x+10\,b\,c^2\,x^2+5\,c^3\,x^3\right)\right)+\right.$$
$$\left.6\times 2^{1/3}\,(b+2\,c\,x)^4\,\left(\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}\right)^{2/3}\,\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},\frac{2}{3}\right\},\left\{\frac{5}{3}\right\},\frac{(b+2\,c\,x)^2}{b^2-4\,a\,c}\right]\right)\Big/\left(128\,c^3\,d\,(d\,(b+2\,c\,x))^{8/3}\,(a+x\,(b+c\,x))^{2/3}\right)$$

Problem 1416: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b\,x+c\,x^2\right)^{4/3}}{(b\,d+2\,c\,d\,x)^{2/3}}\,dx$$

Optimal (type 4, 597 leaves, 6 steps):

$$-\frac{\left(b^2-4ac\right)\left(d\left(b+2cx\right)\right)^{1/3}\left(a+bx+cx^2\right)^{1/3}}{9c^2d}+\frac{\left(d\left(b+2cx\right)\right)^{1/3}\left(a+bx+cx^2\right)^{4/3}}{6cd}+$$

$$\left(\left(b^2-4ac\right)\left(d\left(b+2cx\right)\right)^{1/3}\left(b^2-4ac-\left(b+2cx\right)^2\right)\left(2c^{1/3}d^{2/3}-\frac{2^{1/3}\left(d\left(b+2cx\right)\right)^{2/3}}{\left(a+bx+cx^2\right)^{1/3}}\right)\right.$$

$$\left.\sqrt{\frac{2\times 2^{1/3}c^{2/3}d^{4/3}+\frac{\left(d\left(b+2cx\right)\right)^{4/3}}{\left(a+bx+cx^2\right)^{2/3}}+\frac{2^{2/3}c^{1/3}d^{2/3}\left(d\left(b+2cx\right)\right)^{2/3}}{\left(a+bx+cx^2\right)^{1/3}}}{\left(2^{2/3}c^{1/3}d^{2/3}-\frac{\left(1+\sqrt{3}\right)\left(d\left(b+2cx\right)\right)^{2/3}}{\left(a+bx+cx^2\right)^{1/3}}\right)^2}}\ \text{EllipticF}\left[\text{ArcCos}\left[\frac{2^{2/3}c^{1/3}d^{2/3}-\frac{\left(1-\sqrt{3}\right)\left(d\left(b+2cx\right)\right)^{2/3}}{\left(a+bx+cx^2\right)^{1/3}}}{2^{2/3}c^{1/3}d^{2/3}-\frac{\left(1+\sqrt{3}\right)\left(d\left(b+2cx\right)\right)^{2/3}}{\left(a+bx+cx^2\right)^{1/3}}}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(72\times 3^{1/4}c^{10/3}d^{5/3}\left(a+bx+cx^2\right)^{2/3}\sqrt{-\frac{\left(d\left(b+2cx\right)\right)^{2/3}\left(2^{2/3}c^{1/3}d^{2/3}-\frac{\left(d\left(b+2cx\right)\right)^{2/3}}{\left(a+bx+cx^2\right)^{1/3}}\right)}{\left(a+bx+cx^2\right)^{1/3}\left(2^{2/3}c^{1/3}d^{2/3}-\frac{\left(1+\sqrt{3}\right)\left(d\left(b+2cx\right)\right)^{2/3}}{\left(a+bx+cx^2\right)^{1/3}}\right)^2}}\right)$$

Result (type 5, 171 leaves) :

$$\frac{1}{18c^3d\left(a+x\left(b+cx\right)\right)^{2/3}}\left(d\left(b+2cx\right)\right)^{1/3}\left(c\left(11a^2c-2a\left(b^2-7bcx-7c^2x^2\right)+x\left(-2b^3+b^2cx+6bc^2x^2+3c^3x^3\right)\right)+\right.$$

$$\left.2\times 2^{1/3}\left(b^2-4ac\right)^2\left(\frac{c\left(a+x\left(b+cx\right)\right)}{-b^2+4ac}\right)^{2/3}\text{HypergeometricPFQ}\left[\left\{\frac{1}{6},\frac{2}{3}\right\},\left\{\frac{7}{6}\right\},\frac{\left(b+2cx\right)^2}{b^2-4ac}\right]\right)$$

■ **Problem 1417: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+bx+cx^2\right)^{4/3}}{\left(bd+2cdx\right)^{8/3}}\,dx$$

Optimal (type 4, 581 leaves, 6 steps) :

$$\frac{(d\,(b+2\,c\,x))^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}}{5\,c^2\,d^3}-\frac{3\,\left(a+b\,x+c\,x^2\right)^{4/3}}{10\,c\,d\,(d\,(b+2\,c\,x))^{5/3}}-$$

$$\left((d\,(b+2\,c\,x))^{1/3}\,\left(b^2-4\,a\,c-(b+2\,c\,x)^2\right)\left(2\,c^{1/3}\,d^{2/3}-\frac{2^{1/3}\,(d\,(b+2\,c\,x))^{2/3}}{\left(a+b\,x+c\,x^2\right)^{1/3}}\right)\sqrt{\frac{2\times 2^{1/3}\,c^{2/3}\,d^{4/3}+\frac{(d\,(b+2\,c\,x))^{4/3}}{\left(a+b\,x+c\,x^2\right)^{2/3}}+\frac{2^{2/3}\,c^{1/3}\,d^{2/3}\,(d\,(b+2\,c\,x))^{2/3}}{\left(a+b\,x+c\,x^2\right)^{1/3}}}{\left(2^{2/3}\,c^{1/3}\,d^{2/3}-\frac{\left(1+\sqrt{3}\right)\,(d\,(b+2\,c\,x))^{2/3}}{\left(a+b\,x+c\,x^2\right)^{1/3}}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcCos}\left[\frac{2^{2/3}\,c^{1/3}\,d^{2/3}-\frac{\left(1-\sqrt{3}\right)\,(d\,(b+2\,c\,x))^{2/3}}{\left(a+b\,x+c\,x^2\right)^{1/3}}}{2^{2/3}\,c^{1/3}\,d^{2/3}-\frac{\left(1+\sqrt{3}\right)\,(d\,(b+2\,c\,x))^{2/3}}{\left(a+b\,x+c\,x^2\right)^{1/3}}}\right],\,\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(40\times 3^{1/4}\,c^{10/3}\,d^{11/3}\,\left(a+b\,x+c\,x^2\right)^{2/3}\sqrt{-\frac{(d\,(b+2\,c\,x))^{2/3}\left(2^{2/3}\,c^{1/3}\,d^{2/3}-\frac{(d\,(b+2\,c\,x))^{2/3}}{\left(a+b\,x+c\,x^2\right)^{1/3}}\right)}{\left(a+b\,x+c\,x^2\right)^{1/3}\left(2^{2/3}\,c^{1/3}\,d^{2/3}-\frac{\left(1+\sqrt{3}\right)\,(d\,(b+2\,c\,x))^{2/3}}{\left(a+b\,x+c\,x^2\right)^{1/3}}\right)^2}}\right)$$

Result (type 5, 145 leaves) :

$$\left((b+2\,c\,x)^3\left(c\,(a+x\,(b+c\,x))\left(5+\frac{3\,\left(b^2-4\,a\,c\right)}{(b+2\,c\,x)^2}\right)-8\times 2^{1/3}\,\left(b^2-4\,a\,c\right)\left(\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}\right)^{2/3}\text{HypergeometricPFQ}\left[\left\{\frac{1}{6},\,\frac{2}{3}\right\},\,\left\{\frac{7}{6}\right\},\,\frac{(b+2\,c\,x)^2}{b^2-4\,a\,c}\right]\right)\right)\Big/\left(40\,c^3\,(d\,(b+2\,c\,x))^{8/3}\,(a+x\,(b+c\,x))^{2/3}\right)$$

- **Problem 1418: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+b\,x+c\,x^2\right)^{4/3}}{(b\,d+2\,c\,d\,x)^{14/3}}\,dx$$

Optimal (type 4, 591 leaves, 6 steps) :

$$-\frac{3\left(a+b x+c x^2\right)^{1/3}}{55 c^2 d^3 (d (b+2 c x))^{5/3}}-\frac{3\left(a+b x+c x^2\right)^{4/3}}{22 c d (d (b+2 c x))^{11/3}}+$$

$$\left(3^{3/4} (d (b+2 c x))^{1/3}\left(b^2-4 a c-(b+2 c x)^2\right)\left(2 c^{1/3} d^{2/3}-\frac{2^{1/3} (d (b+2 c x))^{2/3}}{\left(a+b x+c x^2\right)^{1/3}}\right)\sqrt{\frac{2\times 2^{1/3} c^{2/3} d^{4/3}+\frac{(d (b+2 c x))^{4/3}}{\left(a+b x+c x^2\right)^{2/3}}+\frac{2^{2/3} c^{1/3} d^{2/3} (d (b+2 c x))^{2/3}}{\left(a+b x+c x^2\right)^{1/3}}}{\left(2^{2/3} c^{1/3} d^{2/3}-\frac{\left(1+\sqrt{3}\right) (d (b+2 c x))^{2/3}}{\left(a+b x+c x^2\right)^{1/3}}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcCos}\left[\frac{2^{2/3} c^{1/3} d^{2/3}-\frac{\left(1-\sqrt{3}\right) (d (b+2 c x))^{2/3}}{\left(a+b x+c x^2\right)^{1/3}}}{2^{2/3} c^{1/3} d^{2/3}-\frac{\left(1+\sqrt{3}\right) (d (b+2 c x))^{2/3}}{\left(a+b x+c x^2\right)^{1/3}}}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(440 c^{10/3}\left(b^2-4 a c\right) d^{17/3}\left(a+b x+c x^2\right)^{2/3}\sqrt{-\frac{(d (b+2 c x))^{2/3}\left(2^{2/3} c^{1/3} d^{2/3}-\frac{(d (b+2 c x))^{2/3}}{\left(a+b x+c x^2\right)^{1/3}}\right)}{\left(a+b x+c x^2\right)^{1/3}\left(2^{2/3} c^{1/3} d^{2/3}-\frac{\left(1+\sqrt{3}\right) (d (b+2 c x))^{2/3}}{\left(a+b x+c x^2\right)^{1/3}}\right)^2}}\right)$$

Result (type 5, 179 leaves) :

$$\left((d (b+2 c x))^{1/3}\left(-3 c\left(5 a^2 c+2 a\left(b^2+9 b c x+9 c^2 x^2\right)+x\left(2 b^3+15 b^2 c x+26 b c^2 x^2+13 c^3 x^3\right)\right)+\right.\right.$$

$$\left.\left.6\times 2^{1/3} (b+2 c x)^4\left(\frac{c (a+x (b+c x))}{-b^2+4 a c}\right)^{2/3}\text{HypergeometricPFQ}\left[\left\{\frac{1}{6},\frac{2}{3}\right\},\left\{\frac{7}{6}\right\},\frac{(b+2 c x)^2}{b^2-4 a c}\right]\right)\right)\Big/\left(110 c^3 d^5 (b+2 c x)^4 (a+x (b+c x))^{2/3}\right)$$

■ Problem 1419: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x+c x^2\right)^{4/3}}{(b d+2 c d x)^{20/3}}\,dx$$

Optimal (type 4, 637 leaves, 7 steps) :

$$-\frac{3\left(a+b x+c x^2\right)^{1/3}}{187 c^2 d^3 (d (b+2 c x))^{11/3}}+\frac{6\left(a+b x+c x^2\right)^{1/3}}{935 c^2 \left(b^2-4 a c\right) d^5 (d (b+2 c x))^{5/3}}-$$

$$\frac{3\left(a+b x+c x^2\right)^{4/3}}{34 c d (d (b+2 c x))^{17/3}}+\left(3\times 3^{3/4} (d (b+2 c x))^{1/3} \left(b^2-4 a c-(b+2 c x)^2\right)\left(2 c^{1/3} d^{2/3}-\frac{2^{1/3} (d (b+2 c x))^{2/3}}{\left(a+b x+c x^2\right)^{1/3}}\right)\right.$$

$$\left.\sqrt{\frac{2\times 2^{1/3} c^{2/3} d^{4/3}+\frac{(d (b+2 c x))^{4/3}}{\left(a+b x+c x^2\right)^{2/3}}+\frac{2^{2/3} c^{1/3} d^{2/3} (d (b+2 c x))^{2/3}}{\left(a+b x+c x^2\right)^{1/3}}}{\left(2^{2/3} c^{1/3} d^{2/3}-\frac{\left(1+\sqrt{3}\right) (d (b+2 c x))^{2/3}}{\left(a+b x+c x^2\right)^{1/3}}\right)^2}}\ \text{EllipticF}\left[\text{ArcCos}\left[\frac{2^{2/3} c^{1/3} d^{2/3}-\frac{\left(1-\sqrt{3}\right) (d (b+2 c x))^{2/3}}{\left(a+b x+c x^2\right)^{1/3}}}{2^{2/3} c^{1/3} d^{2/3}-\frac{\left(1+\sqrt{3}\right) (d (b+2 c x))^{2/3}}{\left(a+b x+c x^2\right)^{1/3}}}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$

$$\left(7480 c^{10/3} \left(b^2-4 a c\right)^2 d^{23/3} \left(a+b x+c x^2\right)^{2/3}\sqrt{-\frac{(d (b+2 c x))^{2/3}\left(2^{2/3} c^{1/3} d^{2/3}-\frac{(d (b+2 c x))^{2/3}}{\left(a+b x+c x^2\right)^{1/3}}\right)}{\left(a+b x+c x^2\right)^{1/3}\left(2^{2/3} c^{1/3} d^{2/3}-\frac{\left(1+\sqrt{3}\right) (d (b+2 c x))^{2/3}}{\left(a+b x+c x^2\right)^{1/3}}\right)^2}}\right)$$

Result (type 5, 170 leaves):

$$\left(3\left(c (a+x (b+c x))\left(55\left(b^2-4 a c\right)^2-95\left(b^2-4 a c\right)(b+2 c x)^2+16 (b+2 c x)^4\right)+\right.\right.$$

$$\left.\left.24\times 2^{1/3} (b+2 c x)^6\left(-\frac{c (a+x (b+c x))}{b^2-4 a c}\right)^{2/3}\text{HypergeometricPFQ}\left[\left\{\frac{1}{6},\frac{2}{3}\right\},\left\{\frac{7}{6}\right\},\frac{(b+2 c x)^2}{b^2-4 a c}\right]\right)\right)\Bigg/$$

$$\left(7480 c^3 \left(b^2-4 a c\right) d (d (b+2 c x))^{17/3} (a+x (b+c x))^{2/3}\right)$$

- **Problem 1423: Result more than twice size of optimal antiderivative.**

$$\int (b d+2 c d x)^m \left(a+b x+c x^2\right)^3 \, dx$$

Optimal (type 3, 141 leaves, 2 steps):

$$-\frac{\left(b^2-4 a c\right)^3 (b d+2 c d x)^{1+m}}{128 c^4 d (1+m)}+\frac{3\left(b^2-4 a c\right)^2 (b d+2 c d x)^{3+m}}{128 c^4 d^3 (3+m)}-\frac{3\left(b^2-4 a c\right) (b d+2 c d x)^{5+m}}{128 c^4 d^5 (5+m)}+\frac{(b d+2 c d x)^{7+m}}{128 c^4 d^7 (7+m)}$$

Result (type 3, 321 leaves):

$$-\frac{1}{8 c^4 (1+m) (3+m) (5+m) (7+m)} (b+2 c x) (d (b+2 c x))^m \left(3 b^6-6 b^5 c (1+m) x-6 b^4 c \left(a (7+m)-c \left(2+3 m+m^2\right) x^2\right)-4 b^3 c^2 (1+m) x \left(-3 a (7+m)+c \left(6+5 m+m^2\right) x^2\right)-12 b c^3 (1+m) x \left(a^2 \left(35+12 m+m^2\right)+2 a c \left(21+10 m+m^2\right) x^2+c^2 \left(15+8 m+m^2\right) x^4\right)-6 b^2 c^2 \left(-a^2 \left(35+12 m+m^2\right)+2 a c \left(14+23 m+10 m^2+m^3\right) x^2+c^2 \left(27+42 m+17 m^2+2 m^3\right) x^4\right)-4 c^3 \left(a^3 \left(105+71 m+15 m^2+m^3\right)+3 a^2 c \left(35+47 m+13 m^2+m^3\right) x^2+3 a c^2 \left(21+31 m+11 m^2+m^3\right) x^4+c^3 \left(15+23 m+9 m^2+m^3\right) x^6\right)\right)$$

- **Problem 1426: Result more than twice size of optimal antiderivative.**

$$\int \frac{(b d+2 c d x)^m}{a+b x+c x^2} \, dx$$

Optimal (type 5, 67 leaves, 2 steps):

$$-\frac{2 (d (b+2 c x))^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, \frac{(b+2 c x)^2}{b^2-4 a c}\right]}{\left(b^2-4 a c\right) d (1+m)}$$

Result (type 5, 186 leaves):

$$\frac{1}{\sqrt{b^2-4 a c}\; m} (d (b+2 c x))^m \left(\left(\frac{b+2 c x}{b-\sqrt{b^2-4 a c}+2 c x}\right)^{-m} \text{Hypergeometric2F1}\left[-m, -m, 1-m, \frac{\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}-2 c x}\right]-\left(\frac{b+2 c x}{b+\sqrt{b^2-4 a c}+2 c x}\right)^{-m} \text{Hypergeometric2F1}\left[-m, -m, 1-m, \frac{\sqrt{b^2-4 a c}}{b+\sqrt{b^2-4 a c}+2 c x}\right]\right)$$

- **Problem 1427: Unable to integrate problem.**

$$\int \frac{(b d+2 c d x)^m}{\left(a+b x+c x^2\right)^2} \, dx$$

Optimal (type 5, 68 leaves, 2 steps):

$$\frac{8 c (d (b+2 c x))^{1+m} \text{Hypergeometric2F1}\left[2, \frac{1+m}{2}, \frac{3+m}{2}, \frac{(b+2 c x)^2}{b^2-4 a c}\right]}{\left(b^2-4 a c\right)^2 d (1+m)}$$

Result (type 8, 26 leaves):

$$\int \frac{(b d+2 c d x)^m}{\left(a+b x+c x^2\right)^2} \, dx$$

- **Problem 1428: Unable to integrate problem.**

$$\int \frac{(b d+2 c d x)^m}{\left(a+b x+c x^2\right)^3} \, dx$$

Optimal (type 5, 70 leaves, 2 steps):

$$-\frac{32\,c^2\,(d\,(b+2\,c\,x))^{1+m}\,\text{Hypergeometric2F1}\left[3,\,\frac{1+m}{2},\,\frac{3+m}{2},\,\frac{(b+2\,c\,x)^2}{b^2-4\,a\,c}\right]}{\left(b^2-4\,a\,c\right)^3\,d\,(1+m)}$$

Result (type 8, 26 leaves):

$$\int \frac{(b\,d+2\,c\,d\,x)^m}{\left(a+b\,x+c\,x^2\right)^3}\,dx$$

- **Problem 1429: Unable to integrate problem.**

$$\int (b\,d+2\,c\,d\,x)^m\,\left(a+b\,x+c\,x^2\right)^{5/2}\,dx$$

Optimal (type 5, 82 leaves, 3 steps):

$$-\frac{2\,(b\,d+2\,c\,d\,x)^{1+m}\,\left(a+b\,x+c\,x^2\right)^{7/2}\,\text{Hypergeometric2F1}\left[1,\,\frac{8+m}{2},\,\frac{3+m}{2},\,\frac{(b+2\,c\,x)^2}{b^2-4\,a\,c}\right]}{\left(b^2-4\,a\,c\right)\,d\,(1+m)}$$

Result (type 8, 28 leaves):

$$\int (b\,d+2\,c\,d\,x)^m\,\left(a+b\,x+c\,x^2\right)^{5/2}\,dx$$

- **Problem 1430: Unable to integrate problem.**

$$\int (b\,d+2\,c\,d\,x)^m\,\left(a+b\,x+c\,x^2\right)^{3/2}\,dx$$

Optimal (type 5, 98 leaves, 3 steps):

$$-\frac{(b\,d+2\,c\,d\,x)^{1+m}\,\left(4\,a-\frac{b^2}{c}+\frac{(b+2\,c\,x)^2}{c}\right)^{5/2}\,\text{Hypergeometric2F1}\left[1,\,\frac{6+m}{2},\,\frac{3+m}{2},\,\frac{(b+2\,c\,x)^2}{b^2-4\,a\,c}\right]}{16\,\left(b^2-4\,a\,c\right)\,d\,(1+m)}$$

Result (type 8, 28 leaves):

$$\int (b\,d+2\,c\,d\,x)^m\,\left(a+b\,x+c\,x^2\right)^{3/2}\,dx$$

- **Problem 1433: Result more than twice size of optimal antiderivative.**

$$\int \frac{(b\,d+2\,c\,d\,x)^m}{\left(a+b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 5, 94 leaves, 3 steps):

$$-\frac{4\,(b\,d+2\,c\,d\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{m}{2},\frac{3+m}{2},\frac{(b+2\,c\,x)^2}{b^2-4\,a\,c}\right]}{\left(b^2-4\,a\,c\right)\,d\,(1+m)\,\sqrt{4\,a-\frac{b^2}{c}+\frac{(b+2\,c\,x)^2}{c}}}$$

Result (type 5, 193 leaves):

$$\left((b+2\,c\,x)\,(d\,(b+2\,c\,x))^m\left(b^2-4\,a\,c+b\,\sqrt{b^2-4\,a\,c}+2\,c\,\sqrt{b^2-4\,a\,c}\,x\right)\left(-b^2+4\,a\,c+b\,\sqrt{b^2-4\,a\,c}+2\,c\,\sqrt{b^2-4\,a\,c}\,x\right)\right.$$
$$\left.\text{HypergeometricPFQ}\left[\left\{\frac{3}{2},\frac{1}{2}+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2}\right\},\frac{(b+2\,c\,x)^2}{b^2-4\,a\,c}\right]\right)\Big/\left(\left(b^2-4\,a\,c\right)^3\,(1+m)\,\sqrt{a+x\,(b+c\,x)}\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\right)$$

Problem 1451: Result more than twice size of optimal antiderivative.

$$\int (d+e\,x)^4\left(a^2+2\,a\,b\,x+b^2\,x^2\right)\,dx$$

Optimal (type 1, 65 leaves, 3 steps):

$$\frac{(b\,d-a\,e)^2\,(d+e\,x)^5}{5\,e^3}-\frac{b\,(b\,d-a\,e)\,(d+e\,x)^6}{3\,e^3}+\frac{b^2\,(d+e\,x)^7}{7\,e^3}$$

Result (type 1, 148 leaves):

$$a^2\,d^4\,x+a\,d^3\,(b\,d+2\,a\,e)\,x^2+\frac{1}{3}\,d^2\left(b^2\,d^2+8\,a\,b\,d\,e+6\,a^2\,e^2\right)x^3+$$
$$d\,e\left(b^2\,d^2+3\,a\,b\,d\,e+a^2\,e^2\right)x^4+\frac{1}{5}\,e^2\left(6\,b^2\,d^2+8\,a\,b\,d\,e+a^2\,e^2\right)x^5+\frac{1}{3}\,b\,e^3\,(2\,b\,d+a\,e)\,x^6+\frac{1}{7}\,b^2\,e^4\,x^7$$

Problem 1463: Result more than twice size of optimal antiderivative.

$$\int (d+e\,x)^6\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^2\,dx$$

Optimal (type 1, 119 leaves, 3 steps):

$$\frac{(b\,d-a\,e)^4\,(d+e\,x)^7}{7\,e^5}-\frac{b\,(b\,d-a\,e)^3\,(d+e\,x)^8}{2\,e^5}+\frac{2\,b^2\,(b\,d-a\,e)^2\,(d+e\,x)^9}{3\,e^5}-\frac{2\,b^3\,(b\,d-a\,e)\,(d+e\,x)^{10}}{5\,e^5}+\frac{b^4\,(d+e\,x)^{11}}{11\,e^5}$$

Result (type 1, 398 leaves):

$$a^4\,d^6\,x+a^3\,d^5\,(2\,b\,d+3\,a\,e)\,x^2+a^2\,d^4\left(2\,b^2\,d^2+8\,a\,b\,d\,e+5\,a^2\,e^2\right)x^3+$$
$$a\,d^3\left(b^3\,d^3+9\,a\,b^2\,d^2\,e+15\,a^2\,b\,d\,e^2+5\,a^3\,e^3\right)x^4+\frac{1}{5}\,d^2\left(b^4\,d^4+24\,a\,b^3\,d^3\,e+90\,a^2\,b^2\,d^2\,e^2+80\,a^3\,b\,d\,e^3+15\,a^4\,e^4\right)x^5+$$
$$d\,e\left(b^4\,d^4+10\,a\,b^3\,d^3\,e+20\,a^2\,b^2\,d^2\,e^2+10\,a^3\,b\,d\,e^3+a^4\,e^4\right)x^6+\frac{1}{7}\,e^2\left(15\,b^4\,d^4+80\,a\,b^3\,d^3\,e+90\,a^2\,b^2\,d^2\,e^2+24\,a^3\,b\,d\,e^3+a^4\,e^4\right)x^7+$$
$$\frac{1}{2}\,b\,e^3\left(5\,b^3\,d^3+15\,a\,b^2\,d^2\,e+9\,a^2\,b\,d\,e^2+a^3\,e^3\right)x^8+\frac{1}{3}\,b^2\,e^4\left(5\,b^2\,d^2+8\,a\,b\,d\,e+2\,a^2\,e^2\right)x^9+\frac{1}{5}\,b^3\,e^5\,(3\,b\,d+2\,a\,e)\,x^{10}+\frac{1}{11}\,b^4\,e^6\,x^{11}$$

- **Problem 1464: Result more than twice size of optimal antiderivative.**

$$\int (d+e x)^5 \left(a^2+2 a b x+b^2 x^2\right)^2 \, dx$$

Optimal (type 1, 119 leaves, 3 steps):

$$\frac{(b d-a e)^4 (d+e x)^6}{6 e^5}-\frac{4 b (b d-a e)^3 (d+e x)^7}{7 e^5}+\frac{3 b^2 (b d-a e)^2 (d+e x)^8}{4 e^5}-\frac{4 b^3 (b d-a e) (d+e x)^9}{9 e^5}+\frac{b^4 (d+e x)^{10}}{10 e^5}$$

Result (type 1, 350 leaves):

$$a^4 d^5 x+\frac{1}{2} a^3 d^4 (4 b d+5 a e) x^2+\frac{2}{3} a^2 d^3 \left(3 b^2 d^2+10 a b d e+5 a^2 e^2\right) x^3+\frac{1}{2} a d^2 \left(2 b^3 d^3+15 a b^2 d^2 e+20 a^2 b d e^2+5 a^3 e^3\right) x^4+\frac{1}{5} d \left(b^4 d^4+20 a b^3 d^3 e+60 a^2 b^2 d^2 e^2+40 a^3 b d e^3+5 a^4 e^4\right) x^5+\frac{1}{6} e \left(5 b^4 d^4+40 a b^3 d^3 e+60 a^2 b^2 d^2 e^2+20 a^3 b d e^3+a^4 e^4\right) x^6+\frac{2}{7} b e^2 \left(5 b^3 d^3+20 a b^2 d^2 e+15 a^2 b d e^2+2 a^3 e^3\right) x^7+\frac{1}{4} b^2 e^3 \left(5 b^2 d^2+10 a b d e+3 a^2 e^2\right) x^8+\frac{1}{9} b^3 e^4 (5 b d+4 a e) x^9+\frac{1}{10} b^4 e^5 x^{10}$$

- **Problem 1465: Result more than twice size of optimal antiderivative.**

$$\int (d+e x)^4 \left(a^2+2 a b x+b^2 x^2\right)^2 \, dx$$

Optimal (type 1, 119 leaves, 3 steps):

$$\frac{(b d-a e)^4 (a+b x)^5}{5 b^5}+\frac{2 e (b d-a e)^3 (a+b x)^6}{3 b^5}+\frac{6 e^2 (b d-a e)^2 (a+b x)^7}{7 b^5}+\frac{e^3 (b d-a e) (a+b x)^8}{2 b^5}+\frac{e^4 (a+b x)^9}{9 b^5}$$

Result (type 1, 273 leaves):

$$a^4 d^4 x+2 a^3 d^3 (b d+a e) x^2+\frac{2}{3} a^2 d^2 \left(3 b^2 d^2+8 a b d e+3 a^2 e^2\right) x^3+a d \left(b^3 d^3+6 a b^2 d^2 e+6 a^2 b d e^2+a^3 e^3\right) x^4+\frac{1}{5} \left(b^4 d^4+16 a b^3 d^3 e+36 a^2 b^2 d^2 e^2+16 a^3 b d e^3+a^4 e^4\right) x^5+\frac{2}{3} b e \left(b^3 d^3+6 a b^2 d^2 e+6 a^2 b d e^2+a^3 e^3\right) x^6+\frac{2}{7} b^2 e^2 \left(3 b^2 d^2+8 a b d e+3 a^2 e^2\right) x^7+\frac{1}{2} b^3 e^3 (b d+a e) x^8+\frac{1}{9} b^4 e^4 x^9$$

- **Problem 1466: Result more than twice size of optimal antiderivative.**

$$\int (d+e x)^3 \left(a^2+2 a b x+b^2 x^2\right)^2 \, dx$$

Optimal (type 1, 92 leaves, 3 steps):

$$\frac{(b d-a e)^3 (a+b x)^5}{5 b^4}+\frac{e (b d-a e)^2 (a+b x)^6}{2 b^4}+\frac{3 e^2 (b d-a e) (a+b x)^7}{7 b^4}+\frac{e^3 (a+b x)^8}{8 b^4}$$

Result (type 1, 217 leaves):

$$a^4 d^3 x + \frac{1}{2} a^3 d^2 (4 b d + 3 a e) x^2 + a^2 d \left(2 b^2 d^2 + 4 a b d e + a^2 e^2\right) x^3 + \frac{1}{4} a \left(4 b^3 d^3 + 18 a b^2 d^2 e + 12 a^2 b d e^2 + a^3 e^3\right) x^4 + \frac{1}{5} b \left(b^3 d^3 + 12 a b^2 d^2 e + 18 a^2 b d e^2 + 4 a^3 e^3\right) x^5 + \frac{1}{2} b^2 e \left(b^2 d^2 + 4 a b d e + 2 a^2 e^2\right) x^6 + \frac{1}{7} b^3 e^2 (3 b d + 4 a e) x^7 + \frac{1}{8} b^4 e^3 x^8$$

- **Problem 1467: Result more than twice size of optimal antiderivative.**

$$\int (d + e x)^2 \left(a^2 + 2 a b x + b^2 x^2\right)^2 dx$$

Optimal (type 1, 65 leaves, 3 steps):

$$\frac{(b d - a e)^2 (a + b x)^5}{5 b^3} + \frac{e (b d - a e) (a + b x)^6}{3 b^3} + \frac{e^2 (a + b x)^7}{7 b^3}$$

Result (type 1, 148 leaves):

$$a^4 d^2 x + a^3 d (2 b d + a e) x^2 + \frac{1}{3} a^2 \left(6 b^2 d^2 + 8 a b d e + a^2 e^2\right) x^3 + a b \left(b^2 d^2 + 3 a b d e + a^2 e^2\right) x^4 + \frac{1}{5} b^2 \left(b^2 d^2 + 8 a b d e + 6 a^2 e^2\right) x^5 + \frac{1}{3} b^3 e (b d + 2 a e) x^6 + \frac{1}{7} b^4 e^2 x^7$$

- **Problem 1468: Result more than twice size of optimal antiderivative.**

$$\int (d + e x) \left(a^2 + 2 a b x + b^2 x^2\right)^2 dx$$

Optimal (type 1, 38 leaves, 3 steps):

$$\frac{(b d - a e) (a + b x)^5}{5 b^2} + \frac{e (a + b x)^6}{6 b^2}$$

Result (type 1, 84 leaves):

$$\frac{1}{30} x \left(15 a^4 (2 d + e x) + 20 a^3 b x (3 d + 2 e x) + 15 a^2 b^2 x^2 (4 d + 3 e x) + 6 a b^3 x^3 (5 d + 4 e x) + b^4 x^4 (6 d + 5 e x)\right)$$

- **Problem 1475: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a^2 + 2 a b x + b^2 x^2\right)^2}{(d + e x)^6} dx$$

Optimal (type 1, 28 leaves, 2 steps):

$$\frac{(a + b x)^5}{5 (b d - a e) (d + e x)^5}$$

Result (type 1, 140 leaves):

$$-\frac{1}{5 e^5 (d+e x)^5}\left(a^4 e^4+a^3 b e^3 (d+5 e x)+a^2 b^2 e^2 \left(d^2+5 d e x+10 e^2 x^2\right)+a b^3 e \left(d^3+5 d^2 e x+10 d e^2 x^2+10 e^3 x^3\right)+b^4 \left(d^4+5 d^3 e x+10 d^2 e^2 x^2+10 d e^3 x^3+5 e^4 x^4\right)\right)$$

- **Problem 1476: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a^2+2 a b x+b^2 x^2\right)^2}{(d+e x)^7} \, dx$$

Optimal (type 1, 58 leaves, 3 steps):

$$\frac{(a+b x)^5}{6 (b d-a e) (d+e x)^6}+\frac{b (a+b x)^5}{30 (b d-a e)^2 (d+e x)^5}$$

Result (type 1, 144 leaves):

$$-\frac{1}{30 e^5 (d+e x)^6}\left(5 a^4 e^4+4 a^3 b e^3 (d+6 e x)+3 a^2 b^2 e^2 \left(d^2+6 d e x+15 e^2 x^2\right)+2 a b^3 e \left(d^3+6 d^2 e x+15 d e^2 x^2+20 e^3 x^3\right)+b^4 \left(d^4+6 d^3 e x+15 d^2 e^2 x^2+20 d e^3 x^3+15 e^4 x^4\right)\right)$$

- **Problem 1481: Result more than twice size of optimal antiderivative.**

$$\int (d+e x)^8 \left(a^2+2 a b x+b^2 x^2\right)^3 \, dx$$

Optimal (type 1, 173 leaves, 3 steps):

$$\frac{(b d-a e)^6 (d+e x)^9}{9 e^7}-\frac{3 b (b d-a e)^5 (d+e x)^{10}}{5 e^7}+\frac{15 b^2 (b d-a e)^4 (d+e x)^{11}}{11 e^7}-\frac{5 b^3 (b d-a e)^3 (d+e x)^{12}}{3 e^7}+\frac{15 b^4 (b d-a e)^2 (d+e x)^{13}}{13 e^7}-\frac{3 b^5 (b d-a e) (d+e x)^{14}}{7 e^7}+\frac{b^6 (d+e x)^{15}}{15 e^7}$$

Result (type 1, 771 leaves):

$$a^6 d^8 x + a^5 d^7 (3 b d + 4 a e) x^2 + \frac{1}{3} a^4 d^6 (15 b^2 d^2 + 48 a b d e + 28 a^2 e^2) x^3 +$$
$$a^3 d^5 (5 b^3 d^3 + 30 a b^2 d^2 e + 42 a^2 b d e^2 + 14 a^3 e^3) x^4 + \frac{1}{5} a^2 d^4 (15 b^4 d^4 + 160 a b^3 d^3 e + 420 a^2 b^2 d^2 e^2 + 336 a^3 b d e^3 + 70 a^4 e^4) x^5 +$$
$$\frac{1}{3} a d^3 (3 b^5 d^5 + 60 a b^4 d^4 e + 280 a^2 b^3 d^3 e^2 + 420 a^3 b^2 d^2 e^3 + 210 a^4 b d e^4 + 28 a^5 e^5) x^6 +$$
$$\frac{1}{7} d^2 (b^6 d^6 + 48 a b^5 d^5 e + 420 a^2 b^4 d^4 e^2 + 1120 a^3 b^3 d^3 e^3 + 1050 a^4 b^2 d^2 e^4 + 336 a^5 b d e^5 + 28 a^6 e^6) x^7 +$$
$$d e (b^6 d^6 + 21 a b^5 d^5 e + 105 a^2 b^4 d^4 e^2 + 175 a^3 b^3 d^3 e^3 + 105 a^4 b^2 d^2 e^4 + 21 a^5 b d e^5 + a^6 e^6) x^8 +$$
$$\frac{1}{9} e^2 (28 b^6 d^6 + 336 a b^5 d^5 e + 1050 a^2 b^4 d^4 e^2 + 1120 a^3 b^3 d^3 e^3 + 420 a^4 b^2 d^2 e^4 + 48 a^5 b d e^5 + a^6 e^6) x^9 +$$
$$\frac{1}{5} b e^3 (28 b^5 d^5 + 210 a b^4 d^4 e + 420 a^2 b^3 d^3 e^2 + 280 a^3 b^2 d^2 e^3 + 60 a^4 b d e^4 + 3 a^5 e^5) x^{10} +$$
$$\frac{1}{11} b^2 e^4 (70 b^4 d^4 + 336 a b^3 d^3 e + 420 a^2 b^2 d^2 e^2 + 160 a^3 b d e^3 + 15 a^4 e^4) x^{11} + \frac{1}{3} b^3 e^5 (14 b^3 d^3 + 42 a b^2 d^2 e + 30 a^2 b d e^2 + 5 a^3 e^3) x^{12} +$$
$$\frac{1}{13} b^4 e^6 (28 b^2 d^2 + 48 a b d e + 15 a^2 e^2) x^{13} + \frac{1}{7} b^5 e^7 (4 b d + 3 a e) x^{14} + \frac{1}{15} b^6 e^8 x^{15}$$

- **Problem 1482: Result more than twice size of optimal antiderivative.**

$$\int (d + e x)^7 (a^2 + 2 a b x + b^2 x^2)^3 \, dx$$

Optimal (type 1, 173 leaves, 3 steps):

$$\frac{(b d - a e)^6 (d + e x)^8}{8 e^7} - \frac{2 b (b d - a e)^5 (d + e x)^9}{3 e^7} + \frac{3 b^2 (b d - a e)^4 (d + e x)^{10}}{2 e^7} -$$
$$\frac{20 b^3 (b d - a e)^3 (d + e x)^{11}}{11 e^7} + \frac{5 b^4 (b d - a e)^2 (d + e x)^{12}}{4 e^7} - \frac{6 b^5 (b d - a e) (d + e x)^{13}}{13 e^7} + \frac{b^6 (d + e x)^{14}}{14 e^7}$$

Result (type 1, 684 leaves):

$$a^6 d^7 x + \frac{1}{2} a^5 d^6 (6 b d + 7 a e) x^2 + a^4 d^5 \left(5 b^2 d^2 + 14 a b d e + 7 a^2 e^2\right) x^3 +$$
$$\frac{1}{4} a^3 d^4 \left(20 b^3 d^3 + 105 a b^2 d^2 e + 126 a^2 b d e^2 + 35 a^3 e^3\right) x^4 + a^2 d^3 \left(3 b^4 d^4 + 28 a b^3 d^3 e + 63 a^2 b^2 d^2 e^2 + 42 a^3 b d e^3 + 7 a^4 e^4\right) x^5 +$$
$$\frac{1}{2} a d^2 \left(2 b^5 d^5 + 35 a b^4 d^4 e + 140 a^2 b^3 d^3 e^2 + 175 a^3 b^2 d^2 e^3 + 70 a^4 b d e^4 + 7 a^5 e^5\right) x^6 +$$
$$\frac{1}{7} d \left(b^6 d^6 + 42 a b^5 d^5 e + 315 a^2 b^4 d^4 e^2 + 700 a^3 b^3 d^3 e^3 + 525 a^4 b^2 d^2 e^4 + 126 a^5 b d e^5 + 7 a^6 e^6\right) x^7 +$$
$$\frac{1}{8} e \left(7 b^6 d^6 + 126 a b^5 d^5 e + 525 a^2 b^4 d^4 e^2 + 700 a^3 b^3 d^3 e^3 + 315 a^4 b^2 d^2 e^4 + 42 a^5 b d e^5 + a^6 e^6\right) x^8 +$$
$$\frac{1}{3} b e^2 \left(7 b^5 d^5 + 70 a b^4 d^4 e + 175 a^2 b^3 d^3 e^2 + 140 a^3 b^2 d^2 e^3 + 35 a^4 b d e^4 + 2 a^5 e^5\right) x^9 +$$
$$\frac{1}{2} b^2 e^3 \left(7 b^4 d^4 + 42 a b^3 d^3 e + 63 a^2 b^2 d^2 e^2 + 28 a^3 b d e^3 + 3 a^4 e^4\right) x^{10} + \frac{1}{11} b^3 e^4 \left(35 b^3 d^3 + 126 a b^2 d^2 e + 105 a^2 b d e^2 + 20 a^3 e^3\right) x^{11} +$$
$$\frac{1}{4} b^4 e^5 \left(7 b^2 d^2 + 14 a b d e + 5 a^2 e^2\right) x^{12} + \frac{1}{13} b^5 e^6 (7 b d + 6 a e) x^{13} + \frac{1}{14} b^6 e^7 x^{14}$$

- **Problem 1483: Result more than twice size of optimal antiderivative.**

$$\int (d + e x)^6 \left(a^2 + 2 a b x + b^2 x^2\right)^3 dx$$

Optimal (type 1, 171 leaves, 3 steps):

$$\frac{(b d - a e)^6 (a + b x)^7}{7 b^7} + \frac{3 e (b d - a e)^5 (a + b x)^8}{4 b^7} + \frac{5 e^2 (b d - a e)^4 (a + b x)^9}{3 b^7} +$$
$$\frac{2 e^3 (b d - a e)^3 (a + b x)^{10}}{b^7} + \frac{15 e^4 (b d - a e)^2 (a + b x)^{11}}{11 b^7} + \frac{e^5 (b d - a e) (a + b x)^{12}}{2 b^7} + \frac{e^6 (a + b x)^{13}}{13 b^7}$$

Result (type 1, 573 leaves):

$$a^6 d^6 x + 3 a^5 d^5 (b d + a e) x^2 + a^4 d^4 \left(5 b^2 d^2 + 12 a b d e + 5 a^2 e^2\right) x^3 + \frac{5}{2} a^3 d^3 \left(2 b^3 d^3 + 9 a b^2 d^2 e + 9 a^2 b d e^2 + 2 a^3 e^3\right) x^4 +$$
$$3 a^2 d^2 \left(b^4 d^4 + 8 a b^3 d^3 e + 15 a^2 b^2 d^2 e^2 + 8 a^3 b d e^3 + a^4 e^4\right) x^5 + a d \left(b^5 d^5 + 15 a b^4 d^4 e + 50 a^2 b^3 d^3 e^2 + 50 a^3 b^2 d^2 e^3 + 15 a^4 b d e^4 + a^5 e^5\right) x^6 +$$
$$\frac{1}{7} \left(b^6 d^6 + 36 a b^5 d^5 e + 225 a^2 b^4 d^4 e^2 + 400 a^3 b^3 d^3 e^3 + 225 a^4 b^2 d^2 e^4 + 36 a^5 b d e^5 + a^6 e^6\right) x^7 +$$
$$\frac{3}{4} b e \left(b^5 d^5 + 15 a b^4 d^4 e + 50 a^2 b^3 d^3 e^2 + 50 a^3 b^2 d^2 e^3 + 15 a^4 b d e^4 + a^5 e^5\right) x^8 + \frac{5}{3} b^2 e^2 \left(b^4 d^4 + 8 a b^3 d^3 e + 15 a^2 b^2 d^2 e^2 + 8 a^3 b d e^3 + a^4 e^4\right) x^9 +$$
$$b^3 e^3 \left(2 b^3 d^3 + 9 a b^2 d^2 e + 9 a^2 b d e^2 + 2 a^3 e^3\right) x^{10} + \frac{3}{11} b^4 e^4 \left(5 b^2 d^2 + 12 a b d e + 5 a^2 e^2\right) x^{11} + \frac{1}{2} b^5 e^5 (b d + a e) x^{12} + \frac{1}{13} b^6 e^6 x^{13}$$

- **Problem 1484: Result more than twice size of optimal antiderivative.**

$$\int (d + e x)^5 \left(a^2 + 2 a b x + b^2 x^2\right)^3 dx$$

Optimal (type 1, 143 leaves, 3 steps):

$$\frac{(b d-a e)^5 (a+b x)^7}{7 b^6}+\frac{5 e (b d-a e)^4 (a+b x)^8}{8 b^6}+\frac{10 e^2 (b d-a e)^3 (a+b x)^9}{9 b^6}+\frac{e^3 (b d-a e)^2 (a+b x)^{10}}{b^6}+\frac{5 e^4 (b d-a e) (a+b x)^{11}}{11 b^6}+\frac{e^5 (a+b x)^{12}}{12 b^6}$$

Result (type 1, 501 leaves):

$$a^6 d^5 x+\frac{1}{2} a^5 d^4 (6 b d+5 a e) x^2+\frac{5}{3} a^4 d^3 \left(3 b^2 d^2+6 a b d e+2 a^2 e^2\right) x^3+\frac{5}{4} a^3 d^2 \left(4 b^3 d^3+15 a b^2 d^2 e+12 a^2 b d e^2+2 a^3 e^3\right) x^4+$$
$$a^2 d \left(3 b^4 d^4+20 a b^3 d^3 e+30 a^2 b^2 d^2 e^2+12 a^3 b d e^3+a^4 e^4\right) x^5+\frac{1}{6} a \left(6 b^5 d^5+75 a b^4 d^4 e+200 a^2 b^3 d^3 e^2+150 a^3 b^2 d^2 e^3+30 a^4 b d e^4+a^5 e^5\right) x^6+$$
$$\frac{1}{7} b \left(b^5 d^5+30 a b^4 d^4 e+150 a^2 b^3 d^3 e^2+200 a^3 b^2 d^2 e^3+75 a^4 b d e^4+6 a^5 e^5\right) x^7+$$
$$\frac{5}{8} b^2 e \left(b^4 d^4+12 a b^3 d^3 e+30 a^2 b^2 d^2 e^2+20 a^3 b d e^3+3 a^4 e^4\right) x^8+\frac{5}{9} b^3 e^2 \left(2 b^3 d^3+12 a b^2 d^2 e+15 a^2 b d e^2+4 a^3 e^3\right) x^9+$$
$$\frac{1}{2} b^4 e^3 \left(2 b^2 d^2+6 a b d e+3 a^2 e^2\right) x^{10}+\frac{1}{11} b^5 e^4 (5 b d+6 a e) x^{11}+\frac{1}{12} b^6 e^5 x^{12}$$

Problem 1485: Result more than twice size of optimal antiderivative.

$$\int (d+e x)^4 \left(a^2+2 a b x+b^2 x^2\right)^3 \, dx$$

Optimal (type 1, 119 leaves, 3 steps):

$$\frac{(b d-a e)^4 (a+b x)^7}{7 b^5}+\frac{e (b d-a e)^3 (a+b x)^8}{2 b^5}+\frac{2 e^2 (b d-a e)^2 (a+b x)^9}{3 b^5}+\frac{2 e^3 (b d-a e) (a+b x)^{10}}{5 b^5}+\frac{e^4 (a+b x)^{11}}{11 b^5}$$

Result (type 1, 398 leaves):

$$a^6 d^4 x+a^5 d^3 (3 b d+2 a e) x^2+a^4 d^2 \left(5 b^2 d^2+8 a b d e+2 a^2 e^2\right) x^3+$$
$$a^3 d \left(5 b^3 d^3+15 a b^2 d^2 e+9 a^2 b d e^2+a^3 e^3\right) x^4+\frac{1}{5} a^2 \left(15 b^4 d^4+80 a b^3 d^3 e+90 a^2 b^2 d^2 e^2+24 a^3 b d e^3+a^4 e^4\right) x^5+$$
$$a b \left(b^4 d^4+10 a b^3 d^3 e+20 a^2 b^2 d^2 e^2+10 a^3 b d e^3+a^4 e^4\right) x^6+\frac{1}{7} b^2 \left(b^4 d^4+24 a b^3 d^3 e+90 a^2 b^2 d^2 e^2+80 a^3 b d e^3+15 a^4 e^4\right) x^7+$$
$$\frac{1}{2} b^3 e \left(b^3 d^3+9 a b^2 d^2 e+15 a^2 b d e^2+5 a^3 e^3\right) x^8+\frac{1}{3} b^4 e^2 \left(2 b^2 d^2+8 a b d e+5 a^2 e^2\right) x^9+\frac{1}{5} b^5 e^3 (2 b d+3 a e) x^{10}+\frac{1}{11} b^6 e^4 x^{11}$$

Problem 1486: Result more than twice size of optimal antiderivative.

$$\int (d+e x)^3 \left(a^2+2 a b x+b^2 x^2\right)^3 \, dx$$

Optimal (type 1, 92 leaves, 3 steps):

$$\frac{(b d-a e)^3 (a+b x)^7}{7 b^4}+\frac{3 e (b d-a e)^2 (a+b x)^8}{8 b^4}+\frac{e^2 (b d-a e) (a+b x)^9}{3 b^4}+\frac{e^3 (a+b x)^{10}}{10 b^4}$$

Result (type 1, 276 leaves):

$$\frac{1}{840}x\left(210 a^6 \left(4 d^3+6 d^2 e x+4 d e^2 x^2+e^3 x^3\right)+252 a^5 b x \left(10 d^3+20 d^2 e x+15 d e^2 x^2+4 e^3 x^3\right)+210 a^4 b^2 x^2 \left(20 d^3+45 d^2 e x+36 d e^2 x^2+10 e^3 x^3\right)+120 a^3 b^3 x^3 \left(35 d^3+84 d^2 e x+70 d e^2 x^2+20 e^3 x^3\right)+45 a^2 b^4 x^4 \left(56 d^3+140 d^2 e x+120 d e^2 x^2+35 e^3 x^3\right)+10 a b^5 x^5 \left(84 d^3+216 d^2 e x+189 d e^2 x^2+56 e^3 x^3\right)+b^6 x^6 \left(120 d^3+315 d^2 e x+280 d e^2 x^2+84 e^3 x^3\right)\right)$$

- **Problem 1487: Result more than twice size of optimal antiderivative.**

$$\int (d+e x)^2 \left(a^2+2 a b x+b^2 x^2\right)^3 \, dx$$

Optimal (type 1, 65 leaves, 3 steps):

$$\frac{(b d-a e)^2 (a+b x)^7}{7 b^3}+\frac{e (b d-a e) (a+b x)^8}{4 b^3}+\frac{e^2 (a+b x)^9}{9 b^3}$$

Result (type 1, 199 leaves):

$$\frac{1}{252}x\left(84 a^6 \left(3 d^2+3 d e x+e^2 x^2\right)+126 a^5 b x \left(6 d^2+8 d e x+3 e^2 x^2\right)+126 a^4 b^2 x^2 \left(10 d^2+15 d e x+6 e^2 x^2\right)+84 a^3 b^3 x^3 \left(15 d^2+24 d e x+10 e^2 x^2\right)+36 a^2 b^4 x^4 \left(21 d^2+35 d e x+15 e^2 x^2\right)+9 a b^5 x^5 \left(28 d^2+48 d e x+21 e^2 x^2\right)+b^6 x^6 \left(36 d^2+63 d e x+28 e^2 x^2\right)\right)$$

- **Problem 1488: Result more than twice size of optimal antiderivative.**

$$\int (d+e x) \left(a^2+2 a b x+b^2 x^2\right)^3 \, dx$$

Optimal (type 1, 38 leaves, 3 steps):

$$\frac{(b d-a e) (a+b x)^7}{7 b^2}+\frac{e (a+b x)^8}{8 b^2}$$

Result (type 1, 122 leaves):

$$\frac{1}{56}x\left(28 a^6 (2 d+e x)+56 a^5 b x (3 d+2 e x)+70 a^4 b^2 x^2 (4 d+3 e x)+56 a^3 b^3 x^3 (5 d+4 e x)+28 a^2 b^4 x^4 (6 d+5 e x)+8 a b^5 x^5 (7 d+6 e x)+b^6 x^6 (8 d+7 e x)\right)$$

- **Problem 1497: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a^2+2 a b x+b^2 x^2\right)^3}{(d+e x)^8} \, dx$$

Optimal (type 1, 28 leaves, 2 steps):

$$\frac{(a+b x)^7}{7 (b d-a e) (d+e x)^7}$$

Result (type 1, 271 leaves):

$$-\frac{1}{7 e^7 (d+e x)^7}\left(a^6 e^6+a^5 b e^5 (d+7 e x)+a^4 b^2 e^4 \left(d^2+7 d e x+21 e^2 x^2\right)+a^3 b^3 e^3 \left(d^3+7 d^2 e x+21 d e^2 x^2+35 e^3 x^3\right)+a^2 b^4 e^2 \left(d^4+7 d^3 e x+21 d^2 e^2 x^2+35 d e^3 x^3+35 e^4 x^4\right)+a b^5 e \left(d^5+7 d^4 e x+21 d^3 e^2 x^2+35 d^2 e^3 x^3+35 d e^4 x^4+21 e^5 x^5\right)+b^6 \left(d^6+7 d^5 e x+21 d^4 e^2 x^2+35 d^3 e^3 x^3+35 d^2 e^4 x^4+21 d e^5 x^5+7 e^6 x^6\right)\right)$$

Problem 1498: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a^2+2 a b x+b^2 x^2\right)^3}{(d+e x)^9} \, dx$$

Optimal (type 1, 58 leaves, 3 steps):

$$\frac{(a+b x)^7}{8 (b d-a e) (d+e x)^8}+\frac{b (a+b x)^7}{56 (b d-a e)^2 (d+e x)^7}$$

Result (type 1, 277 leaves):

$$-\frac{1}{56 e^7 (d+e x)^8}\left(7 a^6 e^6+6 a^5 b e^5 (d+8 e x)+5 a^4 b^2 e^4 \left(d^2+8 d e x+28 e^2 x^2\right)+4 a^3 b^3 e^3 \left(d^3+8 d^2 e x+28 d e^2 x^2+56 e^3 x^3\right)+3 a^2 b^4 e^2 \left(d^4+8 d^3 e x+28 d^2 e^2 x^2+56 d e^3 x^3+70 e^4 x^4\right)+2 a b^5 e \left(d^5+8 d^4 e x+28 d^3 e^2 x^2+56 d^2 e^3 x^3+70 d e^4 x^4+56 e^5 x^5\right)+b^6 \left(d^6+8 d^5 e x+28 d^4 e^2 x^2+56 d^3 e^3 x^3+70 d^2 e^4 x^4+56 d e^5 x^5+28 e^6 x^6\right)\right)$$

Problem 1499: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a^2+2 a b x+b^2 x^2\right)^3}{(d+e x)^{10}} \, dx$$

Optimal (type 1, 89 leaves, 4 steps):

$$\frac{(a+b x)^7}{9 (b d-a e) (d+e x)^9}+\frac{b (a+b x)^7}{36 (b d-a e)^2 (d+e x)^8}+\frac{b^2 (a+b x)^7}{252 (b d-a e)^3 (d+e x)^7}$$

Result (type 1, 277 leaves):

$$-\frac{1}{252 e^7 (d+e x)^9}\left(28 a^6 e^6+21 a^5 b e^5 (d+9 e x)+15 a^4 b^2 e^4 \left(d^2+9 d e x+36 e^2 x^2\right)+10 a^3 b^3 e^3 \left(d^3+9 d^2 e x+36 d e^2 x^2+84 e^3 x^3\right)+6 a^2 b^4 e^2 \left(d^4+9 d^3 e x+36 d^2 e^2 x^2+84 d e^3 x^3+126 e^4 x^4\right)+3 a b^5 e \left(d^5+9 d^4 e x+36 d^3 e^2 x^2+84 d^2 e^3 x^3+126 d e^4 x^4+126 e^5 x^5\right)+b^6 \left(d^6+9 d^5 e x+36 d^4 e^2 x^2+84 d^3 e^3 x^3+126 d^2 e^4 x^4+126 d e^5 x^5+84 e^6 x^6\right)\right)$$

Problem 1500: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a^2+2 a b x+b^2 x^2\right)^3}{(d+e x)^{11}} \, dx$$

Optimal (type 1, 120 leaves, 5 steps):

$$\frac{(a+b x)^7}{10 (b d-a e) (d+e x)^{10}}+\frac{b (a+b x)^7}{30 (b d-a e)^2 (d+e x)^9}+\frac{b^2 (a+b x)^7}{120 (b d-a e)^3 (d+e x)^8}+\frac{b^3 (a+b x)^7}{840 (b d-a e)^4 (d+e x)^7}$$

Result (type 1, 277 leaves):

$$-\frac{1}{840 e^7 (d+e x)^{10}} \left(84 a^6 e^6+56 a^5 b e^5 (d+10 e x)+35 a^4 b^2 e^4 \left(d^2+10 d e x+45 e^2 x^2\right)+20 a^3 b^3 e^3 \left(d^3+10 d^2 e x+45 d e^2 x^2+120 e^3 x^3\right)+10 a^2 b^4 e^2 \left(d^4+10 d^3 e x+45 d^2 e^2 x^2+120 d e^3 x^3+210 e^4 x^4\right)+4 a b^5 e \left(d^5+10 d^4 e x+45 d^3 e^2 x^2+120 d^2 e^3 x^3+210 d e^4 x^4+252 e^5 x^5\right)+b^6 \left(d^6+10 d^5 e x+45 d^4 e^2 x^2+120 d^3 e^3 x^3+210 d^2 e^4 x^4+252 d e^5 x^5+210 e^6 x^6\right)\right)$$

- **Problem 1526: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d+e x)^7}{\left(a^2+2 a b x+b^2 x^2\right)^3} \, dx$$

Optimal (type 3, 181 leaves, 3 steps):

$$\frac{e^6 (7 b d-6 a e) x}{b^7}+\frac{e^7 x^2}{2 b^6}-\frac{(b d-a e)^7}{5 b^8 (a+b x)^5}-\frac{7 e (b d-a e)^6}{4 b^8 (a+b x)^4}-\frac{7 e^2 (b d-a e)^5}{b^8 (a+b x)^3}-\frac{35 e^3 (b d-a e)^4}{2 b^8 (a+b x)^2}-\frac{35 e^4 (b d-a e)^3}{b^8 (a+b x)}+\frac{21 e^5 (b d-a e)^2 \text{Log}[a+b x]}{b^8}$$

Result (type 3, 389 leaves):

$$\frac{1}{20 b^8 (a+b x)^5} \left(459 a^7 e^7+3 a^6 b e^6 (-406 d+625 e x)+a^5 b^2 e^5 \left(959 d^2-5250 d e x+2700 e^2 x^2\right)+5 a^4 b^3 e^4 \left(-28 d^3+875 d^2 e x-1680 d e^2 x^2+260 e^3 x^3\right)-5 a^3 b^4 e^3 \left(7 d^4+140 d^3 e x-1540 d^2 e^2 x^2+1120 d e^3 x^3+80 e^4 x^4\right)-a^2 b^5 e^2 \left(14 d^5+175 d^4 e x+1400 d^3 e^2 x^2-6300 d^2 e^3 x^3+700 d e^4 x^4+500 e^5 x^5\right)-7 a b^6 e \left(d^6+10 d^5 e x+50 d^4 e^2 x^2+200 d^3 e^3 x^3-300 d^2 e^4 x^4-100 d e^5 x^5+10 e^6 x^6\right)-b^7 \left(4 d^7+35 d^6 e x+140 d^5 e^2 x^2+350 d^4 e^3 x^3+700 d^3 e^4 x^4-140 d e^6 x^6-10 e^7 x^7\right)+420 e^5 (b d-a e)^2 (a+b x)^5 \text{Log}[a+b x]\right)$$

- **Problem 1529: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d+e x)^4}{\left(a^2+2 a b x+b^2 x^2\right)^3} \, dx$$

Optimal (type 1, 28 leaves, 2 steps):

$$-\frac{(d+e x)^5}{5 (b d-a e) (a+b x)^5}$$

Result (type 1, 140 leaves):

$$-\frac{1}{5 b^5 (a+b x)^5} \left(a^4 e^4+a^3 b e^3 (d+5 e x)+a^2 b^2 e^2 \left(d^2+5 d e x+10 e^2 x^2\right)+a b^3 e \left(d^3+5 d^2 e x+10 d e^2 x^2+10 e^3 x^3\right)+b^4 \left(d^4+5 d^3 e x+10 d^2 e^2 x^2+10 d e^3 x^3+5 e^4 x^4\right)\right)$$

- **Problem 1537: Result more than twice size of optimal antiderivative.**

$$\int (d+e x) \left(9+12 x+4 x^2\right)^3 \, dx$$

Optimal (type 1, 31 leaves, 3 steps):

$$\frac{1}{28}(2d-3e)(3+2x)^7+\frac{1}{32}e(3+2x)^8$$

Result (type 1, 81 leaves):

$$729dx+\frac{729}{2}(4d+e)x^2+324(5d+3e)x^3+135(8d+9e)x^4+432(d+2e)x^5+24(4d+15e)x^6+\frac{64}{7}(d+9e)x^7+8ex^8$$

- **Problem 1564: Result more than twice size of optimal antiderivative.**

$$\int\frac{(a^2+2abx+b^2x^2)^{3/2}}{(d+ex)^5}\,dx$$

Optimal (type 2, 48 leaves, 2 steps):

$$\frac{(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{4(bd-ae)(d+ex)^4}$$

Result (type 2, 109 leaves):

$$-\frac{1}{4e^4(a+bx)(d+ex)^4}\sqrt{(a+bx)^2}\left(a^3e^3+a^2be^2(d+4ex)+ab^2e\left(d^2+4dex+6e^2x^2\right)+b^3\left(d^3+4d^2ex+6de^2x^2+4e^3x^3\right)\right)$$

- **Problem 1581: Result more than twice size of optimal antiderivative.**

$$\int\frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^7}\,dx$$

Optimal (type 2, 48 leaves, 2 steps):

$$\frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6(bd-ae)(d+ex)^6}$$

Result (type 2, 218 leaves):

$$-\frac{1}{6e^6(a+bx)(d+ex)^6}\sqrt{(a+bx)^2}\left(a^5e^5+a^4be^4(d+6ex)+a^3b^2e^3\left(d^2+6dex+15e^2x^2\right)+a^2b^3e^2\left(d^3+6d^2ex+15de^2x^2+20e^3x^3\right)+\right.$$
$$\left.ab^4e\left(d^4+6d^3ex+15d^2e^2x^2+20de^3x^3+15e^4x^4\right)+b^5\left(d^5+6d^4ex+15d^3e^2x^2+20d^2e^3x^3+15de^4x^4+6e^5x^5\right)\right)$$

- **Problem 1582: Result more than twice size of optimal antiderivative.**

$$\int\frac{(a^2+2abx+b^2x^2)^{5/2}}{(d+ex)^8}\,dx$$

Optimal (type 2, 98 leaves, 3 steps):

$$\frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{7(bd-ae)(d+ex)^7}+\frac{b(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{42(bd-ae)^2(d+ex)^6}$$

Result (type 2, 223 leaves):

$$-\frac{1}{42 e^6 (a+b x) (d+e x)^7}\sqrt{(a+b x)^2} \left(6 a^5 e^5+5 a^4 b e^4 (d+7 e x)+4 a^3 b^2 e^3 \left(d^2+7 d e x+21 e^2 x^2\right)+3 a^2 b^3 e^2 \left(d^3+7 d^2 e x+21 d e^2 x^2+35 e^3 x^3\right)+2 a b^4 e \left(d^4+7 d^3 e x+21 d^2 e^2 x^2+35 d e^3 x^3+35 e^4 x^4\right)+b^5 \left(d^5+7 d^4 e x+21 d^3 e^2 x^2+35 d^2 e^3 x^3+35 d e^4 x^4+21 e^5 x^5\right)\right)$$

- **Problem 1607: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d+e x)^3}{\left(a^2+2 a b x+b^2 x^2\right)^{5/2}} \, dx$$

Optimal (type 2, 48 leaves, 2 steps):

$$-\frac{(d+e x)^4}{4 (b d-a e) (a+b x)^3 \sqrt{a^2+2 a b x+b^2 x^2}}$$

Result (type 2, 106 leaves):

$$\frac{-a^3 e^3-a^2 b e^2 (d+4 e x)-a b^2 e \left(d^2+4 d e x+6 e^2 x^2\right)-b^3 \left(d^3+4 d^2 e x+6 d e^2 x^2+4 e^3 x^3\right)}{4 b^4 (a+b x)^3 \sqrt{(a+b x)^2}}$$

- **Problem 1731: Result more than twice size of optimal antiderivative.**

$$\int (d+e x)^m \left(a^2+2 a b x+b^2 x^2\right)^3 \, dx$$

Optimal (type 3, 206 leaves, 3 steps):

$$\frac{(b d-a e)^6 (d+e x)^{1+m}}{e^7 (1+m)}-\frac{6 b (b d-a e)^5 (d+e x)^{2+m}}{e^7 (2+m)}+\frac{15 b^2 (b d-a e)^4 (d+e x)^{3+m}}{e^7 (3+m)}-\frac{20 b^3 (b d-a e)^3 (d+e x)^{4+m}}{e^7 (4+m)}+\frac{15 b^4 (b d-a e)^2 (d+e x)^{5+m}}{e^7 (5+m)}-\frac{6 b^5 (b d-a e) (d+e x)^{6+m}}{e^7 (6+m)}+\frac{b^6 (d+e x)^{7+m}}{e^7 (7+m)}$$

Result (type 3, 646 leaves):

$$\frac{1}{e^7 (1+m) (2+m) (3+m) (4+m) (5+m) (6+m) (7+m)} (d+e x)^{1+m} \left(a^6 e^6 \left(5040+8028 m+5104 m^2+1665 m^3+295 m^4+27 m^5+m^6\right)-6 a^5 b e^5 \left(2520+2754 m+1175 m^2+245 m^3+25 m^4+m^5\right) (d-e (1+m) x)+15 a^4 b^2 e^4 \left(840+638 m+179 m^2+22 m^3+m^4\right) \left(2 d^2-2 d e (1+m) x+e^2 \left(2+3 m+m^2\right) x^2\right)+20 a^3 b^3 e^3 \left(210+107 m+18 m^2+m^3\right) \left(-6 d^3+6 d^2 e (1+m) x-3 d e^2 \left(2+3 m+m^2\right) x^2+e^3 \left(6+11 m+6 m^2+m^3\right) x^3\right)+15 a^2 b^4 e^2 \left(42+13 m+m^2\right) \left(24 d^4-24 d^3 e (1+m) x+12 d^2 e^2 \left(2+3 m+m^2\right) x^2-4 d e^3 \left(6+11 m+6 m^2+m^3\right) x^3+e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4\right)+6 a b^5 e (7+m) \left(-120 d^5+120 d^4 e (1+m) x-60 d^3 e^2 \left(2+3 m+m^2\right) x^2+20 d^2 e^3 \left(6+11 m+6 m^2+m^3\right) x^3-5 d e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4+e^5 \left(120+274 m+225 m^2+85 m^3+15 m^4+m^5\right) x^5\right)+b^6 \left(720 d^6-720 d^5 e (1+m) x+360 d^4 e^2 \left(2+3 m+m^2\right) x^2-120 d^3 e^3 \left(6+11 m+6 m^2+m^3\right) x^3+30 d^2 e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4-6 d e^5 \left(120+274 m+225 m^2+85 m^3+15 m^4+m^5\right) x^5+e^6 \left(720+1764 m+1624 m^2+735 m^3+175 m^4+21 m^5+m^6\right) x^6\right)\right)$$

Problem 1732: Result more than twice size of optimal antiderivative.

$$\int (d+e x)^m \left(a^2+2 a b x+b^2 x^2\right)^2 \, dx$$

Optimal (type 3, 142 leaves, 3 steps):

$$\frac{(b d-a e)^4 (d+e x)^{1+m}}{e^5 (1+m)}-\frac{4 b (b d-a e)^3 (d+e x)^{2+m}}{e^5 (2+m)}+\frac{6 b^2 (b d-a e)^2 (d+e x)^{3+m}}{e^5 (3+m)}-\frac{4 b^3 (b d-a e) (d+e x)^{4+m}}{e^5 (4+m)}+\frac{b^4 (d+e x)^{5+m}}{e^5 (5+m)}$$

Result (type 3, 292 leaves):

$$\frac{1}{e^5 (1+m) (2+m) (3+m) (4+m) (5+m)} (d+e x)^{1+m} \left(a^4 e^4 \left(120+154 m+71 m^2+14 m^3+m^4\right)-4 a^3 b e^3 \left(60+47 m+12 m^2+m^3\right) (d-e (1+m) x)+6 a^2 b^2 e^2 \left(20+9 m+m^2\right) \left(2 d^2-2 d e (1+m) x+e^2 \left(2+3 m+m^2\right) x^2\right)+4 a b^3 e (5+m) \left(-6 d^3+6 d^2 e (1+m) x-3 d e^2 \left(2+3 m+m^2\right) x^2+e^3 \left(6+11 m+6 m^2+m^3\right) x^3\right)+b^4 \left(24 d^4-24 d^3 e (1+m) x+12 d^2 e^2 \left(2+3 m+m^2\right) x^2-4 d e^3 \left(6+11 m+6 m^2+m^3\right) x^3+e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4\right)\right)$$

Problem 1734: Unable to integrate problem.

$$\int \frac{(d+e x)^m}{a^2+2 a b x+b^2 x^2} \, dx$$

Optimal (type 5, 51 leaves, 2 steps):

$$\frac{e (d+e x)^{1+m} \text{Hypergeometric2F1}\left[2, 1+m, 2+m, \frac{b (d+e x)}{b d-a e}\right]}{(b d-a e)^2 (1+m)}$$

Result (type 8, 28 leaves):

$$\int \frac{(d+e x)^m}{a^2+2 a b x+b^2 x^2} \, dx$$

Problem 1735: Unable to integrate problem.

$$\int \frac{(d+e x)^m}{\left(a^2+2 a b x+b^2 x^2\right)^2} \, dx$$

Optimal (type 5, 53 leaves, 2 steps):

$$\frac{e^3 (d+e x)^{1+m} \text{Hypergeometric2F1}\left[4, 1+m, 2+m, \frac{b (d+e x)}{b d-a e}\right]}{(b d-a e)^4 (1+m)}$$

Result (type 8, 28 leaves):

$$\int \frac{(d+e x)^m}{\left(a^2+2 a b x+b^2 x^2\right)^2} \, dx$$

Problem 1736: Unable to integrate problem.

$$\int \frac{(d+e x)^m}{\left(a^2+2 a b x+b^2 x^2\right)^3} \, dx$$

Optimal (type 5, 53 leaves, 2 steps):

$$\frac{e^5 (d+e x)^{1+m} \text{Hypergeometric2F1}\left[6, 1+m, 2+m, \frac{b (d+e x)}{b d-a e}\right]}{(b d-a e)^6 (1+m)}$$

Result (type 8, 28 leaves):

$$\int \frac{(d+e x)^m}{\left(a^2+2 a b x+b^2 x^2\right)^3} \, dx$$

Problem 1741: Unable to integrate problem.

$$\int \frac{(d+e x)^m}{\left(a^2+2 a b x+b^2 x^2\right)^{3/2}} \, dx$$

Optimal (type 5, 79 leaves, 2 steps):

$$-\frac{e^2 (a+b x) (d+e x)^{1+m} \text{Hypergeometric2F1}\left[3, 1+m, 2+m, \frac{b (d+e x)}{b d-a e}\right]}{(b d-a e)^3 (1+m) \sqrt{a^2+2 a b x+b^2 x^2}}$$

Result (type 8, 30 leaves):

$$\int \frac{(d+e x)^m}{\left(a^2+2 a b x+b^2 x^2\right)^{3/2}} \, dx$$

Problem 1742: Unable to integrate problem.

$$\int \frac{(d+e x)^m}{\left(a^2+2 a b x+b^2 x^2\right)^{5/2}} \, dx$$

Optimal (type 5, 79 leaves, 2 steps):

$$-\frac{e^4 (a+b x) (d+e x)^{1+m} \text{Hypergeometric2F1}\left[5, 1+m, 2+m, \frac{b (d+e x)}{b d-a e}\right]}{(b d-a e)^5 (1+m) \sqrt{a^2+2 a b x+b^2 x^2}}$$

Result (type 8, 30 leaves):

$$\int \frac{(d+e x)^m}{\left(a^2+2 a b x+b^2 x^2\right)^{5/2}} \, dx$$

Problem 1749: Unable to integrate problem.

$$\int \frac{\left(a^2+2 a b x+b^2 x^2\right)^p}{(d+e x)^2} \, dx$$

Optimal (type 5, 72 leaves, 2 steps):

$$\frac{b (a+b x) \left(a^2+2 a b x+b^2 x^2\right)^p \text{Hypergeometric2F1}\left[2,1+2 p,2 (1+p),-\frac{e (a+b x)}{b d-a e}\right]}{(b d-a e)^2 (1+2 p)}$$

Result (type 8, 28 leaves):

$$\int \frac{\left(a^2+2 a b x+b^2 x^2\right)^p}{(d+e x)^2} \, dx$$

Problem 1750: Unable to integrate problem.

$$\int \frac{\left(a^2+2 a b x+b^2 x^2\right)^p}{(d+e x)^3} \, dx$$

Optimal (type 5, 74 leaves, 2 steps):

$$\frac{b^2 (a+b x) \left(a^2+2 a b x+b^2 x^2\right)^p \text{Hypergeometric2F1}\left[3,1+2 p,2 (1+p),-\frac{e (a+b x)}{b d-a e}\right]}{(b d-a e)^3 (1+2 p)}$$

Result (type 8, 28 leaves):

$$\int \frac{\left(a^2+2 a b x+b^2 x^2\right)^p}{(d+e x)^3} \, dx$$

Problem 1751: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (d+e x)^{3/2} \left(a^2+2 a b x+b^2 x^2\right)^p \, dx$$

Optimal (type 5, 83 leaves, 3 steps):

$$\frac{2 \left(-\frac{e (a+b x)}{b d-a e}\right)^{-2 p} (d+e x)^{5/2} \left(a^2+2 a b x+b^2 x^2\right)^p \text{Hypergeometric2F1}\left[\frac{5}{2},-2 p,\frac{7}{2},\frac{b (d+e x)}{b d-a e}\right]}{5 e}$$

Result (type 6, 202 leaves):

$$\frac{1}{3 e} d \left((a+b x)^2\right)^p \sqrt{d+e x} \left(\left(9 a e^2 x^2 \text{AppellF1}\left[2,-2 p,-\frac{1}{2},3,-\frac{b x}{a},-\frac{e x}{d}\right]\right)\Big/\right.$$
$$\left(6 a d \text{AppellF1}\left[2,-2 p,-\frac{1}{2},3,-\frac{b x}{a},-\frac{e x}{d}\right]+4 b d p x \text{AppellF1}\left[3,1-2 p,-\frac{1}{2},4,-\frac{b x}{a},-\frac{e x}{d}\right]+\right.$$
$$\left.\left.a e x \text{AppellF1}\left[3,-2 p,\frac{1}{2},4,-\frac{b x}{a},-\frac{e x}{d}\right]\right)+2 \left(\frac{e (a+b x)}{-b d+a e}\right)^{-2 p} (d+e x) \text{Hypergeometric2F1}\left[\frac{3}{2},-2 p,\frac{5}{2},\frac{b (d+e x)}{b d-a e}\right]\right)$$

Problem 1759: Result more than twice size of optimal antiderivative.

$$\int (a+b x)^3 \left(a c+(b c+a d) x+b d x^2\right) dx$$

Optimal (type 1, 38 leaves, 3 steps):

$$\frac{(b c-a d) (a+b x)^5}{5 b^2}+\frac{d (a+b x)^6}{6 b^2}$$

Result (type 1, 84 leaves):

$$\frac{1}{30} x \left(15 a^4 (2 c+d x)+20 a^3 b x (3 c+2 d x)+15 a^2 b^2 x^2 (4 c+3 d x)+6 a b^3 x^3 (5 c+4 d x)+b^4 x^4 (6 c+5 d x)\right)$$

Problem 1769: Result more than twice size of optimal antiderivative.

$$\int (a+b x)^3 \left(a c+(b c+a d) x+b d x^2\right)^2 dx$$

Optimal (type 1, 65 leaves, 3 steps):

$$\frac{(b c-a d)^2 (a+b x)^6}{6 b^3}+\frac{2 d (b c-a d) (a+b x)^7}{7 b^3}+\frac{d^2 (a+b x)^8}{8 b^3}$$

Result (type 1, 189 leaves):

$$a^5 c^2 x+\frac{1}{2} a^4 c (5 b c+2 a d) x^2+\frac{1}{3} a^3 \left(10 b^2 c^2+10 a b c d+a^2 d^2\right) x^3+\frac{5}{4} a^2 b \left(2 b^2 c^2+4 a b c d+a^2 d^2\right) x^4+$$
$$a b^2 \left(b^2 c^2+4 a b c d+2 a^2 d^2\right) x^5+\frac{1}{6} b^3 \left(b^2 c^2+10 a b c d+10 a^2 d^2\right) x^6+\frac{1}{7} b^4 d (2 b c+5 a d) x^7+\frac{1}{8} b^5 d^2 x^8$$

Problem 1770: Result more than twice size of optimal antiderivative.

$$\int (a+b x)^2 \left(a c+(b c+a d) x+b d x^2\right)^2 dx$$

Optimal (type 1, 65 leaves, 3 steps):

$$\frac{(b c-a d)^2 (a+b x)^5}{5 b^3}+\frac{d (b c-a d) (a+b x)^6}{3 b^3}+\frac{d^2 (a+b x)^7}{7 b^3}$$

Result (type 1, 148 leaves):

$$a^4 c^2 x+a^3 c (2 b c+a d) x^2+\frac{1}{3} a^2 \left(6 b^2 c^2+8 a b c d+a^2 d^2\right) x^3+$$
$$a b \left(b^2 c^2+3 a b c d+a^2 d^2\right) x^4+\frac{1}{5} b^2 \left(b^2 c^2+8 a b c d+6 a^2 d^2\right) x^5+\frac{1}{3} b^3 d (b c+2 a d) x^6+\frac{1}{7} b^4 d^2 x^7$$

Problem 1783: Result more than twice size of optimal antiderivative.

$$\int (a+b x)^3 \left(a c+(b c+a d) x+b d x^2\right)^3 dx$$

Optimal (type 1, 92 leaves, 3 steps):

$$\frac{(b c-a d)^3 (a+b x)^7}{7 b^4}+\frac{3 d (b c-a d)^2 (a+b x)^8}{8 b^4}+\frac{d^2 (b c-a d) (a+b x)^9}{3 b^4}+\frac{d^3 (a+b x)^{10}}{10 b^4}$$

Result (type 1, 276 leaves):

$$\frac{1}{840} x \left(210 a^6 \left(4 c^3+6 c^2 d x+4 c d^2 x^2+d^3 x^3\right)+252 a^5 b x \left(10 c^3+20 c^2 d x+15 c d^2 x^2+4 d^3 x^3\right)+210 a^4 b^2 x^2 \left(20 c^3+45 c^2 d x+36 c d^2 x^2+10 d^3 x^3\right)+120 a^3 b^3 x^3 \left(35 c^3+84 c^2 d x+70 c d^2 x^2+20 d^3 x^3\right)+45 a^2 b^4 x^4 \left(56 c^3+140 c^2 d x+120 c d^2 x^2+35 d^3 x^3\right)+10 a b^5 x^5 \left(84 c^3+216 c^2 d x+189 c d^2 x^2+56 d^3 x^3\right)+b^6 x^6 \left(120 c^3+315 c^2 d x+280 c d^2 x^2+84 d^3 x^3\right)\right)$$

Problem 1784: Result more than twice size of optimal antiderivative.

$$\int (a+b x)^2 \left(a c+(b c+a d) x+b d x^2\right)^3 \, dx$$

Optimal (type 1, 92 leaves, 3 steps):

$$\frac{(b c-a d)^3 (a+b x)^6}{6 b^4}+\frac{3 d (b c-a d)^2 (a+b x)^7}{7 b^4}+\frac{3 d^2 (b c-a d) (a+b x)^8}{8 b^4}+\frac{d^3 (a+b x)^9}{9 b^4}$$

Result (type 1, 235 leaves):

$$\frac{1}{504} x \left(126 a^5 \left(4 c^3+6 c^2 d x+4 c d^2 x^2+d^3 x^3\right)+126 a^4 b x \left(10 c^3+20 c^2 d x+15 c d^2 x^2+4 d^3 x^3\right)+84 a^3 b^2 x^2 \left(20 c^3+45 c^2 d x+36 c d^2 x^2+10 d^3 x^3\right)+36 a^2 b^3 x^3 \left(35 c^3+84 c^2 d x+70 c d^2 x^2+20 d^3 x^3\right)+9 a b^4 x^4 \left(56 c^3+140 c^2 d x+120 c d^2 x^2+35 d^3 x^3\right)+b^5 x^5 \left(84 c^3+216 c^2 d x+189 c d^2 x^2+56 d^3 x^3\right)\right)$$

Problem 1785: Result more than twice size of optimal antiderivative.

$$\int (a+b x) \left(a c+(b c+a d) x+b d x^2\right)^3 \, dx$$

Optimal (type 1, 92 leaves, 3 steps):

$$\frac{(b c-a d)^3 (a+b x)^5}{5 b^4}+\frac{d (b c-a d)^2 (a+b x)^6}{2 b^4}+\frac{3 d^2 (b c-a d) (a+b x)^7}{7 b^4}+\frac{d^3 (a+b x)^8}{8 b^4}$$

Result (type 1, 217 leaves):

$$a^4 c^3 x+\frac{1}{2} a^3 c^2 (4 b c+3 a d) x^2+a^2 c \left(2 b^2 c^2+4 a b c d+a^2 d^2\right) x^3+\frac{1}{4} a \left(4 b^3 c^3+18 a b^2 c^2 d+12 a^2 b c d^2+a^3 d^3\right) x^4+\frac{1}{5} b \left(b^3 c^3+12 a b^2 c^2 d+18 a^2 b c d^2+4 a^3 d^3\right) x^5+\frac{1}{2} b^2 d \left(b^2 c^2+4 a b c d+2 a^2 d^2\right) x^6+\frac{1}{7} b^3 d^2 (3 b c+4 a d) x^7+\frac{1}{8} b^4 d^3 x^8$$

Problem 1794: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a c+(b c+a d) x+b d x^2\right)^3}{(a+b x)^8} \, dx$$

Optimal (type 1, 28 leaves, 2 steps):

$$-\frac{(c+d x)^4}{4 (b c-a d) (a+b x)^4}$$

Result (type 1, 91 leaves):

$$-\frac{a^3 d^3+a^2 b d^2 (c+4 d x)+a b^2 d \left(c^2+4 c d x+6 d^2 x^2\right)+b^3 \left(c^3+4 c^2 d x+6 c d^2 x^2+4 d^3 x^3\right)}{4 b^4 (a+b x)^4}$$

- **Problem 1828: Result more than twice size of optimal antiderivative.**

$$\int (d+e x)^4 \left(a d e+\left(c d^2+a e^2\right) x+c d e x^2\right) \, dx$$

Optimal (type 1, 39 leaves, 3 steps):

$$\frac{1}{6} \left(a-\frac{c d^2}{e^2}\right) (d+e x)^6+\frac{c d (d+e x)^7}{7 e^2}$$

Result (type 1, 117 leaves):

$$\frac{1}{42} x \left(7 a e \left(6 d^5+15 d^4 e x+20 d^3 e^2 x^2+15 d^2 e^3 x^3+6 d e^4 x^4+e^5 x^5\right)+c d x \left(21 d^5+70 d^4 e x+105 d^3 e^2 x^2+84 d^2 e^3 x^3+35 d e^4 x^4+6 e^5 x^5\right)\right)$$

- **Problem 1829: Result more than twice size of optimal antiderivative.**

$$\int (d+e x)^3 \left(a d e+\left(c d^2+a e^2\right) x+c d e x^2\right) \, dx$$

Optimal (type 1, 39 leaves, 3 steps):

$$\frac{1}{5} \left(a-\frac{c d^2}{e^2}\right) (d+e x)^5+\frac{c d (d+e x)^6}{6 e^2}$$

Result (type 1, 95 leaves):

$$\frac{1}{30} x \left(6 a e \left(5 d^4+10 d^3 e x+10 d^2 e^2 x^2+5 d e^3 x^3+e^4 x^4\right)+c d x \left(15 d^4+40 d^3 e x+45 d^2 e^2 x^2+24 d e^3 x^3+5 e^4 x^4\right)\right)$$

- **Problem 1839: Result more than twice size of optimal antiderivative.**

$$\int (d+e x)^2 \left(a d e+\left(c d^2+a e^2\right) x+c d e x^2\right)^2 \, dx$$

Optimal (type 1, 77 leaves, 3 steps):

$$\frac{\left(c d^2-a e^2\right)^2 (d+e x)^5}{5 e^3}-\frac{c d \left(c d^2-a e^2\right) (d+e x)^6}{3 e^3}+\frac{c^2 d^2 (d+e x)^7}{7 e^3}$$

Result (type 1, 160 leaves):

$$\frac{1}{15} a c d e x^2 \left(15 d^4+40 d^3 e x+45 d^2 e^2 x^2+24 d e^3 x^3+5 e^4 x^4\right)+$$
$$\frac{1}{105} c^2 d^2 x^3 \left(35 d^4+105 d^3 e x+126 d^2 e^2 x^2+70 d e^3 x^3+15 e^4 x^4\right)+a^2 \left(d^4 e^2 x+2 d^3 e^3 x^2+2 d^2 e^4 x^3+d e^5 x^4+\frac{e^6 x^5}{5}\right)$$

- **Problem 1851: Result more than twice size of optimal antiderivative.**

$$\int (d+e x)^2 \left(a d e+\left(c d^2+a e^2\right) x+c d e x^2\right)^3 \, dx$$

Optimal (type 1, 111 leaves, 3 steps):

$$-\frac{\left(c d^2-a e^2\right)^3 (d+e x)^6}{6 e^4}+\frac{3 c d \left(c d^2-a e^2\right)^2 (d+e x)^7}{7 e^4}-\frac{3 c^2 d^2 \left(c d^2-a e^2\right) (d+e x)^8}{8 e^4}+\frac{c^3 d^3 (d+e x)^9}{9 e^4}$$

Result (type 1, 255 leaves):

$$\frac{1}{504} x \left(84 a^3 e^3 \left(6 d^5+15 d^4 e x+20 d^3 e^2 x^2+15 d^2 e^3 x^3+6 d e^4 x^4+e^5 x^5\right)+36 a^2 c d e^2 x \left(21 d^5+70 d^4 e x+105 d^3 e^2 x^2+84 d^2 e^3 x^3+35 d e^4 x^4+6 e^5 x^5\right)+9 a c^2 d^2 e x^2 \left(56 d^5+210 d^4 e x+336 d^3 e^2 x^2+280 d^2 e^3 x^3+120 d e^4 x^4+21 e^5 x^5\right)+c^3 d^3 x^3 \left(126 d^5+504 d^4 e x+840 d^3 e^2 x^2+720 d^2 e^3 x^3+315 d e^4 x^4+56 e^5 x^5\right)\right)$$

- **Problem 1861: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a d e+\left(c d^2+a e^2\right) x+c d e x^2\right)^3}{(d+e x)^8} \, dx$$

Optimal (type 1, 35 leaves, 2 steps):

$$\frac{(a e+c d x)^4}{4 \left(c d^2-a e^2\right) (d+e x)^4}$$

Result (type 1, 100 leaves):

$$-\frac{a^3 e^6+a^2 c d e^4 (d+4 e x)+a c^2 d^2 e^2 \left(d^2+4 d e x+6 e^2 x^2\right)+c^3 d^3 \left(d^3+4 d^2 e x+6 d e^2 x^2+4 e^3 x^3\right)}{4 e^4 (d+e x)^4}$$

- **Problem 1974: Result unnecessarily involves higher level functions.**

$$\int \frac{d+e x}{\left(a d e+\left(c d^2+a e^2\right) x+c d e x^2\right)^{1/3}} \, dx$$

Optimal (type 4, 1485 leaves, 5 steps):

$$\frac{3\left(a d e+\left(c d^2+a e^2\right) x+c d e x^2\right)^{2/3}}{4 c d}+\left(3\left(c d^2-a e^2\right)\sqrt{\left(c d^2+a e^2+2 c d e x\right)^2}\sqrt{\left(a e^2+c d\left(d+2 e x\right)\right)^2}\right)\Big/$$

$$\left(2\times 2^{1/3} c^{5/3} d^{5/3} e^{2/3}\left(c d^2+a e^2+2 c d e x\right)\left(\left(1+\sqrt{3}\right)\left(c d^2-a e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3}\left(\left(a e+c d x\right)\left(d+e x\right)\right)^{1/3}\right)\right)-$$

$$\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\left(c d^2-a e^2\right)^{5/3}\sqrt{\left(c d^2+a e^2+2 c d e x\right)^2}\left(\left(c d^2-a e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3}\left(\left(a e+c d x\right)\left(d+e x\right)\right)^{1/3}\right)\right.$$

$$\sqrt{\left(\left(\left(c d^2-a e^2\right)^{4/3}-2^{2/3} c^{1/3} d^{1/3} e^{1/3}\left(c d^2-a e^2\right)^{2/3}\left(\left(a e+c d x\right)\left(d+e x\right)\right)^{1/3}+2\times 2^{1/3} c^{2/3} d^{2/3} e^{2/3}\left(\left(a e+c d x\right)\left(d+e x\right)\right)^{2/3}\right)\Big/\left(\left(1+\sqrt{3}\right)\left(c d^2-a e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3}\left(\left(a e+c d x\right)\left(d+e x\right)\right)^{1/3}\right)^2\right)}$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\left(c d^2-a e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3}\left(\left(a e+c d x\right)\left(d+e x\right)\right)^{1/3}}{\left(1+\sqrt{3}\right)\left(c d^2-a e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3}\left(\left(a e+c d x\right)\left(d+e x\right)\right)^{1/3}}\right],-7-4\sqrt{3}\right]\right)\Big/\left(4\times 2^{1/3} c^{5/3} d^{5/3}\right.$$

$$\left.e^{2/3}\left(c d^2+a e^2+2 c d e x\right)\sqrt{\frac{\left(c d^2-a e^2\right)^{2/3}\left(\left(c d^2-a e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3}\left(\left(a e+c d x\right)\left(d+e x\right)\right)^{1/3}\right)}{\left(\left(1+\sqrt{3}\right)\left(c d^2-a e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3}\left(\left(a e+c d x\right)\left(d+e x\right)\right)^{1/3}\right)^2}}\sqrt{\left(a e^2+c d\left(d+2 e x\right)\right)^2}\right)+$$

$$\left(3^{3/4}\left(c d^2-a e^2\right)^{5/3}\sqrt{\left(c d^2+a e^2+2 c d e x\right)^2}\left(\left(c d^2-a e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3}\left(\left(a e+c d x\right)\left(d+e x\right)\right)^{1/3}\right)\right.$$

$$\sqrt{\left(\left(\left(c d^2-a e^2\right)^{4/3}-2^{2/3} c^{1/3} d^{1/3} e^{1/3}\left(c d^2-a e^2\right)^{2/3}\left(\left(a e+c d x\right)\left(d+e x\right)\right)^{1/3}+2\times 2^{1/3} c^{2/3} d^{2/3} e^{2/3}\left(\left(a e+c d x\right)\left(d+e x\right)\right)^{2/3}\right)\Big/\left(\left(1+\sqrt{3}\right)\left(c d^2-a e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3}\left(\left(a e+c d x\right)\left(d+e x\right)\right)^{1/3}\right)^2\right)}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\left(c d^2-a e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3}\left(\left(a e+c d x\right)\left(d+e x\right)\right)^{1/3}}{\left(1+\sqrt{3}\right)\left(c d^2-a e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3}\left(\left(a e+c d x\right)\left(d+e x\right)\right)^{1/3}}\right],-7-4\sqrt{3}\right]\right)\Big/\left(2^{5/6} c^{5/3} d^{5/3} e^{2/3}\right.$$

$$\left.\left(c d^2+a e^2+2 c d e x\right)\sqrt{\frac{\left(c d^2-a e^2\right)^{2/3}\left(\left(c d^2-a e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3}\left(\left(a e+c d x\right)\left(d+e x\right)\right)^{1/3}\right)}{\left(\left(1+\sqrt{3}\right)\left(c d^2-a e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3}\left(\left(a e+c d x\right)\left(d+e x\right)\right)^{1/3}\right)^2}}\sqrt{\left(a e^2+c d\left(d+2 e x\right)\right)^2}\right)$$

Result (type 5, 120 leaves):

$$\frac{3\left(d+e x\right)\left(e\left(a e+c d x\right)+\left(c d^2-a e^2\right)\left(\frac{e\left(a e+c d x\right)}{-c d^2+a e^2}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{c d\left(d+e x\right)}{c d^2-a e^2}\right]\right)}{4 c d e\left(\left(a e+c d x\right)\left(d+e x\right)\right)^{1/3}}$$

Problem 1975: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a\, d\, e+\left(c\, d^2+a\, e^2\right) x+c\, d\, e\, x^2\right)^{1/3}}\, dx$$

Optimal (type 4, 1432 leaves, 4 steps):

$$\left(3 \sqrt{\left(c\, d^2+a\, e^2+2\, c\, d\, e\, x\right)^2} \sqrt{\left(a\, e^2+c\, d\, (d+2\, e\, x)\right)^2}\right) \Big/$$

$$\left(2^{1/3} c^{2/3} d^{2/3} e^{2/3} \left(c\, d^2+a\, e^2+2\, c\, d\, e\, x\right) \left(\left(1+\sqrt{3}\right) \left(c\, d^2-a\, e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3} \left((a\, e+c\, d\, x)\, (d+e\, x)\right)^{1/3}\right)\right) -$$

$$\Bigg(3\times 3^{1/4} \sqrt{2-\sqrt{3}} \left(c\, d^2-a\, e^2\right)^{2/3} \sqrt{\left(c\, d^2+a\, e^2+2\, c\, d\, e\, x\right)^2} \left(\left(c\, d^2-a\, e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3} \left((a\, e+c\, d\, x)\, (d+e\, x)\right)^{1/3}\right)$$

$$\sqrt{\left(\left(c\, d^2-a\, e^2\right)^{4/3}-2^{2/3} c^{1/3} d^{1/3} e^{1/3} \left(c\, d^2-a\, e^2\right)^{2/3} \left((a\, e+c\, d\, x)\, (d+e\, x)\right)^{1/3}+2\times 2^{1/3} c^{2/3} d^{2/3} e^{2/3} \left((a\, e+c\, d\, x)\, (d+e\, x)\right)^{2/3}\right) \Big/ \left(\left(1+\sqrt{3}\right) \left(c\, d^2-a\, e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3} \left((a\, e+c\, d\, x)\, (d+e\, x)\right)^{1/3}\right)^2}$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) \left(c\, d^2-a\, e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3} \left((a\, e+c\, d\, x)\, (d+e\, x)\right)^{1/3}}{\left(1+\sqrt{3}\right) \left(c\, d^2-a\, e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3} \left((a\, e+c\, d\, x)\, (d+e\, x)\right)^{1/3}}\right],-7-4 \sqrt{3}\right]\Bigg) \Bigg/ \Bigg(2\times 2^{1/3} c^{2/3} d^{2/3}$$

$$e^{2/3} \left(c\, d^2+a\, e^2+2\, c\, d\, e\, x\right) \sqrt{\frac{\left(c\, d^2-a\, e^2\right)^{2/3} \left(\left(c\, d^2-a\, e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3} \left((a\, e+c\, d\, x)\, (d+e\, x)\right)^{1/3}\right)}{\left(\left(1+\sqrt{3}\right) \left(c\, d^2-a\, e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3} \left((a\, e+c\, d\, x)\, (d+e\, x)\right)^{1/3}\right)^2}} \sqrt{\left(a\, e^2+c\, d\, (d+2\, e\, x)\right)^2}\Bigg) +$$

$$\Bigg(2^{1/6}\, 3^{3/4} \left(c\, d^2-a\, e^2\right)^{2/3} \sqrt{\left(c\, d^2+a\, e^2+2\, c\, d\, e\, x\right)^2} \left(\left(c\, d^2-a\, e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3} \left((a\, e+c\, d\, x)\, (d+e\, x)\right)^{1/3}\right)$$

$$\sqrt{\left(\left(c\, d^2-a\, e^2\right)^{4/3}-2^{2/3} c^{1/3} d^{1/3} e^{1/3} \left(c\, d^2-a\, e^2\right)^{2/3} \left((a\, e+c\, d\, x)\, (d+e\, x)\right)^{1/3}+2\times 2^{1/3} c^{2/3} d^{2/3} e^{2/3} \left((a\, e+c\, d\, x)\, (d+e\, x)\right)^{2/3}\right) \Big/ \left(\left(1+\sqrt{3}\right) \left(c\, d^2-a\, e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3} \left((a\, e+c\, d\, x)\, (d+e\, x)\right)^{1/3}\right)^2}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) \left(c\, d^2-a\, e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3} \left((a\, e+c\, d\, x)\, (d+e\, x)\right)^{1/3}}{\left(1+\sqrt{3}\right) \left(c\, d^2-a\, e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3} \left((a\, e+c\, d\, x)\, (d+e\, x)\right)^{1/3}}\right],-7-4 \sqrt{3}\right]\Bigg) \Bigg/$$

$$\left(c^{2/3} d^{2/3} e^{2/3} \left(c\, d^2+a\, e^2+2\, c\, d\, e\, x\right) \sqrt{\frac{\left(c\, d^2-a\, e^2\right)^{2/3} \left(\left(c\, d^2-a\, e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3} \left((a\, e+c\, d\, x)\, (d+e\, x)\right)^{1/3}\right)}{\left(\left(1+\sqrt{3}\right) \left(c\, d^2-a\, e^2\right)^{2/3}+2^{2/3} c^{1/3} d^{1/3} e^{1/3} \left((a\, e+c\, d\, x)\, (d+e\, x)\right)^{1/3}\right)^2}} \sqrt{\left(a\, e^2+c\, d\, (d+2\, e\, x)\right)^2}\right)$$

Result (type 5, 90 leaves):

$$\frac{3\left(\frac{e(a e+c d x)}{-c d^2+a e^2}\right)^{1/3}(d+e x)\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{c d(d+e x)}{c d^2-a e^2}\right]}{2 e\left((a e+c d x)(d+e x)\right)^{1/3}}$$

- **Problem 2083: Result unnecessarily involves higher level functions.**

$$\int \frac{(d+e x)^{2/3}}{\sqrt{a d e+\left(c d^2+a e^2\right) x+c d e x^2}}\,dx$$

Optimal (type 4, 566 leaves, 5 steps):

$$\frac{3(a e+c d x)(d+e x)^{2/3}}{2 c d\sqrt{a d e+\left(c d^2+a e^2\right) x+c d e x^2}}+\left(3^{3/4}\left(c d^2-a e^2\right)^{2/3}\sqrt{a d e+c d^2 x}\,(d+e x)^{2/3}\right.$$

$$\left(\left(c d^2-a e^2\right)^{1/3}-c^{1/3} d^{2/3}\left(1+\frac{e x}{d}\right)^{1/3}\right)\sqrt{\frac{\left(c d^2-a e^2\right)^{2/3}+c^{1/3} d^{2/3}\left(c d^2-a e^2\right)^{1/3}\left(1+\frac{e x}{d}\right)^{1/3}+c^{2/3} d^{4/3}\left(1+\frac{e x}{d}\right)^{2/3}}{\left(\left(c d^2-a e^2\right)^{1/3}-\left(1+\sqrt{3}\right) c^{1/3} d^{2/3}\left(1+\frac{e x}{d}\right)^{1/3}\right)^2}}$$

$$\left.\text{EllipticF}\left[\text{ArcCos}\left[\frac{\left(c d^2-a e^2\right)^{1/3}-\left(1-\sqrt{3}\right) c^{1/3} d^{2/3}\left(1+\frac{e x}{d}\right)^{1/3}}{\left(c d^2-a e^2\right)^{1/3}-\left(1+\sqrt{3}\right) c^{1/3} d^{2/3}\left(1+\frac{e x}{d}\right)^{1/3}}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(4 c d e\sqrt{d(a e+c d x)}\sqrt{a d e+\left(c d^2+a e^2\right) x+c d e x^2}\sqrt{-\frac{c^{1/3} d^{2/3}\left(1+\frac{e x}{d}\right)^{1/3}\left(\left(c d^2-a e^2\right)^{1/3}-c^{1/3} d^{2/3}\left(1+\frac{e x}{d}\right)^{1/3}\right)}{\left(\left(c d^2-a e^2\right)^{1/3}-\left(1+\sqrt{3}\right) c^{1/3} d^{2/3}\left(1+\frac{e x}{d}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 124 leaves):

$$\frac{3(d+e x)^{2/3}\left(e(a e+c d x)+\left(c d^2-a e^2\right)\sqrt{\frac{e(a e+c d x)}{-c d^2+a e^2}}\,\text{Hypergeometric2F1}\left[\frac{1}{6},\frac{1}{2},\frac{7}{6},\frac{c d(d+e x)}{c d^2-a e^2}\right]\right)}{2 c d e\sqrt{(a e+c d x)(d+e x)}}$$

- **Problem 2089: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d+e x)^m}{\left(a d e+\left(c d^2+a e^2\right) x+c d e x^2\right)^3}\,dx$$

Optimal (type 5, 64 leaves, 2 steps):

$$\frac{e^2(d+e x)^{-2+m}\,\text{Hypergeometric2F1}\left[3,-2+m,-1+m,\frac{c d(d+e x)}{c d^2-a e^2}\right]}{\left(c d^2-a e^2\right)^3(2-m)}$$

Result (type 5, 368 leaves):

$$\frac{1}{\left(-c d^2+a e^2\right)^5}c^2 d^2 (d+e x)^m \left(\frac{6 e^2}{m}+\frac{\left(c d^2 e-a e^3\right)^2}{c^2 d^2 (-2+m) (d+e x)^2}+\frac{3 e^2 \left(c d^2-a e^2\right)}{c d (-1+m) (d+e x)}-\frac{3 e \left(-c d^2+a e^2\right) \left(\frac{c d (d+e x)}{e (a e+c d x)}\right)^{-m} \text{Hypergeometric2F1}\left[1-m,-m,2-m,\frac{-c d^2+a e^2}{e (a e+c d x)}\right]}{(-1+m) (a e+c d x)}-\frac{\left(c d^2-a e^2\right)^2 \left(\frac{c d (d+e x)}{e (a e+c d x)}\right)^{-m} \text{Hypergeometric2F1}\left[2-m,-m,3-m,\frac{-c d^2+a e^2}{e (a e+c d x)}\right]}{(-2+m) (a e+c d x)^2}-\frac{6 e^2 \left(\frac{c d (d+e x)}{e (a e+c d x)}\right)^{-m} \text{Hypergeometric2F1}\left[-m,-m,1-m,\frac{-c d^2+a e^2}{e (a e+c d x)}\right]}{m}\right)$$

Problem 2090: Result more than twice size of optimal antiderivative.

$$\int \frac{(d+e x)^m}{\left(a d e+\left(c d^2+a e^2\right) x+c d e x^2\right)^4} \, dx$$

Optimal (type 5, 65 leaves, 2 steps) :

$$-\frac{e^3 (d+e x)^{-3+m} \text{Hypergeometric2F1}\left[4,-3+m,-2+m,\frac{c d (d+e x)}{c d^2-a e^2}\right]}{\left(c d^2-a e^2\right)^4 (3-m)}$$

Result (type 5, 504 leaves) :

$$\frac{1}{\left(-c d^2+a e^2\right)^7}c^3 d^3 (d+e x)^m \left(-\frac{20 e^3}{m}+\frac{\left(-c d^2 e+a e^3\right)^3}{c^3 d^3 (-3+m) (d+e x)^3}-\frac{4 e^3 \left(c d^2-a e^2\right)^2}{c^2 d^2 (-2+m) (d+e x)^2}+\frac{10 e^3 \left(-c d^2+a e^2\right)}{c d (-1+m) (d+e x)}+\frac{10 e^2 \left(-c d^2+a e^2\right) \left(\frac{c d (d+e x)}{e (a e+c d x)}\right)^{-m} \text{Hypergeometric2F1}\left[1-m,-m,2-m,\frac{-c d^2+a e^2}{e (a e+c d x)}\right]}{(-1+m) (a e+c d x)}+\frac{4 e \left(c d^2-a e^2\right)^2 \left(\frac{c d (d+e x)}{e (a e+c d x)}\right)^{-m} \text{Hypergeometric2F1}\left[2-m,-m,3-m,\frac{-c d^2+a e^2}{e (a e+c d x)}\right]}{(-2+m) (a e+c d x)^2}-\frac{\left(c d^2-a e^2\right)^3 \left(\frac{c d (d+e x)}{e (a e+c d x)}\right)^{-m} \text{Hypergeometric2F1}\left[3-m,-m,4-m,\frac{-c d^2+a e^2}{e (a e+c d x)}\right]}{(-3+m) (a e+c d x)^3}+\frac{20 e^3 \left(\frac{c d (d+e x)}{e (a e+c d x)}\right)^{-m} \text{Hypergeometric2F1}\left[-m,-m,1-m,\frac{-c d^2+a e^2}{e (a e+c d x)}\right]}{m}\right)$$

- **Problem 2092: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (d+e x)^3 \left(a d e+\left(c d^2+a e^2\right) x+c d e x^2\right)^p \, dx$$

Optimal (type 5, 95 leaves, 3 steps):

$$-\frac{(a e+c d x) (d+e x)^4 \left(a d e+\left(c d^2+a e^2\right) x+c d e x^2\right)^p \text{Hypergeometric2F1}\left[1,5+2 p,5+p,\frac{c d (d+e x)}{c d^2-a e^2}\right]}{\left(c d^2-a e^2\right) (4+p)}$$

Result (type 6, 533 leaves):

$$\frac{1}{4 e} d \left((a e+c d x) (d+e x)\right)^p \left(\left(18 a d^2 e^3 x^2 \text{AppellF1}\left[2,-p,-p,3,-\frac{c d x}{a e},-\frac{e x}{d}\right]\right) \Big/ \left(3 a d e \text{AppellF1}\left[2,-p,-p,3,-\frac{c d x}{a e},-\frac{e x}{d}\right]+ p x \left(c d^2 \text{AppellF1}\left[3,1-p,-p,4,-\frac{c d x}{a e},-\frac{e x}{d}\right]+a e^2 \text{AppellF1}\left[3,-p,1-p,4,-\frac{c d x}{a e},-\frac{e x}{d}\right]\right)\right)+ \left(16 a d e^4 x^3 \text{AppellF1}\left[3,-p,-p,4,-\frac{c d x}{a e},-\frac{e x}{d}\right]\right) \Big/ \left(4 a d e \text{AppellF1}\left[3,-p,-p,4,-\frac{c d x}{a e},-\frac{e x}{d}\right]+ p x \left(c d^2 \text{AppellF1}\left[4,1-p,-p,5,-\frac{c d x}{a e},-\frac{e x}{d}\right]+a e^2 \text{AppellF1}\left[4,-p,1-p,5,-\frac{c d x}{a e},-\frac{e x}{d}\right]\right)\right)+ \left(5 a e^5 x^4 \text{AppellF1}\left[4,-p,-p,5,-\frac{c d x}{a e},-\frac{e x}{d}\right]\right) \Big/ \left(5 a d e \text{AppellF1}\left[4,-p,-p,5,-\frac{c d x}{a e},-\frac{e x}{d}\right]+ p x \left(c d^2 \text{AppellF1}\left[5,1-p,-p,6,-\frac{c d x}{a e},-\frac{e x}{d}\right]+a e^2 \text{AppellF1}\left[5,-p,1-p,6,-\frac{c d x}{a e},-\frac{e x}{d}\right]\right)\right)+ \frac{4 d^2 \left(\frac{e (a e+c d x)}{-c d^2+a e^2}\right)^{-p} (d+e x) \text{Hypergeometric2F1}\left[-p,1+p,2+p,\frac{c d (d+e x)}{c d^2-a e^2}\right]}{1+p}\right)$$

- **Problem 2093: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (d+e x)^2 \left(a d e+\left(c d^2+a e^2\right) x+c d e x^2\right)^p \, dx$$

Optimal (type 5, 95 leaves, 3 steps):

$$-\frac{1}{\left(c d^2-a e^2\right) (3+p)} (a e+c d x) (d+e x)^3 \left(a d e+\left(c d^2+a e^2\right) x+c d e x^2\right)^p \text{Hypergeometric2F1}\left[1,2 (2+p),4+p,\frac{c d (d+e x)}{c d^2-a e^2}\right]$$

Result (type 6, 385 leaves):

$$\frac{1}{3e} d\,((a e+c d x)(d+e x))^p \left(\left(9 a d e^3 x^2 \text{AppellF1}\left[2,-p,-p,3,-\frac{c d x}{a e},-\frac{e x}{d}\right]\right) \Big/ \left(3 a d e \text{AppellF1}\left[2,-p,-p,3,-\frac{c d x}{a e},-\frac{e x}{d}\right]+ p x \left(c d^2 \text{AppellF1}\left[3,1-p,-p,4,-\frac{c d x}{a e},-\frac{e x}{d}\right]+a e^2 \text{AppellF1}\left[3,-p,1-p,4,-\frac{c d x}{a e},-\frac{e x}{d}\right]\right)\right)+ \left(4 a e^4 x^3 \text{AppellF1}\left[3,-p,-p,4,-\frac{c d x}{a e},-\frac{e x}{d}\right]\right) \Big/ \left(4 a d e \text{AppellF1}\left[3,-p,-p,4,-\frac{c d x}{a e},-\frac{e x}{d}\right]+ c d^2 p x \text{AppellF1}\left[4,1-p,-p,5,-\frac{c d x}{a e},-\frac{e x}{d}\right]+a e^2 p x \text{AppellF1}\left[4,-p,1-p,5,-\frac{c d x}{a e},-\frac{e x}{d}\right]\right)+ \frac{3 d \left(\frac{e (a e+c d x)}{-c d^2+a e^2}\right)^{-p} (d+e x) \text{Hypergeometric2F1}\left[-p,1+p,2+p,\frac{c d (d+e x)}{c d^2-a e^2}\right]}{1+p}\right)$$

- **Problem 2094: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (d+e x)\left(a d e+\left(c d^2+a e^2\right) x+c d e x^2\right)^p \, dx$$

Optimal (type 5, 95 leaves, 2 steps):

$$-\frac{(a e+c d x)(d+e x)^2 \left(a d e+\left(c d^2+a e^2\right) x+c d e x^2\right)^p \text{Hypergeometric2F1}\left[1,3+2 p,3+p,\frac{c d (d+e x)}{c d^2-a e^2}\right]}{\left(c d^2-a e^2\right)(2+p)}$$

Result (type 6, 237 leaves):

$$\frac{1}{e} d\,((a e+c d x)(d+e x))^p \left(\left(3 a e^3 x^2 \text{AppellF1}\left[2,-p,-p,3,-\frac{c d x}{a e},-\frac{e x}{d}\right]\right) \Big/ \left(6 a d e \text{AppellF1}\left[2,-p,-p,3,-\frac{c d x}{a e},-\frac{e x}{d}\right]+ 2 p x \left(c d^2 \text{AppellF1}\left[3,1-p,-p,4,-\frac{c d x}{a e},-\frac{e x}{d}\right]+a e^2 \text{AppellF1}\left[3,-p,1-p,4,-\frac{c d x}{a e},-\frac{e x}{d}\right]\right)\right)+ \frac{\left(\frac{e (a e+c d x)}{-c d^2+a e^2}\right)^{-p} (d+e x) \text{Hypergeometric2F1}\left[-p,1+p,2+p,\frac{c d (d+e x)}{c d^2-a e^2}\right]}{1+p}\right)$$

- **Problem 2103: Result unnecessarily involves higher level functions.**

$$\int (d+e x)^{-4-2 p}\left(a d e+\left(c d^2+a e^2\right) x+c d e x^2\right)^p \, dx$$

Optimal (type 3, 206 leaves, 3 steps):

$$\frac{2\,c\,d\,(d+e\,x)^{-3-2\,p}\left(a\,d\,e+\left(c\,d^2+a\,e^2\right)x+c\,d\,e\,x^2\right)^{1+p}}{\left(c\,d^2-a\,e^2\right)^2(2+p)\,(3+p)}+$$

$$\frac{2\,c^2\,d^2\,(d+e\,x)^{-2\,(1+p)}\left(a\,d\,e+\left(c\,d^2+a\,e^2\right)x+c\,d\,e\,x^2\right)^{1+p}}{\left(c\,d^2-a\,e^2\right)^3(1+p)\,(2+p)\,(3+p)}+\frac{(d+e\,x)^{-2\,(2+p)}\left(a\,d\,e+\left(c\,d^2+a\,e^2\right)x+c\,d\,e\,x^2\right)^{1+p}}{\left(c\,d^2-a\,e^2\right)(3+p)}$$

Result (type 5, 101 leaves):

$$-\frac{\left(\frac{e\,(a\,e+c\,d\,x)}{-c\,d^2+a\,e^2}\right)^{-p}(d+e\,x)^{-3-2\,p}\,((a\,e+c\,d\,x)\,(d+e\,x))^p\,\text{Hypergeometric2F1}\left[-3-p,\,-p,\,-2-p,\,\frac{c\,d\,(d+e\,x)}{c\,d^2-a\,e^2}\right]}{e\,(3+p)}$$

- **Problem 2202: Result more than twice size of optimal antiderivative.**

$$\int\frac{(d+e\,x)^4}{\left(a+b\,x+c\,x^2\right)^3}\,dx$$

Optimal (type 3, 169 leaves, 4 steps):

$$-\frac{(d+e\,x)^3\,(b\,d-2\,a\,e+(2\,c\,d-b\,e)\,x)}{2\left(b^2-4\,a\,c\right)\left(a+b\,x+c\,x^2\right)^2}+\frac{3\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)\,(b\,d-2\,a\,e+(2\,c\,d-b\,e)\,x)}{\left(b^2-4\,a\,c\right)^2\left(a+b\,x+c\,x^2\right)}-\frac{12\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\text{ArcTanh}\left[\frac{b+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right]}{\left(b^2-4\,a\,c\right)^{5/2}}$$

Result (type 3, 413 leaves):

$$\frac{1}{2}\Bigg(\left(b^4\,e^4\,x+b^3\,e^3\,(a\,e-4\,c\,d\,x)+2\,b^2\,c\,e^2\left(3\,c\,d^2\,x-2\,a\,e\,(d+e\,x)\right)+b\,c\left(-3\,a^2\,e^4+c^2\,d^3\,(d-4\,e\,x)+6\,a\,c\,d\,e^2\,(d+2\,e\,x)\right)+\right.$$
$$\left.2\,c^2\left(c^2\,d^4\,x+a^2\,e^3\,(4\,d+e\,x)-2\,a\,c\,d^2\,e\,(2\,d+3\,e\,x)\right)\right)\Big/\left(c^3\left(-b^2+4\,a\,c\right)\left(a+x\,(b+c\,x)\right)^2\right)+$$
$$\left(b^5\,e^4+2\,b^3\,c\,e^2\left(3\,c\,d^2-4\,a\,e^2\right)-2\,b^4\,c\,e^3\,(2\,d+e\,x)+2\,b\,c^2\left(11\,a^2\,e^4+3\,c^2\,d^3\,(d-4\,e\,x)+6\,a\,c\,d\,e^2\,(d-2\,e\,x)\right)+\right.$$
$$\left.4\,b^2\,c^2\,e\left(-3\,c\,d^2\,(d-e\,x)+a\,e^2\,(5\,d+4\,e\,x)\right)+4\,c^3\left(3\,c^2\,d^4\,x+6\,a\,c\,d^2\,e^2\,x-a^2\,e^3\,(16\,d+5\,e\,x)\right)\right)\Big/$$
$$\left(c^3\left(b^2-4\,a\,c\right)^2(a+x\,(b+c\,x))\right)+\frac{24\left(c\,d^2+e\,(-b\,d+a\,e)\right)^2\text{ArcTan}\left[\frac{b+2\,c\,x}{\sqrt{-b^2+4\,a\,c}}\right]}{\left(-b^2+4\,a\,c\right)^{5/2}}\Bigg)$$

- **Problem 2212: Result more than twice size of optimal antiderivative.**

$$\int\frac{x^6}{\left(a+b\,x+c\,x^2\right)^4}\,dx$$

Optimal (type 3, 146 leaves, 5 steps):

$$\frac{x^5\,(2a+bx)}{3\,(b^2-4ac)\,(a+bx+cx^2)^3}-\frac{5\,a\,x^3\,(2a+bx)}{3\,(b^2-4ac)^2\,(a+bx+cx^2)^2}+\frac{10\,a^2\,x\,(2a+bx)}{(b^2-4ac)^3\,(a+bx+cx^2)}+\frac{40\,a^3\,\text{ArcTanh}\left[\frac{b+2cx}{\sqrt{b^2-4ac}}\right]}{(b^2-4ac)^{7/2}}$$

Result (type 3, 314 leaves) :

$$\frac{b^7-12ab^5c+48a^2b^3c^2-59a^3bc^3-4b^6cx+33ab^4c^2x-72a^2b^2c^3x+26a^3c^4x}{3c^5\,(b^2-4ac)^2\,(a+x\,(b+cx))^2}+\frac{-b^7+12ab^5c-48a^2b^3c^2+74a^3bc^3+b^6cx-12ab^4c^2x+48a^2b^2c^3x-44a^3c^4x}{c^4\,(-b^2+4ac)^3\,(a+x\,(b+cx))}+\frac{b^6x+ab^4\,(b-6cx)+a^3c^2\,(5b-2cx)+a^2b^2c\,(-5b+9cx)}{3c^5\,(-b^2+4ac)\,(a+x\,(b+cx))^3}+\frac{40\,a^3\,\text{ArcTan}\left[\frac{b+2cx}{\sqrt{-b^2+4ac}}\right]}{(-b^2+4ac)^{7/2}}$$

Problem 2214: Result more than twice size of optimal antiderivative.

$$\int\frac{(d+ex)^4}{(a+bx+cx^2)^4}\,dx$$

Optimal (type 3, 259 leaves, 5 steps) :

$$-\frac{(b+2cx)\,(d+ex)^4}{3\,(b^2-4ac)\,(a+bx+cx^2)^3}+\frac{(d+ex)^3\,(5bcd-2b^2e-2ace+5c\,(2cd-be)\,x)}{3\,(b^2-4ac)^2\,(a+bx+cx^2)^2}-\frac{2\,(5c^2d^2+b^2e^2-ce\,(5bd-ae))\,(d+ex)\,(bd-2ae+(2cd-be)\,x)}{(b^2-4ac)^3\,(a+bx+cx^2)}+\frac{8\,(cd^2-bde+ae^2)\,(5c^2d^2+b^2e^2-ce\,(5bd-ae))\,\text{ArcTanh}\left[\frac{b+2cx}{\sqrt{b^2-4ac}}\right]}{(b^2-4ac)^{7/2}}$$

Result (type 3, 572 leaves) :

$$\frac{1}{3}\left(\frac{6\,\left(5c^3d^4+b^2e^3\,(-bd+ae)+2c^2d^2e\,(-5bd+3ae)+ce^2\,(6b^2d^2-6abde+a^2e^2)\right)\,(b+2cx)}{c\,(-b^2+4ac)^3\,(a+x\,(b+cx))}+\right.$$
$$\left(b^4e^4x+b^3e^3\,(ae-4cdx)+2b^2ce^2\,(3cd^2x-2ae\,(d+ex))+bc\,(-3a^2e^4+c^2d^3\,(d-4ex)+6acde^2\,(d+2ex))+\right.$$
$$\left.2c^2\,(c^2d^4x+a^2e^3\,(4d+ex)-2acd^2e\,(2d+3ex))\right)\Big/\left(c^3\,(-b^2+4ac)\,(a+x\,(b+cx))^3\right)+$$
$$\left(b^5e^4-b^4ce^3\,(4d+ex)+bc^2\,(17a^2e^4+5c^2d^3\,(d-4ex)+6acde^2\,(d-2ex))+b^3ce^2\,(-7ae^2+2cd\,(3d-ex))+\right.$$
$$\left.2b^2c^2e\,(ae^2\,(9d+5ex)+cd^2\,(-5d+6ex))+2c^3\,(5c^2d^4x+6acd^2e^2x-a^2e^3\,(24d+7ex))\right)\Big/\left(c^3\,(b^2-4ac)^2\,(a+x\,(b+cx))^2\right)+$$
$$\left.\frac{1}{(-b^2+4ac)^{7/2}}24\,\left(5c^3d^4+b^2e^3\,(-bd+ae)+2c^2d^2e\,(-5bd+3ae)+ce^2\,(6b^2d^2-6abde+a^2e^2)\right)\,\text{ArcTan}\left[\frac{b+2cx}{\sqrt{-b^2+4ac}}\right]\right)$$

- **Problem 2221: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d+e x)^5}{\left(a+b x+c x^2\right)^5}\,dx$$

Optimal (type 3, 388 leaves, 6 steps):

$$-\frac{(b+2 c x)\,(d+e x)^5}{4\left(b^2-4 a c\right)\left(a+b x+c x^2\right)^4}+\frac{(d+e x)^4\left(14 b c d-5 b^2 e-8 a c e+14 c\,(2 c d-b e)\,x\right)}{12\left(b^2-4 a c\right)^2\left(a+b x+c x^2\right)^3}+$$
$$\left((d+e x)^3\left(63 b^2 c d e+28 a c^2 d e-10 b^3 e^2-10 b c\left(7 c d^2+3 a e^2\right)-c\left(140 c^2 d^2+27 b^2 e^2-4 c e\,(35 b d-8 a e)\right) x\right)\right)\Big/$$
$$\left(12\left(b^2-4 a c\right)^3\left(a+b x+c x^2\right)^2\right)+\frac{5\,(2 c d-b e)\left(7 c^2 d^2+b^2 e^2-c e\,(7 b d-3 a e)\right)(d+e x)\,(b d-2 a e+(2 c d-b e)\,x)}{2\left(b^2-4 a c\right)^4\left(a+b x+c x^2\right)}-$$
$$\frac{10\,(2 c d-b e)\left(c d^2-b d e+a e^2\right)\left(7 c^2 d^2+b^2 e^2-c e\,(7 b d-3 a e)\right)\operatorname{ArcTanh}\left[\frac{b+2 c x}{\sqrt{b^2-4 a c}}\right]}{\left(b^2-4 a c\right)^{9/2}}$$

Result (type 3, 985 leaves):

$$\frac{1}{12}\Bigg(\left(30\,(2 c d-b e)\left(7 c^3 d^4-2 c^2 d^2 e\,(7 b d-5 a e)+b^2 e^3\,(-b d+a e)+c e^2\left(8 b^2 d^2-10 a b d e+3 a^2 e^2\right)\right)(b+2 c x)\right)\Big/$$
$$\left(c\left(b^2-4 a c\right)^4\,(a+x\,(b+c x))\right)+$$
$$\frac{1}{c^3\left(-b^2+4 a c\right)^3\,(a+x\,(b+c x))^2}\left(5 b^5 c d e^4+3 b^6 e^5+4 c^3\left(-48 a^3 e^5+35 c^3 d^5 x+50 a c^2 d^3 e^2 x+15 a^2 c d e^4 x\right)+\right.$$
$$b^2 c^2 e\left(129 a^2 e^4-25 c^2 d^3\,(7 d-12 e x)-30 a c d e^2\,(5 d-4 e x)\right)+10 b c^3\left(7 c^2 d^4\,(d-5 e x)+10 a c d^2 e^2\,(d-3 e x)+3 a^2 e^4\,(d-e x)\right)+$$
$$\left.10 b^3 c^2 e^2\left(5 c d^2\,(3 d-2 e x)+a e^2\,(6 d-e x)\right)+b^4 c e^3\left(-41 a e^2+10 c d\,(-5 d+e x)\right)\right)-$$
$$\Big(3\left(b^5 e^5 x+b^4 e^4\,(a e-5 c d x)-5 b^3 c e^3\left(-2 c d^2 x+a e\,(d+e x)\right)-2 b^2 c e^2\left(2 a^2 e^3+5 c^2 d^3 x-5 a c d e\,(d+2 e x)\right)+\right.$$
$$\left.2 c^2\left(a^3 e^5-c^3 d^5 x-5 a^2 c d e^3\,(2 d+e x)+5 a c^2 d^3 e\,(d+2 e x)\right)+b c^2\left(-c^2 d^4\,(d-5 e x)+5 a^2 e^4\,(3 d+e x)-10 a c d^2 e^2\,(d+3 e x)\right)\right)\Big)\Big/$$
$$\left(c^4\left(-b^2+4 a c\right)(a+x\,(b+c x))^4\right)+\frac{1}{c^4\left(b^2-4 a c\right)^2\,(a+x\,(b+c x))^3}\left(-3 b^6 e^5+3 b^5 c e^4\,(5 d+2 e x)+b^4 c e^3\left(27 a e^2-10 c d\,(3 d+e x)\right)-\right.$$
$$10 b^3 c^2 e^2\left(5 a e^2\,(2 d+e x)+c d^2\,(-3 d+2 e x)\right)+4 c^3\left(16 a^3 e^5+7 c^3 d^5 x+10 a c^2 d^3 e^2 x-5 a^2 c d e^3\,(16 d+9 e x)\right)+2 b c^3$$
$$\left.\left(7 c^2 d^4\,(d-5 e x)+10 a c d^2 e^2\,(d-3 e x)+5 a^2 e^4\,(23 d+9 e x)\right)+b^2 c^2 e\left(-83 a^2 e^4+5 c^2 d^3\,(-7 d+12 e x)+10 a c d e^2\,(13 d+12 e x)\right)\right)+$$
$$\frac{1}{\left(-b^2+4 a c\right)^{9/2}}120\,(2 c d-b e)\left(7 c^3 d^4-2 c^2 d^2 e\,(7 b d-5 a e)+b^2 e^3\,(-b d+a e)+c e^2\left(8 b^2 d^2-10 a b d e+3 a^2 e^2\right)\right)\operatorname{ArcTan}\left[\frac{b+2 c x}{\sqrt{-b^2+4 a c}}\right]\Bigg)$$

- **Problem 2309: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(1+2 x)^{7/2}}{2+3 x+5 x^2}\,dx$$

Optimal (type 3, 279 leaves, 13 steps):

$$-\frac{76}{125}\sqrt{1+2x}+\frac{16}{75}(1+2x)^{3/2}+\frac{4}{25}(1+2x)^{5/2}+\frac{1}{125}\sqrt{\frac{2}{155}\left(-168698+42875\sqrt{35}\right)}\,\mathrm{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}-10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]-$$

$$\frac{1}{125}\sqrt{\frac{2}{155}\left(-168698+42875\sqrt{35}\right)}\,\mathrm{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}+10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]-$$

$$\frac{1}{125}\sqrt{\frac{1}{310}\left(168698+42875\sqrt{35}\right)}\,\mathrm{Log}\left[\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right]+$$

$$\frac{1}{125}\sqrt{\frac{1}{310}\left(168698+42875\sqrt{35}\right)}\,\mathrm{Log}\left[\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right]$$

Result (type 3, 142 leaves):

$$\frac{8}{375}\sqrt{1+2x}\left(-11+50x+30x^2\right)-\frac{2\,i\left(-6696\,i+233\sqrt{31}\right)\mathrm{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]}{3875\sqrt{-10-5\,i\sqrt{31}}}+\frac{2\,i\left(6696\,i+233\sqrt{31}\right)\mathrm{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]}{3875\sqrt{5\,i\left(2\,i+\sqrt{31}\right)}}$$

- **Problem 2310: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(1+2x)^{5/2}}{2+3x+5x^2}\,dx$$

Optimal (type 3, 266 leaves, 12 steps):

$$\frac{16}{25}\sqrt{1+2x}+\frac{4}{15}(1+2x)^{3/2}+\frac{1}{25}\sqrt{\frac{2}{155}\left(7162+1225\sqrt{35}\right)}\,\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}-10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]-$$

$$\frac{1}{25}\sqrt{\frac{2}{155}\left(7162+1225\sqrt{35}\right)}\,\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}+10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]+$$

$$\frac{1}{25}\sqrt{\frac{1}{310}\left(-7162+1225\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right]-$$

$$\frac{1}{25}\sqrt{\frac{1}{310}\left(-7162+1225\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right]$$

Result (type 3, 137 leaves) :

$$\frac{4}{75}\sqrt{1+2x}\,(17+10x)+\frac{2\,i\left(589\,i+178\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]}{775\sqrt{-10-5\,i\sqrt{31}}}-\frac{2\,i\left(-589\,i+178\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]}{775\sqrt{5\,i\left(2\,i+\sqrt{31}\right)}}$$

- **Problem 2311: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(1+2x)^{3/2}}{2+3x+5x^2}\,dx$$

Optimal (type 3, 253 leaves, 11 steps) :

$$\frac{4}{5}\sqrt{1+2x}+\frac{1}{5}\sqrt{\frac{2}{155}\left(-178+35\sqrt{35}\right)}\ \text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}-10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]-$$

$$\frac{1}{5}\sqrt{\frac{2}{155}\left(-178+35\sqrt{35}\right)}\ \text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}+10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]+$$

$$\frac{1}{5}\sqrt{\frac{1}{310}\left(178+35\sqrt{35}\right)}\ \text{Log}\left[\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\ \sqrt{1+2x}+5\,(1+2x)\right]-$$

$$\frac{1}{5}\sqrt{\frac{1}{310}\left(178+35\sqrt{35}\right)}\ \text{Log}\left[\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\ \sqrt{1+2x}+5\,(1+2x)\right]$$

Result (type 3, 130 leaves) :

$$\frac{4}{5}\sqrt{1+2x}+\frac{2\left(27\,i+4\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]}{5\sqrt{-155\,i\left(-2\,i+\sqrt{31}\right)}}+\frac{2\left(-27\,i+4\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]}{5\sqrt{155\,i\left(2\,i+\sqrt{31}\right)}}$$

- **Problem 2312: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{1+2x}}{2+3x+5x^2}\,dx$$

Optimal (type 3, 222 leaves, 10 steps) :

$$-\sqrt{\frac{2}{5\left(-2+\sqrt{35}\right)}}\ \text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}-10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]+\sqrt{\frac{2}{5\left(-2+\sqrt{35}\right)}}\ \text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}+10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]+$$

$$\frac{\text{Log}\left[\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\ \sqrt{1+2x}+5\,(1+2x)\right]}{\sqrt{10\left(2+\sqrt{35}\right)}}-\frac{\text{Log}\left[\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\ \sqrt{1+2x}+5\,(1+2x)\right]}{\sqrt{10\left(2+\sqrt{35}\right)}}$$

Result (type 3, 112 leaves) :

$$\frac{2\left(\sqrt{-2+i\sqrt{31}}\left(-2i+\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]+\sqrt{-2-i\sqrt{31}}\left(2i+\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]\right)}{5\sqrt{217}}$$

- **Problem 2313: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\sqrt{1+2x}\left(2+3x+5x^2\right)}\,dx$$

Optimal (type 3, 218 leaves, 10 steps) :

$$-\sqrt{\frac{2}{217}\left(2+\sqrt{35}\right)}\ \text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}-10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]+\sqrt{\frac{2}{217}\left(2+\sqrt{35}\right)}\ \text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}+10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]-$$

$$\frac{\text{Log}\left[\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right]}{\sqrt{14\left(2+\sqrt{35}\right)}}+\frac{\text{Log}\left[\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right]}{\sqrt{14\left(2+\sqrt{35}\right)}}$$

Result (type 3, 95 leaves) :

$$\frac{2i\left(-\sqrt{-2+i\sqrt{31}}\ \text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]+\sqrt{-2-i\sqrt{31}}\ \text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]\right)}{\sqrt{217}}$$

- **Problem 2314: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{(1+2x)^{3/2}\left(2+3x+5x^2\right)}\,dx$$

Optimal (type 3, 253 leaves, 11 steps) :

$$-\frac{4}{7\sqrt{1+2x}}+\frac{1}{7}\sqrt{\frac{2}{217}\left(-178+35\sqrt{35}\right)}\ \text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}-10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]-$$

$$\frac{1}{7}\sqrt{\frac{2}{217}\left(-178+35\sqrt{35}\right)}\ \text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}+10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]-$$

$$\frac{1}{7}\sqrt{\frac{1}{434}\left(178+35\sqrt{35}\right)}\ \text{Log}\left[\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\ \sqrt{1+2x}+5\ (1+2x)\right]+$$

$$\frac{1}{7}\sqrt{\frac{1}{434}\left(178+35\sqrt{35}\right)}\ \text{Log}\left[\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\ \sqrt{1+2x}+5\ (1+2x)\right]$$

Result (type 3, 130 leaves) :

$$-\frac{4}{7\sqrt{1+2x}}-\frac{2\left(2\,i+\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]}{7\sqrt{-\frac{31}{5}\,i\left(-2\,i+\sqrt{31}\right)}}-\frac{2\left(-2\,i+\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]}{7\sqrt{\frac{31}{5}\,i\left(2\,i+\sqrt{31}\right)}}$$

- **Problem 2315: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{(1+2x)^{5/2}\left(2+3x+5x^2\right)}\,dx$$

Optimal (type 3, 266 leaves, 12 steps) :

$$-\frac{4}{21\,(1+2x)^{3/2}}-\frac{16}{49\sqrt{1+2x}}+\frac{1}{49}\sqrt{\frac{2}{217}\left(7162+1225\sqrt{35}\right)}\,\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}-10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]-$$

$$\frac{1}{49}\sqrt{\frac{2}{217}\left(7162+1225\sqrt{35}\right)}\,\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}+10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]-$$

$$\frac{1}{49}\sqrt{\frac{1}{434}\left(-7162+1225\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\,\sqrt{1+2x}+5\,(1+2x)\right]+$$

$$\frac{1}{49}\sqrt{\frac{1}{434}\left(-7162+1225\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\,\sqrt{1+2x}+5\,(1+2x)\right]$$

Result (type 3, 139 leaves):

$$\frac{2\left(-\frac{62\,(19+24x)}{(1+2x)^{3/2}}+\frac{3\,i\left(124\,i+27\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]}{\sqrt{-\frac{1}{5}\,i\left(-2\,i+\sqrt{31}\right)}}-\frac{3\left(124+27\,i\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]}{\sqrt{\frac{1}{5}\,i\left(2\,i+\sqrt{31}\right)}}\right)}{4557}$$

- **Problem 2316: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(1+2x)^{7/2}}{\left(2+3x+5x^2\right)^2}\,dx$$

Optimal (type 3, 296 leaves, 13 steps):

$$\frac{604}{775}\sqrt{1+2x}-\frac{8}{155}(1+2x)^{3/2}-\frac{(5-4x)(1+2x)^{5/2}}{31\left(2+3x+5x^2\right)}+\frac{1}{775}\sqrt{\frac{2}{155}\left(-5682718+968975\sqrt{35}\right)}\,\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}-10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]-$$
$$\frac{1}{775}\sqrt{\frac{2}{155}\left(-5682718+968975\sqrt{35}\right)}\,\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}+10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]+$$
$$\frac{1}{775}\sqrt{\frac{1}{310}\left(5682718+968975\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right]-$$
$$\frac{1}{775}\sqrt{\frac{1}{310}\left(5682718+968975\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right]$$

Result (type 3, 150 leaves) :

$$\frac{\sqrt{1+2x}\left(1003+1132x+2480x^2\right)}{775\left(2+3x+5x^2\right)}+\frac{2\left(25234+3657\,i\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]}{24025\sqrt{-10-5\,i\sqrt{31}}}+\frac{2\left(25234-3657\,i\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]}{24025\sqrt{5\,i\left(2\,i+\sqrt{31}\right)}}$$

- **Problem 2317: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(1+2x)^{5/2}}{\left(2+3x+5x^2\right)^2}\,dx$$

Optimal (type 3, 283 leaves, 12 steps) :

$$-\frac{8}{155}\sqrt{1+2x}-\frac{(5-4x)(1+2x)^{3/2}}{31\left(2+3x+5x^2\right)}-\frac{1}{155}\sqrt{\frac{2}{155}\left(32678+10325\sqrt{35}\right)}\ \text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}-10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]+$$

$$\frac{1}{155}\sqrt{\frac{2}{155}\left(32678+10325\sqrt{35}\right)}\ \text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}+10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]+$$

$$\frac{1}{155}\sqrt{\frac{1}{310}\left(-32678+10325\sqrt{35}\right)}\ \text{Log}\left[\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\ \sqrt{1+2x}+5(1+2x)\right]-$$

$$\frac{1}{155}\sqrt{\frac{1}{310}\left(-32678+10325\sqrt{35}\right)}\ \text{Log}\left[\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\ \sqrt{1+2x}+5(1+2x)\right]$$

Result (type 3, 147 leaves) :

$$-\frac{\sqrt{1+2x}\ (41+54x)}{155\left(2+3x+5x^2\right)}+\frac{2\left(-264\,i+97\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]}{155\sqrt{-155\,i\left(-2\,i+\sqrt{31}\right)}}+\frac{2\left(264\,i+97\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]}{155\sqrt{155\,i\left(2\,i+\sqrt{31}\right)}}$$

- **Problem 2318: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(1+2x)^{3/2}}{\left(2+3x+5x^2\right)^2}\,dx$$

Optimal (type 3, 270 leaves, 11 steps) :

$$-\frac{(5-4x)\sqrt{1+2x}}{31\left(2+3x+5x^2\right)}-\frac{1}{31}\sqrt{\frac{2}{155}\left(218+47\sqrt{35}\right)}\ \text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}-10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]+$$

$$\frac{1}{31}\sqrt{\frac{2}{155}\left(218+47\sqrt{35}\right)}\ \text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}+10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]-$$

$$\frac{1}{31}\sqrt{\frac{1}{310}\left(-218+47\sqrt{35}\right)}\ \text{Log}\left[\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\ \sqrt{1+2x}+5(1+2x)\right]+$$

$$\frac{1}{31}\sqrt{\frac{1}{310}\left(-218+47\sqrt{35}\right)}\ \text{Log}\left[\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\ \sqrt{1+2x}+5(1+2x)\right]$$

Result (type 3, 145 leaves) :

$$\frac{\sqrt{1+2x}\ (-5+4x)}{31\left(2+3x+5x^2\right)}+\frac{2\left(62-39\,i\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]}{961\sqrt{-10-5\,i\sqrt{31}}}+\frac{2\left(62+39\,i\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]}{961\sqrt{5\,i\left(2\,i+\sqrt{31}\right)}}$$

- **Problem 2319: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{1+2x}}{\left(2+3x+5x^2\right)^2}\,dx$$

Optimal (type 3, 270 leaves, 11 steps) :

$$\frac{\sqrt{1+2x}\,(3+10x)}{31\,(2+3x+5x^2)}-\frac{1}{31}\sqrt{\frac{2}{217}\left(218+47\sqrt{35}\right)}\,\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}-10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]+$$

$$\frac{1}{31}\sqrt{\frac{2}{217}\left(218+47\sqrt{35}\right)}\,\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}+10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]+$$

$$\frac{1}{31}\sqrt{\frac{1}{434}\left(-218+47\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\,\sqrt{1+2x}+5\,(1+2x)\right]-$$

$$\frac{1}{31}\sqrt{\frac{1}{434}\left(-218+47\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\,\sqrt{1+2x}+5\,(1+2x)\right]$$

Result (type 3, 143 leaves) :

$$\frac{2}{961}\left(\frac{31\sqrt{1+2x}\,(3+10x)}{4+6x+10x^2}+\frac{\left(-4\,i+\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]}{\sqrt{-\frac{1}{155}\,i\left(-2\,i+\sqrt{31}\right)}}+\frac{\left(4\,i+\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]}{\sqrt{\frac{1}{155}\,i\left(2\,i+\sqrt{31}\right)}}\right)$$

- **Problem 2320: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\sqrt{1+2x}\,\left(2+3x+5x^2\right)^2}\,dx$$

Optimal (type 3, 270 leaves, 11 steps) :

$$\frac{\sqrt{1+2x}\,(37+20x)}{217\left(2+3x+5x^2\right)}-\frac{1}{217}\sqrt{\frac{2}{217}\left(32678+10325\sqrt{35}\right)}\,\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}-10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]+$$

$$\frac{1}{217}\sqrt{\frac{2}{217}\left(32678+10325\sqrt{35}\right)}\,\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}+10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]-$$

$$\frac{1}{217}\sqrt{\frac{1}{434}\left(-32678+10325\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right]+$$

$$\frac{1}{217}\sqrt{\frac{1}{434}\left(-32678+10325\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right]$$

Result (type 3, 147 leaves) :

$$\frac{2\left(\frac{31\sqrt{1+2x}\,(37+20x)}{4+6x+10x^2}+\frac{\left(62-101\,i\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]}{\sqrt{-\frac{1}{5}\,i\left(-2\,i+\sqrt{31}\right)}}+\frac{\left(62+101\,i\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]}{\sqrt{\frac{1}{5}\,i\left(2\,i+\sqrt{31}\right)}}\right)}{6727}$$

Problem 2321: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{(1+2x)^{3/2}\left(2+3x+5x^2\right)^2}\,dx$$

Optimal (type 3, 283 leaves, 12 steps) :

$$-\frac{604}{1519\sqrt{1+2x}}+\frac{37+20x}{217\sqrt{1+2x}\left(2+3x+5x^2\right)}+\frac{\sqrt{\frac{2}{217}\left(-5682718+968975\sqrt{35}\right)}\operatorname{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}-10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]}{1519}-$$

$$\frac{\sqrt{\frac{2}{217}\left(-5682718+968975\sqrt{35}\right)}\operatorname{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}+10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]}{1519}-$$

$$\frac{\sqrt{\frac{1}{434}\left(5682718+968975\sqrt{35}\right)}\operatorname{Log}\left[\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5\left(1+2x\right)\right]}{1519}+$$

$$\frac{\sqrt{\frac{1}{434}\left(5682718+968975\sqrt{35}\right)}\operatorname{Log}\left[\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5\left(1+2x\right)\right]}{1519}$$

Result (type 3, 160 leaves):

$$\frac{2\left(-\frac{31\left(949+1672x+3020x^2\right)}{2\sqrt{1+2x}\left(2+3x+5x^2\right)}-\frac{i\left(-4681i+512\sqrt{31}\right)\operatorname{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]}{\sqrt{-\frac{1}{5}i\left(-2i+\sqrt{31}\right)}}+\frac{i\left(4681i+512\sqrt{31}\right)\operatorname{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]}{\sqrt{\frac{1}{5}i\left(2i+\sqrt{31}\right)}}\right)}{47089}$$

- **Problem 2322: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\left(1+2x\right)^{5/2}\left(2+3x+5x^2\right)^2}\,dx$$

Optimal (type 3, 296 leaves, 13 steps):

$$-\frac{820}{4557\,(1+2x)^{3/2}}-\frac{4680}{10633\sqrt{1+2x}}+\frac{37+20x}{217\,(1+2x)^{3/2}\left(2+3x+5x^2\right)}+$$

$$\frac{5\sqrt{\frac{2}{217}\left(12504542+2632525\sqrt{35}\right)}\;\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}-10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]}{10633}-\frac{5\sqrt{\frac{2}{217}\left(12504542+2632525\sqrt{35}\right)}\;\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}+10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]}{10633}-$$

$$\frac{5\sqrt{\frac{1}{434}\left(-12504542+2632525\sqrt{35}\right)}\;\text{Log}\left[\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5\,(1+2x)\right]}{10633}+$$

$$\frac{5\sqrt{\frac{1}{434}\left(-12504542+2632525\sqrt{35}\right)}\;\text{Log}\left[\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5\,(1+2x)\right]}{10633}$$

Result (type 3, 165 leaves):

$$\frac{1}{988869}$$

$$2\left(-\frac{31\left(34121+112560x+183140x^2+140400x^3\right)}{2\,(1+2x)^{3/2}\left(2+3x+5x^2\right)}+\frac{15\,i\left(7254\,i+967\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]}{\sqrt{-\frac{1}{5}\,i\left(-2\,i+\sqrt{31}\right)}}-\frac{15\,i\left(-7254\,i+967\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]}{\sqrt{\frac{1}{5}\,i\left(2\,i+\sqrt{31}\right)}}\right)$$

- **Problem 2323: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(1+2x)^{9/2}}{\left(2+3x+5x^2\right)^3}\,dx$$

Optimal (type 3, 313 leaves, 13 steps):

$$-\frac{1584\sqrt{1+2x}}{24025}-\frac{(5-4x)(1+2x)^{7/2}}{62\left(2+3x+5x^2\right)^2}-\frac{(1143-1088x)(1+2x)^{3/2}}{9610\left(2+3x+5x^2\right)}-$$

$$\frac{3\sqrt{\frac{1}{310}\left(250141922+64681225\sqrt{35}\right)}\ \text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}-10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]}{24025}+\frac{3\sqrt{\frac{1}{310}\left(250141922+64681225\sqrt{35}\right)}\ \text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}+10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]}{24025}+$$

$$\frac{3\sqrt{\frac{1}{310}\left(-250141922+64681225\sqrt{35}\right)}\ \text{Log}\left[\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right]}{48050}-$$

$$\frac{3\sqrt{\frac{1}{310}\left(-250141922+64681225\sqrt{35}\right)}\ \text{Log}\left[\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right]}{48050}$$

Result (type 3, 161 leaves) :

$$\frac{1}{3723875}\left(-\frac{155\sqrt{1+2x}\left(27977+87291x+144557x^2+86150x^3\right)}{2\left(2+3x+5x^2\right)^2}+\right.$$

$$\left.\frac{3\left(228749-23998\,i\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]}{\sqrt{-\frac{1}{5}i\left(-2i+\sqrt{31}\right)}}+\frac{3\left(228749+23998\,i\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]}{\sqrt{\frac{1}{5}i\left(2i+\sqrt{31}\right)}}\right)$$

■ **Problem 2324: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(1+2x)^{7/2}}{\left(2+3x+5x^2\right)^3}\,dx$$

Optimal (type 3, 300 leaves, 12 steps) :

$$-\frac{(5-4x)(1+2x)^{5/2}}{62\left(2+3x+5x^2\right)^2}-\frac{(957-592x)\sqrt{1+2x}}{9610\left(2+3x+5x^2\right)}-\frac{\sqrt{\frac{1}{310}\left(9651062+1806875\sqrt{35}\right)}\,\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}-10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]}{4805}+$$

$$\frac{\sqrt{\frac{1}{310}\left(9651062+1806875\sqrt{35}\right)}\,\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}+10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]}{4805}-$$

$$\frac{\sqrt{\frac{1}{310}\left(-9651062+1806875\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right]}{9610}+$$

$$\frac{\sqrt{\frac{1}{310}\left(-9651062+1806875\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right]}{9610}$$

Result (type 3, 159 leaves) :

$$\frac{\frac{155\sqrt{1+2x}\left(-2689-4167x-3629x^2+5440x^3\right)}{2\left(2+3x+5x^2\right)^2}+\frac{\left(16864-7353\,i\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]}{\sqrt{-\frac{1}{5}i\left(-2i+\sqrt{31}\right)}}+\frac{\left(16864+7353\,i\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]}{\sqrt{\frac{1}{5}i\left(2i+\sqrt{31}\right)}}}{744775}$$

- **Problem 2325: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(1+2x)^{5/2}}{\left(2+3x+5x^2\right)^3}\,dx$$

Optimal (type 3, 300 leaves, 12 steps) :

$$-\frac{(5-4x)(1+2x)^{3/2}}{62\left(2+3x+5x^2\right)^2}+\frac{3\sqrt{1+2x}\,(11+78x)}{1922\left(2+3x+5x^2\right)}-\frac{3}{961}\sqrt{\frac{1}{310}\left(15082+2705\sqrt{35}\right)}\,\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}-10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]+$$

$$\frac{3}{961}\sqrt{\frac{1}{310}\left(15082+2705\sqrt{35}\right)}\,\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}+10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]+$$

$$\frac{3\sqrt{\frac{1}{310}\left(-15082+2705\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right]}{1922}-$$

$$\frac{3\sqrt{\frac{1}{310}\left(-15082+2705\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right]}{1922}$$

Result (type 3, 155 leaves) :

$$\frac{\sqrt{1+2x}\left(-89+381x+1115x^2+1170x^3\right)}{1922\left(2+3x+5x^2\right)^2}+\frac{3\left(1209-218\,i\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]}{29791\sqrt{-10-5\,i\sqrt{31}}}+\frac{3\left(1209+218\,i\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]}{29791\sqrt{5\,i\left(2\,i+\sqrt{31}\right)}}$$

- **Problem 2326: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(1+2x)^{3/2}}{\left(2+3x+5x^2\right)^3}\,dx$$

Optimal (type 3, 300 leaves, 12 steps) :

$$-\frac{(5-4x)\sqrt{1+2x}}{62\left(2+3x+5x^2\right)^2}+\frac{\sqrt{1+2x}\,(67+120x)}{1922\left(2+3x+5x^2\right)}-\frac{3}{961}\sqrt{\frac{1}{434}\left(15082+2705\sqrt{35}\right)}\,\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}-10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]+$$

$$\frac{3}{961}\sqrt{\frac{1}{434}\left(15082+2705\sqrt{35}\right)}\,\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}+10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]-$$

$$\frac{3\sqrt{\frac{1}{434}\left(-15082+2705\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\,\sqrt{1+2x}+5\,(1+2x)\right]}{1922}+$$

$$\frac{3\sqrt{\frac{1}{434}\left(-15082+2705\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\,\sqrt{1+2x}+5\,(1+2x)\right]}{1922}$$

Result (type 3, 161 leaves) :

$$\frac{\frac{31\sqrt{1+2x}\,\left(-21+565x+695x^2+600x^3\right)}{2\left(2+3x+5x^2\right)^2}+\frac{3\left(124-47i\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]}{\sqrt{-\frac{1}{5}i\left(-2i+\sqrt{31}\right)}}+\frac{3\left(124+47i\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]}{\sqrt{\frac{1}{5}i\left(2i+\sqrt{31}\right)}}}{29791}$$

- **Problem 2327: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{1+2x}}{\left(2+3x+5x^2\right)^3}\,dx$$

Optimal (type 3, 300 leaves, 12 steps) :

$$\frac{\sqrt{1+2x}\,(3+10x)}{62\,(2+3x+5x^2)^2}+\frac{\sqrt{1+2x}\,(599+1790x)}{13454\,(2+3x+5x^2)}-\frac{\sqrt{\frac{1}{434}\left(9651062+1806875\sqrt{35}\right)}\,\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}-10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]}{6727}+$$

$$\frac{\sqrt{\frac{1}{434}\left(9651062+1806875\sqrt{35}\right)}\,\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}+10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]}{6727}+$$

$$\frac{\sqrt{\frac{1}{434}\left(-9651062+1806875\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\,\sqrt{1+2x}+5\,(1+2x)\right]}{13454}-$$

$$\frac{\sqrt{\frac{1}{434}\left(-9651062+1806875\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\,\sqrt{1+2x}+5\,(1+2x)\right]}{13454}$$

Result (type 3, 159 leaves) :

$$\frac{\frac{31\sqrt{1+2x}\,\left(1849+7547x+8365x^2+8950x^3\right)}{2\,\left(2+3x+5x^2\right)^2}+\frac{\left(5549-902\,i\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]}{\sqrt{-\frac{1}{5}i\left(-2i+\sqrt{31}\right)}}+\frac{\left(5549+902\,i\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]}{\sqrt{\frac{1}{5}i\left(2i+\sqrt{31}\right)}}}{208537}$$

- **Problem 2328: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{1+2x}\,\left(2+3x+5x^2\right)^3}\,dx$$

Optimal (type 3, 314 leaves, 12 steps) :

$$\frac{\sqrt{1+2x}\,(37+20x)}{434\,(2+3x+5x^2)^2}+\frac{\sqrt{1+2x}\,(9227+7920x)}{94178\,(2+3x+5x^2)}-\frac{3\sqrt{\frac{1}{434}\left(2+\sqrt{35}\right)}\,\left(7379+264\sqrt{35}\right)\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}-10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]}{47089}+$$

$$\frac{3\sqrt{\frac{1}{434}\left(2+\sqrt{35}\right)}\,\left(7379+264\sqrt{35}\right)\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}+10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]}{47089}-$$

$$\frac{3\sqrt{\frac{1}{434}\left(-250141922+64681225\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\,\sqrt{1+2x}+5\,(1+2x)\right]}{94178}+$$

$$\frac{3\sqrt{\frac{1}{434}\left(-250141922+64681225\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\,\sqrt{1+2x}+5\,(1+2x)\right]}{94178}$$

Result (type 3, 161 leaves) :

$$\frac{1}{1459759}\left(\frac{31\sqrt{1+2x}\,\left(26483+47861x+69895x^2+39600x^3\right)}{2\,(2+3x+5x^2)^2}+\frac{3\left(8184-7907\,i\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]}{\sqrt{-\frac{1}{5}\,i\left(-2\,i+\sqrt{31}\right)}}+\frac{3\left(8184+7907\,i\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]}{\sqrt{\frac{1}{5}\,i\left(2\,i+\sqrt{31}\right)}}\right)$$

- **Problem 2329: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{(1+2x)^{3/2}\,(2+3x+5x^2)^3}\,dx$$

Optimal (type 3, 313 leaves, 13 steps) :

$$-\frac{81090}{329623\sqrt{1+2x}}+\frac{37+20x}{434\sqrt{1+2x}\left(2+3x+5x^2\right)^2}+\frac{5(2329+2080x)}{94178\sqrt{1+2x}\left(2+3x+5x^2\right)}-$$

$$\frac{15\sqrt{\frac{1}{434}\left(-2257111762+387427075\sqrt{35}\right)}\,\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}-10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]}{329623}+$$

$$\frac{15\sqrt{\frac{1}{434}\left(-2257111762+387427075\sqrt{35}\right)}\,\text{ArcTan}\left[\frac{\sqrt{10\left(2+\sqrt{35}\right)}+10\sqrt{1+2x}}{\sqrt{10\left(-2+\sqrt{35}\right)}}\right]}{329623}-$$

$$\frac{15\sqrt{\frac{1}{434}\left(2257111762+387427075\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}-\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right]}{659246}+$$

$$\frac{15\sqrt{\frac{1}{434}\left(2257111762+387427075\sqrt{35}\right)}\,\text{Log}\left[\sqrt{35}+\sqrt{10\left(2+\sqrt{35}\right)}\sqrt{1+2x}+5(1+2x)\right]}{659246}$$

Result (type 3, 170 leaves) :

$$\frac{1}{10218313}\left(-\frac{31\left(429487+1525635x+4077245x^2+4501400x^3+4054500x^4\right)}{2\sqrt{1+2x}\left(2+3x+5x^2\right)^2}-\frac{15\,i\left(-83793\,i+12686\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]}{\sqrt{-\frac{1}{5}i\left(-2i+\sqrt{31}\right)}}+\frac{15\,i\left(83793\,i+12686\sqrt{31}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]}{\sqrt{\frac{1}{5}i\left(2i+\sqrt{31}\right)}}\right)$$

- **Problem 2355: Result more than twice size of optimal antiderivative.**

$$\int (d+e x)^3\left(a+b x+c x^2\right)^{5/2}\,dx$$

Optimal (type 3, 400 leaves, 7 steps) :

$$\frac{5\left(b^2-4ac\right)^2(2cd-be)\left(32c^2d^2+11b^2e^2-4ce(8bd+3ae)\right)(b+2cx)\sqrt{a+bx+cx^2}}{32768c^6}-$$
$$\frac{5\left(b^2-4ac\right)(2cd-be)\left(32c^2d^2+11b^2e^2-4ce(8bd+3ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{12288c^5}+$$
$$\frac{(2cd-be)\left(32c^2d^2+11b^2e^2-4ce(8bd+3ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{768c^4}+\frac{e(d+ex)^2\left(a+bx+cx^2\right)^{7/2}}{9c}+$$
$$\frac{e\left(640c^2d^2+99b^2e^2-2ce(243bd+32ae)+154ce(2cd-be)x\right)\left(a+bx+cx^2\right)^{7/2}}{2016c^3}-$$
$$\frac{5\left(b^2-4ac\right)^3(2cd-be)\left(32c^2d^2+11b^2e^2-4ce(8bd+3ae)\right)\text{ArcTanh}\left[\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right]}{65536c^{13/2}}$$

Result (type 3, 807 leaves):

$$\frac{1}{4128768c^{13/2}}\Big(2\sqrt{c}\sqrt{a+x(b+cx)}\Big(-3465b^8e^3+210b^7ce^2(81d+11ex)-$$
$$84b^6ce\left(-485ae^2+c\left(360d^2+135dex+22e^2x^2\right)\right)+72b^5c^2\left(-7ae^2(375d+49ex)+2c\left(140d^3+140d^2ex+63de^2x^2+11e^3x^3\right)\right)-$$
$$16b^4c^2\left(10143a^2e^3-9ace\left(2240d^2+791dex+124e^2x^2\right)+2c^2x\left(420d^3+504d^2ex+243de^2x^2+44e^3x^3\right)\right)+$$
$$32b^3c^3\left(9a^2e^2(2359d+293ex)+8c^2x^2\left(42d^3+54d^2ex+27de^2x^2+5e^3x^3\right)-4ac\left(1680d^3+1512d^2ex+639de^2x^2+107e^3x^3\right)\right)+$$
$$192b^2c^3\Big(1221a^3e^3-a^2ce\left(5544d^2+1791dex+266e^2x^2\right)+4ac^2x\left(168d^3+180d^2ex+81de^2x^2+14e^3x^3\right)+8c^3x^3$$
$$\left(378d^3+888d^2ex+729de^2x^2+206e^3x^3\right)\Big)+128bc^4\Big(-13a^3e^2(459d+53ex)+6a^2c\left(924d^3+684d^2ex+261de^2x^2+41e^3x^3\right)+$$
$$24ac^2x^2\left(546d^3+1182d^2ex+921de^2x^2+251e^3x^3\right)+16c^3x^4\left(420d^3+1044d^2ex+891de^2x^2+259e^3x^3\right)\Big)+$$
$$256c^4\Big(-256a^4e^3+a^3ce\left(3456d^2+945dex+128e^2x^2\right)+16c^4x^5\left(84d^3+216d^2ex+189de^2x^2+56e^3x^3\right)+$$
$$8ac^3x^3\left(546d^3+1296d^2ex+1071de^2x^2+304e^3x^3\right)+6a^2c^2x\left(924d^3+1728d^2ex+1239de^2x^2+320e^3x^3\right)\Big)\Big)+$$
$$315\left(b^2-4ac\right)^3(-2cd+be)\left(32c^2d^2+11b^2e^2-4ce(8bd+3ae)\right)\text{Log}\left[b+2cx+2\sqrt{c}\sqrt{a+x(b+cx)}\right]\Big)$$

- **Problem 2440: Result unnecessarily involves imaginary or complex numbers.**

$$\int(dx)^{5/2}\sqrt{a+bx+cx^2}\,dx$$

Optimal (type 4, 504 leaves, 8 steps):

$$-\frac{4\left(8 b^4-36 a b^2 c+21 a^2 c^2\right) d^3 x \sqrt{a+b x+c x^2}}{315 c^{7/2} \sqrt{d x}\left(\sqrt{a}+\sqrt{c} x\right)}+$$

$$\frac{2 d^2 \sqrt{d x}\left(b\left(8 b^2+3 a c\right)+3 c\left(8 b^2-7 a c\right) x\right) \sqrt{a+b x+c x^2}}{315 c^3}-\frac{4 b d^2 \sqrt{d x}\left(a+b x+c x^2\right)^{3/2}}{21 c^2}+\frac{2 d (d x)^{3/2}\left(a+b x+c x^2\right)^{3/2}}{9 c}+$$

$$\left(4 a^{1/4}\left(8 b^4-36 a b^2 c+21 a^2 c^2\right) d^3 \sqrt{x}\left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+b x+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/$$

$$\left(315 c^{15/4} \sqrt{d x} \sqrt{a+b x+c x^2}\right)-\left(a^{1/4}\left(16 b^4-72 a b^2 c+42 a^2 c^2+\sqrt{a} b \sqrt{c}\left(8 b^2-27 a c\right)\right) d^3 \sqrt{x}\left(\sqrt{a}+\sqrt{c} x\right)\right.$$

$$\left.\sqrt{\frac{a+b x+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(315 c^{15/4} \sqrt{d x} \sqrt{a+b x+c x^2}\right)$$

Result (type 4, 594 leaves) :

$$\frac{1}{315\,c^4\,x^{5/2}\,\sqrt{a+x\,(b+c\,x)}}\,(d\,x)^{5/2}\left(-\frac{4\,\left(8\,b^4-36\,a\,b^2\,c+21\,a^2\,c^2\right)\,(a+x\,(b+c\,x))}{\sqrt{x}}+2\,c\,\sqrt{x}\,(a+x\,(b+c\,x))\,\left(8\,b^3-6\,b^2\,c\,x+b\,c\,\left(-27\,a+5\,c\,x^2\right)+7\,c^2\,x\,\left(2\,a+5\,c\,x^2\right)\right)+\right.$$

$$\frac{1}{\sqrt{\frac{a}{b+\sqrt{b^2-4\,a\,c}}}}\,\mathrm{i}\,\left(8\,b^4-36\,a\,b^2\,c+21\,a^2\,c^2\right)\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,\sqrt{2+\frac{4\,a}{\left(b+\sqrt{b^2-4\,a\,c}\right)\,x}}\,x$$

$$\sqrt{\frac{2\,a+b\,x-\sqrt{b^2-4\,a\,c}\,x}{b\,x-\sqrt{b^2-4\,a\,c}\,x}}\,\mathrm{EllipticE}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{\frac{a}{b+\sqrt{b^2-4\,a\,c}}}}{\sqrt{x}}\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-\frac{1}{\sqrt{\frac{a}{b+\sqrt{b^2-4\,a\,c}}}}$$

$$\mathrm{i}\,\left(-8\,b^5+44\,a\,b^3\,c-48\,a^2\,b\,c^2+8\,b^4\,\sqrt{b^2-4\,a\,c}-36\,a\,b^2\,c\,\sqrt{b^2-4\,a\,c}+21\,a^2\,c^2\,\sqrt{b^2-4\,a\,c}\right)\,\sqrt{2+\frac{4\,a}{\left(b+\sqrt{b^2-4\,a\,c}\right)\,x}}$$

$$\left.x\,\sqrt{\frac{2\,a+b\,x-\sqrt{b^2-4\,a\,c}\,x}{b\,x-\sqrt{b^2-4\,a\,c}\,x}}\,\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{\frac{a}{b+\sqrt{b^2-4\,a\,c}}}}{\sqrt{x}}\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

- **Problem 2441: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (d+e\,x)^{3/2}\,\sqrt{a+b\,x+c\,x^2}\,dx$$

Optimal (type 4, 581 leaves, 7 steps) :

$$\frac{2\sqrt{d+e\,x}\,\left(3\,c^2\,d^2-4\,b^2\,e^2+c\,e\,(9\,b\,d-5\,a\,e)+12\,c\,e\,(2\,c\,d-b\,e)\,x\right)\sqrt{a+b\,x+c\,x^2}}{105\,c^2\,e}+\frac{2\,e\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)^{3/2}}{7\,c}-$$

$$\left(\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,(2\,c\,d-b\,e)\,\left(3\,c^2\,d^2+8\,b^2\,e^2-c\,e\,(3\,b\,d+29\,a\,e)\right)\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/\left(105\,c^3\,e^2\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\left(4\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\left(3\,c^2\,d^2+2\,b^2\,e^2-c\,e\,(3\,b\,d+5\,a\,e)\right)\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/\left(105\,c^3\,e^2\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 5328 leaves) :

$$\sqrt{d+e\,x}\,\left(\frac{2\,\left(3\,c^2\,d^2+9\,b\,c\,d\,e-4\,b^2\,e^2+10\,a\,c\,e^2\right)}{105\,c^2\,e}+\frac{2\,(8\,c\,d+b\,e)\,x}{35\,c}+\frac{2\,e\,x^2}{7}\right)\sqrt{a+x\,(b+c\,x)}+\frac{1}{105\,c^2\,e^3\,\sqrt{a+b\,x+c\,x^2}}$$

$$2\sqrt{a+x\,(b+c\,x)}\left(-\left((2\,c\,d-b\,e)\,\left(3\,c^2\,d^2-3\,b\,c\,d\,e+8\,b^2\,e^2-29\,a\,c\,e^2\right)\,(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)\right)\right/$$

$$\left(c\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)+$$

$$\frac{1}{c\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\left(c\,d^2-b\,d\,e+a\,e^2\right)\,(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\left(\left(3\,i\,c^3\,d^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\right/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(9\,i\,b\,c^2\,d^2\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(19\,i\,b^2\,c\,d\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(29\,i\,a\,c^2\,d\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(2\,i\sqrt{2}\,b^3\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(29\, i\, a\, b\, c\, e^3 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(2 \sqrt{2} \left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(3\, i \sqrt{2}\, c^3 d^2 \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\left(3\, i \sqrt{2}\, b\, c^2 d\, e \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(2\,i\sqrt{2}\,b^2 c\,e^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(5\,i\sqrt{2}\,a\,c^2 e^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\Bigg)\Bigg)$$

- **Problem 2442: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\sqrt{d+e\,x}\;\sqrt{a+b\,x+c\,x^2}\,dx$$

Optimal (type 4, 513 leaves, 7 steps) :

$$-\frac{2\,(2\,c\,d-b\,e)\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}}{15\,c\,e}+\frac{2\,(d+e\,x)^{3/2}\,\sqrt{a+b\,x+c\,x^2}}{5\,e}-$$

$$\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(c^2\,d^2+b^2\,e^2-c\,e\,(b\,d+3\,a\,e)\right)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/\left(15\,c^2\,e^2\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,(2\,c\,d-b\,e)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/\left(15\,c^2\,e^2\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 3384 leaves) :

$$\left(\frac{2\,(c\,d+b\,e)}{15\,c\,e}+\frac{2\,x}{5}\right)\,\sqrt{d+e\,x}\,\sqrt{a+x\,(b+c\,x)}\,+$$

$$\frac{1}{15\, c\, e^3 \sqrt{a+b\, x+c\, x^2}} \sqrt{a+x\,(b+c\, x)}\; \Bigg(-\frac{4\left(c^2 d^2-b\, c\, d\, e+b^2 e^2-3\, a\, c\, e^2\right)(d+e\, x)^{3/2}\left(c+\frac{c\, d^2}{(d+e\, x)^2}-\frac{b\, d\, e}{(d+e\, x)^2}+\frac{a\, e^2}{(d+e\, x)^2}-\frac{2\, c\, d}{d+e\, x}+\frac{b\, e}{d+e\, x}\right)}{c\sqrt{\frac{(d+e\, x)^2\left(c\left(-1+\frac{d}{d+e\, x}\right)^2+\frac{e\left(b-\frac{b\, d}{d+e\, x}+\frac{a\, e}{d+e\, x}\right)}{d+e\, x}\right)}{e^2}}}+$$

$$\frac{1}{c\sqrt{\frac{(d+e\, x)^2\left(c\left(-1+\frac{d}{d+e\, x}\right)^2+\frac{e\left(b-\frac{b\, d}{d+e\, x}+\frac{a\, e}{d+e\, x}\right)}{d+e\, x}\right)}{e^2}}}\, 2\left(c\, d^2-b\, d\, e+a\, e^2\right)(d+e\, x)\sqrt{c+\frac{c\, d^2}{(d+e\, x)^2}-\frac{b\, d\, e}{(d+e\, x)^2}+\frac{a\, e^2}{(d+e\, x)^2}-\frac{2\, c\, d}{d+e\, x}+\frac{b\, e}{d+e\, x}}$$

$$\Bigg(\Bigg(\mathbb{i}\, c^2 d^2\left(2\, c\, d-b\, e+\sqrt{b^2 e^2-4\, a\, c\, e^2}\right)\sqrt{1-\frac{2\left(c\, d^2-b\, d\, e+a\, e^2\right)}{\left(2\, c\, d-b\, e-\sqrt{b^2 e^2-4\, a\, c\, e^2}\right)(d+e\, x)}}\;\sqrt{1-\frac{2\left(c\, d^2-b\, d\, e+a\, e^2\right)}{\left(2\, c\, d-b\, e+\sqrt{b^2 e^2-4\, a\, c\, e^2}\right)(d+e\, x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\, d^2-b\, d\, e+a\, e^2}{2\, c\, d-b\, e-\sqrt{b^2 e^2-4\, a\, c\, e^2}}}}{\sqrt{d+e\, x}}\right],\ \frac{2\, c\, d-b\, e-\sqrt{b^2 e^2-4\, a\, c\, e^2}}{2\, c\, d-b\, e+\sqrt{b^2 e^2-4\, a\, c\, e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\, d^2-b\, d\, e+a\, e^2}{2\, c\, d-b\, e-\sqrt{b^2 e^2-4\, a\, c\, e^2}}}}{\sqrt{d+e\, x}}\right],\ \frac{2\, c\, d-b\, e-\sqrt{b^2 e^2-4\, a\, c\, e^2}}{2\, c\, d-b\, e+\sqrt{b^2 e^2-4\, a\, c\, e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c\, d^2-b\, d\, e+a\, e^2\right)\sqrt{-\frac{c\, d^2-b\, d\, e+a\, e^2}{2\, c\, d-b\, e-\sqrt{b^2 e^2-4\, a\, c\, e^2}}}\;\sqrt{c+\frac{c\, d^2-b\, d\, e+a\, e^2}{(d+e\, x)^2}+\frac{-2\, c\, d+b\, e}{d+e\, x}}\right)-$$

$$\Bigg(\mathbb{i}\, b\, c\, d\, e\left(2\, c\, d-b\, e+\sqrt{b^2 e^2-4\, a\, c\, e^2}\right)\sqrt{1-\frac{2\left(c\, d^2-b\, d\, e+a\, e^2\right)}{\left(2\, c\, d-b\, e-\sqrt{b^2 e^2-4\, a\, c\, e^2}\right)(d+e\, x)}}\;\sqrt{1-\frac{2\left(c\, d^2-b\, d\, e+a\, e^2\right)}{\left(2\, c\, d-b\, e+\sqrt{b^2 e^2-4\, a\, c\, e^2}\right)(d+e\, x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(i\,b^2\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(3\,i\,a\,c\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(\mathrm{i}\sqrt{2}\,c^2 d\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(\mathrm{i}\,b c e\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)\Bigg)\Bigg)$$

Problem 2443: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a+b x+c x^2}}{\sqrt{d+e x}}\,dx$$

Optimal (type 4, 444 leaves, 6 steps):

$$\frac{2\sqrt{d+e x}\sqrt{a+b x+c x^2}}{3 e}-\left(\sqrt{2}\sqrt{b^2-4 a c}\,(2 c d-b e)\sqrt{d+e x}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Bigg/$$

$$\left(3 c e^2\sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\sqrt{a+b x+c x^2}\right)+\Bigg(4\sqrt{2}\sqrt{b^2-4 a c}\left(c d^2-b d e+a e^2\right)\sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}$$

$$\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\Bigg)\Bigg/\left(3 c e^2\sqrt{d+e x}\sqrt{a+b x+c x^2}\right)$$

Result (type 4, 1847 leaves):

$$\frac{2\sqrt{d+e\,x}\,\sqrt{a+x\,(b+c\,x)}}{3\,e}+\frac{1}{3\,e^3\,\sqrt{a+b\,x+c\,x^2}}\sqrt{a+x\,(b+c\,x)}\,\left(-\frac{2\,(2\,c\,d-b\,e)\,(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)}{c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}+\right.$$

$$\frac{1}{c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\,2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\left(\left(i\,c\,d\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(i\,b\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(\mathbb{i}\sqrt{2}\,c\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left.\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)\right)$$

- **Problem 2444: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b x+c x^2}}{(d+e x)^{3/2}}\, dx$$

Optimal (type 4, 419 leaves, 6 steps) :

$$-\frac{2\sqrt{a+bx+cx^2}}{e\sqrt{d+ex}}+\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right]}{e^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}-$$

$$\left(2\sqrt{2}\sqrt{b^2-4ac}\,(2cd-be)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right]\right)\Big/\left(c\,e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}\right)$$

Result (type 4, 865 leaves) :

$$\frac{1}{e^3\sqrt{d+e\,x}\,\sqrt{a+x\,(b+c\,x)}}\left(4\,c\,e^2\,x^2+4\,e^2\,(a+b\,x)-2\,e^2\,(a+x\,(b+c\,x))-\frac{1}{\sqrt{\frac{c\,d^2+e\,(-b\,d+a\,e)}{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}}}\right.$$

$$i\sqrt{2}\left(2\,c\,d-b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}\right)(d+e\,x)^{3/2}\sqrt{\frac{-2\,a\,e^2+d\sqrt{(b^2-4\,a\,c)\,e^2}+2\,c\,d\,e\,x+e\sqrt{(b^2-4\,a\,c)\,e^2}\,x+b\,e\,(d-e\,x)}{\left(2\,c\,d-b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{\frac{2\,a\,e^2+d\sqrt{(b^2-4\,a\,c)\,e^2}-2\,c\,d\,e\,x+e\sqrt{(b^2-4\,a\,c)\,e^2}\,x+b\,e\,(-d+e\,x)}{\left(-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}\right)(d+e\,x)}}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c\,d^2-b\,d\,e+a\,e^2}{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}}}{\sqrt{d+e\,x}}\right],-\frac{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}{2\,c\,d-b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}\right]+\frac{1}{\sqrt{\frac{c\,d^2+e\,(-b\,d+a\,e)}{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}}}$$

$$i\sqrt{2}\sqrt{(b^2-4\,a\,c)\,e^2}\,(d+e\,x)^{3/2}\sqrt{\frac{-2\,a\,e^2+d\sqrt{(b^2-4\,a\,c)\,e^2}+2\,c\,d\,e\,x+e\sqrt{(b^2-4\,a\,c)\,e^2}\,x+b\,e\,(d-e\,x)}{\left(2\,c\,d-b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{\frac{2\,a\,e^2+d\sqrt{(b^2-4\,a\,c)\,e^2}-2\,c\,d\,e\,x+e\sqrt{(b^2-4\,a\,c)\,e^2}\,x+b\,e\,(-d+e\,x)}{\left(-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}\right)(d+e\,x)}}$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c\,d^2-b\,d\,e+a\,e^2}{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}}}{\sqrt{d+e\,x}}\right],-\frac{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}{2\,c\,d-b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}\right]\right)$$

- **Problem 2445: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{a+b\,x+c\,x^2}}{(d+e\,x)^{5/2}}\,dx$$

Optimal (type 4, 497 leaves, 7 steps):

$$-\frac{2\sqrt{a+b\,x+c\,x^2}}{3\,e\,(d+e\,x)^{3/2}}+\frac{2\,(2\,c\,d-b\,e)\,\sqrt{a+b\,x+c\,x^2}}{3\,e\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{d+e\,x}}-$$

$$\left(\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\,(2\,c\,d-b\,e)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/$$

$$\left(3\,e^2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\left(4\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/$$

$$\left(3\,e^2\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 1914 leaves) :

$$\sqrt{d+e\,x}\,\sqrt{a+x\,(b+c\,x)}\,\left(-\frac{2}{3\,e\,(d+e\,x)^2}-\frac{2\,(-2\,c\,d+b\,e)}{3\,e\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,(d+e\,x)}\right)-$$

$$\frac{1}{3\,e^3\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{a+b\,x+c\,x^2}}\,2\,c\,\sqrt{a+x\,(b+c\,x)}\,\left(\frac{(2\,c\,d-b\,e)\,(d+e\,x)^{3/2}\,\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)}{c\,\sqrt{\frac{(d+e\,x)^2\,\left(c\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\,\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}-\right.$$

$$\frac{1}{c\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\Bigg(\Bigg(\mathbb{i}\,c\,d\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\Bigg[\mathbb{i}\,\text{ArcSinh}\Bigg[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Bigg],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Bigg]-$$

$$\text{EllipticF}\Bigg[\mathbb{i}\,\text{ArcSinh}\Bigg[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Bigg],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Bigg]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(\mathbb{i}\,b\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\Bigg[\mathbb{i}\,\text{ArcSinh}\Bigg[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Bigg],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Bigg]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(\mathbb{i}\,\sqrt{2}\,c\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg/$$

$$\left(\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)\right)$$

- **Problem 2446: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{a+b\,x+c\,x^2}}{(d+e\,x)^{7/2}}\,dx$$

Optimal (type 4, 617 leaves, 8 steps) :

$$-\frac{2\sqrt{a+bx+cx^2}}{5e(d+ex)^{5/2}}+\frac{2(2cd-be)\sqrt{a+bx+cx^2}}{15e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}}+$$

$$\frac{4\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)\sqrt{a+bx+cx^2}}{15e\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}}-\left(2\sqrt{2}\sqrt{b^2-4ac}\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)\right.$$

$$\left.\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right]\right)\Big/$$

$$\left(15e^2\left(cd^2-bde+ae^2\right)^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}\right)+$$

$$\left(2\sqrt{2}\sqrt{b^2-4ac}(2cd-be)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right]\right)\Big/\left(15e^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}\right)$$

Result (type 4, 3493 leaves) :

$$\sqrt{d+ex}\sqrt{a+x(b+cx)}\left(-\frac{2}{5e(d+ex)^3}-\frac{2(-2cd+be)}{15e\left(cd^2-bde+ae^2\right)(d+ex)^2}-\frac{4\left(-c^2d^2+bcde-b^2e^2+3ace^2\right)}{15e\left(cd^2-bde+ae^2\right)^2(d+ex)}\right)-$$

$$\frac{1}{15\,e^3\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\sqrt{a+b\,x+c\,x^2}}$$

$$2\,c\,\sqrt{a+x\,(b+c\,x)}\,\left(\frac{2\left(c^2\,d^2-b\,c\,d\,e+b^2\,e^2-3\,a\,c\,e^2\right)(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)}{c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}-\right.$$

$$\frac{1}{c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\left(\left(\mathbb{i}\,c^2\,d^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\;\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(i\, b\, c\, d\, e \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2} \left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(i\, b^2 e^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2} \left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\left(3\,\mathbb{i}\,a\,c\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(\mathbb{i}\,\sqrt{2}\,c^2\,d\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(\mathbb{i}\,b\,c\,e\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\left.\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right/$$

$$\left.\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)$$

- **Problem 2447: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (d+e\,x)^{3/2}\,\left(a+b\,x+c\,x^2\right)^{3/2}\,dx$$

Optimal (type 4, 816 leaves, 8 steps) :

$$\frac{1}{1155\, c^3 e^3} 2 \sqrt{d+e x}\, \Big(8 c^4 d^4+8 b^4 e^4-c^3 d^2 e\, (19 b d-42 a e)-b^2 c e^3\, (19 b d+21 a e)+$$
$$3 c^2 e^2 \left(2 b^2 d^2+17 a b d e-10 a^2 e^2\right)-3 c e\, (2 c d-b e) \left(c^2 d^2+8 b^2 e^2-c e\, (b d+31 a e)\right) x\Big) \sqrt{a+b x+c x^2}\, +$$
$$\frac{2 \sqrt{d+e x}\, \left(c^2 d^2-6 b^2 e^2+c e\, (13 b d-3 a e)+14 c e\, (2 c d-b e)\, x\right) \left(a+b x+c x^2\right)^{3/2}}{231\, c^2 e}+\frac{2 e \sqrt{d+e x}\, \left(a+b x+c x^2\right)^{5/2}}{11\, c}-$$

$$\left(8 \sqrt{2} \sqrt{b^2-4 a c}\, (2 c d-b e) \left(c^2 d^2-2 b^2 e^2-c e\, (b d-9 a e)\right) \left(c^2 d^2+b^2 e^2-c e\, (b d+3 a e)\right) \sqrt{d+e x} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right) \Bigg/ \left(1155\, c^4 e^4 \sqrt{\frac{c\, (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{a+b x+c x^2}\right)+$$

$$\left(2 \sqrt{2} \sqrt{b^2-4 a c}\, \left(c d^2-b d e+a e^2\right) \left(16 c^4 d^4-8 b^4 e^4-4 c^3 d^2 e\, (8 b d-21 a e)+b^2 c e^3\, (13 b d+51 a e)+3 c^2 e^2 \left(b^2 d^2-28 a b d e-20 a^2 e^2\right)\right)\right.$$

$$\sqrt{\frac{c\, (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right) \Bigg/ \left(1155\, c^4 e^4 \sqrt{d+e x} \sqrt{a+b x+c x^2}\right)$$

Result (type 4, 10 848 leaves) :

$$\frac{1}{a+b x+c x^2}\sqrt{d+e x}\left(1\Big/\left(1155 c^3 e^3\right) 2 \left(8 c^4 d^4-19 b c^3 d^3 e+6 b^2 c^2 d^2 e^2+47 a c^3 d^2 e^2-19 b^3 c d e^3+116 a b c^2 d e^3+8 b^4 e^4-51 a b^2 c e^4+60 a^2 c^2 e^4\right)+\right.$$

$$\frac{4 \left(-3 c^3 d^3+7 b c^2 d^2 e+7 b^2 c d e^2+163 a c^2 d e^2-3 b^3 e^3+16 a b c e^3\right) x}{1155 c^2 e^2}+\frac{2 \left(c^2 d^2+41 b c d e+b^2 e^2+39 a c e^2\right) x^2}{231 c e}+$$

$$\left.\frac{8}{33} (c d+b e) x^3+\frac{2}{11} c e x^4\right) (a+x (b+c x))^{3/2}+\frac{1}{1155 c^3 e^5 \left(a+b x+c x^2\right)^{3/2}}$$

$$2 (a+x (b+c x))^{3/2}\left(-\left(8 (2 c d-b e) \left(c^4 d^4-2 b c^3 d^3 e+6 a c^3 d^2 e^2+b^3 c d e^3-6 a b c^2 d e^3-2 b^4 e^4+15 a b^2 c e^4-27 a^2 c^2 e^4\right)\right.\right.$$

$$\left.\left.(d+e x)^{3/2} \left(c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)\right)\right/\left(c \sqrt{\frac{(d+e x)^2 \left(c \left(-1+\frac{d}{d+e x}\right)^2+\frac{e \left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)+$$

$$\frac{1}{c \sqrt{\frac{(d+e x)^2 \left(c \left(-1+\frac{d}{d+e x}\right)^2+\frac{e \left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}\left(c d^2-b d e+a e^2\right) (d+e x) \sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\left(\left(4 i \sqrt{2} c^5 d^5 \left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}} \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(10\,i\sqrt{2}\,b c^4 d^4 e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(4\,i\sqrt{2}\,b^2 c^3 d^3 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(24\,\mathbb{i}\,\sqrt{2}\,a\,c^4\,d^3\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\ \Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(4\,\mathbb{i}\,\sqrt{2}\,b^3\,c^2\,d^2\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\ \Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(36\,i\sqrt{2}\,a b c^3 d^2 e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(10\,i\sqrt{2}\,b^4 c d e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(72\,i\sqrt{2}\,a b^2 c^2 d e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(108\,i\sqrt{2}\,a^2 c^3 d e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(4\,i\sqrt{2}\,b^5 e^5\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(30\,i\sqrt{2}\,a b^3 c e^5\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(54\,i\sqrt{2}\,a^2 b c^2 e^5\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(8\,i\sqrt{2}\,c^5 d^4\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(16\,\mathrm{i}\,\sqrt{2}\,b\,c^4\,d^3\,e\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(3\,\mathrm{i}\,b^2\,c^3\,d^2\,e^2\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(42\,\mathrm{i}\,\sqrt{2}\,a\,c^4\,d^2\,e^2\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(13\,i\,b^3 c^2 d e^3\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(42\,i\sqrt{2}\,a b c^3 d e^3\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(4\,i\sqrt{2}\,b^4\,c\,e^4\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(51\,i\,a\,b^2\,c^2\,e^4\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(30\,i\sqrt{2}\,a^2\,c^3\,e^4\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left.\left.\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)\right)$$

- **Problem 2448: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\sqrt{d+e\,x}\ \left(a+b\,x+c\,x^2\right)^{3/2}\,dx$$

Optimal (type 4, 712 leaves, 8 steps):

$$\frac{1}{315 c^2 e^3} 2 \sqrt{d+e x} \left(8 c^3 d^3-4 b^3 e^3-3 c^2 d e (5 b d-8 a e)+3 b c e^2 (b d+3 a e)-6 c e \left(c^2 d^2+2 b^2 e^2-c e (b d+7 a e)\right) x\right) \sqrt{a+b x+c x^2} -$$

$$\frac{2 (2 c d-b e) \sqrt{d+e x} \left(a+b x+c x^2\right)^{3/2}}{21 c e}+\frac{2 (d+e x)^{3/2} \left(a+b x+c x^2\right)^{3/2}}{9 e}-$$

$$\left(\sqrt{2} \sqrt{b^2-4 a c} \left(16 c^4 d^4-8 b^4 e^4-4 c^3 d^2 e (8 b d-15 a e)+b^2 c e^3 (7 b d+57 a e)+3 c^2 e^2 \left(3 b^2 d^2-20 a b d e-28 a^2 e^2\right)\right)\right.$$

$$\left.\sqrt{d+e x} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c} e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right) \Big/$$

$$\left(315 c^3 e^4 \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{a+b x+c x^2}\right)+$$

$$\left(8 \sqrt{2} \sqrt{b^2-4 a c} (2 c d-b e) \left(c d^2-b d e+a e^2\right) \left(2 c^2 d^2-b^2 e^2-2 c e (b d-3 a e)\right) \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\right.$$

$$\left.\sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c} e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right) \Big/ \left(315 c^3 e^4 \sqrt{d+e x} \sqrt{a+b x+c x^2}\right)$$

Result (type 4, 7541 leaves) :

$$\frac{1}{a+b x+c x^2} \sqrt{d+e x}$$

$$\left(\frac{2\left(8c^3d^3-15bc^2d^2e+3b^2cde^2+29ac^2de^2-4b^3e^3+24abce^3\right)}{315c^2e^3}+\frac{2\left(-6c^2d^2+11bcde+3b^2e^2+77ace^2\right)x}{315ce^2}+\frac{2(cd+10be)x^2}{63e}+\frac{2cx^3}{9}\right)$$

$$(a+x(b+cx))^{3/2}+\frac{1}{315c^2e^5\left(a+bx+cx^2\right)^{3/2}}$$

$$(a+x(b+cx))^{3/2}\left(-\left(2\left(16c^4d^4-32bc^3d^3e+9b^2c^2d^2e^2+60ac^3d^2e^2+7b^3cde^3-60abc^2de^3-8b^4e^4+57ab^2ce^4-84a^2c^2e^4\right)\right.\right.$$

$$\left.\left.(d+ex)^{3/2}\left(c+\frac{cd^2}{(d+ex)^2}-\frac{bde}{(d+ex)^2}+\frac{ae^2}{(d+ex)^2}-\frac{2cd}{d+ex}+\frac{be}{d+ex}\right)\right)\Big/\left(c\sqrt{\frac{(d+ex)^2\left(c\left(-1+\frac{d}{d+ex}\right)^2+\frac{e\left(b-\frac{bd}{d+ex}+\frac{ae}{d+ex}\right)}{d+ex}\right)}{e^2}}\right)+\right.$$

$$\frac{1}{c\sqrt{\frac{(d+ex)^2\left(c\left(-1+\frac{d}{d+ex}\right)^2+\frac{e\left(b-\frac{bd}{d+ex}+\frac{ae}{d+ex}\right)}{d+ex}\right)}{e^2}}}2\left(cd^2-bde+ae^2\right)(d+ex)\sqrt{c+\frac{cd^2}{(d+ex)^2}-\frac{bde}{(d+ex)^2}+\frac{ae^2}{(d+ex)^2}-\frac{2cd}{d+ex}+\frac{be}{d+ex}}$$

$$\left(\left(4\,i\sqrt{2}\,c^4d^4\left(2cd-be+\sqrt{b^2e^2-4ace^2}\right)\sqrt{1-\frac{2\left(cd^2-bde+ae^2\right)}{\left(2cd-be-\sqrt{b^2e^2-4ace^2}\right)(d+ex)}}\sqrt{1-\frac{2\left(cd^2-bde+ae^2\right)}{\left(2cd-be+\sqrt{b^2e^2-4ace^2}\right)(d+ex)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{cd^2-bde+ae^2}{2cd-be-\sqrt{b^2e^2-4ace^2}}}}{\sqrt{d+ex}}\right],\frac{2cd-be-\sqrt{b^2e^2-4ace^2}}{2cd-be+\sqrt{b^2e^2-4ace^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(8\,i\,\sqrt{2}\,b c^3 d^3 e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(9\,i\,b^2 c^2 d^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(15\,i\sqrt{2}\,a c^3 d^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(7\,i\,b^3 c d e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(15\,i\sqrt{2}\,a b c^2 d e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(2\,i\sqrt{2}\,b^4 e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(57\,\mathrm{i}\,a\,b^2\,c\,e^4\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\ \Bigg(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(21\,\mathrm{i}\,\sqrt{2}\,a^2\,c^2\,e^4\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\ \Bigg(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(8\,i\sqrt{2}\,c^4 d^3\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(12\,i\sqrt{2}\,b c^3 d^2 e\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(24\,\mathbb{i}\,\sqrt{2}\,a\,c^3\,d\,e^2\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(2\,\mathbb{i}\,\sqrt{2}\,b^3\,c\,e^3\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(12\,\mathbb{i}\,\sqrt{2}\,a\,b\,c^2\,e^3\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\Bigg)\Bigg)$$

- **Problem 2449: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\left(a+b\,x+c\,x^2\right)^{3/2}}{\sqrt{d+e\,x}}\,dx$$

Optimal (type 4, 579 leaves, 7 steps) :

$$\frac{2\sqrt{d+e x}\,\left(8 c^2 d^2+b^2 e^2-c e\,(11 b d-10 a e)-3 c e\,(2 c d-b e)\,x\right)\sqrt{a+b x+c x^2}}{35 c e^3}+\frac{2\sqrt{d+e x}\,\left(a+b x+c x^2\right)^{3/2}}{7 e}-$$

$$\left(2\sqrt{2}\,\sqrt{b^2-4 a c}\,(2 c d-b e)\left(4 c^2 d^2-b^2 e^2-4 c e\,(b d-2 a e)\right)\sqrt{d+e x}\,\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}\right]\right)\Big/\left(35 c^2 e^4\sqrt{\frac{c\,(d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}}\,\sqrt{a+b x+c x^2}\right)+$$

$$\left(2\sqrt{2}\,\sqrt{b^2-4 a c}\,\left(c d^2-b d e+a e^2\right)\left(16 c^2 d^2-b^2 e^2-4 c e\,(4 b d-5 a e)\right)\sqrt{\frac{c\,(d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}}\,\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}\right]\right)\Big/\left(35 c^2 e^4\sqrt{d+e x}\,\sqrt{a+b x+c x^2}\right)$$

Result (type 4, 5338 leaves) :

$$\frac{\sqrt{d+e x}\,\left(\frac{2\left(8 c^2 d^2-11 b c d e+b^2 e^2+15 a c e^2\right)}{35 c e^3}+\frac{4\,(-3 c d+4 b e)\,x}{35 e^2}+\frac{2 c x^2}{7 e}\right)\,(a+x\,(b+c x))^{3/2}}{a+b x+c x^2}+\frac{1}{35 c e^5\left(a+b x+c x^2\right)^{3/2}}$$

$$(a+x(b+cx))^{3/2}\left(-\left(4(2cd-be)\left(4c^2d^2-4bcde-b^2e^2+8ace^2\right)(d+ex)^{3/2}\left(c+\frac{cd^2}{(d+ex)^2}-\frac{bde}{(d+ex)^2}+\frac{ae^2}{(d+ex)^2}-\frac{2cd}{d+ex}+\frac{be}{d+ex}\right)\right)\Big/\right.$$

$$\left(c\sqrt{\frac{(d+ex)^2\left(c\left(-1+\frac{d}{d+ex}\right)^2+\frac{e\left(b-\frac{bd}{d+ex}+\frac{ae}{d+ex}\right)}{d+ex}\right)}{e^2}}\right)+$$

$$\frac{1}{c\sqrt{\frac{(d+ex)^2\left(c\left(-1+\frac{d}{d+ex}\right)^2+\frac{e\left(b-\frac{bd}{d+ex}+\frac{ae}{d+ex}\right)}{d+ex}\right)}{e^2}}}2\left(cd^2-bde+ae^2\right)(d+ex)\sqrt{c+\frac{cd^2}{(d+ex)^2}-\frac{bde}{(d+ex)^2}+\frac{ae^2}{(d+ex)^2}-\frac{2cd}{d+ex}+\frac{be}{d+ex}}$$

$$\left(\left(4\,i\sqrt{2}\,c^3d^3\left(2cd-be+\sqrt{b^2e^2-4ace^2}\right)\sqrt{1-\frac{2\left(cd^2-bde+ae^2\right)}{\left(2cd-be-\sqrt{b^2e^2-4ace^2}\right)(d+ex)}}\sqrt{1-\frac{2\left(cd^2-bde+ae^2\right)}{\left(2cd-be+\sqrt{b^2e^2-4ace^2}\right)(d+ex)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{cd^2-bde+ae^2}{2cd-be-\sqrt{b^2e^2-4ace^2}}}}{\sqrt{d+ex}}\right],\frac{2cd-be-\sqrt{b^2e^2-4ace^2}}{2cd-be+\sqrt{b^2e^2-4ace^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{cd^2-bde+ae^2}{2cd-be-\sqrt{b^2e^2-4ace^2}}}}{\sqrt{d+ex}}\right],\frac{2cd-be-\sqrt{b^2e^2-4ace^2}}{2cd-be+\sqrt{b^2e^2-4ace^2}}\right]\right)\right)\Big/$$

$$\left(\left(cd^2-bde+ae^2\right)\sqrt{-\frac{cd^2-bde+ae^2}{2cd-be-\sqrt{b^2e^2-4ace^2}}}\sqrt{c+\frac{cd^2-bde+ae^2}{(d+ex)^2}+\frac{-2cd+be}{d+ex}}\right)-$$

$$\left(6\, i \sqrt{2}\, b\, c^2 d^2 e \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(i \sqrt{2}\, b^2 c\, d\, e^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(8\, i \sqrt{2}\, a c^2 d e^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(i\, b^3 e^3 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2} \left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\left(4\,\mathrm{i}\,\sqrt{2}\,a\,b\,c\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(8\,\mathrm{i}\,\sqrt{2}\,c^3\,d^2\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(8\,\mathrm{i}\,\sqrt{2}\,b\,c^2\,d\,e\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(i\, b^2 c\, e^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(10\, i\sqrt{2}\, a\, c^2 e^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\Bigg)\Bigg)$$

- **Problem 2450: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+b\,x+c\,x^2\right)^{3/2}}{(d+e\,x)^{3/2}}\,dx$$

Optimal (type 4, 515 leaves, 7 steps) :

$$-\frac{2\sqrt{d+e x}\,(8 c d-7 b e-6 c e x)\sqrt{a+b x+c x^2}}{5 e^3}-\frac{2\left(a+b x+c x^2\right)^{3/2}}{e\sqrt{d+e x}}+$$

$$\left(\sqrt{2}\sqrt{b^2-4 a c}\left(16 c^2 d^2+b^2 e^2-4 c e (4 b d-3 a e)\right)\sqrt{d+e x}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}\right]\right)\Big/\left(5 c e^4\sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}}\sqrt{a+b x+c x^2}\right)-$$

$$\left(16\sqrt{2}\sqrt{b^2-4 a c}\,(2 c d-b e)\left(c d^2-b d e+a e^2\right)\sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}\right]\right)\Big/\left(5 c e^4\sqrt{d+e x}\sqrt{a+b x+c x^2}\right)$$

Result (type 4, 3427 leaves) :

$$\frac{\sqrt{d+e x}\,(a+x (b+c x))^{3/2}\left(\frac{2 (-3 c d+2 b e)}{5 e^3}+\frac{2 c x}{5 e^2}-\frac{2\left(c d^2-b d e+a e^2\right)}{e^3 (d+e x)}\right)}{a+b x+c x^2}+$$

$$\frac{1}{5 e^5 \left(a+b x+c x^2\right)^{3/2}} (a+x (b+c x))^{3/2} \left(\frac{2 \left(16 c^2 d^2-16 b c d e+b^2 e^2+12 a c e^2\right) (d+e x)^{3/2} \left(c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)}{c \sqrt{\frac{(d+e x)^2 \left(c \left(-1+\frac{d}{d+e x}\right)^2+\frac{e \left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}-\right.$$

$$\frac{1}{c \sqrt{\frac{(d+e x)^2 \left(c \left(-1+\frac{d}{d+e x}\right)^2+\frac{e \left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}} 2 \left(c d^2-b d e+a e^2\right) (d+e x) \sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\left(\left(4 i \sqrt{2} c^2 d^2 \left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}} \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\left(c d^2-b d e+a e^2\right) \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(4 i \sqrt{2} b c d e \left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(\mathbb{i}\, b^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(3\,\mathbb{i}\sqrt{2}\, a c e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\text{i} \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\text{i} \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(8\,\text{i}\sqrt{2}\,c^2 d\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[\text{i} \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(4\,\text{i}\sqrt{2}\,b c e\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[\text{i} \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\Bigg)\Bigg)$$

- **Problem 2451: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\left(a+b\,x+c\,x^2\right)^{3/2}}{(d+e\,x)^{5/2}}\,dx$$

Optimal (type 4, 499 leaves, 7 steps) :

$$\frac{2\,(8\,c\,d-3\,b\,e+2\,c\,e\,x)\,\sqrt{a+b\,x+c\,x^2}}{3\,e^3\,\sqrt{d+e\,x}}-\frac{2\,\left(a+b\,x+c\,x^2\right)^{3/2}}{3\,e\,(d+e\,x)^{3/2}}-$$

$$\left(8\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\,(2\,c\,d-b\,e)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}\,+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/$$

$$\left(3\,e^4\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\,\left(16\,c^2\,d^2+3\,b^2\,e^2-4\,c\,e\,(4\,b\,d-a\,e)\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}\,+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/\left(3\,c\,e^4\,\sqrt{d+e\,x}\,\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 5751 leaves) :

$$\frac{\sqrt{d+e\,x}\,\,(a+x\,(b+c\,x))^{3/2}\left(\frac{2\,c}{3\,e^3}-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{3\,e^3\,(d+e\,x)^2}+\frac{8\,(2\,c\,d-b\,e)}{3\,e^3\,(d+e\,x)}\right)}{a+b\,x+c\,x^2}-$$

$$\frac{1}{3\,e^5\,\left(a+b\,x+c\,x^2\right)^{3/2}}\,2\,(a+x\,(b+c\,x))^{3/2}\left(\frac{8\,(2\,c\,d-b\,e)\,\,(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)}{\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}-\right.$$

$$\left(4\,i\,\sqrt{2}\,c^2\,d^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)+$$

$$\left(6\,i\,\sqrt{2}\,b\,c\,d^2\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)-$$

$$\left(2\,i\,\sqrt{2}\,b^2\,d\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\ \sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)-$$

$$\left(4\,\mathbb{i}\,\sqrt{2}\,a\,c\,d\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\ \sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\ \sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)+$$

$$\left(2\,\mathbb{i}\,\sqrt{2}\,a\,b\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\ \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)-$$

$$\left(8\,i\sqrt{2}\,c^2 d^2 (d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\ \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)+$$

$$\left(8\,\mathrm{i}\,\sqrt{2}\,b\,c\,d\,e\,(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\;\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)-$$

$$\left(3\,\mathrm{i}\,b^2\,e^2\,(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\;\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)-$$

$$\left(2\,\mathrm{i}\,\sqrt{2}\,a\,c\,e^2\,(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\ \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)\Bigg)$$

- **Problem 2452: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+b x+c x^2\right)^{3/2}}{(d+e x)^{7/2}}\, dx$$

Optimal (type 4, 578 leaves, 7 steps) :

$$-\frac{2\left(8c^2d^3+abe^3-cde\,(7bd-4ae)+e\left(10c^2d^2+b^2e^2-2ce\,(5bd-3ae)\right)x\right)\sqrt{a+bx+cx^2}}{5e^3\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}}-$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}}{5e\,(d+ex)^{5/2}}+\Bigg(\sqrt{2}\sqrt{b^2-4ac}\,\left(16c^2d^2+b^2e^2-4ce\,(4bd-3ae)\right)$$

$$\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right]\Bigg)\Bigg/$$

$$\left(5e^4\left(cd^2-bde+ae^2\right)\sqrt{\frac{c\,(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}\right)-\Bigg(16\sqrt{2}\sqrt{b^2-4ac}\,(2cd-be)\sqrt{\frac{c\,(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

$$\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right]\Bigg)\Bigg/\left(5e^4\sqrt{d+ex}\sqrt{a+bx+cx^2}\right)$$

Result (type 4, 3506 leaves) :

$$\frac{\sqrt{d+ex}\,\left(a+x\,(b+cx)\right)^{3/2}\left(-\frac{2\left(cd^2-bde+ae^2\right)}{5e^3\,(d+ex)^3}+\frac{4\,(2cd-be)}{5e^3\,(d+ex)^2}-\frac{2\left(11c^2d^2-11bcde+b^2e^2+7ace^2\right)}{5e^3\left(cd^2-bde+ae^2\right)(d+ex)}\right)}{a+bx+cx^2}-\frac{1}{5e^5\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$2 c (a + x (b + c x))^{3/2} \left(\frac{\left(-16 c^2 d^2 + 16 b c d e - b^2 e^2 - 12 a c e^2\right) (d + e x)^{3/2} \left(c + \frac{c d^2}{(d+e x)^2} - \frac{b d e}{(d+e x)^2} + \frac{a e^2}{(d+e x)^2} - \frac{2 c d}{d+e x} + \frac{b e}{d+e x}\right)}{c \sqrt{\frac{(d+e x)^2 \left(c \left(-1+\frac{d}{d+e x}\right)^2 + \frac{e \left(b - \frac{b d}{d+e x} + \frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}} + \right.$$

$$\frac{1}{c \sqrt{\frac{(d+e x)^2 \left(c \left(-1+\frac{d}{d+e x}\right)^2 + \frac{e \left(b - \frac{b d}{d+e x} + \frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}} \left(c d^2 - b d e + a e^2\right) (d + e x) \sqrt{c + \frac{c d^2}{(d + e x)^2} - \frac{b d e}{(d + e x)^2} + \frac{a e^2}{(d + e x)^2} - \frac{2 c d}{d + e x} + \frac{b e}{d + e x}}$$

$$\left(\left(4 i \sqrt{2}\, c^2 d^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \right. \right.$$

$$\left(\text{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}} \right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}} \right] - \right.$$

$$\left. \left. \text{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}} \right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}} \right] \right) \right) \Big/$$

$$\left(\left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}} \right) -$$

$$\left(4 i \sqrt{2}\, b c d e \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i b^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(3 i \sqrt{2}\, a c e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(8 i \sqrt{2} c^2 d\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(4 i \sqrt{2} b c e\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left.\left.\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)\right)$$

- **Problem 2453: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\left(a+b\,x+c\,x^2\right)^{3/2}}{(d+e\,x)^{9/2}}\,dx$$

Optimal (type 4, 721 leaves, 8 steps) :

$$\frac{4\,(2\,c\,d-b\,e)\,\left(4\,c^2\,d^2-b^2\,e^2-4\,c\,e\,(b\,d-2\,a\,e)\right)\sqrt{a+b\,x+c\,x^2}}{35\,e^3\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\sqrt{d+e\,x}}-$$

$$\left(2\,\left(8\,c^2\,d^3-c\,d\,e\,(5\,b\,d-4\,a\,e)-b\,e^2\,(2\,b\,d-3\,a\,e)+e\,\left(14\,c^2\,d^2+b^2\,e^2-2\,c\,e\,(7\,b\,d-5\,a\,e)\right)x\right)\sqrt{a+b\,x+c\,x^2}\right)\Big/$$

$$\left(35\,e^3\,\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)^{5/2}\right)-\frac{2\,\left(a+b\,x+c\,x^2\right)^{3/2}}{7\,e\,(d+e\,x)^{7/2}}-$$

$$\left(2\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,(2\,c\,d-b\,e)\,\left(4\,c^2\,d^2-b^2\,e^2-4\,c\,e\,(b\,d-2\,a\,e)\right)\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\right.\right.$$

$$\left.\left.-\frac{2\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\right]\right)\Big/\left(35\,e^4\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}}\,\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\left(2\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(16\,c^2\,d^2-b^2\,e^2-4\,c\,e\,(4\,b\,d-5\,a\,e)\right)\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\right]\right)\Big/\left(35\,e^4\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 5469 leaves) :

$$\frac{1}{a+b\,x+c\,x^2}\sqrt{d+e\,x}\,(a+x\,(b+c\,x))^{3/2}$$

$$\left(-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{7\,e^3\,(d+e\,x)^4}+\frac{16\,(2\,c\,d-b\,e)}{35\,e^3\,(d+e\,x)^3}-\frac{2\left(19\,c^2\,d^2-19\,b\,c\,d\,e+b^2\,e^2+15\,a\,c\,e^2\right)}{35\,e^3\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)^2}+\frac{4\,(-2\,c\,d+b\,e)\left(-4\,c^2\,d^2+4\,b\,c\,d\,e+b^2\,e^2-8\,a\,c\,e^2\right)}{35\,e^3\left(c\,d^2-b\,d\,e+a\,e^2\right)^2(d+e\,x)}\right)+$$

$$\frac{1}{35\,e^5\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\left(a+b\,x+c\,x^2\right)^{3/2}}\,2\,c\,(a+x\,(b+c\,x))^{3/2}$$

$$\left(-\left(2\,(2\,c\,d-b\,e)\left(4\,c^2\,d^2-4\,b\,c\,d\,e-b^2\,e^2+8\,a\,c\,e^2\right)(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)\right)\Big/\right.$$

$$\left(c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)+$$

$$\frac{1}{c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\left(\left(4\,i\,\sqrt{2}\,c^3\,d^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(6\,\mathbb{i}\sqrt{2}\ b c^2 d^2 e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(\mathbb{i}\sqrt{2}\ b^2 c d e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(8\,\text{i}\,\sqrt{2}\,a\,c^2\,d\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\Bigg(\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(\text{i}\,b^3\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(4\,i\sqrt{2}\,a b c e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(8\,i\sqrt{2}\,c^3 d^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(8\,\mathrm{i}\,\sqrt{2}\ b\,c^2\,d\,e\ \sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\ \sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\ \sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(\mathrm{i}\,b^2\,c\,e^2\ \sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\ \sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\ \sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\ \sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(10\,\mathrm{i}\,\sqrt{2}\ a\,c^2\,e^2\ \sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\ \sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)\Bigg)$$

- **Problem 2454: Result unnecessarily involves imaginary or complex numbers.**

$$\int\sqrt{d x}\,\left(a+b x+c x^2\right)^{5/2}\,dx$$

Optimal (type 4, 616 leaves, 9 steps):

$$-\frac{4\left(24 b^6-268 a b^4 c+951 a^2 b^2 c^2-924 a^3 c^3\right) d x\sqrt{a+b x+c x^2}}{9009 c^{7/2}\sqrt{d x}\left(\sqrt{a}+\sqrt{c}\,x\right)}+$$

$$\frac{2\sqrt{d x}\left(b\left(24 b^4-151 a b^2 c+108 a^2 c^2\right)+3 c\left(24 b^4-181 a b^2 c+308 a^2 c^2\right) x\right)\sqrt{a+b x+c x^2}}{9009 c^3}-$$

$$\frac{10\sqrt{d x}\left(3 b\left(6 b^2-19 a c\right)+14 c\left(3 b^2-11 a c\right) x\right)\left(a+b x+c x^2\right)^{3/2}}{9009 c^2}+\frac{10 b\sqrt{d x}\left(a+b x+c x^2\right)^{5/2}}{143 c}+\frac{2 (d x)^{3/2}\left(a+b x+c x^2\right)^{5/2}}{13 d}+$$

$$\left(4 a^{1/4}\left(24 b^6-268 a b^4 c+951 a^2 b^2 c^2-924 a^3 c^3\right) d\sqrt{x}\left(\sqrt{a}+\sqrt{c}\,x\right)\sqrt{\frac{a+b x+c x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Bigg/$$

$$\left(9009 c^{15/4}\sqrt{d x}\sqrt{a+b x+c x^2}\right)-\Bigg(a^{1/4}\left(\sqrt{a}\,b\sqrt{c}\left(24 b^4-241 a b^2 c+708 a^2 c^2\right)+2\left(24 b^6-268 a b^4 c+951 a^2 b^2 c^2-924 a^3 c^3\right)\right) d$$

$$\sqrt{x}\left(\sqrt{a}+\sqrt{c}\,x\right)\sqrt{\frac{a+b x+c x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\Bigg)\Bigg/\left(9009 c^{15/4}\sqrt{d x}\sqrt{a+b x+c x^2}\right)$$

Result (type 4, 708 leaves):

$$\frac{1}{9009\,c^4\sqrt{x}\,\sqrt{a+x\,(b+c\,x)}}$$

$$\sqrt{d\,x}\left(-\frac{4\,(24\,b^6-268\,a\,b^4\,c+951\,a^2\,b^2\,c^2-924\,a^3\,c^3)\,(a+x\,(b+c\,x))}{\sqrt{x}}+2\,c\,\sqrt{x}\,(a+x\,(b+c\,x))\,\left(24\,b^5-18\,b^4\,c\,x+b^3\,c\,(-241\,a+15\,c\,x^2)+\right.\right.$$

$$\left.3\,b^2\,c^2\,x\,(54\,a+371\,c\,x^2)+77\,c^3\,x\,(31\,a^2+28\,a\,c\,x^2+9\,c^2\,x^4)+b\,c^2\,(708\,a^2+3071\,a\,c\,x^2+1701\,c^2\,x^4)\right)+$$

$$\frac{1}{\sqrt{\frac{a}{b+\sqrt{b^2-4\,a\,c}}}}\,i\,(24\,b^6-268\,a\,b^4\,c+951\,a^2\,b^2\,c^2-924\,a^3\,c^3)\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\sqrt{2+\frac{4\,a}{\left(b+\sqrt{b^2-4\,a\,c}\right)x}}\;x$$

$$\sqrt{\frac{2\,a+b\,x-\sqrt{b^2-4\,a\,c}\;x}{b\,x-\sqrt{b^2-4\,a\,c}\;x}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{\frac{a}{b+\sqrt{b^2-4\,a\,c}}}}{\sqrt{x}}\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+\frac{1}{\sqrt{\frac{a}{b+\sqrt{b^2-4\,a\,c}}}}$$

$$i\left(24\,b^7-292\,a\,b^5\,c+1192\,a^2\,b^3\,c^2-1632\,a^3\,b\,c^3-24\,b^6\sqrt{b^2-4\,a\,c}+268\,a\,b^4\,c\,\sqrt{b^2-4\,a\,c}-951\,a^2\,b^2\,c^2\,\sqrt{b^2-4\,a\,c}+924\,a^3\,c^3\,\sqrt{b^2-4\,a\,c}\right)$$

$$\left.\sqrt{2+\frac{4\,a}{\left(b+\sqrt{b^2-4\,a\,c}\right)x}}\;x\,\sqrt{\frac{2\,a+b\,x-\sqrt{b^2-4\,a\,c}\;x}{b\,x-\sqrt{b^2-4\,a\,c}\;x}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{\frac{a}{b+\sqrt{b^2-4\,a\,c}}}}{\sqrt{x}}\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

- **Problem 2455: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{(a+b\,x+c\,x^2)^{5/2}}{\sqrt{d+e\,x}}\,dx$$

Optimal (type 4, 847 leaves, 8 steps) :

$$\frac{1}{693\, c^2\, e^5} 2 \sqrt{d+e\,x}\, \left(128\, c^4\, d^4 - 4\, b^4\, e^4 - 4\, c^3\, d^2\, e\, (76\, b\, d - 69\, a\, e) - b^2\, c\, e^3\, (7\, b\, d - 27\, a\, e) + 3\, c^2\, e^2\, \left(65\, b^2\, d^2 - 124\, a\, b\, d\, e + 60\, a^2\, e^2\right) - 12\, c\, e\, (2\, c\, d - b\, e)\, \left(4\, c^2\, d^2 - b^2\, e^2 - 4\, c\, e\, (b\, d - 2\, a\, e)\right)\, x\right) \sqrt{a+b\,x+c\,x^2} + \frac{10 \sqrt{d+e\,x}\, \left(16\, c^2\, d^2 + 3\, b^2\, e^2 - c\, e\, (23\, b\, d - 18\, a\, e) - 7\, c\, e\, (2\, c\, d - b\, e)\, x\right)\, \left(a+b\,x+c\,x^2\right)^{3/2}}{693\, c\, e^3} + \frac{2 \sqrt{d+e\,x}\, \left(a+b\,x+c\,x^2\right)^{5/2}}{11\, e} -$$

$$\left(\sqrt{2}\, \sqrt{b^2-4\,a\,c}\, (2\, c\, d - b\, e)\, \left(128\, c^4\, d^4 + 8\, b^4\, e^4 + b^2\, c\, e^3\, (29\, b\, d - 93\, a\, e) - 4\, c^3\, d^2\, e\, (64\, b\, d - 93\, a\, e) + 3\, c^2\, e^2\, \left(33\, b^2\, d^2 - 124\, a\, b\, d\, e + 124\, a^2\, e^2\right)\right) \right.$$

$$\left. \sqrt{d+e\,x}\, \sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right], -\frac{2\sqrt{b^2-4\,a\,c}\, e}{2\, c\, d - \left(b+\sqrt{b^2-4\,a\,c}\right) e}\right]\right) \Big/$$

$$\left(693\, c^3\, e^6\, \sqrt{\frac{c\,(d+e\,x)}{2\, c\, d - \left(b+\sqrt{b^2-4\,a\,c}\right) e}}\, \sqrt{a+b\,x+c\,x^2}\right) + \left(4\sqrt{2}\, \sqrt{b^2-4\,a\,c}\, \left(c\, d^2 - b\, d\, e + a\, e^2\right) \right.$$

$$\left(128\, c^4\, d^4 + 2\, b^4\, e^4 - 4\, c^3\, d^2\, e\, (64\, b\, d - 69\, a\, e) + b^2\, c\, e^3\, (5\, b\, d - 21\, a\, e) + 3\, c^2\, e^2\, \left(41\, b^2\, d^2 - 92\, a\, b\, d\, e + 60\, a^2\, e^2\right)\right) \sqrt{\frac{c\,(d+e\,x)}{2\, c\, d - \left(b+\sqrt{b^2-4\,a\,c}\right) e}}$$

$$\left. \sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right], -\frac{2\sqrt{b^2-4\,a\,c}\, e}{2\, c\, d - \left(b+\sqrt{b^2-4\,a\,c}\right) e}\right]\right) \Big/ \left(693\, c^3\, e^6\, \sqrt{d+e\,x}\, \sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 10 879 leaves) :

$$\frac{1}{\left(a+b x+c x^2\right)^2}$$

$$\sqrt{d+e x}\left(1\Big/\left(693 c^2 e^5\right) 2\left(128 c^4 d^4-304 b c^3 d^3 e+195 b^2 c^2 d^2 e^2+356 a c^3 d^2 e^2-7 b^3 c d e^3-487 a b c^2 d e^3-4 b^4 e^4+42 a b^2 c e^4+333 a^2 c^2 e^4\right)+\right.$$

$$\frac{2\left(-96 c^3 d^3+224 b c^2 d^2 e-139 b^2 c d e^2-262 a c^2 d e^2+3 b^3 e^3+347 a b c e^3\right) x}{693 c e^4}+\frac{2\left(80 c^2 d^2-185 b c d e+113 b^2 e^2+216 a c e^2\right) x^2}{693 e^3}+$$

$$\left.\frac{2 c(-10 c d+23 b e) x^3}{99 e^2}+\frac{2 c^2 x^4}{11 e}\right)(a+x(b+c x))^{5/2}-\frac{1}{693 c^2 e^7\left(a+b x+c x^2\right)^{5/2}} 2(a+x(b+c x))^{5/2}$$

$$\left(\left((2 c d-b e)\left(128 c^4 d^4-256 b c^3 d^3 e+99 b^2 c^2 d^2 e^2+372 a c^3 d^2 e^2+29 b^3 c d e^3-372 a b c^2 d e^3+8 b^4 e^4-93 a b^2 c e^4+372 a^2 c^2 e^4\right)\right.\right.$$

$$\left.(d+e x)^{3/2}\left(c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)\right)\Bigg/\left(c\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)-$$

$$\frac{1}{c\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}\left(c d^2-b d e+a e^2\right)(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\left(\left(64\, i \sqrt{2}\, c^5 d^5\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(160\,i\sqrt{2}\,b c^4 d^4 e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(227\,i\,b^2 c^3 d^3 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(186\,\mathbb{i}\sqrt{2}\,a c^4 d^3 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(41\,\mathbb{i}\,b^3 c^2 d^2 e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(279\,\mathbb{i}\sqrt{2}\,a b c^3 d^2 e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(13\,\mathbb{i}\,b^4 c d e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(2 \sqrt{2} \left(c d^2-b d e+a e^2\right) \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right) +$$

$$\left(93\, i\, a b^2 c^2 d e^4 \left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2} \left(c d^2-b d e+a e^2\right) \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right) +$$

$$\left(186\, i \sqrt{2}\, a^2 c^3 d e^4 \left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(2\,\mathbb{i}\sqrt{2}\,b^5 e^5\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(93\,\mathbb{i}\,a b^3 c e^5\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(93\,i\sqrt{2}\,a^2 b c^2 e^5\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(128\,i\sqrt{2}\,c^5 d^4\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(256\,\mathbb{i}\sqrt{2}\,b c^4 d^3 e\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(123\,\mathbb{i}\sqrt{2}\,b^2 c^3 d^2 e^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(276\, i \sqrt{2}\, a\, c^4 d^2 e^2 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(5\, i \sqrt{2}\, b^3 c^2 d\, e^3 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(276\, i \sqrt{2}\, a\, b\, c^3 d\, e^3 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\Bigg(2 i \sqrt{2}\, b^4 c e^4 \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}$$

$$\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\Bigg) \Big/$$

$$\left(\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\Bigg(21 i \sqrt{2}\, a b^2 c^2 e^4 \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}$$

$$\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\Bigg) \Big/$$

$$\left(\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\Bigg(180 i \sqrt{2}\, a^2 c^3 e^4 \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\Bigg)\Bigg)$$

- **Problem 2456: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\left(a+b\,x+c\,x^2\right)^{5/2}}{(d+e\,x)^{3/2}}\,dx$$

Optimal (type 4, 716 leaves, 8 steps) :

$$-\frac{1}{63\,c\,e^5}\left(\frac{2\sqrt{d+e\,x}\,\left(128\,c^3\,d^3-b^3\,e^3+3\,b\,c\,e^2\,(37\,b\,d-36\,a\,e)-12\,c^2\,d\,e\,(20\,b\,d-11\,a\,e)-3\,c\,e\,\left(32\,c^2\,d^2+b^2\,e^2-4\,c\,e\,(8\,b\,d-7\,a\,e)\right)\,x\right)\sqrt{a+b\,x+c\,x^2}-10\sqrt{d+e\,x}\,(16\,c\,d-15\,b\,e-14\,c\,e\,x)\,\left(a+b\,x+c\,x^2\right)^{3/2}}{63\,e^3}-\frac{2\left(a+b\,x+c\,x^2\right)^{5/2}}{e\sqrt{d+e\,x}}+\right.$$

$$\left(2\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(128\,c^4\,d^4-b^4\,e^4-4\,c^3\,d^2\,e\,(64\,b\,d-57\,a\,e)-b^2\,c\,e^3\,(7\,b\,d-15\,a\,e)+3\,c^2\,e^2\,\left(45\,b^2\,d^2-76\,a\,b\,d\,e+28\,a^2\,e^2\right)\right)\right.$$

$$\left.\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\sqrt{b^2-4\,a\,c}\;e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/$$

$$\left(63\,c^2\,e^6\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)-$$

$$\left(2\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,(2\,c\,d-b\,e)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\left(128\,c^2\,d^2-b^2\,e^2-4\,c\,e\,(32\,b\,d-33\,a\,e)\right)\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\right.$$

$$\left.\left.\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\sqrt{b^2-4\,a\,c}\;e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/\left(63\,c^2\,e^6\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)\right)$$

Result (type 4, 7946 leaves) :

$$\frac{1}{\left(a+b x+c x^2\right)^2}\sqrt{d+e x}\,(a+x\,(b+c x))^{5/2}$$

$$\left(\frac{2\left(-65 c^3 d^3+114 b c^2 d^2 e-48 b^2 c d e^2-86 a c^2 d e^2+b^3 e^3+57 a b c e^3\right)}{63 c e^5}+\frac{2\left(33 c^2 d^2-50 b c d e+15 b^2 e^2+28 a c e^2\right) x}{63 e^4}+\right.$$

$$\left.\frac{2 c\,(-17 c d+19 b e)\,x^2}{63 e^3}+\frac{2 c^2 x^3}{9 e^2}-\frac{2\left(c d^2-b d e+a e^2\right)^2}{e^5\,(d+e x)}\right)-\frac{1}{63 c e^7\left(a+b x+c x^2\right)^{5/2}}\,2\,(a+x\,(b+c x))^{5/2}$$

$$\left(-\left(2\left(128 c^4 d^4-256 b c^3 d^3 e+135 b^2 c^2 d^2 e^2+228 a c^3 d^2 e^2-7 b^3 c d e^3-228 a b c^2 d e^3-b^4 e^4+15 a b^2 c e^4+84 a^2 c^2 e^4\right)(d+e x)^{3/2}\right.\right.$$

$$\left.\left.\left(c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)\right)\middle/\left(c\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)+\right.$$

$$\frac{1}{c\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}\left(c d^2-b d e+a e^2\right)(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\left(\left(64\, i \sqrt{2}\, c^4 d^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(128\,i\sqrt{2}\,b c^3 d^3 e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(135\,i\,b^2 c^2 d^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(114\,\mathbb{i}\,\sqrt{2}\,a c^3 d^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(7\,\mathbb{i}\,b^3 c d e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg) \Bigg/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(114\, i \sqrt{2}\, a b c^2 d e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg) \Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(i\, b^4 e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(15\,i\,a\,b^2\,c\,e^4\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(42\,i\,\sqrt{2}\,a^2\,c^2\,e^4\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(128\,\mathbb{i}\,\sqrt{2}\,c^4\,d^3\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\ \sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(192\,\mathbb{i}\,\sqrt{2}\,b\,c^3\,d^2\,e\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\ \sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(63\, i \sqrt{2}\, b^2 c^2 d e^2 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(132\, i \sqrt{2}\, a c^3 d e^2 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i\, b^3 c e^3 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(66\,\mathrm{i}\sqrt{2}\,a\,b\,c^2\,e^3\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left.\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)$$

- **Problem 2457: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\left(a+b\,x+c\,x^2\right)^{5/2}}{(d+e\,x)^{5/2}}\,dx$$

Optimal (type 4, 622 leaves, 8 steps) :

$$\frac{2\sqrt{d+e x}\left(128 c^2 d^2+51 b^2 e^2-4 c e (44 b d-5 a e)-48 c e (2 c d-b e) x\right)\sqrt{a+b x+c x^2}}{21 e^5}+\frac{10 (16 c d-7 b e+2 c e x)\left(a+b x+c x^2\right)^{3/2}}{21 e^3\sqrt{d+e x}}-$$

$$\frac{2\left(a+b x+c x^2\right)^{5/2}}{3 e (d+e x)^{3/2}}-\left(\sqrt{2}\sqrt{b^2-4 a c}\,(2 c d-b e)\left(128 c^2 d^2+3 b^2 e^2-4 c e (32 b d-29 a e)\right)\sqrt{d+e x}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}\right]\right)\Bigg/\left(21 c e^6\sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}}\sqrt{a+b x+c x^2}\right)+$$

$$\left(4\sqrt{2}\sqrt{b^2-4 a c}\left(c d^2-b d e+a e^2\right)\left(128 c^2 d^2+27 b^2 e^2-4 c e (32 b d-5 a e)\right)\sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}\right]\right)\Bigg/\left(21 c e^6\sqrt{d+e x}\sqrt{a+b x+c x^2}\right)$$

Result (type 4, 5407 leaves) :

$$\frac{1}{\left(a+b x+c x^2\right)^2}\sqrt{d+e x}\,(a+x (b+c x))^{5/2}$$

$$\left(\frac{2\left(37 c^2 d^2-43 b c d e+9 b^2 e^2+16 a c e^2\right)}{21 e^5}+\frac{2 c (-4 c d+3 b e) x}{7 e^4}+\frac{2 c^2 x^2}{7 e^3}-\frac{2\left(c d^2-b d e+a e^2\right)^2}{3 e^5 (d+e x)^2}-\frac{14 (-2 c d+b e)\left(c d^2-b d e+a e^2\right)}{3 e^5 (d+e x)}\right)-$$

$$\frac{1}{21 e^7\left(a+b x+c x^2\right)^{5/2}}2\,(a+x (b+c x))^{5/2}$$

$$\left(\left((2 c d-b e) \left(128 c^2 d^2-128 b c d e+3 b^2 e^2+116 a c e^2\right) (d+e x)^{3/2} \left(c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)\right)\Big/\right.$$

$$\left(c \sqrt{\frac{(d+e x)^2 \left(c \left(-1+\frac{d}{d+e x}\right)^2+\frac{e \left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)-$$

$$\frac{1}{c \sqrt{\frac{(d+e x)^2 \left(c \left(-1+\frac{d}{d+e x}\right)^2+\frac{e \left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}} \left(c d^2-b d e+a e^2\right) (d+e x) \sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\left(\left(64 i \sqrt{2}\, c^3 d^3 \left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}\right.\right.$$

$$\sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}} \left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right) \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(96\,i\,\sqrt{2}\,b\,c^2\,d^2\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(67\,i\,b^2\,c\,d\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(58\,i\,\sqrt{2}\,a\,c^2\,d\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(3\,i\,b^3\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(29\, i\, \sqrt{2}\, a\, b\, c\, e^3 \left(2\, c\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}\right) \sqrt{1 - \frac{2\, \left(c\, d^2 - b\, d\, e + a\, e^2\right)}{\left(2\, c\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}\right)\, (d + e\, x)}}\right.$$

$$\sqrt{1 - \frac{2\, \left(c\, d^2 - b\, d\, e + a\, e^2\right)}{\left(2\, c\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}\right)\, (d + e\, x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\, \sqrt{-\frac{c\, d^2 - b\, d\, e + a\, e^2}{2\, c\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}}}{\sqrt{d + e\, x}}\right], \frac{2\, c\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}{2\, c\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\, \sqrt{-\frac{c\, d^2 - b\, d\, e + a\, e^2}{2\, c\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}}}{\sqrt{d + e\, x}}\right], \frac{2\, c\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}{2\, c\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}\right]\right)\right) \Big/$$

$$\left(\left(c\, d^2 - b\, d\, e + a\, e^2\right) \sqrt{-\frac{c\, d^2 - b\, d\, e + a\, e^2}{2\, c\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}}\, \sqrt{c + \frac{c\, d^2 - b\, d\, e + a\, e^2}{(d + e\, x)^2} + \frac{-2\, c\, d + b\, e}{d + e\, x}}\right) +$$

$$\left(128\, i\, \sqrt{2}\, c^3 d^2 \sqrt{1 - \frac{2\, \left(c\, d^2 - b\, d\, e + a\, e^2\right)}{\left(2\, c\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}\right)\, (d + e\, x)}}\, \sqrt{1 - \frac{2\, \left(c\, d^2 - b\, d\, e + a\, e^2\right)}{\left(2\, c\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}\right)\, (d + e\, x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\, \sqrt{-\frac{c\, d^2 - b\, d\, e + a\, e^2}{2\, c\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}}}{\sqrt{d + e\, x}}\right], \frac{2\, c\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}{2\, c\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c\, d^2 - b\, d\, e + a\, e^2}{2\, c\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}}\, \sqrt{c + \frac{c\, d^2 - b\, d\, e + a\, e^2}{(d + e\, x)^2} + \frac{-2\, c\, d + b\, e}{d + e\, x}}\right) -$$

$$\left(128\, i\, \sqrt{2}\, b\, c^2 d\, e \sqrt{1 - \frac{2\, \left(c\, d^2 - b\, d\, e + a\, e^2\right)}{\left(2\, c\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}\right)\, (d + e\, x)}}\, \sqrt{1 - \frac{2\, \left(c\, d^2 - b\, d\, e + a\, e^2\right)}{\left(2\, c\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}\right)\, (d + e\, x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(27\,i\sqrt{2}\,b^2 c e^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(20\,i\sqrt{2}\,a c^2 e^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\Bigg)\Bigg)$$

- **Problem 2458: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+b\,x+c\,x^2\right)^{5/2}}{(d+e\,x)^{7/2}}\,dx$$

Optimal (type 4, 603 leaves, 8 steps) :

$$-\frac{2\left(128 c^2 d^2+15 b^2 e^2-4 c e (28 b d-9 a e)+16 c e (2 c d-b e) x\right)\sqrt{a+b x+c x^2}}{15 e^5\sqrt{d+e x}}+\frac{2 (16 c d-5 b e+6 c e x)\left(a+b x+c x^2\right)^{3/2}}{15 e^3 (d+e x)^{3/2}}-$$

$$\frac{2\left(a+b x+c x^2\right)^{5/2}}{5 e (d+e x)^{5/2}}+\left(2\sqrt{2}\sqrt{b^2-4 a c}\left(128 c^2 d^2+23 b^2 e^2-4 c e (32 b d-9 a e)\right)\sqrt{d+e x}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Bigg/\left(15 e^6\sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\sqrt{a+b x+c x^2}\right)-$$

$$\left(2\sqrt{2}\sqrt{b^2-4 a c}\,(2 c d-b e)\left(128 c^2 d^2+15 b^2 e^2-4 c e (32 b d-17 a e)\right)\sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Bigg/\left(15 c e^6\sqrt{d+e x}\sqrt{a+b x+c x^2}\right)$$

Result (type 4, 8961 leaves) :

$$\frac{1}{\left(a+b x+c x^2\right)^2}\sqrt{d+e x}\,(a+x (b+c x))^{5/2}$$

$$\left(\frac{2 c (-19 c d+11 b e)}{15 e^5}+\frac{2 c^2 x}{5 e^4}-\frac{2\left(c d^2-b d e+a e^2\right)^2}{5 e^5 (d+e x)^3}-\frac{22 (-2 c d+b e)\left(c d^2-b d e+a e^2\right)}{15 e^5 (d+e x)^2}-\frac{2\left(128 c^2 d^2-128 b c d e+23 b^2 e^2+36 a c e^2\right)}{15 e^5 (d+e x)}\right)-$$

$$\frac{1}{15 e^7\left(a+b x+c x^2\right)^{5/2}}\, 2\, (a+x (b+c x))^{5/2}$$

$$-\left(2\left(128\,c^2 d^2-128\,b\,c\,d\,e+23\,b^2 e^2+36\,a\,c\,e^2\right)(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)\right)\Big/$$

$$\left(\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)+\Bigg(64\,\mathrm{i}\,\sqrt{2}\,c^3 d^4\left(2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)(d+e\,x)$$

$$\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}\;\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\;\Bigg(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\;\frac{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\;\frac{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\;\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)-$$

$$\Bigg(128\,\mathrm{i}\,\sqrt{2}\,b\,c^2 d^3 e\left(2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\ \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)+$$

$$\left(151\,\mathbb{i}\,b^2 c d^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/\Bigg(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)$$

$$\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\ \sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\ \Bigg)+$$

$$\Bigg(82\,i\,\sqrt{2}\,a\,c^2\,d^2\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\ \sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\Bigg(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\ \sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\ \Bigg)-$$

$$\left(23\, i\, b^3 d\, e^3 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x) \sqrt{c + \frac{c d^2}{(d+e x)^2} - \frac{b d e}{(d+e x)^2} + \frac{a e^2}{(d+e x)^2} - \frac{2 c d}{d+e x} + \frac{b e}{d+e x}}\right.$$

$$\sqrt{1 - \frac{2\left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right)(d + e x)}}\ \sqrt{1 - \frac{2\left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right)(d + e x)}}$$

$$\left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/ \left(\sqrt{2}\left(c d^2 - b d e + a e^2\right)\right.$$

$$\left.\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}\ \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\ \sqrt{\frac{(d + e x)^2 \left(c \left(-1 + \frac{d}{d+e x}\right)^2 + \frac{e\left(b - \frac{b d}{d+e x} + \frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right) -$$

$$\left(82\, i \sqrt{2}\, a b c d e^3 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x) \sqrt{c + \frac{c d^2}{(d+e x)^2} - \frac{b d e}{(d+e x)^2} + \frac{a e^2}{(d+e x)^2} - \frac{2 c d}{d+e x} + \frac{b e}{d+e x}}\right.$$

$$\sqrt{1 - \frac{2\left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right)(d + e x)}}\ \sqrt{1 - \frac{2\left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right)(d + e x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\ \sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)+$$

$$\left(23\,i\,a\,b^2\,e^4\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\ \sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/\left(\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\right.$$

$$\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\;\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\Bigg)+$$

$$\Bigg(18\,i\,\sqrt{2}\,a^2\,c\,e^4\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\;\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\;\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)+$$

$$\Bigg(128\,i\,\sqrt{2}\,c^3\,d^3\,(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}\;\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\ \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)-$$

$$\left(192\, i\sqrt{2}\, b c^2 d^2 e\,(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\ \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)+$$

$$\left(79\, i\sqrt{2}\, b^2 c d e^2\,(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)+$$

$$\left(68\,i\,\sqrt{2}\,a\,c^2\,d\,e^2\,(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)-$$

$$\left(15\,i\,b^3\,e^3\,(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)-$$

$$\left(34\, i \sqrt{2}\, a\, b\, c\, e^3\, (d+e\,x) \sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}} \sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\ \sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)$$

- **Problem 2459: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+b\,x+c\,x^2\right)^{5/2}}{(d+e\,x)^{9/2}}\,dx$$

Optimal (type 4, 731 leaves, 8 steps) :

$$\left(2 c \left(128 c^2 d^3-4 c d e (44 b d-29 a e)+3 b e^2 (17 b d-16 a e)+e \left(32 c^2 d^2+3 b^2 e^2-4 c e (8 b d-5 a e)\right) x\right) \sqrt{a+b x+c x^2}\right)\Big/$$
$$\left(21 e^5 \left(c d^2-b d e+a e^2\right) \sqrt{d+e x}\right)-\left(2 \left(16 c^2 d^3+3 a b e^3-c d e (13 b d-4 a e)+e \left(22 c^2 d^2+3 b^2 e^2-2 c e (11 b d-5 a e)\right) x\right) \left(a+b x+c x^2\right)^{3/2}\right)\Big/$$
$$\left(21 e^3 \left(c d^2-b d e+a e^2\right) (d+e x)^{5/2}\right)-\frac{2 \left(a+b x+c x^2\right)^{5/2}}{7 e (d+e x)^{7/2}}-$$
$$\left(\sqrt{2} \sqrt{b^2-4 a c} (2 c d-b e) \left(128 c^2 d^2+3 b^2 e^2-4 c e (32 b d-29 a e)\right) \sqrt{d+e x} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticE}\left[\right.\right.$$
$$\left.\left.\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Big/\left(21 e^6 \left(c d^2-b d e+a e^2\right) \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{a+b x+c x^2}\right)+$$
$$\left(4 \sqrt{2} \sqrt{b^2-4 a c} \left(128 c^2 d^2+27 b^2 e^2-4 c e (32 b d-5 a e)\right) \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$
$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Big/\left(21 e^6 \sqrt{d+e x} \sqrt{a+b x+c x^2}\right)$$

Result (type 4, 5482 leaves) :

$$\frac{1}{\left(a+b x+c x^2\right)^2}$$
$$\sqrt{d+e x} (a+x (b+c x))^{5/2} \left(\frac{2 c^2}{3 e^5}-\frac{2 \left(c d^2-b d e+a e^2\right)^2}{7 e^5 (d+e x)^4}-\frac{6 (-2 c d+b e) \left(c d^2-b d e+a e^2\right)}{7 e^5 (d+e x)^3}-\frac{2 \left(52 c^2 d^2-52 b c d e+9 b^2 e^2+16 a c e^2\right)}{21 e^5 (d+e x)^2}-\right.$$

$$\left.\frac{2\,(-2\,c\,d+b\,e)\left(79\,c^2\,d^2-79\,b\,c\,d\,e+3\,b^2\,e^2+67\,a\,c\,e^2\right)}{21\,e^5\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)}\right)-\frac{1}{21\,e^7\left(c\,d^2-b\,d\,e+a\,e^2\right)\left(a+b\,x+c\,x^2\right)^{5/2}}\,2\,c\,(a+x\,(b+c\,x))^{5/2}$$

$$\left(\left((2\,c\,d-b\,e)\left(128\,c^2\,d^2-128\,b\,c\,d\,e+3\,b^2\,e^2+116\,a\,c\,e^2\right)(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)\right)\Bigg/\right.$$

$$\left(c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)-$$

$$\frac{1}{c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\left(\left(64\,i\,\sqrt{2}\,c^3\,d^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(96\,\mathrm{i}\sqrt{2}\,b c^2 d^2 e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\mathrm{EllipticE}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(67\,\mathrm{i}\,b^2 c d e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\mathrm{EllipticE}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(58 i \sqrt{2}\, a c^2 d e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(3 i b^3 e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(29\,\mathbb{i}\,\sqrt{2}\,a\,b\,c\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(128\,\mathbb{i}\,\sqrt{2}\,c^3\,d^2\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(128\, i \sqrt{2}\, b\, c^2\, d\, e \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(27\, i \sqrt{2}\, b^2\, c\, e^2 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(20\, i \sqrt{2}\, a\, c^2\, e^2 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\Bigg)$$

- **Problem 2460: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\left(a+b\,x+c\,x^2\right)^{5/2}}{(d+e\,x)^{11/2}}\,dx$$

Optimal (type 4, 923 leaves, 8 steps):

$$-\left(2\left(128\,c^4\,d^5-2\,a\,b^3\,e^5-4\,c^3\,d^3\,e\,(60\,b\,d-49\,a\,e)-b\,c\,e^3\left(b^2\,d^2+9\,a\,b\,d\,e-24\,a^2\,e^2\right)+3\,c^2\,d\,e^2\left(37\,b^2\,d^2-52\,a\,b\,d\,e+12\,a^2\,e^2\right)+e\left(160\,c^4\,d^4-2\,b^4\,e^4-4\,c^3\,d^2\,e\,(80\,b\,d-69\,a\,e)-b^2\,c\,e^3\,(11\,b\,d-27\,a\,e)+3\,c^2\,e^2\left(57\,b^2\,d^2-92\,a\,b\,d\,e+28\,a^2\,e^2\right)\right)x\right)\sqrt{a+b\,x+c\,x^2}\right)\Big/\left(63\,e^5\left(c\,d^2-b\,d\,e+a\,e^2\right)^2(d+e\,x)^{3/2}\right)-$$

$$\left(2\left(16\,c^2\,d^3-b\,e^2\,(2\,b\,d-5\,a\,e)-c\,d\,e\,(11\,b\,d-4\,a\,e)+e\left(26\,c^2\,d^2+3\,b^2\,e^2-2\,c\,e\,(13\,b\,d-7\,a\,e)\right)x\right)\left(a+b\,x+c\,x^2\right)^{3/2}\right)\Big/$$

$$\left(63\,e^3\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)^{7/2}\right)-\frac{2\left(a+b\,x+c\,x^2\right)^{5/2}}{9\,e\,(d+e\,x)^{9/2}}+$$

$$\left(2\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(128\,c^4\,d^4-b^4\,e^4-4\,c^3\,d^2\,e\,(64\,b\,d-57\,a\,e)-b^2\,c\,e^3\,(7\,b\,d-15\,a\,e)+3\,c^2\,e^2\left(45\,b^2\,d^2-76\,a\,b\,d\,e+28\,a^2\,e^2\right)\right)\right.$$

$$\left.\sqrt{d+e\,x}\,\sqrt{-\frac{c\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4\,a\,c}\;e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\right]\right)\Big/$$

$$\left(63\,e^6\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}}\,\sqrt{a+b\,x+c\,x^2}\right)-$$

$$\Bigg(2\sqrt{2}\sqrt{b^2-4ac}\,(2cd-be)\left(128c^2d^2-b^2e^2-4ce(32bd-33ae)\right)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right]\Bigg)\Bigg/\left(63e^6\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}\right)$$

Result (type 4, 8108 leaves) :

$$\frac{1}{\left(a+bx+cx^2\right)^2}\sqrt{d+ex}\,(a+x(b+cx))^{5/2}\Bigg(-\frac{2\left(cd^2-bde+ae^2\right)^2}{9e^5(d+ex)^5}-\frac{38(-2cd+be)\left(cd^2-bde+ae^2\right)}{63e^5(d+ex)^4}-$$

$$\frac{2\left(88c^2d^2-88bcde+15b^2e^2+28ace^2\right)}{63e^5(d+ex)^3}-\frac{2(-2cd+be)\left(61c^2d^2-61bcde+b^2e^2+57ace^2\right)}{63e^5\left(cd^2-bde+ae^2\right)(d+ex)^2}-$$

$$\left(2\left(193c^4d^4-386bc^3d^3e+207b^2c^2d^2e^2+330ac^3d^2e^2-14b^3cde^3-330abc^2de^3-2b^4e^4+30ab^2ce^4+105a^2c^2e^4\right)\right)\Big/$$

$$\left(63e^5\left(cd^2-bde+ae^2\right)^2(d+ex)\right)\Bigg)+\frac{1}{63e^7\left(cd^2-bde+ae^2\right)^2\left(a+bx+cx^2\right)^{5/2}}$$

$$2c\,(a+x(b+cx))^{5/2}\Bigg(\Big(2\left(128c^4d^4-256bc^3d^3e+135b^2c^2d^2e^2+228ac^3d^2e^2-7b^3cde^3-228abc^2de^3-b^4e^4+15ab^2ce^4+84a^2c^2e^4\right)$$

$$(d+ex)^{3/2}\left(c+\frac{cd^2}{(d+ex)^2}-\frac{bde}{(d+ex)^2}+\frac{ae^2}{(d+ex)^2}-\frac{2cd}{d+ex}+\frac{be}{d+ex}\right)\Big)\Bigg/\left(c\sqrt{\frac{(d+ex)^2\left(c\left(-1+\frac{d}{d+ex}\right)^2+\frac{e\left(b-\frac{bd}{d+ex}+\frac{ae}{d+ex}\right)}{d+ex}\right)}{e^2}}\right)-$$

$$\frac{1}{c\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\left(\left(64\,\mathbb{i}\,\sqrt{2}\,c^4\,d^4\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\right/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(128\,\mathbb{i}\,\sqrt{2}\,b\,c^3\,d^3\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(135\,i\,b^2 c^2 d^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(114\,i\sqrt{2}\,a c^3 d^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(7\,\mathbb{i}\,b^3 c d e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(114\,\mathbb{i}\sqrt{2}\,a b c^2 d e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(i\,b^4\,e^4\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\ \sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(15\,i\,a\,b^2\,c\,e^4\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\ \Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(42\,i\,\sqrt{2}\,a^2\,c^2\,e^4\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(128\,i\,\sqrt{2}\,c^4\,d^3\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\left(192 i \sqrt{2} b c^3 d^2 e \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(63 i \sqrt{2} b^2 c^2 d e^2 \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(132 i \sqrt{2} a c^3 d e^2 \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i\,b^3 c e^3\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(66\,i\sqrt{2}\,a b c^2 e^3\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\Bigg)\Bigg)$$

- **Problem 2461: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{(d+e\,x)^{7/2}}{\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 600 leaves, 8 steps) :

$$\frac{2 e \left(71 c^2 d^2+24 b^2 e^2-c e (71 b d+25 a e)\right) \sqrt{d+e x} \sqrt{a+b x+c x^2}}{105 c^3}+\frac{12 e (2 c d-b e) (d+e x)^{3/2} \sqrt{a+b x+c x^2}}{35 c^2}+$$

$$\frac{2 e (d+e x)^{5/2} \sqrt{a+b x+c x^2}}{7 c}+\left(8 \sqrt{2} \sqrt{b^2-4 a c} (2 c d-b e) \left(11 c^2 d^2+6 b^2 e^2-c e (11 b d+13 a e)\right) \sqrt{d+e x} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Bigg/\left(105 c^4 \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{a+b x+c x^2}\right)-$$

$$\left(2 \sqrt{2} \sqrt{b^2-4 a c} \left(c d^2-b d e+a e^2\right) \left(71 c^2 d^2+24 b^2 e^2-c e (71 b d+25 a e)\right) \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Bigg/\left(105 c^4 \sqrt{d+e x} \sqrt{a+b x+c x^2}\right)$$

Result (type 4, 5340 leaves) :

$$\frac{\sqrt{d+e x} \left(a+b x+c x^2\right) \left(-\frac{2 e \left(-122 c^2 d^2+89 b c d e-24 b^2 e^2+25 a c e^2\right)}{105 c^3}-\frac{4 e^2 (-11 c d+3 b e) x}{35 c^2}+\frac{2 e^3 x^2}{7 c}\right)}{\sqrt{a+x (b+c x)}}+\frac{1}{105 c^3 e \sqrt{a+x (b+c x)}}$$

$$\sqrt{a+b x+c x^2}\left(\left(16\,(2 c d-b e)\left(11 c^2 d^2-11 b c d e+6 b^2 e^2-13 a c e^2\right)(d+e x)^{3/2}\left(c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)\right)\Big/\right.$$

$$\left(c\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)-$$

$$\frac{1}{c\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}\,2\left(c d^2-b d e+a e^2\right)(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\left(\left(44\,i\sqrt{2}\,c^3 d^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(66\,\mathrm{i}\,\sqrt{2}\,b\,c^2\,d^2\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(46\,\mathrm{i}\,\sqrt{2}\,b^2\,c\,d\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(52\,i\,\sqrt{2}\;a\,c^2\,d\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(12\,i\,\sqrt{2}\;b^3\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(26\,i\,\sqrt{2}\,a\,b\,c\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(71\,i\,c^3\,d^2\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(71\,i\,b\,c^2\,d\,e\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(12\,i\sqrt{2}\,b^2 c e^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(25\,i\,a c^2 e^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left.\left.\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)\right)$$

- **Problem 2462: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(d+e\,x)^{5/2}}{\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 509 leaves, 7 steps) :

$$\frac{8 e (2 c d - b e) \sqrt{d + e x} \sqrt{a + b x + c x^2}}{15 c^2} + \frac{2 e (d + e x)^{3/2} \sqrt{a + b x + c x^2}}{5 c} +$$

$$\left(\sqrt{2} \sqrt{b^2 - 4 a c} \left(23 c^2 d^2 + 8 b^2 e^2 - c e (23 b d + 9 a e)\right) \sqrt{d + e x} \sqrt{-\frac{c \left(a + b x + c x^2\right)}{b^2 - 4 a c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2 - 4 a c}\, e}{2 c d - \left(b + \sqrt{b^2 - 4 a c}\right) e}\right]\right) \Bigg/ \left(15 c^3 \sqrt{\frac{c (d + e x)}{2 c d - \left(b + \sqrt{b^2 - 4 a c}\right) e}} \sqrt{a + b x + c x^2}\right) -$$

$$\left(8 \sqrt{2} \sqrt{b^2 - 4 a c} (2 c d - b e) \left(c d^2 - b d e + a e^2\right) \sqrt{\frac{c (d + e x)}{2 c d - \left(b + \sqrt{b^2 - 4 a c}\right) e}} \sqrt{-\frac{c \left(a + b x + c x^2\right)}{b^2 - 4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2 - 4 a c}\, e}{2 c d - \left(b + \sqrt{b^2 - 4 a c}\right) e}\right]\right) \Bigg/ \left(15 c^3 \sqrt{d + e x} \sqrt{a + b x + c x^2}\right)$$

Result (type 4, 868 leaves) :

$$\frac{\sqrt{d+e x}\left(-\frac{2 e(-11 c d+4 b e)}{15 c^2}+\frac{2 e^2 x}{5 c}\right)\left(a+b x+c x^2\right)}{\sqrt{a+x(b+c x)}}+\frac{1}{15 c^3 e \sqrt{a+x(b+c x)} \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}$$

$$2 (d+e x)^{3/2} \sqrt{a+b x+c x^2}\left(\left(23 c^2 d^2+8 b^2 e^2-c e(23 b d+9 a e)\right)\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)-\right.$$

$$\frac{1}{2 \sqrt{2} \sqrt{\frac{c d^2+e(-b d+a e)}{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}} \sqrt{d+e x}} i \sqrt{1-\frac{2\left(c d^2+e(-b d+a e)\right)}{\left(2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}\right)(d+e x)}}$$

$$\sqrt{1+\frac{2\left(c d^2+e(-b d+a e)\right)}{\left(-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}\right)(d+e x)}}\left(\left(2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}\right)\left(23 c^2 d^2+8 b^2 e^2-c e(23 b d+9 a e)\right)\right.$$

$$\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{\frac{c d^2-b d e+a e^2}{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}}}{\sqrt{d+e x}}\right],-\frac{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}{2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}}\right]+\left(-30 c^3 d^3+8 b^2 e^2\left(b e-\sqrt{\left(b^2-4 a c\right) e^2}\right)-\right.$$

$$c^2 d\left(-45 b d e-34 a e^2+23 d \sqrt{\left(b^2-4 a c\right) e^2}\right)+c e\left(-31 b^2 d e-17 a b e^2+23 b d \sqrt{\left(b^2-4 a c\right) e^2}+9 a e \sqrt{\left(b^2-4 a c\right) e^2}\right)\Big)$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{\frac{c d^2-b d e+a e^2}{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}}}{\sqrt{d+e x}}\right],-\frac{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}{2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}}\right]\right)\right)$$

- **Problem 2463: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e x)^{3/2}}{\sqrt{a+b x+c x^2}} \, dx$$

Optimal (type 4, 439 leaves, 6 steps):

$$\frac{2 e \sqrt{d+e x} \sqrt{a+b x+c x^2}}{3 c}+$$

$$\left(2 \sqrt{2} \sqrt{b^2-4 a c} (2 c d-b e) \sqrt{d+e x} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2-4 a c} e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right) \Big/$$

$$\left(3 c^2 \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{a+b x+c x^2}\right)-\left(2 \sqrt{2} \sqrt{b^2-4 a c} \left(c d^2-b d e+a e^2\right) \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\right.$$

$$\left.\sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2-4 a c} e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right) \Big/ \left(3 c^2 \sqrt{d+e x} \sqrt{a+b x+c x^2}\right)$$

Result (type 4, 735 leaves) :

$$\frac{2\,e\,\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)}{3\,c\,\sqrt{a+x\,(b+c\,x)}}+$$

$$\frac{1}{3\,c^2\,e\,\sqrt{a+x\,(b+c\,x)}\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\,2\,(d+e\,x)^{3/2}\,\sqrt{a+b\,x+c\,x^2}\,\left(2\,(2\,c\,d-b\,e)\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)+\right.$$

$$\frac{1}{\sqrt{2}\,\sqrt{\frac{c\,d^2+e\,(-b\,d+a\,e)}{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}}\,\sqrt{d+e\,x}}\,i\,\sqrt{1-\frac{2\left(c\,d^2+e\,(-b\,d+a\,e)\right)}{\left(2\,c\,d-b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}\right)(d+e\,x)}}\,\sqrt{1+\frac{2\left(c\,d^2+e\,(-b\,d+a\,e)\right)}{\left(-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}\right)(d+e\,x)}}$$

$$\left((-2\,c\,d+b\,e)\left(2\,c\,d-b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{\frac{c\,d^2-b\,d\,e+a\,e^2}{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}}}{\sqrt{d+e\,x}}\right],-\frac{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}{2\,c\,d-b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}\right]+\right.$$

$$\left(3\,c^2\,d^2+b\,e\left(b\,e-\sqrt{(b^2-4\,a\,c)\,e^2}\right)+c\left(-3\,b\,d\,e-a\,e^2+2\,d\,\sqrt{(b^2-4\,a\,c)\,e^2}\right)\right)$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{\frac{c\,d^2-b\,d\,e+a\,e^2}{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}}}{\sqrt{d+e\,x}}\right],-\frac{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}{2\,c\,d-b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}\right]\right)\right)$$

- **Problem 2464: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{d+e\,x}}{\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 188 leaves, 2 steps):

$$\frac{\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\right]}{c\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}}\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 365 leaves) :

$$\left(i\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\sqrt{\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{d+ex}\right],\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)e}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{d+ex}\right],\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)e}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,ce\sqrt{\frac{c}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+x(b+cx)}\right)$$

- **Problem 2465: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}\,dx$$

Optimal (type 4, 189 leaves, 2 steps) :

$$\frac{1}{c\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}$$

$$2\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right]$$

Result (type 4, 308 leaves) :

$$\left(i\,(d+e\,x)\,\sqrt{2-\frac{4\,\left(c\,d^2+e\,(-b\,d+a\,e)\right)}{\left(2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1+\frac{2\,\left(c\,d^2+e\,(-b\,d+a\,e)\right)}{\left(-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{\frac{c\,d^2-b\,d\,e+a\,e^2}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}}}{\sqrt{d+e\,x}}\right],\,-\frac{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}{2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}\right]\right)\Big/\left(e\,\sqrt{\frac{c\,d^2+e\,(-b\,d+a\,e)}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}}\,\sqrt{a+x\,(b+c\,x)}\right)$$

- **Problem 2466: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{(d+e\,x)^{3/2}\,\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 248 leaves, 4 steps) :

$$-\frac{2\,e\,\sqrt{a+b\,x+c\,x^2}}{\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{d+e\,x}}+\frac{\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]}{\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 408 leaves) :

$$\left(-\frac{4 e^2 (a + x (b + c x))}{\sqrt{d + e x}} + \right.$$

$$1 \Big/ \left(\sqrt{\frac{c}{-2 c d + \left(b + \sqrt{b^2 - 4 a c}\right) e}}\right) i \sqrt{2} \left(2 c d + \left(-b + \sqrt{b^2 - 4 a c}\right) e\right) \sqrt{\frac{e \left(b + \sqrt{b^2 - 4 a c} + 2 c x\right)}{-2 c d + \left(b + \sqrt{b^2 - 4 a c}\right) e}} \sqrt{1 - \frac{2 c (d + e x)}{2 c d + \left(-b + \sqrt{b^2 - 4 a c}\right) e}}$$

$$\left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{-2 c d + \left(b + \sqrt{b^2 - 4 a c}\right) e}} \sqrt{d + e x}\right], \frac{2 c d - \left(b + \sqrt{b^2 - 4 a c}\right) e}{2 c d + \left(-b + \sqrt{b^2 - 4 a c}\right) e}\right] - \text{EllipticF}\left[\right.\right.$$

$$\left.\left.\left. i \,\text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{-2 c d + \left(b + \sqrt{b^2 - 4 a c}\right) e}} \sqrt{d + e x}\right], \frac{2 c d - \left(b + \sqrt{b^2 - 4 a c}\right) e}{2 c d + \left(-b + \sqrt{b^2 - 4 a c}\right) e}\right]\right)\right) \Big/ \left(2 e \left(c d^2 + e (-b d + a e)\right) \sqrt{a + x (b + c x)}\right)$$

- **Problem 2467: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(d + e x)^{5/2} \sqrt{a + b x + c x^2}} \, dx$$

Optimal (type 4, 523 leaves, 7 steps) :

$$-\frac{2 e \sqrt{a+b x+c x^2}}{3 \left(c d^2-b d e+a e^2\right) (d+e x)^{3/2}}-\frac{4 e (2 c d-b e) \sqrt{a+b x+c x^2}}{3 \left(c d^2-b d e+a e^2\right)^2 \sqrt{d+e x}}+$$

$$\left(2 \sqrt{2} \sqrt{b^2-4 a c} (2 c d-b e) \sqrt{d+e x} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right) \Bigg/$$

$$\left(3 \left(c d^2-b d e+a e^2\right)^2 \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{a+b x+c x^2}\right) -$$

$$\left(2 \sqrt{2} \sqrt{b^2-4 a c} \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right) \Bigg/$$

$$\left(3 \left(c d^2-b d e+a e^2\right) \sqrt{d+e x} \sqrt{a+b x+c x^2}\right)$$

Result (type 4, 812 leaves) :

$$\frac{\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)\left(-\frac{2\,e}{3\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)^2}+\frac{4\,e\,(-2\,c\,d+b\,e)}{3\left(c\,d^2-b\,d\,e+a\,e^2\right)^2(d+e\,x)}\right)}{\sqrt{a+x\,(b+c\,x)}}+$$

$$\Bigg(2\,(d+e\,x)^{3/2}\sqrt{a+b\,x+c\,x^2}\,\Bigg(2\,(2\,c\,d-b\,e)\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)+$$

$$\frac{1}{\sqrt{2}\,\sqrt{\frac{c\,d^2+e\,(-b\,d+a\,e)}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}}}\,\sqrt{d+e\,x}}\,i\,\sqrt{1-\frac{2\left(c\,d^2+e\,(-b\,d+a\,e)\right)}{\left(2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}\right)(d+e\,x)}}\,\sqrt{1+\frac{2\left(c\,d^2+e\,(-b\,d+a\,e)\right)}{\left(-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}\right)(d+e\,x)}}$$

$$\Bigg((-2\,c\,d+b\,e)\left(2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}\right)\text{EllipticE}\Bigg[i\,\text{ArcSinh}\Bigg[\frac{\sqrt{2}\,\sqrt{\frac{c\,d^2-b\,d\,e+a\,e^2}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}}}}{\sqrt{d+e\,x}}\Bigg],\,-\frac{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}}{2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}}\Bigg]+$$

$$\left(3\,c^2\,d^2+b\,e\left(b\,e-\sqrt{\left(b^2-4\,a\,c\right)e^2}\right)+c\left(-3\,b\,d\,e-a\,e^2+2\,d\sqrt{\left(b^2-4\,a\,c\right)e^2}\right)\right)$$

$$\text{EllipticF}\Bigg[i\,\text{ArcSinh}\Bigg[\frac{\sqrt{2}\,\sqrt{\frac{c\,d^2-b\,d\,e+a\,e^2}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}}}}{\sqrt{d+e\,x}}\Bigg],\,-\frac{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}}{2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}}\Bigg]\Bigg)\Bigg)\Bigg)\Bigg/$$

$$\left(3\,e\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\sqrt{a+x\,(b+c\,x)}\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)$$

- **Problem 2468: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(d+e\,x)^{7/2}\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 629 leaves, 8 steps) :

$$-\frac{2 e \sqrt{a+b x+c x^2}}{5 \left(c d^2-b d e+a e^2\right) (d+e x)^{5/2}}-\frac{8 e (2 c d-b e) \sqrt{a+b x+c x^2}}{15 \left(c d^2-b d e+a e^2\right)^2 (d+e x)^{3/2}}-$$

$$\frac{2 e \left(23 c^2 d^2+8 b^2 e^2-c e (23 b d+9 a e)\right) \sqrt{a+b x+c x^2}}{15 \left(c d^2-b d e+a e^2\right)^3 \sqrt{d+e x}}+\left(\sqrt{2} \sqrt{b^2-4 a c} \left(23 c^2 d^2+8 b^2 e^2-c e (23 b d+9 a e)\right)\right.$$

$$\left.\sqrt{d+e x} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Big/$$

$$\left(15 \left(c d^2-b d e+a e^2\right)^3 \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{a+b x+c x^2}\right)-\left(8 \sqrt{2} \sqrt{b^2-4 a c} (2 c d-b e) \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\right.$$

$$\left.\sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Big/\left(15 \left(c d^2-b d e+a e^2\right)^2 \sqrt{d+e x} \sqrt{a+b x+c x^2}\right)$$

Result (type 4, 983 leaves) :

$$\frac{1}{\sqrt{a+x\,(b+c\,x)}}\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)\left(-\frac{2\,e}{5\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)^3}+\frac{8\,e\,(-2\,c\,d+b\,e)}{15\left(c\,d^2-b\,d\,e+a\,e^2\right)^2(d+e\,x)^2}+\frac{2\,e\left(-23\,c^2\,d^2+23\,b\,c\,d\,e-8\,b^2\,e^2+9\,a\,c\,e^2\right)}{15\left(c\,d^2-b\,d\,e+a\,e^2\right)^3(d+e\,x)}\right)+$$

$$\left(2\,(d+e\,x)^{3/2}\sqrt{a+b\,x+c\,x^2}\left(\left(23\,c^2\,d^2+8\,b^2\,e^2-c\,e\,(23\,b\,d+9\,a\,e)\right)\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)-\right.\right.$$

$$\frac{1}{2\sqrt{2}\sqrt{\frac{c\,d^2+e\,(-b\,d+a\,e)}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}}}\sqrt{d+e\,x}}\,i\,\sqrt{1-\frac{2\left(c\,d^2+e\,(-b\,d+a\,e)\right)}{\left(2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}\right)(d+e\,x)}}$$

$$\sqrt{1+\frac{2\left(c\,d^2+e\,(-b\,d+a\,e)\right)}{\left(-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}\right)(d+e\,x)}}\left(\left(2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}\right)\left(23\,c^2\,d^2+8\,b^2\,e^2-c\,e\,(23\,b\,d+9\,a\,e)\right)\right.$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c\,d^2-b\,d\,e+a\,e^2}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}}}}{\sqrt{d+e\,x}}\right],-\frac{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}}{2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}}\right]+\left(-30\,c^3\,d^3+8\,b^2\,e^2\left(b\,e-\sqrt{\left(b^2-4\,a\,c\right)e^2}\right)-\right.$$

$$c^2\,d\left(-45\,b\,d\,e-34\,a\,e^2+23\,d\sqrt{\left(b^2-4\,a\,c\right)e^2}\right)+c\,e\left(-31\,b^2\,d\,e-17\,a\,b\,e^2+23\,b\,d\sqrt{\left(b^2-4\,a\,c\right)e^2}+9\,a\,e\sqrt{\left(b^2-4\,a\,c\right)e^2}\right)\Big)$$

$$\left.\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c\,d^2-b\,d\,e+a\,e^2}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}}}}{\sqrt{d+e\,x}}\right],-\frac{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}}{2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}}\right]\right)\right)\right)\Big/$$

$$\left(15\,e\left(c\,d^2-b\,d\,e+a\,e^2\right)^3\sqrt{a+x\,(b+c\,x)}\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)$$

Problem 2469: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(d+e x)^{7/2}}{\left(a+b x+c x^2\right)^{3/2}} \, dx$$

Optimal (type 4, 641 leaves, 8 steps):

$$-\frac{2 (d+e x)^{5/2} (b d-2 a e+(2 c d-b e) x)}{\left(b^2-4 a c\right) \sqrt{a+b x+c x^2}}+\frac{4 e \left(3 c^2 d^2+2 b^2 e^2-c e (3 b d+5 a e)\right) \sqrt{d+e x} \sqrt{a+b x+c x^2}}{3 c^2 \left(b^2-4 a c\right)}+$$

$$\frac{2 e (2 c d-b e) (d+e x)^{3/2} \sqrt{a+b x+c x^2}}{c \left(b^2-4 a c\right)}+\left(\sqrt{2} (2 c d-b e) \left(3 c^2 d^2+8 b^2 e^2-c e (3 b d+29 a e)\right) \sqrt{d+e x} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right) \Bigg/ \left(3 c^3 \sqrt{b^2-4 a c} \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{a+b x+c x^2}\right)-$$

$$\left(4 \sqrt{2} \left(c d^2-b d e+a e^2\right) \left(3 c^2 d^2+2 b^2 e^2-c e (3 b d+5 a e)\right) \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right) \Bigg/ \left(3 c^3 \sqrt{b^2-4 a c} \sqrt{d+e x} \sqrt{a+b x+c x^2}\right)$$

Result (type 4, 5433 leaves):

$$\frac{1}{(a+x (b+c x))^{3/2}} \sqrt{d+e x} \left(a+b x+c x^2\right)^2$$

$$\left(\frac{2 e^3}{3 c^2}+\left(2 \left(b c^2 d^3-6 a c^2 d^2 e+3 a b c d e^2-a b^2 e^3+2 a^2 c e^3+2 c^3 d^3 x-3 b c^2 d^2 e x+3 b^2 c d e^2 x-6 a c^2 d e^2 x-b^3 e^3 x+3 a b c e^3 x\right)\right)\Big/\right.$$

$$\left.\left(c^2 \left(-b^2+4 a c\right) \left(a+b x+c x^2\right)\right)\right)+\frac{1}{3 c^2 \left(-b^2+4 a c\right) e \left(a+x (b+c x)\right)^{3/2}}$$

$$2 \left(a+b x+c x^2\right)^{3/2} \left(-\left((2 c d-b e) \left(3 c^2 d^2-3 b c d e+8 b^2 e^2-29 a c e^2\right) (d+e x)^{3/2} \left(c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)\right)\Big/\right.$$

$$\left(c \sqrt{\frac{(d+e x)^2 \left(c \left(-1+\frac{d}{d+e x}\right)^2+\frac{e \left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)+$$

$$\frac{1}{c \sqrt{\frac{(d+e x)^2 \left(c \left(-1+\frac{d}{d+e x}\right)^2+\frac{e \left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}} \left(c d^2-b d e+a e^2\right) (d+e x) \sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\left(\left(3\, i\, c^3 d^3 \left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}} \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(9\,\mathbb{i}\,b\,c^2\,d^2\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(19\,\mathbb{i}\,b^2\,c\,d\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\Big[\mathbb{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Big],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Big]\Bigg)\Bigg)\Bigg/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(29\,\mathbb{i}\,a\,c^2\,d\,e^2\,\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}$$

$$\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\Bigg(\text{EllipticE}\Big[\mathbb{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Big],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Big]-$$

$$\text{EllipticF}\Big[\mathbb{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Big],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Big]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(2\,\mathbb{i}\,\sqrt{2}\,b^3\,e^3\,\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}$$

$$\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\Bigg(\text{EllipticE}\Big[\mathbb{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Big],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Big]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(29\,i\,a b c e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(3\,i\sqrt{2}\,c^3 d^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(3\,\mathrm{i}\,\sqrt{2}\,b\,c^2\,d\,e\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\ \sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(2\,\mathrm{i}\,\sqrt{2}\,b^2\,c\,e^2\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\ \sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(5\,\mathrm{i}\,\sqrt{2}\,a\,c^2\,e^2\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\ \sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\Bigg)\Bigg)$$

- **Problem 2470: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{(d+e\,x)^{5/2}}{\left(a+b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 533 leaves, 7 steps) :

$$-\frac{2 (d+e x)^{3/2} (b d-2 a e+(2 c d-b e) x)}{\left(b^2-4 a c\right) \sqrt{a+b x+c x^2}}+\frac{2 e (2 c d-b e) \sqrt{d+e x} \sqrt{a+b x+c x^2}}{c \left(b^2-4 a c\right)}+$$

$$\left(2 \sqrt{2} \left(c^2 d^2+b^2 e^2-c e (b d+3 a e)\right) \sqrt{d+e x} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Big/$$

$$\left(c^2 \sqrt{b^2-4 a c} \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{a+b x+c x^2}\right)-$$

$$\left(2 \sqrt{2} (2 c d-b e) \left(c d^2-b d e+a e^2\right) \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Big/\left(c^2 \sqrt{b^2-4 a c} \sqrt{d+e x} \sqrt{a+b x+c x^2}\right)$$

Result (type 4, 3447 leaves) :

$$\frac{2 \sqrt{d+e x} \left(b c d^2-4 a c d e+a b e^2+2 c^2 d^2 x-2 b c d e x+b^2 e^2 x-2 a c e^2 x\right) \left(a+b x+c x^2\right)}{c \left(-b^2+4 a c\right) (a+x (b+c x))^{3/2}}+$$

$$\frac{1}{c\left(-b^2+4\,a\,c\right)\,e\,\left(a+x\,(b+c\,x)\right)^{3/2}}\left(a+b\,x+c\,x^2\right)^{3/2}\left(-\frac{4\left(c^2\,d^2-b\,c\,d\,e+b^2\,e^2-3\,a\,c\,e^2\right)(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)}{c\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}+\right.$$

$$\frac{1}{c\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\,2\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\left(\left(\mathbb{i}\,c^2\,d^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(\mathbb{i}\,b\,c\,d\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(i\,b^2\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(3\,i\,a\,c\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i\sqrt{2}\,c^2 d\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(i\,b\,c\,e\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\Bigg)\Bigg)$$

Problem 2471: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d+e\,x)^{3/2}}{\left(a+b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 457 leaves, 6 steps) :

$$-\frac{2\,\sqrt{d+e\,x}\,\left(b\,d-2\,a\,e+(2\,c\,d-b\,e)\,x\right)}{\left(b^2-4\,a\,c\right)\,\sqrt{a+b\,x+c\,x^2}}+\frac{\sqrt{2}\,(2\,c\,d-b\,e)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]}{c\,\sqrt{b^2-4\,a\,c}\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}}-$$

$$\left(4\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/\left(c\,\sqrt{b^2-4\,a\,c}\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 600 leaves) :

$$\frac{1}{2\left(b^2-4ac\right)\sqrt{a+x(b+cx)}}\left(-\frac{4e(-2cd+be)(a+x(b+cx))}{c\sqrt{d+ex}}-4\sqrt{d+ex}\,(-2ae+2cdx+b(d-ex))-\right.$$

$$\frac{1}{ce\sqrt{\frac{cd^2+e(-bd+ae)}{-2cd+be+\sqrt{\left(b^2-4ac\right)e^2}}}}\,\mathrm{i}\,(d+ex)\sqrt{1-\frac{2\left(cd^2+e(-bd+ae)\right)}{\left(2cd-be+\sqrt{\left(b^2-4ac\right)e^2}\right)(d+ex)}}\sqrt{2+\frac{4\left(cd^2+e(-bd+ae)\right)}{\left(-2cd+be+\sqrt{\left(b^2-4ac\right)e^2}\right)(d+ex)}}$$

$$\left(-(-2cd+be)\left(2cd-be+\sqrt{\left(b^2-4ac\right)e^2}\right)\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{cd^2-bde+ae^2}{-2cd+be+\sqrt{\left(b^2-4ac\right)e^2}}}}{\sqrt{d+ex}}\right],-\frac{-2cd+be+\sqrt{\left(b^2-4ac\right)e^2}}{2cd-be+\sqrt{\left(b^2-4ac\right)e^2}}\right]+\right.$$

$$\left(-b^2e^2+4ace^2-2cd\sqrt{\left(b^2-4ac\right)e^2}+be\sqrt{\left(b^2-4ac\right)e^2}\right)$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{cd^2-bde+ae^2}{-2cd+be+\sqrt{\left(b^2-4ac\right)e^2}}}}{\sqrt{d+ex}}\right],-\frac{-2cd+be+\sqrt{\left(b^2-4ac\right)e^2}}{2cd-be+\sqrt{\left(b^2-4ac\right)e^2}}\right]\right)\right)$$

- **Problem 2472: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{d+ex}}{\left(a+bx+cx^2\right)^{3/2}}\,dx$$

Optimal (type 4, 426 leaves, 6 steps) :

$$-\frac{2\,(b+2\,c\,x)\,\sqrt{d+e\,x}}{(b^2-4\,a\,c)\,\sqrt{a+b\,x+c\,x^2}}+\frac{2\,\sqrt{2}\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]}{\sqrt{b^2-4\,a\,c}\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}}-$$

$$\left(2\,\sqrt{2}\,(2\,c\,d-b\,e)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/$$

$$\left(c\,\sqrt{b^2-4\,a\,c}\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 996 leaves) :

$$-\frac{2\,(b+2\,c\,x)\,\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)}{\left(b^2-4\,a\,c\right)\,(a+x\,(b+c\,x))^{3/2}}+\left((d+e\,x)^{3/2}\,\left(a+b\,x+c\,x^2\right)^{3/2}\right.$$

$$\left(-4\,\sqrt{\frac{c\,d^2+e\,(-b\,d+a\,e)}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}}\,\left(c\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\,\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)+\frac{1}{\sqrt{d+e\,x}}\,\mathrm{i}\,\sqrt{2}\,\left(2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}\right)\right.$$

$$\sqrt{\frac{\sqrt{\left(b^2-4\,a\,c\right)\,e^2}-\frac{2\,a\,e^2}{d+e\,x}-2\,c\,d\,\left(-1+\frac{d}{d+e\,x}\right)+b\,e\,\left(-1+\frac{2\,d}{d+e\,x}\right)}{2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}}\,\sqrt{\frac{\sqrt{\left(b^2-4\,a\,c\right)\,e^2}+\frac{2\,a\,e^2}{d+e\,x}+2\,c\,d\,\left(-1+\frac{d}{d+e\,x}\right)+b\,\left(e-\frac{2\,d\,e}{d+e\,x}\right)}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}}$$

$$\mathrm{EllipticE}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{\frac{c\,d^2-b\,d\,e+a\,e^2}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}}}{\sqrt{d+e\,x}}\right],-\frac{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}{2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}\right]-\frac{1}{\sqrt{d+e\,x}}\,\mathrm{i}\,\sqrt{2}\,\sqrt{\left(b^2-4\,a\,c\right)\,e^2}$$

$$\sqrt{\frac{\sqrt{\left(b^2-4\,a\,c\right)\,e^2}-\frac{2\,a\,e^2}{d+e\,x}-2\,c\,d\,\left(-1+\frac{d}{d+e\,x}\right)+b\,e\,\left(-1+\frac{2\,d}{d+e\,x}\right)}{2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}}\,\sqrt{\frac{\sqrt{\left(b^2-4\,a\,c\right)\,e^2}+\frac{2\,a\,e^2}{d+e\,x}+2\,c\,d\,\left(-1+\frac{d}{d+e\,x}\right)+b\,\left(e-\frac{2\,d\,e}{d+e\,x}\right)}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}}$$

$$\left.\left.\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{\frac{c\,d^2-b\,d\,e+a\,e^2}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}}}{\sqrt{d+e\,x}}\right],-\frac{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}{2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}\right]\right)\right)\Bigg/$$

$$\left(\left(-b^2+4\,a\,c\right)\,e\,\sqrt{\frac{c\,d^2+e\,(-b\,d+a\,e)}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}}\,(a+x\,(b+c\,x))^{3/2}\,\sqrt{\frac{(d+e\,x)^2\,\left(c\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\,\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)$$

- **Problem 2473: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{1}{\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 480 leaves, 6 steps):

$$-\frac{2\sqrt{d+e x}\,\left(b c d-b^2 e+2 a c e+c\,(2 c d-b e)\,x\right)}{\left(b^2-4 a c\right)\left(c d^2-b d e+a e^2\right)\sqrt{a+b x+c x^2}}+$$

$$\frac{\sqrt{2}\,(2 c d-b e)\sqrt{d+e x}\sqrt{-\frac{c\,(a+b x+c x^2)}{b^2-4 a c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}\right]}{\sqrt{b^2-4 a c}\left(c d^2-b d e+a e^2\right)\sqrt{\frac{c\,(d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}}\sqrt{a+b x+c x^2}}-$$

$$\frac{4\sqrt{2}\sqrt{\frac{c\,(d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}}\sqrt{-\frac{c\,(a+b x+c x^2)}{b^2-4 a c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}\right]}{\sqrt{b^2-4 a c}\sqrt{d+e x}\sqrt{a+b x+c x^2}}$$

Result (type 4, 1939 leaves) :

$$\frac{2\sqrt{d+e x}\,\left(b c d-b^2 e+2 a c e+2 c^2 d x-b c e x\right)\left(a+b x+c x^2\right)}{\left(-b^2+4 a c\right)\left(c d^2-b d e+a e^2\right)\left(a+x\,(b+c x)\right)^{3/2}}-$$

$$\frac{1}{\left(-b^2+4 a c\right)e\left(c d^2-b d e+a e^2\right)\left(a+x\,(b+c x)\right)^{3/2}}\,2 c\left(a+b x+c x^2\right)^{3/2}\left(\frac{(2 c d-b e)\,(d+e x)^{3/2}\left(c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)}{c\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}-\right.$$

$$\frac{1}{c\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}\left(c d^2-b d e+a e^2\right)(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\left(\left(\mathbb{i}\, c d\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Bigg/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(i\,b e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Bigg/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i\sqrt{2}\,c\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg/$$

$$\left(\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)$$

- **Problem 2474: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(d+e\,x)^{3/2}\,\left(a+b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 607 leaves, 7 steps) :

$$-\frac{2\left(b c d-b^2 e+2 a c e+c (2 c d-b e) x\right)}{\left(b^2-4 a c\right)\left(c d^2-b d e+a e^2\right)\sqrt{d+e x}\sqrt{a+b x+c x^2}}-\frac{4 e\left(c^2 d^2+b^2 e^2-c e (b d+3 a e)\right)\sqrt{a+b x+c x^2}}{\left(b^2-4 a c\right)\left(c d^2-b d e+a e^2\right)^2\sqrt{d+e x}}+$$

$$\left(2\sqrt{2}\left(c^2 d^2+b^2 e^2-c e (b d+3 a e)\right)\sqrt{d+e x}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}\right]\right)\Bigg/$$

$$\left(\sqrt{b^2-4 a c}\left(c d^2-b d e+a e^2\right)^2\sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}}\sqrt{a+b x+c x^2}\right)-$$

$$\left(2\sqrt{2}\,(2 c d-b e)\sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}\right]\right)\Bigg/$$

$$\left(\sqrt{b^2-4 a c}\left(c d^2-b d e+a e^2\right)\sqrt{d+e x}\sqrt{a+b x+c x^2}\right)$$

Result (type 4, 3554 leaves) :

$$\frac{1}{(a+x (b+c x))^{3/2}}\sqrt{d+e x}\left(a+b x+c x^2\right)^2$$

$$\left(-\frac{2 e^3}{\left(c d^2-b d e+a e^2\right)^2 (d+e x)}-\frac{2\left(-b c^2 d^2+2 b^2 c d e-4 a c^2 d e-b^3 e^2+3 a b c e^2-2 c^3 d^2 x+2 b c^2 d e x-b^2 c e^2 x+2 a c^2 e^2 x\right)}{\left(-b^2+4 a c\right)\left(c d^2-b d e+a e^2\right)^2\left(a+b x+c x^2\right)}\right)-$$

$$\frac{1}{\left(-b^2+4 a c\right) e\left(c d^2-b d e+a e^2\right)^2 (a+x (b+c x))^{3/2}}$$

$$2\,c\,\left(a+b\,x+c\,x^2\right)^{3/2}\left(\frac{2\,\left(c^2\,d^2-b\,c\,d\,e+b^2\,e^2-3\,a\,c\,e^2\right)\,(d+e\,x)^{3/2}\,\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)}{c\,\sqrt{\frac{(d+e\,x)^2\,\left(c\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\,\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}-\right.$$

$$\frac{1}{c\,\sqrt{\frac{(d+e\,x)^2\,\left(c\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\,\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\left(c\,d^2-b\,d\,e+a\,e^2\right)\,(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\left(\left(\mathrm{i}\,c^2\,d^2\,\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.\right.$$

$$\left(\mathrm{EllipticE}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\left.\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\right/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(\mathrm{i}\,b\,c\,d\,e\,\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i\,b^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(3\,i\,a c e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i\sqrt{2}\,c^2 d\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(i\,b\,c\,e\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left.\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)\right)$$

- **Problem 2475: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{1}{(d+e\,x)^{5/2}\,\left(a+b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 744 leaves, 8 steps) :

$$-\frac{2\left(b c d-b^2 e+2 a c e+c(2 c d-b e) x\right)}{\left(b^2-4 a c\right)\left(c d^2-b d e+a e^2\right)(d+e x)^{3/2} \sqrt{a+b x+c x^2}}-$$

$$\frac{4 e\left(3 c^2 d^2+2 b^2 e^2-c e(3 b d+5 a e)\right) \sqrt{a+b x+c x^2}}{3\left(b^2-4 a c\right)\left(c d^2-b d e+a e^2\right)^2(d+e x)^{3/2}}-\frac{2 e(2 c d-b e)\left(3 c^2 d^2+8 b^2 e^2-c e(3 b d+29 a e)\right) \sqrt{a+b x+c x^2}}{3\left(b^2-4 a c\right)\left(c d^2-b d e+a e^2\right)^3 \sqrt{d+e x}}+$$

$$\left(\sqrt{2}\,(2 c d-b e)\left(3 c^2 d^2+8 b^2 e^2-c e(3 b d+29 a e)\right) \sqrt{d+e x} \sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Big/$$

$$\left(3 \sqrt{b^2-4 a c}\left(c d^2-b d e+a e^2\right)^3 \sqrt{\frac{c(d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{a+b x+c x^2}\right)-$$

$$\left(4 \sqrt{2}\left(3 c^2 d^2+2 b^2 e^2-c e(3 b d+5 a e)\right) \sqrt{\frac{c(d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Big/\left(3 \sqrt{b^2-4 a c}\left(c d^2-b d e+a e^2\right)^2 \sqrt{d+e x} \sqrt{a+b x+c x^2}\right)$$

Result (type 4, 5565 leaves) :

$$\frac{1}{(a+x(b+cx))^{3/2}}$$

$$\sqrt{d+ex}\,\left(a+bx+cx^2\right)^2\left(-\frac{2e^3}{3\left(cd^2-bde+ae^2\right)^2(d+ex)^2}+\frac{10e^3(-2cd+be)}{3\left(cd^2-bde+ae^2\right)^3(d+ex)}-\left(2\left(-bc^3d^3+3b^2c^2d^2e-6ac^3d^2e-3b^3cde^2+9abc^2de^2+b^4e^3-4ab^2ce^3+2a^2c^2e^3-2c^4d^3x+3bc^3d^2ex-3b^2c^2de^2x+6ac^3de^2x+b^3ce^3x-3abc^2e^3x\right)\right)\Big/\left(\left(-b^2+4ac\right)\left(cd^2-bde+ae^2\right)^3\left(a+bx+cx^2\right)\right)\right)+\frac{1}{3\left(-b^2+4ac\right)e\left(cd^2-bde+ae^2\right)^3(a+x(b+cx))^{3/2}}\,2c\left(a+bx+cx^2\right)^{3/2}$$

$$\left(-\left((2cd-be)\left(3c^2d^2-3bcde+8b^2e^2-29ace^2\right)(d+ex)^{3/2}\left(c+\frac{cd^2}{(d+ex)^2}-\frac{bde}{(d+ex)^2}+\frac{ae^2}{(d+ex)^2}-\frac{2cd}{d+ex}+\frac{be}{d+ex}\right)\right)\Big/\right.$$

$$\left(c\sqrt{\frac{(d+ex)^2\left(c\left(-1+\frac{d}{d+ex}\right)^2+\frac{e\left(b-\frac{bd}{d+ex}+\frac{ae}{d+ex}\right)}{d+ex}\right)}{e^2}}\right)+$$

$$\frac{1}{c\sqrt{\frac{(d+ex)^2\left(c\left(-1+\frac{d}{d+ex}\right)^2+\frac{e\left(b-\frac{bd}{d+ex}+\frac{ae}{d+ex}\right)}{d+ex}\right)}{e^2}}}\left(cd^2-bde+ae^2\right)(d+ex)\sqrt{c+\frac{cd^2}{(d+ex)^2}-\frac{bde}{(d+ex)^2}+\frac{ae^2}{(d+ex)^2}-\frac{2cd}{d+ex}+\frac{be}{d+ex}}$$

$$\left(\left(3\,i\,c^3d^3\left(2cd-be+\sqrt{b^2e^2-4ace^2}\right)\sqrt{1-\frac{2\left(cd^2-bde+ae^2\right)}{\left(2cd-be-\sqrt{b^2e^2-4ace^2}\right)(d+ex)}}\sqrt{1-\frac{2\left(cd^2-bde+ae^2\right)}{\left(2cd-be+\sqrt{b^2e^2-4ace^2}\right)(d+ex)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{cd^2-bde+ae^2}{2cd-be-\sqrt{b^2e^2-4ace^2}}}}{\sqrt{d+ex}}\right],\frac{2cd-be-\sqrt{b^2e^2-4ace^2}}{2cd-be+\sqrt{b^2e^2-4ace^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(9\,i\,b c^2 d^2 e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(19\,i\,b^2 c d e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(29\,i\,a\,c^2\,d\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(2\,i\,\sqrt{2}\,b^3\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\ \text{ArcSinh}\left[\frac{\sqrt{2}\ \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg) \Big/$$

$$\Bigg(\left(c d^2-b d e+a e^2\right) \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\Bigg)+$$

$$\Bigg(29\ \mathbb{i}\ a b c e^3 \left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}$$

$$\sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}\ \Bigg(\text{EllipticE}\left[\mathbb{i}\ \text{ArcSinh}\left[\frac{\sqrt{2}\ \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\ \text{ArcSinh}\left[\frac{\sqrt{2}\ \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg) \Big/$$

$$\Bigg(2 \sqrt{2}\ \left(c d^2-b d e+a e^2\right) \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\Bigg)+$$

$$\Bigg(3\ \mathbb{i} \sqrt{2}\ c^3 d^2 \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}\ \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}$$

$$\text{EllipticF}\left[\mathbb{i}\ \text{ArcSinh}\left[\frac{\sqrt{2}\ \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg) \Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(3 i \sqrt{2}\, b c^2 d e \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(2 i \sqrt{2}\, b^2 c e^2 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(5 i \sqrt{2}\, a c^2 e^2 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)\Bigg)$$

- **Problem 2476: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{(d+e x)^{7/2}}{\left(a+b x+c x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 659 leaves, 7 steps) :

$$-\frac{2\,(d+e\,x)^{5/2}\,(b\,d-2\,a\,e+(2\,c\,d-b\,e)\,x)}{3\,(b^2-4\,a\,c)\,(a+b\,x+c\,x^2)^{3/2}}+$$

$$\left(2\,\sqrt{d+e\,x}\,\left(8\,b\,c\,d\,(c\,d^2+3\,a\,e^2)-4\,a\,c\,e\,(3\,c\,d^2+5\,a\,e^2)-b^2\,(9\,c\,d^2\,e-a\,e^3)+(2\,c\,d-b\,e)\,(8\,c^2\,d^2-b^2\,e^2-4\,c\,e\,(2\,b\,d-3\,a\,e))\,x\right)\right)\Big/$$

$$\left(3\,c\,(b^2-4\,a\,c)^2\,\sqrt{a+b\,x+c\,x^2}\right)-\left(2\,\sqrt{2}\,(2\,c\,d-b\,e)\,(4\,c^2\,d^2-b^2\,e^2-4\,c\,e\,(b\,d-2\,a\,e))\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/\left(3\,c^2\,(b^2-4\,a\,c)^{3/2}\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\left(2\,\sqrt{2}\,(c\,d^2-b\,d\,e+a\,e^2)\,(16\,c^2\,d^2-b^2\,e^2-4\,c\,e\,(4\,b\,d-5\,a\,e))\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/\left(3\,c^2\,(b^2-4\,a\,c)^{3/2}\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 5598 leaves) :

$$\frac{1}{(a+x\,(b+c\,x))^{5/2}}\sqrt{d+e\,x}\,(a+b\,x+c\,x^2)^3$$

$$\Big(\big(2\,(b\,c^2\,d^3-6\,a\,c^2\,d^2\,e+3\,a\,b\,c\,d\,e^2-a\,b^2\,e^3+2\,a^2\,c\,e^3+2\,c^3\,d^3\,x-3\,b\,c^2\,d^2\,e\,x+3\,b^2\,c\,d\,e^2\,x-6\,a\,c^2\,d\,e^2\,x-b^3\,e^3\,x+3\,a\,b\,c\,e^3\,x)\big)\Big/$$

$$\left(3\,c^2\,(-b^2+4\,a\,c)\,(a+b\,x+c\,x^2)^2\right)+\big(2\,(8\,b\,c^3\,d^3-13\,b^2\,c^2\,d^2\,e+4\,a\,c^3\,d^2\,e+3\,b^3\,c\,d\,e^2+12\,a\,b\,c^2\,d\,e^2-b^4\,e^3+7\,a\,b^2\,c\,e^3-28\,a^2\,c^2\,e^3+$$

$$16\,c^4\,d^3\,x-24\,b\,c^3\,d^2\,e\,x+4\,b^2\,c^2\,d\,e^2\,x+32\,a\,c^3\,d\,e^2\,x+2\,b^3\,c\,e^3\,x-16\,a\,b\,c^2\,e^3\,x)\big)\Big/\left(3\,c^2\,(-b^2+4\,a\,c)^2\,(a+b\,x+c\,x^2)\right)\Big)+$$

$$\frac{1}{3\,c\,\left(-b^2+4\,a\,c\right)^2\,e\,\left(a+x\,(b+c\,x)\right)^{5/2}}\left(a+b\,x+c\,x^2\right)^{5/2}\Bigg(-\Bigg(4\,(2\,c\,d-b\,e)\,\left(4\,c^2\,d^2-4\,b\,c\,d\,e-b^2\,e^2+8\,a\,c\,e^2\right)\,(d+e\,x)^{3/2}$$

$$\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)\Bigg)\Bigg/\left(c\,\sqrt{\frac{(d+e\,x)^2\,\left(c\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\,\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)+$$

$$\frac{1}{c\,\sqrt{\frac{(d+e\,x)^2\,\left(c\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\,\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\,2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\Bigg(\Bigg(4\,i\,\sqrt{2}\,c^3\,d^3\,\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(6\,i\,\sqrt{2}\,b\,c^2\,d^2\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(i\,\sqrt{2}\,b^2\,c\,d\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(8\,i\,\sqrt{2}\,a\,c^2\,d\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(i\,b^3\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(4\,\mathrm{i}\,\sqrt{2}\;a\,b\,c\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(8\,\mathrm{i}\,\sqrt{2}\;c^3\,d^2\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(8\,\mathrm{i}\,\sqrt{2}\;b\,c^2\,d\,e\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(i\,b^2 c\,e^2\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(10\,i\,\sqrt{2}\,a\,c^2 e^2\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left.\left.\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)\right)$$

- **Problem 2477: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(d+e\,x)^{5/2}}{\left(a+b\,x+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 590 leaves, 7 steps) :

$$-\frac{2\,(d+e\,x)^{3/2}\,(b\,d-2\,a\,e+(2\,c\,d-b\,e)\,x)}{3\,(b^2-4\,a\,c)\,(a+b\,x+c\,x^2)^{3/2}}-\frac{2\sqrt{d+e\,x}\,\left(7\,b^2\,d\,e+4\,a\,c\,d\,e-8\,b\,(c\,d^2+a\,e^2)-(16\,c^2\,d^2+b^2\,e^2-4\,c\,e\,(4\,b\,d-3\,a\,e))\,x\right)}{3\,(b^2-4\,a\,c)^2\sqrt{a+b\,x+c\,x^2}}-$$

$$\left(\sqrt{2}\,(16\,c^2\,d^2+b^2\,e^2-4\,c\,e\,(4\,b\,d-3\,a\,e))\sqrt{d+e\,x}\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\right]\right)\Big/$$

$$\left(3\,c\,(b^2-4\,a\,c)^{3/2}\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}}\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\left(16\sqrt{2}\,(2\,c\,d-b\,e)\,(c\,d^2-b\,d\,e+a\,e^2)\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}}\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\right]\right)\Big/\left(3\,c\,(b^2-4\,a\,c)^{3/2}\sqrt{d+e\,x}\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 3577 leaves) :

$$\frac{1}{(a+x\,(b+c\,x))^{5/2}}\sqrt{d+e\,x}\,(a+b\,x+c\,x^2)^3\left(\frac{2\,(b\,c\,d^2-4\,a\,c\,d\,e+a\,b\,e^2+2\,c^2\,d^2\,x-2\,b\,c\,d\,e\,x+b^2\,e^2\,x-2\,a\,c\,e^2\,x)}{3\,c\,(-b^2+4\,a\,c)\,(a+b\,x+c\,x^2)^2}+\right.$$

$$\left.\frac{2\,(8\,b\,c^2\,d^2-9\,b^2\,c\,d\,e+4\,a\,c^2\,d\,e+b^3\,e^2+4\,a\,b\,c\,e^2+16\,c^3\,d^2\,x-16\,b\,c^2\,d\,e\,x+b^2\,c\,e^2\,x+12\,a\,c^2\,e^2\,x)}{3\,c\,(-b^2+4\,a\,c)^2\,(a+b\,x+c\,x^2)}\right)+\frac{1}{3\,(-b^2+4\,a\,c)^2\,e\,(a+x\,(b+c\,x))^{5/2}}$$

$$\left(a+b x+c x^2\right)^{5/2}\left(-\frac{2\left(16 c^2 d^2-16 b c d e+b^2 e^2+12 a c e^2\right)(d+e x)^{3/2}\left(c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)}{c \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}+\right.$$

$$\frac{1}{c \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}} 2\left(c d^2-b d e+a e^2\right)(d+e x) \sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\left(\left(4 i \sqrt{2} c^2 d^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\left.\text{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\right/$$

$$\left(\left(c d^2-b d e+a e^2\right) \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(4 i \sqrt{2} b c d e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(\mathrm{i}\, b^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(3\,\mathrm{i}\sqrt{2}\, a c e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(8\,\mathbb{i}\,\sqrt{2}\,c^2\,d\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(4\,\mathbb{i}\,\sqrt{2}\,b\,c\,e\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left.\left.\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)\right)$$

- **Problem 2478: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(d+e\,x)^{3/2}}{\left(a+b\,x+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 542 leaves, 7 steps):

$$-\frac{2\,\sqrt{d+e\,x}\,\left(b\,d-2\,a\,e+(2\,c\,d-b\,e)\,x\right)}{3\,\left(b^2-4\,a\,c\right)\,\left(a+b\,x+c\,x^2\right)^{3/2}}+\frac{2\,\sqrt{d+e\,x}\,\left(8\,b\,c\,d-5\,b^2\,e+4\,a\,c\,e+8\,c\,(2\,c\,d-b\,e)\,x\right)}{3\,\left(b^2-4\,a\,c\right)^2\,\sqrt{a+b\,x+c\,x^2}}-$$

$$\frac{8\,\sqrt{2}\,(2\,c\,d-b\,e)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\;e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]}{3\,\left(b^2-4\,a\,c\right)^{3/2}\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\;\sqrt{a+b\,x+c\,x^2}}+$$

$$\left(2\,\sqrt{2}\,\left(16\,c^2\,d^2+3\,b^2\,e^2-4\,c\,e\,(4\,b\,d-a\,e)\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\;\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\;e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/\left(3\,c\,\left(b^2-4\,a\,c\right)^{3/2}\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 5802 leaves):

$$\frac{\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)^3\left(\frac{2\,(-b\,d+2\,a\,e-2\,c\,d\,x+b\,e\,x)}{3\,(b^2-4\,a\,c)\,(a+b\,x+c\,x^2)^2}-\frac{2\,(-8\,b\,c\,d+5\,b^2\,e-4\,a\,c\,e-16\,c^2\,d\,x+8\,b\,c\,e\,x)}{3\,(b^2-4\,a\,c)^2\,(a+b\,x+c\,x^2)}\right)}{(a+x\,(b+c\,x))^{5/2}}-$$

$$\frac{1}{3\,(-b^2+4\,a\,c)^2\,e\,(a+x\,(b+c\,x))^{5/2}}\,2\,\left(a+b\,x+c\,x^2\right)^{5/2}\left(\frac{8\,(2\,c\,d-b\,e)\,(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)}{\sqrt{\frac{(d+e\,x)^2\left(c\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\,\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}-\right.$$

$$\left(4\,i\,\sqrt{2}\,c^2\,d^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}\right.$$

$$\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\;\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left((c\,d^2-b\,d\,e+a\,e^2)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\;\sqrt{\frac{(d+e\,x)^2\left(c\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\,\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)+$$

$$\left(6\,i\sqrt{2}\,b\,c\,d^2 e\left(2cd-be+\sqrt{b^2e^2-4ace^2}\right)(d+ex)\sqrt{c+\frac{cd^2}{(d+ex)^2}-\frac{bde}{(d+ex)^2}+\frac{ae^2}{(d+ex)^2}-\frac{2cd}{d+ex}+\frac{be}{d+ex}}\right.$$

$$\sqrt{1-\frac{2\left(cd^2-bde+ae^2\right)}{\left(2cd-be-\sqrt{b^2e^2-4ace^2}\right)(d+ex)}}\ \sqrt{1-\frac{2\left(cd^2-bde+ae^2\right)}{\left(2cd-be+\sqrt{b^2e^2-4ace^2}\right)(d+ex)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{cd^2-bde+ae^2}{2cd-be-\sqrt{b^2e^2-4ace^2}}}}{\sqrt{d+ex}}\right],\frac{2cd-be-\sqrt{b^2e^2-4ace^2}}{2cd-be+\sqrt{b^2e^2-4ace^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{cd^2-bde+ae^2}{2cd-be-\sqrt{b^2e^2-4ace^2}}}}{\sqrt{d+ex}}\right],\frac{2cd-be-\sqrt{b^2e^2-4ace^2}}{2cd-be+\sqrt{b^2e^2-4ace^2}}\right]\right)\right)\Big/$$

$$\left(\left(cd^2-bde+ae^2\right)\sqrt{-\frac{cd^2-bde+ae^2}{2cd-be-\sqrt{b^2e^2-4ace^2}}}\sqrt{c+\frac{cd^2-bde+ae^2}{(d+ex)^2}+\frac{-2cd+be}{d+ex}}\sqrt{\frac{(d+ex)^2\left(c\left(-1+\frac{d}{d+ex}\right)^2+\frac{e\left(b-\frac{bd}{d+ex}+\frac{ae}{d+ex}\right)}{d+ex}\right)}{e^2}}\right)-$$

$$\left(2\,i\sqrt{2}\,b^2 d\,e^2\left(2cd-be+\sqrt{b^2e^2-4ace^2}\right)(d+ex)\sqrt{c+\frac{cd^2}{(d+ex)^2}-\frac{bde}{(d+ex)^2}+\frac{ae^2}{(d+ex)^2}-\frac{2cd}{d+ex}+\frac{be}{d+ex}}\right.$$

$$\sqrt{1-\frac{2\left(cd^2-bde+ae^2\right)}{\left(2cd-be-\sqrt{b^2e^2-4ace^2}\right)(d+ex)}}\ \sqrt{1-\frac{2\left(cd^2-bde+ae^2\right)}{\left(2cd-be+\sqrt{b^2e^2-4ace^2}\right)(d+ex)}}$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)-$$

$$\left(4\,\mathrm{i}\sqrt{2}\,a c d e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left((c d^2 - b d e + a e^2) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}} \sqrt{\frac{(d + e x)^2 \left(c \left(-1 + \frac{d}{d+e x}\right)^2 + \frac{e \left(b - \frac{b d}{d+e x} + \frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}} + \right.$$

$$\left(2 i \sqrt{2} a b e^3 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x) \sqrt{c + \frac{c d^2}{(d + e x)^2} - \frac{b d e}{(d + e x)^2} + \frac{a e^2}{(d + e x)^2} - \frac{2 c d}{d + e x} + \frac{b e}{d + e x}}\right.$$

$$\sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}$$

$$\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right)\right/$$

$$\left((c d^2 - b d e + a e^2) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}} \sqrt{\frac{(d + e x)^2 \left(c \left(-1 + \frac{d}{d+e x}\right)^2 + \frac{e \left(b - \frac{b d}{d+e x} + \frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right) -$$

$$\left(8 i \sqrt{2} c^2 d^2 (d + e x) \sqrt{c + \frac{c d^2}{(d + e x)^2} - \frac{b d e}{(d + e x)^2} + \frac{a e^2}{(d + e x)^2} - \frac{2 c d}{d + e x} + \frac{b e}{d + e x}} \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\ \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)+$$

$$\Bigg(8\,i\sqrt{2}\,b c d e\,(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\ \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)-$$

$$\Bigg(3\,i\,b^2 e^2\,(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)-$$

$$\left(2\,\mathrm{i}\sqrt{2}\,a c e^2 (d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)\right)$$

- **Problem 2479: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{d+e x}}{\left(a+b x+c x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 605 leaves, 7 steps) :

$$-\frac{2\,(b+2\,c\,x)\,\sqrt{d+e\,x}}{3\,\left(b^2-4\,a\,c\right)\,\left(a+b\,x+c\,x^2\right)^{3/2}}-\frac{2\,\sqrt{d+e\,x}\,\left(9\,b^2\,c\,d\,e-4\,a\,c^2\,d\,e-b^3\,e^2-4\,b\,c\,\left(2\,c\,d^2+a\,e^2\right)-c\,\left(16\,c^2\,d^2+b^2\,e^2-4\,c\,e\,(4\,b\,d-3\,a\,e)\right)\,x\right)}{3\,\left(b^2-4\,a\,c\right)^2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{a+b\,x+c\,x^2}}-$$

$$\left(\sqrt{2}\,\left(16\,c^2\,d^2+b^2\,e^2-4\,c\,e\,(4\,b\,d-3\,a\,e)\right)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/$$

$$\left(3\,\left(b^2-4\,a\,c\right)^{3/2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\left(16\,\sqrt{2}\,(2\,c\,d-b\,e)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/$$

$$\left(3\,\left(b^2-4\,a\,c\right)^{3/2}\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 3560 leaves):

$$\frac{1}{(a+x\,(b+c\,x))^{5/2}}\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)^3$$

$$\left(-\frac{2\,(b+2\,c\,x)}{3\,\left(b^2-4\,a\,c\right)\,\left(a+b\,x+c\,x^2\right)^2}+\frac{2\,\left(8\,b\,c^2\,d^2-9\,b^2\,c\,d\,e+4\,a\,c^2\,d\,e+b^3\,e^2+4\,a\,b\,c\,e^2+16\,c^3\,d^2\,x-16\,b\,c^2\,d\,e\,x+b^2\,c\,e^2\,x+12\,a\,c^2\,e^2\,x\right)}{3\,\left(-b^2+4\,a\,c\right)^2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\left(a+b\,x+c\,x^2\right)}\right)+$$

$$\frac{1}{3\,\left(-b^2+4\,a\,c\right)^2\,e\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,(a+x\,(b+c\,x))^{5/2}}$$

$$2 c \left(a+b x+c x^2\right)^{5/2} \Bigg(\frac{\left(-16 c^2 d^2+16 b c d e-b^2 e^2-12 a c e^2\right) (d+e x)^{3/2} \left(c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)}{c \sqrt{\frac{(d+e x)^2 \left(c \left(-1+\frac{d}{d+e x}\right)^2+\frac{e \left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}+$$

$$\frac{1}{c \sqrt{\frac{(d+e x)^2 \left(c \left(-1+\frac{d}{d+e x}\right)^2+\frac{e \left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}} \left(c d^2-b d e+a e^2\right) (d+e x) \sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\Bigg(\Bigg(4 i \sqrt{2}\, c^2 d^2 \left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}} \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}$$

$$\Bigg(\text{EllipticE}\Big[i\, \text{ArcSinh}\Big[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\Big], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\Big]-$$

$$\text{EllipticF}\Big[i\, \text{ArcSinh}\Big[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\Big], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\Big]\Bigg)\Bigg)\Bigg/$$

$$\Bigg(\left(c d^2-b d e+a e^2\right) \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\Bigg)-$$

$$\Bigg(4 i \sqrt{2}\, b c d e \left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i\, b^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(3\, i\sqrt{2}\, a c e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(8 i \sqrt{2}\, c^2 d\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(4 i \sqrt{2}\, b c e\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left.\left.\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)\right)$$

- **Problem 2480: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{d+e\,x}\;\left(a+b\,x+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 725 leaves, 7 steps) :

$$-\frac{2\sqrt{d+e\,x}\,\left(b\,c\,d-b^2\,e+2\,a\,c\,e+c\,(2\,c\,d-b\,e)\,x\right)}{3\,\left(b^2-4\,a\,c\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\left(a+b\,x+c\,x^2\right)^{3/2}}-\Big(2\sqrt{d+e\,x}$$

$$\left(3\,a\,c\,e\,(2\,c\,d-b\,e)^2-\left(b\,c\,d-b^2\,e+2\,a\,c\,e\right)\,\left(8\,c^2\,d^2-2\,b^2\,e^2-5\,c\,e\,(b\,d-2\,a\,e)\right)-2\,c\,(2\,c\,d-b\,e)\,\left(4\,c^2\,d^2-b^2\,e^2-4\,c\,e\,(b\,d-2\,a\,e)\right)\,x\right)\Big)\Big/$$

$$\left(3\,\left(b^2-4\,a\,c\right)^2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,\sqrt{a+b\,x+c\,x^2}\right)-\Bigg(2\sqrt{2}\,(2\,c\,d-b\,e)\,\left(4\,c^2\,d^2-b^2\,e^2-4\,c\,e\,(b\,d-2\,a\,e)\right)\,\sqrt{d+e\,x}$$

$$\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4\,a\,c}\;e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\Bigg)\Bigg/$$

$$\left(3\,\left(b^2-4\,a\,c\right)^{3/2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\;\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\Bigg(2\sqrt{2}\,\left(16\,c^2\,d^2-b^2\,e^2-4\,c\,e\,(4\,b\,d-5\,a\,e)\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\;\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4\,a\,c}\;e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\Bigg)\Bigg/\left(3\,\left(b^2-4\,a\,c\right)^{3/2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{d+e\,x}\;\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 5566 leaves) :

$$\frac{1}{\left(a+x\,(b+c\,x)\right)^{5/2}}$$

$$\sqrt{d+e x}\,\left(a+b x+c x^2\right)^3\left(\frac{2\left(b c d-b^2 e+2 a c e+2 c^2 d x-b c e x\right)}{3\left(-b^2+4 a c\right)\left(c d^2-b d e+a e^2\right)\left(a+b x+c x^2\right)^2}+\left(2\left(8 b c^3 d^3-13 b^2 c^2 d^2 e+4 a c^3 d^2 e+3 b^3 c d e^2+12 a b c^2 d e^2+2 b^4 e^3-17 a b^2 c e^3+20 a^2 c^2 e^3+16 c^4 d^3 x-24 b c^3 d^2 e x+4 b^2 c^2 d e^2 x+32 a c^3 d e^2 x+2 b^3 c e^3 x-16 a b c^2 e^3 x\right)\right)\Big/\left(3\left(-b^2+4 a c\right)^2\left(c d^2-b d e+a e^2\right)^2\left(a+b x+c x^2\right)\right)\right)+\frac{1}{3\left(-b^2+4 a c\right)^2 e\left(c d^2-b d e+a e^2\right)^2\left(a+x\left(b+c x\right)\right)^{5/2}}$$

$$2 c\left(a+b x+c x^2\right)^{5/2}\left(-\left(2\left(2 c d-b e\right)\left(4 c^2 d^2-4 b c d e-b^2 e^2+8 a c e^2\right)\left(d+e x\right)^{3/2}\left(c+\frac{c d^2}{\left(d+e x\right)^2}-\frac{b d e}{\left(d+e x\right)^2}+\frac{a e^2}{\left(d+e x\right)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)\right)\Big/\left(c\sqrt{\frac{\left(d+e x\right)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)+\right.$$

$$\frac{1}{c\sqrt{\frac{\left(d+e x\right)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}\left(c d^2-b d e+a e^2\right)\left(d+e x\right)\sqrt{c+\frac{c d^2}{\left(d+e x\right)^2}-\frac{b d e}{\left(d+e x\right)^2}+\frac{a e^2}{\left(d+e x\right)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\left(\left(4 i\sqrt{2}\,c^3 d^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)\left(d+e x\right)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\left(d+e x\right)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(6\,i\,\sqrt{2}\,b\,c^2 d^2 e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(i\,\sqrt{2}\,b^2 c\,d\,e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(8\,i\sqrt{2}\,a c^2 d e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(i\,b^3 e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(4\,i\sqrt{2}\,a b c e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(8\,i\sqrt{2}\,c^3 d^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(8\, i \sqrt{2}\ b c^2 d e \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(i\, b^2 c e^2 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(10\, i \sqrt{2}\ a c^2 e^2 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)\Bigg)\Bigg)$$

- **Problem 2481: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{1}{(d+e x)^{3/2}\left(a+b x+c x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 918 leaves, 8 steps) :

$$-\frac{2\left(b c d-b^2 e+2 a c e+c\,(2 c d-b e)\,x\right)}{3\left(b^2-4 a c\right)\left(c d^2-b d e+a e^2\right)\sqrt{d+e x}\left(a+b x+c x^2\right)^{3/2}}-$$

$$\left(2\left(5 a c e\,(2 c d-b e)^2-\left(b c d-b^2 e+2 a c e\right)\left(8 c^2 d^2-4 b^2 e^2-c e\,(3 b d-14 a e)\right)-4 c\,(2 c d-b e)\left(2 c^2 d^2-b^2 e^2-2 c e\,(b d-3 a e)\right)x\right)\right)\Big/$$

$$\left(3\left(b^2-4 a c\right)^2\left(c d^2-b d e+a e^2\right)^2\sqrt{d+e x}\sqrt{a+b x+c x^2}\right)+$$

$$\left(2 e\left(16 c^4 d^4-8 b^4 e^4-4 c^3 d^2 e\,(8 b d-15 a e)+b^2 c e^3\,(7 b d+57 a e)+3 c^2 e^2\left(3 b^2 d^2-20 a b d e-28 a^2 e^2\right)\right)\sqrt{a+b x+c x^2}\right)\Big/$$

$$\left(3\left(b^2-4 a c\right)^2\left(c d^2-b d e+a e^2\right)^3\sqrt{d+e x}\right)-$$

$$\Bigg(\sqrt{2}\left(16 c^4 d^4-8 b^4 e^4-4 c^3 d^2 e\,(8 b d-15 a e)+b^2 c e^3\,(7 b d+57 a e)+3 c^2 e^2\left(3 b^2 d^2-20 a b d e-28 a^2 e^2\right)\right)$$

$$\sqrt{d+e\,x}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\;e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\Bigg)\Bigg/$$

$$\left(3\,(b^2-4\,a\,c)^{3/2}\,(c\,d^2-b\,d\,e+a\,e^2)^3\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\Bigg(8\,\sqrt{2}\,(2\,c\,d-b\,e)\,\left(2\,c^2\,d^2-b^2\,e^2-2\,c\,e\,(b\,d-3\,a\,e)\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\;e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\Bigg)\Bigg/\left(3\,(b^2-4\,a\,c)^{3/2}\,(c\,d^2-b\,d\,e+a\,e^2)^2\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 7870 leaves) :

$$\frac{1}{(a+x\,(b+c\,x))^{5/2}}\sqrt{d+e\,x}\,(a+b\,x+c\,x^2)^3$$

$$\Bigg(-\frac{2\,e^5}{(c\,d^2-b\,d\,e+a\,e^2)^3\,(d+e\,x)}-\frac{2\,(-b\,c^2\,d^2+2\,b^2\,c\,d\,e-4\,a\,c^2\,d\,e-b^3\,e^2+3\,a\,b\,c\,e^2-2\,c^3\,d^2\,x+2\,b\,c^2\,d\,e\,x-b^2\,c\,e^2\,x+2\,a\,c^2\,e^2\,x)}{3\,(-b^2+4\,a\,c)\,(c\,d^2-b\,d\,e+a\,e^2)^2\,(a+b\,x+c\,x^2)^2}-$$

$$\big(2\,(-8\,b\,c^4\,d^4+17\,b^2\,c^3\,d^3\,e-4\,a\,c^4\,d^3\,e-6\,b^3\,c^2\,d^2\,e^2-24\,a\,b\,c^3\,d^2\,e^2-8\,b^4\,c\,d\,e^3+69\,a\,b^2\,c^2\,d\,e^3-84\,a^2\,c^3\,d\,e^3+5\,b^5\,e^4-37\,a\,b^3\,c\,e^4+60\,a^2\,b\,c^2$$
$$e^4-16\,c^5\,d^4\,x+32\,b\,c^4\,d^3\,e\,x-9\,b^2\,c^3\,d^2\,e^2\,x-60\,a\,c^4\,d^2\,e^2\,x-7\,b^3\,c^2\,d\,e^3\,x+60\,a\,b\,c^3\,d\,e^3\,x+5\,b^4\,c\,e^4\,x-33\,a\,b^2\,c^2\,e^4\,x+36\,a^2\,c^3\,e^4\,x)\big)\Big/$$

$$\left(3\,(-b^2+4\,a\,c)^2\,(c\,d^2-b\,d\,e+a\,e^2)^3\,(a+b\,x+c\,x^2)\right)\Bigg)-\frac{1}{3\,(-b^2+4\,a\,c)^2\,e\,(c\,d^2-b\,d\,e+a\,e^2)^3\,(a+x\,(b+c\,x))^{5/2}}$$

$$2\,c\,\left(a+b\,x+c\,x^2\right)^{5/2}\left(\left(\left(16\,c^4\,d^4-32\,b\,c^3\,d^3\,e+9\,b^2\,c^2\,d^2\,e^2+60\,a\,c^3\,d^2\,e^2+7\,b^3\,c\,d\,e^3-60\,a\,b\,c^2\,d\,e^3-8\,b^4\,e^4+57\,a\,b^2\,c\,e^4-84\,a^2\,c^2\,e^4\right)\right.\right.$$

$$\left.\left.(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)\right)\middle/\left(c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)-\right.$$

$$\frac{1}{c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\left(\left(\left(4\,\mathbb{i}\,\sqrt{2}\,c^4\,d^4\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.\right.\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\right/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(8\,i\,\sqrt{2}\,b\,c^3\,d^3\,e\left(2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(9\,i\,b^2\,c^2\,d^2\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(15\, i \sqrt{2}\, a c^3 d^2 e^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(7\, i\, b^3 c d e^3 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(2 \sqrt{2} \left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\left(15\, i \sqrt{2}\, a\, b\, c^2\, d\, e^3 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2\left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right)(d + e x)}}\right.$$

$$\sqrt{1 - \frac{2\left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right)(d + e x)}} \left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\left(2\, i \sqrt{2}\, b^4 e^4 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2\left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right)(d + e x)}}\right.$$

$$\sqrt{1 - \frac{2\left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right)(d + e x)}} \left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(57\,i\,a\,b^2\,c\,e^4\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(21\,i\sqrt{2}\,a^2\,c^2\,e^4\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(8\,i\sqrt{2}\,c^4 d^3\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(12\,i\sqrt{2}\,b c^3 d^2 e\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(24\,i\sqrt{2}\,a c^3 d e^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(2\, i \sqrt{2}\, b^3 c\, e^3 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(12\, i \sqrt{2}\, a\, b\, c^2 e^3 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left.\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)\right)$$

- **Problem 2482: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{3+5x}}{\sqrt{2+5x-12x^2}}\,dx$$

Optimal (type 4, 30 leaves, 2 steps) :

$$-\frac{1}{3}\sqrt{19}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{2\sqrt{2-3x}}{\sqrt{11}}\right],\frac{55}{76}\right]$$

Result (type 4, 86 leaves) :

$$\frac{\sqrt{19}\sqrt{-1-4x}\sqrt{2-3x}\left(-\text{EllipticE}\left[\text{ArcSin}\left[\frac{2\sqrt{3+5x}}{\sqrt{7}}\right],\frac{21}{76}\right]+\text{EllipticF}\left[\text{ArcSin}\left[\frac{2\sqrt{3+5x}}{\sqrt{7}}\right],\frac{21}{76}\right]\right)}{3\sqrt{2+5x-12x^2}}$$

Problem 2483: Result unnecessarily involves higher level functions.

$$\int (d+e\,x)^2\left(a+b\,x+c\,x^2\right)^{4/3}dx$$

Optimal (type 4, 638 leaves, 6 steps) :

$$-\frac{3\left(b^2-4ac\right)\left(17c^2d^2+5b^2e^2-ce\,(17bd+3ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{1/3}}{935\,c^4}+$$
$$\frac{3\left(17c^2d^2+5b^2e^2-ce\,(17bd+3ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{4/3}}{374\,c^3}+\frac{15e\,(2cd-be)\left(a+bx+cx^2\right)^{7/3}}{119\,c^2}+$$
$$\frac{3e\,(d+ex)\left(a+bx+cx^2\right)^{7/3}}{17c}+\left(2^{1/3}\,3^{3/4}\sqrt{2+\sqrt{3}}\left(b^2-4ac\right)^2\left(17c^2d^2+5b^2e^2-ce\,(17bd+3ae)\right)\right.$$
$$\left(\left(b^2-4ac\right)^{1/3}+2^{2/3}c^{1/3}\left(a+bx+cx^2\right)^{1/3}\right)\sqrt{\frac{\left(b^2-4ac\right)^{2/3}-2^{2/3}c^{1/3}\left(b^2-4ac\right)^{1/3}\left(a+bx+cx^2\right)^{1/3}+2\times2^{1/3}c^{2/3}\left(a+bx+cx^2\right)^{2/3}}{\left(\left(1+\sqrt{3}\right)\left(b^2-4ac\right)^{1/3}+2^{2/3}c^{1/3}\left(a+bx+cx^2\right)^{1/3}\right)^2}}$$
$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\left(b^2-4ac\right)^{1/3}+2^{2/3}c^{1/3}\left(a+bx+cx^2\right)^{1/3}}{\left(1+\sqrt{3}\right)\left(b^2-4ac\right)^{1/3}+2^{2/3}c^{1/3}\left(a+bx+cx^2\right)^{1/3}}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$
$$\left(935\,c^{13/3}\,(b+2cx)\sqrt{\frac{\left(b^2-4ac\right)^{1/3}\left(\left(b^2-4ac\right)^{1/3}+2^{2/3}c^{1/3}\left(a+bx+cx^2\right)^{1/3}\right)}{\left(\left(1+\sqrt{3}\right)\left(b^2-4ac\right)^{1/3}+2^{2/3}c^{1/3}\left(a+bx+cx^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 413 leaves) :

$$-\frac{1}{26180\,c^5\,(a+x\,(b+c\,x))^{2/3}}$$

$$3\Big(2\,c\,(a+x\,(b+c\,x))\,\big(70\,b^5\,e^2-7\,b^4\,c\,e\,(34\,d+5\,e\,x)+b^2\,c^2\,\big(a\,e\,(1547\,d+211\,e\,x)-c\,x\,\big(119\,d^2+85\,d\,e\,x+20\,e^2\,x^2\big)\big)+b^3\,c\,\big(-497\,a\,e^2+c\,\big(238\,d^2+119\,d\,e\,x+25\,e^2\,x^2\big)\big)-b\,c^2\,\big(-823\,a^2\,e^2+15\,c^2\,x^2\,\big(119\,d^2+170\,d\,e\,x+66\,e^2\,x^2\big)+a\,c\,\big(1547\,d^2+646\,d\,e\,x+125\,e^2\,x^2\big)\big)-2\,c^3\,\big(a^2\,e\,(935\,d+112\,e\,x)+5\,c^2\,x^3\,\big(119\,d^2+187\,d\,e\,x+77\,e^2\,x^2\big)+a\,c\,x\,\big(1547\,d^2+1870\,d\,e\,x+665\,e^2\,x^2\big)\big)\big)-7\times 2^{1/3}\,\big(b^2-4\,a\,c\big)^2\,\big(17\,c^2\,d^2+5\,b^2\,e^2-c\,e\,(17\,b\,d+3\,a\,e)\big)\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\Big)\left(\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{2/3}\text{Hypergeometric2F1}\Big[\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{-b+\sqrt{b^2-4\,a\,c}-2\,c\,x}{2\,\sqrt{b^2-4\,a\,c}}\Big]\Big)$$

- **Problem 2484: Result unnecessarily involves higher level functions.**

$$\int (d+e\,x)\,\big(a+b\,x+c\,x^2\big)^{4/3}\,dx$$

Optimal (type 4, 539 leaves, 5 steps):

$$-\frac{3\,\big(b^2-4\,a\,c\big)\,(2\,c\,d-b\,e)\,(b+2\,c\,x)\,\big(a+b\,x+c\,x^2\big)^{1/3}}{110\,c^3}+\frac{3\,(2\,c\,d-b\,e)\,(b+2\,c\,x)\,\big(a+b\,x+c\,x^2\big)^{4/3}}{44\,c^2}+\frac{3\,e\,\big(a+b\,x+c\,x^2\big)^{7/3}}{14\,c}+\Bigg(3^{3/4}\,\sqrt{2+\sqrt{3}}\,\big(b^2-4\,a\,c\big)^2\,(2\,c\,d-b\,e)\,\Big(\big(b^2-4\,a\,c\big)^{1/3}+2^{2/3}\,c^{1/3}\,\big(a+b\,x+c\,x^2\big)^{1/3}\Big)\sqrt{\frac{\big(b^2-4\,a\,c\big)^{2/3}-2^{2/3}\,c^{1/3}\,\big(b^2-4\,a\,c\big)^{1/3}\,\big(a+b\,x+c\,x^2\big)^{1/3}+2\times 2^{1/3}\,c^{2/3}\,\big(a+b\,x+c\,x^2\big)^{2/3}}{\Big(\big(1+\sqrt{3}\big)\,\big(b^2-4\,a\,c\big)^{1/3}+2^{2/3}\,c^{1/3}\,\big(a+b\,x+c\,x^2\big)^{1/3}\Big)^2}}\;\text{EllipticF}\Big[\text{ArcSin}\Big[\frac{\big(1-\sqrt{3}\big)\,\big(b^2-4\,a\,c\big)^{1/3}+2^{2/3}\,c^{1/3}\,\big(a+b\,x+c\,x^2\big)^{1/3}}{\big(1+\sqrt{3}\big)\,\big(b^2-4\,a\,c\big)^{1/3}+2^{2/3}\,c^{1/3}\,\big(a+b\,x+c\,x^2\big)^{1/3}}\Big],\,-7-4\,\sqrt{3}\Big]\Bigg)\Bigg/\Bigg(55\times 2^{2/3}\,c^{10/3}\,(b+2\,c\,x)\,\sqrt{\frac{\big(b^2-4\,a\,c\big)^{1/3}\,\Big(\big(b^2-4\,a\,c\big)^{1/3}+2^{2/3}\,c^{1/3}\,\big(a+b\,x+c\,x^2\big)^{1/3}\Big)}{\Big(\big(1+\sqrt{3}\big)\,\big(b^2-4\,a\,c\big)^{1/3}+2^{2/3}\,c^{1/3}\,\big(a+b\,x+c\,x^2\big)^{1/3}\Big)^2}}\Bigg)$$

Result (type 5, 273 leaves):

$$-\frac{1}{3080\,c^4\,(a+x\,(b+c\,x))^{2/3}}\,3\,\Big(2\,c\,(a+x\,(b+c\,x))\,\big(-14\,b^4\,e+7\,b^3\,c\,(4\,d+e\,x)+b^2\,c\,(91\,a\,e-c\,x\,(14\,d+5\,e\,x))-2\,b\,c^2\,\big(15\,c\,x^2\,(7\,d+5\,e\,x)+a\,(91\,d+19\,e\,x)\big)-2\,c^2\,\big(55\,a^2\,e+5\,c^2\,x^3\,(14\,d+11\,e\,x)+2\,a\,c\,x\,(91\,d+55\,e\,x)\big)\big)+7\times 2^{1/3}\,\big(b^2-4\,a\,c\big)^2\,(-2\,c\,d+b\,e)\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\Big)\left(\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{2/3}\text{Hypergeometric2F1}\Big[\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{-b+\sqrt{b^2-4\,a\,c}-2\,c\,x}{2\,\sqrt{b^2-4\,a\,c}}\Big]\Big)$$

Problem 2485: Result unnecessarily involves higher level functions.

$$\int \left(a+b\,x+c\,x^2\right)^{4/3}\,dx$$

Optimal (type 4, 490 leaves, 4 steps):

$$-\frac{3\,\left(b^2-4\,a\,c\right)\,(b+2\,c\,x)\,\left(a+b\,x+c\,x^2\right)^{1/3}}{55\,c^2}+\frac{3\,(b+2\,c\,x)\,\left(a+b\,x+c\,x^2\right)^{4/3}}{22\,c}+\left(2^{1/3}\,3^{3/4}\,\sqrt{2+\sqrt{3}}\,\left(b^2-4\,a\,c\right)^2\,\left(\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)\sqrt{\frac{\left(b^2-4\,a\,c\right)^{2/3}-2^{2/3}\,c^{1/3}\,\left(b^2-4\,a\,c\right)^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}+2\times 2^{1/3}\,c^{2/3}\,\left(a+b\,x+c\,x^2\right)^{2/3}}{\left(\left(1+\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)^2}}\;\text{EllipticF}\Big[\text{ArcSin}\Big[\frac{\left(1-\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}}{\left(1+\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}}\Big],-7-4\,\sqrt{3}\Big]\right)\Bigg/\left(55\,c^{7/3}\,(b+2\,c\,x)\,\sqrt{\frac{\left(b^2-4\,a\,c\right)^{1/3}\,\left(\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)}{\left(\left(1+\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 179 leaves):

$$\frac{1}{220\,c^3\,(a+x\,(b+c\,x))^{2/3}}3\left(2\,c\,(b+2\,c\,x)\,(a+x\,(b+c\,x))\,\left(-2\,b^2+5\,b\,c\,x+c\,\left(13\,a+5\,c\,x^2\right)\right)+\right.$$
$$\left.2^{1/3}\,\left(b^2-4\,a\,c\right)^2\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\,\left(\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,\frac{2}{3},\,\frac{4}{3},\,\frac{-b+\sqrt{b^2-4\,a\,c}-2\,c\,x}{2\,\sqrt{b^2-4\,a\,c}}\right]\right)$$

- **Problem 2486: Unable to integrate problem.**

$$\int\frac{\left(a+b\,x+c\,x^2\right)^{4/3}}{d+e\,x}\,dx$$

Optimal (type 6, 180 leaves, 2 steps) :

$$\frac{3\,\left(a+b\,x+c\,x^2\right)^{4/3}\,\text{AppellF1}\left[-\frac{8}{3},\,-\frac{4}{3},\,-\frac{4}{3},\,-\frac{5}{3},\,\frac{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)\,e}{2\,c\,(d+e\,x)},\,\frac{2\,d-\frac{\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}{c}}{2\,(d+e\,x)}\right]}{2^{1/3}\,e\,\left(\frac{e\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}{c\,(d+e\,x)}\right)^{4/3}\,\left(\frac{e\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}{c\,(d+e\,x)}\right)^{4/3}}$$

Result (type 8, 24 leaves) :

$$\int\frac{\left(a+b\,x+c\,x^2\right)^{4/3}}{d+e\,x}\,dx$$

- **Problem 2487: Unable to integrate problem.**

$$\int\frac{\left(a+b\,x+c\,x^2\right)^{4/3}}{(d+e\,x)^2}\,dx$$

Optimal (type 6, 189 leaves, 2 steps) :

$$\frac{12\times 2^{2/3}\,\left(a+b\,x+c\,x^2\right)^{4/3}\,\text{AppellF1}\left[-\frac{5}{3},\,-\frac{4}{3},\,-\frac{4}{3},\,-\frac{2}{3},\,\frac{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)\,e}{2\,c\,(d+e\,x)},\,\frac{2\,d-\frac{\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}{c}}{2\,(d+e\,x)}\right]}{5\,e\,\left(\frac{e\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}{c\,(d+e\,x)}\right)^{4/3}\,\left(\frac{e\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}{c\,(d+e\,x)}\right)^{4/3}\,(d+e\,x)}$$

Result (type 8, 24 leaves) :

$$\int\frac{\left(a+b\,x+c\,x^2\right)^{4/3}}{(d+e\,x)^2}\,dx$$

Problem 2488: Unable to integrate problem.

$$\int \frac{\left(a+b x+c x^2\right)^{4/3}}{(d+e x)^3} \, dx$$

Optimal (type 6, 187 leaves, 2 steps):

$$\frac{6\times 2^{2/3} \left(a+b x+c x^2\right)^{4/3} \text{AppellF1}\left[-\frac{2}{3},-\frac{4}{3},-\frac{4}{3},\frac{1}{3},\frac{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}{2 c (d+e x)},\frac{2 d-\frac{\left(b+\sqrt{b^2-4 a c}\right) e}{c}}{2 (d+e x)}\right]}{e \left(\frac{e \left(b-\sqrt{b^2-4 a c}+2 c x\right)}{c (d+e x)}\right)^{4/3} \left(\frac{e \left(b+\sqrt{b^2-4 a c}+2 c x\right)}{c (d+e x)}\right)^{4/3} (d+e x)^2}$$

Result (type 8, 24 leaves):

$$\int \frac{\left(a+b x+c x^2\right)^{4/3}}{(d+e x)^3} \, dx$$

Problem 2489: Result unnecessarily involves higher level functions.

$$\int \frac{(d+e x)^3}{\left(a+b x+c x^2\right)^{7/3}} \, dx$$

Optimal (type 4, 1224 leaves, 6 steps):

$$-\frac{3\,(d+e\,x)^2\,(b\,d-2\,a\,e+(2\,c\,d-b\,e)\,x)}{4\,\left(b^2-4\,a\,c\right)\,\left(a+b\,x+c\,x^2\right)^{4/3}}+$$

$$\left(3\,\left(10\,b\,c\,d\,\left(c\,d^2+3\,a\,e^2\right)-8\,a\,c\,e\,\left(2\,c\,d^2+3\,a\,e^2\right)-b^2\,\left(11\,c\,d^2\,e-a\,e^3\right)+(2\,c\,d-b\,e)\,\left(10\,c^2\,d^2-b^2\,e^2-2\,c\,e\,(5\,b\,d-7\,a\,e)\right)\,x\right)\right)\Big/$$

$$\left(4\,c\,\left(b^2-4\,a\,c\right)^2\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)-\frac{3\,(2\,c\,d-b\,e)\,\left(5\,c^2\,d^2-b^2\,e^2-c\,e\,(5\,b\,d-9\,a\,e)\right)\,(b+2\,c\,x)}{2\times 2^{1/3}\,c^{5/3}\,\left(b^2-4\,a\,c\right)^2\,\left(\left(1+\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)}+$$

$$\left(3\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,(2\,c\,d-b\,e)\,\left(5\,c^2\,d^2-b^2\,e^2-c\,e\,(5\,b\,d-9\,a\,e)\right)\,\left(\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)\right.$$

$$\sqrt{\frac{\left(b^2-4\,a\,c\right)^{2/3}-2^{2/3}\,c^{1/3}\,\left(b^2-4\,a\,c\right)^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}+2\times 2^{1/3}\,c^{2/3}\,\left(a+b\,x+c\,x^2\right)^{2/3}}{\left(\left(1+\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)^2}}$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}}{\left(1+\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(4\times 2^{1/3}\,c^{5/3}\,\left(b^2-4\,a\,c\right)^{5/3}\,(b+2\,c\,x)\,\sqrt{\frac{\left(b^2-4\,a\,c\right)^{1/3}\,\left(\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)}{\left(\left(1+\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)^2}}\right)-$$

$$\left(3^{3/4}\,(2\,c\,d-b\,e)\,\left(5\,c^2\,d^2-b^2\,e^2-c\,e\,(5\,b\,d-9\,a\,e)\right)\,\left(\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)\right.$$

$$\sqrt{\frac{\left(b^2-4\,a\,c\right)^{2/3}-2^{2/3}\,c^{1/3}\,\left(b^2-4\,a\,c\right)^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}+2\times 2^{1/3}\,c^{2/3}\,\left(a+b\,x+c\,x^2\right)^{2/3}}{\left(\left(1+\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)^2}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}}{\left(1+\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(2^{5/6}\,c^{5/3}\,\left(b^2-4\,a\,c\right)^{5/3}\,(b+2\,c\,x)\,\sqrt{\frac{\left(b^2-4\,a\,c\right)^{1/3}\,\left(\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)}{\left(\left(1+\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 400 leaves) :

$$-\frac{1}{16\,c^2\,\left(b^2-4\,a\,c\right)^2\,\left(a+x\,\left(b+c\,x\right)\right)^{4/3}}$$

$$3\,\Bigg(4\,\left(b^2-4\,a\,c\right)\,\left(-b^3\,e^3\,x+b^2\,e^2\,\left(-a\,e+3\,c\,d\,x\right)+2\,c\,\left(a^2\,e^3+c^2\,d^3\,x-3\,a\,c\,d\,e\,\left(d+e\,x\right)\right)+b\,c\,\left(c\,d^2\,\left(d-3\,e\,x\right)+3\,a\,e^2\,\left(d+e\,x\right)\right)\right)-$$

$$4\,\left(a+x\,\left(b+c\,x\right)\right)\,\left(-b^4\,e^3+b^3\,c\,e^2\,\left(3\,d+2\,e\,x\right)+4\,c^2\,\left(-8\,a^2\,e^3+5\,c^2\,d^3\,x+9\,a\,c\,d\,e^2\,x\right)+2\,b\,c^2\,\left(5\,c\,d^2\,\left(d-3\,e\,x\right)+9\,a\,e^2\,\left(d-e\,x\right)\right)+b^2\,c\,e\,\left(7\,a\,e^2+3\,c\,d\,\left(-5\,d+2\,e\,x\right)\right)\right)+2^{2/3}\,\left(-2\,c\,d+b\,e\right)\,\left(-5\,c^2\,d^2+b^2\,e^2+c\,e\,\left(5\,b\,d-9\,a\,e\right)\right)$$

$$\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\,\left(\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{1/3}\,\left(a+x\,\left(b+c\,x\right)\right)\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,\frac{2}{3},\,\frac{5}{3},\,\frac{-b+\sqrt{b^2-4\,a\,c}-2\,c\,x}{2\,\sqrt{b^2-4\,a\,c}}\right]\Bigg)$$

- **Problem 2490: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(d+e\,x\right)^2}{\left(a+b\,x+c\,x^2\right)^{7/3}}\,dx$$

Optimal (type 4, 1153 leaves, 6 steps) :

$$-\frac{3\,(d+e\,x)\,\left(b\,d-2\,a\,e+(2\,c\,d-b\,e)\,x\right)}{4\,\left(b^2-4\,a\,c\right)\,\left(a+b\,x+c\,x^2\right)^{4/3}}-\frac{3\,\left(4\,b^2\,d\,e+4\,a\,c\,d\,e-5\,b\,\left(c\,d^2+a\,e^2\right)-\left(10\,c^2\,d^2+b^2\,e^2-2\,c\,e\,(5\,b\,d-3\,a\,e)\right)\,x\right)}{2\,\left(b^2-4\,a\,c\right)^2\,\left(a+b\,x+c\,x^2\right)^{1/3}}-$$

$$\frac{3\,\left(10\,c^2\,d^2+b^2\,e^2-2\,c\,e\,(5\,b\,d-3\,a\,e)\right)\,(b+2\,c\,x)}{2\times 2^{1/3}\,c^{2/3}\,\left(b^2-4\,a\,c\right)^2\,\left(\left(1+\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)}+$$

$$\left(3\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,\left(10\,c^2\,d^2+b^2\,e^2-2\,c\,e\,(5\,b\,d-3\,a\,e)\right)\,\left(\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)\right.$$

$$\sqrt{\frac{\left(b^2-4\,a\,c\right)^{2/3}-2^{2/3}\,c^{1/3}\,\left(b^2-4\,a\,c\right)^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}+2\times 2^{1/3}\,c^{2/3}\,\left(a+b\,x+c\,x^2\right)^{2/3}}{\left(\left(1+\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)^2}}$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}}{\left(1+\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(4\times 2^{1/3}\,c^{2/3}\,\left(b^2-4\,a\,c\right)^{5/3}\,(b+2\,c\,x)\,\sqrt{\frac{\left(b^2-4\,a\,c\right)^{1/3}\,\left(\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)}{\left(\left(1+\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)^2}}\right)-$$

$$\left(3^{3/4}\,\left(10\,c^2\,d^2+b^2\,e^2-2\,c\,e\,(5\,b\,d-3\,a\,e)\right)\,\left(\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)\right.$$

$$\sqrt{\frac{\left(b^2-4\,a\,c\right)^{2/3}-2^{2/3}\,c^{1/3}\,\left(b^2-4\,a\,c\right)^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}+2\times 2^{1/3}\,c^{2/3}\,\left(a+b\,x+c\,x^2\right)^{2/3}}{\left(\left(1+\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)^2}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}}{\left(1+\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(2^{5/6}\,c^{2/3}\,\left(b^2-4\,a\,c\right)^{5/3}\,(b+2\,c\,x)\,\sqrt{\frac{\left(b^2-4\,a\,c\right)^{1/3}\,\left(\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)}{\left(\left(1+\sqrt{3}\right)\,\left(b^2-4\,a\,c\right)^{1/3}+2^{2/3}\,c^{1/3}\,\left(a+b\,x+c\,x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 278 leaves) :

$$-\frac{1}{16 c \left(b^2-4 a c\right)^2 (a+x (b+c x))^{4/3}} 3 \left(-4 \left(10 c^2 d^2+b^2 e^2+2 c e (-5 b d+3 a e)\right) (b+2 c x) (a+x (b+c x))+ 4 \left(b^2-4 a c\right) \left(a b e^2+2 c^2 d^2 x+b^2 e^2 x+b c d (d-2 e x)-2 a c e (2 d+e x)\right)+2^{2/3} \left(10 c^2 d^2+b^2 e^2+2 c e (-5 b d+3 a e)\right) \left(b-\sqrt{b^2-4 a c}+2 c x\right) \left(\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}\right)^{1/3} (a+x (b+c x)) \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{-b+\sqrt{b^2-4 a c}-2 c x}{2 \sqrt{b^2-4 a c}}\right]\right)$$

- **Problem 2491: Result unnecessarily involves higher level functions.**

$$\int \frac{d+e x}{\left(a+b x+c x^2\right)^{7/3}} \, dx$$

Optimal (type 4, 1043 leaves, 6 steps) :

$$-\frac{3\,(b\,d-2\,a\,e+(2\,c\,d-b\,e)\,x)}{4\,(b^2-4\,a\,c)\,(a+b\,x+c\,x^2)^{4/3}}+\frac{15\,(2\,c\,d-b\,e)\,(b+2\,c\,x)}{4\,(b^2-4\,a\,c)^2\,(a+b\,x+c\,x^2)^{1/3}}-$$

$$\frac{15\,c^{1/3}\,(2\,c\,d-b\,e)\,(b+2\,c\,x)}{2\times 2^{1/3}\,(b^2-4\,a\,c)^2\,\left(\left(1+\sqrt{3}\right)\,(b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}\right)}+\Bigg(15\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\;c^{1/3}\,(2\,c\,d-b\,e)$$

$$\left((b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}\right)\sqrt{\frac{(b^2-4\,a\,c)^{2/3}-2^{2/3}\,c^{1/3}\,(b^2-4\,a\,c)^{1/3}\,(a+b\,x+c\,x^2)^{1/3}+2\times 2^{1/3}\,c^{2/3}\,(a+b\,x+c\,x^2)^{2/3}}{\left(\left(1+\sqrt{3}\right)\,(b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}\right)^2}}$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,(b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}}{\left(1+\sqrt{3}\right)\,(b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}}\right],\,-7-4\,\sqrt{3}\right]\Bigg)\Bigg/$$

$$\left(4\times 2^{1/3}\,(b^2-4\,a\,c)^{5/3}\,(b+2\,c\,x)\,\sqrt{\frac{(b^2-4\,a\,c)^{1/3}\,\left((b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}\right)}{\left(\left(1+\sqrt{3}\right)\,(b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}\right)^2}}\right)-$$

$$\Bigg(5\times 3^{3/4}\,c^{1/3}\,(2\,c\,d-b\,e)\,\left((b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}\right)$$

$$\sqrt{\frac{(b^2-4\,a\,c)^{2/3}-2^{2/3}\,c^{1/3}\,(b^2-4\,a\,c)^{1/3}\,(a+b\,x+c\,x^2)^{1/3}+2\times 2^{1/3}\,c^{2/3}\,(a+b\,x+c\,x^2)^{2/3}}{\left(\left(1+\sqrt{3}\right)\,(b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}\right)^2}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,(b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}}{\left(1+\sqrt{3}\right)\,(b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}}\right],\,-7-4\,\sqrt{3}\right]\Bigg)\Bigg/$$

$$\left(2^{5/6}\,(b^2-4\,a\,c)^{5/3}\,(b+2\,c\,x)\,\sqrt{\frac{(b^2-4\,a\,c)^{1/3}\,\left((b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}\right)}{\left(\left(1+\sqrt{3}\right)\,(b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}\right)^2}}\right)$$

Result (type 5, 200 leaves) :

$$\left(3\left(20\,(2\,c\,d-b\,e)\,(b+2\,c\,x)+\frac{4\left(b^2-4\,a\,c\right)\,(-b\,d+2\,a\,e-2\,c\,d\,x+b\,e\,x)}{a+x\,(b+c\,x)}+5\times 2^{2/3}\,(-2\,c\,d+b\,e)\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\right.\right.$$

$$\left.\left.\left(\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\,\frac{2}{3},\,\frac{5}{3},\,\frac{-b+\sqrt{b^2-4\,a\,c}-2\,c\,x}{2\,\sqrt{b^2-4\,a\,c}}\right]\right)\right)\bigg/\left(16\left(b^2-4\,a\,c\right)^2\,(a+x\,(b+c\,x))^{1/3}\right)$$

- **Problem 2492: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\left(a+b\,x+c\,x^2\right)^{7/3}}\,dx$$

Optimal (type 4, 993 leaves, 6 steps) :

$$-\frac{3\,(b+2\,c\,x)}{4\,(b^2-4\,a\,c)\,(a+b\,x+c\,x^2)^{4/3}}+\frac{15\,c\,(b+2\,c\,x)}{2\,(b^2-4\,a\,c)^2\,(a+b\,x+c\,x^2)^{1/3}}-$$

$$\frac{15\,c^{4/3}\,(b+2\,c\,x)}{2^{1/3}\,(b^2-4\,a\,c)^2\,\left(\left(1+\sqrt{3}\right)\,(b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}\right)}+\Bigg(15\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\;c^{4/3}$$

$$\left((b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}\right)\sqrt{\frac{(b^2-4\,a\,c)^{2/3}-2^{2/3}\,c^{1/3}\,(b^2-4\,a\,c)^{1/3}\,(a+b\,x+c\,x^2)^{1/3}+2\times 2^{1/3}\,c^{2/3}\,(a+b\,x+c\,x^2)^{2/3}}{\left(\left(1+\sqrt{3}\right)\,(b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}\right)^2}}$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,(b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}}{\left(1+\sqrt{3}\right)\,(b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}}\right],\,-7-4\,\sqrt{3}\right]\Bigg)\Bigg/$$

$$\left(2\times 2^{1/3}\,(b^2-4\,a\,c)^{5/3}\,(b+2\,c\,x)\,\sqrt{\frac{(b^2-4\,a\,c)^{1/3}\,\left((b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}\right)}{\left(\left(1+\sqrt{3}\right)\,(b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}\right)^2}}\right)-$$

$$\Bigg(5\times 2^{1/6}\,3^{3/4}\,c^{4/3}\,\left((b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}\right)$$

$$\sqrt{\frac{(b^2-4\,a\,c)^{2/3}-2^{2/3}\,c^{1/3}\,(b^2-4\,a\,c)^{1/3}\,(a+b\,x+c\,x^2)^{1/3}+2\times 2^{1/3}\,c^{2/3}\,(a+b\,x+c\,x^2)^{2/3}}{\left(\left(1+\sqrt{3}\right)\,(b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}\right)^2}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,(b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}}{\left(1+\sqrt{3}\right)\,(b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}}\right],\,-7-4\,\sqrt{3}\right]\Bigg)\Bigg/$$

$$\left((b^2-4\,a\,c)^{5/3}\,(b+2\,c\,x)\,\sqrt{\frac{(b^2-4\,a\,c)^{1/3}\,\left((b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}\right)}{\left(\left(1+\sqrt{3}\right)\,(b^2-4\,a\,c)^{1/3}+2^{2/3}\,c^{1/3}\,(a+b\,x+c\,x^2)^{1/3}\right)^2}}\right)$$

Result (type 5, 173 leaves) :

$$\left(3 \left(20 c (b+2 c x)-\frac{2 \left(b^2-4 a c\right) (b+2 c x)}{a+x (b+c x)}-5\times 2^{2/3} c \left(b-\sqrt{b^2-4 a c}+2 c x\right) \left(\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{-b+\sqrt{b^2-4 a c}-2 c x}{2 \sqrt{b^2-4 a c}}\right]\right)\right) \Big/ \left(8 \left(b^2-4 a c\right)^2 (a+x (b+c x))^{1/3}\right)$$

- **Problem 2496: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(d+e x) \left(c^2 d^2-b c d e+b^2 e^2+3 b c e^2 x+3 c^2 e^2 x^2\right)^{1/3}} \, dx$$

Optimal (type 3, 242 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2 (c d-b e-c e x)}{\sqrt{3} (2 c d-b e)^{1/3} \left(c^2 d^2-b c d e+b^2 e^2+3 b c e^2 x+3 c^2 e^2 x^2\right)^{1/3}}\right]}{\sqrt{3} e (2 c d-b e)^{2/3}}-\frac{\text{Log}[d+e x]}{2 e (2 c d-b e)^{2/3}}+\frac{\text{Log}\left[3 c e^2 (c d-b e)-3 c^2 e^3 x-3 c e^2 (2 c d-b e)^{1/3} \left(c^2 d^2-b c d e+b^2 e^2+3 b c e^2 x+3 c^2 e^2 x^2\right)^{1/3}\right]}{2 e (2 c d-b e)^{2/3}}$$

Result (type 6, 317 leaves):

$$-\left(3^{1/3} \left(\frac{3 b c e^2-\sqrt{3} \sqrt{-c^2 e^2 (-2 c d+b e)^2}+6 c^2 e^2 x}{c^2 e (d+e x)}\right)^{1/3} \left(\frac{3 b c e^2+\sqrt{3} \sqrt{-c^2 e^2 (-2 c d+b e)^2}+6 c^2 e^2 x}{c^2 e (d+e x)}\right)^{1/3} \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},-\frac{-6 c^2 d e+3 b c e^2+\sqrt{3} \sqrt{-c^2 e^2 (-2 c d+b e)^2}}{6 c^2 e (d+e x)},\frac{6 c^2 d e-3 b c e^2+\sqrt{3} \sqrt{-c^2 e^2 (-2 c d+b e)^2}}{6 c^2 e (d+e x)}\right]\right) \Big/ \left(2\times 2^{2/3} e \left(b^2 e^2+b c e (-d+3 e x)+c^2 \left(d^2+3 e^2 x^2\right)\right)^{1/3}\right)$$

- **Problem 2497: Result unnecessarily involves higher level functions.**

$$\int \frac{(2+3 x)^3}{\left(52-54 x+27 x^2\right)^{1/3}} \, dx$$

Optimal (type 4, 635 leaves, 7 steps):

$$\frac{1}{30}(2+3x)^2\left(52-54x+27x^2\right)^{2/3}+\frac{1}{7}(27+8x)\left(52-54x+27x^2\right)^{2/3}+\frac{9000\times 5^{1/3}(1-x)}{7\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)}-$$

$$\left(25\times 5^{5/6}\sqrt{2+\sqrt{3}}\left(30-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)\sqrt{\frac{900+30\times 10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}+10^{2/3}\left(2700+(-54+54x)^2\right)^{2/3}}{\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{30\left(1+\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}{30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Bigg/$$

$$\left(189\sqrt{2}\,3^{1/4}(1-x)\sqrt{-\frac{30-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}{\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(50\times 5^{5/6}\left(30-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)\sqrt{\frac{900+30\times 10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}+10^{2/3}\left(2700+(-54+54x)^2\right)^{2/3}}{\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{30\left(1+\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}{30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Bigg/$$

$$\left(189\times 3^{3/4}(1-x)\sqrt{-\frac{30-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}{\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 124 leaves) :

$$\frac{1}{210\left(52-54x+27x^2\right)^{1/3}}\left(43576-28404x+8406x^2+5346x^3+1701x^4+\right.$$

$$\left.250\times 3^{1/3}\,10^{2/3}\left(9\,i+5\sqrt{3}-9\,i\,x\right)^{1/3}\left(-5\,i-3\sqrt{3}+3\sqrt{3}\,x\right)\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{-9\,i+5\sqrt{3}+9\,i\,x}{10\sqrt{3}}\right]\right)$$

- **Problem 2498: Result unnecessarily involves higher level functions.**

$$\int\frac{(2+3x)^2}{\left(52-54x+27x^2\right)^{1/3}}\,dx$$

Optimal (type 4, 628 leaves, 8 steps) :

$$\frac{25}{42}\left(52-54x+27x^2\right)^{2/3}+\frac{1}{21}(2+3x)\left(52-54x+27x^2\right)^{2/3}+\frac{2700\times 5^{1/3}(1-x)}{7\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)}-$$

$$\left(5\times 5^{5/6}\sqrt{2+\sqrt{3}}\left(30-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)\sqrt{\frac{900+30\times 10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}+10^{2/3}\left(2700+(-54+54x)^2\right)^{2/3}}{\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{30\left(1+\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}{30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Bigg/$$

$$\left(126\sqrt{2}\,3^{1/4}(1-x)\sqrt{-\frac{30-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}{\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(5\times 5^{5/6}\left(30-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)\sqrt{\frac{900+30\times 10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}+10^{2/3}\left(2700+(-54+54x)^2\right)^{2/3}}{\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{30\left(1+\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}{30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Bigg/$$

$$\left(63\times 3^{3/4}(1-x)\sqrt{-\frac{30-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}{\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 119 leaves) :

$$\frac{1}{42\left(52-54x+27x^2\right)^{1/3}}$$

$$\left(1508-1254x+459x^2+162x^3+15\times 3^{1/3}\,10^{2/3}\left(9i+5\sqrt{3}-9ix\right)^{1/3}\left(-5i-3\sqrt{3}+3\sqrt{3}\,x\right)\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{-9i+5\sqrt{3}+9ix}{10\sqrt{3}}\right]\right)$$

- **Problem 2499: Result unnecessarily involves higher level functions.**

$$\int\frac{2+3x}{\left(52-54x+27x^2\right)^{1/3}}\,dx$$

Optimal (type 4, 603 leaves, 6 steps) :

$$\frac{1}{12}\left(52-54x+27x^2\right)^{2/3}+\frac{90\times 5^{1/3}(1-x)}{30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}-$$

$$\left(5^{5/6}\sqrt{2+\sqrt{3}}\left(30-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)\sqrt{\frac{900+30\times 10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}+10^{2/3}\left(2700+(-54+54x)^2\right)^{2/3}}{\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{30\left(1+\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}{30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(108\sqrt{2}\,3^{1/4}(1-x)\sqrt{-\frac{30-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}{\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(5^{5/6}\left(30-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)\sqrt{\frac{900+30\times 10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}+10^{2/3}\left(2700+(-54+54x)^2\right)^{2/3}}{\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{30\left(1+\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}{30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(54\times 3^{3/4}(1-x)\sqrt{-\frac{30-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}{\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 113 leaves):

$$\frac{1}{12\left(52-54x+27x^2\right)^{1/3}}$$

$$\left(52-54x+27x^2+3^{1/3}\,10^{2/3}\left(9\,i+5\sqrt{3}-9\,i\,x\right)^{1/3}\left(-5\,i-3\sqrt{3}+3\sqrt{3}\,x\right)\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{-9\,i+5\sqrt{3}+9\,i\,x}{10\sqrt{3}}\right]\right)$$

- **Problem 2500: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\frac{1}{(2+3x)\left(52-54x+27x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 108 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{2/3}\,(8-3\,x)}{\sqrt{3}\,5^{1/3}\,\left(52-54\,x+27\,x^2\right)^{1/3}}\right]}{3\,\sqrt{3}\,10^{2/3}}-\frac{\text{Log}[2+3\,x]}{6\times 10^{2/3}}+\frac{\text{Log}\left[216-81\,x-27\times 10^{1/3}\,\left(52-54\,x+27\,x^2\right)^{1/3}\right]}{6\times 10^{2/3}}$$

Result (type 6, 288 leaves) :

$$-\left((2+3\,x)\,\left(-9-5\,i\,\sqrt{3}+9\,x\right)\,\left(-9+5\,i\,\sqrt{3}+9\,x\right)\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{15-5\,i\,\sqrt{3}}{6+9\,x},\frac{15+5\,i\,\sqrt{3}}{6+9\,x}\right]\right)\Big/$$
$$\left(2\,\left(52-54\,x+27\,x^2\right)^{4/3}\,\left((6+9\,x)\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{15-5\,i\,\sqrt{3}}{6+9\,x},\frac{15+5\,i\,\sqrt{3}}{6+9\,x}\right]+\right.\right.$$
$$\left.\left.\left(3+i\,\sqrt{3}\right)\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{4}{3},\frac{8}{3},\frac{15-5\,i\,\sqrt{3}}{6+9\,x},\frac{15+5\,i\,\sqrt{3}}{6+9\,x}\right]+\left(3-i\,\sqrt{3}\right)\,\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},\frac{1}{3},\frac{8}{3},\frac{15-5\,i\,\sqrt{3}}{6+9\,x},\frac{15+5\,i\,\sqrt{3}}{6+9\,x}\right]\right)\right)$$

- **Problem 2501: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(2+3\,x)^2\,\left(52-54\,x+27\,x^2\right)^{1/3}}\,dx$$

Optimal (type 4, 719 leaves, 8 steps) :

$$-\frac{\left(52-54\,x+27\,x^2\right)^{2/3}}{300\,(2+3\,x)}+\frac{9\,(1-x)}{10\times 5^{2/3}\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54\,x)^2\right)^{1/3}\right)}-\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{2/3}\,(8-3\,x)}{\sqrt{3}\,5^{1/3}\left(52-54\,x+27\,x^2\right)^{1/3}}\right]}{30\,\sqrt{3}\,10^{2/3}}-$$

$$\left(\sqrt{2+\sqrt{3}}\left(30-10^{1/3}\left(2700+(-54+54\,x)^2\right)^{1/3}\right)\sqrt{\frac{900+30\times 10^{1/3}\left(2700+(-54+54\,x)^2\right)^{1/3}+10^{2/3}\left(2700+(-54+54\,x)^2\right)^{2/3}}{\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54\,x)^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{30\left(1+\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54\,x)^2\right)^{1/3}}{30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54\,x)^2\right)^{1/3}}\right],\,-7+4\,\sqrt{3}\right]\right)\Big/$$

$$\left(10\,800\,\sqrt{2}\,3^{1/4}\,5^{1/6}\,(1-x)\sqrt{-\frac{30-10^{1/3}\left(2700+(-54+54\,x)^2\right)^{1/3}}{\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54\,x)^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(\left(30-10^{1/3}\left(2700+(-54+54\,x)^2\right)^{1/3}\right)\sqrt{\frac{900+30\times 10^{1/3}\left(2700+(-54+54\,x)^2\right)^{1/3}+10^{2/3}\left(2700+(-54+54\,x)^2\right)^{2/3}}{\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54\,x)^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{30\left(1+\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54\,x)^2\right)^{1/3}}{30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54\,x)^2\right)^{1/3}}\right],\,-7+4\,\sqrt{3}\right]\right)\Big/$$

$$\left(5400\times 3^{3/4}\,5^{1/6}\,(1-x)\sqrt{-\frac{30-10^{1/3}\left(2700+(-54+54\,x)^2\right)^{1/3}}{\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54\,x)^2\right)^{1/3}\right)^2}}\right)-\frac{\text{Log}[2+3\,x]}{60\times 10^{2/3}}+\frac{\text{Log}\left[216-81\,x-27\times 10^{1/3}\left(52-54\,x+27\,x^2\right)^{1/3}\right]}{60\times 10^{2/3}}$$

Result (type 6, 402 leaves) :

$$\left(-\frac{60\left(52-54\,x+27\,x^2\right)^2}{2+3\,x}-\left(900\,(2+3\,x)\left(-9-5\,i\,\sqrt{3}+9\,x\right)\left(-9+5\,i\,\sqrt{3}+9\,x\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{15-5\,i\,\sqrt{3}}{6+9\,x},\frac{15+5\,i\,\sqrt{3}}{6+9\,x}\right]\right)\Big/\right.$$

$$\left((6+9\,x)\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{15-5\,i\,\sqrt{3}}{6+9\,x},\frac{15+5\,i\,\sqrt{3}}{6+9\,x}\right]+\left(3+i\,\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{4}{3},\frac{8}{3},\frac{15-5\,i\,\sqrt{3}}{6+9\,x},\frac{15+5\,i\,\sqrt{3}}{6+9\,x}\right]+\right.$$

$$\left.\left(3-i\,\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},\frac{1}{3},\frac{8}{3},\frac{15-5\,i\,\sqrt{3}}{6+9\,x},\frac{15+5\,i\,\sqrt{3}}{6+9\,x}\right]\right)+3^{5/6}\,10^{2/3}\left(9\,i+5\,\sqrt{3}-9\,i\,x\right)^{1/3}$$

$$\left.\left(-9-5\,i\,\sqrt{3}+9\,x\right)\left(52-54\,x+27\,x^2\right)\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{-9\,i+5\,\sqrt{3}+9\,i\,x}{10\,\sqrt{3}}\right]\right)\Big/\left(18\,000\left(52-54\,x+27\,x^2\right)^{4/3}\right)$$

- **Problem 2502: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(2+3x)^3 \left(52-54x+27x^2\right)^{1/3}} \, dx$$

Optimal (type 4, 744 leaves, 9 steps) :

$$-\frac{\left(52-54x+27x^2\right)^{2/3}}{600\,(2+3x)^2}-\frac{\left(52-54x+27x^2\right)^{2/3}}{1500\,(2+3x)}+\frac{9\,(1-x)}{50\times 5^{2/3}\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)}-\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{2/3}\,(8-3x)}{\sqrt{3}\,5^{1/3}\,\left(52-54x+27x^2\right)^{1/3}}\right]}{300\sqrt{3}\,10^{2/3}}-$$

$$\left(\sqrt{2+\sqrt{3}}\left(30-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)\sqrt{\frac{900+30\times 10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}+10^{2/3}\left(2700+(-54+54x)^2\right)^{2/3}}{\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{30\left(1+\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}{30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(54\,000\sqrt{2}\,3^{1/4}\,5^{1/6}\,(1-x)\sqrt{-\frac{30-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}{\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(\left(30-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)\sqrt{\frac{900+30\times 10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}+10^{2/3}\left(2700+(-54+54x)^2\right)^{2/3}}{\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{30\left(1+\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}{30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(27\,000\times 3^{3/4}\,5^{1/6}\,(1-x)\sqrt{-\frac{30-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}}{\left(30\left(1-\sqrt{3}\right)-10^{1/3}\left(2700+(-54+54x)^2\right)^{1/3}\right)^2}}\right)-\frac{\text{Log}[2+3x]}{600\times 10^{2/3}}+\frac{\text{Log}\left[216-81x-27\times 10^{1/3}\left(52-54x+27x^2\right)^{1/3}\right]}{600\times 10^{2/3}}$$

Result (type 6, 407 leaves) :

$$\left(-\frac{90\,(3+2x)\,(52-54x+27x^2)^2}{(2+3x)^2}-\left(450\,(2+3x)\left(-9-5\,i\sqrt{3}+9x\right)\left(-9+5\,i\sqrt{3}+9x\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{15-5\,i\sqrt{3}}{6+9x},\frac{15+5\,i\sqrt{3}}{6+9x}\right]\right)\Big/\right.$$
$$\left((6+9x)\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{15-5\,i\sqrt{3}}{6+9x},\frac{15+5\,i\sqrt{3}}{6+9x}\right]+\left(3+i\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{4}{3},\frac{8}{3},\frac{15-5\,i\sqrt{3}}{6+9x},\frac{15+5\,i\sqrt{3}}{6+9x}\right]+\right.$$
$$\left.\left(3-i\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},\frac{1}{3},\frac{8}{3},\frac{15-5\,i\sqrt{3}}{6+9x},\frac{15+5\,i\sqrt{3}}{6+9x}\right]\right)+3^{5/6}\,10^{2/3}\left(9\,i+5\sqrt{3}-9\,i\,x\right)^{1/3}$$
$$\left.\left(-9-5\,i\sqrt{3}+9x\right)\left(52-54x+27x^2\right)\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{-9\,i+5\sqrt{3}+9\,i\,x}{10\sqrt{3}}\right]\right)\Big/\left(90000\left(52-54x+27x^2\right)^{4/3}\right)$$

- **Problem 2503: Result unnecessarily involves higher level functions.**

$$\int\frac{(2+3x)^3}{\left(28+54x+27x^2\right)^{1/3}}\,dx$$

Optimal (type 4, 589 leaves, 7 steps) :

$$\frac{1}{30}(2+3x)^2\left(28+54x+27x^2\right)^{2/3}-\frac{1}{35}(1+8x)\left(28+54x+27x^2\right)^{2/3}+$$
$$\frac{72\,(1+x)}{7\left(6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)}-\left(\sqrt{2\left(2+\sqrt{3}\right)}\left(6-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)\right.$$
$$\left.\sqrt{\frac{1+\left(28+54x+27x^2\right)^{1/3}+\left(28+54x+27x^2\right)^{2/3}}{\left(6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{6\left(1+\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}{6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$
$$\left(63\times3^{1/4}\,(1+x)\sqrt{-\frac{6-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}{\left(6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)^2}}\right)+\left(4\left(6-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)\right.$$
$$\left.\sqrt{\frac{1+\left(28+54x+27x^2\right)^{1/3}+\left(28+54x+27x^2\right)^{2/3}}{\left(6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{6\left(1+\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}{6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$
$$\left(63\times3^{3/4}\,(1+x)\sqrt{-\frac{6-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}{\left(6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 120 leaves) :

$$\frac{1}{210\left(28+54x+27x^2\right)^{1/3}}\left(616+2196x+4302x^2+4374x^3+1701x^4-10\times 2^{2/3}3^{1/3}\left(-9i+\sqrt{3}-9ix\right)^{1/3}\left(-i+3\sqrt{3}+3\sqrt{3}x\right)\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{9i+\sqrt{3}+9ix}{2\sqrt{3}}\right]\right)$$

- **Problem 2504: Result unnecessarily involves higher level functions.**

$$\int\frac{(2+3x)^2}{\left(28+54x+27x^2\right)^{1/3}}\,dx$$

Optimal (type 4, 585 leaves, 7 steps):

$$-\frac{5}{42}\left(28+54x+27x^2\right)^{2/3}+\frac{1}{21}(2+3x)\left(28+54x+27x^2\right)^{2/3}-\frac{108(1+x)}{7\left(6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)}+\left(\sqrt{2+\sqrt{3}}\left(6-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)\sqrt{\frac{1+\left(28+54x+27x^2\right)^{1/3}+\left(28+54x+27x^2\right)^{2/3}}{\left(6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{6\left(1+\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}{6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/\left(21\sqrt{2}\,3^{1/4}(1+x)\sqrt{-\frac{6-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}{\left(6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)^2}}\right)-\left(2\left(6-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)\sqrt{\frac{1+\left(28+54x+27x^2\right)^{1/3}+\left(28+54x+27x^2\right)^{2/3}}{\left(6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{6\left(1+\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}{6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/\left(21\times 3^{3/4}(1+x)\sqrt{-\frac{6-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}{\left(6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 115 leaves):

$$\frac{1}{42\left(28+54x+27x^2\right)^{1/3}}\left(-28+114x+297x^2+162x^3+3\times 2^{2/3}3^{1/3}\left(-9i+\sqrt{3}-9ix\right)^{1/3}\left(-i+3\sqrt{3}+3\sqrt{3}x\right)\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{9i+\sqrt{3}+9ix}{2\sqrt{3}}\right]\right)$$

- **Problem 2505: Result unnecessarily involves higher level functions.**

$$\int \frac{2+3x}{\left(28+54x+27x^2\right)^{1/3}}\,dx$$

Optimal (type 4, 560 leaves, 6 steps):

$$\frac{1}{12}\left(28+54x+27x^2\right)^{2/3}+\frac{18\,(1+x)}{6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}-$$

$$\left(\sqrt{2+\sqrt{3}}\left(6-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)\sqrt{\frac{1+\left(28+54x+27x^2\right)^{1/3}+\left(28+54x+27x^2\right)^{2/3}}{\left(6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{6\left(1+\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}{6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(18\sqrt{2}\;3^{1/4}\,(1+x)\sqrt{-\frac{6-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}{\left(6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(\left(6-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)\sqrt{\frac{1+\left(28+54x+27x^2\right)^{1/3}+\left(28+54x+27x^2\right)^{2/3}}{\left(6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{6\left(1+\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}{6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/\left(9\times3^{3/4}\,(1+x)\sqrt{-\frac{6-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}{\left(6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 109 leaves):

$$\frac{1}{12\left(28+54x+27x^2\right)^{1/3}}\left(28+54x+27x^2+2^{2/3}\,3^{1/3}\left(-9\,i+\sqrt{3}-9\,i\,x\right)^{1/3}\left(i-3\sqrt{3}-3\sqrt{3}\,x\right)\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{9\,i+\sqrt{3}+9\,i\,x}{2\sqrt{3}}\right]\right)$$

- **Problem 2506: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(2+3x)\left(28+54x+27x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 103 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{2/3}(4+3x)}{\sqrt{3}\left(28+54x+27x^2\right)^{1/3}}\right]}{3\times 2^{2/3}\sqrt{3}}-\frac{\text{Log}[2+3x]}{6\times 2^{2/3}}+\frac{\text{Log}\left[-108-81x+27\times 2^{1/3}\left(28+54x+27x^2\right)^{1/3}\right]}{6\times 2^{2/3}}$$

Result (type 6, 294 leaves):

$$-\left(5(2+3x)\left(9-i\sqrt{3}+9x\right)\left(9+i\sqrt{3}+9x\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},-\frac{3+i\sqrt{3}}{6+9x},\frac{-3+i\sqrt{3}}{6+9x}\right]\right)\Big/$$
$$\left(2\left(28+54x+27x^2\right)^{4/3}\left(15(2+3x)\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},-\frac{3+i\sqrt{3}}{6+9x},\frac{-3+i\sqrt{3}}{6+9x}\right]+\right.\right.$$
$$\left.\left.i\left(3i+\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{4}{3},\frac{8}{3},-\frac{3+i\sqrt{3}}{6+9x},\frac{-3+i\sqrt{3}}{6+9x}\right]+\left(-3-i\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},\frac{1}{3},\frac{8}{3},-\frac{3+i\sqrt{3}}{6+9x},\frac{-3+i\sqrt{3}}{6+9x}\right]\right)\right)$$

- **Problem 2507: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{(2+3x)^2\left(28+54x+27x^2\right)^{1/3}}\,dx$$

Optimal (type 4, 671 leaves, 9 steps):

$$-\frac{\left(28+54x+27x^2\right)^{2/3}}{12\,(2+3x)}-\frac{9\,(1+x)}{2\left(6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)}+$$

$$\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{2/3}\,(4+3x)}{\sqrt{3}\,\left(28+54x+27x^2\right)^{1/3}}\right]}{6\times 2^{2/3}\sqrt{3}}+\left(\sqrt{2+\sqrt{3}}\,\left(6-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)\right.$$

$$\left.\sqrt{\frac{1+\left(28+54x+27x^2\right)^{1/3}+\left(28+54x+27x^2\right)^{2/3}}{\left(6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{6\left(1+\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}{6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(72\sqrt{2}\,3^{1/4}\,(1+x)\sqrt{-\frac{6-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}{\left(6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)^2}}\right)-\left(\left(6-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)\right.$$

$$\left.\sqrt{\frac{1+\left(28+54x+27x^2\right)^{1/3}+\left(28+54x+27x^2\right)^{2/3}}{\left(6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{6\left(1+\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}{6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(36\times 3^{3/4}\,(1+x)\sqrt{-\frac{6-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}}{\left(6\left(1-\sqrt{3}\right)-2^{1/3}\left(108+(54+54x)^2\right)^{1/3}\right)^2}}\right)+\frac{\text{Log}[2+3x]}{12\times 2^{2/3}}-\frac{\text{Log}\left[-108-81x+27\times 2^{1/3}\left(28+54x+27x^2\right)^{1/3}\right]}{12\times 2^{2/3}}$$

Result (type 6, 405 leaves) :

$$\frac{1}{432\left(28+54x+27x^2\right)^{4/3}}$$

$$\left(-\frac{36\left(28+54x+27x^2\right)^2}{2+3x}+\left(540\,(2+3x)\left(9-i\sqrt{3}+9x\right)\left(9+i\sqrt{3}+9x\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},-\frac{3+i\sqrt{3}}{6+9x},\frac{-3+i\sqrt{3}}{6+9x}\right]\right)\Big/\right.$$

$$\left(15\,(2+3x)\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},-\frac{3+i\sqrt{3}}{6+9x},\frac{-3+i\sqrt{3}}{6+9x}\right]+\right.$$

$$\left.i\left(3i+\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{4}{3},\frac{8}{3},-\frac{3+i\sqrt{3}}{6+9x},\frac{-3+i\sqrt{3}}{6+9x}\right]+\left(-3-i\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},\frac{1}{3},\frac{8}{3},-\frac{3+i\sqrt{3}}{6+9x},\frac{-3+i\sqrt{3}}{6+9x}\right]\right)+$$

$$\left.3\times 2^{2/3}\,3^{5/6}\left(-9i+\sqrt{3}-9ix\right)^{1/3}\left(9-i\sqrt{3}+9x\right)\left(28+54x+27x^2\right)\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{9i+\sqrt{3}+9ix}{2\sqrt{3}}\right]\right)$$

Problem 2508: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(2+3x)^3 \left(28+54x+27x^2\right)^{1/3}} \, dx$$

Optimal (type 4, 696 leaves, 9 steps):

$$-\frac{\left(28+54x+27x^2\right)^{2/3}}{24 (2+3x)^2}+\frac{\left(28+54x+27x^2\right)^{2/3}}{12 (2+3x)}+\frac{9 (1+x)}{2 \left(6 \left(1-\sqrt{3}\right)-2^{1/3} \left(108+(54+54x)^2\right)^{1/3}\right)}-\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{2/3} (4+3x)}{\sqrt{3} \left(28+54x+27x^2\right)^{1/3}}\right]}{12\times 2^{2/3} \sqrt{3}}-\left(\sqrt{2+\sqrt{3}} \left(6-2^{1/3} \left(108+(54+54x)^2\right)^{1/3}\right) \sqrt{\frac{1+\left(28+54x+27x^2\right)^{1/3}+\left(28+54x+27x^2\right)^{2/3}}{\left(6 \left(1-\sqrt{3}\right)-2^{1/3} \left(108+(54+54x)^2\right)^{1/3}\right)^2}} \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{6 \left(1+\sqrt{3}\right)-2^{1/3} \left(108+(54+54x)^2\right)^{1/3}}{6 \left(1-\sqrt{3}\right)-2^{1/3} \left(108+(54+54x)^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right) \Big/ \left(72 \sqrt{2}\, 3^{1/4} (1+x) \sqrt{-\frac{6-2^{1/3} \left(108+(54+54x)^2\right)^{1/3}}{\left(6 \left(1-\sqrt{3}\right)-2^{1/3} \left(108+(54+54x)^2\right)^{1/3}\right)^2}}\right)+\left(\left(6-2^{1/3} \left(108+(54+54x)^2\right)^{1/3}\right) \sqrt{\frac{1+\left(28+54x+27x^2\right)^{1/3}+\left(28+54x+27x^2\right)^{2/3}}{\left(6 \left(1-\sqrt{3}\right)-2^{1/3} \left(108+(54+54x)^2\right)^{1/3}\right)^2}} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{6 \left(1+\sqrt{3}\right)-2^{1/3} \left(108+(54+54x)^2\right)^{1/3}}{6 \left(1-\sqrt{3}\right)-2^{1/3} \left(108+(54+54x)^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right) \Big/ \left(36\times 3^{3/4} (1+x) \sqrt{-\frac{6-2^{1/3} \left(108+(54+54x)^2\right)^{1/3}}{\left(6 \left(1-\sqrt{3}\right)-2^{1/3} \left(108+(54+54x)^2\right)^{1/3}\right)^2}}\right)-\frac{\text{Log}[2+3x]}{24\times 2^{2/3}}+\frac{\text{Log}\left[-108-81x+27\times 2^{1/3} \left(28+54x+27x^2\right)^{1/3}\right]}{24\times 2^{2/3}}$$

Result (type 6, 412 leaves):

$$\frac{1}{432\,(28+54x+27x^2)^{4/3}}\left(\frac{54\,(1+2x)\,(28+54x+27x^2)^2}{(2+3x)^2}-\left(270\,(2+3x)\,\left(9-i\sqrt{3}+9x\right)\,\left(9+i\sqrt{3}+9x\right)\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},-\frac{3+i\sqrt{3}}{6+9x},\frac{-3+i\sqrt{3}}{6+9x}\right]\right)\Big/\right.$$
$$\left(15\,(2+3x)\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},-\frac{3+i\sqrt{3}}{6+9x},\frac{-3+i\sqrt{3}}{6+9x}\right]+\right.$$
$$\left.i\left(3i+\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{4}{3},\frac{8}{3},-\frac{3+i\sqrt{3}}{6+9x},\frac{-3+i\sqrt{3}}{6+9x}\right]+\left(-3-i\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},\frac{1}{3},\frac{8}{3},-\frac{3+i\sqrt{3}}{6+9x},\frac{-3+i\sqrt{3}}{6+9x}\right]\right)+$$
$$\left.3\,i\,2^{2/3}\,3^{5/6}\left(-9i+\sqrt{3}-9ix\right)^{1/3}\left(9i+\sqrt{3}+9ix\right)\left(28+54x+27x^2\right)\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{9i+\sqrt{3}+9ix}{2\sqrt{3}}\right]\right)$$

- **Problem 2509: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{(d+e\,x)\,\left(-c^2d^2+b\,c\,d\,e+2\,b^2e^2+9\,b\,c\,e^2x+9\,c^2e^2x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 564 leaves, 2 steps):

$$-\left(\sqrt{3}\left(-c\,e\,(c\,d-2\,b\,e)+3\,c^2e^2x\right)^{1/3}\left(c\,e\,(c\,d+b\,e)+3\,c^2e^2x\right)^{1/3}\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2^{1/3}\left(-c\,e\,(c\,d-2\,b\,e)+3\,c^2e^2x\right)^{2/3}}{\sqrt{3}\,c^{1/3}e^{1/3}\,(2\,c\,d-b\,e)^{1/3}\left(c\,e\,(c\,d+b\,e)+3\,c^2e^2x\right)^{1/3}}\right]\right)\Big/$$
$$\left(2\times2^{1/3}c^{2/3}e^{5/3}\,(2\,c\,d-b\,e)^{2/3}\left(-(c\,d-2\,b\,e)\,(c\,d+b\,e)+9\,b\,c\,e^2x+9\,c^2e^2x^2\right)^{1/3}\right)-$$
$$\frac{\left(-c\,e\,(c\,d-2\,b\,e)+3\,c^2e^2x\right)^{1/3}\left(c\,e\,(c\,d+b\,e)+3\,c^2e^2x\right)^{1/3}\text{Log}[d+e\,x]}{2\times2^{1/3}c^{2/3}e^{5/3}\,(2\,c\,d-b\,e)^{2/3}\left(-(c\,d-2\,b\,e)\,(c\,d+b\,e)+9\,b\,c\,e^2x+9\,c^2e^2x^2\right)^{1/3}}+$$
$$\left(3\left(-c\,e\,(c\,d-2\,b\,e)+3\,c^2e^2x\right)^{1/3}\left(c\,e\,(c\,d+b\,e)+3\,c^2e^2x\right)^{1/3}\right.$$
$$\left.\text{Log}\left[-\frac{\left(\frac{3}{2}\right)^{1/3}\left(-c\,e\,(c\,d-2\,b\,e)+3\,c^2e^2x\right)^{2/3}}{c^{1/3}e^{1/3}\,(2\,c\,d-b\,e)^{1/3}}-6^{1/3}\left(c\,e\,(c\,d+b\,e)+3\,c^2e^2x\right)^{1/3}\right]\right)\Big/$$
$$\left(4\times2^{1/3}c^{2/3}e^{5/3}\,(2\,c\,d-b\,e)^{2/3}\left(-(c\,d-2\,b\,e)\,(c\,d+b\,e)+9\,b\,c\,e^2x+9\,c^2e^2x^2\right)^{1/3}\right)$$

Result (type 6, 290 leaves):

$$-\left(3^{1/3}\left(\frac{3bce^2-\sqrt{c^2e^2(-2cd+be)^2}+6c^2e^2x}{c^2e(d+ex)}\right)^{1/3}\left(\frac{3bce^2+\sqrt{c^2e^2(-2cd+be)^2}+6c^2e^2x}{c^2e(d+ex)}\right)^{1/3}\right.$$

$$\left.\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},-\frac{-6c^2de+3bce^2+\sqrt{c^2e^2(-2cd+be)^2}}{6c^2e(d+ex)},\frac{6c^2de-3bce^2+\sqrt{c^2e^2(-2cd+be)^2}}{6c^2e(d+ex)}\right]\right)\Big/$$

$$\left(2\times 2^{2/3}e\left(2b^2e^2+bce(d+9ex)-c^2\left(d^2-9e^2x^2\right)\right)^{1/3}\right)$$

Problem 2510: Result unnecessarily involves higher level functions.

$$\int (d+ex)^3\left(a+bx+cx^2\right)^{1/4}dx$$

Optimal (type 4, 374 leaves, 5 steps):

$$\frac{(2cd-be)\left(28c^2d^2+13b^2e^2-4ce(7bd+6ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{1/4}}{168c^4}+$$

$$\frac{2e(d+ex)^2\left(a+bx+cx^2\right)^{5/4}}{9c}+\frac{e\left(616c^2d^2+117b^2e^2-2ce(243bd+56ae)+130ce(2cd-be)x\right)\left(a+bx+cx^2\right)^{5/4}}{630c^3}-$$

$$\frac{1}{336\sqrt{2}\,c^{17/4}(b+2cx)}\left(b^2-4ac\right)^{5/4}(2cd-be)\left(28c^2d^2+13b^2e^2-4ce(7bd+6ae)\right)$$

$$\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\left(a+bx+cx^2\right)^{1/4}}{\left(b^2-4ac\right)^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 5, 376 leaves):

$$\frac{1}{672(a+x(b+cx))^{3/4}}\left(\frac{1}{15c^4}4(a+x(b+cx))\left(-195b^4e^3+6b^3ce^2(135d+13ex)-\right.\right.$$

$$4b^2ce\left(-207ae^2+c\left(315d^2+81dex+13e^2x^2\right)\right)+8bc^2\left(-ae^2(333d+31ex)+c\left(105d^3+63d^2ex+27de^2x^2+5e^3x^3\right)\right)+$$

$$\left.16c^2\left(-28a^2e^3+ace\left(189d^2+45dex+7e^2x^2\right)+c^2x\left(105d^3+189d^2ex+135de^2x^2+35e^3x^3\right)\right)\right)+$$

$$\frac{1}{c^5}2^{1/4}\left(b^2-4ac\right)(-2cd+be)\left(28c^2d^2+13b^2e^2-4ce(7bd+6ae)\right)\left(b-\sqrt{b^2-4ac}+2cx\right)$$

$$\left.\left(\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{-b+\sqrt{b^2-4ac}-2cx}{2\sqrt{b^2-4ac}}\right]\right)$$

Problem 2511: Result unnecessarily involves higher level functions.

$$\int (d+e x)^2 \left(a+b x+c x^2\right)^{1/4} \, dx$$

Optimal (type 4, 319 leaves, 5 steps):

$$\frac{\left(28 c^2 d^2+9 b^2 e^2-4 c e (7 b d+2 a e)\right) (b+2 c x) \left(a+b x+c x^2\right)^{1/4}}{84 c^3}+\frac{9 e (2 c d-b e) \left(a+b x+c x^2\right)^{5/4}}{35 c^2}+$$
$$\frac{2 e (d+e x) \left(a+b x+c x^2\right)^{5/4}}{7 c}-\frac{1}{168 \sqrt{2}\, c^{13/4} (b+2 c x)}\left(b^2-4 a c\right)^{5/4} \left(28 c^2 d^2+9 b^2 e^2-4 c e (7 b d+2 a e)\right)$$
$$\sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right) \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}} \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right) \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2}\, c^{1/4} \left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 5, 274 leaves):

$$\frac{1}{1680 c^4 (a+x (b+c x))^{3/4}} \Bigg(4 c (a+x (b+c x))$$
$$\left(45 b^3 e^2-2 b^2 c e (70 d+9 e x)+4 b c \left(-37 a e^2+c \left(35 d^2+14 d e x+3 e^2 x^2\right)\right)+8 c^2 \left(a e (42 d+5 e x)+c x \left(35 d^2+42 d e x+15 e^2 x^2\right)\right)\right)-$$
$$5\times 2^{1/4} \left(b^2-4 a c\right) \left(28 c^2 d^2+9 b^2 e^2-4 c e (7 b d+2 a e)\right) \left(b-\sqrt{b^2-4 a c}+2 c x\right)$$
$$\left(\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{-b+\sqrt{b^2-4 a c}-2 c x}{2 \sqrt{b^2-4 a c}}\right]\Bigg)$$

Problem 2512: Result unnecessarily involves higher level functions.

$$\int (d+e x) \left(a+b x+c x^2\right)^{1/4} \, dx$$

Optimal (type 4, 241 leaves, 4 steps):

$$\frac{(2 c d-b e) (b+2 c x) \left(a+b x+c x^2\right)^{1/4}}{6 c^2}+\frac{2 e \left(a+b x+c x^2\right)^{5/4}}{5 c}-\frac{1}{12 \sqrt{2}\, c^{9/4} (b+2 c x)}$$
$$\left(b^2-4 a c\right)^{5/4} (2 c d-b e) \sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right) \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}} \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right) \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2}\, c^{1/4} \left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 5, 194 leaves):

$$\frac{1}{120\,c^3\,(a+x\,(b+c\,x))^{3/4}}\left(4\,c\,(a+x\,(b+c\,x))\,\left(-5\,b^2\,e+2\,b\,c\,(5\,d+e\,x)+4\,c\,(3\,a\,e+c\,x\,(5\,d+3\,e\,x))\right)+\right.$$
$$\left.5\times 2^{1/4}\,\left(b^2-4\,a\,c\right)\,(-2\,c\,d+b\,e)\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\,\left(\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{3/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{5}{4},\,\frac{-b+\sqrt{b^2-4\,a\,c}-2\,c\,x}{2\,\sqrt{b^2-4\,a\,c}}\right]\right)$$

- **Problem 2513: Result unnecessarily involves higher level functions.**

$$\int\left(a+b\,x+c\,x^2\right)^{1/4}\,dx$$

Optimal (type 4, 201 leaves, 3 steps) :

$$\frac{(b+2\,c\,x)\,\left(a+b\,x+c\,x^2\right)^{1/4}}{3\,c}-\frac{1}{6\,\sqrt{2}\,c^{5/4}\,(b+2\,c\,x)}$$
$$\left(b^2-4\,a\,c\right)^{5/4}\,\sqrt{\frac{(b+2\,c\,x)^2}{\left(b^2-4\,a\,c\right)\,\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)^2}}\,\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\,\left(a+b\,x+c\,x^2\right)^{1/4}}{\left(b^2-4\,a\,c\right)^{1/4}}\right],\,\frac{1}{2}\right]$$

Result (type 5, 155 leaves) :

$$\frac{1}{12\,c^2\,(a+x\,(b+c\,x))^{3/4}}\left(4\,c\,(b+2\,c\,x)\,(a+x\,(b+c\,x))-\right.$$
$$\left.2^{1/4}\,\left(b^2-4\,a\,c\right)\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\,\left(\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{3/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{5}{4},\,\frac{-b+\sqrt{b^2-4\,a\,c}-2\,c\,x}{2\,\sqrt{b^2-4\,a\,c}}\right]\right)$$

- **Problem 2514: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+b\,x+c\,x^2\right)^{1/4}}{d+e\,x}\,dx$$

Optimal (type 4, 881 leaves, 19 steps) :

$$\frac{2\left(a+b x+c x^2\right)^{1/4}}{e}-\frac{\left(-b^2+4 a c\right)^{3/4}\left(c d^2-b d e+a e^2\right)^{1/4}\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{3/4}\text{ArcTan}\left[\frac{\left(-b^2+4 a c\right)^{1/4}\sqrt{e}\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}}{\sqrt{2}\, c^{1/4}\left(c d^2-b d e+a e^2\right)^{1/4}}\right]}{c^{3/4} e^{3/2}\left(a+b x+c x^2\right)^{3/4}}-$$

$$\frac{\left(-b^2+4 a c\right)^{3/4}\left(c d^2-b d e+a e^2\right)^{1/4}\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{3/4}\text{ArcTanh}\left[\frac{\left(-b^2+4 a c\right)^{1/4}\sqrt{e}\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}}{\sqrt{2}\, c^{1/4}\left(c d^2-b d e+a e^2\right)^{1/4}}\right]}{c^{3/4} e^{3/2}\left(a+b x+c x^2\right)^{3/4}}-\frac{1}{\sqrt{2}\, c^{1/4} e^2 (b+2 c x)}$$

$$\left(b^2-4 a c\right)^{1/4}(2 c d-b e)\sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right)\left(1+\frac{2\sqrt{c}\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\, c^{1/4}\left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right],\frac{1}{2}\right]-$$

$$\left(\left(b^2-4 a c\right)(2 c d-b e)\sqrt{\frac{(b+2 c x)^2}{b^2-4 a c}}\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{3/4}\text{EllipticPi}\left[-\frac{\sqrt{-b^2+4 a c}\, e}{2\sqrt{c}\sqrt{c d^2-b d e+a e^2}},\text{ArcSin}\left[\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}\right],-1\right]\right)\Big/$$

$$\left(\sqrt{2}\, c e^2 (b+2 c x)\left(a+b x+c x^2\right)^{3/4}\right)-$$

$$\left(\left(b^2-4 a c\right)(2 c d-b e)\sqrt{\frac{(b+2 c x)^2}{b^2-4 a c}}\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{3/4}\text{EllipticPi}\left[\frac{\sqrt{-b^2+4 a c}\, e}{2\sqrt{c}\sqrt{c d^2-b d e+a e^2}},\text{ArcSin}\left[\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}\right],-1\right]\right)\Big/$$

$$\left(\sqrt{2}\, c e^2 (b+2 c x)\left(a+b x+c x^2\right)^{3/4}\right)$$

Result (type 6, 178 leaves) :

$$\frac{2\sqrt{2}\,(a+x(b+c x))^{1/4}\,\text{AppellF1}\left[-\frac{1}{2},-\frac{1}{4},-\frac{1}{4},\frac{1}{2},\frac{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}{2 c (d+e x)},\frac{2 c d-b e+\sqrt{b^2-4 a c}\, e}{2 c d+2 c e x}\right]}{e\left(\frac{e\left(b-\sqrt{b^2-4 a c}+2 c x\right)}{c (d+e x)}\right)^{1/4}\left(\frac{e\left(b+\sqrt{b^2-4 a c}+2 c x\right)}{c (d+e x)}\right)^{1/4}}$$

Problem 2515: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x+c x^2\right)^{1/4}}{(d+e x)^2}\, dx$$

Optimal (type 4, 944 leaves, 19 steps) :

$$-\frac{\left(a+b x+c x^2\right)^{1/4}}{e\,(d+e x)}+\frac{\left(-b^2+4 a c\right)^{3/4} (2 c d-b e) \left(-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{3/4} \text{ArcTan}\left[\frac{\left(-b^2+4 a c\right)^{1/4} \sqrt{e} \left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}}{\sqrt{2}\, c^{1/4} \left(c d^2-b d e+a e^2\right)^{1/4}}\right]}{4 c^{3/4} e^{3/2} \left(c d^2-b d e+a e^2\right)^{3/4} \left(a+b x+c x^2\right)^{3/4}}+$$

$$\frac{\left(-b^2+4 a c\right)^{3/4} (2 c d-b e) \left(-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{3/4} \text{ArcTanh}\left[\frac{\left(-b^2+4 a c\right)^{1/4} \sqrt{e} \left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}}{\sqrt{2}\, c^{1/4} \left(c d^2-b d e+a e^2\right)^{1/4}}\right]}{4 c^{3/4} e^{3/2} \left(c d^2-b d e+a e^2\right)^{3/4} \left(a+b x+c x^2\right)^{3/4}}+\frac{1}{\sqrt{2}\, e^2\, (b+2 c x)}$$

$$c^{3/4} \left(b^2-4 a c\right)^{1/4} \sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right) \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}} \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right) \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2}\, c^{1/4} \left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right], \frac{1}{2}\right]+$$

$$\left(\left(b^2-4 a c\right) (2 c d-b e)^2 \sqrt{\frac{(b+2 c x)^2}{b^2-4 a c}} \left(-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{3/4} \text{EllipticPi}\left[-\frac{\sqrt{-b^2+4 a c}\; e}{2 \sqrt{c} \sqrt{c d^2-b d e+a e^2}}, \text{ArcSin}\left[\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}\right], -1\right]\right) \Big/$$

$$\left(4 \sqrt{2}\, c\, e^2 \left(c d^2-b d e+a e^2\right) (b+2 c x) \left(a+b x+c x^2\right)^{3/4}\right)+$$

$$\left(\left(b^2-4 a c\right) (2 c d-b e)^2 \sqrt{\frac{(b+2 c x)^2}{b^2-4 a c}} \left(-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{3/4} \text{EllipticPi}\left[\frac{\sqrt{-b^2+4 a c}\; e}{2 \sqrt{c} \sqrt{c d^2-b d e+a e^2}}, \text{ArcSin}\left[\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}\right], -1\right]\right) \Big/$$

$$\left(4 \sqrt{2}\, c\, e^2 \left(c d^2-b d e+a e^2\right) (b+2 c x) \left(a+b x+c x^2\right)^{3/4}\right)$$

Result (type 6, 185 leaves) :

$$-\frac{2 \sqrt{2}\, (a+x (b+c x))^{1/4}\, \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{4}, -\frac{1}{4}, \frac{3}{2}, \frac{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}{2 c (d+e x)}, \frac{2 c d-b e+\sqrt{b^2-4 a c}\; e}{2 c d+2 c e x}\right]}{e \left(\frac{e \left(b-\sqrt{b^2-4 a c}+2 c x\right)}{c (d+e x)}\right)^{1/4} \left(\frac{e \left(b+\sqrt{b^2-4 a c}+2 c x\right)}{c (d+e x)}\right)^{1/4} (d+e x)}$$

- **Problem 2516: Result unnecessarily involves higher level functions.**

$$\int (d+e x)^3 \left(a+b x+c x^2\right)^{3/4} \, dx$$

Optimal (type 4, 703 leaves, 7 steps) :

$$\frac{(2 c d-b e) \left(12 c^2 d^2+5 b^2 e^2-4 c e (3 b d+2 a e)\right) (b+2 c x) \left(a+b x+c x^2\right)^{3/4}}{120 c^4}+$$

$$\frac{2 e (d+e x)^2 \left(a+b x+c x^2\right)^{7/4}}{11 c}+\frac{e \left(312 c^2 d^2+55 b^2 e^2-2 c e (121 b d+24 a e)+70 c e (2 c d-b e) x\right) \left(a+b x+c x^2\right)^{7/4}}{462 c^3}-$$

$$\frac{\sqrt{b^2-4 a c} (2 c d-b e) \left(12 c^2 d^2+5 b^2 e^2-4 c e (3 b d+2 a e)\right) (b+2 c x) \left(a+b x+c x^2\right)^{1/4}}{80 c^{9/2} \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)}+\frac{1}{80 \sqrt{2} c^{19/4} (b+2 c x)}$$

$$\left(b^2-4 a c\right)^{7/4} (2 c d-b e) \left(12 c^2 d^2+5 b^2 e^2-4 c e (3 b d+2 a e)\right) \sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right) \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}} \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)$$

$$\text{EllipticE}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} c^{1/4} \left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right], \frac{1}{2}\right]-\frac{1}{160 \sqrt{2} c^{19/4} (b+2 c x)}\left(b^2-4 a c\right)^{7/4} (2 c d-b e) \left(12 c^2 d^2+5 b^2 e^2-4 c e (3 b d+2 a e)\right)$$

$$\sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right) \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}} \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right) \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} c^{1/4} \left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 5, 377 leaves) :

$$\frac{1}{480 (a+x (b+c x))^{1/4}} \left(\frac{1}{77 c^4} 4 (a+x (b+c x)) \left(-385 b^4 e^3+22 b^3 c e^2 (77 d+15 e x)-\right.\right.$$

$$12 b^2 c e \left(-143 a e^2+c \left(231 d^2+121 d e x+25 e^2 x^2\right)\right)+8 b c^2 \left(-3 a e^2 (253 d+47 e x)+c \left(231 d^3+297 d^2 e x+165 d e^2 x^2+35 e^3 x^3\right)\right)+$$

$$\left.16 c^2 \left(-60 a^2 e^3+3 a c e \left(165 d^2+77 d e x+15 e^2 x^2\right)+c^2 x \left(231 d^3+495 d^2 e x+385 d e^2 x^2+105 e^3 x^3\right)\right)\right)+\frac{1}{c^5}$$

$$2^{3/4} \left(b^2-4 a c\right) (-2 c d+b e) \left(12 c^2 d^2+5 b^2 e^2-4 c e (3 b d+2 a e)\right) \left(b-\sqrt{b^2-4 a c}+2 c x\right) \left(\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}\right)^{1/4}$$

$$\left.\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{-b+\sqrt{b^2-4 a c}-2 c x}{2 \sqrt{b^2-4 a c}}\right]\right)$$

Problem 2517: Result unnecessarily involves higher level functions.

$$\int (d+e x)^2 \left(a+b x+c x^2\right)^{3/4} \, dx$$

Optimal (type 4, 630 leaves, 7 steps):

$$\frac{\left(36 c^2 d^2+11 b^2 e^2-4 c e (9 b d+2 a e)\right) (b+2 c x) \left(a+b x+c x^2\right)^{3/4}}{180 c^3}+\frac{11 e (2 c d-b e) \left(a+b x+c x^2\right)^{7/4}}{63 c^2}+$$

$$\frac{2 e (d+e x) \left(a+b x+c x^2\right)^{7/4}}{9 c}-\frac{\sqrt{b^2-4 a c} \left(36 c^2 d^2+11 b^2 e^2-4 c e (9 b d+2 a e)\right) (b+2 c x) \left(a+b x+c x^2\right)^{1/4}}{120 c^{7/2} \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)}+\frac{1}{120 \sqrt{2} c^{15/4} (b+2 c x)}$$

$$\left(b^2-4 a c\right)^{7/4} \left(36 c^2 d^2+11 b^2 e^2-4 c e (9 b d+2 a e)\right) \sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right) \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}} \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)$$

$$\text{EllipticE}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} c^{1/4} \left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right], \frac{1}{2}\right]-\frac{1}{240 \sqrt{2} c^{15/4} (b+2 c x)} \left(b^2-4 a c\right)^{7/4} \left(36 c^2 d^2+11 b^2 e^2-4 c e (9 b d+2 a e)\right)$$

$$\sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right) \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}} \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right) \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} c^{1/4} \left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 5, 275 leaves):

$$\frac{1}{5040 c^4 (a+x (b+c x))^{1/4}} \Bigg(4 c (a+x (b+c x))$$

$$\left(77 b^3 e^2-6 b^2 c e (42 d+11 e x)+12 b c \left(-23 a e^2+c \left(21 d^2+18 d e x+5 e^2 x^2\right)\right)+8 c^2 \left(3 a e (30 d+7 e x)+c x \left(63 d^2+90 d e x+35 e^2 x^2\right)\right)\right)-$$

$$7\times 2^{3/4} \left(b^2-4 a c\right) \left(36 c^2 d^2+11 b^2 e^2-4 c e (9 b d+2 a e)\right) \left(b-\sqrt{b^2-4 a c}+2 c x\right)$$

$$\left(\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{-b+\sqrt{b^2-4 a c}-2 c x}{2 \sqrt{b^2-4 a c}}\right]\Bigg)$$

- **Problem 2518: Result unnecessarily involves higher level functions.**

$$\int (d+e x) \left(a+b x+c x^2\right)^{3/4} dx$$

Optimal (type 4, 510 leaves, 6 steps):

$$\frac{(2cd-be)(b+2cx)\left(a+bx+cx^2\right)^{3/4}}{10c^2}+\frac{2e\left(a+bx+cx^2\right)^{7/4}}{7c}-\frac{3\sqrt{b^2-4ac}\,(2cd-be)(b+2cx)\left(a+bx+cx^2\right)^{1/4}}{20c^{5/2}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}+$$

$$\frac{1}{20\sqrt{2}\,c^{11/4}(b+2cx)}3\left(b^2-4ac\right)^{7/4}(2cd-be)\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}$$

$$\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\left(a+bx+cx^2\right)^{1/4}}{\left(b^2-4ac\right)^{1/4}}\right],\frac{1}{2}\right]-\frac{1}{40\sqrt{2}\,c^{11/4}(b+2cx)}$$

$$3\left(b^2-4ac\right)^{7/4}(2cd-be)\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\left(a+bx+cx^2\right)^{1/4}}{\left(b^2-4ac\right)^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 5, 195 leaves) :

$$\frac{1}{280c^3(a+x(b+cx))^{1/4}}\Bigg(4c(a+x(b+cx))\left(-7b^2e+2bc(7d+3ex)+4c(5ae+cx(7d+5ex))\right)+$$

$$7\times 2^{3/4}\left(b^2-4ac\right)(-2cd+be)\left(b-\sqrt{b^2-4ac}+2cx\right)\left(\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{-b+\sqrt{b^2-4ac}-2cx}{2\sqrt{b^2-4ac}}\right]\Bigg)$$

- **Problem 2519: Result unnecessarily involves higher level functions.**

$$\int\left(a+bx+cx^2\right)^{3/4}dx$$

Optimal (type 4, 452 leaves, 5 steps) :

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/4}}{5c}-\frac{3\sqrt{b^2-4ac}\,(b+2cx)\left(a+bx+cx^2\right)^{1/4}}{10c^{3/2}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}+\frac{1}{10\sqrt{2}\,c^{7/4}(b+2cx)}3\left(b^2-4ac\right)^{7/4}$$

$$\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\left(a+bx+cx^2\right)^{1/4}}{\left(b^2-4ac\right)^{1/4}}\right],\frac{1}{2}\right]-\frac{1}{20\sqrt{2}\,c^{7/4}(b+2cx)}$$

$$3\left(b^2-4ac\right)^{7/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\left(a+bx+cx^2\right)^{1/4}}{\left(b^2-4ac\right)^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 5, 155 leaves) :

$$\frac{1}{20c^2\,(a+x\,(b+cx))^{1/4}}\left(4c\,(b+2cx)\,(a+x\,(b+cx))-\right.$$

$$\left.2^{3/4}\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}+2cx\right)\left(\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{-b+\sqrt{b^2-4ac}-2cx}{2\sqrt{b^2-4ac}}\right]\right)$$

- **Problem 2520: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+bx+cx^2\right)^{3/4}}{d+ex}\,dx$$

Optimal (type 4, 1209 leaves, 20 steps) :

$$\frac{2\left(a+b x+c x^2\right)^{3/4}}{3 e}-\frac{(2 c d-b e)\,(b+2 c x)\,\left(a+b x+c x^2\right)^{1/4}}{\sqrt{c}\,\sqrt{b^2-4 a c}\,e^2\left(1+\frac{2\sqrt{c}\,\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)}+$$

$$\frac{\left(-b^2+4 a c\right)^{1/4}\left(c d^2-b d e+a e^2\right)^{3/4}\left(-\frac{c\,(a+b x+c x^2)}{b^2-4 a c}\right)^{1/4}\text{ArcTan}\left[\frac{\left(-b^2+4 a c\right)^{1/4}\sqrt{e}\,\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}}{\sqrt{2}\,c^{1/4}\left(c d^2-b d e+a e^2\right)^{1/4}}\right]}{c^{1/4}\,e^{5/2}\left(a+b x+c x^2\right)^{1/4}}-$$

$$\frac{\left(-b^2+4 a c\right)^{1/4}\left(c d^2-b d e+a e^2\right)^{3/4}\left(-\frac{c\,(a+b x+c x^2)}{b^2-4 a c}\right)^{1/4}\text{ArcTanh}\left[\frac{\left(-b^2+4 a c\right)^{1/4}\sqrt{e}\,\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}}{\sqrt{2}\,c^{1/4}\left(c d^2-b d e+a e^2\right)^{1/4}}\right]}{c^{1/4}\,e^{5/2}\left(a+b x+c x^2\right)^{1/4}}+\frac{1}{\sqrt{2}\,c^{3/4}\,e^2\,(b+2 c x)}$$

$$\left(b^2-4 a c\right)^{3/4}(2 c d-b e)\sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right)\left(1+\frac{2\sqrt{c}\,\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}}\left(1+\frac{2\sqrt{c}\,\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right],\frac{1}{2}\right]-$$

$$\frac{1}{2\sqrt{2}\,c^{3/4}\,e^2\,(b+2 c x)}\left(b^2-4 a c\right)^{3/4}(2 c d-b e)\sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right)\left(1+\frac{2\sqrt{c}\,\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}}\left(1+\frac{2\sqrt{c}\,\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)$$

$$\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right],\frac{1}{2}\right]-\left(\sqrt{-b^2+4 a c}\,(2 c d-b e)\sqrt{c d^2-b d e+a e^2}\sqrt{\frac{(b+2 c x)^2}{b^2-4 a c}}\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{1/4}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\sqrt{-b^2+4 a c}\,e}{2\sqrt{c}\,\sqrt{c d^2-b d e+a e^2}},\text{ArcSin}\left[\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}\right],-1\right]\right)\Big/\left(\sqrt{2}\,\sqrt{c}\,e^3\,(b+2 c x)\left(a+b x+c x^2\right)^{1/4}\right)+$$

$$\left(\sqrt{-b^2+4 a c}\,(2 c d-b e)\sqrt{c d^2-b d e+a e^2}\sqrt{\frac{(b+2 c x)^2}{b^2-4 a c}}\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{1/4}\right.$$

$$\left.\text{EllipticPi}\left[\frac{\sqrt{-b^2+4 a c}\,e}{2\sqrt{c}\,\sqrt{c d^2-b d e+a e^2}},\text{ArcSin}\left[\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}\right],-1\right]\right)\Big/\left(\sqrt{2}\,\sqrt{c}\,e^3\,(b+2 c x)\left(a+b x+c x^2\right)^{1/4}\right)$$

Result (type 6, 180 leaves) :

$$\frac{4\sqrt{2}\,(a+x\,(b+c\,x))^{3/4}\,\text{AppellF1}\left[-\frac{3}{2},-\frac{3}{4},-\frac{3}{4},-\frac{1}{2},\frac{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}{2\,c\,(d+e\,x)},\frac{2\,c\,d-b\,e+\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d+2\,c\,e\,x}\right]}{3\,e\left(\frac{e\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}{c\,(d+e\,x)}\right)^{3/4}\left(\frac{e\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}{c\,(d+e\,x)}\right)^{3/4}}$$

- **Problem 2521: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+b\,x+c\,x^2\right)^{3/4}}{(d+e\,x)^2}\,dx$$

Optimal (type 4, 1220 leaves, 20 steps):

$$-\frac{\left(a+b x+c x^2\right)^{3/4}}{e\,(d+e x)}+\frac{3 \sqrt{c}\,(b+2 c x)\left(a+b x+c x^2\right)^{1/4}}{\sqrt{b^2-4 a c}\;e^2\left(1+\frac{2 \sqrt{c}\,\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)}-\frac{3\left(-b^2+4 a c\right)^{1/4}(2 c d-b e)\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{1/4}\text{ArcTan}\left[\frac{\left(-b^2+4 a c\right)^{1/4}\sqrt{e}\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}}{\sqrt{2}\,c^{1/4}\left(c d^2-b d e+a e^2\right)^{1/4}}\right]}{4 c^{1/4} e^{5/2}\left(c d^2-b d e+a e^2\right)^{1/4}\left(a+b x+c x^2\right)^{1/4}}+$$

$$\frac{3\left(-b^2+4 a c\right)^{1/4}(2 c d-b e)\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{1/4}\text{ArcTanh}\left[\frac{\left(-b^2+4 a c\right)^{1/4}\sqrt{e}\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}}{\sqrt{2}\,c^{1/4}\left(c d^2-b d e+a e^2\right)^{1/4}}\right]}{4 c^{1/4} e^{5/2}\left(c d^2-b d e+a e^2\right)^{1/4}\left(a+b x+c x^2\right)^{1/4}}-\frac{1}{\sqrt{2}\,e^2\,(b+2 c x)}$$

$$3 c^{1/4}\left(b^2-4 a c\right)^{3/4}\sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right)\left(1+\frac{2 \sqrt{c}\,\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}}\left(1+\frac{2 \sqrt{c}\,\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right],\frac{1}{2}\right]+$$

$$\frac{1}{2 \sqrt{2}\,e^2\,(b+2 c x)}3 c^{1/4}\left(b^2-4 a c\right)^{3/4}\sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right)\left(1+\frac{2 \sqrt{c}\,\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}}\left(1+\frac{2 \sqrt{c}\,\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)$$

$$\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right],\frac{1}{2}\right]+\left(3 \sqrt{-b^2+4 a c}\,(2 c d-b e)^2\sqrt{\frac{(b+2 c x)^2}{b^2-4 a c}}\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{1/4}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\sqrt{-b^2+4 a c}\;e}{2 \sqrt{c}\,\sqrt{c d^2-b d e+a e^2}},\text{ArcSin}\left[\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}\right],-1\right]\right)\Big/\left(4 \sqrt{2}\,\sqrt{c}\;e^3\sqrt{c d^2-b d e+a e^2}\,(b+2 c x)\left(a+b x+c x^2\right)^{1/4}\right)-$$

$$\left(3 \sqrt{-b^2+4 a c}\,(2 c d-b e)^2\sqrt{\frac{(b+2 c x)^2}{b^2-4 a c}}\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{1/4}\text{EllipticPi}\left[\frac{\sqrt{-b^2+4 a c}\;e}{2 \sqrt{c}\,\sqrt{c d^2-b d e+a e^2}},\text{ArcSin}\left[\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}\right],-1\right]\right)\Big/$$

$$\left(4 \sqrt{2}\,\sqrt{c}\;e^3\sqrt{c d^2-b d e+a e^2}\,(b+2 c x)\left(a+b x+c x^2\right)^{1/4}\right)$$

Result (type 6, 185 leaves):

$$\frac{4 \sqrt{2}\,(a+x\,(b+c x))^{3/4}\,\text{AppellF1}\left[-\frac{1}{2},-\frac{3}{4},-\frac{3}{4},\frac{1}{2},\frac{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}{2 c\,(d+e x)},\frac{2 c d-b e+\sqrt{b^2-4 a c}\;e}{2 c d+2 c e x}\right]}{e\left(\frac{e\left(b-\sqrt{b^2-4 a c}+2 c x\right)}{c\,(d+e x)}\right)^{3/4}\left(\frac{e\left(b+\sqrt{b^2-4 a c}+2 c x\right)}{c\,(d+e x)}\right)^{3/4}(d+e x)}$$

- **Problem 2522: Result unnecessarily involves higher level functions.**

$$\int (d+e\,x)^3\,\left(a+b\,x+c\,x^2\right)^{5/4}\,dx$$

Optimal (type 4, 448 leaves, 6 steps):

$$-\frac{5\,\left(b^2-4\,a\,c\right)\,(2\,c\,d-b\,e)\,\left(44\,c^2\,d^2+17\,b^2\,e^2-4\,c\,e\,(11\,b\,d+6\,a\,e)\right)\,(b+2\,c\,x)\,\left(a+b\,x+c\,x^2\right)^{1/4}}{7392\,c^5}+$$
$$\frac{(2\,c\,d-b\,e)\,\left(44\,c^2\,d^2+17\,b^2\,e^2-4\,c\,e\,(11\,b\,d+6\,a\,e)\right)\,(b+2\,c\,x)\,\left(a+b\,x+c\,x^2\right)^{5/4}}{616\,c^4}+\frac{2\,e\,(d+e\,x)^2\,\left(a+b\,x+c\,x^2\right)^{9/4}}{13\,c}+$$
$$\frac{e\,\left(1320\,c^2\,d^2+221\,b^2\,e^2-2\,c\,e\,(507\,b\,d+88\,a\,e)+306\,c\,e\,(2\,c\,d-b\,e)\,x\right)\,\left(a+b\,x+c\,x^2\right)^{9/4}}{2574\,c^3}+$$
$$\left(5\,\left(b^2-4\,a\,c\right)^{9/4}\,(2\,c\,d-b\,e)\,\left(44\,c^2\,d^2+17\,b^2\,e^2-4\,c\,e\,(11\,b\,d+6\,a\,e)\right)\,\sqrt{\frac{(b+2\,c\,x)^2}{\left(b^2-4\,a\,c\right)\,\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)^2}}\right.$$
$$\left.\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\,\left(a+b\,x+c\,x^2\right)^{1/4}}{\left(b^2-4\,a\,c\right)^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(14784\,\sqrt{2}\,c^{21/4}\,(b+2\,c\,x)\right)$$

Result (type 5, 590 leaves):

$$\frac{1}{1153152\,c^6\,(a+x\,(b+c\,x))^{3/4}}$$
$$\left(-4\,c\,(a+x\,(b+c\,x))\,\left(-3315\,b^6\,e^3+78\,b^5\,c\,e^2\,(195\,d+17\,e\,x)-52\,b^4\,c\,e\,\left(-498\,a\,e^2+c\,\left(495\,d^2+117\,d\,e\,x+17\,e^2\,x^2\right)\right)-16\right.\right.$$
$$b^2\,c^2\,\left(3419\,a^2\,e^3-2\,a\,c\,e\,\left(5148\,d^2+1131\,d\,e\,x+158\,e^2\,x^2\right)+c^2\,x\,\left(429\,d^3+429\,d^2\,e\,x+195\,d\,e^2\,x^2+35\,e^3\,x^3\right)\right)+$$
$$8\,b^3\,c^2\,\left(-26\,a\,e^2\,(513\,d+43\,e\,x)+c\,\left(2145\,d^3+1287\,d^2\,e\,x+507\,d\,e^2\,x^2+85\,e^3\,x^3\right)\right)-64\,c^3\,\left(-308\,a^3\,e^3+a^2\,c\,e\,\left(3003\,d^2+585\,d\,e\,x+77\,e^2\,x^2\right)+\right.$$
$$\left.3\,c^3\,x^3\,\left(429\,d^3+1001\,d^2\,e\,x+819\,d\,e^2\,x^2+231\,e^3\,x^3\right)+2\,a\,c^2\,x\,\left(1716\,d^3+3003\,d^2\,e\,x+2106\,d\,e^2\,x^2+539\,e^3\,x^3\right)\right)+$$
$$\left.32\,b\,c^3\,\left(a^2\,e^2\,(5421\,d+431\,e\,x)-4\,a\,c\,\left(858\,d^3+429\,d^2\,e\,x+156\,d\,e^2\,x^2+25\,e^3\,x^3\right)-3\,c^2\,x^2\,\left(1287\,d^3+2717\,d^2\,e\,x+2093\,d\,e^2\,x^2+567\,e^3\,x^3\right)\right)\right)+$$
$$195\times 2^{1/4}\,\left(b^2-4\,a\,c\right)^2\,(2\,c\,d-b\,e)\,\left(44\,c^2\,d^2+17\,b^2\,e^2-4\,c\,e\,(11\,b\,d+6\,a\,e)\right)\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)$$
$$\left.\left(\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{3/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{-b+\sqrt{b^2-4\,a\,c}-2\,c\,x}{2\,\sqrt{b^2-4\,a\,c}}\right]\right)$$

Problem 2523: Result unnecessarily involves higher level functions.

$$\int (d+e x)^2 \left(a+b x+c x^2\right)^{5/4} \, dx$$

Optimal (type 4, 384 leaves, 6 steps):

$$-\frac{5 \left(b^2-4 a c\right) \left(44 c^2 d^2+13 b^2 e^2-4 c e (11 b d+2 a e)\right) (b+2 c x) \left(a+b x+c x^2\right)^{1/4}}{3696 c^4}+\frac{\left(44 c^2 d^2+13 b^2 e^2-4 c e (11 b d+2 a e)\right) (b+2 c x) \left(a+b x+c x^2\right)^{5/4}}{308 c^3}+\frac{13 e (2 c d-b e) \left(a+b x+c x^2\right)^{9/4}}{99 c^2}+\frac{2 e (d+e x) \left(a+b x+c x^2\right)^{9/4}}{11 c}+\frac{1}{7392 \sqrt{2}\, c^{17/4} (b+2 c x)} 5 \left(b^2-4 a c\right)^{9/4} \left(44 c^2 d^2+13 b^2 e^2-4 c e (11 b d+2 a e)\right) \sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right) \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}} \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right) \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2}\, c^{1/4} \left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 5, 416 leaves):

$$\frac{1}{44352 c^5 (a+x (b+c x))^{3/4}} \left(-4 c (a+x (b+c x)) \left(195 b^5 e^2-6 b^4 c e (110 d+13 e x)+8 b^2 c^2 \left(2 a e (264 d+29 e x)-c x \left(33 d^2+22 d e x+5 e^2 x^2\right)\right)+4 b^3 c \left(-342 a e^2+c \left(165 d^2+66 d e x+13 e^2 x^2\right)\right)-32 c^3 \left(a^2 e (154 d+15 e x)+4 a c x \left(66 d^2+77 d e x+27 e^2 x^2\right)+c^2 x^3 \left(99 d^2+154 d e x+63 e^2 x^2\right)\right)-16 b c^2 \left(-139 a^2 e^2+8 a c \left(33 d^2+11 d e x+2 e^2 x^2\right)+c^2 x^2 \left(297 d^2+418 d e x+161 e^2 x^2\right)\right)\right)+15\times 2^{1/4} \left(b^2-4 a c\right)^2 \left(44 c^2 d^2+13 b^2 e^2-4 c e (11 b d+2 a e)\right) \left(b-\sqrt{b^2-4 a c}+2 c x\right) \left(\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{-b+\sqrt{b^2-4 a c}-2 c x}{2 \sqrt{b^2-4 a c}}\right]\right)$$

Problem 2524: Result unnecessarily involves higher level functions.

$$\int (d+e x) \left(a+b x+c x^2\right)^{5/4} \, dx$$

Optimal (type 4, 285 leaves, 5 steps):

$$-\frac{5\left(b^2-4ac\right)(2cd-be)(b+2cx)\left(a+bx+cx^2\right)^{1/4}}{168c^3}+\frac{(2cd-be)(b+2cx)\left(a+bx+cx^2\right)^{5/4}}{14c^2}+\frac{2e\left(a+bx+cx^2\right)^{9/4}}{9c}+\frac{1}{336\sqrt{2}\,c^{13/4}(b+2cx)}$$

$$5\left(b^2-4ac\right)^{9/4}(2cd-be)\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\left(a+bx+cx^2\right)^{1/4}}{\left(b^2-4ac\right)^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 5, 270 leaves):

$$\frac{1}{2016c^4(a+x(b+cx))^{3/4}}\left(-4c(a+x(b+cx))\left(-15b^4e+6b^3c(5d+ex)-4b^2c(-24ae+cx(3d+ex))-16c^2\left(7a^2e+c^2x^3(9d+7ex)+2acx(12d+7ex)\right)-8bc^2\left(4a(6d+ex)+cx^2(27d+19ex)\right)\right)-15\times2^{1/4}\left(b^2-4ac\right)^2(-2cd+be)\left(b-\sqrt{b^2-4ac}+2cx\right)\left(\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{-b+\sqrt{b^2-4ac}-2cx}{2\sqrt{b^2-4ac}}\right]\right)$$

- **Problem 2525: Result unnecessarily involves higher level functions.**

$$\int\left(a+bx+cx^2\right)^{5/4}dx$$

Optimal (type 4, 236 leaves, 4 steps):

$$-\frac{5\left(b^2-4ac\right)(b+2cx)\left(a+bx+cx^2\right)^{1/4}}{84c^2}+\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/4}}{7c}+\frac{1}{168\sqrt{2}\,c^{9/4}(b+2cx)}$$

$$5\left(b^2-4ac\right)^{9/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\left(a+bx+cx^2\right)^{1/4}}{\left(b^2-4ac\right)^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 5, 181 leaves):

$$\frac{1}{336c^3(a+x(b+cx))^{3/4}}\left(4c(b+2cx)(a+x(b+cx))\left(-5b^2+12bcx+4c\left(8a+3cx^2\right)\right)+5\times2^{1/4}\left(b^2-4ac\right)^2\left(b-\sqrt{b^2-4ac}+2cx\right)\left(\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{-b+\sqrt{b^2-4ac}-2cx}{2\sqrt{b^2-4ac}}\right]\right)$$

Problem 2526: Unable to integrate problem.

$$\int \frac{\left(a+b x+c x^2\right)^{5/4}}{d+e x}\,dx$$

Optimal (type 4, 1014 leaves, 20 steps):

$$\frac{\left(12 c^2 d^2+b^2 e^2-2 c e (7 b d-6 a e)-2 c e (2 c d-b e) x\right) \left(a+b x+c x^2\right)^{1/4}}{6 c e^3}+$$

$$\frac{2 \left(a+b x+c x^2\right)^{5/4}}{5 e}-\frac{\left(-b^2+4 a c\right)^{3/4} \left(c d^2-b d e+a e^2\right)^{5/4} \left(-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{3/4} \text{ArcTan}\left[\frac{\left(-b^2+4 a c\right)^{1/4} \sqrt{e} \left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}}{\sqrt{2} c^{1/4} \left(c d^2-b d e+a e^2\right)^{1/4}}\right]}{c^{3/4} e^{7/2} \left(a+b x+c x^2\right)^{3/4}}-$$

$$\frac{\left(-b^2+4 a c\right)^{3/4} \left(c d^2-b d e+a e^2\right)^{5/4} \left(-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{3/4} \text{ArcTanh}\left[\frac{\left(-b^2+4 a c\right)^{1/4} \sqrt{e} \left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}}{\sqrt{2} c^{1/4} \left(c d^2-b d e+a e^2\right)^{1/4}}\right]}{c^{3/4} e^{7/2} \left(a+b x+c x^2\right)^{3/4}}-\frac{1}{12 \sqrt{2} c^{5/4} e^4 (b+2 c x)}$$

$$\left(b^2-4 a c\right)^{1/4} (2 c d-b e) \left(12 c^2 d^2-b^2 e^2-4 c e (3 b d-4 a e)\right) \sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right) \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}} \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)$$

$$\text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} c^{1/4} \left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right],\frac{1}{2}\right]-\left(\left(b^2-4 a c\right) (2 c d-b e) \left(c d^2-b d e+a e^2\right) \sqrt{\frac{(b+2 c x)^2}{b^2-4 a c}} \left(-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{3/4}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\sqrt{-b^2+4 a c}\, e}{2 \sqrt{c} \sqrt{c d^2-b d e+a e^2}},\text{ArcSin}\left[\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}\right],-1\right]\right)\Big/\left(\sqrt{2} c e^4 (b+2 c x) \left(a+b x+c x^2\right)^{3/4}\right)-$$

$$\left(\left(b^2-4 a c\right) (2 c d-b e) \left(c d^2-b d e+a e^2\right) \sqrt{\frac{(b+2 c x)^2}{b^2-4 a c}} \left(-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{3/4}\right.$$

$$\left.\text{EllipticPi}\left[\frac{\sqrt{-b^2+4 a c}\, e}{2 \sqrt{c} \sqrt{c d^2-b d e+a e^2}},\text{ArcSin}\left[\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}\right],-1\right]\right)\Big/\left(\sqrt{2} c e^4 (b+2 c x) \left(a+b x+c x^2\right)^{3/4}\right)$$

Result (type 8, 24 leaves):

$$\int \frac{\left(a+b x+c x^2\right)^{5/4}}{d+e x}\,dx$$

Problem 2527: Unable to integrate problem.

$$\int \frac{\left(a+b x+c x^2\right)^{5/4}}{(d+e x)^2} \, dx$$

Optimal (type 4, 975 leaves, 20 steps):

$$-\frac{5 (3 c d-2 b e-c e x) \left(a+b x+c x^2\right)^{1/4}}{3 e^3}-\frac{\left(a+b x+c x^2\right)^{5/4}}{e (d+e x)}+$$

$$\left(5 \left(-b^2+4 a c\right)^{3/4} (2 c d-b e) \left(c d^2-b d e+a e^2\right)^{1/4} \left(-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{3/4} \text{ArcTan}\left[\frac{\left(-b^2+4 a c\right)^{1/4} \sqrt{e} \left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}}{\sqrt{2} c^{1/4} \left(c d^2-b d e+a e^2\right)^{1/4}}\right]\right)\Big/$$

$$\left(4 c^{3/4} e^{7/2} \left(a+b x+c x^2\right)^{3/4}\right)+$$

$$\left(5 \left(-b^2+4 a c\right)^{3/4} (2 c d-b e) \left(c d^2-b d e+a e^2\right)^{1/4} \left(-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{3/4} \text{ArcTanh}\left[\frac{\left(-b^2+4 a c\right)^{1/4} \sqrt{e} \left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}}{\sqrt{2} c^{1/4} \left(c d^2-b d e+a e^2\right)^{1/4}}\right]\right)\Big/$$

$$\left(4 c^{3/4} e^{7/2} \left(a+b x+c x^2\right)^{3/4}\right)+\frac{1}{6 \sqrt{2} c^{1/4} e^4 (b+2 c x)}5 \left(b^2-4 a c\right)^{1/4} \left(6 c^2 d^2+b^2 e^2-2 c e (3 b d-a e)\right)$$

$$\sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right) \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}} \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right) \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} c^{1/4} \left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right], \frac{1}{2}\right]+$$

$$\left(5 \left(b^2-4 a c\right) (2 c d-b e)^2 \sqrt{\frac{(b+2 c x)^2}{b^2-4 a c}} \left(-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{3/4} \text{EllipticPi}\left[-\frac{\sqrt{-b^2+4 a c}\, e}{2 \sqrt{c} \sqrt{c d^2-b d e+a e^2}}, \text{ArcSin}\left[\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}\right], -1\right]\right)\Big/$$

$$\left(4 \sqrt{2} c e^4 (b+2 c x) \left(a+b x+c x^2\right)^{3/4}\right)+$$

$$\left(5 \left(b^2-4 a c\right) (2 c d-b e)^2 \sqrt{\frac{(b+2 c x)^2}{b^2-4 a c}} \left(-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{3/4} \text{EllipticPi}\left[\frac{\sqrt{-b^2+4 a c}\, e}{2 \sqrt{c} \sqrt{c d^2-b d e+a e^2}}, \text{ArcSin}\left[\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}\right], -1\right]\right)\Big/$$

$$\left(4 \sqrt{2} c e^4 (b+2 c x) \left(a+b x+c x^2\right)^{3/4}\right)$$

Result (type 8, 24 leaves):

$$\int \frac{\left(a+b x+c x^2\right)^{5/4}}{(d+e x)^2} \, dx$$

Problem 2528: Result unnecessarily involves higher level functions.

$$\int \frac{(d+e x)^3}{\left(a+b x+c x^2\right)^{1/4}} \, dx$$

Optimal (type 4, 637 leaves, 6 steps):

$$\frac{2 e (d+e x)^2 \left(a+b x+c x^2\right)^{3/4}}{7 c}+\frac{e \left(360 c^2 d^2+77 b^2 e^2-2 c e (147 b d+40 a e)+66 c e (2 c d-b e) x\right) \left(a+b x+c x^2\right)^{3/4}}{210 c^3}+\frac{(2 c d-b e) \left(20 c^2 d^2+11 b^2 e^2-4 c e (5 b d+6 a e)\right) (b+2 c x) \left(a+b x+c x^2\right)^{1/4}}{20 c^{7/2} \sqrt{b^2-4 a c} \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)}-\frac{1}{20 \sqrt{2} c^{15/4} (b+2 c x)} \left(b^2-4 a c\right)^{3/4} (2 c d-b e) \left(20 c^2 d^2+11 b^2 e^2-4 c e (5 b d+6 a e)\right) \sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right) \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}} \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right) \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} c^{1/4} \left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right], \frac{1}{2}\right]+\frac{1}{40 \sqrt{2} c^{15/4} (b+2 c x)} \left(b^2-4 a c\right)^{3/4} (2 c d-b e) \left(20 c^2 d^2+11 b^2 e^2-4 c e (5 b d+6 a e)\right) \sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right) \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}} \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right) \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} c^{1/4} \left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 5, 232 leaves):

$$\frac{1}{840 c^4 (a+x (b+c x))^{1/4}} \left(4 c e (a+x (b+c x)) \left(77 b^2 e^2-2 c e (147 b d+40 a e+33 b e x)+12 c^2 \left(35 d^2+21 d e x+5 e^2 x^2\right)\right)+7 \times 2^{3/4} (2 c d-b e) \left(20 c^2 d^2+11 b^2 e^2-4 c e (5 b d+6 a e)\right) \left(b-\sqrt{b^2-4 a c}+2 c x\right) \left(\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{-b+\sqrt{b^2-4 a c}-2 c x}{2 \sqrt{b^2-4 a c}}\right]\right)$$

Problem 2529: Result unnecessarily involves higher level functions.

$$\int \frac{(d+e x)^2}{\left(a+b x+c x^2\right)^{1/4}} \, dx$$

Optimal (type 4, 573 leaves, 6 steps):

$$\frac{7 e (2 c d-b e) \left(a+b x+c x^2\right)^{3/4}}{15 c^2}+\frac{2 e (d+e x) \left(a+b x+c x^2\right)^{3/4}}{5 c}+\frac{\left(20 c^2 d^2+7 b^2 e^2-4 c e (5 b d+2 a e)\right) (b+2 c x) \left(a+b x+c x^2\right)^{1/4}}{10 c^{5/2} \sqrt{b^2-4 a c} \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)}-$$

$$\frac{1}{10 \sqrt{2} c^{11/4} (b+2 c x)}\left(b^2-4 a c\right)^{3/4} \left(20 c^2 d^2+7 b^2 e^2-4 c e (5 b d+2 a e)\right) \sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right) \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}} \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)$$

$$\text{EllipticE}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} c^{1/4} \left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right], \frac{1}{2}\right]+\frac{1}{20 \sqrt{2} c^{11/4} (b+2 c x)}\left(b^2-4 a c\right)^{3/4} \left(20 c^2 d^2+7 b^2 e^2-4 c e (5 b d+2 a e)\right)$$

$$\sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right) \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}} \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right) \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} c^{1/4} \left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 5, 185 leaves) :

$$\frac{1}{60 c^3 (a+x (b+c x))^{1/4}}\Bigg(4 c e (20 c d-7 b e+6 c e x) (a+x (b+c x))+2^{3/4} \left(20 c^2 d^2+7 b^2 e^2-4 c e (5 b d+2 a e)\right)$$

$$\left(b-\sqrt{b^2-4 a c}+2 c x\right) \left(\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{-b+\sqrt{b^2-4 a c}-2 c x}{2 \sqrt{b^2-4 a c}}\right]\Bigg)$$

- **Problem 2530: Result unnecessarily involves higher level functions.**

$$\int \frac{d+e x}{\left(a+b x+c x^2\right)^{1/4}} \, dx$$

Optimal (type 4, 469 leaves, 5 steps) :

$$\frac{2e\left(a+bx+cx^2\right)^{3/4}}{3c}+\frac{(2cd-be)(b+2cx)\left(a+bx+cx^2\right)^{1/4}}{c^{3/2}\sqrt{b^2-4ac}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}-\frac{1}{\sqrt{2}\,c^{7/4}(b+2cx)}\left(b^2-4ac\right)^{3/4}(2cd-be)$$

$$\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\left(a+bx+cx^2\right)^{1/4}}{\left(b^2-4ac\right)^{1/4}}\right],\frac{1}{2}\right]+\frac{1}{2\sqrt{2}\,c^{7/4}(b+2cx)}$$

$$\left(b^2-4ac\right)^{3/4}(2cd-be)\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\left(a+bx+cx^2\right)^{1/4}}{\left(b^2-4ac\right)^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 5, 152 leaves):

$$\frac{1}{12c\,(a+x(b+cx))^{1/4}}\left(8e\,(a+x(b+cx))-\right.$$

$$\left.1/c2\times 2^{3/4}(-2cd+be)\left(b-\sqrt{b^2-4ac}+2cx\right)\left(\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{-b+\sqrt{b^2-4ac}-2cx}{2\sqrt{b^2-4ac}}\right]\right)$$

■ Problem 2531: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a+bx+cx^2\right)^{1/4}}\,dx$$

Optimal (type 4, 418 leaves, 4 steps):

$$\frac{2(b+2cx)\left(a+bx+cx^2\right)^{1/4}}{\sqrt{c}\sqrt{b^2-4ac}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}-\frac{1}{c^{3/4}(b+2cx)}\sqrt{2}\left(b^2-4ac\right)^{3/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}$$

$$\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\left(a+bx+cx^2\right)^{1/4}}{\left(b^2-4ac\right)^{1/4}}\right],\frac{1}{2}\right]+\frac{1}{\sqrt{2}\,c^{3/4}(b+2cx)}$$

$$\left(b^2-4ac\right)^{3/4}\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\left(a+bx+cx^2\right)^{1/4}}{\left(b^2-4ac\right)^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 5, 126 leaves) :

$$\frac{2^{3/4}\left(b-\sqrt{b^2-4ac}+2cx\right)\left(\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{-b+\sqrt{b^2-4ac}-2cx}{2\sqrt{b^2-4ac}}\right]}{3c\,(a+x\,(b+cx))^{1/4}}$$

- **Problem 2532: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{(d+ex)\,(a+bx+cx^2)^{1/4}}\,dx$$

Optimal (type 4, 733 leaves, 14 steps) :

$$\frac{(-b^2+4ac)^{1/4}\left(-\frac{c(a+bx+cx^2)}{b^2-4ac}\right)^{1/4}\text{ArcTan}\left[\frac{(-b^2+4ac)^{1/4}\sqrt{e}\left(1-\frac{(b+2cx)^2}{b^2-4ac}\right)^{1/4}}{\sqrt{2}\,c^{1/4}\,(cd^2-bde+ae^2)^{1/4}}\right]}{c^{1/4}\sqrt{e}\,(cd^2-bde+ae^2)^{1/4}\,(a+bx+cx^2)^{1/4}}-\frac{(-b^2+4ac)^{1/4}\left(-\frac{c(a+bx+cx^2)}{b^2-4ac}\right)^{1/4}\text{ArcTanh}\left[\frac{(-b^2+4ac)^{1/4}\sqrt{e}\left(1-\frac{(b+2cx)^2}{b^2-4ac}\right)^{1/4}}{\sqrt{2}\,c^{1/4}\,(cd^2-bde+ae^2)^{1/4}}\right]}{c^{1/4}\sqrt{e}\,(cd^2-bde+ae^2)^{1/4}\,(a+bx+cx^2)^{1/4}}-$$

$$\left(\sqrt{-b^2+4ac}\,(2cd-be)\sqrt{\frac{(b+2cx)^2}{b^2-4ac}}\left(-\frac{c(a+bx+cx^2)}{b^2-4ac}\right)^{1/4}\text{EllipticPi}\left[-\frac{\sqrt{-b^2+4ac}\,e}{2\sqrt{c}\sqrt{cd^2-bde+ae^2}},\text{ArcSin}\left[\left(1-\frac{(b+2cx)^2}{b^2-4ac}\right)^{1/4}\right],-1\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{c}\,e\sqrt{cd^2-bde+ae^2}\,(b+2cx)\,(a+bx+cx^2)^{1/4}\right)+$$

$$\left(\sqrt{-b^2+4ac}\,(2cd-be)\sqrt{\frac{(b+2cx)^2}{b^2-4ac}}\left(-\frac{c(a+bx+cx^2)}{b^2-4ac}\right)^{1/4}\text{EllipticPi}\left[\frac{\sqrt{-b^2+4ac}\,e}{2\sqrt{c}\sqrt{cd^2-bde+ae^2}},\text{ArcSin}\left[\left(1-\frac{(b+2cx)^2}{b^2-4ac}\right)^{1/4}\right],-1\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{c}\,e\sqrt{cd^2-bde+ae^2}\,(b+2cx)\,(a+bx+cx^2)^{1/4}\right)$$

Result (type 6, 178 leaves) :

$$-\frac{1}{e\,(a+x\,(b+cx))^{1/4}}$$

$$\sqrt{2}\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{c\,(d+ex)}\right)^{1/4}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{c\,(d+ex)}\right)^{1/4}\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},\frac{1}{4},\frac{3}{2},\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)e}{2c\,(d+ex)},\frac{2cd-be+\sqrt{b^2-4ac}\,e}{2cd+2cex}\right]$$

- **Problem 2533: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{(d+ex)^2\,(a+bx+cx^2)^{1/4}}\,dx$$

Optimal (type 4, 1280 leaves, 20 steps) :

$$-\frac{e\left(a+b x+c x^2\right)^{3/4}}{\left(c d^2-b d e+a e^2\right)(d+e x)}+\frac{\sqrt{c}\,(b+2 c x)\left(a+b x+c x^2\right)^{1/4}}{\sqrt{b^2-4 a c}\left(c d^2-b d e+a e^2\right)\left(1+\frac{2\sqrt{c}\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)}+$$

$$\frac{\left(-b^2+4 a c\right)^{1/4}(2 c d-b e)\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{1/4}\text{ArcTan}\left[\frac{\left(-b^2+4 a c\right)^{1/4}\sqrt{e}\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}}{\sqrt{2}\,c^{1/4}\left(c d^2-b d e+a e^2\right)^{1/4}}\right]}{4 c^{1/4}\sqrt{e}\left(c d^2-b d e+a e^2\right)^{5/4}\left(a+b x+c x^2\right)^{1/4}}-$$

$$\frac{\left(-b^2+4 a c\right)^{1/4}(2 c d-b e)\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{1/4}\text{ArcTanh}\left[\frac{\left(-b^2+4 a c\right)^{1/4}\sqrt{e}\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}}{\sqrt{2}\,c^{1/4}\left(c d^2-b d e+a e^2\right)^{1/4}}\right]}{4 c^{1/4}\sqrt{e}\left(c d^2-b d e+a e^2\right)^{5/4}\left(a+b x+c x^2\right)^{1/4}}-$$

$$\left(c^{1/4}\left(b^2-4 a c\right)^{3/4}\sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right)\left(1+\frac{2\sqrt{c}\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)(b+2 c x)\right)+$$

$$\left(c^{1/4}\left(b^2-4 a c\right)^{3/4}\sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right)\left(1+\frac{2\sqrt{c}\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)(b+2 c x)\right)-$$

$$\left(\sqrt{-b^2+4 a c}\,(2 c d-b e)^2\sqrt{\frac{(b+2 c x)^2}{b^2-4 a c}}\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{1/4}\text{EllipticPi}\left[-\frac{\sqrt{-b^2+4 a c}\,e}{2\sqrt{c}\sqrt{c d^2-b d e+a e^2}},\text{ArcSin}\left[\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}\right],-1\right]\right)\Big/$$

$$\left(4\sqrt{2}\sqrt{c}\,e\left(c d^2-b d e+a e^2\right)^{3/2}(b+2 c x)\left(a+b x+c x^2\right)^{1/4}\right)+$$

$$\left(\sqrt{-b^2+4 a c}\,(2 c d-b e)^2\sqrt{\frac{(b+2 c x)^2}{b^2-4 a c}}\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{1/4}\text{EllipticPi}\left[\frac{\sqrt{-b^2+4 a c}\,e}{2\sqrt{c}\sqrt{c d^2-b d e+a e^2}},\text{ArcSin}\left[\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}\right],-1\right]\right)\Big/$$

$$\left(4\sqrt{2}\sqrt{c}\,e\left(c d^2-b d e+a e^2\right)^{3/2}(b+2 c x)\left(a+b x+c x^2\right)^{1/4}\right)$$

Result (type 6, 187 leaves) :

$$-\left(\sqrt{2}\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{c\,(d+ex)}\right)^{1/4}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{c\,(d+ex)}\right)^{1/4}\right.$$

$$\left.\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},\frac{1}{4},\frac{5}{2},\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)e}{2c\,(d+ex)},\frac{2cd-be+\sqrt{b^2-4ac}\,e}{2cd+2cex}\right]\right)\Big/\left(3e\,(d+ex)\,(a+x\,(b+cx))^{1/4}\right)$$

- **Problem 2534: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{(d+ex)^3\left(a+bx+cx^2\right)^{1/4}}\,dx$$

Optimal (type 4, 1465 leaves, 21 steps) :

$$-\frac{e\left(a+bx+cx^2\right)^{3/4}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2}-\frac{5e\,(2cd-be)\left(a+bx+cx^2\right)^{3/4}}{8\left(cd^2-bde+ae^2\right)^2(d+ex)}+\frac{5\sqrt{c}\,(2cd-be)\,(b+2cx)\left(a+bx+cx^2\right)^{1/4}}{8\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)^2\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}+$$

$$\left(\left(-b^2+4ac\right)^{1/4}\left(12c^2d^2+5b^2e^2-4ce\,(3bd+2ae)\right)\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{1/4}\text{ArcTan}\left[\frac{\left(-b^2+4ac\right)^{1/4}\sqrt{e}\left(1-\frac{(b+2cx)^2}{b^2-4ac}\right)^{1/4}}{\sqrt{2}\,c^{1/4}\left(cd^2-bde+ae^2\right)^{1/4}}\right]\right)\Big/$$

$$\left(32c^{1/4}\sqrt{e}\left(cd^2-bde+ae^2\right)^{9/4}\left(a+bx+cx^2\right)^{1/4}\right)-$$

$$\left(\left(-b^2+4ac\right)^{1/4}\left(12c^2d^2+5b^2e^2-4ce\,(3bd+2ae)\right)\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{1/4}\text{ArcTanh}\left[\frac{\left(-b^2+4ac\right)^{1/4}\sqrt{e}\left(1-\frac{(b+2cx)^2}{b^2-4ac}\right)^{1/4}}{\sqrt{2}\,c^{1/4}\left(cd^2-bde+ae^2\right)^{1/4}}\right]\right)\Big/$$

$$\left(32c^{1/4}\sqrt{e}\left(cd^2-bde+ae^2\right)^{9/4}\left(a+bx+cx^2\right)^{1/4}\right)-\left(5c^{1/4}\left(b^2-4ac\right)^{3/4}(2cd-be)\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\right.$$

$$\left.\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\left(a+bx+cx^2\right)^{1/4}}{\left(b^2-4ac\right)^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(8\sqrt{2}\left(cd^2-bde+ae^2\right)^2(b+2cx)\right)+$$

$$\left(5\, c^{1/4} \left(b^2-4\, a\, c\right)^{3/4} (2\, c\, d-b\, e) \sqrt{\frac{(b+2\, c\, x)^2}{\left(b^2-4\, a\, c\right) \left(1+\frac{2 \sqrt{c} \sqrt{a+b\, x+c\, x^2}}{\sqrt{b^2-4\, a\, c}}\right)^2}} \left(1+\frac{2 \sqrt{c} \sqrt{a+b\, x+c\, x^2}}{\sqrt{b^2-4\, a\, c}}\right)\right.$$

$$\left.\text{EllipticF}\left[2\, \text{ArcTan}\left[\frac{\sqrt{2}\, c^{1/4} \left(a+b\, x+c\, x^2\right)^{1/4}}{\left(b^2-4\, a\, c\right)^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(16 \sqrt{2} \left(c\, d^2-b\, d\, e+a\, e^2\right)^2 (b+2\, c\, x)\right) -$$

$$\left(\sqrt{-b^2+4\, a\, c}\; (2\, c\, d-b\, e) \left(12\, c^2 d^2+5\, b^2 e^2-4\, c\, e\, (3\, b\, d+2\, a\, e)\right) \sqrt{\frac{(b+2\, c\, x)^2}{b^2-4\, a\, c}} \left(-\frac{c \left(a+b\, x+c\, x^2\right)}{b^2-4\, a\, c}\right)^{1/4} \text{EllipticPi}\left[\right.\right.$$

$$\left.\left.-\frac{\sqrt{-b^2+4\, a\, c}\; e}{2 \sqrt{c} \sqrt{c\, d^2-b\, d\, e+a\, e^2}}, \text{ArcSin}\left[\left(1-\frac{(b+2\, c\, x)^2}{b^2-4\, a\, c}\right)^{1/4}\right], -1\right]\right) \Big/ \left(32 \sqrt{2} \sqrt{c}\; e \left(c\, d^2-b\, d\, e+a\, e^2\right)^{5/2} (b+2\, c\, x) \left(a+b\, x+c\, x^2\right)^{1/4}\right) +$$

$$\left(\sqrt{-b^2+4\, a\, c}\; (2\, c\, d-b\, e) \left(12\, c^2 d^2+5\, b^2 e^2-4\, c\, e\, (3\, b\, d+2\, a\, e)\right) \sqrt{\frac{(b+2\, c\, x)^2}{b^2-4\, a\, c}} \left(-\frac{c \left(a+b\, x+c\, x^2\right)}{b^2-4\, a\, c}\right)^{1/4}\right.$$

$$\left.\text{EllipticPi}\left[\frac{\sqrt{-b^2+4\, a\, c}\; e}{2 \sqrt{c} \sqrt{c\, d^2-b\, d\, e+a\, e^2}}, \text{ArcSin}\left[\left(1-\frac{(b+2\, c\, x)^2}{b^2-4\, a\, c}\right)^{1/4}\right], -1\right]\right) \Big/ \left(32 \sqrt{2} \sqrt{c}\; e \left(c\, d^2-b\, d\, e+a\, e^2\right)^{5/2} (b+2\, c\, x) \left(a+b\, x+c\, x^2\right)^{1/4}\right)$$

Result (type 6, 187 leaves):

$$-\left(\sqrt{2} \left(\frac{e \left(b-\sqrt{b^2-4\, a\, c}+2\, c\, x\right)}{c\, (d+e\, x)}\right)^{1/4} \left(\frac{e \left(b+\sqrt{b^2-4\, a\, c}+2\, c\, x\right)}{c\, (d+e\, x)}\right)^{1/4}\right.$$

$$\left.\text{AppellF1}\left[\frac{5}{2}, \frac{1}{4}, \frac{1}{4}, \frac{7}{2}, \frac{2\, c\, d-\left(b+\sqrt{b^2-4\, a\, c}\right) e}{2\, c\, (d+e\, x)}, \frac{2\, c\, d-b\, e+\sqrt{b^2-4\, a\, c}\; e}{2\, c\, d+2\, c\, e\, x}\right]\right) \Big/ \left(5\, e\, (d+e\, x)^2 \left(a+x\, (b+c\, x)\right)^{1/4}\right)$$

- **Problem 2535: Result unnecessarily involves higher level functions.**

$$\int \frac{(d+e\, x)^3}{\left(a+b\, x+c\, x^2\right)^{3/4}}\, dx$$

Optimal (type 4, 307 leaves, 4 steps):

$$\frac{2 e (d+e x)^2 \left(a+b x+c x^2\right)^{1/4}}{5 c}+\frac{e \left(56 c^2 d^2+15 b^2 e^2-2 c e (25 b d+8 a e)+6 c e (2 c d-b e) x\right) \left(a+b x+c x^2\right)^{1/4}}{10 c^3}+\frac{1}{4 \sqrt{2} c^{13/4} (b+2 c x)}\left(b^2-4 a c\right)^{1/4} (2 c d-b e) \left(4 c^2 d^2+3 b^2 e^2-4 c e (b d+2 a e)\right) \sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right) \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}} \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right) \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} c^{1/4} \left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 5, 230 leaves):

$$\frac{1}{40 c^4 (a+x (b+c x))^{3/4}}\left(4 c e (a+x (b+c x)) \left(15 b^2 e^2-2 c e (25 b d+8 a e+3 b e x)+4 c^2 \left(15 d^2+5 d e x+e^2 x^2\right)\right)+5\times 2^{1/4} (2 c d-b e) \left(4 c^2 d^2+3 b^2 e^2-4 c e (b d+2 a e)\right) \left(b-\sqrt{b^2-4 a c}+2 c x\right) \left(\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{-b+\sqrt{b^2-4 a c}-2 c x}{2 \sqrt{b^2-4 a c}}\right]\right)$$

- **Problem 2536: Result unnecessarily involves higher level functions.**

$$\int \frac{(d+e x)^2}{\left(a+b x+c x^2\right)^{3/4}} \, dx$$

Optimal (type 4, 262 leaves, 4 steps):

$$\frac{5 e (2 c d-b e) \left(a+b x+c x^2\right)^{1/4}}{3 c^2}+\frac{2 e (d+e x) \left(a+b x+c x^2\right)^{1/4}}{3 c}+\frac{1}{6 \sqrt{2} c^{9/4} (b+2 c x)}\left(b^2-4 a c\right)^{1/4} \left(12 c^2 d^2+5 b^2 e^2-4 c e (3 b d+2 a e)\right) \sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right) \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}} \left(1+\frac{2 \sqrt{c} \sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right) \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} c^{1/4} \left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 5, 186 leaves):

$$\frac{1}{12\,c^3\,(a+x\,(b+c\,x))^{3/4}}\left(4\,c\,e\,(a+x\,(b+c\,x))\,(-5\,b\,e+2\,c\,(6\,d+e\,x))+2^{1/4}\left(12\,c^2\,d^2+5\,b^2\,e^2-4\,c\,e\,(3\,b\,d+2\,a\,e)\right)\right.$$
$$\left.\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\left(\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{-b+\sqrt{b^2-4\,a\,c}-2\,c\,x}{2\,\sqrt{b^2-4\,a\,c}}\right]\right)$$

- **Problem 2537: Result unnecessarily involves higher level functions.**

$$\int\frac{d+e\,x}{\left(a+b\,x+c\,x^2\right)^{3/4}}\,dx$$

Optimal (type 4, 200 leaves, 3 steps):

$$\frac{2\,e\,\left(a+b\,x+c\,x^2\right)^{1/4}}{c}+\frac{1}{\sqrt{2}\,c^{5/4}\,(b+2\,c\,x)}$$
$$\left(b^2-4\,a\,c\right)^{1/4}(2\,c\,d-b\,e)\sqrt{\frac{(b+2\,c\,x)^2}{\left(b^2-4\,a\,c\right)\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)^2}}\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\left(a+b\,x+c\,x^2\right)^{1/4}}{\left(b^2-4\,a\,c\right)^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 5, 152 leaves):

$$\frac{1}{4\,c\,(a+x\,(b+c\,x))^{3/4}}\left(8\,e\,(a+x\,(b+c\,x))-\right.$$
$$\left.1/c2\times2^{1/4}\,(-2\,c\,d+b\,e)\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\left(\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{-b+\sqrt{b^2-4\,a\,c}-2\,c\,x}{2\,\sqrt{b^2-4\,a\,c}}\right]\right)$$

- **Problem 2538: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{\left(a+b\,x+c\,x^2\right)^{3/4}}\,dx$$

Optimal (type 4, 170 leaves, 2 steps):

$$\frac{1}{c^{1/4}\,(b+2\,c\,x)}\sqrt{2}\,\left(b^2-4\,a\,c\right)^{1/4}\sqrt{\frac{(b+2\,c\,x)^2}{\left(b^2-4\,a\,c\right)\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)^2}}\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\,\left(a+b\,x+c\,x^2\right)^{1/4}}{\left(b^2-4\,a\,c\right)^{1/4}}\right],\,\frac{1}{2}\right]$$

Result (type 5, 123 leaves):

$$\frac{2^{1/4}\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\left(\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{5}{4},\,\frac{-b+\sqrt{b^2-4\,a\,c}-2\,c\,x}{2\,\sqrt{b^2-4\,a\,c}}\right]}{c\,(a+x\,(b+c\,x))^{3/4}}$$

- **Problem 2539: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{(d+e\,x)\,\left(a+b\,x+c\,x^2\right)^{3/4}}\,dx$$

Optimal (type 4, 709 leaves, 15 steps):

$$-\frac{\left(-b^2+4\,a\,c\right)^{3/4}\sqrt{e}\,\left(-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}\right)^{3/4}\text{ArcTan}\left[\frac{\left(-b^2+4\,a\,c\right)^{1/4}\sqrt{e}\,\left(1-\frac{(b+2\,c\,x)^2}{b^2-4\,a\,c}\right)^{1/4}}{\sqrt{2}\,c^{1/4}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^{1/4}}\right]}{c^{3/4}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^{3/4}\,\left(a+b\,x+c\,x^2\right)^{3/4}}-$$

$$\frac{\left(-b^2+4\,a\,c\right)^{3/4}\sqrt{e}\,\left(-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}\right)^{3/4}\text{ArcTanh}\left[\frac{\left(-b^2+4\,a\,c\right)^{1/4}\sqrt{e}\,\left(1-\frac{(b+2\,c\,x)^2}{b^2-4\,a\,c}\right)^{1/4}}{\sqrt{2}\,c^{1/4}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^{1/4}}\right]}{c^{3/4}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^{3/4}\,\left(a+b\,x+c\,x^2\right)^{3/4}}-$$

$$\left(\left(b^2-4\,a\,c\right)\,(2\,c\,d-b\,e)\,\sqrt{\frac{(b+2\,c\,x)^2}{b^2-4\,a\,c}}\,\left(-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}\right)^{3/4}\text{EllipticPi}\left[-\frac{\sqrt{-b^2+4\,a\,c}\,e}{2\,\sqrt{c}\,\sqrt{c\,d^2-b\,d\,e+a\,e^2}},\,\text{ArcSin}\left[\left(1-\frac{(b+2\,c\,x)^2}{b^2-4\,a\,c}\right)^{1/4}\right],\,-1\right]\right)\Big/$$

$$\left(\sqrt{2}\,c\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,(b+2\,c\,x)\,\left(a+b\,x+c\,x^2\right)^{3/4}\right)-$$

$$\left(\left(b^2-4\,a\,c\right)\,(2\,c\,d-b\,e)\,\sqrt{\frac{(b+2\,c\,x)^2}{b^2-4\,a\,c}}\,\left(-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}\right)^{3/4}\text{EllipticPi}\left[\frac{\sqrt{-b^2+4\,a\,c}\,e}{2\,\sqrt{c}\,\sqrt{c\,d^2-b\,d\,e+a\,e^2}},\,\text{ArcSin}\left[\left(1-\frac{(b+2\,c\,x)^2}{b^2-4\,a\,c}\right)^{1/4}\right],\,-1\right]\right)\Big/$$

$$\left(\sqrt{2}\,c\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,(b+2\,c\,x)\,\left(a+b\,x+c\,x^2\right)^{3/4}\right)$$

Result (type 6, 180 leaves):

$$-\frac{1}{3\sqrt{2}\ e\ (a+x\ (b+c\ x))^{3/4}}\left(\frac{e\left(b-\sqrt{b^2-4\ a\ c}+2\ c\ x\right)}{c\ (d+e\ x)}\right)^{3/4}\left(\frac{e\left(b+\sqrt{b^2-4\ a\ c}+2\ c\ x\right)}{c\ (d+e\ x)}\right)^{3/4}\text{AppellF1}\left[\frac{3}{2},\frac{3}{4},\frac{3}{4},\frac{5}{2},\frac{2\ c\ d-\left(b+\sqrt{b^2-4\ a\ c}\right)e}{2\ c\ (d+e\ x)},\frac{2\ c\ d-b\ e+\sqrt{b^2-4\ a\ c}\ e}{2\ c\ d+2\ c\ e\ x}\right]$$

- **Problem 2540: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{(d+e\ x)^2\left(a+b\ x+c\ x^2\right)^{3/4}}\,dx$$

Optimal (type 4, 970 leaves, 19 steps):

$$-\frac{e\left(a+b\ x+c\ x^2\right)^{1/4}}{\left(c\ d^2-b\ d\ e+a\ e^2\right)(d+e\ x)}-\frac{3\left(-b^2+4\ a\ c\right)^{3/4}\sqrt{e}\ (2\ c\ d-b\ e)\left(-\frac{c\left(a+b\ x+c\ x^2\right)}{b^2-4\ a\ c}\right)^{3/4}\text{ArcTan}\left[\frac{\left(-b^2+4\ a\ c\right)^{1/4}\sqrt{e}\left(1-\frac{(b+2\ c\ x)^2}{b^2-4\ a\ c}\right)^{1/4}}{\sqrt{2}\ c^{1/4}\left(c\ d^2-b\ d\ e+a\ e^2\right)^{1/4}}\right]}{4\ c^{3/4}\left(c\ d^2-b\ d\ e+a\ e^2\right)^{7/4}\left(a+b\ x+c\ x^2\right)^{3/4}}-$$

$$\frac{3\left(-b^2+4\ a\ c\right)^{3/4}\sqrt{e}\ (2\ c\ d-b\ e)\left(-\frac{c\left(a+b\ x+c\ x^2\right)}{b^2-4\ a\ c}\right)^{3/4}\text{ArcTanh}\left[\frac{\left(-b^2+4\ a\ c\right)^{1/4}\sqrt{e}\left(1-\frac{(b+2\ c\ x)^2}{b^2-4\ a\ c}\right)^{1/4}}{\sqrt{2}\ c^{1/4}\left(c\ d^2-b\ d\ e+a\ e^2\right)^{1/4}}\right]}{4\ c^{3/4}\left(c\ d^2-b\ d\ e+a\ e^2\right)^{7/4}\left(a+b\ x+c\ x^2\right)^{3/4}}-$$

$$\left(c^{3/4}\left(b^2-4\ a\ c\right)^{1/4}\sqrt{\frac{(b+2\ c\ x)^2}{\left(b^2-4\ a\ c\right)\left(1+\frac{2\sqrt{c}\ \sqrt{a+b\ x+c\ x^2}}{\sqrt{b^2-4\ a\ c}}\right)^2}}\left(1+\frac{2\sqrt{c}\ \sqrt{a+b\ x+c\ x^2}}{\sqrt{b^2-4\ a\ c}}\right)\text{EllipticF}\left[2\ \text{ArcTan}\left[\frac{\sqrt{2}\ c^{1/4}\left(a+b\ x+c\ x^2\right)^{1/4}}{\left(b^2-4\ a\ c\right)^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(\sqrt{2}\left(c\ d^2-b\ d\ e+a\ e^2\right)(b+2\ c\ x)\right)-$$

$$\left(3\left(b^2-4\ a\ c\right)(2\ c\ d-b\ e)^2\sqrt{\frac{(b+2\ c\ x)^2}{b^2-4\ a\ c}}\left(-\frac{c\left(a+b\ x+c\ x^2\right)}{b^2-4\ a\ c}\right)^{3/4}\text{EllipticPi}\left[-\frac{\sqrt{-b^2+4\ a\ c}\ e}{2\sqrt{c}\ \sqrt{c\ d^2-b\ d\ e+a\ e^2}},\text{ArcSin}\left[\left(1-\frac{(b+2\ c\ x)^2}{b^2-4\ a\ c}\right)^{1/4}\right],-1\right]\right)\Big/$$

$$\left(4\sqrt{2}\ c\left(c\ d^2-b\ d\ e+a\ e^2\right)^2(b+2\ c\ x)\left(a+b\ x+c\ x^2\right)^{3/4}\right)-$$

$$\left(3\left(b^2-4\ a\ c\right)(2\ c\ d-b\ e)^2\sqrt{\frac{(b+2\ c\ x)^2}{b^2-4\ a\ c}}\left(-\frac{c\left(a+b\ x+c\ x^2\right)}{b^2-4\ a\ c}\right)^{3/4}\text{EllipticPi}\left[\frac{\sqrt{-b^2+4\ a\ c}\ e}{2\sqrt{c}\ \sqrt{c\ d^2-b\ d\ e+a\ e^2}},\text{ArcSin}\left[\left(1-\frac{(b+2\ c\ x)^2}{b^2-4\ a\ c}\right)^{1/4}\right],-1\right]\right)\Big/$$

$$\left(4\sqrt{2}\ c\left(c\ d^2-b\ d\ e+a\ e^2\right)^2(b+2\ c\ x)\left(a+b\ x+c\ x^2\right)^{3/4}\right)$$

Result (type 6, 187 leaves):

$$-\left(\left(\frac{e\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}{c\,(d+e\,x)}\right)^{3/4}\left(\frac{e\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}{c\,(d+e\,x)}\right)^{3/4}\text{AppellF1}\left[\frac{5}{2},\frac{3}{4},\frac{3}{4},\frac{7}{2},\frac{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,(d+e\,x)},\frac{2\,c\,d-b\,e+\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d+2\,c\,e\,x}\right]\right)\Big/$$
$$\left(5\sqrt{2}\,e\,(d+e\,x)\,(a+x\,(b+c\,x))^{3/4}\right)$$

- **Problem 2541: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{(d+e\,x)^3\left(a+b\,x+c\,x^2\right)^{3/4}}\,dx$$

Optimal (type 4, 1134 leaves, 20 steps) :

$$-\frac{e\left(a+b x+c x^2\right)^{1/4}}{2\left(c d^2-b d e+a e^2\right)(d+e x)^2}-\frac{7 e (2 c d-b e)\left(a+b x+c x^2\right)^{1/4}}{8\left(c d^2-b d e+a e^2\right)^2 (d+e x)}-$$

$$\left(3\left(-b^2+4 a c\right)^{3/4}\sqrt{e}\left(20 c^2 d^2+7 b^2 e^2-4 c e (5 b d+2 a e)\right)\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{3/4}\text{ArcTan}\left[\frac{\left(-b^2+4 a c\right)^{1/4}\sqrt{e}\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}}{\sqrt{2}\, c^{1/4}\left(c d^2-b d e+a e^2\right)^{1/4}}\right]\right)\Big/$$

$$\left(32 c^{3/4}\left(c d^2-b d e+a e^2\right)^{11/4}\left(a+b x+c x^2\right)^{3/4}\right)-$$

$$\left(3\left(-b^2+4 a c\right)^{3/4}\sqrt{e}\left(20 c^2 d^2+7 b^2 e^2-4 c e (5 b d+2 a e)\right)\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{3/4}\text{ArcTanh}\left[\frac{\left(-b^2+4 a c\right)^{1/4}\sqrt{e}\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}}{\sqrt{2}\, c^{1/4}\left(c d^2-b d e+a e^2\right)^{1/4}}\right]\right)\Big/$$

$$\left(32 c^{3/4}\left(c d^2-b d e+a e^2\right)^{11/4}\left(a+b x+c x^2\right)^{3/4}\right)-\left(7 c^{3/4}\left(b^2-4 a c\right)^{1/4}(2 c d-b e)\sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right)\left(1+\frac{2\sqrt{c}\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}}\right.$$

$$\left.\left(1+\frac{2\sqrt{c}\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\, c^{1/4}\left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(8\sqrt{2}\left(c d^2-b d e+a e^2\right)^2 (b+2 c x)\right)-$$

$$\left(3\left(b^2-4 a c\right)(2 c d-b e)\left(20 c^2 d^2+7 b^2 e^2-4 c e (5 b d+2 a e)\right)\sqrt{\frac{(b+2 c x)^2}{b^2-4 a c}}\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{3/4}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\sqrt{-b^2+4 a c}\; e}{2\sqrt{c}\sqrt{c d^2-b d e+a e^2}},\text{ArcSin}\left[\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}\right],-1\right]\right)\Big/\left(32\sqrt{2}\, c\left(c d^2-b d e+a e^2\right)^3 (b+2 c x)\left(a+b x+c x^2\right)^{3/4}\right)-$$

$$\left(3\left(b^2-4 a c\right)(2 c d-b e)\left(20 c^2 d^2+7 b^2 e^2-4 c e (5 b d+2 a e)\right)\sqrt{\frac{(b+2 c x)^2}{b^2-4 a c}}\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{3/4}\right.$$

$$\left.\text{EllipticPi}\left[\frac{\sqrt{-b^2+4 a c}\; e}{2\sqrt{c}\sqrt{c d^2-b d e+a e^2}},\text{ArcSin}\left[\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}\right],-1\right]\right)\Big/\left(32\sqrt{2}\, c\left(c d^2-b d e+a e^2\right)^3 (b+2 c x)\left(a+b x+c x^2\right)^{3/4}\right)$$

Result (type 6, 187 leaves) :

$$-\left(\left(\frac{e\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}{c\,(d+e\,x)}\right)^{3/4}\left(\frac{e\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}{c\,(d+e\,x)}\right)^{3/4}\text{AppellF1}\left[\frac{7}{2},\frac{3}{4},\frac{3}{4},\frac{9}{2},\frac{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,(d+e\,x)},\frac{2\,c\,d-b\,e+\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d+2\,c\,e\,x}\right]\right)\Big/\left(7\sqrt{2}\,e\,(d+e\,x)^2\,(a+x\,(b+c\,x))^{3/4}\right)$$

- **Problem 2542: Result unnecessarily involves higher level functions.**

$$\int\frac{(d+e\,x)^3}{\left(a+b\,x+c\,x^2\right)^{5/4}}\,dx$$

Optimal (type 4, 662 leaves, 6 steps) :

$$-\frac{4\,(d+e\,x)^2\,(b\,d-2\,a\,e+(2\,c\,d-b\,e)\,x)}{(b^2-4\,a\,c)\,(a+b\,x+c\,x^2)^{1/4}}+\frac{2\,e\,\left(24\,c^2\,d^2+7\,b^2\,e^2-2\,c\,e\,(9\,b\,d+8\,a\,e)+6\,c\,e\,(2\,c\,d-b\,e)\,x\right)\,(a+b\,x+c\,x^2)^{3/4}}{3\,c^2\,(b^2-4\,a\,c)}+$$

$$\frac{(2\,c\,d-b\,e)\,\left(4\,c^2\,d^2+7\,b^2\,e^2-4\,c\,e\,(b\,d+6\,a\,e)\right)\,(b+2\,c\,x)\,(a+b\,x+c\,x^2)^{1/4}}{c^{5/2}\,(b^2-4\,a\,c)^{3/2}\,\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)}-$$

$$\left((2\,c\,d-b\,e)\,\left(4\,c^2\,d^2+7\,b^2\,e^2-4\,c\,e\,(b\,d+6\,a\,e)\right)\,\sqrt{\frac{(b+2\,c\,x)^2}{(b^2-4\,a\,c)\,\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)^2}}\right.$$

$$\left.\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\,(a+b\,x+c\,x^2)^{1/4}}{(b^2-4\,a\,c)^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/\left(\sqrt{2}\,c^{11/4}\,(b^2-4\,a\,c)^{1/4}\,(b+2\,c\,x)\right)+$$

$$\left((2\,c\,d-b\,e)\,\left(4\,c^2\,d^2+7\,b^2\,e^2-4\,c\,e\,(b\,d+6\,a\,e)\right)\,\sqrt{\frac{(b+2\,c\,x)^2}{(b^2-4\,a\,c)\,\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)^2}}\,\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\,(a+b\,x+c\,x^2)^{1/4}}{(b^2-4\,a\,c)^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/\left(2\,\sqrt{2}\,c^{11/4}\,(b^2-4\,a\,c)^{1/4}\,(b+2\,c\,x)\right)$$

Result (type 5, 284 leaves) :

$$-\frac{1}{6\,c^3\,\left(b^2-4\,a\,c\right)\,\left(a+x\,\left(b+c\,x\right)\right)^{1/4}}\left(-4\,c\,\left(b^2-4\,a\,c\right)\,e^3\,\left(a+x\,\left(b+c\,x\right)\right)+\right.$$
$$24\,c\,\left(-b^3\,e^3\,x+b^2\,e^2\,\left(-a\,e+3\,c\,d\,x\right)+2\,c\,\left(a^2\,e^3+c^2\,d^3\,x-3\,a\,c\,d\,e\,\left(d+e\,x\right)\right)+b\,c\,\left(c\,d^2\,\left(d-3\,e\,x\right)+3\,a\,e^2\,\left(d+e\,x\right)\right)\right)+$$
$$2^{3/4}\,\left(-2\,c\,d+b\,e\right)\,\left(4\,c^2\,d^2+7\,b^2\,e^2-4\,c\,e\,\left(b\,d+6\,a\,e\right)\right)\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)$$
$$\left.\left(\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{7}{4},\,\frac{-b+\sqrt{b^2-4\,a\,c}-2\,c\,x}{2\,\sqrt{b^2-4\,a\,c}}\right]\right)$$

- **Problem 2543: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(d+e\,x\right)^2}{\left(a+b\,x+c\,x^2\right)^{5/4}}\,dx$$

Optimal (type 4, 594 leaves, 6 steps):

$$-\frac{4\,(d+e\,x)\,(b\,d-2\,a\,e+(2\,c\,d-b\,e)\,x)}{(b^2-4\,a\,c)\,(a+b\,x+c\,x^2)^{1/4}}+\frac{4\,e\,(2\,c\,d-b\,e)\,(a+b\,x+c\,x^2)^{3/4}}{c\,(b^2-4\,a\,c)}+$$

$$\frac{2\,(4\,c^2\,d^2+3\,b^2\,e^2-4\,c\,e\,(b\,d+2\,a\,e))\,(b+2\,c\,x)\,(a+b\,x+c\,x^2)^{1/4}}{c^{3/2}\,(b^2-4\,a\,c)^{3/2}\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)}-\left(\sqrt{2}\,(4\,c^2\,d^2+3\,b^2\,e^2-4\,c\,e\,(b\,d+2\,a\,e))\,\sqrt{\frac{(b+2\,c\,x)^2}{(b^2-4\,a\,c)\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)^2}}\right.$$

$$\left.\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\,(a+b\,x+c\,x^2)^{1/4}}{(b^2-4\,a\,c)^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(c^{7/4}\,(b^2-4\,a\,c)^{1/4}\,(b+2\,c\,x)\right)+$$

$$\left((4\,c^2\,d^2+3\,b^2\,e^2-4\,c\,e\,(b\,d+2\,a\,e))\,\sqrt{\frac{(b+2\,c\,x)^2}{(b^2-4\,a\,c)\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)^2}}\,\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\,(a+b\,x+c\,x^2)^{1/4}}{(b^2-4\,a\,c)^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(\sqrt{2}\,c^{7/4}\,(b^2-4\,a\,c)^{1/4}\,(b+2\,c\,x)\right)$$

Result (type 5, 221 leaves) :

$$\left(24\,(a\,b\,e^2+2\,c^2\,d^2\,x+b^2\,e^2\,x+b\,c\,d\,(d-2\,e\,x)-2\,a\,c\,e\,(2\,d+e\,x))-1/c2\times 2^{3/4}\,(4\,c^2\,d^2+3\,b^2\,e^2-4\,c\,e\,(b\,d+2\,a\,e))\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\right.$$

$$\left.\left(\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{-b+\sqrt{b^2-4\,a\,c}-2\,c\,x}{2\,\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(6\,c\,(-b^2+4\,a\,c)\,(a+x\,(b+c\,x))^{1/4}\right)$$

- **Problem 2544: Result unnecessarily involves higher level functions.**

$$\int\frac{d+e\,x}{(a+b\,x+c\,x^2)^{5/4}}\,dx$$

Optimal (type 4, 490 leaves, 5 steps) :

$$-\frac{4\,(b\,d-2\,a\,e+(2\,c\,d-b\,e)\,x)}{(b^2-4\,a\,c)\,(a+b\,x+c\,x^2)^{1/4}}+\frac{4\,(2\,c\,d-b\,e)\,(b+2\,c\,x)\,(a+b\,x+c\,x^2)^{1/4}}{\sqrt{c}\,(b^2-4\,a\,c)^{3/2}\left(1+\frac{2\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)}-$$

$$\left(2\sqrt{2}\,(2\,c\,d-b\,e)\sqrt{\frac{(b+2\,c\,x)^2}{(b^2-4\,a\,c)\left(1+\frac{2\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)^2}}\left(1+\frac{2\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\,(a+b\,x+c\,x^2)^{1/4}}{(b^2-4\,a\,c)^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(c^{3/4}\,(b^2-4\,a\,c)^{1/4}\,(b+2\,c\,x)\right)+$$

$$\left(\sqrt{2}\,(2\,c\,d-b\,e)\sqrt{\frac{(b+2\,c\,x)^2}{(b^2-4\,a\,c)\left(1+\frac{2\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)^2}}\left(1+\frac{2\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\,(a+b\,x+c\,x^2)^{1/4}}{(b^2-4\,a\,c)^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(c^{3/4}\,(b^2-4\,a\,c)^{1/4}\,(b+2\,c\,x)\right)$$

Result (type 5, 167 leaves):

$$-\left(2\left(6\,c\,(-2\,a\,e+2\,c\,d\,x+b\,(d-e\,x))+2^{3/4}\,(-2\,c\,d+b\,e)\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\right.\right.$$

$$\left.\left.\left(\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{-b+\sqrt{b^2-4\,a\,c}-2\,c\,x}{2\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/\left(3\,c\,(b^2-4\,a\,c)\,(a+x\,(b+c\,x))^{1/4}\right)$$

- **Problem 2545: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{(a+b\,x+c\,x^2)^{5/4}}\,dx$$

Optimal (type 4, 451 leaves, 5 steps):

$$-\frac{4\,(b+2\,c\,x)}{\left(b^2-4\,a\,c\right)\,\left(a+b\,x+c\,x^2\right)^{1/4}}+\frac{8\,\sqrt{c}\,(b+2\,c\,x)\,\left(a+b\,x+c\,x^2\right)^{1/4}}{\left(b^2-4\,a\,c\right)^{3/2}\,\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)}-\frac{1}{\left(b^2-4\,a\,c\right)^{1/4}\,(b+2\,c\,x)}4\,\sqrt{2}\,c^{1/4}\,\sqrt{\frac{(b+2\,c\,x)^2}{\left(b^2-4\,a\,c\right)\,\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)^2}}$$

$$\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\,\left(a+b\,x+c\,x^2\right)^{1/4}}{\left(b^2-4\,a\,c\right)^{1/4}}\right],\,\frac{1}{2}\right]+\frac{1}{\left(b^2-4\,a\,c\right)^{1/4}\,(b+2\,c\,x)}$$

$$2\,\sqrt{2}\,c^{1/4}\,\sqrt{\frac{(b+2\,c\,x)^2}{\left(b^2-4\,a\,c\right)\,\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)^2}}\,\left(1+\frac{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{b^2-4\,a\,c}}\right)\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\,\left(a+b\,x+c\,x^2\right)^{1/4}}{\left(b^2-4\,a\,c\right)^{1/4}}\right],\,\frac{1}{2}\right]$$

Result (type 5, 161 leaves):

$$\left(4\,\left(-3\,(b+2\,c\,x)+2^{3/4}\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\,\left(\frac{b^2-4\,a\,c+b\,\sqrt{b^2-4\,a\,c}+2\,c\,\sqrt{b^2-4\,a\,c}\,x}{b^2-4\,a\,c}\right)^{1/4}\right.\right.$$

$$\left.\left.\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{7}{4},\,\frac{-b+\sqrt{b^2-4\,a\,c}-2\,c\,x}{2\,\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/\left(3\,\left(b^2-4\,a\,c\right)\,(a+x\,(b+c\,x))^{1/4}\right)$$

- **Problem 2546: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{(d+e\,x)\,\left(a+b\,x+c\,x^2\right)^{5/4}}\,dx$$

Optimal (type 4, 1299 leaves, 20 steps):

$$-\frac{4\left(b c d-b^2 e+2 a c e+c\,(2 c d-b e)\,x\right)}{\left(b^2-4 a c\right)\left(c d^2-b d e+a e^2\right)\left(a+b x+c x^2\right)^{1/4}}+\frac{4 \sqrt{c}\,(2 c d-b e)\,(b+2 c x)\left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{3/2}\left(c d^2-b d e+a e^2\right)\left(1+\frac{2 \sqrt{c}\,\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)}+$$

$$\frac{\left(-b^2+4 a c\right)^{1/4} e^{3/2}\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{1/4} \text{ArcTan}\left[\frac{\left(-b^2+4 a c\right)^{1/4} \sqrt{e}\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}}{\sqrt{2}\, c^{1/4}\left(c d^2-b d e+a e^2\right)^{1/4}}\right]}{c^{1/4}\left(c d^2-b d e+a e^2\right)^{5/4}\left(a+b x+c x^2\right)^{1/4}}-$$

$$\frac{\left(-b^2+4 a c\right)^{1/4} e^{3/2}\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{1/4} \text{ArcTanh}\left[\frac{\left(-b^2+4 a c\right)^{1/4} \sqrt{e}\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}}{\sqrt{2}\, c^{1/4}\left(c d^2-b d e+a e^2\right)^{1/4}}\right]}{c^{1/4}\left(c d^2-b d e+a e^2\right)^{5/4}\left(a+b x+c x^2\right)^{1/4}}-$$

$$\left(2 \sqrt{2}\, c^{1/4}\,(2 c d-b e) \sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right)\left(1+\frac{2 \sqrt{c}\,\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}}\left(1+\frac{2 \sqrt{c}\,\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right) \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{\sqrt{2}\, c^{1/4}\left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right], \frac{1}{2}\right]\right)\Big/$$

$$\left(\left(b^2-4 a c\right)^{1/4}\left(c d^2-b d e+a e^2\right)(b+2 c x)\right)+$$

$$\left(\sqrt{2}\, c^{1/4}\,(2 c d-b e) \sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right)\left(1+\frac{2 \sqrt{c}\,\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}}\left(1+\frac{2 \sqrt{c}\,\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right) \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2}\, c^{1/4}\left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right], \frac{1}{2}\right]\right)\Big/$$

$$\left(\left(b^2-4 a c\right)^{1/4}\left(c d^2-b d e+a e^2\right)(b+2 c x)\right)-$$

$$\left(\sqrt{-b^2+4 a c}\; e\,(2 c d-b e) \sqrt{\frac{(b+2 c x)^2}{b^2-4 a c}}\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{1/4} \text{EllipticPi}\left[-\frac{\sqrt{-b^2+4 a c}\; e}{2 \sqrt{c}\,\sqrt{c d^2-b d e+a e^2}}, \text{ArcSin}\left[\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}\right], -1\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{c}\left(c d^2-b d e+a e^2\right)^{3/2}(b+2 c x)\left(a+b x+c x^2\right)^{1/4}\right)+$$

$$\left(\sqrt{-b^2+4 a c}\; e\,(2 c d-b e) \sqrt{\frac{(b+2 c x)^2}{b^2-4 a c}}\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{1/4} \text{EllipticPi}\left[\frac{\sqrt{-b^2+4 a c}\; e}{2 \sqrt{c}\,\sqrt{c d^2-b d e+a e^2}}, \text{ArcSin}\left[\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}\right], -1\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{c}\left(c d^2-b d e+a e^2\right)^{3/2}(b+2 c x)\left(a+b x+c x^2\right)^{1/4}\right)$$

Result (type 6, 180 leaves) :

$$-\left(\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{5/4}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{c(d+ex)}\right)^{5/4}\text{AppellF1}\left[\frac{5}{2},\frac{5}{4},\frac{5}{4},\frac{7}{2},\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)e}{2c(d+ex)},\frac{2cd-be+\sqrt{b^2-4ac}\,e}{2cd+2cex}\right]\right)\Big/$$
$$\left(10\sqrt{2}\,e\,(a+x(b+cx))^{5/4}\right)$$

- **Problem 2547: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{(d+ex)^2\left(a+bx+cx^2\right)^{5/4}}\,dx$$

Optimal (type 4, 1485 leaves, 21 steps):

$$-\frac{4\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(d+ex)\left(a+bx+cx^2\right)^{1/4}}-$$
$$\frac{e\left(8c^2d^2+5b^2e^2-4ce(2bd+3ae)\right)\left(a+bx+cx^2\right)^{3/4}}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^2(d+ex)}+\frac{\sqrt{c}\left(8c^2d^2+5b^2e^2-4ce(2bd+3ae)\right)(b+2cx)\left(a+bx+cx^2\right)^{1/4}}{\left(b^2-4ac\right)^{3/2}\left(cd^2-bde+ae^2\right)^2\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)}+$$

$$\frac{5\left(-b^2+4ac\right)^{1/4}e^{3/2}(2cd-be)\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{1/4}\text{ArcTan}\left[\frac{\left(-b^2+4ac\right)^{1/4}\sqrt{e}\left(1-\frac{(b+2cx)^2}{b^2-4ac}\right)^{1/4}}{\sqrt{2}\,c^{1/4}\left(cd^2-bde+ae^2\right)^{1/4}}\right]}{4c^{1/4}\left(cd^2-bde+ae^2\right)^{9/4}\left(a+bx+cx^2\right)^{1/4}}-$$

$$\frac{5\left(-b^2+4ac\right)^{1/4}e^{3/2}(2cd-be)\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{1/4}\text{ArcTanh}\left[\frac{\left(-b^2+4ac\right)^{1/4}\sqrt{e}\left(1-\frac{(b+2cx)^2}{b^2-4ac}\right)^{1/4}}{\sqrt{2}\,c^{1/4}\left(cd^2-bde+ae^2\right)^{1/4}}\right]}{4c^{1/4}\left(cd^2-bde+ae^2\right)^{9/4}\left(a+bx+cx^2\right)^{1/4}}-$$

$$\left(c^{1/4}\left(8c^2d^2+5b^2e^2-4ce(2bd+3ae)\right)\sqrt{\frac{(b+2cx)^2}{\left(b^2-4ac\right)\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{\sqrt{b^2-4ac}}\right)\right.$$

$$\left.\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\left(a+bx+cx^2\right)^{1/4}}{\left(b^2-4ac\right)^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(\sqrt{2}\left(b^2-4ac\right)^{1/4}\left(cd^2-bde+ae^2\right)^2(b+2cx)\right)+$$

$$\left(c^{1/4}\left(8 c^2 d^2+5 b^2 e^2-4 c e (2 b d+3 a e)\right)\sqrt{\frac{(b+2 c x)^2}{\left(b^2-4 a c\right)\left(1+\frac{2\sqrt{c}\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)^2}}\left(1+\frac{2\sqrt{c}\sqrt{a+b x+c x^2}}{\sqrt{b^2-4 a c}}\right)\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{2}\,c^{1/4}\left(a+b x+c x^2\right)^{1/4}}{\left(b^2-4 a c\right)^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(2\sqrt{2}\left(b^2-4 a c\right)^{1/4}\left(c d^2-b d e+a e^2\right)^2(b+2 c x)\right)-\left(5\sqrt{-b^2+4 a c}\;e\right.$$

$$\left.(2 c d-b e)^2\sqrt{\frac{(b+2 c x)^2}{b^2-4 a c}}\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{1/4}\text{EllipticPi}\left[-\frac{\sqrt{-b^2+4 a c}\;e}{2\sqrt{c}\sqrt{c d^2-b d e+a e^2}},\text{ArcSin}\left[\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}\right],-1\right]\right)\Big/$$

$$\left(4\sqrt{2}\sqrt{c}\left(c d^2-b d e+a e^2\right)^{5/2}(b+2 c x)\left(a+b x+c x^2\right)^{1/4}\right)+\left(5\sqrt{-b^2+4 a c}\;e\,(2 c d-b e)^2\sqrt{\frac{(b+2 c x)^2}{b^2-4 a c}}\left(-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}\right)^{1/4}\right.$$

$$\left.\text{EllipticPi}\left[\frac{\sqrt{-b^2+4 a c}\;e}{2\sqrt{c}\sqrt{c d^2-b d e+a e^2}},\text{ArcSin}\left[\left(1-\frac{(b+2 c x)^2}{b^2-4 a c}\right)^{1/4}\right],-1\right]\right)\Big/\left(4\sqrt{2}\sqrt{c}\left(c d^2-b d e+a e^2\right)^{5/2}(b+2 c x)\left(a+b x+c x^2\right)^{1/4}\right)$$

Result (type 6, 187 leaves) :

$$-\left(\left(\frac{e\left(b-\sqrt{b^2-4 a c}+2 c x\right)}{c\,(d+e x)}\right)^{5/4}\left(\frac{e\left(b+\sqrt{b^2-4 a c}+2 c x\right)}{c\,(d+e x)}\right)^{5/4}\text{AppellF1}\left[\frac{7}{2},\frac{5}{4},\frac{5}{4},\frac{9}{2},\frac{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}{2 c\,(d+e x)},\frac{2 c d-b e+\sqrt{b^2-4 a c}\;e}{2 c d+2 c e x}\right]\right)\Big/$$

$$\left(14\sqrt{2}\;e\,(d+e x)\,(a+x\,(b+c x))^{5/4}\right)$$

Problem 2549: Result more than twice size of optimal antiderivative.

$$\int (d+e x)^m\left(a+b x+c x^2\right)^4\,dx$$

Optimal (type 3, 485 leaves, 2 steps) :

$$\frac{\left(c d^2-b d e+a e^2\right)^4 (d+e x)^{1+m}}{e^9 (1+m)}-\frac{4 (2 c d-b e) \left(c d^2-b d e+a e^2\right)^3 (d+e x)^{2+m}}{e^9 (2+m)}+$$
$$\frac{2 \left(c d^2-b d e+a e^2\right)^2 \left(14 c^2 d^2+3 b^2 e^2-2 c e (7 b d-a e)\right) (d+e x)^{3+m}}{e^9 (3+m)}-$$
$$\frac{4 (2 c d-b e) \left(c d^2-b d e+a e^2\right) \left(7 c^2 d^2+b^2 e^2-c e (7 b d-3 a e)\right) (d+e x)^{4+m}}{e^9 (4+m)}+\frac{1}{e^9 (5+m)}$$
$$\left(70 c^4 d^4+b^4 e^4-4 b^2 c e^3 (5 b d-3 a e)-20 c^3 d^2 e (7 b d-3 a e)+6 c^2 e^2 \left(15 b^2 d^2-10 a b d e+a^2 e^2\right)\right) (d+e x)^{5+m}-$$
$$\frac{4 c (2 c d-b e) \left(7 c^2 d^2+b^2 e^2-c e (7 b d-3 a e)\right) (d+e x)^{6+m}}{e^9 (6+m)}+$$
$$\frac{2 c^2 \left(14 c^2 d^2+3 b^2 e^2-2 c e (7 b d-a e)\right) (d+e x)^{7+m}}{e^9 (7+m)}-\frac{4 c^3 (2 c d-b e) (d+e x)^{8+m}}{e^9 (8+m)}+\frac{c^4 (d+e x)^{9+m}}{e^9 (9+m)}$$

Result (type 3, 1708 leaves) :

$$\frac{1}{e^9 (1+m) (2+m) (3+m) (4+m) (5+m) (6+m) (7+m) (8+m) (9+m)}$$
$$(d+e x)^{1+m} \left(c^4 \left(40320 d^8-40320 d^7 e (1+m) x+20160 d^6 e^2 \left(2+3 m+m^2\right) x^2-6720 d^5 e^3 \left(6+11 m+6 m^2+m^3\right) x^3+\right.\right.$$
$$1680 d^4 e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4-336 d^3 e^5 \left(120+274 m+225 m^2+85 m^3+15 m^4+m^5\right) x^5+$$
$$56 d^2 e^6 \left(720+1764 m+1624 m^2+735 m^3+175 m^4+21 m^5+m^6\right) x^6-8 d e^7 \left(5040+13068 m+13132 m^2+6769 m^3+1960 m^4+322 m^5+28 m^6+m^7\right) x^7+$$
$$\left.e^8 \left(40320+109584 m+118124 m^2+67284 m^3+22449 m^4+4536 m^5+546 m^6+36 m^7+m^8\right) x^8\right)+e^4 \left(3024+1650 m+335 m^2+30 m^3+m^4\right)$$
$$\left(a^4 e^4 \left(120+154 m+71 m^2+14 m^3+m^4\right)+4 a^3 b e^3 \left(60+47 m+12 m^2+m^3\right) (-d+e (1+m) x)+6 a^2 b^2 e^2 \left(20+9 m+m^2\right)\right.$$
$$\left(2 d^2-2 d e (1+m) x+e^2 \left(2+3 m+m^2\right) x^2\right)+4 a b^3 e (5+m) \left(-6 d^3+6 d^2 e (1+m) x-3 d e^2 \left(2+3 m+m^2\right) x^2+e^3 \left(6+11 m+6 m^2+m^3\right) x^3\right)+$$
$$\left.b^4 \left(24 d^4-24 d^3 e (1+m) x+12 d^2 e^2 \left(2+3 m+m^2\right) x^2-4 d e^3 \left(6+11 m+6 m^2+m^3\right) x^3+e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4\right)\right)+$$
$$4 c e^3 \left(504+191 m+24 m^2+m^3\right) \left(a^3 e^3 \left(120+74 m+15 m^2+m^3\right) \left(2 d^2-2 d e (1+m) x+e^2 \left(2+3 m+m^2\right) x^2\right)+\right.$$
$$3 a^2 b e^2 \left(30+11 m+m^2\right) \left(-6 d^3+6 d^2 e (1+m) x-3 d e^2 \left(2+3 m+m^2\right) x^2+e^3 \left(6+11 m+6 m^2+m^3\right) x^3\right)+$$
$$3 a b^2 e (6+m) \left(24 d^4-24 d^3 e (1+m) x+12 d^2 e^2 \left(2+3 m+m^2\right) x^2-4 d e^3 \left(6+11 m+6 m^2+m^3\right) x^3+e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4\right)+$$
$$b^3 \left(-120 d^5+120 d^4 e (1+m) x-60 d^3 e^2 \left(2+3 m+m^2\right) x^2+20 d^2 e^3 \left(6+11 m+6 m^2+m^3\right) x^3-\right.$$
$$\left.\left.5 d e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4+e^5 \left(120+274 m+225 m^2+85 m^3+15 m^4+m^5\right) x^5\right)\right)+6 c^2 e^2 \left(72+17 m+m^2\right)$$
$$\left(a^2 e^2 \left(42+13 m+m^2\right) \left(24 d^4-24 d^3 e (1+m) x+12 d^2 e^2 \left(2+3 m+m^2\right) x^2-4 d e^3 \left(6+11 m+6 m^2+m^3\right) x^3+e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4\right)+\right.$$
$$2 a b e (7+m) \left(-120 d^5+120 d^4 e (1+m) x-60 d^3 e^2 \left(2+3 m+m^2\right) x^2+20 d^2 e^3 \left(6+11 m+6 m^2+m^3\right) x^3-\right.$$
$$\left.5 d e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4+e^5 \left(120+274 m+225 m^2+85 m^3+15 m^4+m^5\right) x^5\right)+$$
$$b^2 \left(720 d^6-720 d^5 e (1+m) x+360 d^4 e^2 \left(2+3 m+m^2\right) x^2-120 d^3 e^3 \left(6+11 m+6 m^2+m^3\right) x^3+30 d^2 e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4-\right.$$
$$\left.\left.6 d e^5 \left(120+274 m+225 m^2+85 m^3+15 m^4+m^5\right) x^5+e^6 \left(720+1764 m+1624 m^2+735 m^3+175 m^4+21 m^5+m^6\right) x^6\right)\right)+4 c^3 e (9+m)$$
$$\left(a e (8+m) \left(720 d^6-720 d^5 e (1+m) x+360 d^4 e^2 \left(2+3 m+m^2\right) x^2-120 d^3 e^3 \left(6+11 m+6 m^2+m^3\right) x^3+30 d^2 e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4-\right.\right.$$
$$\left.6 d e^5 \left(120+274 m+225 m^2+85 m^3+15 m^4+m^5\right) x^5+e^6 \left(720+1764 m+1624 m^2+735 m^3+175 m^4+21 m^5+m^6\right) x^6\right)+$$
$$b \left(-5040 d^7+5040 d^6 e (1+m) x-2520 d^5 e^2 \left(2+3 m+m^2\right) x^2+840 d^4 e^3 \left(6+11 m+6 m^2+m^3\right) x^3-210 d^3 e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4+\right.$$
$$42 d^2 e^5 \left(120+274 m+225 m^2+85 m^3+15 m^4+m^5\right) x^5-7 d e^6 \left(720+1764 m+1624 m^2+735 m^3+175 m^4+21 m^5+m^6\right) x^6+$$
$$\left.\left.\left.e^7 \left(5040+13068 m+13132 m^2+6769 m^3+1960 m^4+322 m^5+28 m^6+m^7\right) x^7\right)\right)\right)$$

Problem 2550: Result more than twice size of optimal antiderivative.

$$\int (d+e x)^m \left(a+b x+c x^2\right)^3 \, dx$$

Optimal (type 3, 305 leaves, 2 steps):

$$\frac{\left(c d^2-b d e+a e^2\right)^3 (d+e x)^{1+m}}{e^7 (1+m)}-\frac{3 (2 c d-b e) \left(c d^2-b d e+a e^2\right)^2 (d+e x)^{2+m}}{e^7 (2+m)}+$$
$$\frac{3 \left(c d^2-b d e+a e^2\right) \left(5 c^2 d^2+b^2 e^2-c e (5 b d-a e)\right) (d+e x)^{3+m}}{e^7 (3+m)}-\frac{(2 c d-b e) \left(10 c^2 d^2+b^2 e^2-2 c e (5 b d-3 a e)\right) (d+e x)^{4+m}}{e^7 (4+m)}+$$
$$\frac{3 c \left(5 c^2 d^2+b^2 e^2-c e (5 b d-a e)\right) (d+e x)^{5+m}}{e^7 (5+m)}-\frac{3 c^2 (2 c d-b e) (d+e x)^{6+m}}{e^7 (6+m)}+\frac{c^3 (d+e x)^{7+m}}{e^7 (7+m)}$$

Result (type 3, 791 leaves):

$$\frac{1}{e^7 (1+m) (2+m) (3+m) (4+m) (5+m) (6+m) (7+m)}$$
$$(d+e x)^{1+m} \left(c^3 \left(720 d^6-720 d^5 e (1+m) x+360 d^4 e^2 \left(2+3 m+m^2\right) x^2-120 d^3 e^3 \left(6+11 m+6 m^2+m^3\right) x^3+30 d^2 e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4-\right.\right.$$
$$\left.6 d e^5 \left(120+274 m+225 m^2+85 m^3+15 m^4+m^5\right) x^5+e^6 \left(720+1764 m+1624 m^2+735 m^3+175 m^4+21 m^5+m^6\right) x^6\right)+$$
$$e^3 \left(210+107 m+18 m^2+m^3\right) \left(a^3 e^3 \left(24+26 m+9 m^2+m^3\right)+3 a^2 b e^2 \left(12+7 m+m^2\right) (-d+e (1+m) x)+\right.$$
$$\left.3 a b^2 e (4+m) \left(2 d^2-2 d e (1+m) x+e^2 \left(2+3 m+m^2\right) x^2\right)+b^3 \left(-6 d^3+6 d^2 e (1+m) x-3 d e^2 \left(2+3 m+m^2\right) x^2+e^3 \left(6+11 m+6 m^2+m^3\right) x^3\right)\right)+$$
$$3 c e^2 \left(42+13 m+m^2\right) \left(a^2 e^2 \left(20+9 m+m^2\right) \left(2 d^2-2 d e (1+m) x+e^2 \left(2+3 m+m^2\right) x^2\right)+\right.$$
$$2 a b e (5+m) \left(-6 d^3+6 d^2 e (1+m) x-3 d e^2 \left(2+3 m+m^2\right) x^2+e^3 \left(6+11 m+6 m^2+m^3\right) x^3\right)+$$
$$\left.b^2 \left(24 d^4-24 d^3 e (1+m) x+12 d^2 e^2 \left(2+3 m+m^2\right) x^2-4 d e^3 \left(6+11 m+6 m^2+m^3\right) x^3+e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4\right)\right)+$$
$$3 c^2 e (7+m) \left(a e (6+m) \left(24 d^4-24 d^3 e (1+m) x+12 d^2 e^2 \left(2+3 m+m^2\right) x^2-4 d e^3 \left(6+11 m+6 m^2+m^3\right) x^3+e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4\right)+\right.$$
$$b \left(-120 d^5+120 d^4 e (1+m) x-60 d^3 e^2 \left(2+3 m+m^2\right) x^2+20 d^2 e^3 \left(6+11 m+6 m^2+m^3\right) x^3-\right.$$
$$\left.\left.\left.5 d e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4+e^5 \left(120+274 m+225 m^2+85 m^3+15 m^4+m^5\right) x^5\right)\right)\right)$$

Problem 2554: Unable to integrate problem.

$$\int \frac{(d+e x)^m}{\left(a+b x+c x^2\right)^2} \, dx$$

Optimal (type 5, 425 leaves, 5 steps):

$$-\frac{(d+e x)^{1+m}\left(b c d-b^2 e+2 a c e+c\,(2 c d-b e)\,x\right)}{\left(b^2-4 a c\right)\left(c d^2-b d e+a e^2\right)\left(a+b x+c x^2\right)}+\left(c\left(4 c^2 d^2+b\left(b+\sqrt{b^2-4 a c}\right) e^2 m-2 c e\left(2 b d-2 a e\,(1-m)+\sqrt{b^2-4 a c}\;d m\right)\right)(d+e x)^{1+m}\right.$$

$$\left.\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{2 c\,(d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\right]\right)\Big/\left(\left(b^2-4 a c\right)^{3/2}\left(2 c d-\left(b-\sqrt{b^2-4 a c}\right) e\right)\left(c d^2-b d e+a e^2\right)(1+m)\right)-$$

$$\left(c\left(e\,(2 c d-b e)\,m+\frac{4 c^2 d^2-4 c e\,(b d-a e\,(1-m))+b^2 e^2 m}{\sqrt{b^2-4 a c}}\right)(d+e x)^{1+m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{2 c\,(d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Big/$$

$$\left(\left(b^2-4 a c\right)\left(2 c d-\left(b+\sqrt{b^2-4 a c}\right) e\right)\left(c d^2-b d e+a e^2\right)(1+m)\right)$$

Result (type 8, 22 leaves):

$$\int \frac{(d+e x)^m}{\left(a+b x+c x^2\right)^2}\,dx$$

Problem 2555: Unable to integrate problem.

$$\int (d+e x)^m\left(a+b x+c x^2\right)^{5/2}\,dx$$

Optimal (type 6, 189 leaves, 2 steps):

$$\frac{(d+e x)^{1+m}\left(a+b x+c x^2\right)^{5/2}\,\text{AppellF1}\left[1+m,-\frac{5}{2},-\frac{5}{2},2+m,\frac{2 c\,(d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e},\frac{2 c\,(d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]}{e\,(1+m)\left(1-\frac{2 c\,(d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\right)^{5/2}\left(1-\frac{2 c\,(d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right)^{5/2}}$$

Result (type 8, 24 leaves):

$$\int (d+e x)^m\left(a+b x+c x^2\right)^{5/2}\,dx$$

Problem 2556: Unable to integrate problem.

$$\int (d+e x)^m\left(a+b x+c x^2\right)^{3/2}\,dx$$

Optimal (type 6, 189 leaves, 2 steps):

$$\frac{(d+e x)^{1+m} \left(a+b x+c x^2\right)^{3/2} \text{AppellF1}\left[1+m,-\frac{3}{2},-\frac{3}{2},2+m,\frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e},\frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]}{e (1+m) \left(1-\frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\right)^{3/2} \left(1-\frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right)^{3/2}}$$

Result (type 8, 24 leaves):

$$\int (d+e x)^m \left(a+b x+c x^2\right)^{3/2} \, dx$$

- **Problem 2561: Result more than twice size of optimal antiderivative.**

$$\int (d x)^m \left(a+b x+c x^2\right)^p \, dx$$

Optimal (type 6, 137 leaves, 2 steps):

$$\frac{1}{d (1+m)} (d x)^{1+m} \left(1+\frac{2 c x}{b-\sqrt{b^2-4 a c}}\right)^{-p} \left(1+\frac{2 c x}{b+\sqrt{b^2-4 a c}}\right)^{-p} \left(a+b x+c x^2\right)^p \text{AppellF1}\left[1+m,-p,-p,2+m,-\frac{2 c x}{b-\sqrt{b^2-4 a c}},-\frac{2 c x}{b+\sqrt{b^2-4 a c}}\right]$$

Result (type 6, 439 leaves):

$$\left(2^{-1-p} c \left(b+\sqrt{b^2-4 a c}\right) (2+m) x (d x)^m \left(\frac{b-\sqrt{b^2-4 a c}}{2 c}+x\right)^{-p} \left(\frac{b-\sqrt{b^2-4 a c}+2 c x}{c}\right)^{1+p}\right.$$
$$\left.\left(2 a+\left(b-\sqrt{b^2-4 a c}\right) x\right)^2 (a+x (b+c x))^{-1+p} \text{AppellF1}\left[1+m,-p,-p,2+m,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/$$
$$\left(\left(-b+\sqrt{b^2-4 a c}\right) (1+m) \left(b+\sqrt{b^2-4 a c}+2 c x\right) \left(-2 a (2+m) \text{AppellF1}\left[1+m,-p,-p,2+m,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]+\right.\right.$$
$$p x \left(\left(-b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[2+m,1-p,-p,3+m,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]-\right.$$
$$\left.\left.\left.\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[2+m,-p,1-p,3+m,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)$$

- **Problem 2563: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (d+e x)^3 \left(a+b x+c x^2\right)^p \, dx$$

Optimal (type 5, 327 leaves, 3 steps):

$$\frac{e\,(d+e\,x)^2\,\left(a+b\,x+c\,x^2\right)^{1+p}}{2\,c\,(2+p)}-\left(e\,\left(b\,e\,(2\,c\,d-b\,e)\,(2+p)\,(3+p)-2\,c\,(3+2\,p)\,\left(c\,d^2\,(5+2\,p)-e\,(a\,e+b\,d\,(2+p))\right)\right)-2\,c\,e\,(2\,c\,d-b\,e)\,(1+p)\,(3+p)\,x\right)\,\left(a+b\,x+c\,x^2\right)^{1+p}\Big/\left(4\,c^3\,(1+p)\,(2+p)\,(3+2\,p)\right)-\left(2^{-1+p}\,(2\,c\,d-b\,e)\,\left(b^2\,e^2\,(3+p)+2\,c^2\,d^2\,(3+2\,p)-2\,c\,e\,(3\,a\,e+b\,d\,(3+2\,p))\right)\,\left(-\frac{b-\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{-1-p}\left(a+b\,x+c\,x^2\right)^{1+p}\,\text{Hypergeometric2F1}\left[-p,\,1+p,\,2+p,\,\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{2\,\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(c^3\,\sqrt{b^2-4\,a\,c}\,(1+p)\,(3+2\,p)\right)$$

Result (type 6, 1326 leaves) :

$$\left(9\times 2^{-2-p}\,c\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,d^2\,e\,x^2\,\left(\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+x\right)^{-p}\left(\frac{b-\sqrt{b^2-4\,a\,c}+2\,c\,x}{c}\right)^{1+p}\left(2\,a+\left(b-\sqrt{b^2-4\,a\,c}\right)\,x\right)^2\,(a+x\,(b+c\,x))^{-1+p}\,\text{AppellF1}\left[2,\,-p,\,-p,\,3,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(\left(-b+\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\,\left(-6\,a\,\text{AppellF1}\left[2,\,-p,\,-p,\,3,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+p\,x\,\left(\left(-b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[3,\,1-p,\,-p,\,4,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]-\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[3,\,-p,\,1-p,\,4,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+$$

$$\left(2\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,d\,e^2\,x^3\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\,\left(2\,a+\left(b-\sqrt{b^2-4\,a\,c}\right)\,x\right)^2\,(a+x\,(b+c\,x))^{-1+p}\,\text{AppellF1}\left[3,\,-p,\,-p,\,4,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(\left(-b+\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\,\left(-8\,a\,\text{AppellF1}\left[3,\,-p,\,-p,\,4,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.\right.$$

$$p\,x\left(\left(-b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[4,\,1-p,\,-p,\,5,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\left.\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[4,\,-p,\,1-p,\,5,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+$$

$$\left(5\times 2^{-3-p}\,c\left(b+\sqrt{b^2-4\,a\,c}\right)e^3\,x^4\left(\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+x\right)^{-p}\left(\frac{b-\sqrt{b^2-4\,a\,c}+2\,c\,x}{c}\right)^{1+p}\left(2\,a+\left(b-\sqrt{b^2-4\,a\,c}\right)x\right)^2\right.$$

$$\left.(a+x\,(b+c\,x))^{-1+p}\,\text{AppellF1}\left[4,\,-p,\,-p,\,5,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\left(-b+\sqrt{b^2-4\,a\,c}\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\left(-10\,a\,\text{AppellF1}\left[4,\,-p,\,-p,\,5,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.\right.$$

$$p\,x\left(\left(-b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[5,\,1-p,\,-p,\,6,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\left.\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[5,\,-p,\,1-p,\,6,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+$$

$$\frac{1}{c\,(1+p)}2^{-1+p}\,d^3\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\left(\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{-p}(a+x\,(b+c\,x))^p$$

$$\text{Hypergeometric2F1}\left[-p,\,1+p,\,2+p,\,\frac{-b+\sqrt{b^2-4\,a\,c}-2\,c\,x}{2\,\sqrt{b^2-4\,a\,c}}\right]$$

- **Problem 2564: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (d+e\,x)^2\left(a+b\,x+c\,x^2\right)^p\,dx$$

Optimal (type 5, 248 leaves, 3 steps) :

$$\frac{e\,(2\,c\,d-b\,e)\,(2+p)\,\left(a+b\,x+c\,x^2\right)^{1+p}}{2\,c^2\,(1+p)\,(3+2\,p)}+\frac{e\,(d+e\,x)\,\left(a+b\,x+c\,x^2\right)^{1+p}}{c\,(3+2\,p)}-$$

$$\left(2^p\,\left(b^2\,e^2\,(2+p)+2\,c^2\,d^2\,(3+2\,p)-2\,c\,e\,(a\,e+b\,d\,(3+2\,p))\right)\,\left(-\frac{b-\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{-1-p}\right.$$

$$\left.\left(a+b\,x+c\,x^2\right)^{1+p}\,\text{Hypergeometric2F1}\left[-p,\,1+p,\,2+p,\,\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{2\,\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(c^2\,\sqrt{b^2-4\,a\,c}\,(1+p)\,(3+2\,p)\right)$$

Result (type 6, 1001 leaves) :

$$\left(3\times 2^{-1-p}\,c\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,d\,e\,x^2\,\left(\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+x\right)^{-p}\,\left(\frac{b-\sqrt{b^2-4\,a\,c}+2\,c\,x}{c}\right)^{1+p}\right.$$

$$\left.\left(2\,a+\left(b-\sqrt{b^2-4\,a\,c}\right)\,x\right)^2\,(a+x\,(b+c\,x))^{-1+p}\,\text{AppellF1}\left[2,\,-p,\,-p,\,3,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\left(-b+\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\,\left(-6\,a\,\text{AppellF1}\left[2,\,-p,\,-p,\,3,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.\right.$$

$$p\,x\,\left(\left(-b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[3,\,1-p,\,-p,\,4,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\left.\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[3,\,-p,\,1-p,\,4,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+$$

$$\left(2\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,e^2\,x^3\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\,\left(2\,a+\left(b-\sqrt{b^2-4\,a\,c}\right)\,x\right)^2\,(a+x\,(b+c\,x))^{-1+p}\right.$$

$$\left.\text{AppellF1}\left[3,\,-p,\,-p,\,4,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(3\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\,\left(-8\,a\,\text{AppellF1}\left[3,\,-p,\,-p,\,4,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.\right.$$

$$p\,x\left(\left(-b+\sqrt{b^2-4ac}\right)\text{AppellF1}\left[4,1-p,-p,5,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]-\right.$$

$$\left.\left(b+\sqrt{b^2-4ac}\right)\text{AppellF1}\left[4,-p,1-p,5,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\right)\Bigg)\Bigg)+$$

$$\frac{1}{2c(1+p)}d^2\left(b-\sqrt{b^2-4ac}+2cx\right)\left(a+bx+cx^2\right)^p\left(1+\frac{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x}{-\frac{-b-\sqrt{b^2-4ac}}{2c}+\frac{-b+\sqrt{b^2-4ac}}{2c}}\right)^{-p}$$

$$\text{Hypergeometric2F1}\left[-p,1+p,2+p,-\frac{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x}{-\frac{-b-\sqrt{b^2-4ac}}{2c}+\frac{-b+\sqrt{b^2-4ac}}{2c}}\right]$$

- **Problem 2565: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int(d+ex)\left(a+bx+cx^2\right)^p\,dx$$

Optimal (type 5, 160 leaves, 2 steps) :

$$\frac{e\left(a+bx+cx^2\right)^{1+p}}{2c(1+p)}-\frac{1}{c\sqrt{b^2-4ac}\,(1+p)}$$

$$2^p(2cd-be)\left(-\frac{b-\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx+cx^2\right)^{1+p}\text{Hypergeometric2F1}\left[-p,1+p,2+p,\frac{b+\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right]$$

Result (type 6, 476 leaves) :

$$\frac{1}{4}\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\,(a+x\,(b+c\,x))^p$$

$$\left(\left(3\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,e\,x^2\,\left(2\,a+\left(b-\sqrt{b^2-4\,a\,c}\right)\,x\right)^2\,\text{AppellF1}\left[2,\,-p,\,-p,\,3,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\right.$$

$$\left(\left(-b+\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\,(a+x\,(b+c\,x))\right.$$

$$\left(-6\,a\,\text{AppellF1}\left[2,\,-p,\,-p,\,3,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+p\,x\,\left(\left(-b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[3,\,1-p,\,-p,\,4,\right.\right.\right.$$

$$\left.\left.\left.\left.-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]-\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\left[3,\,-p,\,1-p,\,4,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+$$

$$\left.\frac{2^{1+p}\,d\,\left(\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{-p}\,\text{Hypergeometric2F1}\left[-p,\,1+p,\,2+p,\,\frac{1}{2}-\frac{b}{2\,\sqrt{b^2-4\,a\,c}}-\frac{c\,x}{\sqrt{b^2-4\,a\,c}}\right]}{c+c\,p}\right)$$

- **Problem 2570: Result more than twice size of optimal antiderivative.**

$$\int (d+e\,x)^{3/2}\,\left(a+b\,x+c\,x^2\right)^p\,dx$$

Optimal (type 6, 185 leaves, 2 steps):

$$\frac{1}{5\,e}2\,(d+e\,x)^{5/2}\,\left(a+b\,x+c\,x^2\right)^p\,\left(1-\frac{2\,c\,(d+e\,x)}{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)\,e}\right)^{-p}$$

$$\left(1-\frac{2\,c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right)^{-p}\,\text{AppellF1}\left[\frac{5}{2},\,-p,\,-p,\,\frac{7}{2},\,\frac{2\,c\,(d+e\,x)}{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)\,e},\,\frac{2\,c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]$$

Result (type 6, 590 leaves):

$$-\left(\left(7\left(2\,c\,d+e\left(-b+\sqrt{\frac{b^2-4\,a\,c}{e^2}}\,e\right)\right)\left(2\,c\,d-e\left(b+\sqrt{\frac{b^2-4\,a\,c}{e^2}}\,e\right)\right)\left(b-\sqrt{\frac{b^2-4\,a\,c}{e^2}}\,e+2\,c\,x\right)\left(b+\sqrt{\frac{b^2-4\,a\,c}{e^2}}\,e+2\,c\,x\right)\right.\right.$$

$$\left.(d+e\,x)^{5/2}\,(a+x\,(b+c\,x))^{-1+p}\,\text{AppellF1}\left[\frac{5}{2},-p,-p,\frac{7}{2},\frac{2\,c\,(d+e\,x)}{2\,c\,d-e\left(b+\sqrt{\frac{b^2-4\,a\,c}{e^2}}\,e\right)},\frac{2\,c\,(d+e\,x)}{2\,c\,d+e\left(-b+\sqrt{\frac{b^2-4\,a\,c}{e^2}}\,e\right)}\right]\right)\Big/$$

$$\left(40\,c^2\,e\left(-7\left(c\,d^2+e\,(-b\,d+a\,e)\right)\text{AppellF1}\left[\frac{5}{2},-p,-p,\frac{7}{2},\frac{2\,c\,(d+e\,x)}{2\,c\,d-e\left(b+\sqrt{\frac{b^2-4\,a\,c}{e^2}}\,e\right)},\frac{2\,c\,(d+e\,x)}{2\,c\,d+e\left(-b+\sqrt{\frac{b^2-4\,a\,c}{e^2}}\,e\right)}\right]+\right.\right.$$

$$p\,(d+e\,x)\left(\left(2\,c\,d+e\left(-b+\sqrt{\frac{b^2-4\,a\,c}{e^2}}\,e\right)\right)\text{AppellF1}\left[\frac{7}{2},1-p,-p,\frac{9}{2},\frac{2\,c\,(d+e\,x)}{2\,c\,d-e\left(b+\sqrt{\frac{b^2-4\,a\,c}{e^2}}\,e\right)},\frac{2\,c\,(d+e\,x)}{2\,c\,d+e\left(-b+\sqrt{\frac{b^2-4\,a\,c}{e^2}}\,e\right)}\right]+\right.$$

$$\left.\left.\left.\left.\left(2\,c\,d-e\left(b+\sqrt{\frac{b^2-4\,a\,c}{e^2}}\,e\right)\right)\text{AppellF1}\left[\frac{7}{2},-p,1-p,\frac{9}{2},\frac{2\,c\,(d+e\,x)}{2\,c\,d-e\left(b+\sqrt{\frac{b^2-4\,a\,c}{e^2}}\,e\right)},\frac{2\,c\,(d+e\,x)}{2\,c\,d+e\left(-b+\sqrt{\frac{b^2-4\,a\,c}{e^2}}\,e\right)}\right]\right)\right)\right)\right)$$

- **Problem 2578: Attempted integration timed out after 120 seconds.**

$$\int (d+e\,x)^{-4-2\,p}\left(a+b\,x+c\,x^2\right)^p\,dx$$

Optimal (type 5, 442 leaves, 3 steps) :

$$-\frac{e\,(d+e\,x)^{-3-2\,p}\,\left(a+b\,x+c\,x^2\right)^{1+p}}{\left(c\,d^2-b\,d\,e+a\,e^2\right)\,(3+2\,p)}-\frac{e\,(2\,c\,d-b\,e)\,(2+p)\,(d+e\,x)^{-2\,(1+p)}\,\left(a+b\,x+c\,x^2\right)^{1+p}}{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,(1+p)\,(3+2\,p)}+$$

$$\left(\left(b^2\,e^2\,(2+p)+2\,c^2\,d^2\,(3+2\,p)-2\,c\,e\,(a\,e+b\,d\,(3+2\,p))\right)\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\,\left(\frac{\left(2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)\,e\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}{\left(2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e\right)\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}\right)^{-p}\right.$$

$$\left.(d+e\,x)^{-1-2\,p}\,\left(a+b\,x+c\,x^2\right)^p\,\text{Hypergeometric2F1}\left[-1-2\,p,\,-p,\,-2\,p,\,-\frac{4\,c\,\sqrt{b^2-4\,a\,c}\,(d+e\,x)}{\left(2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e\right)\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}\right]\right)\Big/$$

$$\left(2\,\left(2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)\,e\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,(1+2\,p)\,(3+2\,p)\right)$$

Result (type 1, 1 leaves) :

???

- **Problem 2579: Result more than twice size of optimal antiderivative.**

$$\int (d+e\,x)^{-5-2\,p}\,\left(a+b\,x+c\,x^2\right)^p\,dx$$

Optimal (type 5, 577 leaves, 4 steps) :

$$-\frac{e\,(2cd-be)\,(3+p)\,(d+ex)^{-3-2p}\,(a+bx+cx^2)^{1+p}}{2\,(cd^2-bde+ae^2)^2\,(2+p)\,(3+2p)}-$$

$$\left(e\left(b^2e^2(6+5p+p^2)+2c^2d^2(9+8p+2p^2)-2ce\left(ae(3+2p)+bd(9+8p+2p^2)\right)\right)(d+ex)^{-2(1+p)}(a+bx+cx^2)^{1+p}\right)\Big/$$

$$\left(4\,(cd^2-bde+ae^2)^3\,(1+p)\,(2+p)\,(3+2p)\right)-\frac{e\,(d+ex)^{-2(2+p)}\,(a+bx+cx^2)^{1+p}}{2\,(cd^2-bde+ae^2)\,(2+p)}+$$

$$\left((2cd-be)\left(b^2e^2(3+p)+2c^2d^2(3+2p)-2ce\,(3ae+bd\,(3+2p))\right)\left(b-\sqrt{b^2-4ac}+2cx\right)\right.$$

$$\left(\frac{\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\left(b-\sqrt{b^2-4ac}+2cx\right)}\right)^{-p}(d+ex)^{-1-2p}\,(a+bx+cx^2)^p$$

$$\left.\text{Hypergeometric2F1}\left[-1-2p,\,-p,\,-2p,\,-\frac{4c\sqrt{b^2-4ac}\,(d+ex)}{\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\left(b-\sqrt{b^2-4ac}+2cx\right)}\right]\right)\Big/$$

$$\left(4\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(cd^2-bde+ae^2)^3\,(1+2p)\,(3+2p)\right)$$

Result (type 5, 3457 leaves) :

$$-\left(\left(2^{-2p}\left(-\frac{-b-\sqrt{b^2-4ac}}{2c}+x\right)^{-p}\left(-\frac{-b+\sqrt{b^2-4ac}}{2c}+x\right)^{-p}\right.\right.$$

$$\left(\frac{b-\sqrt{b^2-4ac}+2cx}{c}\right)^p\left(\frac{b+\sqrt{b^2-4ac}+2cx}{c}\right)^p(d+ex)^{-4-2p}\left(\frac{-be-\sqrt{b^2-4ac}\,e-2cex}{2cd-be-\sqrt{b^2-4ac}\,e}\right)^{-p}$$

$$\left(\frac{-be+\sqrt{b^2-4ac}\,e-2cex}{2cd-be+\sqrt{b^2-4ac}\,e}\right)^{-p}(a+bx+cx^2)^p\left(1-\frac{2c\,(d+ex)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)^{1+p}\left(1-\frac{2c\,(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^p$$

$$\left(-3\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)^4p\left(b+\sqrt{b^2-4ac}+2cx\right)\text{Gamma}[-p]\,\text{Gamma}[-2\,(1+p)]\,\text{Hypergeometric2F1}\left[1,\,-p,\,-3-2p,\right.\right.$$

$$\frac{4\,c\,\sqrt{b^2-4\,a\,c}\,\,(d+e\,x)}{\left(2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e\right)\,\left(b+\sqrt{b^2-4\,a\,c}\,+2\,c\,x\right)}\Bigg]-11\,\left(2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e\right)^4\,p^2\,\left(b+\sqrt{b^2-4\,a\,c}\,+2\,c\,x\right)$$

$$\text{Gamma}[-p]\,\text{Gamma}[-2\,(1+p)]\,\text{Hypergeometric2F1}\Big[1,\,-p,\,-3-2\,p,\,\frac{4\,c\,\sqrt{b^2-4\,a\,c}\,\,(d+e\,x)}{\left(2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e\right)\,\left(b+\sqrt{b^2-4\,a\,c}\,+2\,c\,x\right)}\Big]-$$

$$12\,\left(2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e\right)^4\,p^3\,\left(b+\sqrt{b^2-4\,a\,c}\,+2\,c\,x\right)\,\text{Gamma}[-p]\,\text{Gamma}[-2\,(1+p)]\,\text{Hypergeometric2F1}\Big[1,\,-p,$$

$$-3-2\,p,\,\frac{4\,c\,\sqrt{b^2-4\,a\,c}\,\,(d+e\,x)}{\left(2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e\right)\,\left(b+\sqrt{b^2-4\,a\,c}\,+2\,c\,x\right)}\Big]-4\,\left(2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e\right)^4\,p^4\,\left(b+\sqrt{b^2-4\,a\,c}\,+2\,c\,x\right)$$

$$\text{Gamma}[-p]\,\text{Gamma}[-2\,(1+p)]\,\text{Hypergeometric2F1}\Big[1,\,-p,\,-3-2\,p,\,\frac{4\,c\,\sqrt{b^2-4\,a\,c}\,\,(d+e\,x)}{\left(2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e\right)\,\left(b+\sqrt{b^2-4\,a\,c}\,+2\,c\,x\right)}\Big]-$$

$$12\,c^2\,\left(2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e\right)^2\,p\,\left(b+\sqrt{b^2-4\,a\,c}\,+2\,c\,x\right)\,(d+e\,x)^2\,\text{Gamma}[-p]\,\text{Gamma}[-2\,(1+p)]$$

$$\text{Hypergeometric2F1}\Big[1,\,-p,\,-1-2\,p,\,\frac{4\,c\,\sqrt{b^2-4\,a\,c}\,\,(d+e\,x)}{\left(2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e\right)\,\left(b+\sqrt{b^2-4\,a\,c}\,+2\,c\,x\right)}\Big]-$$

$$24\,c^2\,\left(2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e\right)^2\,p^2\,\left(b+\sqrt{b^2-4\,a\,c}\,+2\,c\,x\right)\,(d+e\,x)^2\,\text{Gamma}[-p]\,\text{Gamma}[-2\,(1+p)]$$

$$\text{Hypergeometric2F1}\Big[1,\,-p,\,-1-2\,p,\,\frac{4\,c\,\sqrt{b^2-4\,a\,c}\,\,(d+e\,x)}{\left(2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e\right)\,\left(b+\sqrt{b^2-4\,a\,c}\,+2\,c\,x\right)}\Big]-$$

$$24\,c^3\,\left(2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e\right)\,p\,\left(b+\sqrt{b^2-4\,a\,c}\,+2\,c\,x\right)\,(d+e\,x)^3\,\text{Gamma}[-p]\,\text{Gamma}[-2\,(1+p)]$$

$$\text{Hypergeometric2F1}\Big[1,\,-p,\,-2\,p,\,\frac{4\,c\,\sqrt{b^2-4\,a\,c}\,\,(d+e\,x)}{\left(2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e\right)\,\left(b+\sqrt{b^2-4\,a\,c}\,+2\,c\,x\right)}\Big]-$$

$$6\,c\,\left(2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e\right)^3\,p\,\left(b+\sqrt{b^2-4\,a\,c}\,+2\,c\,x\right)\,(d+e\,x)\,\text{Gamma}[-p]\,\text{Gamma}[-2\,(1+p)]$$

$$\text{Hypergeometric2F1}\left[1,-p,-2(1+p),\frac{4c\sqrt{b^2-4ac}\,(d+ex)}{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\left(b+\sqrt{b^2-4ac}+2cx\right)}\right]-$$

$$18c\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)^3 p^2\left(b+\sqrt{b^2-4ac}+2cx\right)(d+ex)\,\text{Gamma}[-p]\,\text{Gamma}[-2(1+p)]$$

$$\text{Hypergeometric2F1}\left[1,-p,-2(1+p),\frac{4c\sqrt{b^2-4ac}\,(d+ex)}{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\left(b+\sqrt{b^2-4ac}+2cx\right)}\right]-$$

$$12c\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)^3 p^3\left(b+\sqrt{b^2-4ac}+2cx\right)(d+ex)\,\text{Gamma}[-p]\,\text{Gamma}[-2(1+p)]$$

$$\text{Hypergeometric2F1}\left[1,-p,-2(1+p),\frac{4c\sqrt{b^2-4ac}\,(d+ex)}{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\left(b+\sqrt{b^2-4ac}+2cx\right)}\right]-$$

$$36c^2\sqrt{b^2-4ac}\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)^2 p\,(d+ex)^2\,\text{Gamma}[-3-2p]\,\text{Gamma}[1-p]\,\text{Hypergeometric2F1}\Big[2,1-p,$$

$$-1-2p,\frac{4c\sqrt{b^2-4ac}\,(d+ex)}{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\left(b+\sqrt{b^2-4ac}+2cx\right)}\Big]-96c^2\sqrt{b^2-4ac}\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)^2 p^2(d+ex)^2$$

$$\text{Gamma}[-3-2p]\,\text{Gamma}[1-p]\,\text{Hypergeometric2F1}\left[2,1-p,-1-2p,\frac{4c\sqrt{b^2-4ac}\,(d+ex)}{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\left(b+\sqrt{b^2-4ac}+2cx\right)}\right]-$$

$$48c^2\sqrt{b^2-4ac}\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)^2 p^3(d+ex)^2\,\text{Gamma}[-3-2p]\,\text{Gamma}[1-p]\,\text{Hypergeometric2F1}\Big[2,1-p,$$

$$-1-2p,\frac{4c\sqrt{b^2-4ac}\,(d+ex)}{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\left(b+\sqrt{b^2-4ac}+2cx\right)}\Big]-264c^4\sqrt{b^2-4ac}\,(d+ex)^4\,\text{Gamma}[-3-2p]\,\text{Gamma}[1-p]$$

$$\text{Hypergeometric2F1}\left[2,1-p,1-2p,\frac{4c\sqrt{b^2-4ac}\,(d+ex)}{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\left(b+\sqrt{b^2-4ac}+2cx\right)}\right]-176c^4\sqrt{b^2-4ac}\,p\,(d+ex)^4$$

$$\text{Gamma}[-3-2p]\,\text{Gamma}[1-p]\,\text{Hypergeometric2F1}\left[2,1-p,1-2p,\frac{4c\sqrt{b^2-4ac}\,(d+ex)}{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\left(b+\sqrt{b^2-4ac}+2cx\right)}\right]-$$

$$216\, c^3 \sqrt{b^2-4 a c}\, \left(2 c d+\left(-b+\sqrt{b^2-4 a c}\right) e\right) p\, (d+e x)^3\, \text{Gamma}[-3-2 p]\, \text{Gamma}[1-p]$$

$$\text{Hypergeometric2F1}\left[2,1-p,-2 p,\frac{4 c \sqrt{b^2-4 a c}\, (d+e x)}{\left(2 c d+\left(-b+\sqrt{b^2-4 a c}\right) e\right) \left(b+\sqrt{b^2-4 a c}+2 c x\right)}\right]-$$

$$144\, c^3 \sqrt{b^2-4 a c}\, \left(2 c d+\left(-b+\sqrt{b^2-4 a c}\right) e\right) p^2\, (d+e x)^3\, \text{Gamma}[-3-2 p]\, \text{Gamma}[1-p]$$

$$\text{Hypergeometric2F1}\left[2,1-p,-2 p,\frac{4 c \sqrt{b^2-4 a c}\, (d+e x)}{\left(2 c d+\left(-b+\sqrt{b^2-4 a c}\right) e\right) \left(b+\sqrt{b^2-4 a c}+2 c x\right)}\right]-144\, c^4 \sqrt{b^2-4 a c}\, (d+e x)^4$$

$$\text{Gamma}[-3-2 p]\, \text{Gamma}[1-p]\, \text{HypergeometricPFQ}\left[\{2,2,1-p\},\{1,1-2 p\},\frac{4 c \sqrt{b^2-4 a c}\, (d+e x)}{\left(2 c d+\left(-b+\sqrt{b^2-4 a c}\right) e\right) \left(b+\sqrt{b^2-4 a c}+2 c x\right)}\right]-$$

$$96\, c^4 \sqrt{b^2-4 a c}\, p\, (d+e x)^4\, \text{Gamma}[-3-2 p]\, \text{Gamma}[1-p]\, \text{HypergeometricPFQ}\Big[\{2,2,1-p\},\{1,1-2 p\},$$

$$\frac{4 c \sqrt{b^2-4 a c}\, (d+e x)}{\left(2 c d+\left(-b+\sqrt{b^2-4 a c}\right) e\right) \left(b+\sqrt{b^2-4 a c}+2 c x\right)}\Big]-72\, c^3 \sqrt{b^2-4 a c}\, \left(2 c d+\left(-b+\sqrt{b^2-4 a c}\right) e\right) p\, (d+e x)^3\, \text{Gamma}[-3-2 p]$$

$$\text{Gamma}[1-p]\, \text{HypergeometricPFQ}\left[\{2,2,1-p\},\{1,-2 p\},\frac{4 c \sqrt{b^2-4 a c}\, (d+e x)}{\left(2 c d+\left(-b+\sqrt{b^2-4 a c}\right) e\right) \left(b+\sqrt{b^2-4 a c}+2 c x\right)}\right]-$$

$$48\, c^3 \sqrt{b^2-4 a c}\, \left(2 c d+\left(-b+\sqrt{b^2-4 a c}\right) e\right) p^2\, (d+e x)^3\, \text{Gamma}[-3-2 p]\, \text{Gamma}[1-p]$$

$$\text{HypergeometricPFQ}\left[\{2,2,1-p\},\{1,-2 p\},\frac{4 c \sqrt{b^2-4 a c}\, (d+e x)}{\left(2 c d+\left(-b+\sqrt{b^2-4 a c}\right) e\right) \left(b+\sqrt{b^2-4 a c}+2 c x\right)}\right]-$$

$$24\, c^4 \sqrt{b^2-4 a c}\, (d+e x)^4\, \text{Gamma}[-3-2 p]\, \text{Gamma}[1-p]\, \text{HypergeometricPFQ}\Big[\{2,2,2,1-p\},\{1,1,1-2 p\},$$

$$\frac{4 c \sqrt{b^2-4 a c}\, (d+e x)}{\left(2 c d+\left(-b+\sqrt{b^2-4 a c}\right) e\right) \left(b+\sqrt{b^2-4 a c}+2 c x\right)}\Big]-16\, c^4 \sqrt{b^2-4 a c}\, p\, (d+e x)^4\, \text{Gamma}[-3-2 p]\, \text{Gamma}[1-p]$$

$$\text{HypergeometricPFQ}\left[\{2,2,2,1-p\},\{1,1,1-2p\},\frac{4c\sqrt{b^2-4ac}\,(d+ex)}{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\left(b+\sqrt{b^2-4ac}+2cx\right)}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(e\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)^4(-4-2p)\,p\,(1+p)(1+2p)(3+2p)\left(b+\sqrt{b^2-4ac}+2cx\right)\text{Gamma}[-p]\,\text{Gamma}[-2(1+p)]\right)$$

- **Problem 2580: Attempted integration timed out after 120 seconds.**

$$\int (d+ex)^{-6-2p}\left(a+bx+cx^2\right)^p\,dx$$

Optimal (type 5, 809 leaves, 5 steps):

$$-\frac{e\,(d+ex)^{-5-2p}\left(a+bx+cx^2\right)^{1+p}}{\left(cd^2-bde+ae^2\right)(5+2p)}-$$

$$\left(e\left(b^2e^2\left(12+7p+p^2\right)+2c^2d^2\left(18+11p+2p^2\right)-2ce\left(3ae(2+p)+bd\left(18+11p+2p^2\right)\right)\right)(d+ex)^{-3-2p}\left(a+bx+cx^2\right)^{1+p}\right)\Big/$$

$$\left(2\left(cd^2-bde+ae^2\right)^3(2+p)(3+2p)(5+2p)\right)-$$

$$\left(e\,(2cd-be)(3+p)\left(b^2e^2\left(8+6p+p^2\right)+2c^2d^2\left(8+7p+2p^2\right)-2ce\left(ae(8+5p)+bd\left(8+7p+2p^2\right)\right)\right)(d+ex)^{-2(1+p)}\left(a+bx+cx^2\right)^{1+p}\right)\Big/$$

$$\left(4\left(cd^2-bde+ae^2\right)^4(1+p)(2+p)(3+2p)(5+2p)\right)-\frac{e\,(2cd-be)(4+p)(d+ex)^{-2(2+p)}\left(a+bx+cx^2\right)^{1+p}}{2\left(cd^2-bde+ae^2\right)^2(2+p)(5+2p)}+$$

$$\Bigg(\left(b^4e^4\left(12+7p+p^2\right)+4c^4d^4\left(15+16p+4p^2\right)-8c^3d^2e(5+2p)(3ae+bd(3+2p))-4b^2ce^3(3+p)(3ae+bd(5+2p))+\right.$$

$$\left.12c^2e^2\left(a^2e^2+2abde(5+2p)+b^2d^2\left(10+9p+2p^2\right)\right)\right)\left(b-\sqrt{b^2-4ac}+2cx\right)\left(\frac{\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\left(b-\sqrt{b^2-4ac}+2cx\right)}\right)^{-p}$$

$$(d+ex)^{-1-2p}\left(a+bx+cx^2\right)^p\,\text{Hypergeometric2F1}\left[-1-2p,-p,-2p,-\frac{4c\sqrt{b^2-4ac}\,(d+ex)}{\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\left(b-\sqrt{b^2-4ac}+2cx\right)}\right]\Bigg)\Bigg/$$

$$\left(4\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\left(cd^2-bde+ae^2\right)^4(1+2p)(3+2p)(5+2p)\right)$$

Result (type 1, 1 leaves):

???

- **Problem 2581: Attempted integration timed out after 120 seconds.**

$$\int (d+e x)^m \left(a+b x+c x^2\right)^{-2-\frac{m}{2}} \, dx$$

Optimal (type 5, 440 leaves, 3 steps):

$$\frac{e (d+e x)^{1+m} \left(a+b x+c x^2\right)^{-1-\frac{m}{2}}}{\left(c d^2-b d e+a e^2\right) (1+m)}+\frac{e (2 c d-b e) m (d+e x)^{2+m} \left(a+b x+c x^2\right)^{-1-\frac{m}{2}}}{2 \left(c d^2-b d e+a e^2\right)^2 (1+m) (2+m)}-$$

$$\left(\left(b^2 e^2 m+4 c^2 d^2 (1+m)+4 c e (a e-b d (1+m))\right) \left(b-\sqrt{b^2-4 a c}+2 c x\right) \left(\frac{\left(2 c d-\left(b-\sqrt{b^2-4 a c}\right) e\right) \left(b+\sqrt{b^2-4 a c}+2 c x\right)}{\left(2 c d-\left(b+\sqrt{b^2-4 a c}\right) e\right) \left(b-\sqrt{b^2-4 a c}+2 c x\right)}\right)^{\frac{4+m}{2}}\right.$$

$$\left.(d+e x)^{3+m} \left(a+b x+c x^2\right)^{-2-\frac{m}{2}} \text{Hypergeometric2F1}\left[3+m,\frac{4+m}{2},4+m,-\frac{4 c \sqrt{b^2-4 a c} (d+e x)}{\left(2 c d-\left(b+\sqrt{b^2-4 a c}\right) e\right) \left(b-\sqrt{b^2-4 a c}+2 c x\right)}\right]\right)\Big/$$

$$\left(4 \left(2 c d-\left(b-\sqrt{b^2-4 a c}\right) e\right) \left(c d^2-b d e+a e^2\right)^2 (1+m) (3+m)\right)$$

Result (type 1, 1 leaves):

???

- **Problem 2582: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(1+x)^{1/3} \left(1-x+x^2\right)^{1/3}} \, dx$$

Optimal (type 3, 102 leaves, 2 steps):

$$\frac{\left(1+x^3\right)^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 x}{\left(1+x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (1+x)^{1/3} \left(1-x+x^2\right)^{1/3}}-\frac{\left(1+x^3\right)^{1/3} \text{Log}\left[-x+\left(1+x^3\right)^{1/3}\right]}{2 (1+x)^{1/3} \left(1-x+x^2\right)^{1/3}}$$

Result (type 6, 281 leaves):

$$\left(45\left(-i+\sqrt{3}+2 i x\right)(1+x)^{2/3}\left(-1+i\sqrt{3}+2x\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{2i(1+x)}{3i+\sqrt{3}},-\frac{2i(1+x)}{-3i+\sqrt{3}}\right]\right)\Big/\left(4\left(1-x+x^2\right)^{4/3}\left(30 i\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{2i(1+x)}{3i+\sqrt{3}},-\frac{2i(1+x)}{-3i+\sqrt{3}}\right]+\left(3i+\sqrt{3}\right)(1+x)\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{4}{3},\frac{8}{3},\frac{2i(1+x)}{3i+\sqrt{3}},-\frac{2i(1+x)}{-3i+\sqrt{3}}\right]-\left(-3i+\sqrt{3}\right)(1+x)\,\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},\frac{1}{3},\frac{8}{3},\frac{2i(1+x)}{3i+\sqrt{3}},-\frac{2i(1+x)}{-3i+\sqrt{3}}\right]\right)\right)$$

- **Problem 2583: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{1}{(1+x)^{2/3}\left(1-x+x^2\right)^{2/3}}\,dx$$

Optimal (type 5, 45 leaves, 2 steps):

$$\frac{x\left(1+x^3\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-x^3\right]}{(1+x)^{2/3}\left(1-x+x^2\right)^{2/3}}$$

Result (type 5, 148 leaves):

$$-\left(\left(i+\sqrt{3}-2 i x\right)(1+x)^{1/3}\left(-\frac{6i+\left(-3i+\sqrt{3}\right)(1+x)}{-6i+\left(3i+\sqrt{3}\right)(1+x)}\right)^{2/3}\left(-6i+\left(3i+\sqrt{3}\right)(1+x)\right)^2\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{2\sqrt{3}(1+x)}{-6i+\left(3i+\sqrt{3}\right)(1+x)}\right]\right)\Big/\left(4\left(3i+\sqrt{3}\right)\left(1-x+x^2\right)^{5/3}\right)$$

- **Problem 2584: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int(1+x)^p\left(1-x+x^2\right)^p\,dx$$

Optimal (type 5, 41 leaves, 2 steps):

$$x(1+x)^p\left(1-x+x^2\right)^p\left(1+x^3\right)^{-p}\text{Hypergeometric2F1}\left[\frac{1}{3},-p,\frac{4}{3},-x^3\right]$$

Result (type 6, 132 leaves):

$$\frac{\left(\frac{i+\sqrt{3}-2ix}{3i+\sqrt{3}}\right)^{-p}\left(\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}\right)^{-p}(1+x)^{1+p}\left(1-x+x^2\right)^p\text{AppellF1}\left[1+p,-p,-p,2+p,\frac{2i(1+x)}{3i+\sqrt{3}},-\frac{2i(1+x)}{-3i+\sqrt{3}}\right]}{1+p}$$

- **Problem 2585: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\frac{1}{(1-x)^{1/3}\left(1+x+x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 109 leaves, 2 steps):

$$-\frac{\left(1-x^3\right)^{1/3}\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,(1-x)^{1/3}\left(1+x+x^2\right)^{1/3}}+\frac{\left(1-x^3\right)^{1/3}\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]}{2\,(1-x)^{1/3}\left(1+x+x^2\right)^{1/3}}$$

Result (type 6, 280 leaves):

$$\left(45\,(1-x)^{2/3}\left(i+\sqrt{3}+2\,i\,x\right)\left(1+i\sqrt{3}+2\,x\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},-\frac{2\,i\,(-1+x)}{3\,i+\sqrt{3}},\frac{2\,i\,(-1+x)}{-3\,i+\sqrt{3}}\right]\right)\Bigg/$$
$$\left(4\left(1+x+x^2\right)^{4/3}\left(-30\,i\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},-\frac{2\,i\,(-1+x)}{3\,i+\sqrt{3}},\frac{2\,i\,(-1+x)}{-3\,i+\sqrt{3}}\right]+(-1+x)\right.\right.$$
$$\left.\left.\left(\left(3\,i+\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{4}{3},\frac{8}{3},-\frac{2\,i\,(-1+x)}{3\,i+\sqrt{3}},\frac{2\,i\,(-1+x)}{-3\,i+\sqrt{3}}\right]-\left(-3\,i+\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},\frac{1}{3},\frac{8}{3},-\frac{2\,i\,(-1+x)}{3\,i+\sqrt{3}},\frac{2\,i\,(-1+x)}{-3\,i+\sqrt{3}}\right]\right)\right)\right)$$

- **Problem 2586: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{1}{(1-x)^{2/3}\left(1+x+x^2\right)^{2/3}}\,dx$$

Optimal (type 5, 45 leaves, 2 steps):

$$\frac{x\left(1-x^3\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right]}{(1-x)^{2/3}\left(1+x+x^2\right)^{2/3}}$$

Result (type 5, 163 leaves):

$$\left((1-x)^{1/3}\left(i+\sqrt{3}+2\,i\,x\right)\left(\frac{3\,i+\sqrt{3}-\left(-3\,i+\sqrt{3}\right)x}{3\,i-\sqrt{3}+\left(3\,i+\sqrt{3}\right)x}\right)^{2/3}\left(3\,i-\sqrt{3}+\left(3\,i+\sqrt{3}\right)x\right)^2\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{2\sqrt{3}\,(1-x)}{6\,i+\left(3\,i+\sqrt{3}\right)(-1+x)}\right]\right)\Bigg/$$
$$\left(4\left(3\,i+\sqrt{3}\right)\left(1+x+x^2\right)^{5/3}\right)$$

- **Problem 2587: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int(1-x)^p\left(1+x+x^2\right)^p\,dx$$

Optimal (type 5, 41 leaves, 2 steps):

$$(1-x)^p\,x\left(1+x+x^2\right)^p\left(1-x^3\right)^{-p}\text{Hypergeometric2F1}\left[\frac{1}{3},-p,\frac{4}{3},x^3\right]$$

Result (type 6, 133 leaves):

$$\frac{1}{1+p}(1-x)^p\left(\frac{-i+\sqrt{3}-2\,i\,x}{-3\,i+\sqrt{3}}\right)^{-p}\left(\frac{i+\sqrt{3}+2\,i\,x}{3\,i+\sqrt{3}}\right)^{-p}(-1+x)\left(1+x+x^2\right)^p\text{AppellF1}\left[1+p,-p,-p,2+p,\frac{2\,i\,(-1+x)}{-3\,i+\sqrt{3}},-\frac{2\,i\,(-1+x)}{3\,i+\sqrt{3}}\right]$$

- **Problem 2588: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(b e - c e x)^{1/3} \left(b^2 + b c x + c^2 x^2\right)^{1/3}} \, dx$$

Optimal (type 3, 196 leaves, 2 steps):

$$-\frac{\left(b^3 e - c^3 e x^3\right)^{1/3} \text{ArcTan}\left[\frac{1-\frac{2 c e^{1/3} x}{\left(b^3 e-c^3 e x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c\, e^{1/3} (b e - c e x)^{1/3} \left(b^2 + b c x + c^2 x^2\right)^{1/3}} + \frac{\left(b^3 e - c^3 e x^3\right)^{1/3} \text{Log}\left[c e^{1/3} x + \left(b^3 e - c^3 e x^3\right)^{1/3}\right]}{2 c\, e^{1/3} (b e - c e x)^{1/3} \left(b^2 + b c x + c^2 x^2\right)^{1/3}}$$

Result (type 6, 241 leaves):

$$-\frac{1}{2 c e \left(b^2 + b c x + c^2 x^2\right)^{1/3}} 3 \left(e (b - c x)\right)^{2/3} \left(\frac{b c - \sqrt{3} \sqrt{-b^2 c^2} + 2 c^2 x}{3 b c - \sqrt{3} \sqrt{-b^2 c^2}}\right)^{1/3}$$

$$\left(\frac{b c + \sqrt{3} \sqrt{-b^2 c^2} + 2 c^2 x}{3 b c + \sqrt{3} \sqrt{-b^2 c^2}}\right)^{1/3} \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \frac{2 c (b - c x)}{3 b c + \sqrt{3} \sqrt{-b^2 c^2}}, \frac{2 c (b - c x)}{3 b c - \sqrt{3} \sqrt{-b^2 c^2}}\right]$$

- **Problem 2589: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(b e - c e x)^{2/3} \left(b^2 + b c x + c^2 x^2\right)^{2/3}} \, dx$$

Optimal (type 5, 71 leaves, 3 steps):

$$\frac{x \left(1 - \frac{c^3 x^3}{b^3}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{c^3 x^3}{b^3}\right]}{(b e - c e x)^{2/3} \left(b^2 + b c x + c^2 x^2\right)^{2/3}}$$

Result (type 5, 232 leaves):

$$-\frac{1}{c e \left(b^2 + b c x + c^2 x^2\right)^{2/3}} 3 \left(e (b - c x)\right)^{1/3} \left(\frac{b c + \sqrt{3} \sqrt{-b^2 c^2} + 2 c^2 x}{3 b c + \sqrt{3} \sqrt{-b^2 c^2}}\right)^{2/3}$$

$$\left(1 + \frac{2 c (-b + c x)}{3 b c - \sqrt{3} \sqrt{-b^2 c^2}}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{4 \sqrt{3}\, c \sqrt{-b^2 c^2} (-b + c x)}{\left(3 b c + \sqrt{3} \sqrt{-b^2 c^2}\right) \left(-b c + \sqrt{3} \sqrt{-b^2 c^2} - 2 c^2 x\right)}\right]$$

- **Problem 2590: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (b e - c e x)^p \left(b^2 + b c x + c^2 x^2\right)^p \, dx$$

Optimal (type 5, 67 leaves, 3 steps):

$$x\,(b\,e-c\,e\,x)^p\,\left(b^2+b\,c\,x+c^2\,x^2\right)^p\,\left(1-\frac{c^3\,x^3}{b^3}\right)^{-p}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,-p,\,\frac{4}{3},\,\frac{c^3\,x^3}{b^3}\right]$$

Result (type 6, 243 leaves) :

$$\frac{1}{c\,(1+p)}(e\,(b-c\,x))^p\,(-b+c\,x)\,\left(\frac{b\,c-\sqrt{3}\,\sqrt{-b^2\,c^2}+2\,c^2\,x}{3\,b\,c-\sqrt{3}\,\sqrt{-b^2\,c^2}}\right)^{-p}\,\left(\frac{b\,c+\sqrt{3}\,\sqrt{-b^2\,c^2}+2\,c^2\,x}{3\,b\,c+\sqrt{3}\,\sqrt{-b^2\,c^2}}\right)^{-p}$$
$$\left(b^2+b\,c\,x+c^2\,x^2\right)^p\,\text{AppellF1}\left[1+p,\,-p,\,-p,\,2+p,\,\frac{2\,c\,(b-c\,x)}{3\,b\,c+\sqrt{3}\,\sqrt{-b^2\,c^2}},\,\frac{2\,c\,(b-c\,x)}{3\,b\,c-\sqrt{3}\,\sqrt{-b^2\,c^2}}\right]$$

Test results for the 2646 problems in "1.2.1.3 (d+e x)^m (f+g x) (a+b x+c x^2)^p.m"

- **Problem 433: Result unnecessarily involves imaginary or complex numbers.**

$$\int (e\,x)^{7/2}\,(A+B\,x)\,\sqrt{a+c\,x^2}\,dx$$

Optimal (type 4, 427 leaves, 10 steps) :

$$\frac{2\,a^2\,e^3\,\sqrt{e\,x}\,(325\,A+539\,B\,x)\,\sqrt{a+c\,x^2}}{15\,015\,c^2}+\frac{28\,a^3\,B\,e^4\,x\,\sqrt{a+c\,x^2}}{195\,c^{5/2}\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}-\frac{10\,a\,A\,e^3\,\sqrt{e\,x}\,\left(a+c\,x^2\right)^{3/2}}{77\,c^2}-\frac{14\,a\,B\,e^2\,(e\,x)^{3/2}\,\left(a+c\,x^2\right)^{3/2}}{117\,c^2}+$$
$$\frac{2\,A\,e\,(e\,x)^{5/2}\,\left(a+c\,x^2\right)^{3/2}}{11\,c}+\frac{2\,B\,(e\,x)^{7/2}\,\left(a+c\,x^2\right)^{3/2}}{13\,c}-\frac{28\,a^{13/4}\,B\,e^4\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{195\,c^{11/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}+$$
$$\frac{2\,a^{11/4}\,\left(539\,\sqrt{a}\,B+325\,A\,\sqrt{c}\right)\,e^4\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{15\,015\,c^{11/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 270 leaves) :

$$\left(2 e^4 \left(\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} \left(a+c x^2\right) \left(3234 a^3 B+315 c^3 x^5 (13 A+11 B x)+10 a c^2 x^3 (117 A+77 B x)-2 a^2 c x (975 A+539 B x)\right)-\right.\right.$$

$$3234 a^{7/2} B \sqrt{c} \sqrt{1+\frac{a}{c x^2}}\, x^{3/2}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+$$

$$\left.\left. 6 a^3 \left(539 \sqrt{a}\, B+325\, i A \sqrt{c}\right) \sqrt{c} \sqrt{1+\frac{a}{c x^2}}\, x^{3/2}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right) \Big/ \left(45045 \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}\, c^3 \sqrt{e x} \sqrt{a+c x^2}\right)$$

- **Problem 434: Result unnecessarily involves imaginary or complex numbers.**

$$\int (e x)^{5/2} (A+B x) \sqrt{a+c x^2}\, dx$$

Optimal (type 4, 397 leaves, 9 steps):

$$-\frac{4 a^2 A e^3 x \sqrt{a+c x^2}}{15 c^{3/2} \sqrt{e x} \left(\sqrt{a}+\sqrt{c}\, x\right)}+\frac{2 a e^2 \sqrt{e x}\, (25 a B-77 A c x) \sqrt{a+c x^2}}{1155 c^2}-\frac{10 a B e^2 \sqrt{e x} \left(a+c x^2\right)^{3/2}}{77 c^2}+$$

$$\frac{2 A e (e x)^{3/2} \left(a+c x^2\right)^{3/2}}{9 c}+\frac{2 B (e x)^{5/2} \left(a+c x^2\right)^{3/2}}{11 c}+\frac{4 a^{9/4} A e^3 \sqrt{x} \left(\sqrt{a}+\sqrt{c}\, x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{15 c^{7/4} \sqrt{e x} \sqrt{a+c x^2}}+$$

$$\frac{2 a^{9/4} \left(25 \sqrt{a}\, B-77 A \sqrt{c}\right) e^3 \sqrt{x} \left(\sqrt{a}+\sqrt{c}\, x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{1155 c^{9/4} \sqrt{e x} \sqrt{a+c x^2}}$$

Result (type 4, 257 leaves):

$$-\left(2 e^3 \left(\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} \left(a+c x^2\right) \left(-35 c^2 x^4 (11 A+9 B x)+6 a^2 (77 A+25 B x)-2 a c x^2 (77 A+45 B x)\right)-\right.\right.$$

$$462 a^{5/2} A \sqrt{c} \sqrt{1+\frac{a}{c x^2}} x^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+$$

$$\left.\left.6 a^{5/2} \left(-25 i \sqrt{a} B+77 A \sqrt{c}\right) \sqrt{1+\frac{a}{c x^2}} x^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right) \Big/ \left(3465 \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} c^2 \sqrt{e x} \sqrt{a+c x^2}\right)$$

- **Problem 435: Result unnecessarily involves imaginary or complex numbers.**

$$\int (e x)^{3/2} (A+B x) \sqrt{a+c x^2} \, dx$$

Optimal (type 4, 363 leaves, 8 steps):

$$-\frac{2 a e \sqrt{e x} (5 A+7 B x) \sqrt{a+c x^2}}{105 c}-\frac{4 a^2 B e^2 x \sqrt{a+c x^2}}{15 c^{3/2} \sqrt{e x} \left(\sqrt{a}+\sqrt{c} x\right)}+\frac{2 A e \sqrt{e x} \left(a+c x^2\right)^{3/2}}{7 c}+$$

$$\frac{2 B (e x)^{3/2} \left(a+c x^2\right)^{3/2}}{9 c}+\frac{4 a^{9/4} B e^2 \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{15 c^{7/4} \sqrt{e x} \sqrt{a+c x^2}}-$$

$$\frac{2 a^{7/4} \left(7 \sqrt{a} B+5 A \sqrt{c}\right) e^2 \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{105 c^{7/4} \sqrt{e x} \sqrt{a+c x^2}}$$

Result (type 4, 251 leaves):

$$-\left(2 e^2 \left(\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} \left(a + c x^2\right) \left(42 a^2 B - 5 c^2 x^3 (9 A + 7 B x) - 2 a c x (15 A + 7 B x)\right) - 42 a^{5/2} B \sqrt{c} \sqrt{1 + \frac{a}{c x^2}} x^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] + 6 a^2 \left(7 \sqrt{a} B + 5 i A \sqrt{c}\right) \sqrt{c} \sqrt{1 + \frac{a}{c x^2}} x^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right)\right) \Bigg/ \left(315 \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} c^2 \sqrt{e x} \sqrt{a + c x^2}\right)$$

- **Problem 436: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{e x} (A + B x) \sqrt{a + c x^2} \, dx$$

Optimal (type 4, 328 leaves, 7 steps) :

$$\frac{4 a A e x \sqrt{a + c x^2}}{5 \sqrt{c} \sqrt{e x} \left(\sqrt{a} + \sqrt{c} x\right)} - \frac{2 \sqrt{e x} (5 a B - 21 A c x) \sqrt{a + c x^2}}{105 c} + \frac{2 B \sqrt{e x} \left(a + c x^2\right)^{3/2}}{7 c} - \frac{4 a^{5/4} A e \sqrt{x} \left(\sqrt{a} + \sqrt{c} x\right) \sqrt{\frac{a + c x^2}{\left(\sqrt{a} + \sqrt{c} x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{5 c^{3/4} \sqrt{e x} \sqrt{a + c x^2}} - \frac{2 a^{5/4} \left(5 \sqrt{a} B - 21 A \sqrt{c}\right) e \sqrt{x} \left(\sqrt{a} + \sqrt{c} x\right) \sqrt{\frac{a + c x^2}{\left(\sqrt{a} + \sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{105 c^{5/4} \sqrt{e x} \sqrt{a + c x^2}}$$

Result (type 4, 236 leaves) :

$$\left(2\,e\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}\,\left(a+c\,x^2\right)\left(3\,c\,x^2\,(7\,A+5\,B\,x)+2\,a\,(21\,A+5\,B\,x)\right)-42\,a^{3/2}\,A\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+\right.\right.$$

$$\left.\left.2\,a^{3/2}\left(-5\,i\,\sqrt{a}\,B+21\,A\,\sqrt{c}\right)\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Big/\left(105\,\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}\,c\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}\right)$$

■ Problem 437: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A+B\,x)\,\sqrt{a+c\,x^2}}{\sqrt{e\,x}}\,dx$$

Optimal (type 4, 297 leaves, 6 steps):

$$\frac{2\,\sqrt{e\,x}\,(5\,A+3\,B\,x)\,\sqrt{a+c\,x^2}}{15\,e}+\frac{4\,a\,B\,x\,\sqrt{a+c\,x^2}}{5\,\sqrt{c}\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}-\frac{4\,a^{5/4}\,B\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{5\,c^{3/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}+$$

$$\frac{2\,a^{3/4}\left(3\,\sqrt{a}\,B+5\,A\,\sqrt{c}\right)\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{15\,c^{3/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 227 leaves):

$$\left(2\,\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}\,\left(a+c\,x^2\right)\left(6\,a\,B+c\,x\,(5\,A+3\,B\,x)\right)-12\,a^{3/2}\,B\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+\right.$$

$$\left.4\,a\left(3\,\sqrt{a}\,B+5\,i\,A\,\sqrt{c}\right)\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\Big/\left(15\,\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}\,c\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}\right)$$

Problem 438: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A+B x)\sqrt{a+c x^2}}{(e x)^{3/2}}\,dx$$

Optimal (type 4, 300 leaves, 6 steps):

$$-\frac{2 (3 A-B x)\sqrt{a+c x^2}}{3 e\sqrt{e x}}+\frac{4 A\sqrt{c}\, x\sqrt{a+c x^2}}{e\sqrt{e x}\left(\sqrt{a}+\sqrt{c}\, x\right)}-\frac{4 a^{1/4} A c^{1/4}\sqrt{x}\left(\sqrt{a}+\sqrt{c}\, x\right)\sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{e\sqrt{e x}\sqrt{a+c x^2}}+$$
$$\frac{2 a^{1/4}\left(\sqrt{a}\, B+3 A\sqrt{c}\right)\sqrt{x}\left(\sqrt{a}+\sqrt{c}\, x\right)\sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{3 c^{1/4} e\sqrt{e x}\sqrt{a+c x^2}}$$

Result (type 4, 215 leaves):

$$\left(x\left(2\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}\,(3 A+B x)\left(a+c x^2\right)-12\sqrt{a}\, A\sqrt{c}\sqrt{1+\frac{a}{c x^2}}\, x^{3/2}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+\right.\right.$$
$$\left.\left.4\sqrt{a}\left(i\sqrt{a}\, B+3 A\sqrt{c}\right)\sqrt{1+\frac{a}{c x^2}}\, x^{3/2}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Big/\left(3\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}\,(e x)^{3/2}\sqrt{a+c x^2}\right)$$

Problem 439: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A+B x)\sqrt{a+c x^2}}{(e x)^{5/2}}\,dx$$

Optimal (type 4, 298 leaves, 6 steps):

$$-\frac{2\,(A+3\,B\,x)\,\sqrt{a+c\,x^2}}{3\,e\,(e\,x)^{3/2}}+\frac{4\,B\,\sqrt{c}\,x\,\sqrt{a+c\,x^2}}{e^2\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}-\frac{4\,a^{1/4}\,B\,c^{1/4}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{e^2\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}+$$

$$\frac{2\,\left(3\,\sqrt{a}\,B+A\,\sqrt{c}\right)\,c^{1/4}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{3\,a^{1/4}\,e^2\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 214 leaves) :

$$\left(x\left(-2\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,(A-3\,B\,x)\,\left(a+c\,x^2\right)-12\,\sqrt{a}\,B\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{5/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+\right.\right.$$

$$\left.\left.4\,\left(3\,\sqrt{a}\,B+i\,A\,\sqrt{c}\right)\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{5/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Bigg/\left(3\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,(e\,x)^{5/2}\,\sqrt{a+c\,x^2}\right)$$

- **Problem 440: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+B\,x)\,\sqrt{a+c\,x^2}}{(e\,x)^{7/2}}\,dx$$

Optimal (type 4, 338 leaves, 7 steps) :

$$-\frac{4\,A\,c\,\sqrt{a+c\,x^2}}{5\,a\,e^3\,\sqrt{e\,x}}-\frac{2\,(3\,A+5\,B\,x)\,\sqrt{a+c\,x^2}}{15\,e\,(e\,x)^{5/2}}+\frac{4\,A\,c^{3/2}\,x\,\sqrt{a+c\,x^2}}{5\,a\,e^3\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}-$$

$$\frac{4\,A\,c^{5/4}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{5\,a^{3/4}\,e^3\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}+$$

$$\frac{2\,\left(5\,\sqrt{a}\,B+3\,A\,\sqrt{c}\right)\,c^{3/4}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{15\,a^{3/4}\,e^3\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 217 leaves) :

$$\left(x\left(-2\,\sqrt{a}\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,(3\,A+5\,B\,x)\,\left(a+c\,x^2\right)-12\,A\,c^{3/2}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{7/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],\,-1\right]+\right.\right.$$

$$\left.\left.4\,\left(5\,i\,\sqrt{a}\,B+3\,A\,\sqrt{c}\right)\,c\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{7/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],\,-1\right]\right)\right)\bigg/\left(15\,\sqrt{a}\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,(e\,x)^{7/2}\,\sqrt{a+c\,x^2}\right)$$

- **Problem 441: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+B\,x)\,\sqrt{a+c\,x^2}}{(e\,x)^{9/2}}\,dx$$

Optimal (type 4, 368 leaves, 8 steps) :

$$-\frac{4 A c \sqrt{a+c x^2}}{21 a e^3 (e x)^{3/2}}-\frac{4 B c \sqrt{a+c x^2}}{5 a e^4 \sqrt{e x}}-\frac{2 (5 A+7 B x) \sqrt{a+c x^2}}{35 e (e x)^{7/2}}+$$

$$\frac{4 B c^{3/2} x \sqrt{a+c x^2}}{5 a e^4 \sqrt{e x} \left(\sqrt{a}+\sqrt{c} x\right)}-\frac{4 B c^{5/4} \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{5 a^{3/4} e^4 \sqrt{e x} \sqrt{a+c x^2}}+$$

$$\frac{2 \left(21 \sqrt{a} B-5 A \sqrt{c}\right) c^{5/4} \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{105 a^{5/4} e^4 \sqrt{e x} \sqrt{a+c x^2}}$$

Result (type 4, 236 leaves) :

$$-\left(2 \sqrt{e x} \left(\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} \left(a+c x^2\right) \left(10 A c x^2+3 a (5 A+7 B x)\right)+42 \sqrt{a} B c^{3/2} \sqrt{1+\frac{a}{c x^2}} x^{9/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]+\right.\right.$$

$$\left.\left. 2 i \left(21 i \sqrt{a} B+5 A \sqrt{c}\right) c^{3/2} \sqrt{1+\frac{a}{c x^2}} x^{9/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right)\right) \Big/ \left(105 a \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} e^5 x^4 \sqrt{a+c x^2}\right)$$

■ **Problem 442: Result unnecessarily involves imaginary or complex numbers.**

$$\int (e x)^{5/2} (A+B x) \left(a+c x^2\right)^{3/2} dx$$

Optimal (type 4, 438 leaves, 10 steps) :

$$-\frac{8 a^3 A e^3 x \sqrt{a+c x^2}}{65 c^{3/2} \sqrt{e x} \left(\sqrt{a}+\sqrt{c}\, x\right)}+\frac{4 a^2 e^2 \sqrt{e x}\, (65 a B-231 A c x) \sqrt{a+c x^2}}{15015 c^2}+\frac{2 a e^2 \sqrt{e x}\, (13 a B-77 A c x) \left(a+c x^2\right)^{3/2}}{3003 c^2}-\frac{2 a B e^2 \sqrt{e x} \left(a+c x^2\right)^{5/2}}{33 c^2}+$$

$$\frac{2 A e (e x)^{3/2} \left(a+c x^2\right)^{5/2}}{13 c}+\frac{2 B (e x)^{5/2} \left(a+c x^2\right)^{5/2}}{15 c}+\frac{8 a^{13/4} A e^3 \sqrt{x} \left(\sqrt{a}+\sqrt{c}\, x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\ \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{65 c^{7/4} \sqrt{e x} \sqrt{a+c x^2}}+$$

$$\frac{4 a^{13/4} \left(65 \sqrt{a}\, B-231 A \sqrt{c}\right) e^3 \sqrt{x} \left(\sqrt{a}+\sqrt{c}\, x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{15015 c^{9/4} \sqrt{e x} \sqrt{a+c x^2}}$$

Result (type 4, 276 leaves) :

$$-\left(2 e^3 \left(\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} \left(a+c x^2\right) \left(-77 c^3 x^6 (15 A+13 B x)-4 a^2 c x^2 (77 A+39 B x)+4 a^3 (231 A+65 B x)-7 a c^2 x^4 (275 A+221 B x)\right)-\right.\right.$$

$$924 a^{7/2} A \sqrt{c} \sqrt{1+\frac{a}{c x^2}}\ x^{3/2}\ \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]+$$

$$\left.\left.4 a^{7/2} \left(-65 i \sqrt{a}\, B+231 A \sqrt{c}\right) \sqrt{1+\frac{a}{c x^2}}\ x^{3/2}\ \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right)\right) \Big/ \left(15015 \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}\ c^2 \sqrt{e x} \sqrt{a+c x^2}\right)$$

- **Problem 443: Result unnecessarily involves imaginary or complex numbers.**

$$\int (e x)^{3/2} (A+B x) \left(a+c x^2\right)^{3/2} dx$$

Optimal (type 4, 400 leaves, 9 steps) :

$$-\frac{4 a^2 e \sqrt{e x}\, (65 A+77 B x) \sqrt{a+c x^2}}{5005 c}-\frac{8 a^3 B e^2 x \sqrt{a+c x^2}}{65 c^{3/2} \sqrt{e x} \left(\sqrt{a}+\sqrt{c}\, x\right)}-\frac{2 a e \sqrt{e x}\, (39 A+77 B x) \left(a+c x^2\right)^{3/2}}{3003 c}+$$

$$\frac{2 A e \sqrt{e x} \left(a+c x^2\right)^{5/2}}{11 c}+\frac{2 B (e x)^{3/2} \left(a+c x^2\right)^{5/2}}{13 c}+\frac{8 a^{13/4} B e^2 \sqrt{x} \left(\sqrt{a}+\sqrt{c}\, x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{65 c^{7/4} \sqrt{e x} \sqrt{a+c x^2}}-$$

$$\frac{4 a^{11/4} \left(77 \sqrt{a}\, B+65 A \sqrt{c}\right) e^2 \sqrt{x} \left(\sqrt{a}+\sqrt{c}\, x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{5005 c^{7/4} \sqrt{e x} \sqrt{a+c x^2}}$$

Result (type 4, 270 leaves) :

$$-\left(2 e^2 \left(\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} \left(a+c x^2\right) \left(924 a^3 B-105 c^3 x^5 (13 A+11 B x)-4 a^2 c x (195 A+77 B x)-5 a c^2 x^3 (507 A+385 B x)\right)-\right.\right.$$

$$924 a^{7/2} B \sqrt{c} \sqrt{1+\frac{a}{c x^2}}\, x^{3/2}\, \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]+$$

$$\left.\left.12 a^3 \left(77 \sqrt{a}\, B+65 i A \sqrt{c}\right) \sqrt{c} \sqrt{1+\frac{a}{c x^2}}\, x^{3/2}\, \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right)\right) \Bigg/ \left(15015 \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}\, c^2 \sqrt{e x} \sqrt{a+c x^2}\right)$$

- **Problem 444: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{e x}\, (A+B x) \left(a+c x^2\right)^{3/2} dx$$

Optimal (type 4, 366 leaves, 8 steps) :

$$\frac{8 a^2 A e x \sqrt{a+c x^2}}{15 \sqrt{c} \sqrt{e x} \left(\sqrt{a}+\sqrt{c} x\right)}-\frac{4 a \sqrt{e x} (15 a B-77 A c x) \sqrt{a+c x^2}}{1155 c}-\frac{2 \sqrt{e x} (9 a B-77 A c x) \left(a+c x^2\right)^{3/2}}{693 c}+$$

$$\frac{2 B \sqrt{e x} \left(a+c x^2\right)^{5/2}}{11 c}-\frac{8 a^{9/4} A e \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{15 c^{3/4} \sqrt{e x} \sqrt{a+c x^2}}-$$

$$\frac{4 a^{9/4} \left(15 \sqrt{a} B-77 A \sqrt{c}\right) e \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{1155 c^{5/4} \sqrt{e x} \sqrt{a+c x^2}}$$

Result (type 4, 254 leaves) :

$$\left(2 e \left(\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} \left(a+c x^2\right) \left(35 c^2 x^4 (11 A+9 B x)+12 a^2 (77 A+15 B x)+a c x^2 (847 A+585 B x)\right)-\right.\right.$$

$$924 a^{5/2} A \sqrt{c} \sqrt{1+\frac{a}{c x^2}} x^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]+$$

$$\left.\left.12 a^{5/2} \left(-15 i \sqrt{a} B+77 A \sqrt{c}\right) \sqrt{1+\frac{a}{c x^2}} x^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right)\right) \Big/ \left(3465 \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} c \sqrt{e x} \sqrt{a+c x^2}\right)$$

- **Problem 445: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B x) \left(a+c x^2\right)^{3/2}}{\sqrt{e x}} \, dx$$

Optimal (type 4, 333 leaves, 7 steps) :

$$\frac{4\,a\,\sqrt{e\,x}\,(15\,A+7\,B\,x)\,\sqrt{a+c\,x^2}}{105\,e}+\frac{8\,a^2\,B\,x\,\sqrt{a+c\,x^2}}{15\,\sqrt{c}\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}+$$

$$\frac{2\,\sqrt{e\,x}\,(9\,A+7\,B\,x)\,\left(a+c\,x^2\right)^{3/2}}{63\,e}-\frac{8\,a^{9/4}\,B\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{15\,c^{3/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}+$$

$$\frac{4\,a^{7/4}\,\left(7\,\sqrt{a}\,B+15\,A\,\sqrt{c}\right)\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{105\,c^{3/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 248 leaves) :

$$\left(2\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,\left(a+c\,x^2\right)\,\left(84\,a^2\,B+5\,c^2\,x^3\,(9\,A+7\,B\,x)+a\,c\,x\,(135\,A+77\,B\,x)\right)-168\,a^{5/2}\,B\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],\,-1\right]+\right.$$

$$\left.24\,a^2\,\left(7\,\sqrt{a}\,B+15\,i\,A\,\sqrt{c}\right)\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],\,-1\right]\right)\Big/\left(315\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,c\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}\right)$$

- **Problem 446: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+B\,x)\,\left(a+c\,x^2\right)^{3/2}}{(e\,x)^{3/2}}\,dx$$

Optimal (type 4, 341 leaves, 7 steps) :

$$\frac{24\,a\,A\,\sqrt{c}\,x\,\sqrt{a+c\,x^2}}{5\,e\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}+\frac{4\,\sqrt{e\,x}\,\left(5\,a\,B+21\,A\,c\,x\right)\,\sqrt{a+c\,x^2}}{35\,e^2}-\frac{2\,\left(7\,A-B\,x\right)\,\left(a+c\,x^2\right)^{3/2}}{7\,e\,\sqrt{e\,x}}-$$

$$\frac{24\,a^{5/4}\,A\,c^{1/4}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{5\,e\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}+$$

$$\frac{4\,a^{5/4}\,\left(5\,\sqrt{a}\,B+21\,A\,\sqrt{c}\right)\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{35\,c^{1/4}\,e\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 232 leaves) :

$$\left(x\left(2\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,\left(a+c\,x^2\right)\,\left(c\,x^2\,\left(7\,A+5\,B\,x\right)+a\,\left(49\,A+15\,B\,x\right)\right)-168\,a^{3/2}\,A\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+\right.\right.$$

$$\left.\left.8\,a^{3/2}\,\left(5\,i\,\sqrt{a}\,B+21\,A\,\sqrt{c}\right)\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Bigg/\left(35\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,\left(e\,x\right)^{3/2}\,\sqrt{a+c\,x^2}\right)$$

- **Problem 447: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(A+B\,x\right)\,\left(a+c\,x^2\right)^{3/2}}{\left(e\,x\right)^{5/2}}\,dx$$

Optimal (type 4, 341 leaves, 7 steps) :

$$\frac{24\,a\,B\,\sqrt{c}\,x\,\sqrt{a+c\,x^2}}{5\,e^2\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}-\frac{4\,(9\,a\,B-5\,A\,c\,x)\,\sqrt{a+c\,x^2}}{15\,e^2\,\sqrt{e\,x}}-\frac{2\,(5\,A-3\,B\,x)\,\left(a+c\,x^2\right)^{3/2}}{15\,e\,(e\,x)^{3/2}}-$$

$$\frac{24\,a^{5/4}\,B\,c^{1/4}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{5\,e^2\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}+$$

$$\frac{4\,a^{3/4}\,\left(9\,\sqrt{a}\,B+5\,A\,\sqrt{c}\right)\,c^{1/4}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{15\,e^2\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 233 leaves) :

$$\left(x\left(2\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,\left(a+c\,x^2\right)\,\left(-5\,a\,A+21\,a\,B\,x+5\,A\,c\,x^2+3\,B\,c\,x^3\right)-72\,a^{3/2}\,B\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{5/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+\right.\right.$$

$$\left.\left.8\,a\,\left(9\,\sqrt{a}\,B+5\,i\,A\,\sqrt{c}\right)\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{5/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Big/\left(15\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,(e\,x)^{5/2}\,\sqrt{a+c\,x^2}\right)$$

- **Problem 448: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+B\,x)\,\left(a+c\,x^2\right)^{3/2}}{(e\,x)^{7/2}}\,dx$$

Optimal (type 4, 339 leaves, 7 steps) :

$$-\frac{4 c (9 A-5 B x) \sqrt{a+c x^2}}{15 e^3 \sqrt{e x}}+\frac{24 A c^{3/2} x \sqrt{a+c x^2}}{5 e^3 \sqrt{e x} \left(\sqrt{a}+\sqrt{c} x\right)}-\frac{2 (3 A+5 B x) \left(a+c x^2\right)^{3/2}}{15 e (e x)^{5/2}}-$$

$$\frac{24 a^{1/4} A c^{5/4} \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{5 e^3 \sqrt{e x} \sqrt{a+c x^2}}+$$

$$\frac{4 a^{1/4} \left(5 \sqrt{a} B+9 A \sqrt{c}\right) c^{3/4} \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{15 e^3 \sqrt{e x} \sqrt{a+c x^2}}$$

Result (type 4, 233 leaves) :

$$\left(x \left(-2 \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} \left(a+c x^2\right) \left(-5 c x^2 (3 A+B x)+a (3 A+5 B x)\right)-72 \sqrt{a} A c^{3/2} \sqrt{1+\frac{a}{c x^2}} x^{7/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]+\right.\right.$$

$$\left.\left.8 \sqrt{a} \left(5 i \sqrt{a} B+9 A \sqrt{c}\right) c \sqrt{1+\frac{a}{c x^2}} x^{7/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right)\right) \Big/ \left(15 \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} (e x)^{7/2} \sqrt{a+c x^2}\right)$$

Problem 449: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A+B x) \left(a+c x^2\right)^{3/2}}{(e x)^{9/2}} \, dx$$

Optimal (type 4, 339 leaves, 7 steps) :

$$-\frac{4\,c\,(5\,A+21\,B\,x)\,\sqrt{a+c\,x^2}}{35\,e^3\,(e\,x)^{3/2}}+\frac{24\,B\,c^{3/2}\,x\,\sqrt{a+c\,x^2}}{5\,e^4\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}-\frac{2\,(5\,A+7\,B\,x)\,\left(a+c\,x^2\right)^{3/2}}{35\,e\,(e\,x)^{7/2}}-$$

$$\frac{24\,a^{1/4}\,B\,c^{5/4}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{5\,e^4\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}+$$

$$\frac{4\,\left(21\,\sqrt{a}\,B+5\,A\,\sqrt{c}\right)\,c^{5/4}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{35\,a^{1/4}\,e^4\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 238 leaves) :

$$\left(2\,\sqrt{e\,x}\,\left(-\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,\left(a+c\,x^2\right)\,\left(5\,c\,x^2\,(3\,A-7\,B\,x)+a\,(5\,A+7\,B\,x)\right)-84\,\sqrt{a}\,B\,c^{3/2}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{9/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],\,-1\right]+\right.\right.$$

$$\left.\left.4\,\left(21\,\sqrt{a}\,B+5\,i\,A\,\sqrt{c}\right)\,c^{3/2}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{9/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],\,-1\right]\right)\right)\Big/\left(35\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,e^5\,x^4\,\sqrt{a+c\,x^2}\right)$$

- **Problem 450: Result unnecessarily involves imaginary or complex numbers.**

$$\int (e\,x)^{3/2}\,(A+B\,x)\,\left(a+c\,x^2\right)^{5/2}\,dx$$

Optimal (type 4, 437 leaves, 10 steps) :

$$-\frac{8 a^3 e \sqrt{e x}\,(221 A+231 B x)\sqrt{a+c x^2}}{51051 c}-\frac{16 a^4 B e^2 x \sqrt{a+c x^2}}{221 c^{3/2}\sqrt{e x}\left(\sqrt{a}+\sqrt{c}\,x\right)}-$$

$$\frac{4 a^2 e \sqrt{e x}\,(221 A+385 B x)\left(a+c x^2\right)^{3/2}}{51051 c}-\frac{2 a e \sqrt{e x}\,(221 A+495 B x)\left(a+c x^2\right)^{5/2}}{36465 c}+\frac{2 A e \sqrt{e x}\left(a+c x^2\right)^{7/2}}{15 c}+$$

$$\frac{2 B (e x)^{3/2}\left(a+c x^2\right)^{7/2}}{17 c}+\frac{16 a^{17/4} B e^2 \sqrt{x}\left(\sqrt{a}+\sqrt{c}\,x\right)\sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{221 c^{7/4}\sqrt{e x}\sqrt{a+c x^2}}-$$

$$\frac{8 a^{15/4}\left(231\sqrt{a}\,B+221 A\sqrt{c}\right) e^2 \sqrt{x}\left(\sqrt{a}+\sqrt{c}\,x\right)\sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{51051 c^{7/4}\sqrt{e x}\sqrt{a+c x^2}}$$

Result (type 4, 289 leaves) :

$$-\frac{1}{255255\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}\,c^2\sqrt{e x}\sqrt{a+c x^2}}$$

$$2 e^2\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}\left(a+c x^2\right)\left(9240 a^4 B-1001 c^4 x^7 (17 A+15 B x)-40 a^3 c x (221 A+77 B x)-28 a c^3 x^5 (1768 A+1485 B x)-\right.\right.$$

$$\left.a^2 c^2 x^3 (45747 A+34265 B x)\right)-9240 a^{9/2} B\sqrt{c}\sqrt{1+\frac{a}{c x^2}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+$$

$$\left.40 a^4\left(231\sqrt{a}\,B+221\,i\,A\sqrt{c}\right)\sqrt{c}\sqrt{1+\frac{a}{c x^2}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)$$

- **Problem 451: Result unnecessarily involves imaginary or complex numbers.**

$$\int\sqrt{e x}\,(A+B x)\left(a+c x^2\right)^{5/2}\,dx$$

Optimal (type 4, 404 leaves, 9 steps) :

$$\frac{16 a^3 A e x \sqrt{a+c x^2}}{39 \sqrt{c} \sqrt{e x} \left(\sqrt{a}+\sqrt{c} x\right)}-\frac{8 a^2 \sqrt{e x} (13 a B-77 A c x) \sqrt{a+c x^2}}{3003 c}-\frac{4 a \sqrt{e x} (39 a B-385 A c x) \left(a+c x^2\right)^{3/2}}{9009 c}-$$

$$\frac{2 \sqrt{e x} (13 a B-165 A c x) \left(a+c x^2\right)^{5/2}}{2145 c}+\frac{2 B \sqrt{e x} \left(a+c x^2\right)^{7/2}}{15 c}-\frac{16 a^{13/4} A e \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{39 c^{3/4} \sqrt{e x} \sqrt{a+c x^2}}-$$

$$\frac{8 a^{13/4} \left(13 \sqrt{a} B-77 A \sqrt{c}\right) e \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{3003 c^{5/4} \sqrt{e x} \sqrt{a+c x^2}}$$

Result (type 4, 273 leaves) :

$$\left(2 e \left(\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} \left(a+c x^2\right) \left(231 c^3 x^6 (15 A+13 B x)+120 a^3 (77 A+13 B x)+28 a c^2 x^4 (385 A+312 B x)+a^2 c x^2 (11935 A+8073 B x)\right)-\right.\right.$$

$$9240 a^{7/2} A \sqrt{c} \sqrt{1+\frac{a}{c x^2}} x^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]+$$

$$\left.\left.120 a^{7/2} \left(-13 i \sqrt{a} B+77 A \sqrt{c}\right) \sqrt{1+\frac{a}{c x^2}} x^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right)\right) \Bigg/ \left(45045 \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} c \sqrt{e x} \sqrt{a+c x^2}\right)$$

- **Problem 452: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B x) \left(a+c x^2\right)^{5/2}}{\sqrt{e x}} \, dx$$

Optimal (type 4, 369 leaves, 8 steps) :

$$\frac{8 a^2 \sqrt{e x}\ (195 A + 77 B x) \sqrt{a + c x^2}}{3003 e} + \frac{16 a^3 B x \sqrt{a + c x^2}}{39 \sqrt{c} \sqrt{e x} \left(\sqrt{a} + \sqrt{c}\ x\right)} + \frac{20 a \sqrt{e x}\ (117 A + 77 B x) \left(a + c x^2\right)^{3/2}}{9009 e} +$$

$$\frac{2 \sqrt{e x}\ (13 A + 11 B x) \left(a + c x^2\right)^{5/2}}{143 e} - \frac{16 a^{13/4} B \sqrt{x} \left(\sqrt{a} + \sqrt{c}\ x\right) \sqrt{\frac{a + c x^2}{\left(\sqrt{a} + \sqrt{c}\ x\right)^2}}\ \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{39 c^{3/4} \sqrt{e x} \sqrt{a + c x^2}} +$$

$$\frac{8 a^{11/4} \left(77 \sqrt{a}\ B + 195 A \sqrt{c}\right) \sqrt{x} \left(\sqrt{a} + \sqrt{c}\ x\right) \sqrt{\frac{a + c x^2}{\left(\sqrt{a} + \sqrt{c}\ x\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{3003 c^{3/4} \sqrt{e x} \sqrt{a + c x^2}}$$

Result (type 4, 267 leaves) :

$$\left(2 \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} \left(a + c x^2\right) \left(1848 a^3 B + 63 c^3 x^5 (13 A + 11 B x) + 4 a c^2 x^3 (702 A + 539 B x) + a^2 c x (4329 A + 2387 B x)\right) - \right.$$

$$3696 a^{7/2} B \sqrt{c} \sqrt{1 + \frac{a}{c x^2}}\ x^{3/2}\ \text{EllipticE}\left[i\ \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] +$$

$$\left. 48 a^3 \left(77 \sqrt{a}\ B + 195 i A \sqrt{c}\right) \sqrt{c} \sqrt{1 + \frac{a}{c x^2}}\ x^{3/2}\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right) \Bigg/ \left(9009 \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}\ c \sqrt{e x} \sqrt{a + c x^2}\right)$$

- **Problem 453: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B x) \left(a + c x^2\right)^{5/2}}{(e x)^{3/2}}\, dx$$

Optimal (type 4, 379 leaves, 8 steps) :

$$\frac{16 a^2 A \sqrt{c}\, x \sqrt{a+c x^2}}{3 e \sqrt{e x} \left(\sqrt{a}+\sqrt{c}\, x\right)} + \frac{8 a \sqrt{e x}\, (15 a B + 77 A c x) \sqrt{a+c x^2}}{231 e^2} + \frac{20 \sqrt{e x}\, (9 a B + 77 A c x) \left(a+c x^2\right)^{3/2}}{693 e^2} -$$

$$\frac{2 (11 A - B x) \left(a+c x^2\right)^{5/2}}{11 e \sqrt{e x}} - \frac{16 a^{9/4} A c^{1/4} \sqrt{x} \left(\sqrt{a}+\sqrt{c}\, x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{3 e \sqrt{e x} \sqrt{a+c x^2}} +$$

$$\frac{8 a^{9/4} \left(15 \sqrt{a}\, B + 77 A \sqrt{c}\right) \sqrt{x} \left(\sqrt{a}+\sqrt{c}\, x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{231 c^{1/4} e \sqrt{e x} \sqrt{a+c x^2}}$$

Result (type 4, 253 leaves) :

$$\left(x \left(2 \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} \left(a+c x^2\right) \left(7 c^2 x^4 (11 A + 9 B x) + 4 a c x^2 (77 A + 54 B x) + 3 a^2 (385 A + 111 B x)\right) - \right.\right.$$

$$3696 a^{5/2} A \sqrt{c} \sqrt{1+\frac{a}{c x^2}}\, x^{3/2}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] +$$

$$\left.\left. 48 a^{5/2} \left(15 i \sqrt{a}\, B + 77 A \sqrt{c}\right) \sqrt{1+\frac{a}{c x^2}}\, x^{3/2}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right)\right) \Big/ \left(693 \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}\, (e x)^{3/2} \sqrt{a+c x^2}\right)$$

■ Problem 454: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A + B x) \left(a+c x^2\right)^{5/2}}{(e x)^{5/2}}\, dx$$

Optimal (type 4, 378 leaves, 8 steps) :

$$\frac{8\,a\,c\,\sqrt{e\,x}\,(5\,A+7\,B\,x)\,\sqrt{a+c\,x^2}}{21\,e^3}+\frac{16\,a^2\,B\,\sqrt{c}\,x\,\sqrt{a+c\,x^2}}{3\,e^2\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}-\frac{20\,(7\,a\,B-3\,A\,c\,x)\,\left(a+c\,x^2\right)^{3/2}}{63\,e^2\,\sqrt{e\,x}}-$$

$$\frac{2\,(3\,A-B\,x)\,\left(a+c\,x^2\right)^{5/2}}{9\,e\,(e\,x)^{3/2}}-\frac{16\,a^{9/4}\,B\,c^{1/4}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{3\,e^2\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}+$$

$$\frac{8\,a^{7/4}\,\left(7\,\sqrt{a}\,B+5\,A\,\sqrt{c}\right)\,c^{1/4}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{21\,e^2\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 253 leaves) :

$$\left(x\left(2\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,\left(a+c\,x^2\right)\,\left(-21\,a^2\,(A-5\,B\,x)+c^2\,x^4\,(9\,A+7\,B\,x)+4\,a\,c\,x^2\,(12\,A+7\,B\,x)\right)-\right.\right.$$

$$336\,a^{5/2}\,B\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{5/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],\,-1\right]+$$

$$\left.\left.48\,a^2\,\left(7\,\sqrt{a}\,B+5\,i\,A\,\sqrt{c}\right)\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{5/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],\,-1\right]\right)\right)\Bigg/\left(63\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,(e\,x)^{5/2}\,\sqrt{a+c\,x^2}\right)$$

- **Problem 455: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+B\,x)\,\left(a+c\,x^2\right)^{5/2}}{(e\,x)^{7/2}}\,dx$$

Optimal (type 4, 376 leaves, 8 steps) :

$$-\frac{8\,a\,c\,(63\,A-25\,B\,x)\,\sqrt{a+c\,x^2}}{105\,e^3\,\sqrt{e\,x}}+\frac{48\,a\,A\,c^{3/2}\,x\,\sqrt{a+c\,x^2}}{5\,e^3\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}-\frac{4\,(25\,a\,B-21\,A\,c\,x)\,\left(a+c\,x^2\right)^{3/2}}{105\,e^2\,(e\,x)^{3/2}}-$$

$$\frac{2\,(7\,A-5\,B\,x)\,\left(a+c\,x^2\right)^{5/2}}{35\,e\,(e\,x)^{5/2}}-\frac{48\,a^{5/4}\,A\,c^{5/4}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{5\,e^3\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}+$$

$$\frac{8\,a^{5/4}\,\left(25\,\sqrt{a}\,B+63\,A\,\sqrt{c}\right)\,c^{3/4}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{105\,e^3\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 254 leaves) :

$$\left(x\left(-2\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,\left(a+c\,x^2\right)\,\left(7\,a^2\,(3\,A+5\,B\,x)-3\,c^2\,x^4\,(7\,A+5\,B\,x)-4\,a\,c\,x^2\,(63\,A+20\,B\,x)\right)-\right.\right.$$

$$1008\,a^{3/2}\,A\,c^{3/2}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{7/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],\,-1\right]+$$

$$\left.\left.16\,a^{3/2}\,\left(25\,i\,\sqrt{a}\,B+63\,A\,\sqrt{c}\right)\,c\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{7/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],\,-1\right]\right)\right)\Big/\left(105\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,(e\,x)^{7/2}\,\sqrt{a+c\,x^2}\right)$$

- **Problem 456: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+B\,x)\,\left(a+c\,x^2\right)^{5/2}}{(e\,x)^{9/2}}\,dx$$

Optimal (type 4, 377 leaves, 8 steps) :

$$\frac{48\,a\,B\,c^{3/2}\,x\,\sqrt{a+c\,x^2}}{5\,e^4\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}-\frac{8\,c\,(63\,a\,B-25\,A\,c\,x)\,\sqrt{a+c\,x^2}}{105\,e^4\,\sqrt{e\,x}}-\frac{4\,(21\,a\,B+25\,A\,c\,x)\,\left(a+c\,x^2\right)^{3/2}}{105\,e^2\,(e\,x)^{5/2}}-$$

$$\frac{2\,(5\,A-7\,B\,x)\,\left(a+c\,x^2\right)^{5/2}}{35\,e\,(e\,x)^{7/2}}-\frac{48\,a^{5/4}\,B\,c^{5/4}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{5\,e^4\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}+$$

$$\frac{8\,a^{3/4}\,\left(63\,\sqrt{a}\,B+25\,A\,\sqrt{c}\right)\,c^{5/4}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{105\,e^4\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 259 leaves) :

$$\left(2\,\sqrt{e\,x}\,\left(-\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,\left(a+c\,x^2\right)\,\left(4\,a\,c\,x^2\,(20\,A-63\,B\,x)-7\,c^2\,x^4\,(5\,A+3\,B\,x)+3\,a^2\,(5\,A+7\,B\,x)\right)-\right.\right.$$

$$504\,a^{3/2}\,B\,c^{3/2}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{9/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],\,-1\right]+$$

$$\left.\left.8\,a\,\left(63\,\sqrt{a}\,B+25\,i\,A\,\sqrt{c}\right)\,c^{3/2}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{9/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],\,-1\right]\right)\right)\Big/\left(105\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,e^5\,x^4\,\sqrt{a+c\,x^2}\right)$$

- **Problem 457: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+B\,x)\,\left(a+c\,x^2\right)^{5/2}}{(e\,x)^{11/2}}\,dx$$

Optimal (type 4, 375 leaves, 8 steps) :

$$-\frac{8 c^2 (7 A-5 B x) \sqrt{a+c x^2}}{21 e^5 \sqrt{e x}}+\frac{16 A c^{5/2} x \sqrt{a+c x^2}}{3 e^5 \sqrt{e x} \left(\sqrt{a}+\sqrt{c} x\right)}-\frac{4 c (7 A+15 B x) \left(a+c x^2\right)^{3/2}}{63 e^3 (e x)^{5/2}}-$$

$$\frac{2 (7 A+9 B x) \left(a+c x^2\right)^{5/2}}{63 e (e x)^{9/2}}-\frac{16 a^{1/4} A c^{9/4} \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{3 e^5 \sqrt{e x} \sqrt{a+c x^2}}+$$

$$\frac{8 a^{1/4} \left(5 \sqrt{a} B+7 A \sqrt{c}\right) c^{7/4} \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{21 e^5 \sqrt{e x} \sqrt{a+c x^2}}$$

Result (type 4, 259 leaves) :

$$\left(\sqrt{e x} \left(-2 \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} \left(a+c x^2\right) \left(-21 c^2 x^4 (3 A+B x)+a^2 (7 A+9 B x)+4 a c x^2 (7 A+12 B x)\right)-\right.\right.$$

$$336 \sqrt{a} A c^{5/2} \sqrt{1+\frac{a}{c x^2}} x^{11/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]+$$

$$\left.\left.48 \sqrt{a} \left(5 i \sqrt{a} B+7 A \sqrt{c}\right) c^2 \sqrt{1+\frac{a}{c x^2}} x^{11/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right)\right) \Big/ \left(63 \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} e^6 x^5 \sqrt{a+c x^2}\right)$$

- **Problem 458: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(e x)^{7/2} (A+B x)}{\sqrt{a+c x^2}} \, dx$$

Optimal (type 4, 388 leaves, 9 steps) :

$$-\frac{10\,a\,A\,e^3\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}{21\,c^2}-\frac{14\,a\,B\,e^2\,(e\,x)^{3/2}\,\sqrt{a+c\,x^2}}{45\,c^2}+\frac{2\,A\,e\,(e\,x)^{5/2}\,\sqrt{a+c\,x^2}}{7\,c}+\frac{2\,B\,(e\,x)^{7/2}\,\sqrt{a+c\,x^2}}{9\,c}+$$

$$\frac{14\,a^2\,B\,e^4\,x\,\sqrt{a+c\,x^2}}{15\,c^{5/2}\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}-\frac{14\,a^{9/4}\,B\,e^4\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{15\,c^{11/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}+$$

$$\frac{a^{7/4}\,\left(49\,\sqrt{a}\,B+25\,A\,\sqrt{c}\right)\,e^4\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{105\,c^{11/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 251 leaves) :

$$\left(2\,e^4\left(\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,\left(a+c\,x^2\right)\,\left(147\,a^2\,B+5\,c^2\,x^3\,(9\,A+7\,B\,x)-a\,c\,x\,(75\,A+49\,B\,x)\right)-147\,a^{5/2}\,B\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+\right.\right.$$

$$\left.\left.3\,a^2\,\left(49\,\sqrt{a}\,B+25\,i\,A\,\sqrt{c}\right)\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Big/\left(315\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,c^3\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}\right)$$

- **Problem 459: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(e\,x)^{5/2}\,(A+B\,x)}{\sqrt{a+c\,x^2}}\,dx$$

Optimal (type 4, 356 leaves, 8 steps) :

$$-\frac{10\,a\,B\,e^2\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}{21\,c^2}+\frac{2\,A\,e\,(e\,x)^{3/2}\,\sqrt{a+c\,x^2}}{5\,c}+\frac{2\,B\,(e\,x)^{5/2}\,\sqrt{a+c\,x^2}}{7\,c}-$$

$$\frac{6\,a\,A\,e^3\,x\,\sqrt{a+c\,x^2}}{5\,c^{3/2}\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}+\frac{6\,a^{5/4}\,A\,e^3\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{5\,c^{7/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}+$$

$$\frac{a^{5/4}\,\left(25\,\sqrt{a}\,B-63\,A\,\sqrt{c}\right)\,e^3\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{105\,c^{9/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 236 leaves) :

$$-\left(2\,e^3\,\left(\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,\left(a+c\,x^2\right)\,\left(-3\,c\,x^2\,(7\,A+5\,B\,x)+a\,(63\,A+25\,B\,x)\right)-63\,a^{3/2}\,A\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+\right.\right.$$

$$\left.\left.a^{3/2}\,\left(-25\,i\,\sqrt{a}\,B+63\,A\,\sqrt{c}\right)\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Big/\left(105\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,c^2\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}\right)$$

■ Problem 460: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(e\,x)^{3/2}\,(A+B\,x)}{\sqrt{a+c\,x^2}}\,dx$$

Optimal (type 4, 326 leaves, 7 steps) :

$$\frac{2 A e \sqrt{e x} \sqrt{a+c x^2}}{3 c}+\frac{2 B (e x)^{3/2} \sqrt{a+c x^2}}{5 c}-\frac{6 a B e^2 x \sqrt{a+c x^2}}{5 c^{3/2} \sqrt{e x} \left(\sqrt{a}+\sqrt{c} x\right)}+$$

$$\frac{6 a^{5/4} B e^2 \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{5 c^{7/4} \sqrt{e x} \sqrt{a+c x^2}}-$$

$$\frac{a^{3/4} \left(9 \sqrt{a} B+5 A \sqrt{c}\right) e^2 \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{15 c^{7/4} \sqrt{e x} \sqrt{a+c x^2}}$$

Result (type 4, 229 leaves) :

$$-\left(2 e^2 \left(\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} \left(a+c x^2\right) (9 a B-c x (5 A+3 B x))-9 a^{3/2} B \sqrt{c} \sqrt{1+\frac{a}{c x^2}} x^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]+\right.\right.$$

$$\left.\left. a \left(9 \sqrt{a} B+5 i A \sqrt{c}\right) \sqrt{c} \sqrt{1+\frac{a}{c x^2}} x^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right)\right) \Big/ \left(15 \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} c^2 \sqrt{e x} \sqrt{a+c x^2}\right)$$

Problem 461: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{e x} (A+B x)}{\sqrt{a+c x^2}} \, dx$$

Optimal (type 4, 287 leaves, 6 steps) :

$$\frac{2 B \sqrt{e x} \sqrt{a+c x^2}}{3 c}+\frac{2 A e x \sqrt{a+c x^2}}{\sqrt{c} \sqrt{e x} \left(\sqrt{a}+\sqrt{c} x\right)}-\frac{2 a^{1/4} A e \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{c^{3/4} \sqrt{e x} \sqrt{a+c x^2}}-$$

$$\frac{a^{1/4} \left(\sqrt{a} B-3 A \sqrt{c}\right) e \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{3 c^{5/4} \sqrt{e x} \sqrt{a+c x^2}}$$

Result (type 4, 216 leaves) :

$$\left(2 e \left(\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} (3 A+B x) \left(a+c x^2\right)-3 \sqrt{a} A \sqrt{c} \sqrt{1+\frac{a}{c x^2}} x^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]+\right.\right.$$

$$\left.\left.\sqrt{a} \left(-i \sqrt{a} B+3 A \sqrt{c}\right) \sqrt{1+\frac{a}{c x^2}} x^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right)\right) \Big/ \left(3 \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} c \sqrt{e x} \sqrt{a+c x^2}\right)$$

- **Problem 462: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+B x}{\sqrt{e x} \sqrt{a+c x^2}} \, dx$$

Optimal (type 4, 253 leaves, 5 steps) :

$$\frac{2 B x \sqrt{a+c x^2}}{\sqrt{c} \sqrt{e x} \left(\sqrt{a}+\sqrt{c} x\right)}-\frac{2 a^{1/4} B \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{c^{3/4} \sqrt{e x} \sqrt{a+c x^2}}+$$

$$\frac{a^{1/4} \left(B+\frac{A \sqrt{c}}{\sqrt{a}}\right) \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{c^{3/4} \sqrt{e x} \sqrt{a+c x^2}}$$

Result (type 4, 207 leaves) :

$$\left(2 B \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} \left(a + c x^2\right) - 2 \sqrt{a} B \sqrt{c} \sqrt{1 + \frac{a}{c x^2}} x^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] + 2 \left(\sqrt{a} B + i A \sqrt{c}\right) \sqrt{c} \sqrt{1 + \frac{a}{c x^2}} x^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right) \Big/ \left(\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} c \sqrt{e x} \sqrt{a + c x^2}\right)$$

- **Problem 463: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B x}{(e x)^{3/2} \sqrt{a + c x^2}} \, dx$$

Optimal (type 4, 293 leaves, 6 steps):

$$-\frac{2 A \sqrt{a + c x^2}}{a e \sqrt{e x}} + \frac{2 A \sqrt{c} x \sqrt{a + c x^2}}{a e \sqrt{e x} \left(\sqrt{a} + \sqrt{c} x\right)} - \frac{2 A c^{1/4} \sqrt{x} \left(\sqrt{a} + \sqrt{c} x\right) \sqrt{\frac{a + c x^2}{\left(\sqrt{a} + \sqrt{c} x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{a^{3/4} e \sqrt{e x} \sqrt{a + c x^2}} + \frac{\left(\sqrt{a} B + A \sqrt{c}\right) \sqrt{x} \left(\sqrt{a} + \sqrt{c} x\right) \sqrt{\frac{a + c x^2}{\left(\sqrt{a} + \sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{a^{3/4} c^{1/4} e \sqrt{e x} \sqrt{a + c x^2}}$$

Result (type 4, 152 leaves):

$$\left(2 \sqrt{1 + \frac{a}{c x^2}} x^{5/2} \left(-A \sqrt{c} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] + \left(i \sqrt{a} B + A \sqrt{c}\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right)\right) \Big/ \left(\sqrt{a} \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} (e x)^{3/2} \sqrt{a + c x^2}\right)$$

- **Problem 464: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+B x}{(e x)^{5/2} \sqrt{a+c x^2}} \, dx$$

Optimal (type 4, 327 leaves, 7 steps):

$$-\frac{2 A \sqrt{a+c x^2}}{3 a e (e x)^{3/2}}-\frac{2 B \sqrt{a+c x^2}}{a e^2 \sqrt{e x}}+\frac{2 B \sqrt{c}\, x \sqrt{a+c x^2}}{a e^2 \sqrt{e x} \left(\sqrt{a}+\sqrt{c}\, x\right)}-\frac{2 B c^{1/4} \sqrt{x} \left(\sqrt{a}+\sqrt{c}\, x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{a^{3/4} e^2 \sqrt{e x} \sqrt{a+c x^2}}+\frac{\left(3 \sqrt{a}\, B-A \sqrt{c}\right) c^{1/4} \sqrt{x} \left(\sqrt{a}+\sqrt{c}\, x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{3 a^{5/4} e^2 \sqrt{e x} \sqrt{a+c x^2}}$$

Result (type 4, 212 leaves):

$$\left(x \left(-2 A \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} \left(a+c x^2\right)-6 \sqrt{a}\, B \sqrt{c} \sqrt{1+\frac{a}{c x^2}}\, x^{5/2}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]+2 \left(3 \sqrt{a}\, B-i A \sqrt{c}\right) \sqrt{c} \sqrt{1+\frac{a}{c x^2}}\, x^{5/2}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right)\right) \Bigg/ \left(3 a \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} (e x)^{5/2} \sqrt{a+c x^2}\right)$$

- **Problem 465: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+B x}{(e x)^{7/2} \sqrt{a+c x^2}} \, dx$$

Optimal (type 4, 363 leaves, 8 steps):

$$-\frac{2 A \sqrt{a+c x^2}}{5 a e (e x)^{5/2}}-\frac{2 B \sqrt{a+c x^2}}{3 a e^2 (e x)^{3/2}}+\frac{6 A c \sqrt{a+c x^2}}{5 a^2 e^3 \sqrt{e x}}-\frac{6 A c^{3/2} x \sqrt{a+c x^2}}{5 a^2 e^3 \sqrt{e x} \left(\sqrt{a}+\sqrt{c} x\right)}+$$

$$\frac{6 A c^{5/4} \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{5 a^{7/4} e^3 \sqrt{e x} \sqrt{a+c x^2}}-$$

$$\frac{\left(5 \sqrt{a} B+9 A \sqrt{c}\right) c^{3/4} \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{15 a^{7/4} e^3 \sqrt{e x} \sqrt{a+c x^2}}$$

Result (type 4, 217 leaves) :

$$\left(x \left(-2 \sqrt{a} \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} (3 A+5 B x) \left(a+c x^2\right)+18 A c^{3/2} \sqrt{1+\frac{a}{c x^2}} x^{7/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]-\right.\right.$$

$$\left.\left.2 \left(5 i \sqrt{a} B+9 A \sqrt{c}\right) c \sqrt{1+\frac{a}{c x^2}} x^{7/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right)\right) \Big/ \left(15 a^{3/2} \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} (e x)^{7/2} \sqrt{a+c x^2}\right)$$

- **Problem 466: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(e x)^{7/2} (A+B x)}{\left(a+c x^2\right)^{3/2}} \, dx$$

Optimal (type 4, 360 leaves, 8 steps) :

$$-\frac{e\,(e\,x)^{5/2}\,(A+B\,x)}{c\,\sqrt{a+c\,x^2}}+\frac{5\,A\,e^3\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}{3\,c^2}+\frac{7\,B\,e^2\,(e\,x)^{3/2}\,\sqrt{a+c\,x^2}}{5\,c^2}-$$

$$\frac{21\,a\,B\,e^4\,x\,\sqrt{a+c\,x^2}}{5\,c^{5/2}\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}+\frac{21\,a^{5/4}\,B\,e^4\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{5\,c^{11/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}-$$

$$\frac{a^{3/4}\,\left(63\,\sqrt{a}\,B+25\,A\,\sqrt{c}\right)\,e^4\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{30\,c^{11/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 240 leaves) :

$$-\left(e^4\left(\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,\left(63\,a^2\,B-2\,c^2\,x^3\,(5\,A+3\,B\,x)+a\,c\,x\,(-25\,A+42\,B\,x)\right)-63\,a^{3/2}\,B\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+\right.\right.$$

$$\left.\left.a\,\left(63\,\sqrt{a}\,B+25\,i\,A\,\sqrt{c}\right)\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Bigg/\left(15\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,c^3\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}\right)$$

Problem 467: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(e\,x)^{5/2}\,(A+B\,x)}{\left(a+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 326 leaves, 7 steps) :

$$-\frac{e\,(e\,x)^{3/2}\,(A+B\,x)}{c\,\sqrt{a+c\,x^2}}+\frac{5\,B\,e^2\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}{3\,c^2}+\frac{3\,A\,e^3\,x\,\sqrt{a+c\,x^2}}{c^{3/2}\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}-$$

$$\frac{3\,a^{1/4}\,A\,e^3\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{c^{7/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}-$$

$$\frac{a^{1/4}\,\left(5\,\sqrt{a}\,B-9\,A\,\sqrt{c}\right)\,e^3\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{6\,c^{9/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 228 leaves) :

$$\left(e^3\left(\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,\left(9\,a\,A+5\,a\,B\,x+6\,A\,c\,x^2+2\,B\,c\,x^3\right)-9\,\sqrt{a}\,A\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],\,-1\right]+\right.\right.$$

$$\left.\left.\sqrt{a}\,\left(-5\,i\,\sqrt{a}\,B+9\,A\,\sqrt{c}\right)\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],\,-1\right]\right)\right)\Big/\left(3\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,c^2\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}\right)$$

- **Problem 468: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(e\,x)^{3/2}\,(A+B\,x)}{\left(a+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 296 leaves, 6 steps) :

$$-\frac{e\sqrt{e\,x}\,(A+B\,x)}{c\sqrt{a+c\,x^2}}+\frac{3\,B\,e^2\,x\sqrt{a+c\,x^2}}{c^{3/2}\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}-\frac{3\,a^{1/4}\,B\,e^2\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{c^{7/4}\sqrt{e\,x}\,\sqrt{a+c\,x^2}}+$$

$$\frac{\left(3\sqrt{a}\,B+A\sqrt{c}\right)e^2\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{1/4}\,c^{7/4}\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 217 leaves) :

$$\left(e^2\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}\,(3\,a\,B+c\,x\,(-A+2\,B\,x))-3\sqrt{a}\,B\sqrt{c}\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+\right.\right.$$

$$\left.\left.\left(3\sqrt{a}\,B+i\,A\sqrt{c}\right)\sqrt{c}\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Big/\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}\,c^2\sqrt{e\,x}\,\sqrt{a+c\,x^2}\right)$$

■ Problem 469: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{e\,x}\,(A+B\,x)}{\left(a+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 298 leaves, 6 steps) :

$$-\frac{\sqrt{e\,x}\,(a\,B-A\,c\,x)}{a\,c\sqrt{a+c\,x^2}}-\frac{A\,e\,x\sqrt{a+c\,x^2}}{a\sqrt{c}\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}+\frac{A\,e\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{a^{3/4}\,c^{3/4}\sqrt{e\,x}\,\sqrt{a+c\,x^2}}+$$

$$\frac{\left(\sqrt{a}\,B-A\sqrt{c}\right)e\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{3/4}\,c^{5/4}\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 204 leaves) :

$$\left(\mathrm{i}\, e \left(-\sqrt{a} \sqrt{\frac{\mathrm{i}\sqrt{a}}{\sqrt{c}}} (A+B x)+A\sqrt{c} \sqrt{1+\frac{a}{c x^2}}\, x^{3/2}\, \text{EllipticE}\left[\mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{\frac{\mathrm{i}\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]-\right.\right.$$

$$\left.\left.\left(-\mathrm{i}\sqrt{a}\, B+A\sqrt{c}\right) \sqrt{1+\frac{a}{c x^2}}\, x^{3/2}\, \text{EllipticF}\left[\mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{\frac{\mathrm{i}\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right) \Big/ \left(\left(\frac{\mathrm{i}\sqrt{a}}{\sqrt{c}}\right)^{3/2} c^{3/2} \sqrt{e x} \sqrt{a+c x^2}\right)$$

- **Problem 470: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+B x}{\sqrt{e x}\, \left(a+c x^2\right)^{3/2}}\, dx$$

Optimal (type 4, 290 leaves, 6 steps):

$$\frac{\sqrt{e x}\, (A+B x)}{a e \sqrt{a+c x^2}}-\frac{B x \sqrt{a+c x^2}}{a \sqrt{c} \sqrt{e x} \left(\sqrt{a}+\sqrt{c}\, x\right)}+\frac{B \sqrt{x} \left(\sqrt{a}+\sqrt{c}\, x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{a^{3/4} c^{3/4} \sqrt{e x} \sqrt{a+c x^2}}-$$

$$\frac{\left(\sqrt{a}\, B-A \sqrt{c}\right) \sqrt{x} \left(\sqrt{a}+\sqrt{c}\, x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{2 a^{5/4} c^{3/4} \sqrt{e x} \sqrt{a+c x^2}}$$

Result (type 4, 211 leaves):

$$\left(\sqrt{\frac{\mathrm{i}\sqrt{a}}{\sqrt{c}}} (-a B+A c x)+\sqrt{a}\, B \sqrt{c} \sqrt{1+\frac{a}{c x^2}}\, x^{3/2}\, \text{EllipticE}\left[\mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{\frac{\mathrm{i}\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+\right.$$

$$\left.\mathrm{i}\left(\mathrm{i}\sqrt{a}\, B+A\sqrt{c}\right)\sqrt{c} \sqrt{1+\frac{a}{c x^2}}\, x^{3/2}\, \text{EllipticF}\left[\mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{\frac{\mathrm{i}\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right) \Big/ \left(a \sqrt{\frac{\mathrm{i}\sqrt{a}}{\sqrt{c}}}\, c \sqrt{e x} \sqrt{a+c x^2}\right)$$

- **Problem 471: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+B x}{(e x)^{3/2} \left(a+c x^2\right)^{3/2}} \, dx$$

Optimal (type 4, 327 leaves, 7 steps):

$$\frac{A+B x}{a e \sqrt{e x} \sqrt{a+c x^2}}-\frac{3 A \sqrt{a+c x^2}}{a^2 e \sqrt{e x}}+\frac{3 A \sqrt{c} \, x \sqrt{a+c x^2}}{a^2 e \sqrt{e x} \left(\sqrt{a}+\sqrt{c} \, x\right)}-\frac{3 A c^{1/4} \sqrt{x} \left(\sqrt{a}+\sqrt{c} \, x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} \, x\right)^2}} \, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{a^{7/4} e \sqrt{e x} \sqrt{a+c x^2}}+$$

$$\frac{\left(\sqrt{a} \, B+3 A \sqrt{c}\right) \sqrt{x} \left(\sqrt{a}+\sqrt{c} \, x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} \, x\right)^2}} \, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{7/4} c^{1/4} e \sqrt{e x} \sqrt{a+c x^2}}$$

Result (type 4, 201 leaves):

$$\Bigg(x \Bigg(\sqrt{a} \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} \, (A+B x)-3 A \sqrt{c} \sqrt{1+\frac{a}{c x^2}} \, x^{3/2} \, \text{EllipticE}\left[i \, \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]+$$

$$\left(i \sqrt{a} \, B+3 A \sqrt{c}\right) \sqrt{1+\frac{a}{c x^2}} \, x^{3/2} \, \text{EllipticF}\left[i \, \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\Bigg)\Bigg) \Bigg/ \left(a^{3/2} \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} \, (e x)^{3/2} \sqrt{a+c x^2}\right)$$

- **Problem 472: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+B x}{(e x)^{5/2} \left(a+c x^2\right)^{3/2}} \, dx$$

Optimal (type 4, 357 leaves, 8 steps):

$$\frac{A+B x}{a e (e x)^{3/2}\sqrt{a+c x^2}}-\frac{5 A\sqrt{a+c x^2}}{3 a^2 e (e x)^{3/2}}-\frac{3 B\sqrt{a+c x^2}}{a^2 e^2\sqrt{e x}}+\frac{3 B\sqrt{c}\, x\sqrt{a+c x^2}}{a^2 e^2\sqrt{e x}\left(\sqrt{a}+\sqrt{c}\, x\right)}-$$

$$\frac{3 B c^{1/4}\sqrt{x}\left(\sqrt{a}+\sqrt{c}\, x\right)\sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{a^{7/4} e^2\sqrt{e x}\sqrt{a+c x^2}}+$$

$$\frac{\left(9\sqrt{a}\, B-5 A\sqrt{c}\right) c^{1/4}\sqrt{x}\left(\sqrt{a}+\sqrt{c}\, x\right)\sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{6 a^{9/4} e^2\sqrt{e x}\sqrt{a+c x^2}}$$

Result (type 4, 219 leaves) :

$$\left(x\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}\left(-2 a A+3 a B x-5 A c x^2\right)-9\sqrt{a}\, B\sqrt{c}\sqrt{1+\frac{a}{c x^2}}\, x^{5/2}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+\right.\right.$$

$$\left.\left.\left(9\sqrt{a}\, B-5 i A\sqrt{c}\right)\sqrt{c}\sqrt{1+\frac{a}{c x^2}}\, x^{5/2}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Big/\left(3 a^2\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}\,(e x)^{5/2}\sqrt{a+c x^2}\right)$$

- **Problem 473: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B x}{(e x)^{7/2}\left(a+c x^2\right)^{3/2}}\, dx$$

Optimal (type 4, 393 leaves, 9 steps) :

$$\frac{A+B x}{a e (e x)^{5/2} \sqrt{a+c x^2}}-\frac{7 A \sqrt{a+c x^2}}{5 a^2 e (e x)^{5/2}}-\frac{5 B \sqrt{a+c x^2}}{3 a^2 e^2 (e x)^{3/2}}+\frac{21 A c \sqrt{a+c x^2}}{5 a^3 e^3 \sqrt{e x}}-$$

$$\frac{21 A c^{3/2} x \sqrt{a+c x^2}}{5 a^3 e^3 \sqrt{e x} \left(\sqrt{a}+\sqrt{c} x\right)}+\frac{21 A c^{5/4} \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{5 a^{11/4} e^3 \sqrt{e x} \sqrt{a+c x^2}}-$$

$$\frac{\left(25 \sqrt{a} B+63 A \sqrt{c}\right) c^{3/4} \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{30 a^{11/4} e^3 \sqrt{e x} \sqrt{a+c x^2}}$$

Result (type 4, 226 leaves):

$$\left(x \left(-\sqrt{a} \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} \left(6 a A+10 a B x+21 A c x^2+25 B c x^3\right)+63 A c^{3/2} \sqrt{1+\frac{a}{c x^2}} x^{7/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]-\right.\right.$$

$$\left.\left.\left(25 i \sqrt{a} B+63 A \sqrt{c}\right) c \sqrt{1+\frac{a}{c x^2}} x^{7/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right)\right) \Big/ \left(15 a^{5/2} \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} (e x)^{7/2} \sqrt{a+c x^2}\right)$$

- **Problem 474: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(e x)^{13/2} (A+B x)}{\left(a+c x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 428 leaves, 10 steps):

$$-\frac{e\,(e\,x)^{11/2}\,(A+B\,x)}{3\,c\,\left(a+c\,x^2\right)^{3/2}}-\frac{e^3\,(e\,x)^{7/2}\,(11\,A+13\,B\,x)}{6\,c^2\,\sqrt{a+c\,x^2}}-\frac{65\,a\,B\,e^6\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}{14\,c^4}+\frac{77\,A\,e^5\,(e\,x)^{3/2}\,\sqrt{a+c\,x^2}}{30\,c^3}+\frac{39\,B\,e^4\,(e\,x)^{5/2}\,\sqrt{a+c\,x^2}}{14\,c^3}-$$

$$\frac{77\,a\,A\,e^7\,x\,\sqrt{a+c\,x^2}}{10\,c^{7/2}\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}+\frac{77\,a^{5/4}\,A\,e^7\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{10\,c^{15/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}+$$

$$\frac{a^{5/4}\left(325\,\sqrt{a}\,B-539\,A\,\sqrt{c}\right)e^7\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{140\,c^{17/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 284 leaves) :

$$\frac{1}{210\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,c^4\,\sqrt{e\,x}\,\left(a+c\,x^2\right)^{3/2}}\,e^7\left(-\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\left(-12\,c^3\,x^6\,(7\,A+5\,B\,x)+35\,a^2\,c\,x^2\,(77\,A+39\,B\,x)+4\,a\,c^2\,x^4\,(231\,A+65\,B\,x)+3\,a^3\,(539\,A+325\,B\,x)\right)+\right.$$

$$1617\,a^{3/2}\,A\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\left(a+c\,x^2\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]-$$

$$\left.3\,a^{3/2}\left(-325\,i\,\sqrt{a}\,B+539\,A\,\sqrt{c}\right)\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\left(a+c\,x^2\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)$$

- **Problem 475: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(e\,x)^{11/2}\,(A+B\,x)}{\left(a+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 398 leaves, 9 steps) :

$$-\frac{e\,(e\,x)^{9/2}\,(A+B\,x)}{3\,c\,\left(a+c\,x^2\right)^{3/2}}-\frac{e^3\,(e\,x)^{5/2}\,(9\,A+11\,B\,x)}{6\,c^2\,\sqrt{a+c\,x^2}}+\frac{5\,A\,e^5\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}{2\,c^3}+\frac{77\,B\,e^4\,(e\,x)^{3/2}\,\sqrt{a+c\,x^2}}{30\,c^3}-$$

$$\frac{77\,a\,B\,e^6\,x\,\sqrt{a+c\,x^2}}{10\,c^{7/2}\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}+\frac{77\,a^{5/4}\,B\,e^6\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{10\,c^{15/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}-$$

$$\frac{a^{3/4}\,\left(77\,\sqrt{a}\,B+25\,A\,\sqrt{c}\right)\,e^6\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{20\,c^{15/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 277 leaves) :

$$\left(e^6\,\left(\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,\left(-231\,a^3\,B+5\,a^2\,c\,x\,(15\,A-77\,B\,x)+3\,a\,c^2\,x^3\,(35\,A-44\,B\,x)+4\,c^3\,x^5\,(5\,A+3\,B\,x)\right)+\right.\right.$$

$$231\,a^{3/2}\,B\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\left(a+c\,x^2\right)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],\,-1\right]-$$

$$\left.\left.3\,i\,a\,\left(-77\,i\,\sqrt{a}\,B+25\,A\,\sqrt{c}\right)\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\left(a+c\,x^2\right)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],\,-1\right]\right)\right)\Big/\left(30\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,c^4\,\sqrt{e\,x}\,\left(a+c\,x^2\right)^{3/2}\right)$$

- **Problem 476: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(e\,x)^{9/2}\,(A+B\,x)}{\left(a+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 368 leaves, 8 steps) :

$$-\frac{e\,(e\,x)^{7/2}\,(A+B\,x)}{3\,c\,\left(a+c\,x^2\right)^{3/2}}-\frac{e^3\,(e\,x)^{3/2}\,(7\,A+9\,B\,x)}{6\,c^2\,\sqrt{a+c\,x^2}}+\frac{5\,B\,e^4\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}{2\,c^3}+$$

$$\frac{7\,A\,e^5\,x\,\sqrt{a+c\,x^2}}{2\,c^{5/2}\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}-\frac{7\,a^{1/4}\,A\,e^5\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,c^{11/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}-$$

$$\frac{a^{1/4}\,\left(5\,\sqrt{a}\,B-7\,A\,\sqrt{c}\right)\,e^5\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{4\,c^{13/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 263 leaves) :

$$\left(e^5\left(\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,\left(4\,c^2\,x^4\,(3\,A+B\,x)+7\,a\,c\,x^2\,(5\,A+3\,B\,x)+3\,a^2\,(7\,A+5\,B\,x)\right)-\right.\right.$$

$$21\,\sqrt{a}\,A\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\left(a+c\,x^2\right)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+$$

$$\left.\left.3\,\sqrt{a}\,\left(-5\,i\,\sqrt{a}\,B+7\,A\,\sqrt{c}\right)\,\sqrt{1+\frac{a}{c\,x^2}}\,x^{3/2}\,\left(a+c\,x^2\right)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Big/\left(6\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,c^3\,\sqrt{e\,x}\,\left(a+c\,x^2\right)^{3/2}\right)$$

- **Problem 477: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(e\,x)^{7/2}\,(A+B\,x)}{\left(a+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 339 leaves, 7 steps) :

$$-\frac{e\,(e\,x)^{5/2}\,(A+B\,x)}{3\,c\,\left(a+c\,x^2\right)^{3/2}}-\frac{e^3\,\sqrt{e\,x}\,\,(5\,A+7\,B\,x)}{6\,c^2\,\sqrt{a+c\,x^2}}+\frac{7\,B\,e^4\,x\,\sqrt{a+c\,x^2}}{2\,c^{5/2}\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}-$$

$$\frac{7\,a^{1/4}\,B\,e^4\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{2\,c^{11/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}+$$

$$\frac{\left(21\,\sqrt{a}\,\,B+5\,A\,\sqrt{c}\,\right)\,e^4\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{12\,a^{1/4}\,c^{11/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 251 leaves) :

$$\left(e^4\left(\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,\left(21\,a^2\,B-5\,a\,c\,x\,(A-7\,B\,x)+c^2\,x^3\,(-7\,A+12\,B\,x)\right)-21\,\sqrt{a}\,\,B\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,\,x^{3/2}\,\left(a+c\,x^2\right)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],\,-1\right]+\right.\right.$$

$$\left.\left.\left(21\,\sqrt{a}\,\,B+5\,i\,A\,\sqrt{c}\,\right)\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,\,x^{3/2}\,\left(a+c\,x^2\right)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],\,-1\right]\right)\right)\Big/\left(6\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,\,c^3\,\sqrt{e\,x}\,\left(a+c\,x^2\right)^{3/2}\right)$$

■ Problem 478: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(e\,x)^{5/2}\,(A+B\,x)}{\left(a+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 347 leaves, 7 steps) :

$$-\frac{e\,(e\,x)^{3/2}\,(A+B\,x)}{3\,c\,\left(a+c\,x^2\right)^{3/2}}-\frac{e^2\,\sqrt{e\,x}\,\,(5\,a\,B-3\,A\,c\,x)}{6\,a\,c^2\,\sqrt{a+c\,x^2}}-\frac{A\,e^3\,x\,\sqrt{a+c\,x^2}}{2\,a\,c^{3/2}\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,\,x\right)}+$$

$$\frac{A\,e^3\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,\,x\right)^2}}\,\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{2\,a^{3/4}\,c^{7/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}+$$

$$\frac{\left(5\,\sqrt{a}\,\,B-3\,A\,\sqrt{c}\right)\,e^3\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,\,x\right)\,\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,\,x\right)^2}}\,\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\,\frac{1}{2}\right]}{12\,a^{3/4}\,c^{9/4}\,\sqrt{e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 243 leaves) :

$$\left(i\,e^3\left(-\sqrt{a}\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}\,\left(a\,(3\,A+5\,B\,x)+c\,x^2\,(5\,A+7\,B\,x)\right)+3\,A\,\sqrt{c}\,\sqrt{1+\frac{a}{c\,x^2}}\,\,x^{3/2}\,\left(a+c\,x^2\right)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],\,-1\right]-\right.\right.$$

$$\left.\left.\left(-5\,i\,\sqrt{a}\,\,B+3\,A\,\sqrt{c}\right)\,\sqrt{1+\frac{a}{c\,x^2}}\,\,x^{3/2}\,\left(a+c\,x^2\right)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\,\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],\,-1\right]\right)\right)\Bigg/\left(6\,\left(\frac{i\,\sqrt{a}}{\sqrt{c}}\right)^{3/2}\,c^{5/2}\,\sqrt{e\,x}\,\left(a+c\,x^2\right)^{3/2}\right)$$

- **Problem 479: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(e\,x)^{3/2}\,(A+B\,x)}{\left(a+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 341 leaves, 7 steps) :

$$-\frac{e\sqrt{e x}\,(A+B x)}{3 c\,(a+c x^2)^{3/2}}+\frac{e\sqrt{e x}\,(A+3 B x)}{6 a c\sqrt{a+c x^2}}-\frac{B e^2 x\sqrt{a+c x^2}}{2 a c^{3/2}\sqrt{e x}\,(\sqrt{a}+\sqrt{c}\,x)}+\frac{B e^2\sqrt{x}\,(\sqrt{a}+\sqrt{c}\,x)\sqrt{\frac{a+c x^2}{(\sqrt{a}+\sqrt{c}\,x)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{2 a^{3/4} c^{7/4}\sqrt{e x}\sqrt{a+c x^2}}-$$

$$\frac{(3\sqrt{a}\,B-A\sqrt{c})\,e^2\sqrt{x}\,(\sqrt{a}+\sqrt{c}\,x)\sqrt{\frac{a+c x^2}{(\sqrt{a}+\sqrt{c}\,x)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{12 a^{5/4} c^{7/4}\sqrt{e x}\sqrt{a+c x^2}}$$

Result (type 4, 249 leaves) :

$$\left(e^2\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}\,(-3 a^2 B+A c^2 x^3-a c x\,(A+5 B x))+3\sqrt{a}\,B\sqrt{c}\,\sqrt{1+\frac{a}{c x^2}}\,x^{3/2}\,(a+c x^2)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+\right.\right.$$

$$\left.\left.i\,(3 i\sqrt{a}\,B+A\sqrt{c})\sqrt{c}\,\sqrt{1+\frac{a}{c x^2}}\,x^{3/2}\,(a+c x^2)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Big/\left(6 a\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}\,c^2\sqrt{e x}\,(a+c x^2)^{3/2}\right)$$

- **Problem 480: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{e x}\,(A+B x)}{(a+c x^2)^{5/2}}\,dx$$

Optimal (type 4, 342 leaves, 7 steps) :

$$-\frac{\sqrt{e x}\,(a B-A c x)}{3 a c\,(a+c x^2)^{3/2}}+\frac{\sqrt{e x}\,(a B+3 A c x)}{6 a^2 c\sqrt{a+c x^2}}-\frac{A e x\sqrt{a+c x^2}}{2 a^2\sqrt{c}\sqrt{e x}\,(\sqrt{a}+\sqrt{c}\,x)}+\frac{A e\sqrt{x}\,(\sqrt{a}+\sqrt{c}\,x)\sqrt{\frac{a+c x^2}{(\sqrt{a}+\sqrt{c}\,x)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{2 a^{7/4} c^{3/4}\sqrt{e x}\sqrt{a+c x^2}}+$$

$$\frac{(\sqrt{a}\,B-3 A\sqrt{c})\,e\sqrt{x}\,(\sqrt{a}+\sqrt{c}\,x)\sqrt{\frac{a+c x^2}{(\sqrt{a}+\sqrt{c}\,x)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{12 a^{7/4} c^{5/4}\sqrt{e x}\sqrt{a+c x^2}}$$

Result (type 4, 239 leaves) :

$$\left(e\left(-\sqrt{a}\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}\left(c x^2 (A-B x)+a (3 A+B x)\right)+3 A\sqrt{c}\sqrt{1+\frac{a}{c x^2}}\, x^{3/2}\left(a+c x^2\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]-\right.\right.$$

$$\left.\left.\left(-i\sqrt{a}\, B+3 A\sqrt{c}\right)\sqrt{1+\frac{a}{c x^2}}\, x^{3/2}\left(a+c x^2\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Big/\left(6 a^{3/2}\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}\, c\sqrt{e x}\left(a+c x^2\right)^{3/2}\right)$$

- **Problem 481: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B x}{\sqrt{e x}\left(a+c x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 335 leaves, 7 steps):

$$\frac{\sqrt{e x}(A+B x)}{3 a e\left(a+c x^2\right)^{3/2}}+\frac{\sqrt{e x}(5 A+3 B x)}{6 a^2 e\sqrt{a+c x^2}}-\frac{B x\sqrt{a+c x^2}}{2 a^2\sqrt{c}\sqrt{e x}\left(\sqrt{a}+\sqrt{c}\, x\right)}+\frac{B\sqrt{x}\left(\sqrt{a}+\sqrt{c}\, x\right)\sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{2 a^{7/4} c^{3/4}\sqrt{e x}\sqrt{a+c x^2}}-$$

$$\frac{\left(3\sqrt{a}\, B-5 A\sqrt{c}\right)\sqrt{x}\left(\sqrt{a}+\sqrt{c}\, x\right)\sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{12 a^{9/4} c^{3/4}\sqrt{e x}\sqrt{a+c x^2}}$$

Result (type 4, 249 leaves):

$$\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}\left(-3 a^2 B+5 A c^2 x^3+a c x (7 A-B x)\right)+3\sqrt{a}\, B\sqrt{c}\sqrt{1+\frac{a}{c x^2}}\, x^{3/2}\left(a+c x^2\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+\right.$$

$$\left. i\left(3 i\sqrt{a}\, B+5 A\sqrt{c}\right)\sqrt{c}\sqrt{1+\frac{a}{c x^2}}\, x^{3/2}\left(a+c x^2\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\Big/\left(6 a^2\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}\, c\sqrt{e x}\left(a+c x^2\right)^{3/2}\right)$$

- **Problem 482: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+B x}{(e x)^{3/2} \left(a+c x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 373 leaves, 8 steps):

$$\frac{A+B x}{3 a e \sqrt{e x} \left(a+c x^2\right)^{3/2}}+\frac{7 A+5 B x}{6 a^2 e \sqrt{e x} \sqrt{a+c x^2}}-\frac{7 A \sqrt{a+c x^2}}{2 a^3 e \sqrt{e x}}+$$
$$\frac{7 A \sqrt{c}\, x \sqrt{a+c x^2}}{2 a^3 e \sqrt{e x} \left(\sqrt{a}+\sqrt{c}\, x\right)}-\frac{7 A c^{1/4} \sqrt{x} \left(\sqrt{a}+\sqrt{c}\, x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{11/4} e \sqrt{e x} \sqrt{a+c x^2}}+$$
$$\frac{\left(5 \sqrt{a}\, B+21 A \sqrt{c}\right) \sqrt{x} \left(\sqrt{a}+\sqrt{c}\, x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{12 a^{11/4} c^{1/4} e \sqrt{e x} \sqrt{a+c x^2}}$$

Result (type 4, 237 leaves):

$$\left(x \left(\sqrt{a} \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} \left(c x^2 (7 A+5 B x)+a (9 A+7 B x)\right)-21 A \sqrt{c} \sqrt{1+\frac{a}{c x^2}}\, x^{3/2} \left(a+c x^2\right) \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]+\right.\right.$$
$$\left.\left.\left(5 i \sqrt{a}\, B+21 A \sqrt{c}\right) \sqrt{1+\frac{a}{c x^2}}\, x^{3/2} \left(a+c x^2\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]\right)\right) \Big/ \left(6 a^{5/2} \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}\, (e x)^{3/2} \left(a+c x^2\right)^{3/2}\right)$$

- **Problem 483: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+B x}{(e x)^{5/2} \left(a+c x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 402 leaves, 9 steps):

$$\frac{A+B\,x}{3\,a\,e\,(e\,x)^{3/2}\left(a+c\,x^2\right)^{3/2}}+\frac{9\,A+7\,B\,x}{6\,a^2\,e\,(e\,x)^{3/2}\sqrt{a+c\,x^2}}-\frac{5\,A\sqrt{a+c\,x^2}}{2\,a^3\,e\,(e\,x)^{3/2}}-\frac{7\,B\sqrt{a+c\,x^2}}{2\,a^3\,e^2\sqrt{e\,x}}+$$

$$\frac{7\,B\sqrt{c}\;x\sqrt{a+c\,x^2}}{2\,a^3\,e^2\sqrt{e\,x}\left(\sqrt{a}+\sqrt{c}\;x\right)}-\frac{7\,B\,c^{1/4}\sqrt{x}\left(\sqrt{a}+\sqrt{c}\;x\right)\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\;x\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{11/4}\,e^2\sqrt{e\,x}\sqrt{a+c\,x^2}}+$$

$$\frac{\left(7\sqrt{a}\;B-5\,A\sqrt{c}\right)c^{1/4}\sqrt{x}\left(\sqrt{a}+\sqrt{c}\;x\right)\sqrt{\frac{a+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\;x\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{4\,a^{13/4}\,e^2\sqrt{e\,x}\sqrt{a+c\,x^2}}$$

Result (type 4, 253 leaves):

$$\left(x\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}\left(-15\,A\,c^2\,x^4+7\,a\,c\,x^2\,(-3\,A+B\,x)+a^2\,(-4\,A+9\,B\,x)\right)-21\sqrt{a}\;B\sqrt{c}\sqrt{1+\frac{a}{c\,x^2}}\;x^{5/2}\left(a+c\,x^2\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]+\right.\right.$$

$$\left.\left.3\left(7\sqrt{a}\;B-5\,i\,A\sqrt{c}\right)\sqrt{c}\sqrt{1+\frac{a}{c\,x^2}}\;x^{5/2}\left(a+c\,x^2\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right)\Big/\left(6\,a^3\sqrt{\frac{i\sqrt{a}}{\sqrt{c}}}\;(e\,x)^{5/2}\left(a+c\,x^2\right)^{3/2}\right)$$

- **Problem 484: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B\,x}{(e\,x)^{7/2}\left(a+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 432 leaves, 10 steps):

$$\frac{A+B x}{3 a e (e x)^{5/2} \left(a+c x^2\right)^{3/2}}+\frac{11 A+9 B x}{6 a^2 e (e x)^{5/2} \sqrt{a+c x^2}}-\frac{77 A \sqrt{a+c x^2}}{30 a^3 e (e x)^{5/2}}-\frac{5 B \sqrt{a+c x^2}}{2 a^3 e^2 (e x)^{3/2}}+\frac{77 A c \sqrt{a+c x^2}}{10 a^4 e^3 \sqrt{e x}}-$$

$$\frac{77 A c^{3/2} x \sqrt{a+c x^2}}{10 a^4 e^3 \sqrt{e x} \left(\sqrt{a}+\sqrt{c} x\right)}+\frac{77 A c^{5/4} \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{10 a^{15/4} e^3 \sqrt{e x} \sqrt{a+c x^2}}-$$

$$\frac{\left(25 \sqrt{a} B+77 A \sqrt{c}\right) c^{3/4} \sqrt{x} \left(\sqrt{a}+\sqrt{c} x\right) \sqrt{\frac{a+c x^2}{\left(\sqrt{a}+\sqrt{c} x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{a^{1/4}}\right],\frac{1}{2}\right]}{20 a^{15/4} e^3 \sqrt{e x} \sqrt{a+c x^2}}$$

Result (type 4, 260 leaves) :

$$\left(x \left(-\sqrt{a} \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} \left(4 a^2 (3 A+5 B x)+3 a c x^2 (33 A+35 B x)+c^2 x^4 (77 A+75 B x)\right)+\right.\right.$$

$$231 A c^{3/2} \sqrt{1+\frac{a}{c x^2}} x^{7/2} \left(a+c x^2\right) \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]-$$

$$\left.\left.3 \left(25 i \sqrt{a} B+77 A \sqrt{c}\right) c \sqrt{1+\frac{a}{c x^2}} x^{7/2} \left(a+c x^2\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right)\right) \Big/ \left(30 a^{7/2} \sqrt{\frac{i \sqrt{a}}{\sqrt{c}}} (e x)^{7/2} \left(a+c x^2\right)^{3/2}\right)$$

- **Problem 527: Result more than twice size of optimal antiderivative.**

$$\int (A+B x) \left(a^2+2 a b x+b^2 x^2\right)^2 \, dx$$

Optimal (type 1, 38 leaves, 3 steps) :

$$\frac{(A b-a B) (a+b x)^5}{5 b^2}+\frac{B (a+b x)^6}{6 b^2}$$

Result (type 1, 84 leaves) :

$$\frac{1}{30} x \left(15 a^4 (2 A+B x)+20 a^3 b x (3 A+2 B x)+15 a^2 b^2 x^2 (4 A+3 B x)+6 a b^3 x^3 (5 A+4 B x)+b^4 x^4 (6 A+5 B x)\right)$$

Problem 543: Result more than twice size of optimal antiderivative.

$$\int x\,(A+B x)\,\left(a^2+2 a b x+b^2 x^2\right)^3\,dx$$

Optimal (type 1, 61 leaves, 3 steps):

$$-\frac{a\,(A b-a B)\,(a+b x)^7}{7 b^3}+\frac{(A b-2 a B)\,(a+b x)^8}{8 b^3}+\frac{B\,(a+b x)^9}{9 b^3}$$

Result (type 1, 140 leaves):

$$\frac{1}{2} a^6 A x^2+\frac{1}{3} a^5\,(6 A b+a B)\,x^3+\frac{3}{4} a^4 b\,(5 A b+2 a B)\,x^4+a^3 b^2\,(4 A b+3 a B)\,x^5+\frac{5}{6} a^2 b^3\,(3 A b+4 a B)\,x^6+\frac{3}{7} a b^4\,(2 A b+5 a B)\,x^7+\frac{1}{8} b^5\,(A b+6 a B)\,x^8+\frac{1}{9} b^6 B x^9$$

Problem 544: Result more than twice size of optimal antiderivative.

$$\int (A+B x)\,\left(a^2+2 a b x+b^2 x^2\right)^3\,dx$$

Optimal (type 1, 38 leaves, 3 steps):

$$\frac{(A b-a B)\,(a+b x)^7}{7 b^2}+\frac{B\,(a+b x)^8}{8 b^2}$$

Result (type 1, 122 leaves):

$$\frac{1}{56} x\,\left(28 a^6\,(2 A+B x)+56 a^5 b x\,(3 A+2 B x)+70 a^4 b^2 x^2\,(4 A+3 B x)+56 a^3 b^3 x^3\,(5 A+4 B x)+28 a^2 b^4 x^4\,(6 A+5 B x)+8 a b^5 x^5\,(7 A+6 B x)+b^6 x^6\,(8 A+7 B x)\right)$$

Problem 553: Result more than twice size of optimal antiderivative.

$$\int \frac{(A+B x)\,\left(a^2+2 a b x+b^2 x^2\right)^3}{x^9}\,dx$$

Optimal (type 1, 44 leaves, 3 steps):

$$-\frac{A\,(a+b x)^7}{8 a x^8}+\frac{(A b-8 a B)\,(a+b x)^7}{56 a^2 x^7}$$

Result (type 1, 123 leaves):

$$-\frac{1}{56 x^8}\left(28 b^6 x^6\,(A+2 B x)+56 a b^5 x^5\,(2 A+3 B x)+70 a^2 b^4 x^4\,(3 A+4 B x)+56 a^3 b^3 x^3\,(4 A+5 B x)+28 a^4 b^2 x^2\,(5 A+6 B x)+8 a^5 b x\,(6 A+7 B x)+a^6\,(7 A+8 B x)\right)$$

Problem 563: Result more than twice size of optimal antiderivative.

$$\int x^3\,(d+e x)\,\left(1+2 x+x^2\right)^5\,dx$$

Optimal (type 1, 69 leaves, 3 steps):

$$-\frac{1}{11}(d-e)(1+x)^{11}+\frac{1}{12}(3d-4e)(1+x)^{12}-\frac{3}{13}(d-2e)(1+x)^{13}+\frac{1}{14}(d-4e)(1+x)^{14}+\frac{1}{15}e(1+x)^{15}$$

Result (type 1, 153 leaves):

$$\frac{d x^4}{4}+\frac{1}{5}(10d+e)x^5+\frac{5}{6}(9d+2e)x^6+\frac{15}{7}(8d+3e)x^7+\frac{15}{4}(7d+4e)x^8+\frac{14}{3}(6d+5e)x^9+\frac{21}{5}(5d+6e)x^{10}+\frac{30}{11}(4d+7e)x^{11}+\frac{5}{4}(3d+8e)x^{12}+\frac{5}{13}(2d+9e)x^{13}+\frac{1}{14}(d+10e)x^{14}+\frac{e x^{15}}{15}$$

Problem 564: Result more than twice size of optimal antiderivative.

$$\int x^2(d+e x)\left(1+2x+x^2\right)^5 dx$$

Optimal (type 1, 55 leaves, 3 steps):

$$\frac{1}{11}(d-e)(1+x)^{11}-\frac{1}{12}(2d-3e)(1+x)^{12}+\frac{1}{13}(d-3e)(1+x)^{13}+\frac{1}{14}e(1+x)^{14}$$

Result (type 1, 148 leaves):

$$\frac{d x^3}{3}+\frac{1}{4}(10d+e)x^4+(9d+2e)x^5+\frac{5}{2}(8d+3e)x^6+\frac{30}{7}(7d+4e)x^7+\frac{21}{4}(6d+5e)x^8+\frac{14}{3}(5d+6e)x^9+3(4d+7e)x^{10}+\frac{15}{11}(3d+8e)x^{11}+\frac{5}{12}(2d+9e)x^{12}+\frac{1}{13}(d+10e)x^{13}+\frac{e x^{14}}{14}$$

Problem 565: Result more than twice size of optimal antiderivative.

$$\int x(d+e x)\left(1+2x+x^2\right)^5 dx$$

Optimal (type 1, 39 leaves, 3 steps):

$$-\frac{1}{11}(d-e)(1+x)^{11}+\frac{1}{12}(d-2e)(1+x)^{12}+\frac{1}{13}e(1+x)^{13}$$

Result (type 1, 147 leaves):

$$\frac{d x^2}{2}+\frac{1}{3}(10d+e)x^3+\frac{5}{4}(9d+2e)x^4+3(8d+3e)x^5+5(7d+4e)x^6+6(6d+5e)x^7+\frac{21}{4}(5d+6e)x^8+\frac{10}{3}(4d+7e)x^9+\frac{3}{2}(3d+8e)x^{10}+\frac{5}{11}(2d+9e)x^{11}+\frac{1}{12}(d+10e)x^{12}+\frac{e x^{13}}{13}$$

Problem 566: Result more than twice size of optimal antiderivative.

$$\int (d+e x)\left(1+2x+x^2\right)^5 dx$$

Optimal (type 1, 25 leaves, 3 steps):

$$\frac{1}{11} (d - e) (1 + x)^{11} + \frac{1}{12} e (1 + x)^{12}$$

Result (type 1, 113 leaves):

$$\frac{1}{132} e x^2 \left(66 + 440 x + 1485 x^2 + 3168 x^3 + 4620 x^4 + 4752 x^5 + 3465 x^6 + 1760 x^7 + 594 x^8 + 120 x^9 + 11 x^{10}\right) + d \left(x + 5 x^2 + 15 x^3 + 30 x^4 + 42 x^5 + 42 x^6 + 30 x^7 + 15 x^8 + 5 x^9 + x^{10} + \frac{x^{11}}{11}\right)$$

Problem 579: Result more than twice size of optimal antiderivative.

$$\int \frac{(d + e x) \left(1 + 2 x + x^2\right)^5}{x^{13}} \, dx$$

Optimal (type 1, 31 leaves, 3 steps):

$$-\frac{d (1 + x)^{11}}{12 x^{12}} + \frac{(d - 12 e) (1 + x)^{11}}{132 x^{11}}$$

Result (type 1, 114 leaves):

$$-\frac{1}{132 x^{12}} \left(12 e x \left(1 + 11 x + 55 x^2 + 165 x^3 + 330 x^4 + 462 x^5 + 462 x^6 + 330 x^7 + 165 x^8 + 55 x^9 + 11 x^{10}\right) + d \left(11 + 120 x + 594 x^2 + 1760 x^3 + 3465 x^4 + 4752 x^5 + 4620 x^6 + 3168 x^7 + 1485 x^8 + 440 x^9 + 66 x^{10}\right)\right)$$

Problem 580: Result more than twice size of optimal antiderivative.

$$\int \frac{(d + e x) \left(1 + 2 x + x^2\right)^5}{x^{14}} \, dx$$

Optimal (type 1, 52 leaves, 4 steps):

$$-\frac{d (1 + x)^{11}}{13 x^{13}} + \frac{(2 d - 13 e) (1 + x)^{11}}{156 x^{12}} - \frac{(2 d - 13 e) (1 + x)^{11}}{1716 x^{11}}$$

Result (type 1, 115 leaves):

$$-\frac{1}{1716 x^{13}} \left(13 e x \left(11 + 120 x + 594 x^2 + 1760 x^3 + 3465 x^4 + 4752 x^5 + 4620 x^6 + 3168 x^7 + 1485 x^8 + 440 x^9 + 66 x^{10}\right) + 2 d \left(66 + 715 x + 3510 x^2 + 10\,296 x^3 + 20\,020 x^4 + 27\,027 x^5 + 25\,740 x^6 + 17\,160 x^7 + 7722 x^8 + 2145 x^9 + 286 x^{10}\right)\right)$$

Problem 581: Result more than twice size of optimal antiderivative.

$$\int \frac{(d + e x) \left(1 + 2 x + x^2\right)^5}{x^{15}} \, dx$$

Optimal (type 1, 71 leaves, 5 steps):

$$-\frac{d\,(1+x)^{11}}{14\,x^{14}}+\frac{(3\,d-14\,e)\,(1+x)^{11}}{182\,x^{13}}-\frac{(3\,d-14\,e)\,(1+x)^{11}}{1092\,x^{12}}+\frac{(3\,d-14\,e)\,(1+x)^{11}}{12\,012\,x^{11}}$$

Result (type 1, 149 leaves):

$$-\frac{d}{14\,x^{14}}-\frac{10\,d+e}{13\,x^{13}}-\frac{5\,(9\,d+2\,e)}{12\,x^{12}}-\frac{15\,(8\,d+3\,e)}{11\,x^{11}}-\frac{3\,(7\,d+4\,e)}{x^{10}}-\frac{14\,(6\,d+5\,e)}{3\,x^{9}}-\frac{21\,(5\,d+6\,e)}{4\,x^{8}}-\frac{30\,(4\,d+7\,e)}{7\,x^{7}}-\frac{5\,(3\,d+8\,e)}{2\,x^{6}}-\frac{2\,d+9\,e}{x^{5}}-\frac{d+10\,e}{4\,x^{4}}-\frac{e}{3\,x^{3}}$$

- **Problem 596: Result more than twice size of optimal antiderivative.**

$$\int x^3\,(1+x)\,\left(1+2\,x+x^2\right)^5\,dx$$

Optimal (type 1, 37 leaves, 3 steps):

$$-\frac{1}{12}\,(1+x)^{12}+\frac{3}{13}\,(1+x)^{13}-\frac{3}{14}\,(1+x)^{14}+\frac{1}{15}\,(1+x)^{15}$$

Result (type 1, 83 leaves):

$$\frac{x^4}{4}+\frac{11\,x^5}{5}+\frac{55\,x^6}{6}+\frac{165\,x^7}{7}+\frac{165\,x^8}{4}+\frac{154\,x^9}{3}+\frac{231\,x^{10}}{5}+30\,x^{11}+\frac{55\,x^{12}}{4}+\frac{55\,x^{13}}{13}+\frac{11\,x^{14}}{14}+\frac{x^{15}}{15}$$

- **Problem 597: Result more than twice size of optimal antiderivative.**

$$\int x^2\,(1+x)\,\left(1+2\,x+x^2\right)^5\,dx$$

Optimal (type 1, 28 leaves, 3 steps):

$$\frac{1}{12}\,(1+x)^{12}-\frac{2}{13}\,(1+x)^{13}+\frac{1}{14}\,(1+x)^{14}$$

Result (type 1, 79 leaves):

$$\frac{x^3}{3}+\frac{11\,x^4}{4}+11\,x^5+\frac{55\,x^6}{2}+\frac{330\,x^7}{7}+\frac{231\,x^8}{4}+\frac{154\,x^9}{3}+33\,x^{10}+15\,x^{11}+\frac{55\,x^{12}}{12}+\frac{11\,x^{13}}{13}+\frac{x^{14}}{14}$$

- **Problem 598: Result more than twice size of optimal antiderivative.**

$$\int x\,(1+x)\,\left(1+2\,x+x^2\right)^5\,dx$$

Optimal (type 1, 19 leaves, 3 steps):

$$-\frac{1}{12}\,(1+x)^{12}+\frac{1}{13}\,(1+x)^{13}$$

Result (type 1, 77 leaves):

$$\frac{x^2}{2}+\frac{11\,x^3}{3}+\frac{55\,x^4}{4}+33\,x^5+55\,x^6+66\,x^7+\frac{231\,x^8}{4}+\frac{110\,x^9}{3}+\frac{33\,x^{10}}{2}+5\,x^{11}+\frac{11\,x^{12}}{12}+\frac{x^{13}}{13}$$

Problem 612: Result more than twice size of optimal antiderivative.

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{13}}\,dx$$

Optimal (type 1, 12 leaves, 2 steps):

$$-\frac{(1+x)^{12}}{12x^{12}}$$

Result (type 1, 75 leaves):

$$-\frac{1}{12x^{12}}-\frac{1}{x^{11}}-\frac{11}{2x^{10}}-\frac{55}{3x^9}-\frac{165}{4x^8}-\frac{66}{x^7}-\frac{77}{x^6}-\frac{66}{x^5}-\frac{165}{4x^4}-\frac{55}{3x^3}-\frac{11}{2x^2}-\frac{1}{x}$$

Problem 613: Result more than twice size of optimal antiderivative.

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{14}}\,dx$$

Optimal (type 1, 25 leaves, 3 steps):

$$-\frac{(1+x)^{12}}{13x^{13}}+\frac{(1+x)^{12}}{156x^{12}}$$

Result (type 1, 77 leaves):

$$-\frac{1}{13x^{13}}-\frac{11}{12x^{12}}-\frac{5}{x^{11}}-\frac{33}{2x^{10}}-\frac{110}{3x^9}-\frac{231}{4x^8}-\frac{66}{x^7}-\frac{55}{x^6}-\frac{33}{x^5}-\frac{55}{4x^4}-\frac{11}{3x^3}-\frac{1}{2x^2}$$

Problem 614: Result more than twice size of optimal antiderivative.

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{15}}\,dx$$

Optimal (type 1, 37 leaves, 4 steps):

$$-\frac{(1+x)^{12}}{14x^{14}}+\frac{(1+x)^{12}}{91x^{13}}-\frac{(1+x)^{12}}{1092x^{12}}$$

Result (type 1, 79 leaves):

$$-\frac{1}{14x^{14}}-\frac{11}{13x^{13}}-\frac{55}{12x^{12}}-\frac{15}{x^{11}}-\frac{33}{x^{10}}-\frac{154}{3x^9}-\frac{231}{4x^8}-\frac{330}{7x^7}-\frac{55}{2x^6}-\frac{11}{x^5}-\frac{11}{4x^4}-\frac{1}{3x^3}$$

Problem 841: Result more than twice size of optimal antiderivative.

$$\int x^m (1+x)\left(1+2x+x^2\right)^5\,dx$$

Optimal (type 3, 143 leaves, 3 steps):

$$\frac{x^{1+m}}{1+m}+\frac{11\,x^{2+m}}{2+m}+\frac{55\,x^{3+m}}{3+m}+\frac{165\,x^{4+m}}{4+m}+\frac{330\,x^{5+m}}{5+m}+\frac{462\,x^{6+m}}{6+m}+\frac{462\,x^{7+m}}{7+m}+\frac{330\,x^{8+m}}{8+m}+\frac{165\,x^{9+m}}{9+m}+\frac{55\,x^{10+m}}{10+m}+\frac{11\,x^{11+m}}{11+m}+\frac{x^{12+m}}{12+m}$$

Result (type 3, 357 leaves):

$$-\Big(\big(x^m\,\big(39\,916\,800+39\,916\,800\,m\,(1+x)+19\,958\,400\,m\,(1+m)\,(1+x)^2+6\,652\,800\,m\,(1+m)\,(2+m)\,(1+x)^3+1\,663\,200\,m\,(1+m)\,(2+m)\,(3+m)\,(1+x)^4+332\,640\,m\,(1+m)\,(2+m)\,(3+m)\,(4+m)\,(1+x)^5+55\,440\,m\,(1+m)\,(2+m)\,(3+m)\,(4+m)\,(5+m)\,(1+x)^6+7920\,m\,(1+m)\,(2+m)\,(3+m)\,(4+m)\,(5+m)\,(6+m)\,(1+x)^7+990\,m\,(1+m)\,(2+m)\,(3+m)\,(4+m)\,(5+m)\,(6+m)\,(7+m)\,(1+x)^8+110\,m\,(1+m)\,(2+m)\,(3+m)\,(4+m)\,(5+m)\,(6+m)\,(7+m)\,(8+m)\,(1+x)^9+11\,m\,(1+m)\,(2+m)\,(3+m)\,(4+m)\,(5+m)\,(6+m)\,(7+m)\,(8+m)\,(9+m)\,(1+x)^{10}+m\,(1+m)\,(2+m)\,(3+m)\,(4+m)\,(5+m)\,(6+m)\,(7+m)\,(8+m)\,(9+m)\,(10+m)\,(1+x)^{11}-(1+m)\,(2+m)\,(3+m)\,(4+m)\,(5+m)\,(6+m)\,(7+m)\,(8+m)\,(9+m)\,(10+m)\,(11+m)\,(1+x)^{12}\big)\big)\Big/\big((1+m)\,(2+m)\,(3+m)\,(4+m)\,(5+m)\,(6+m)\,(7+m)\,(8+m)\,(9+m)\,(10+m)\,(11+m)\,(12+m)\big)\Big)$$

■ Problem 842: Result more than twice size of optimal antiderivative.

$$\int x^m\,(d+e\,x)\,\left(1+2\,x+x^2\right)^5\,dx$$

Optimal (type 3, 209 leaves, 3 steps):

$$\frac{d\,x^{1+m}}{1+m}+\frac{(10\,d+e)\,x^{2+m}}{2+m}+\frac{5\,(9\,d+2\,e)\,x^{3+m}}{3+m}+\frac{15\,(8\,d+3\,e)\,x^{4+m}}{4+m}+\frac{30\,(7\,d+4\,e)\,x^{5+m}}{5+m}+\frac{42\,(6\,d+5\,e)\,x^{6+m}}{6+m}+\frac{42\,(5\,d+6\,e)\,x^{7+m}}{7+m}+\frac{30\,(4\,d+7\,e)\,x^{8+m}}{8+m}+\frac{15\,(3\,d+8\,e)\,x^{9+m}}{9+m}+\frac{5\,(2\,d+9\,e)\,x^{10+m}}{10+m}+\frac{(d+10\,e)\,x^{11+m}}{11+m}+\frac{e\,x^{12+m}}{12+m}$$

Result (type 3, 499 leaves):

$$\Big(x^m\,\big(3\,628\,800\,(e\,(1+m)-d\,(12+m))+3\,628\,800\,m\,(e\,(1+m)-d\,(12+m))\,(1+x)+1\,814\,400\,m\,(1+m)\,(e\,(1+m)-d\,(12+m))\,(1+x)^2+604\,800\,m\,(1+m)\,(2+m)\,(e\,(1+m)-d\,(12+m))\,(1+x)^3+151\,200\,m\,(1+m)\,(2+m)\,(3+m)\,(e\,(1+m)-d\,(12+m))\,(1+x)^4+30\,240\,m\,(1+m)\,(2+m)\,(3+m)\,(4+m)\,(e\,(1+m)-d\,(12+m))\,(1+x)^5+5040\,m\,(1+m)\,(2+m)\,(3+m)\,(4+m)\,(5+m)\,(e\,(1+m)-d\,(12+m))\,(1+x)^6+720\,m\,(1+m)\,(2+m)\,(3+m)\,(4+m)\,(5+m)\,(6+m)\,(e\,(1+m)-d\,(12+m))\,(1+x)^7+90\,m\,(1+m)\,(2+m)\,(3+m)\,(4+m)\,(5+m)\,(6+m)\,(7+m)\,(e\,(1+m)-d\,(12+m))\,(1+x)^8+10\,m\,(1+m)\,(2+m)\,(3+m)\,(4+m)\,(5+m)\,(6+m)\,(7+m)\,(8+m)\,(e\,(1+m)-d\,(12+m))\,(1+x)^9+m\,(1+m)\,(2+m)\,(3+m)\,(4+m)\,(5+m)\,(6+m)\,(7+m)\,(8+m)\,(9+m)\,(e\,(1+m)-d\,(12+m))\,(1+x)^{10}+(1+m)\,(2+m)\,(3+m)\,(4+m)\,(5+m)\,(6+m)\,(7+m)\,(8+m)\,(9+m)\,(10+m)\,(-2\,e\,(6+m)+d\,(12+m))\,(1+x)^{11}+e\,(1+m)\,(2+m)\,(3+m)\,(4+m)\,(5+m)\,(6+m)\,(7+m)\,(8+m)\,(9+m)\,(10+m)\,(11+m)\,(1+x)^{12}\big)\Big)\Big/\big((1+m)\,(2+m)\,(3+m)\,(4+m)\,(5+m)\,(6+m)\,(7+m)\,(8+m)\,(9+m)\,(10+m)\,(11+m)\,(12+m)\big)$$

■ Problem 980: Result more than twice size of optimal antiderivative.

$$\int \frac{1-x}{x\,\sqrt{1+3\,x+x^2}}\,dx$$

Optimal (type 3, 19 leaves, 2 steps):

$$-2\,\text{ArcTanh}\left[\frac{1+x}{\sqrt{1+3\,x+x^2}}\right]$$

Result (type 3, 47 leaves) :

$$\mathrm{Log}[x]-\mathrm{Log}\left[3+2\,x+2\,\sqrt{1+3\,x+x^2}\,\right]-\mathrm{Log}\left[2+3\,x+2\,\sqrt{1+3\,x+x^2}\,\right]$$

Problem 1029: Result unnecessarily involves imaginary or complex numbers.

$$\int\sqrt{x}\,(A+B\,x)\,\sqrt{a+b\,x+c\,x^2}\,dx$$

Optimal (type 4, 454 leaves, 6 steps) :

$$-\frac{2\left(5\,a\,b\,B\,c-2\left(b^2-3\,a\,c\right)(4\,b\,B-7\,A\,c)\right)\sqrt{x}\,\sqrt{a+b\,x+c\,x^2}}{105\,c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x\right)}-$$

$$\frac{2\,\sqrt{x}\,\left(4\,b^2\,B-7\,A\,b\,c+5\,a\,B\,c+3\,c\,(4\,b\,B-7\,A\,c)\,x\right)\sqrt{a+b\,x+c\,x^2}}{105\,c^2}+\frac{2\,B\,\sqrt{x}\,\left(a+b\,x+c\,x^2\right)^{3/2}}{7\,c}+\frac{1}{105\,c^{11/4}\,\sqrt{a+b\,x+c\,x^2}}$$

$$2\,a^{1/4}\left(5\,a\,b\,B\,c-2\left(b^2-3\,a\,c\right)(4\,b\,B-7\,A\,c)\right)\left(\sqrt{a}+\sqrt{c}\,x\right)\sqrt{\frac{a+b\,x+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\;\mathrm{EllipticE}\left[2\,\mathrm{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]-$$

$$\frac{1}{105\,c^{11/4}\,\sqrt{a+b\,x+c\,x^2}}a^{1/4}\left(5\,a\,b\,B\,c-2\left(b^2-3\,a\,c\right)(4\,b\,B-7\,A\,c)-\sqrt{a}\,\sqrt{c}\,\left(4\,b^2\,B-7\,A\,b\,c-10\,a\,B\,c\right)\right)$$

$$\left(\sqrt{a}+\sqrt{c}\,x\right)\sqrt{\frac{a+b\,x+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\;\mathrm{EllipticF}\left[2\,\mathrm{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]$$

Result (type 4, 638 leaves) :

$$\frac{2\sqrt{x}\,\sqrt{a+x(b+c x)}\,\left(-4 b^2 B+b c(7 A+3 B x)+c(10 a B+3 c x(7 A+5 B x))\right)}{105 c^2}-\frac{1}{210 c^3\sqrt{\frac{a}{b+\sqrt{b^2-4 a c}}}\,\sqrt{x}\,\sqrt{a+x(b+c x)}}$$

$$\Bigg(-4\left(8 b^3 B-14 A b^2 c-29 a b B c+42 a A c^2\right)\sqrt{\frac{a}{b+\sqrt{b^2-4 a c}}}\,(a+x(b+c x))+i\left(8 b^3 B-14 A b^2 c-29 a b B c+42 a A c^2\right)\left(-b+\sqrt{b^2-4 a c}\right)$$

$$\sqrt{1+\frac{2 a}{\left(b+\sqrt{b^2-4 a c}\right) x}}\;x^{3/2}\sqrt{\frac{4 a+2 b x-2\sqrt{b^2-4 a c}\;x}{b x-\sqrt{b^2-4 a c}\;x}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{a}{b+\sqrt{b^2-4 a c}}}}{\sqrt{x}}\right],\frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]+$$

$$i\left(8 b^4 B+2 a c^2\left(10 a B-21 A\sqrt{b^2-4 a c}\right)-2 b^3\left(7 A c+4 B\sqrt{b^2-4 a c}\right)+a b c\left(56 A c+29 B\sqrt{b^2-4 a c}\right)+b^2\left(-37 a B c+14 A c\sqrt{b^2-4 a c}\right)\right)$$

$$\sqrt{1+\frac{2 a}{\left(b+\sqrt{b^2-4 a c}\right) x}}\;x^{3/2}\sqrt{\frac{4 a+2 b x-2\sqrt{b^2-4 a c}\;x}{b x-\sqrt{b^2-4 a c}\;x}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{a}{b+\sqrt{b^2-4 a c}}}}{\sqrt{x}}\right],\frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\Bigg)$$

- **Problem 1030: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+B x)\sqrt{a+b x+c x^2}}{\sqrt{x}}\,dx$$

Optimal (type 4, 373 leaves, 5 steps):

$$-\frac{2\left(2 b^2 B-5 A b c-6 a B c\right)\sqrt{x}\,\sqrt{a+b x+c x^2}}{15 c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x\right)}+\frac{2\sqrt{x}\,(b B+5 A c+3 B c x)\sqrt{a+b x+c x^2}}{15 c}+\frac{1}{15 c^{7/4}\sqrt{a+b x+c x^2}}$$

$$2 a^{1/4}\left(2 b^2 B-5 A b c-6 a B c\right)\left(\sqrt{a}+\sqrt{c}\,x\right)\sqrt{\frac{a+b x+c x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]-\frac{1}{15 c^{7/4}\sqrt{a+b x+c x^2}}$$

$$a^{1/4}\left(b+2\sqrt{a}\,\sqrt{c}\right)\left(2 b B-3\sqrt{a}\,B\sqrt{c}-5 A c\right)\left(\sqrt{a}+\sqrt{c}\,x\right)\sqrt{\frac{a+b x+c x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]$$

Result (type 4, 550 leaves):

$$\frac{1}{30\sqrt{a+x\,(b+c\,x)}}\left(\frac{4\sqrt{x}\,(b\,B+5\,A\,c+3\,B\,c\,x)\,(a+x\,(b+c\,x))}{c}+\frac{1}{c^2}x\left(-\frac{4\left(2\,b^2\,B-5\,A\,b\,c-6\,a\,B\,c\right)(a+x\,(b+c\,x))}{x^{3/2}}+\frac{1}{\sqrt{\frac{a}{b+\sqrt{b^2-4\,a\,c}}}}\,i\left(2\,b^2\,B-5\,A\,b\,c-6\,a\,B\,c\right)\right.\right.$$

$$\left(-b+\sqrt{b^2-4\,a\,c}\right)\sqrt{2+\frac{4\,a}{\left(b+\sqrt{b^2-4\,a\,c}\right)x}}\sqrt{\frac{2\,a+b\,x-\sqrt{b^2-4\,a\,c}\;x}{b\,x-\sqrt{b^2-4\,a\,c}\;x}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{a}{b+\sqrt{b^2-4\,a\,c}}}}{\sqrt{x}}\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+$$

$$\frac{1}{\sqrt{\frac{a}{b+\sqrt{b^2-4\,a\,c}}}}\,i\left(2\,b^3\,B-b^2\left(5\,A\,c+2\,B\sqrt{b^2-4\,a\,c}\right)+2\,a\,c\left(10\,A\,c+3\,B\sqrt{b^2-4\,a\,c}\right)+b\left(-8\,a\,B\,c+5\,A\,c\sqrt{b^2-4\,a\,c}\right)\right)$$

$$\left.\left.\sqrt{2+\frac{4\,a}{\left(b+\sqrt{b^2-4\,a\,c}\right)x}}\sqrt{\frac{2\,a+b\,x-\sqrt{b^2-4\,a\,c}\;x}{b\,x-\sqrt{b^2-4\,a\,c}\;x}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{a}{b+\sqrt{b^2-4\,a\,c}}}}{\sqrt{x}}\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)\right)$$

- **Problem 1031: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+B\,x)\sqrt{a+b\,x+c\,x^2}}{x^{3/2}}\,dx$$

Optimal (type 4, 341 leaves, 5 steps):

$$-\frac{2\,(3\,A-B\,x)\sqrt{a+b\,x+c\,x^2}}{3\sqrt{x}}+\frac{2\,(b\,B+6\,A\,c)\sqrt{x}\sqrt{a+b\,x+c\,x^2}}{3\sqrt{c}\left(\sqrt{a}+\sqrt{c}\;x\right)}-$$

$$\frac{2\,a^{1/4}\,(b\,B+6\,A\,c)\left(\sqrt{a}+\sqrt{c}\;x\right)\sqrt{\frac{a+b\,x+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\;x\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{3\,c^{3/4}\sqrt{a+b\,x+c\,x^2}}+\frac{1}{3\,a^{1/4}\,c^{3/4}\sqrt{a+b\,x+c\,x^2}}$$

$$\left(b+2\sqrt{a}\sqrt{c}\right)\left(\sqrt{a}\;B+3\,A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}\;x\right)\sqrt{\frac{a+b\,x+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\;x\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]$$

Result (type 4, 491 leaves):

$$\frac{1}{6\sqrt{a+x(b+cx)}}\left(\frac{4(bB+6Ac)(a+x(b+cx))}{c\sqrt{x}}+\frac{4(-3A+Bx)(a+x(b+cx))}{\sqrt{x}}-\right.$$

$$1\Bigg/\left(c\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}\right)\mathrm{i}(bB+6Ac)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{2+\frac{4a}{\left(b+\sqrt{b^2-4ac}\right)x}}\ x\ \sqrt{\frac{2a+bx-\sqrt{b^2-4ac}\ x}{bx-\sqrt{b^2-4ac}\ x}}$$

$$\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}}{\sqrt{x}}\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]+1\Bigg/\left(c\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}\right)\mathrm{i}\left(-b^2B+4aBc+bB\sqrt{b^2-4ac}+6Ac\sqrt{b^2-4ac}\right)$$

$$\left.\sqrt{2+\frac{4a}{\left(b+\sqrt{b^2-4ac}\right)x}}\ x\ \sqrt{\frac{2a+bx-\sqrt{b^2-4ac}\ x}{bx-\sqrt{b^2-4ac}\ x}}\ \text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}}{\sqrt{x}}\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]\right)$$

- **Problem 1032: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{5/2}}\,dx$$

Optimal (type 4, 353 leaves, 5 steps):

$$-\frac{2(aA+(Ab+3aB)x)\sqrt{a+bx+cx^2}}{3ax^{3/2}}+\frac{2(Ab+6aB)\sqrt{c}\sqrt{x}\sqrt{a+bx+cx^2}}{3a\left(\sqrt{a}+\sqrt{c}\,x\right)}-$$

$$\frac{2(Ab+6aB)c^{1/4}\left(\sqrt{a}+\sqrt{c}\,x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]}{3a^{3/4}\sqrt{a+bx+cx^2}}+\frac{1}{3a^{3/4}c^{1/4}\sqrt{a+bx+cx^2}}$$

$$\left((Ab+6aB)\sqrt{c}+\sqrt{a}(3bB+2Ac)\right)\left(\sqrt{a}+\sqrt{c}\,x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]$$

Result (type 4, 499 leaves) :

$$\frac{1}{6\,a\,x^{3/2}\,\sqrt{a+x\,(b+c\,x)}}\left(-4\,(A\,b\,x+a\,(A+3\,B\,x))\,(a+x\,(b+c\,x))+\frac{1}{\sqrt{\frac{a}{b+\sqrt{b^2-4\,a\,c}}}}\,x\left(4\,(A\,b+6\,a\,B)\,\sqrt{\frac{a}{b+\sqrt{b^2-4\,a\,c}}}\,(a+x\,(b+c\,x))+\mathrm{i}\,(A\,b+6\,a\,B)\left(b-\sqrt{b^2-4\,a\,c}\right)\right.\right.$$

$$\sqrt{1+\frac{2\,a}{\left(b+\sqrt{b^2-4\,a\,c}\right)\,x}}\;x^{3/2}\,\sqrt{\frac{4\,a+2\,b\,x-2\,\sqrt{b^2-4\,a\,c}\;x}{b\,x-\sqrt{b^2-4\,a\,c}\;x}}\;\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{\frac{a}{b+\sqrt{b^2-4\,a\,c}}}}{\sqrt{x}}\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+$$

$$\mathrm{i}\left(6\,a\,B\,\sqrt{b^2-4\,a\,c}+A\left(-b^2+4\,a\,c+b\,\sqrt{b^2-4\,a\,c}\right)\right)\sqrt{1+\frac{2\,a}{\left(b+\sqrt{b^2-4\,a\,c}\right)\,x}}\;x^{3/2}$$

$$\left.\left.\sqrt{\frac{4\,a+2\,b\,x-2\,\sqrt{b^2-4\,a\,c}\;x}{b\,x-\sqrt{b^2-4\,a\,c}\;x}}\;\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{\frac{a}{b+\sqrt{b^2-4\,a\,c}}}}{\sqrt{x}}\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)\right)$$

- **Problem 1033: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+B\,x)\,\sqrt{a+b\,x+c\,x^2}}{x^{7/2}}\,dx$$

Optimal (type 4, 421 leaves, 6 steps) :

$$\frac{2\left(2 A b^2-5 a b B-6 a A c\right)\sqrt{a+b x+c x^2}}{15 a^2 \sqrt{x}}-\frac{2\left(3 a A+(A b+5 a B) x\right)\sqrt{a+b x+c x^2}}{15 a x^{5/2}}+$$

$$\frac{2\sqrt{c}\left(5 a b B-2 A\left(b^2-3 a c\right)\right)\sqrt{x}\sqrt{a+b x+c x^2}}{15 a^2\left(\sqrt{a}+\sqrt{c}\, x\right)}-\frac{1}{15 a^{7/4}\sqrt{a+b x+c x^2}}$$

$$2 c^{1/4}\left(5 a b B-2 A\left(b^2-3 a c\right)\right)\left(\sqrt{a}+\sqrt{c}\, x\right)\sqrt{\frac{a+b x+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]-\frac{1}{15 a^{7/4}\sqrt{a+b x+c x^2}}$$

$$\left(b+2\sqrt{a}\sqrt{c}\right)\left(2 A b-5 a B-3\sqrt{a}\, A\sqrt{c}\right) c^{1/4}\left(\sqrt{a}+\sqrt{c}\, x\right)\sqrt{\frac{a+b x+c x^2}{\left(\sqrt{a}+\sqrt{c}\, x\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]$$

Result (type 4, 576 leaves) :

$$\frac{1}{30 a^2 x^{5/2}\sqrt{a+x\,(b+c x)}}\Bigg(-4\left(a+x\,(b+c x)\right)\left(-2 A b^2 x^2+a^2\,(3 A+5 B x)+a x\,(5 b B x+A\,(b+6 c x))\right)+$$

$$\frac{1}{\sqrt{\frac{a}{b+\sqrt{b^2-4 a c}}}}\, x^2\Bigg(4\left(-2 A b^2+5 a b B+6 a A c\right)\sqrt{\frac{a}{b+\sqrt{b^2-4 a c}}}\,\left(a+x\,(b+c x)\right)+i\left(-b+\sqrt{b^2-4 a c}\right)\left(-5 a b B+2 A\left(b^2-3 a c\right)\right)$$

$$\sqrt{1+\frac{2 a}{\left(b+\sqrt{b^2-4 a c}\right) x}}\; x^{3/2}\sqrt{\frac{4 a+2 b x-2\sqrt{b^2-4 a c}\; x}{b x-\sqrt{b^2-4 a c}\; x}}\ \text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{a}{b+\sqrt{b^2-4 a c}}}}{\sqrt{x}}\right],\frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]-$$

$$i\left(5 a B\left(b^2-4 a c-b\sqrt{b^2-4 a c}\right)+2 A\left(-b^3+4 a b c+b^2\sqrt{b^2-4 a c}-3 a c\sqrt{b^2-4 a c}\right)\right)\sqrt{1+\frac{2 a}{\left(b+\sqrt{b^2-4 a c}\right) x}}$$

$$x^{3/2}\sqrt{\frac{4 a+2 b x-2\sqrt{b^2-4 a c}\; x}{b x-\sqrt{b^2-4 a c}\; x}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{a}{b+\sqrt{b^2-4 a c}}}}{\sqrt{x}}\right],\frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\Bigg)\Bigg)$$

- **Problem 1034: Result unnecessarily involves imaginary or complex numbers.**

$$\int (2-5x)\, x^{7/2} \sqrt{2+5x+3x^2}\, dx$$

Optimal (type 4, 251 leaves, 9 steps):

$$\frac{1543648\sqrt{x}\,(2+3x)}{6567561\sqrt{2+5x+3x^2}} - \frac{8\sqrt{x}\,(397265+502911x)\sqrt{2+5x+3x^2}}{2189187} + \frac{157160\sqrt{x}\,\left(2+5x+3x^2\right)^{3/2}}{243243} - \frac{21620\, x^{3/2}\left(2+5x+3x^2\right)^{3/2}}{34749} + \frac{656\, x^{5/2}\left(2+5x+3x^2\right)^{3/2}}{1287} - \frac{10}{39} x^{7/2}\left(2+5x+3x^2\right)^{3/2} - \frac{1543648\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{6567561\sqrt{2+5x+3x^2}} + \frac{349240\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{2189187\sqrt{2+5x+3x^2}}$$

Result (type 4, 178 leaves):

$$\left(2\left(1543648+2811400x+670548x^2-141444x^3+58374x^4+2892348x^5+671895x^6-10195794x^7-7577955x^8\right) + 1543648\, i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\, x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right] - 495928\, i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\, x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right) \Big/ \left(6567561\sqrt{x}\sqrt{2+5x+3x^2}\right)$$

- **Problem 1035: Result unnecessarily involves imaginary or complex numbers.**

$$\int (2-5x)\, x^{5/2} \sqrt{2+5x+3x^2}\, dx$$

Optimal (type 4, 228 leaves, 8 steps):

$$-\frac{261784\sqrt{x}\,(2+3x)}{841995\sqrt{2+5x+3x^2}}+\frac{8\sqrt{x}\,(57860+74313x)\sqrt{2+5x+3x^2}}{280665}-\frac{4420\sqrt{x}\,\left(2+5x+3x^2\right)^{3/2}}{6237}+\frac{532}{891}x^{3/2}\left(2+5x+3x^2\right)^{3/2}-$$

$$\frac{10}{33}x^{5/2}\left(2+5x+3x^2\right)^{3/2}+\frac{261784\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{841995\sqrt{2+5x+3x^2}}-\frac{13016\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{56133\sqrt{2+5x+3x^2}}$$

Result (type 4, 170 leaves):

$$\left(-523568-918440x-198168x^2+39780x^3+947916x^4+271350x^5-\right.$$

$$3129840x^6-2296350x^7-261784\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]+$$

$$\left.66544\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)\Big/\left(841995\sqrt{x}\sqrt{2+5x+3x^2}\right)$$

- **Problem 1036: Result unnecessarily involves imaginary or complex numbers.**

$$\int (2-5x)\,x^{3/2}\sqrt{2+5x+3x^2}\,dx$$

Optimal (type 4, 205 leaves, 7 steps):

$$\frac{2360\sqrt{x}\,(2+3x)}{5103\sqrt{2+5x+3x^2}}-\frac{4\sqrt{x}\,(779+1035x)\sqrt{2+5x+3x^2}}{1701}+\frac{136}{189}\sqrt{x}\left(2+5x+3x^2\right)^{3/2}-\frac{10}{27}x^{3/2}\left(2+5x+3x^2\right)^{3/2}-$$

$$\frac{2360\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{5103\sqrt{2+5x+3x^2}}+\frac{668\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{1701\sqrt{2+5x+3x^2}}$$

Result (type 4, 165 leaves):

$$\frac{1}{5103\sqrt{x}\sqrt{2+5x+3x^2}}\left(4720+7792x+1380x^2+7920x^3+2970x^4-23652x^5-17010x^6+\right.$$

$$\left.2360\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-356\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

- **Problem 1037: Result unnecessarily involves imaginary or complex numbers.**

$$\int (2-5x)\sqrt{x}\sqrt{2+5x+3x^2}\,dx$$

Optimal (type 4, 182 leaves, 6 steps):

$$-\frac{2476\sqrt{x}\,(2+3x)}{2835\sqrt{2+5x+3x^2}}+\frac{4}{945}\sqrt{x}\,(430+639x)\sqrt{2+5x+3x^2}-\frac{10}{21}\sqrt{x}\,\left(2+5x+3x^2\right)^{3/2}+$$

$$\frac{2476\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{2835\sqrt{2+5x+3x^2}}-\frac{164\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{189\sqrt{2+5x+3x^2}}$$

Result (type 4, 163 leaves):

$$\frac{1}{2835\sqrt{x}\sqrt{2+5x+3x^2}}\left(-2\left(2476+3730x-3354x^2-1935x^3+8748x^4+6075x^5\right)-\right.$$

$$\left.2476\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]+16\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

- **Problem 1038: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{\sqrt{x}}\,dx$$

Optimal (type 4, 159 leaves, 5 steps):

$$\frac{88\sqrt{x}\,(2+3x)}{27\sqrt{2+5x+3x^2}}+\frac{2}{9}(1-9x)\sqrt{x}\sqrt{2+5x+3x^2}-$$

$$\frac{88\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{27\sqrt{2+5x+3x^2}}+\frac{34\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{9\sqrt{2+5x+3x^2}}$$

Result (type 4, 158 leaves) :

$$\frac{1}{27\sqrt{x}\sqrt{2+5x+3x^2}}\left(2\left(88+226x+93x^2-126x^3-81x^4\right)+\right.$$

$$\left.88\,\text{i}\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]+14\,\text{i}\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

Problem 1039: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{3/2}}\,dx$$

Optimal (type 4, 159 leaves, 5 steps) :

$$\frac{22\sqrt{x}\,(2+3x)}{9\sqrt{2+5x+3x^2}}-\frac{2(6+5x)\sqrt{2+5x+3x^2}}{3\sqrt{x}}-$$

$$\frac{22\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{9\sqrt{2+5x+3x^2}}+\frac{10\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{3\sqrt{2+5x+3x^2}}$$

Result (type 4, 153 leaves) :

$$\frac{1}{9\sqrt{x}\,\sqrt{2+5x+3x^2}}\left(-2\left(14+65x+96x^2+45x^3\right)+\right.$$

$$\left.22\,i\,\sqrt{2}\,\sqrt{1+\frac{1}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]+8\,i\,\sqrt{2}\,\sqrt{1+\frac{1}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

Problem 1040: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{5/2}}\,dx$$

Optimal (type 4, 157 leaves, 5 steps):

$$-\frac{50\sqrt{x}\,(2+3x)}{3\sqrt{2+5x+3x^2}}-\frac{4\,(1-5x)\sqrt{2+5x+3x^2}}{3\,x^{3/2}}+$$

$$\frac{50\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{3\sqrt{2+5x+3x^2}}-\frac{21\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2+5x+3x^2}}$$

Result (type 4, 153 leaves):

$$\frac{1}{3\,x^{3/2}\sqrt{2+5x+3x^2}}\left(-2\left(4+40x+81x^2+45x^3\right)-\right.$$

$$\left.50\,i\,\sqrt{2}\,\sqrt{1+\frac{1}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{5/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-13\,i\,\sqrt{2}\,\sqrt{1+\frac{1}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{5/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

Problem 1041: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{7/2}}\,dx$$

Optimal (type 4, 180 leaves, 6 steps):

$$-\frac{139\sqrt{x}\,(2+3x)}{15\sqrt{2+5x+3x^2}}-\frac{4\,(3-10x)\sqrt{2+5x+3x^2}}{15\,x^{5/2}}+\frac{139\sqrt{2+5x+3x^2}}{15\sqrt{x}}+\frac{139\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{15\sqrt{2+5x+3x^2}}-\frac{11\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2+5x+3x^2}}$$

Result (type 4, 153 leaves):

$$\frac{1}{15\,x^{5/2}\sqrt{2+5x+3x^2}}\left(4\left(-6+5x+41x^2+30x^3\right)-139\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{7/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-26\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{7/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

Problem 1042: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{9/2}}\,dx$$

Optimal (type 4, 205 leaves, 7 steps):

$$\frac{62\sqrt{x}\,(2+3x)}{21\sqrt{2+5x+3x^2}}-\frac{4\,(1-3x)\sqrt{2+5x+3x^2}}{7\,x^{7/2}}+\frac{43\sqrt{2+5x+3x^2}}{21\,x^{3/2}}-\frac{62\sqrt{2+5x+3x^2}}{21\sqrt{x}}-\frac{62\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{21\sqrt{2+5x+3x^2}}+\frac{43\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{7\sqrt{2}\sqrt{2+5x+3x^2}}$$

Result (type 4, 155 leaves):

$$\frac{1}{42\, x^{7/2}\sqrt{2+5x+3x^2}}\left(-48+24x+460x^2+646x^3+258x^4+\right.$$

$$\left.124\, i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\, x^{9/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]+5\, i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\, x^{9/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

Problem 1043: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{11/2}}\,dx$$

Optimal (type 4, 228 leaves, 8 steps):

$$-\frac{1331\sqrt{x}\,(2+3x)}{630\sqrt{2+5x+3x^2}}-\frac{4(7-20x)\sqrt{2+5x+3x^2}}{63\,x^{9/2}}+\frac{97\sqrt{2+5x+3x^2}}{105\,x^{5/2}}-\frac{79\sqrt{2+5x+3x^2}}{63\,x^{3/2}}+\frac{1331\sqrt{2+5x+3x^2}}{630\sqrt{x}}+$$

$$\frac{1331\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{315\sqrt{2}\sqrt{2+5x+3x^2}}-\frac{79\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{21\sqrt{2}\sqrt{2+5x+3x^2}}$$

Result (type 4, 160 leaves):

$$\frac{1}{630\, x^{9/2}\sqrt{2+5x+3x^2}}\left(-560+200x+4324x^2+3730x^3-2204x^4-2370x^5-\right.$$

$$\left.1331\, i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\, x^{11/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]+146\, i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\, x^{11/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

Problem 1044: Result unnecessarily involves imaginary or complex numbers.

$$\int (2-5x)\, x^{5/2}\left(2+5x+3x^2\right)^{3/2}dx$$

Optimal (type 4, 256 leaves, 9 steps):

$$-\frac{497824\sqrt{x}\,(2+3x)}{32837805\sqrt{2+5x+3x^2}}-\frac{8\sqrt{x}\,(190465+205407x)\sqrt{2+5x+3x^2}}{10945935}+\frac{8\sqrt{x}\,(27010+32921x)\left(2+5x+3x^2\right)^{3/2}}{243243}-\frac{4660\sqrt{x}\left(2+5x+3x^2\right)^{5/2}}{11583}+\frac{136}{351}x^{3/2}\left(2+5x+3x^2\right)^{5/2}-\frac{2}{9}x^{5/2}\left(2+5x+3x^2\right)^{5/2}+\frac{497824\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{32837805\sqrt{2+5x+3x^2}}-\frac{61736\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{2189187\sqrt{2+5x+3x^2}}$$

Result (type 4, 183 leaves) :

$$\left(-497824\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-2\left(497824+318520x-273876x^2+91620x^3-37601118x^4-83323080x^5+69664455x^6+337486905x^7+320800095x^8+98513415x^9+214108\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)\right)\Big/\left(32837805\sqrt{x}\sqrt{2+5x+3x^2}\right)$$

- **Problem 1045: Result unnecessarily involves imaginary or complex numbers.**

$$\int (2-5x)\,x^{3/2}\left(2+5x+3x^2\right)^{3/2}dx$$

Optimal (type 4, 233 leaves, 8 steps) :

$$\frac{55112\sqrt{x}\,(2+3x)}{729729\sqrt{2+5x+3x^2}}+\frac{8\sqrt{x}\,(6908+6381x)\sqrt{2+5x+3x^2}}{243243}-\frac{4\sqrt{x}\,(6959+8575x)\left(2+5x+3x^2\right)^{3/2}}{27027}+\frac{556\sqrt{x}\left(2+5x+3x^2\right)^{5/2}}{1287}-\frac{10}{39}x^{3/2}\left(2+5x+3x^2\right)^{5/2}-\frac{55112\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{729729\sqrt{2+5x+3x^2}}+\frac{25448\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{243243\sqrt{2+5x+3x^2}}$$

Result (type 4, 178 leaves) :

$$\left(-2\left(-55112-61436x+8508x^2-1171602x^3-2497986x^4+1830195x^5+8989785x^6+8374023x^7+2525985x^8\right)+\right.$$
$$55112\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]+$$
$$\left.21232\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)\Big/\left(729729\sqrt{x}\sqrt{2+5x+3x^2}\right)$$

- **Problem 1046: Result unnecessarily involves imaginary or complex numbers.**

$$\int(2-5x)\sqrt{x}\left(2+5x+3x^2\right)^{3/2}dx$$

Optimal (type 4, 210 leaves, 7 steps) :

$$-\frac{424\sqrt{x}\,(2+3x)}{1155\sqrt{2+5x+3x^2}}-\frac{4}{385}\sqrt{x}\,(55+39x)\sqrt{2+5x+3x^2}+\frac{4}{231}\sqrt{x}\,(65+84x)\left(2+5x+3x^2\right)^{3/2}-\frac{10}{33}\sqrt{x}\left(2+5x+3x^2\right)^{5/2}+$$
$$\frac{424\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{1155\sqrt{2+5x+3x^2}}-\frac{36\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{77\sqrt{2+5x+3x^2}}$$

Result (type 4, 173 leaves) :

$$\frac{1}{1155\sqrt{x}\sqrt{2+5x+3x^2}}\left(-424\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-\right.$$
$$\left.2\left(424+520x-3106x^2-6140x^3+3497x^4+17775x^5+16065x^6+4725x^7+58\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)\right)$$

Problem 1047: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{\sqrt{x}}\,dx$$

Optimal (type 4, 187 leaves, 6 steps):

$$\frac{860\sqrt{x}\,(2+3x)}{243\sqrt{2+5x+3x^2}}+\frac{4}{81}\sqrt{x}\,(82+45x)\sqrt{2+5x+3x^2}-\frac{2}{9}\sqrt{x}\,(1+5x)\left(2+5x+3x^2\right)^{3/2}-$$
$$\frac{860\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{243\sqrt{2+5x+3x^2}}+\frac{356\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{81\sqrt{2+5x+3x^2}}$$

Result (type 4, 165 leaves):

$$\frac{1}{243\sqrt{x}\sqrt{2+5x+3x^2}}\left(1720+6052x+6420x^2-1746x^3-9990x^4-8586x^5-2430x^6+\right.$$
$$\left.860\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]+208\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

Problem 1048: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{3/2}}\,dx$$

Optimal (type 4, 187 leaves, 6 steps):

$$\frac{5848\sqrt{x}\,(2+3x)}{315\sqrt{2+5x+3x^2}}+\frac{2}{105}\sqrt{x}\,(1045+531x)\sqrt{2+5x+3x^2}-\frac{2\,(14+5x)\left(2+5x+3x^2\right)^{3/2}}{7\sqrt{x}}-$$
$$\frac{5848\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{315\sqrt{2+5x+3x^2}}+\frac{482\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{21\sqrt{2+5x+3x^2}}$$

Result (type 4, 163 leaves):

$$\frac{1}{315\sqrt{x}\sqrt{2+5x+3x^2}}\left(-2\left(-3328-7390x+177x^2+9855x^3+7641x^4+2025x^5\right)+5848\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]+1382\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

Problem 1049: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{5/2}}\,dx$$

Optimal (type 4, 183 leaves, 6 steps):

$$-\frac{34\sqrt{x}\,(2+3x)}{3\sqrt{2+5x+3x^2}}+\frac{2\,(2-x)\sqrt{2+5x+3x^2}}{\sqrt{x}}-\frac{2\,(2+3x)\left(2+5x+3x^2\right)^{3/2}}{3x^{3/2}}+\frac{34\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{3\sqrt{2+5x+3x^2}}-\frac{14\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2+5x+3x^2}}$$

Result (type 4, 163 leaves):

$$\frac{1}{3x^{3/2}\sqrt{2+5x+3x^2}}\left(-34\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{5/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-2\left(8+74x+195x^2+219x^3+117x^4+27x^5+4\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{5/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)\right)$$

Problem 1050: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{7/2}}\,dx$$

Optimal (type 4, 185 leaves, 6 steps):

$$-\frac{1418\sqrt{x}\,(2+3x)}{15\sqrt{2+5x+3x^2}}+\frac{2\,(89-35x)\sqrt{2+5x+3x^2}}{5\sqrt{x}}-\frac{4\,(3-5x)\,\left(2+5x+3x^2\right)^{3/2}}{15\,x^{5/2}}+$$
$$\frac{1418\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{15\sqrt{2+5x+3x^2}}-\frac{117\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2+5x+3x^2}}$$

Result (type 4, 163 leaves):

$$\frac{1}{15\,x^{5/2}\sqrt{2+5x+3x^2}}\left(-2\left(24+80x+906x^2+2230x^3+1605x^4+225x^5\right)-\right.$$
$$\left.1418\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{7/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-337\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{7/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

- **Problem 1051: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{9/2}}\,dx$$

Optimal (type 4, 187 leaves, 6 steps):

$$-\frac{633\sqrt{x}\,(2+3x)}{7\sqrt{2+5x+3x^2}}+\frac{3\,(22+133x)\sqrt{2+5x+3x^2}}{7\,x^{3/2}}-\frac{4\,(1-2x)\,\left(2+5x+3x^2\right)^{3/2}}{7\,x^{7/2}}+$$
$$\frac{633\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{7\sqrt{2+5x+3x^2}}-\frac{783\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{7\sqrt{2+5x+3x^2}}$$

Result (type 4, 163 leaves):

$$\frac{1}{7\,x^{7/2}\sqrt{2+5x+3x^2}}\left(-2\left(8+24x-72x^2-19x^3+384x^4+315x^5\right)-\right.$$

$$\left.633\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{9/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-150\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{9/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

Problem 1052: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{11/2}}\,dx$$

Optimal (type 4, 210 leaves, 7 steps):

$$-\frac{5438\sqrt{x}\,(2+3x)}{315\sqrt{2+5x+3x^2}}+\frac{5438\sqrt{2+5x+3x^2}}{315\sqrt{x}}+\frac{(1446+4055x)\sqrt{2+5x+3x^2}}{315\,x^{5/2}}-\frac{4\,(7-15x)\left(2+5x+3x^2\right)^{3/2}}{63\,x^{9/2}}+$$

$$\frac{5438\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{315\sqrt{2+5x+3x^2}}-\frac{899\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{21\sqrt{2}\sqrt{2+5x+3x^2}}$$

Result (type 4, 160 leaves):

$$\frac{1}{630\,x^{9/2}\sqrt{2+5x+3x^2}}\left(-1120-3200x+7424x^2+44\,480x^3+64\,706x^4+29\,730x^5-\right.$$

$$\left.10\,876\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{11/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-2609\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{11/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

Problem 1053: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{13/2}}\,dx$$

Optimal (type 4, 233 leaves, 8 steps) :

$$\frac{3229\sqrt{x}\,(2+3x)}{1386\sqrt{2+5x+3x^2}}+\frac{1357\sqrt{2+5x+3x^2}}{693\,x^{3/2}}-\frac{3229\sqrt{2+5x+3x^2}}{1386\sqrt{x}}+\frac{(634+1367x)\sqrt{2+5x+3x^2}}{231\,x^{7/2}}-\frac{4\,(9-20x)\,(2+5x+3x^2)^{3/2}}{99\,x^{11/2}}-$$

$$\frac{3229\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{693\sqrt{2}\sqrt{2+5x+3x^2}}+\frac{1357\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{231\sqrt{2}\sqrt{2+5x+3x^2}}$$

Result (type 4, 165 leaves) :

$$\frac{1}{1386\,x^{11/2}\sqrt{2+5x+3x^2}}\left(-2016-5600x+11360x^2+61744x^3+86914x^4+48256x^5+8142x^6+\right.$$

$$\left.3229\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{13/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]+842\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{13/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

■ Problem 1054: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(2-5x)\,(2+5x+3x^2)^{3/2}}{x^{15/2}}\,dx$$

Optimal (type 4, 256 leaves, 9 steps) :

$$-\frac{6907\sqrt{x}\,(2+3x)}{10010\sqrt{2+5x+3x^2}}+\frac{204\sqrt{2+5x+3x^2}}{385\,x^{5/2}}-\frac{1231\sqrt{2+5x+3x^2}}{2002\,x^{3/2}}+\frac{6907\sqrt{2+5x+3x^2}}{10010\sqrt{x}}+\frac{(1834+3445x)\sqrt{2+5x+3x^2}}{1001\,x^{9/2}}-$$

$$\frac{4\,(11-25x)\,(2+5x+3x^2)^{3/2}}{143\,x^{13/2}}+\frac{6907\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{5005\sqrt{2}\sqrt{2+5x+3x^2}}-\frac{3693\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{2002\sqrt{2}\sqrt{2+5x+3x^2}}$$

Result (type 4, 170 leaves) :

$$\left(-24640-67200\,x+125440\,x^2+654400\,x^3+840316\,x^4+\right.$$

$$361120\,x^5-29726\,x^6-36930\,x^7-13814\,i\,\sqrt{2}\,\sqrt{1+\frac{1}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{15/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-$$

$$\left.4651\,i\,\sqrt{2}\,\sqrt{1+\frac{1}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{15/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)\Big/\left(20020\,x^{13/2}\,\sqrt{2+5\,x+3\,x^2}\right)$$

- **Problem 1055: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B\,x}{\sqrt{e\,x}\,\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 300 leaves, 5 steps) :

$$\frac{2\,B\,x\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{c}\,\sqrt{e\,x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)}-\frac{2\,a^{1/4}\,B\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+b\,x+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{c^{3/4}\,\sqrt{e\,x}\,\sqrt{a+b\,x+c\,x^2}}+$$

$$\frac{a^{1/4}\,\left(B+\frac{A\,\sqrt{c}}{\sqrt{a}}\right)\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x\right)\,\sqrt{\frac{a+b\,x+c\,x^2}{\left(\sqrt{a}+\sqrt{c}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{x}}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]}{c^{3/4}\,\sqrt{e\,x}\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 444 leaves) :

$$-\left(x^2\left(-\frac{4B\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}\,(a+x\,(b+c\,x))}{x^2}+1\Big/\left(\sqrt{x}\right)\,\mathrm{i}\,B\left(-b+\sqrt{b^2-4ac}\right)\sqrt{2+\frac{4a}{\left(b+\sqrt{b^2-4ac}\right)x}}\,\sqrt{\frac{2a+b\,x-\sqrt{b^2-4ac}\,x}{b\,x-\sqrt{b^2-4ac}\,x}}\right.\right.$$

$$\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}}{\sqrt{x}}\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]-1\Big/\left(\sqrt{x}\right)\mathrm{i}\left(-b\,B+2\,A\,c+B\sqrt{b^2-4ac}\right)\sqrt{2+\frac{4a}{\left(b+\sqrt{b^2-4ac}\right)x}}$$

$$\left.\left.\sqrt{\frac{2a+b\,x-\sqrt{b^2-4ac}\,x}{b\,x-\sqrt{b^2-4ac}\,x}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}}{\sqrt{x}}\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]\right)\right)\Big/\left(2\,c\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}\,\sqrt{e\,x}\,\sqrt{a+x\,(b+c\,x)}\right)$$

- **Problem 1056: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(2-5x)\,x^{7/2}}{\sqrt{2+5x+3x^2}}\,dx$$

Optimal (type 4, 223 leaves, 8 steps) :

$$-\frac{68920\sqrt{x}\,(2+3x)}{15309\sqrt{2+5x+3x^2}}+\frac{11320\sqrt{x}\,\sqrt{2+5x+3x^2}}{5103}-\frac{820}{567}x^{3/2}\sqrt{2+5x+3x^2}+\frac{508}{567}x^{5/2}\sqrt{2+5x+3x^2}-\frac{10}{27}x^{7/2}\sqrt{2+5x+3x^2}+$$

$$\frac{68920\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{15309\sqrt{2+5x+3x^2}}-\frac{11320\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{5103\sqrt{2+5x+3x^2}}$$

Result (type 4, 168 leaves) :

$$\frac{1}{15309\sqrt{x}\sqrt{2+5x+3x^2}}\left(-2\left(68920+138340x+40620x^2-9306x^3+4590x^4-6399x^5+8505x^6\right)-\right.$$

$$\left.68920\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]+34960\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

Problem 1057: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(2-5x)\,x^{5/2}}{\sqrt{2+5x+3x^2}}\,dx$$

Optimal (type 4, 200 leaves, 7 steps):

$$\frac{13688\sqrt{x}\,(2+3x)}{2835\sqrt{2+5x+3x^2}}-\frac{412}{189}\sqrt{x}\sqrt{2+5x+3x^2}+\frac{128}{105}x^{3/2}\sqrt{2+5x+3x^2}-\frac{10}{21}x^{5/2}\sqrt{2+5x+3x^2}-$$

$$\frac{13688\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{2835\sqrt{2+5x+3x^2}}+\frac{412\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{189\sqrt{2+5x+3x^2}}$$

Result (type 4, 160 leaves):

$$\frac{1}{2835\sqrt{x}\sqrt{2+5x+3x^2}}\left(27376+56080x+17076x^2-3960x^3+3618x^4-4050x^5+\right.$$

$$\left.13688\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-7508\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

Problem 1058: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(2-5x)\,x^{3/2}}{\sqrt{2+5x+3x^2}}\,dx$$

Optimal (type 4, 177 leaves, 6 steps):

$$-\frac{412\sqrt{x}\,(2+3x)}{81\sqrt{2+5x+3x^2}}+\frac{52}{27}\sqrt{x}\,\sqrt{2+5x+3x^2}-\frac{2}{3}x^{3/2}\sqrt{2+5x+3x^2}+\frac{412\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{81\sqrt{2+5x+3x^2}}-\frac{52\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{27\sqrt{2+5x+3x^2}}$$

Result (type 4, 158 leaves):

$$\frac{1}{81\sqrt{x}\,\sqrt{2+5x+3x^2}}\left(-2\left(412+874x+282x^2-99x^3+81x^4\right)-412\,i\sqrt{2}\,\sqrt{1+\frac{1}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]+256\,i\sqrt{2}\,\sqrt{1+\frac{1}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

- **Problem 1059: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(2-5x)\sqrt{x}}{\sqrt{2+5x+3x^2}}\,dx$$

Optimal (type 4, 154 leaves, 5 steps):

$$\frac{136\sqrt{x}\,(2+3x)}{27\sqrt{2+5x+3x^2}}-\frac{10}{9}\sqrt{x}\,\sqrt{2+5x+3x^2}-\frac{136\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{27\sqrt{2+5x+3x^2}}+\frac{10\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{9\sqrt{2+5x+3x^2}}$$

Result (type 4, 150 leaves):

$$\frac{1}{27\sqrt{x}\sqrt{2+5x+3x^2}}\left(272+620x+258x^2-90x^3+\right.$$

$$\left.136\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-106\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

- **Problem 1060: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{2-5x}{\sqrt{x}\sqrt{2+5x+3x^2}}\,dx$$

Optimal (type 4, 129 leaves, 4 steps) :

$$-\frac{10\sqrt{x}\,(2+3x)}{3\sqrt{2+5x+3x^2}}+\frac{10\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{3\sqrt{2+5x+3x^2}}+\frac{2\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2+5x+3x^2}}$$

Result (type 4, 150 leaves) :

$$-\frac{1}{3\sqrt{2+5x+3x^2}}$$

$$2x^{3/2}\left(5\left(3+\frac{2}{x^2}+\frac{5}{x}\right)+\frac{5\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]}{\sqrt{x}}-\frac{8\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]}{\sqrt{x}}\right)$$

- **Problem 1061: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{2-5x}{x^{3/2}\sqrt{2+5x+3x^2}}\,dx$$

Optimal (type 4, 146 leaves, 5 steps) :

$$\frac{2\sqrt{x}\,(2+3x)}{\sqrt{2+5x+3x^2}}-\frac{2\sqrt{2+5x+3x^2}}{\sqrt{x}}-\frac{2\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2+5x+3x^2}}-\frac{5\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2+5x+3x^2}}$$

Result (type 4, 90 leaves):

$$\frac{i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}\,x\left(2\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-7\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)}{\sqrt{2+5x+3x^2}}$$

Problem 1062: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{2-5x}{x^{5/2}\sqrt{2+5x+3x^2}}\,dx$$

Optimal (type 4, 175 leaves, 6 steps):

$$-\frac{25\sqrt{x}\,(2+3x)}{3\sqrt{2+5x+3x^2}}-\frac{2\sqrt{2+5x+3x^2}}{3x^{3/2}}+\frac{25\sqrt{2+5x+3x^2}}{3\sqrt{x}}+\frac{25\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{3\sqrt{2+5x+3x^2}}-\frac{\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2+5x+3x^2}}$$

Result (type 4, 148 leaves):

$$\frac{1}{3x^{3/2}\sqrt{2+5x+3x^2}}\left(-2\left(2+5x+3x^2\right)-25\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{5/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]+22\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{5/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

Problem 1063: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{2-5x}{x^{7/2}\sqrt{2+5x+3x^2}}\,dx$$

Optimal (type 4, 196 leaves, 7 steps):

$$\frac{66\sqrt{x}\,(2+3x)}{5\sqrt{2+5x+3x^2}}-\frac{2\sqrt{2+5x+3x^2}}{5x^{5/2}}+\frac{3\sqrt{2+5x+3x^2}}{x^{3/2}}-\frac{66\sqrt{2+5x+3x^2}}{5\sqrt{x}}-$$

$$\frac{66\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{5\sqrt{2+5x+3x^2}}+\frac{9\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2}\sqrt{2+5x+3x^2}}$$

Result (type 4, 150 leaves) :

$$\frac{1}{10x^{5/2}\sqrt{2+5x+3x^2}}\left(-8+40x+138x^2+90x^3+\right.$$

$$\left.132\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{7/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-87\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{7/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

- **Problem 1064: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(2-5x)\,x^{7/2}}{\left(2+5x+3x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 197 leaves, 7 steps) :

$$-\frac{24\sqrt{x}\,(2+3x)}{\sqrt{2+5x+3x^2}}+\frac{2x^{5/2}\,(74+95x)}{3\sqrt{2+5x+3x^2}}+20\sqrt{x}\sqrt{2+5x+3x^2}-\frac{64}{3}x^{3/2}\sqrt{2+5x+3x^2}+$$

$$\frac{24\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2+5x+3x^2}}-\frac{20\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2+5x+3x^2}}$$

Result (type 4, 156 leaves) :

$$\frac{1}{3\sqrt{x}\,\sqrt{2+5x+3x^2}}\left(-2\left(72+120x+22x^2-4x^3+x^4\right)-72\,i\,\sqrt{2}\,\sqrt{1+\frac{1}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]+12\,i\,\sqrt{2}\,\sqrt{1+\frac{1}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

Problem 1065: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(2-5x)\,x^{5/2}}{\left(2+5x+3x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 182 leaves, 6 steps):

$$\frac{1804\sqrt{x}\,(2+3x)}{81\sqrt{2+5x+3x^2}}+\frac{2x^{3/2}\,(74+95x)}{3\sqrt{2+5x+3x^2}}-\frac{580}{27}\sqrt{x}\,\sqrt{2+5x+3x^2}-\frac{1804\sqrt{2}\,(1+x)\,\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{81\sqrt{2+5x+3x^2}}+\frac{580\sqrt{2}\,(1+x)\,\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{27\sqrt{2+5x+3x^2}}$$

Result (type 4, 150 leaves):

$$\frac{1}{81\sqrt{x}\,\sqrt{2+5x+3x^2}}\left(3608+5540x+708x^2-90x^3+1804\,i\,\sqrt{2}\,\sqrt{1+\frac{1}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-64\,i\,\sqrt{2}\,\sqrt{1+\frac{1}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

Problem 1066: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(2-5x)\,x^{3/2}}{\left(2+5x+3x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 159 leaves, 5 steps):

$$-\frac{200\sqrt{x}\,(2+3x)}{9\sqrt{2+5x+3x^2}}+\frac{2\sqrt{x}\,(74+95x)}{3\sqrt{2+5x+3x^2}}+$$

$$\frac{200\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{9\sqrt{2+5x+3x^2}}-\frac{74\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{3\sqrt{2+5x+3x^2}}$$

Result (type 4, 145 leaves):

$$\frac{1}{9\sqrt{x}\sqrt{2+5x+3x^2}}\left(-400-556x-30x^2-\right.$$

$$\left.200\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-22\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

- **Problem 1067: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(2-5x)\sqrt{x}}{\left(2+5x+3x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 155 leaves, 5 steps):

$$\frac{74\sqrt{x}\,(2+3x)}{3\sqrt{2+5x+3x^2}}-\frac{2\sqrt{x}\,(30+37x)}{\sqrt{2+5x+3x^2}}-\frac{74\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{3\sqrt{2+5x+3x^2}}+\frac{30\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2+5x+3x^2}}$$

Result (type 4, 140 leaves):

$$\frac{1}{3\sqrt{x}\sqrt{2+5x+3x^2}}$$

$$\left(148+190x+74\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]+16\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

- **Problem 1068: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{2-5x}{\sqrt{x}\,\left(2+5x+3x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 151 leaves, 5 steps):

$$-\frac{30\sqrt{x}\,(2+3x)}{\sqrt{2+5x+3x^2}}+\frac{2\sqrt{x}\,(38+45x)}{\sqrt{2+5x+3x^2}}+\frac{30\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2+5x+3x^2}}-\frac{37\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2+5x+3x^2}}$$

Result (type 4, 137 leaves):

$$\frac{1}{\sqrt{x}\,\sqrt{2+5x+3x^2}}\left(-60-74x-30\,i\,\sqrt{2}\,\sqrt{1+\frac{1}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-7\,i\,\sqrt{2}\,\sqrt{1+\frac{1}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

- **Problem 1069: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{2-5x}{x^{3/2}\,\left(2+5x+3x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 172 leaves, 6 steps):

$$\frac{39\sqrt{x}\,(2+3x)}{\sqrt{2+5x+3x^2}}+\frac{2\,(38+45x)}{\sqrt{x}\,\sqrt{2+5x+3x^2}}-\frac{39\sqrt{2+5x+3x^2}}{\sqrt{x}}-$$
$$\frac{39\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2+5x+3x^2}}+\frac{45\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2+5x+3x^2}}$$

Result (type 4, 137 leaves):

$$\frac{1}{\sqrt{x}\,\sqrt{2+5x+3x^2}}\left(76+90x+39\,i\sqrt{2}\,\sqrt{1+\frac{1}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]+6\,i\sqrt{2}\,\sqrt{1+\frac{1}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

- **Problem 1070: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{2-5x}{x^{5/2}\left(2+5x+3x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 201 leaves, 7 steps) :

$$-\frac{170\sqrt{x}\,(2+3x)}{3\sqrt{2+5x+3x^2}}+\frac{2\,(38+45x)}{x^{3/2}\sqrt{2+5x+3x^2}}-\frac{115\sqrt{2+5x+3x^2}}{3x^{3/2}}+\frac{170\sqrt{2+5x+3x^2}}{3\sqrt{x}}+$$
$$\frac{170\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{3\sqrt{2+5x+3x^2}}-\frac{115\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2}\,\sqrt{2+5x+3x^2}}$$

Result (type 4, 145 leaves) :

$$\frac{1}{6x^{3/2}\sqrt{2+5x+3x^2}}\left(-4-610x-690x^2-\right.$$
$$\left.340\,i\sqrt{2}\,\sqrt{1+\frac{1}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{5/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-5\,i\sqrt{2}\,\sqrt{1+\frac{1}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{5/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

- **Problem 1071: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{2-5x}{x^{7/2}\left(2+5x+3x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 224 leaves, 8 steps) :

$$\frac{2693\sqrt{x}\,(2+3x)}{30\sqrt{2+5x+3x^2}}+\frac{2\,(38+45x)}{x^{5/2}\sqrt{2+5x+3x^2}}-\frac{191\sqrt{2+5x+3x^2}}{5x^{5/2}}+\frac{157\sqrt{2+5x+3x^2}}{3x^{3/2}}-\frac{2693\sqrt{2+5x+3x^2}}{30\sqrt{x}}-$$

$$\frac{2693\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{15\sqrt{2}\sqrt{2+5x+3x^2}}+\frac{157\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2}\sqrt{2+5x+3x^2}}$$

Result (type 4, 150 leaves) :

$$\frac{1}{30x^{5/2}\sqrt{2+5x+3x^2}}\left(-12+110x+4412x^2+4710x^3+\right.$$

$$\left.2693\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{7/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-338\,i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{7/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

- **Problem 1072: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(2-5x)\,x^{13/2}}{\left(2+5x+3x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 256 leaves, 9 steps) :

$$\frac{2x^{11/2}\,(74+95x)}{9\left(2+5x+3x^2\right)^{3/2}}-\frac{1521056\sqrt{x}\,(2+3x)}{76545\sqrt{2+5x+3x^2}}-\frac{4x^{7/2}\,(1484+1685x)}{27\sqrt{2+5x+3x^2}}+$$

$$\frac{211144\sqrt{x}\sqrt{2+5x+3x^2}}{5103}-\frac{167336x^{3/2}\sqrt{2+5x+3x^2}}{2835}+\frac{45820}{567}x^{5/2}\sqrt{2+5x+3x^2}+$$

$$\frac{1521056\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{76545\sqrt{2+5x+3x^2}}-\frac{211144\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{5103\sqrt{2+5x+3x^2}}$$

Result (type 4, 187 leaves) :

$$\left(-2\left(3042112+8876240x+5504080x^2-2967300x^3-2106756x^4+262710x^5-70956x^6+18225x^7\right)-\right.$$

$$1521056\,i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\left(2+5x+3x^2\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-$$

$$\left.1646104\,i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\left(2+5x+3x^2\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)\Big/\left(76545\sqrt{x}\left(2+5x+3x^2\right)^{3/2}\right)$$

- **Problem 1073: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(2-5x)\,x^{11/2}}{\left(2+5x+3x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 233 leaves, 8 steps):

$$\frac{2x^{9/2}(74+95x)}{9\left(2+5x+3x^2\right)^{3/2}}+\frac{33608\sqrt{x}\,(2+3x)}{729\sqrt{2+5x+3x^2}}-\frac{8x^{5/2}(773+905x)}{27\sqrt{2+5x+3x^2}}-\frac{16040}{243}\sqrt{x}\sqrt{2+5x+3x^2}+\frac{2348}{27}x^{3/2}\sqrt{2+5x+3x^2}-$$

$$\frac{33608\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{729\sqrt{2+5x+3x^2}}+\frac{16040\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{243\sqrt{2+5x+3x^2}}$$

Result (type 4, 179 leaves):

$$\frac{1}{729\sqrt{x}\left(2+5x+3x^2\right)^{3/2}}\left(134\,432+479\,680\,x+534\,680\,x^2+161\,784\,x^3-21\,276\,x^4+\right.$$

$$2484\,x^5-486\,x^6+33\,608\,\mathrm{i}\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\left(2+5x+3x^2\right)\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]+$$

$$\left.14\,512\,\mathrm{i}\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\left(2+5x+3x^2\right)\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

- **Problem 1074: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(2-5x)\,x^{9/2}}{\left(2+5x+3x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 210 leaves, 7 steps):

$$\frac{2x^{7/2}(74+95x)}{9\left(2+5x+3x^2\right)^{3/2}}-\frac{17\,512\sqrt{x}\,(2+3x)}{243\sqrt{2+5x+3x^2}}-\frac{4x^{3/2}(536+645x)}{9\sqrt{2+5x+3x^2}}+\frac{7540}{81}\sqrt{x}\sqrt{2+5x+3x^2}+$$

$$\frac{17\,512\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{243\sqrt{2+5x+3x^2}}-\frac{7540\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{81\sqrt{2+5x+3x^2}}$$

Result (type 4, 177 leaves):

$$\frac{1}{243 \sqrt{x} \left(2+5 x+3 x^2\right)^{3/2}} \left(-2 \left(35024+129880 x+155660 x^2+58590 x^3-1512 x^4+135 x^5\right)-17512 i \sqrt{2+\frac{2}{x}} \sqrt{3+\frac{2}{x}} x^{3/2} \left(2+5 x+3 x^2\right) \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right], \frac{3}{2}\right]-\right.$$

$$\left.5108 i \sqrt{2+\frac{2}{x}} \sqrt{3+\frac{2}{x}} x^{3/2} \left(2+5 x+3 x^2\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right], \frac{3}{2}\right]\right)$$

- **Problem 1075: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(2-5 x) x^{7/2}}{\left(2+5 x+3 x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 187 leaves, 6 steps):

$$\frac{2 x^{5/2} (74+95 x)}{9 \left(2+5 x+3 x^2\right)^{3/2}}+\frac{8020 \sqrt{x} (2+3 x)}{81 \sqrt{2+5 x+3 x^2}}-\frac{40 \sqrt{x} (167+206 x)}{27 \sqrt{2+5 x+3 x^2}}-$$

$$\frac{8020 \sqrt{2} (1+x) \sqrt{\frac{2+3 x}{1+x}} \text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right], -\frac{1}{2}\right]}{81 \sqrt{2+5 x+3 x^2}}+\frac{3340 \sqrt{2} (1+x) \sqrt{\frac{2+3 x}{1+x}} \text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right], -\frac{1}{2}\right]}{27 \sqrt{2+5 x+3 x^2}}$$

Result (type 4, 169 leaves):

$$\frac{1}{81\sqrt{x}\,(2+5x+3x^2)^{3/2}}\left(32080+120320x+147100x^2+58212x^3-270x^4+8020\,i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}(2+5x+3x^2)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]+\right.$$

$$\left.2000\,i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}(2+5x+3x^2)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

Problem 1076: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(2-5x)\,x^{5/2}}{(2+5x+3x^2)^{5/2}}\,dx$$

Optimal (type 4, 187 leaves, 6 steps) :

$$\frac{2x^{3/2}(74+95x)}{9(2+5x+3x^2)^{3/2}}-\frac{3464\sqrt{x}\,(2+3x)}{27\sqrt{2+5x+3x^2}}+\frac{4\sqrt{x}\,(715+866x)}{9\sqrt{2+5x+3x^2}}+$$

$$\frac{3464\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{27\sqrt{2+5x+3x^2}}-\frac{1430\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{9\sqrt{2+5x+3x^2}}$$

Result (type 4, 167 leaves) :

$$\frac{1}{27\sqrt{x}\,(2+5x+3x^2)^{3/2}}\left(-2\left(6928+26060x+32020x^2+12825x^3\right)-3464\,i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}(2+5x+3x^2)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-\right.$$

$$\left.826\,i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}(2+5x+3x^2)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

Problem 1077: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(2-5x)\,x^{3/2}}{\left(2+5x+3x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 187 leaves, 6 steps):

$$\frac{2\sqrt{x}\,(74+95x)}{9\left(2+5x+3x^2\right)^{3/2}}+\frac{1450\sqrt{x}\,(2+3x)}{9\sqrt{2+5x+3x^2}}-\frac{2\sqrt{x}\,(1831+2175x)}{9\sqrt{2+5x+3x^2}}-$$
$$\frac{1450\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{9\sqrt{2+5x+3x^2}}+\frac{598\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{3\sqrt{2+5x+3x^2}}$$

Result (type 4, 164 leaves):

$$\frac{1}{9\sqrt{x}\,\left(2+5x+3x^2\right)^{3/2}}\left(5800+21824x+26830x^2+10764x^3+1450\,i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\left(2+5x+3x^2\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]+\right.$$
$$\left.344\,i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\left(2+5x+3x^2\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

Problem 1078: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(2-5x)\,\sqrt{x}}{\left(2+5x+3x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 179 leaves, 6 steps):

$$-\frac{2\sqrt{x}\,(30+37x)}{3\left(2+5x+3x^2\right)^{3/2}}-\frac{198\sqrt{x}\,(2+3x)}{\sqrt{2+5x+3x^2}}+\frac{2\sqrt{x}\,(250+297x)}{\sqrt{2+5x+3x^2}}+$$
$$\frac{198\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2+5x+3x^2}}-\frac{245\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2+5x+3x^2}}$$

Result (type 4, 165 leaves):

$$-\frac{2\left(1188+4470x+5494x^2+2205x^3\right)}{3\sqrt{x}\left(2+5x+3x^2\right)^{3/2}}-\frac{198\,i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}\,x\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]}{\sqrt{2+5x+3x^2}}-\frac{47\,i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}\,x\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]}{\sqrt{2+5x+3x^2}}$$

- **Problem 1079: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{2-5x}{\sqrt{x}\left(2+5x+3x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 185 leaves, 6 steps):

$$\frac{2\sqrt{x}\,(38+45x)}{3\left(2+5x+3x^2\right)^{3/2}}+\frac{715\sqrt{x}\,(2+3x)}{3\sqrt{2+5x+3x^2}}-\frac{5\sqrt{x}\,(361+429x)}{3\sqrt{2+5x+3x^2}}-\frac{715\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{3\sqrt{2+5x+3x^2}}+\frac{295\sqrt{2}\,(1+x)\sqrt{\frac{2+3x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2+5x+3x^2}}$$

Result (type 4, 167 leaves):

$$\frac{1}{3\sqrt{x}\left(2+5x+3x^2\right)^{3/2}}\left(2\left(1430+5383x+6615x^2+2655x^3\right)+715\,i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\left(2+5x+3x^2\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]+170\,i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}\,x^{3/2}\left(2+5x+3x^2\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

- **Problem 1080: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{2-5x}{x^{3/2}\left(2+5x+3x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 208 leaves, 7 steps):

$$\frac{2\,(38+45\,x)}{3\,\sqrt{x}\,\left(2+5\,x+3\,x^2\right)^{3/2}}-\frac{838\,\sqrt{x}\,(2+3\,x)}{3\,\sqrt{2+5\,x+3\,x^2}}-\frac{1717+2085\,x}{3\,\sqrt{x}\,\sqrt{2+5\,x+3\,x^2}}+\frac{838\,\sqrt{2+5\,x+3\,x^2}}{3\,\sqrt{x}}+$$

$$\frac{838\,\sqrt{2}\,(1+x)\,\sqrt{\frac{2+3\,x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{3\,\sqrt{2+5\,x+3\,x^2}}-\frac{695\,(1+x)\,\sqrt{\frac{2+3\,x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2}\,\sqrt{2+5\,x+3\,x^2}}$$

Result (type 4, 167 leaves) :

$$\frac{1}{6\,\sqrt{x}\,\left(2+5\,x+3\,x^2\right)^{3/2}}\left(-2\,\left(3358+12\,665\,x+15\,576\,x^2+6255\,x^3\right)-1676\,i\,\sqrt{2+\frac{2}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\left(2+5\,x+3\,x^2\right)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]-\right.$$

$$\left.409\,i\,\sqrt{2+\frac{2}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{3/2}\,\left(2+5\,x+3\,x^2\right)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\frac{3}{2}\right]\right)$$

- **Problem 1081: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{2-5\,x}{x^{5/2}\,\left(2+5\,x+3\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 225 leaves, 8 steps) :

$$\frac{2\,(38+45\,x)}{3\,x^{3/2}\,\left(2+5\,x+3\,x^2\right)^{3/2}}+\frac{625\,\sqrt{x}\,(2+3\,x)}{2\,\sqrt{2+5\,x+3\,x^2}}-\frac{3\,(181+225\,x)}{x^{3/2}\,\sqrt{2+5\,x+3\,x^2}}+\frac{265\,\sqrt{2+5\,x+3\,x^2}}{x^{3/2}}-\frac{625\,\sqrt{2+5\,x+3\,x^2}}{2\,\sqrt{x}}-$$

$$\frac{625\,(1+x)\,\sqrt{\frac{2+3\,x}{1+x}}\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2}\,\sqrt{2+5\,x+3\,x^2}}+\frac{795\,(1+x)\,\sqrt{\frac{2+3\,x}{1+x}}\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],-\frac{1}{2}\right]}{\sqrt{2}\,\sqrt{2+5\,x+3\,x^2}}$$

Result (type 4, 169 leaves) :

$$\frac{1}{6\,x^{3/2}\,\left(2+5\,x+3\,x^2\right)^{3/2}}\left(-4+7590\,x+28\,806\,x^2+35\,550\,x^3+14\,310\,x^4+1875\,i\,\sqrt{2+\frac{2}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{5/2}\,\left(2+5\,x+3\,x^2\right)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\,\frac{3}{2}\right]+\right.$$

$$\left.510\,i\,\sqrt{2+\frac{2}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{5/2}\,\left(2+5\,x+3\,x^2\right)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\,\frac{3}{2}\right]\right)$$

- **Problem 1082: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{2-5\,x}{x^{7/2}\,\left(2+5\,x+3\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 256 leaves, 9 steps) :

$$\frac{2\,(38+45\,x)}{3\,x^{5/2}\,\left(2+5\,x+3\,x^2\right)^{3/2}}-\frac{9521\,\sqrt{x}\,\,(2+3\,x)}{30\,\sqrt{2+5\,x+3\,x^2}}-\frac{1541+1965\,x}{3\,x^{5/2}\,\sqrt{2+5\,x+3\,x^2}}+\frac{1252\,\sqrt{2+5\,x+3\,x^2}}{5\,x^{5/2}}-\frac{1733\,\sqrt{2+5\,x+3\,x^2}}{6\,x^{3/2}}+$$

$$\frac{9521\,\sqrt{2+5\,x+3\,x^2}}{30\,\sqrt{x}}+\frac{9521\,(1+x)\,\sqrt{\frac{2+3\,x}{1+x}}\,\,\text{EllipticE}\left[\text{ArcTan}\left[\sqrt{x}\right],\,-\frac{1}{2}\right]}{15\,\sqrt{2}\,\sqrt{2+5\,x+3\,x^2}}-\frac{1733\,(1+x)\,\sqrt{\frac{2+3\,x}{1+x}}\,\,\text{EllipticF}\left[\text{ArcTan}\left[\sqrt{x}\right],\,-\frac{1}{2}\right]}{2\,\sqrt{2}\,\sqrt{2+5\,x+3\,x^2}}$$

Result (type 4, 177 leaves) :

$$\frac{1}{60\,x^{5/2}\,\left(2+5\,x+3\,x^2\right)^{3/2}}$$

$$\left(-2\,\left(12-130\,x+39\,836\,x^2+154\,195\,x^3+192\,342\,x^4+77\,985\,x^5\right)-19\,042\,i\,\sqrt{2+\frac{2}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{7/2}\,\left(2+5\,x+3\,x^2\right)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\,\frac{3}{2}\right]-\right.$$

$$\left.6953\,i\,\sqrt{2+\frac{2}{x}}\,\sqrt{3+\frac{2}{x}}\,x^{7/2}\,\left(2+5\,x+3\,x^2\right)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right],\,\frac{3}{2}\right]\right)$$

Problem 1087: Result unnecessarily involves higher level functions.

$$\int \frac{(e\,x)^m\,(A+B\,x)}{\left(a+b\,x+c\,x^2\right)^2}\,dx$$

Optimal (type 5, 318 leaves, 5 steps):

$$\frac{(e\,x)^{1+m}\left(A\,b^2-a\,b\,B-2\,a\,A\,c+(A\,b-2\,a\,B)\,c\,x\right)}{a\left(b^2-4\,a\,c\right)e\left(a+b\,x+c\,x^2\right)}-$$

$$\left(c\left(A\,b\left(b+\sqrt{b^2-4\,a\,c}\right)m-2\,a\left(b\,B-2\,A\,c\,(1-m)+B\sqrt{b^2-4\,a\,c}\;m\right)\right)(e\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\,1+m,\,2+m,\,-\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(a\left(b^2-4\,a\,c\right)^{3/2}\left(b-\sqrt{b^2-4\,a\,c}\right)e\,(1+m)\right)-\frac{c\left((A\,b-2\,a\,B)\,m+\frac{2\,a\,(b\,B-2\,A\,c\,(1-m))-A\,b^2\,m}{\sqrt{b^2-4\,a\,c}}\right)(e\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\,1+m,\,2+m,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}\right]}{a\left(b^2-4\,a\,c\right)\left(b+\sqrt{b^2-4\,a\,c}\right)e\,(1+m)}$$

Result (type 6, 583 leaves):

$$\frac{1}{4\,c\,(2+m)\,(a+x\,(b+c\,x))^3}$$

$$a\,x\,(e\,x)^m\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\left(\left(A\,(2+m)^2\,\text{AppellF1}\left[1+m,\,2,\,2,\,2+m,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\right.$$

$$\left((1+m)\left(a\,(2+m)\,\text{AppellF1}\left[1+m,\,2,\,2,\,2+m,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]-x\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[2+m,\,2,\,3,\,3+m,\right.\right.\right.\right.$$

$$\left.\left.\left.\left.-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[2+m,\,3,\,2,\,3+m,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)-$$

$$\left(B\,(3+m)\,x\,\text{AppellF1}\left[2+m,\,2,\,2,\,3+m,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(-a\,(3+m)\,\text{AppellF1}\left[2+m,\,2,\,2,\,3+m,\right.\right.$$

$$-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\left(b+\sqrt{b^2-4\,a\,c}\right)x\,\text{AppellF1}\left[3+m,\,2,\,3,\,4+m,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+$$

$$\left.\left.\left(b-\sqrt{b^2-4\,a\,c}\right)x\,\text{AppellF1}\left[3+m,\,3,\,2,\,4+m,\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)$$

Problem 1088: Result more than twice size of optimal antiderivative.

$$\int (e\,x)^m\,(A+B\,x)\,\left(a+b\,x+c\,x^2\right)^{5/2}\,dx$$

Optimal (type 6, 281 leaves, 5 steps):

$$\frac{A\,(e\,x)^{1+m}\left(a+b\,x+c\,x^2\right)^{5/2}\text{AppellF1}\left[1+m,-\frac{5}{2},-\frac{5}{2},2+m,-\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}\right]}{e\,(1+m)\left(1+\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}}\right)^{5/2}\left(1+\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}\right)^{5/2}}+$$
$$\frac{B\,(e\,x)^{2+m}\left(a+b\,x+c\,x^2\right)^{5/2}\text{AppellF1}\left[2+m,-\frac{5}{2},-\frac{5}{2},3+m,-\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}\right]}{e^2\,(2+m)\left(1+\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}}\right)^{5/2}\left(1+\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}\right)^{5/2}}$$

Result (type 6, 4573 leaves):

$$\left(a^2\,A\left(b-\sqrt{b^2-4\,a\,c}\right)\left(b+\sqrt{b^2-4\,a\,c}\right)(2+m)\,x\,(e\,x)^m\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\right.$$
$$\left.\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\left(a+x\,(b+c\,x)\right)^2\text{AppellF1}\left[1+m,-\frac{1}{2},-\frac{1}{2},2+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left(4\,c^2\,(1+m)\left(a+b\,x+c\,x^2\right)^{5/2}\left(4\,a\,(2+m)\,\text{AppellF1}\left[1+m,-\frac{1}{2},-\frac{1}{2},2+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.\right.$$
$$\left(b+\sqrt{b^2-4\,a\,c}\right)x\,\text{AppellF1}\left[2+m,-\frac{1}{2},\frac{1}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+$$
$$\left.\left.\left(b-\sqrt{b^2-4\,a\,c}\right)x\,\text{AppellF1}\left[2+m,\frac{1}{2},-\frac{1}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)+$$
$$\left(a\,A\,b\left(b-\sqrt{b^2-4\,a\,c}\right)\left(b+\sqrt{b^2-4\,a\,c}\right)(3+m)\,x^2\,(e\,x)^m\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\right.$$
$$\left.\left(a+x\,(b+c\,x)\right)^2\text{AppellF1}\left[2+m,-\frac{1}{2},-\frac{1}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(2\,c^2\,(2+m)\,\left(a+b\,x+c\,x^2\right)^{5/2}\left(4\,a\,(3+m)\,\text{AppellF1}\left[2+m,-\frac{1}{2},-\frac{1}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.\right.$$

$$\left(b+\sqrt{b^2-4\,a\,c}\right)x\,\text{AppellF1}\left[3+m,-\frac{1}{2},\frac{1}{2},4+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+$$

$$\left.\left.\left(b-\sqrt{b^2-4\,a\,c}\right)x\,\text{AppellF1}\left[3+m,\frac{1}{2},-\frac{1}{2},4+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)+$$

$$\left(a^2\,B\left(b-\sqrt{b^2-4\,a\,c}\right)\left(b+\sqrt{b^2-4\,a\,c}\right)(3+m)\,x^2\,(e\,x)^m\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\right.$$

$$\left.(a+x\,(b+c\,x))^2\,\text{AppellF1}\left[2+m,-\frac{1}{2},-\frac{1}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\bigg/$$

$$\left(4\,c^2\,(2+m)\,\left(a+b\,x+c\,x^2\right)^{5/2}\left(4\,a\,(3+m)\,\text{AppellF1}\left[2+m,-\frac{1}{2},-\frac{1}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.\right.$$

$$\left(b+\sqrt{b^2-4\,a\,c}\right)x\,\text{AppellF1}\left[3+m,-\frac{1}{2},\frac{1}{2},4+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+$$

$$\left.\left.\left(b-\sqrt{b^2-4\,a\,c}\right)x\,\text{AppellF1}\left[3+m,\frac{1}{2},-\frac{1}{2},4+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)+$$

$$\left(A\,b^2\left(b-\sqrt{b^2-4\,a\,c}\right)\left(b+\sqrt{b^2-4\,a\,c}\right)(4+m)\,x^3\,(e\,x)^m\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\right.$$

$$\left.(a+x\,(b+c\,x))^2\,\text{AppellF1}\left[3+m,-\frac{1}{2},-\frac{1}{2},4+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\bigg/$$

$$\left(4\,c^2\,(3+m)\,\left(a+b\,x+c\,x^2\right)^{5/2}\left(4\,a\,(4+m)\,\text{AppellF1}\left[3+m,-\frac{1}{2},-\frac{1}{2},4+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.\right.$$

$$\left(b+\sqrt{b^2-4\,a\,c}\right)x\,\text{AppellF1}\left[4+m,-\frac{1}{2},\frac{1}{2},5+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+$$

$$\left(b-\sqrt{b^2-4ac}\right) x\,\text{AppellF1}\left[4+m,\frac{1}{2},-\frac{1}{2},5+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\Bigg)\Bigg)+$$

$$\Bigg(a\,b\,B\left(b-\sqrt{b^2-4ac}\right)\left(b+\sqrt{b^2-4ac}\right)(4+m)\,x^3\,(e\,x)^m\left(b-\sqrt{b^2-4ac}+2cx\right)\left(b+\sqrt{b^2-4ac}+2cx\right)$$

$$(a+x(b+cx))^2\,\text{AppellF1}\left[3+m,-\frac{1}{2},-\frac{1}{2},4+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\Bigg)\Big/$$

$$\Bigg(2c^2(3+m)\left(a+bx+cx^2\right)^{5/2}\Bigg(4a(4+m)\,\text{AppellF1}\left[3+m,-\frac{1}{2},-\frac{1}{2},4+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]+$$

$$\left(b+\sqrt{b^2-4ac}\right) x\,\text{AppellF1}\left[4+m,-\frac{1}{2},\frac{1}{2},5+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]+$$

$$\left(b-\sqrt{b^2-4ac}\right) x\,\text{AppellF1}\left[4+m,\frac{1}{2},-\frac{1}{2},5+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\Bigg)\Bigg)+$$

$$\Bigg(a\,A\left(b-\sqrt{b^2-4ac}\right)\left(b+\sqrt{b^2-4ac}\right)(4+m)\,x^3\,(e\,x)^m\left(b-\sqrt{b^2-4ac}+2cx\right)\left(b+\sqrt{b^2-4ac}+2cx\right)$$

$$(a+x(b+cx))^2\,\text{AppellF1}\left[3+m,-\frac{1}{2},-\frac{1}{2},4+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\Bigg)\Big/$$

$$\Bigg(2c(3+m)\left(a+bx+cx^2\right)^{5/2}\Bigg(4a(4+m)\,\text{AppellF1}\left[3+m,-\frac{1}{2},-\frac{1}{2},4+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]+$$

$$\left(b+\sqrt{b^2-4ac}\right) x\,\text{AppellF1}\left[4+m,-\frac{1}{2},\frac{1}{2},5+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]+$$

$$\left(b-\sqrt{b^2-4ac}\right) x\,\text{AppellF1}\left[4+m,\frac{1}{2},-\frac{1}{2},5+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\Bigg)\Bigg)+$$

$$\Bigg(b^2\,B\left(b-\sqrt{b^2-4ac}\right)\left(b+\sqrt{b^2-4ac}\right)(5+m)\,x^4\,(e\,x)^m\left(b-\sqrt{b^2-4ac}+2cx\right)\left(b+\sqrt{b^2-4ac}+2cx\right)$$

$$\left.\left.(a+x\,(b+c\,x))^2\,\text{AppellF1}\left[4+m,-\frac{1}{2},-\frac{1}{2},5+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right/$$

$$\left(4\,c^2\,(4+m)\,\left(a+b\,x+c\,x^2\right)^{5/2}\left(4\,a\,(5+m)\,\text{AppellF1}\left[4+m,-\frac{1}{2},-\frac{1}{2},5+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.\right.$$

$$\left(b+\sqrt{b^2-4\,a\,c}\right)\,x\,\text{AppellF1}\left[5+m,-\frac{1}{2},\frac{1}{2},6+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+$$

$$\left.\left.\left(b-\sqrt{b^2-4\,a\,c}\right)\,x\,\text{AppellF1}\left[5+m,\frac{1}{2},-\frac{1}{2},6+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)+$$

$$\left(A\,b\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,(5+m)\,x^4\,(e\,x)^m\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\right.$$

$$\left.\left.(a+x\,(b+c\,x))^2\,\text{AppellF1}\left[4+m,-\frac{1}{2},-\frac{1}{2},5+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right/$$

$$\left(2\,c\,(4+m)\,\left(a+b\,x+c\,x^2\right)^{5/2}\left(4\,a\,(5+m)\,\text{AppellF1}\left[4+m,-\frac{1}{2},-\frac{1}{2},5+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.\right.$$

$$\left(b+\sqrt{b^2-4\,a\,c}\right)\,x\,\text{AppellF1}\left[5+m,-\frac{1}{2},\frac{1}{2},6+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+$$

$$\left.\left.\left(b-\sqrt{b^2-4\,a\,c}\right)\,x\,\text{AppellF1}\left[5+m,\frac{1}{2},-\frac{1}{2},6+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)+$$

$$\left(a\,B\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,(5+m)\,x^4\,(e\,x)^m\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\right.$$

$$\left.\left.(a+x\,(b+c\,x))^2\,\text{AppellF1}\left[4+m,-\frac{1}{2},-\frac{1}{2},5+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right/$$

$$\left(2\,c\,(4+m)\,\left(a+b\,x+c\,x^2\right)^{5/2}\left(4\,a\,(5+m)\,\text{AppellF1}\left[4+m,-\frac{1}{2},-\frac{1}{2},5+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.\right.$$

$$\left(b+\sqrt{b^2-4ac}\right) x \,\text{AppellF1}\left[5+m,-\frac{1}{2},\frac{1}{2},6+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]+$$

$$\left.\left.\left(b-\sqrt{b^2-4ac}\right) x \,\text{AppellF1}\left[5+m,\frac{1}{2},-\frac{1}{2},6+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\right)\right)+$$

$$\left(A\left(b-\sqrt{b^2-4ac}\right)\left(b+\sqrt{b^2-4ac}\right)(6+m)\,x^5\,(e\,x)^m\left(b-\sqrt{b^2-4ac}+2cx\right)\left(b+\sqrt{b^2-4ac}+2cx\right)(a+x\,(b+c\,x))^2\right.$$

$$\left.\text{AppellF1}\left[5+m,-\frac{1}{2},-\frac{1}{2},6+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(4\,(5+m)\left(a+b\,x+c\,x^2\right)^{5/2}\left(4\,a\,(6+m)\,\text{AppellF1}\left[5+m,-\frac{1}{2},-\frac{1}{2},6+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]+\right.\right.$$

$$\left(b+\sqrt{b^2-4ac}\right) x \,\text{AppellF1}\left[6+m,-\frac{1}{2},\frac{1}{2},7+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]+$$

$$\left.\left.\left(b-\sqrt{b^2-4ac}\right) x \,\text{AppellF1}\left[6+m,\frac{1}{2},-\frac{1}{2},7+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\right)\right)+$$

$$\left(b\,B\left(b-\sqrt{b^2-4ac}\right)\left(b+\sqrt{b^2-4ac}\right)(6+m)\,x^5\,(e\,x)^m\left(b-\sqrt{b^2-4ac}+2cx\right)\left(b+\sqrt{b^2-4ac}+2cx\right)\right.$$

$$\left.(a+x\,(b+c\,x))^2\,\text{AppellF1}\left[5+m,-\frac{1}{2},-\frac{1}{2},6+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(2\,c\,(5+m)\left(a+b\,x+c\,x^2\right)^{5/2}\left(4\,a\,(6+m)\,\text{AppellF1}\left[5+m,-\frac{1}{2},-\frac{1}{2},6+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]+\right.\right.$$

$$\left(b+\sqrt{b^2-4ac}\right) x \,\text{AppellF1}\left[6+m,-\frac{1}{2},\frac{1}{2},7+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]+$$

$$\left.\left.\left(b-\sqrt{b^2-4ac}\right) x \,\text{AppellF1}\left[6+m,\frac{1}{2},-\frac{1}{2},7+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\right)\right)+$$

$$\left(B\left(b-\sqrt{b^2-4ac}\right)\left(b+\sqrt{b^2-4ac}\right)(7+m)\,x^6\,(e\,x)^m\left(b-\sqrt{b^2-4ac}+2cx\right)\left(b+\sqrt{b^2-4ac}+2cx\right)(a+x\,(b+c\,x))^2\right.$$

$$\left.\text{AppellF1}\left[6+m,-\frac{1}{2},-\frac{1}{2},7+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(4\,(6+m)\left(a+b\,x+c\,x^2\right)^{5/2}\left(4\,a\,(7+m)\,\text{AppellF1}\left[6+m,-\frac{1}{2},-\frac{1}{2},7+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]+\right.\right.$$

$$\left(b+\sqrt{b^2-4ac}\right)x\,\text{AppellF1}\left[7+m,-\frac{1}{2},\frac{1}{2},8+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]+$$

$$\left.\left.\left(b-\sqrt{b^2-4ac}\right)x\,\text{AppellF1}\left[7+m,\frac{1}{2},-\frac{1}{2},8+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\right)\right)$$

- **Problem 1089: Result more than twice size of optimal antiderivative.**

$$\int (e\,x)^m\,(A+B\,x)\left(a+b\,x+c\,x^2\right)^{3/2}\,dx$$

Optimal (type 6, 281 leaves, 5 steps):

$$\frac{A\,(e\,x)^{1+m}\left(a+b\,x+c\,x^2\right)^{3/2}\text{AppellF1}\left[1+m,-\frac{3}{2},-\frac{3}{2},2+m,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right]}{e\,(1+m)\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)^{3/2}}+$$

$$\frac{B\,(e\,x)^{2+m}\left(a+b\,x+c\,x^2\right)^{3/2}\text{AppellF1}\left[2+m,-\frac{3}{2},-\frac{3}{2},3+m,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right]}{e^2\,(2+m)\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)^{3/2}}$$

Result (type 6, 2211 leaves):

$$\left(a\,A\left(b-\sqrt{b^2-4ac}\right)\left(b+\sqrt{b^2-4ac}\right)(2+m)\,x\,(e\,x)^m\left(b-\sqrt{b^2-4ac}+2cx\right)\right.$$

$$\left.\left(b+\sqrt{b^2-4ac}+2cx\right)\text{AppellF1}\left[1+m,-\frac{1}{2},-\frac{1}{2},2+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/$$

$$\left(4\,c^2\,(1+m)\,\sqrt{a+x\,(b+c\,x)}\,\left(4\,a\,(2+m)\,\text{AppellF1}\left[1+m,-\frac{1}{2},-\frac{1}{2},2+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.\right.$$

$$\left(b+\sqrt{b^2-4\,a\,c}\right)\,x\,\text{AppellF1}\left[2+m,-\frac{1}{2},\frac{1}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+$$

$$\left.\left.\left(b-\sqrt{b^2-4\,a\,c}\right)\,x\,\text{AppellF1}\left[2+m,\frac{1}{2},-\frac{1}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)+$$

$$\left(A\,b\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,(3+m)\,x^2\,(e\,x)^m\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\right.$$

$$\left.\text{AppellF1}\left[2+m,-\frac{1}{2},-\frac{1}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(4\,c^2\,(2+m)\,\sqrt{a+x\,(b+c\,x)}\,\left(4\,a\,(3+m)\,\text{AppellF1}\left[2+m,-\frac{1}{2},-\frac{1}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.\right.$$

$$\left(b+\sqrt{b^2-4\,a\,c}\right)\,x\,\text{AppellF1}\left[3+m,-\frac{1}{2},\frac{1}{2},4+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+$$

$$\left.\left.\left(b-\sqrt{b^2-4\,a\,c}\right)\,x\,\text{AppellF1}\left[3+m,\frac{1}{2},-\frac{1}{2},4+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)+$$

$$\left(a\,B\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,(3+m)\,x^2\,(e\,x)^m\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\right.$$

$$\left.\text{AppellF1}\left[2+m,-\frac{1}{2},-\frac{1}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(4\,c^2\,(2+m)\,\sqrt{a+x\,(b+c\,x)}\,\left(4\,a\,(3+m)\,\text{AppellF1}\left[2+m,-\frac{1}{2},-\frac{1}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.\right.$$

$$\left(b+\sqrt{b^2-4\,a\,c}\right)\,x\,\text{AppellF1}\left[3+m,-\frac{1}{2},\frac{1}{2},4+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+$$

$$\left(b-\sqrt{b^2-4ac}\right) x\,\text{AppellF1}\left[3+m,\frac{1}{2},-\frac{1}{2},4+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\Bigg)\Bigg)+$$

$$\Bigg(b B\left(b-\sqrt{b^2-4ac}\right)\left(b+\sqrt{b^2-4ac}\right)(4+m)x^3(ex)^m\left(b-\sqrt{b^2-4ac}+2cx\right)\left(b+\sqrt{b^2-4ac}+2cx\right)$$

$$\text{AppellF1}\left[3+m,-\frac{1}{2},-\frac{1}{2},4+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\Bigg)\Bigg/$$

$$\Bigg(4c^2(3+m)\sqrt{a+x(b+cx)}\Bigg(4a(4+m)\,\text{AppellF1}\left[3+m,-\frac{1}{2},-\frac{1}{2},4+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]+$$

$$\left(b+\sqrt{b^2-4ac}\right) x\,\text{AppellF1}\left[4+m,-\frac{1}{2},\frac{1}{2},5+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]+$$

$$\left(b-\sqrt{b^2-4ac}\right) x\,\text{AppellF1}\left[4+m,\frac{1}{2},-\frac{1}{2},5+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\Bigg)\Bigg)+$$

$$\Bigg(A\left(b-\sqrt{b^2-4ac}\right)\left(b+\sqrt{b^2-4ac}\right)(4+m)x^3(ex)^m\left(b-\sqrt{b^2-4ac}+2cx\right)\left(b+\sqrt{b^2-4ac}+2cx\right)$$

$$\text{AppellF1}\left[3+m,-\frac{1}{2},-\frac{1}{2},4+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\Bigg)\Bigg/$$

$$\Bigg(4c(3+m)\sqrt{a+x(b+cx)}\Bigg(4a(4+m)\,\text{AppellF1}\left[3+m,-\frac{1}{2},-\frac{1}{2},4+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]+$$

$$\left(b+\sqrt{b^2-4ac}\right) x\,\text{AppellF1}\left[4+m,-\frac{1}{2},\frac{1}{2},5+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]+$$

$$\left(b-\sqrt{b^2-4ac}\right) x\,\text{AppellF1}\left[4+m,\frac{1}{2},-\frac{1}{2},5+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\Bigg)\Bigg)+$$

$$\Bigg(B\left(b-\sqrt{b^2-4ac}\right)\left(b+\sqrt{b^2-4ac}\right)(5+m)x^4(ex)^m\left(b-\sqrt{b^2-4ac}+2cx\right)\left(b+\sqrt{b^2-4ac}+2cx\right)$$

$$\text{AppellF1}\left[4+m,-\frac{1}{2},-\frac{1}{2},5+m,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]\Bigg)\Bigg/$$

$$\left(4 c (4+m) \sqrt{a+x (b+c x)} \left(4 a (5+m) \text{AppellF1}\left[4+m,-\frac{1}{2},-\frac{1}{2},5+m,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]+\right.\right.$$

$$\left(b+\sqrt{b^2-4 a c}\right) x \text{AppellF1}\left[5+m,-\frac{1}{2},\frac{1}{2},6+m,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]+$$

$$\left.\left.\left(b-\sqrt{b^2-4 a c}\right) x \text{AppellF1}\left[5+m,\frac{1}{2},-\frac{1}{2},6+m,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)$$

- **Problem 1090: Result more than twice size of optimal antiderivative.**

$$\int (e x)^m (A+B x) \sqrt{a+b x+c x^2}\, dx$$

Optimal (type 6, 281 leaves, 5 steps):

$$\frac{A (e x)^{1+m} \sqrt{a+b x+c x^2}\ \text{AppellF1}\left[1+m,-\frac{1}{2},-\frac{1}{2},2+m,-\frac{2 c x}{b-\sqrt{b^2-4 a c}},-\frac{2 c x}{b+\sqrt{b^2-4 a c}}\right]}{e (1+m) \sqrt{1+\frac{2 c x}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x}{b+\sqrt{b^2-4 a c}}}}+$$

$$\frac{B (e x)^{2+m} \sqrt{a+b x+c x^2}\ \text{AppellF1}\left[2+m,-\frac{1}{2},-\frac{1}{2},3+m,-\frac{2 c x}{b-\sqrt{b^2-4 a c}},-\frac{2 c x}{b+\sqrt{b^2-4 a c}}\right]}{e^2 (2+m) \sqrt{1+\frac{2 c x}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x}{b+\sqrt{b^2-4 a c}}}}$$

Result (type 6, 644 leaves):

$$\frac{1}{4\,c^2\,(2+m)\,\sqrt{a+x\,(b+c\,x)}}\left(b-\sqrt{b^2-4\,a\,c}\right)\left(b+\sqrt{b^2-4\,a\,c}\right)x\,(e\,x)^m\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)$$

$$\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\left(\left(A\,(2+m)^2\,\text{AppellF1}\left[1+m,-\frac{1}{2},-\frac{1}{2},2+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\right.$$

$$\left((1+m)\left(4\,a\,(2+m)\,\text{AppellF1}\left[1+m,-\frac{1}{2},-\frac{1}{2},2+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b+\sqrt{b^2-4\,a\,c}\right)x\,\text{AppellF1}\left[2+m,-\frac{1}{2},\frac{1}{2},3+m,\right.\right.\right.$$

$$\left.\left.\left.-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)x\,\text{AppellF1}\left[2+m,\frac{1}{2},-\frac{1}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)+$$

$$\left(B\,(3+m)\,x\,\text{AppellF1}\left[2+m,-\frac{1}{2},-\frac{1}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(4\,a\,(3+m)\,\text{AppellF1}\left[2+m,-\frac{1}{2},-\frac{1}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b+\sqrt{b^2-4\,a\,c}\right)x\,\text{AppellF1}\left[3+m,-\frac{1}{2},\frac{1}{2},4+m,\right.\right.$$

$$\left.\left.-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)x\,\text{AppellF1}\left[3+m,\frac{1}{2},-\frac{1}{2},4+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)$$

- **Problem 1091: Result more than twice size of optimal antiderivative.**

$$\int\frac{(e\,x)^m\,(A+B\,x)}{\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 6, 281 leaves, 5 steps):

$$\frac{A\,(e\,x)^{1+m}\sqrt{1+\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}}}\sqrt{1+\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}}\,\text{AppellF1}\left[1+m,\frac{1}{2},\frac{1}{2},2+m,-\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}\right]}{e\,(1+m)\,\sqrt{a+b\,x+c\,x^2}}+$$

$$\frac{B\,(e\,x)^{2+m}\sqrt{1+\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}}}\sqrt{1+\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}}\,\text{AppellF1}\left[2+m,\frac{1}{2},\frac{1}{2},3+m,-\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}\right]}{e^2\,(2+m)\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 6, 614 leaves):

$$\frac{1}{c\,(2+m)\,(a+x\,(b+c\,x))^{3/2}}$$

$$a\,x\,(e\,x)^m\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\left(-\left(A\,(2+m)^2\,\text{AppellF1}\left[1+m,\frac{1}{2},\frac{1}{2},2+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\right.$$

$$\left((1+m)\left(-4\,a\,(2+m)\,\text{AppellF1}\left[1+m,\frac{1}{2},\frac{1}{2},2+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b+\sqrt{b^2-4\,a\,c}\right)x\,\text{AppellF1}\left[2+m,\frac{1}{2},\frac{3}{2},3+m,\right.\right.\right.$$

$$\left.\left.\left.-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)x\,\text{AppellF1}\left[2+m,\frac{3}{2},\frac{1}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)-$$

$$\left(B\,(3+m)\,x\,\text{AppellF1}\left[2+m,\frac{1}{2},\frac{1}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(-4\,a\,(3+m)\,\text{AppellF1}\left[2+m,\frac{1}{2},\frac{1}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b+\sqrt{b^2-4\,a\,c}\right)x\,\text{AppellF1}\left[3+m,\frac{1}{2},\frac{3}{2},4+m,\right.\right.$$

$$\left.\left.\left.-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)x\,\text{AppellF1}\left[3+m,\frac{3}{2},\frac{1}{2},4+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)$$

- **Problem 1092: Result more than twice size of optimal antiderivative.**

$$\int\frac{(e\,x)^m\,(A+B\,x)}{\left(a+b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 6, 281 leaves, 5 steps):

$$\frac{A\,(e\,x)^{1+m}\left(1+\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}}\right)^{3/2}\left(1+\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}\right)^{3/2}\text{AppellF1}\left[1+m,\frac{3}{2},\frac{3}{2},2+m,-\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}\right]}{e\,(1+m)\left(a+b\,x+c\,x^2\right)^{3/2}}+$$

$$\frac{B\,(e\,x)^{2+m}\left(1+\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}}\right)^{3/2}\left(1+\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}\right)^{3/2}\text{AppellF1}\left[2+m,\frac{3}{2},\frac{3}{2},3+m,-\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}\right]}{e^2\,(2+m)\left(a+b\,x+c\,x^2\right)^{3/2}}$$

Result (type 6, 616 leaves):

$$\frac{1}{c\,(2+m)\,(a+x\,(b+c\,x))^{5/2}}$$

$$a\,x\,(e\,x)^m\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)\left(\left(A\,(2+m)^2\,\text{AppellF1}\left[1+m,\frac{3}{2},\frac{3}{2},2+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\right.$$
$$\left((1+m)\left(4\,a\,(2+m)\,\text{AppellF1}\left[1+m,\frac{3}{2},\frac{3}{2},2+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]-3\,x\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[2+m,\frac{3}{2},\frac{5}{2},3+m,\right.\right.\right.\right.$$
$$\left.\left.\left.\left.-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[2+m,\frac{5}{2},\frac{3}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+$$
$$\left(B\,(3+m)\,x\,\text{AppellF1}\left[2+m,\frac{3}{2},\frac{3}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(4\,a\,(3+m)\,\text{AppellF1}\left[2+m,\frac{3}{2},\frac{3}{2},3+m,\right.\right.$$
$$\left.-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]-3\,x\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[3+m,\frac{3}{2},\frac{5}{2},4+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.$$
$$\left.\left.\left.\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[3+m,\frac{5}{2},\frac{3}{2},4+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)$$

- **Problem 1093: Result more than twice size of optimal antiderivative.**

$$\int\frac{(e\,x)^m\,(A+B\,x)}{\left(a+b\,x+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 6, 281 leaves, 5 steps):

$$\frac{A\,(e\,x)^{1+m}\left(1+\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}}\right)^{5/2}\left(1+\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}\right)^{5/2}\text{AppellF1}\left[1+m,\frac{5}{2},\frac{5}{2},2+m,-\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}\right]}{e\,(1+m)\left(a+b\,x+c\,x^2\right)^{5/2}}+$$
$$\frac{B\,(e\,x)^{2+m}\left(1+\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}}\right)^{5/2}\left(1+\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}\right)^{5/2}\text{AppellF1}\left[2+m,\frac{5}{2},\frac{5}{2},3+m,-\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}\right]}{e^2\,(2+m)\left(a+b\,x+c\,x^2\right)^{5/2}}$$

Result (type 6, 576 leaves):

$$\frac{1}{(2+m)\,(a+x\,(b+c\,x))^{5/2}}\,4\,a\,x\,(e\,x)^m\left(\left(A\,(2+m)^2\,\text{AppellF1}\left[1+m,\frac{5}{2},\frac{5}{2},2+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left((1+m)\left(4\,a\,(2+m)\,\text{AppellF1}\left[1+m,\frac{5}{2},\frac{5}{2},2+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]-5\,x\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[2+m,\frac{5}{2},\frac{7}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[2+m,\frac{7}{2},\frac{5}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)+\left(B\,(3+m)\,x\,\text{AppellF1}\left[2+m,\frac{5}{2},\frac{5}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(4\,a\,(3+m)\,\text{AppellF1}\left[2+m,\frac{5}{2},\frac{5}{2},3+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]-5\,x\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[3+m,\frac{5}{2},\frac{7}{2},4+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[3+m,\frac{7}{2},\frac{5}{2},4+m,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)$$

- **Problem 1094: Result more than twice size of optimal antiderivative.**

$$\int (e\,x)^m\,(A+B\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$$

Optimal (type 6, 277 leaves, 5 steps) :

$$\frac{1}{e\,(1+m)}A\,(e\,x)^{1+m}\left(1+\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}}\right)^{-p}\left(1+\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}\right)^{-p}\left(a+b\,x+c\,x^2\right)^p\,\text{AppellF1}\left[1+m,-p,-p,2+m,-\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}\right]+\frac{1}{e^2\,(2+m)}B\,(e\,x)^{2+m}\left(1+\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}}\right)^{-p}\left(1+\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}\right)^{-p}\left(a+b\,x+c\,x^2\right)^p\,\text{AppellF1}\left[2+m,-p,-p,3+m,-\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}},-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}\right]$$

Result (type 6, 725 leaves):

$$\frac{1}{\left(-b+\sqrt{b^2-4ac}\right)(2+m)\left(b+\sqrt{b^2-4ac}+2cx\right)} 2^{-1-p}\,c\left(b+\sqrt{b^2-4ac}\right)x\,(e\,x)^m\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x\right)^{-p}\left(\frac{b-\sqrt{b^2-4ac}+2cx}{c}\right)^{1+p}\left(2a+\left(b-\sqrt{b^2-4ac}\right)x\right)^2\,(a+x\,(b+c\,x))^{-1+p}$$

$$\left(-\left(A\,(2+m)^2\,\text{AppellF1}\left[1+m,-p,-p,2+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/\left((1+m)\left(2a\,(2+m)\,\text{AppellF1}\left[1+m,-p,-p,2+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]+p\,x\left(\left(b-\sqrt{b^2-4ac}\right)\text{AppellF1}\left[2+m,1-p,-p,3+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]+\left(b+\sqrt{b^2-4ac}\right)\text{AppellF1}\left[2+m,-p,1-p,3+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\right)\right)\right)+$$

$$\left(B\,(3+m)\,x\,\text{AppellF1}\left[2+m,-p,-p,3+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/\left(-2a\,(3+m)\,\text{AppellF1}\left[2+m,-p,-p,3+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]+p\,x\left(\left(-b+\sqrt{b^2-4ac}\right)\text{AppellF1}\left[3+m,1-p,-p,4+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]-\left(b+\sqrt{b^2-4ac}\right)\text{AppellF1}\left[3+m,-p,1-p,4+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right]\right)\right)\right)$$

- **Problem 1095: Result unnecessarily involves higher level functions.**

$$\int x^3\,(A+B\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$$

Optimal (type 5, 442 leaves, 4 steps):

$$-\frac{(b B (4+p)-A c (5+2 p)) x^2 \left(a+b x+c x^2\right)^{1+p}}{2 c^2 (2+p) (5+2 p)}+\frac{B x^3 \left(a+b x+c x^2\right)^{1+p}}{c (5+2 p)}+$$
$$\left(\left(2 a c (3+2 p) (b B (4+p)-A c (5+2 p))+b (2+p) \left(6 a B c (2+p)-b^2 B \left(12+7 p+p^2\right)+A b c \left(15+11 p+2 p^2\right)\right)-\right.\right.$$
$$\left.\left.2 c (1+p) \left(6 a B c (2+p)-b^2 B \left(12+7 p+p^2\right)+A b c \left(15+11 p+2 p^2\right)\right) x\right) \left(a+b x+c x^2\right)^{1+p}\right) / \left(4 c^4 (1+p) (2+p) (3+2 p) (5+2 p)\right)-$$
$$\left(2^{-1+p} \left(12 a^2 B c^2-12 a b^2 B c (3+p)+6 a A b c^2 (5+2 p)+b^4 B \left(12+7 p+p^2\right)-A b^3 c \left(15+11 p+2 p^2\right)\right) \left(-\frac{b-\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}\right)^{-1-p}\right.$$
$$\left.\left(a+b x+c x^2\right)^{1+p} \text{Hypergeometric2F1}\left[-p, 1+p, 2+p, \frac{b+\sqrt{b^2-4 a c}+2 c x}{2 \sqrt{b^2-4 a c}}\right]\right) / \left(c^4 \sqrt{b^2-4 a c} (1+p) (3+2 p) (5+2 p)\right)$$

Result (type 6, 588 leaves) :

$$-\frac{1}{80 c}\left(b+\sqrt{b^2-4 a c}\right) x^4 \left(b-\sqrt{b^2-4 a c}+2 c x\right) \left(2 a+\left(b-\sqrt{b^2-4 a c}\right) x\right)$$
$$(a+x (b+c x))^{-1+p} \left(-\left(25 A \text{AppellF1}\left[4, -p, -p, 5, -\frac{2 c x}{b+\sqrt{b^2-4 a c}}, \frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]\right) \right/$$
$$\left(10 a \text{AppellF1}\left[4, -p, -p, 5, -\frac{2 c x}{b+\sqrt{b^2-4 a c}}, \frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]+p x \left(\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[5, 1-p, -p, 6,\right.\right.\right.$$
$$\left.\left.\left.-\frac{2 c x}{b+\sqrt{b^2-4 a c}}, \frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]+\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[5, -p, 1-p, 6, -\frac{2 c x}{b+\sqrt{b^2-4 a c}}, \frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)+$$
$$\left(24 B x \text{AppellF1}\left[5, -p, -p, 6, -\frac{2 c x}{b+\sqrt{b^2-4 a c}}, \frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]\right) / \left(-12 a \text{AppellF1}\left[5, -p, -p, 6, -\frac{2 c x}{b+\sqrt{b^2-4 a c}},\right.\right.$$
$$\left.\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]+p x \left(\left(-b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[6, 1-p, -p, 7, -\frac{2 c x}{b+\sqrt{b^2-4 a c}}, \frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]-\right.$$
$$\left.\left.\left.\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[6, -p, 1-p, 7, -\frac{2 c x}{b+\sqrt{b^2-4 a c}}, \frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)$$

- **Problem 1096: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int x^2 (A+B x) \left(a+b x+c x^2\right)^p \, dx$$

Optimal (type 5, 287 leaves, 3 steps) :

$$\frac{B x^2 \left(a+b x+c x^2\right)^{1+p}}{2 c (2+p)}-\left(\left(2 a B c (3+2 p)+b (2+p) (2 A c (2+p)-b B (3+p))-2 c (1+p) (2 A c (2+p)-b B (3+p)) x\right) \left(a+b x+c x^2\right)^{1+p}\right) \Big/ \left(4 c^3 (1+p) (2+p) (3+2 p)\right)-\left(2^{-1+p} \left(6 a b B c-4 a A c^2+2 A b^2 c (2+p)-b^3 B (3+p)\right) \left(-\frac{b-\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}\right)^{-1-p} \left(a+b x+c x^2\right)^{1+p} \text{Hypergeometric2F1}\left[-p,1+p,2+p,\frac{b+\sqrt{b^2-4 a c}+2 c x}{2 \sqrt{b^2-4 a c}}\right]\right) \Big/ \left(c^3 \sqrt{b^2-4 a c} (1+p) (3+2 p)\right)$$

Result (type 6, 587 leaves) :

$$-\frac{1}{48 c}\left(b+\sqrt{b^2-4 a c}\right) x^3 \left(b-\sqrt{b^2-4 a c}+2 c x\right) \left(2 a+\left(b-\sqrt{b^2-4 a c}\right) x\right) (a+x (b+c x))^{-1+p} \left(-\left(16 A \text{AppellF1}\left[3,-p,-p,4,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \left(8 a \text{AppellF1}\left[3,-p,-p,4,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]+p x \left(\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[4,1-p,-p,5,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]+\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[4,-p,1-p,5,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)-\left(15 B x \text{AppellF1}\left[4,-p,-p,5,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \left(10 a \text{AppellF1}\left[4,-p,-p,5,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]+p x \left(\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[5,1-p,-p,6,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]+\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[5,-p,1-p,6,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)$$

- **Problem 1097: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int x (A+B x) \left(a+b x+c x^2\right)^p \, dx$$

Optimal (type 5, 211 leaves, 2 steps) :

$$-\frac{(b B (2+p)-A c (3+2 p)-2 B c (1+p) x) \left(a+b x+c x^2\right)^{1+p}}{2 c^2 (1+p) (3+2 p)}+$$
$$\left(2^p \left(2 a B c-b^2 B (2+p)+A b c (3+2 p)\right) \left(-\frac{b-\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}\right)^{-1-p} \left(a+b x+c x^2\right)^{1+p}\right.$$
$$\left.\text{Hypergeometric2F1}\left[-p,1+p,2+p,\frac{b+\sqrt{b^2-4 a c}+2 c x}{2 \sqrt{b^2-4 a c}}\right]\right)\Big/\left(c^2 \sqrt{b^2-4 a c} (1+p) (3+2 p)\right)$$

Result (type 6, 588 leaves) :

$$-\frac{1}{24 c}\left(b+\sqrt{b^2-4 a c}\right) x^2 \left(b-\sqrt{b^2-4 a c}+2 c x\right) \left(2 a+\left(b-\sqrt{b^2-4 a c}\right) x\right)$$
$$(a+x (b+c x))^{-1+p} \left(-\left(9 A\, \text{AppellF1}\left[2,-p,-p,3,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]\right)\Big/\right.$$
$$\left(6 a\, \text{AppellF1}\left[2,-p,-p,3,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]+p x \left(\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[3,1-p,-p,4,\right.\right.\right.$$
$$\left.\left.\left.-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]+\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[3,-p,1-p,4,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)+$$
$$\left(8 B x\, \text{AppellF1}\left[3,-p,-p,4,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]\right)\Big/\left(-8 a\, \text{AppellF1}\left[3,-p,-p,4,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]+\right.$$
$$p x \left(\left(-b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[4,1-p,-p,5,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]-\right.$$
$$\left.\left.\left.\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[4,-p,1-p,5,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)$$

- **Problem 1098: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (A+B x) \left(a+b x+c x^2\right)^p \, dx$$

Optimal (type 5, 158 leaves, 2 steps) :

$$\frac{B\left(a+b x+c x^2\right)^{1+p}}{2 c (1+p)}+\frac{1}{c\sqrt{b^2-4 a c}\,(1+p)}$$

$$2^p (b B-2 A c)\left(-\frac{b-\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}\right)^{-1-p}\left(a+b x+c x^2\right)^{1+p} \text{Hypergeometric2F1}\left[-p,1+p,2+p,\frac{b+\sqrt{b^2-4 a c}+2 c x}{2\sqrt{b^2-4 a c}}\right]$$

Result (type 6, 476 leaves):

$$\frac{1}{4}\left(b-\sqrt{b^2-4 a c}+2 c x\right)(a+x (b+c x))^p$$

$$\left(\left(3 B\left(b+\sqrt{b^2-4 a c}\right) x^2\left(2 a+\left(b-\sqrt{b^2-4 a c}\right) x\right)^2 \text{AppellF1}\left[2,-p,-p,3,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]\right)\Big/\right.$$

$$\left(\left(-b+\sqrt{b^2-4 a c}\right)\left(b+\sqrt{b^2-4 a c}+2 c x\right)(a+x (b+c x))\right.$$

$$\left(-6 a \text{AppellF1}\left[2,-p,-p,3,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]+p x\left(\left(-b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[3,1-p,-p,4,\right.\right.\right.$$

$$\left.\left.\left.\left.-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]-\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[3,-p,1-p,4,-\frac{2 c x}{b+\sqrt{b^2-4 a c}},\frac{2 c x}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)+$$

$$\left.\frac{2^{1+p} A\left(\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}\right)^{-p} \text{Hypergeometric2F1}\left[-p,1+p,2+p,\frac{1}{2}-\frac{b}{2\sqrt{b^2-4 a c}}-\frac{c x}{\sqrt{b^2-4 a c}}\right]}{c+c p}\right)$$

- **Problem 1100: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+B x)\left(a+b x+c x^2\right)^p}{x^2}\,dx$$

Optimal (type 6, 315 leaves, 5 steps):

$$-\frac{A\,(a+b\,x+c\,x^2)^{1+p}}{a\,x}+\frac{1}{a\,p}2^{-1+2\,p}\,(a\,B+A\,b\,p)\,\left(\frac{b-\sqrt{b^2-4\,a\,c}+2\,c\,x}{c\,x}\right)^{-p}\,\left(\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{c\,x}\right)^{-p}$$

$$(a+b\,x+c\,x^2)^p\,\text{AppellF1}\Big[-2\,p,\,-p,\,-p,\,1-2\,p,\,-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c\,x},\,-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c\,x}\Big]-\frac{1}{a\,\sqrt{b^2-4\,a\,c}\,(1+p)}$$

$$2^{1+p}\,A\,c\,(1+2\,p)\,\left(-\frac{b-\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}\right)^{-1-p}\,(a+b\,x+c\,x^2)^{1+p}\,\text{Hypergeometric2F1}\Big[-p,\,1+p,\,2+p,\,\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{2\,\sqrt{b^2-4\,a\,c}}\Big]$$

Result (type 6, 733 leaves) :

$$\frac{1}{4\,(-1+2\,p)}\,\Big(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\Big)\,(a+x\,(b+c\,x))^{-1+p}$$

$$\Bigg(\Bigg(4\,A\,(-1+p)\,\Big(-b+\sqrt{b^2-4\,a\,c}-2\,c\,x\Big)\,\text{AppellF1}\Big[1-2\,p,\,-p,\,-p,\,2-2\,p,\,-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c\,x},\,\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c\,x}\Big]\Bigg)\Big/$$

$$\Bigg(-4\,c\,(-1+p)\,x\,\text{AppellF1}\Big[1-2\,p,\,-p,\,-p,\,2-2\,p,\,-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c\,x},\,\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c\,x}\Big]+$$

$$\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,p\,\text{AppellF1}\Big[2-2\,p,\,1-p,\,-p,\,3-2\,p,\,-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c\,x},\,\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c\,x}\Big]+$$

$$\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,p\,\text{AppellF1}\Big[2-2\,p,\,-p,\,1-p,\,3-2\,p,\,-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c\,x},\,\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c\,x}\Big]\Bigg)-$$

$$\Bigg(B\,(1-2\,p)^2\,x\,\Big(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\Big)\,\text{AppellF1}\Big[-2\,p,\,-p,\,-p,\,1-2\,p,\,-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c\,x},\,\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c\,x}\Big]\Bigg)\Big/$$

$$\Bigg(p\,\Bigg(\Big(b+\sqrt{b^2-4\,a\,c}\Big)\,p\,\text{AppellF1}\Big[1-2\,p,\,1-p,\,-p,\,2-2\,p,\,-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c\,x},\,\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c\,x}\Big]+$$

$$\Big(b-\sqrt{b^2-4\,a\,c}\Big)\,p\,\text{AppellF1}\Big[1-2\,p,\,-p,\,1-p,\,2-2\,p,\,-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c\,x},\,\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c\,x}\Big]+$$

$$2\,c\,(1-2\,p)\,x\,\text{AppellF1}\Big[-2\,p,\,-p,\,-p,\,1-2\,p,\,-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c\,x},\,\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c\,x}\Big]\Bigg)\Bigg)\Bigg)$$

Problem 1127: Result more than twice size of optimal antiderivative.

$$\int (A+B x) (d+e x)^m \left(b x+c x^2\right)^3 \, dx$$

Optimal (type 3, 484 leaves, 2 steps):

$$-\frac{d^3 (B d-A e) (c d-b e)^3 (d+e x)^{1+m}}{e^8 (1+m)}+\frac{d^2 (c d-b e)^2 (B d (7 c d-4 b e)-3 A e (2 c d-b e)) (d+e x)^{2+m}}{e^8 (2+m)}+$$
$$\frac{3 d (c d-b e) \left(A e \left(5 c^2 d^2-5 b c d e+b^2 e^2\right)-B d \left(7 c^2 d^2-8 b c d e+2 b^2 e^2\right)\right) (d+e x)^{3+m}}{e^8 (3+m)}+\frac{1}{e^8 (4+m)}$$
$$\left(B d \left(35 c^3 d^3-60 b c^2 d^2 e+30 b^2 c d e^2-4 b^3 e^3\right)-A e \left(20 c^3 d^3-30 b c^2 d^2 e+12 b^2 c d e^2-b^3 e^3\right)\right) (d+e x)^{4+m}+$$
$$\frac{\left(3 A c e \left(5 c^2 d^2-5 b c d e+b^2 e^2\right)-B \left(35 c^3 d^3-45 b c^2 d^2 e+15 b^2 c d e^2-b^3 e^3\right)\right) (d+e x)^{5+m}}{e^8 (5+m)}-$$
$$\frac{3 c \left(A c e (2 c d-b e)-B \left(7 c^2 d^2-6 b c d e+b^2 e^2\right)\right) (d+e x)^{6+m}}{e^8 (6+m)}-\frac{c^2 (7 B c d-3 b B e-A c e) (d+e x)^{7+m}}{e^8 (7+m)}+\frac{B c^3 (d+e x)^{8+m}}{e^8 (8+m)}$$

Result (type 3, 1043 leaves):

$$\frac{1}{e^8 (1+m) (2+m) (3+m) (4+m) (5+m) (6+m) (7+m) (8+m)}$$
$$(d+e x)^m \left(-6 d^4 \left(A e (8+m) \left(-120 c^3 d^3+60 b c^2 d^2 e (7+m)-12 b^2 c d e^2 \left(42+13 m+m^2\right)+b^3 e^3 \left(210+107 m+18 m^2+m^3\right)\right)+\right.\right.$$
$$\left.4 B d \left(210 c^3 d^3-90 b c^2 d^2 e (8+m)+15 b^2 c d e^2 \left(56+15 m+m^2\right)-b^3 e^3 \left(336+146 m+21 m^2+m^3\right)\right)\right)+$$
$$6 d^3 e m \left(A e (8+m) \left(-120 c^3 d^3+60 b c^2 d^2 e (7+m)-12 b^2 c d e^2 \left(42+13 m+m^2\right)+b^3 e^3 \left(210+107 m+18 m^2+m^3\right)\right)+\right.$$
$$\left.4 B d \left(210 c^3 d^3-90 b c^2 d^2 e (8+m)+15 b^2 c d e^2 \left(56+15 m+m^2\right)-b^3 e^3 \left(336+146 m+21 m^2+m^3\right)\right)\right) x-$$
$$3 d^2 e^2 m (1+m) \left(A e (8+m) \left(-120 c^3 d^3+60 b c^2 d^2 e (7+m)-12 b^2 c d e^2 \left(42+13 m+m^2\right)+b^3 e^3 \left(210+107 m+18 m^2+m^3\right)\right)+\right.$$
$$\left.4 B d \left(210 c^3 d^3-90 b c^2 d^2 e (8+m)+15 b^2 c d e^2 \left(56+15 m+m^2\right)-b^3 e^3 \left(336+146 m+21 m^2+m^3\right)\right)\right) x^2+$$
$$d e^3 m (1+m) (2+m) \left(A e (8+m) \left(-120 c^3 d^3+60 b c^2 d^2 e (7+m)-12 b^2 c d e^2 \left(42+13 m+m^2\right)+b^3 e^3 \left(210+107 m+18 m^2+m^3\right)\right)+\right.$$
$$\left.4 B d \left(210 c^3 d^3-90 b c^2 d^2 e (8+m)+15 b^2 c d e^2 \left(56+15 m+m^2\right)-b^3 e^3 \left(336+146 m+21 m^2+m^3\right)\right)\right) x^3+$$
$$e^4 (1+m) (2+m) (3+m) \left(A e (8+m) \left(30 c^3 d^3 m-15 b c^2 d^2 e m (7+m)+3 b^2 c d e^2 m \left(42+13 m+m^2\right)+b^3 e^3 \left(210+107 m+18 m^2+m^3\right)\right)+\right.$$
$$\left.B d m \left(-210 c^3 d^3+90 b c^2 d^2 e (8+m)-15 b^2 c d e^2 \left(56+15 m+m^2\right)+b^3 e^3 \left(336+146 m+21 m^2+m^3\right)\right)\right) x^4+$$
$$e^5 (1+m) (2+m) (3+m) (4+m) \left(b^3 B e^3 \left(336+146 m+21 m^2+m^3\right)+3 b^2 c e^2 \left(56+15 m+m^2\right) (B d m+A e (6+m))+\right.$$
$$\left.3 b c^2 d e m (8+m) (-6 B d+A e (7+m))-6 c^3 d^2 m (-7 B d+A e (8+m))\right) x^5+$$
$$c e^6 (1+m) (2+m) (3+m) (4+m) (5+m) \left(3 b^2 B e^2 \left(56+15 m+m^2\right)+3 b c e (8+m) (B d m+A e (7+m))+c^2 d m (-7 B d+A e (8+m))\right) x^6+$$
$$c^2 e^7 (1+m) (2+m) (3+m) (4+m) (5+m) (6+m) (B c d m+3 b B e (8+m)+A c e (8+m)) x^7+$$
$$\left.B c^3 e^8 (1+m) (2+m) (3+m) (4+m) (5+m) (6+m) (7+m) x^8\right)$$

Problem 1255: Result unnecessarily involves imaginary or complex numbers.

$$\int (A+B x) \sqrt{d+e x} \sqrt{b x+c x^2} \, dx$$

Optimal (type 4, 433 leaves, 9 steps):

$$\frac{2\sqrt{d+e x}\,\left(7 A c e\,(c d+b e)-B\left(4 c^2 d^2-2 b c d e+4 b^2 e^2\right)+3 c e\,(B c d-4 b B e+7 A c e)\,x\right)\sqrt{b x+c x^2}}{105\, c^2 e^2}+$$

$$\frac{2 B\sqrt{d+e x}\,\left(b x+c x^2\right)^{3/2}}{7 c}+\left(2\sqrt{-b}\,\left(5 c\,(3 b B-7 A c)\,d e\,(2 c d-b e)+(B c d-4 b B e+7 A c e)\left(8 c^2 d^2-3 b c d e-2 b^2 e^2\right)\right)\right.$$

$$\left.\sqrt{x}\,\sqrt{1+\frac{c x}{b}}\,\sqrt{d+e x}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]\right)\Big/\left(105\, c^{5/2} e^3\sqrt{1+\frac{e x}{d}}\,\sqrt{b x+c x^2}\right)+$$

$$\left(2\sqrt{-b}\,d\,(c d-b e)\,\left(7 A c e\,(2 c d-b e)-B\left(8 c^2 d^2-b c d e-4 b^2 e^2\right)\right)\sqrt{x}\,\sqrt{1+\frac{c x}{b}}\,\sqrt{1+\frac{e x}{d}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]\right)\Big/$$

$$\left(105\, c^{5/2} e^3\sqrt{d+e x}\,\sqrt{b x+c x^2}\right)$$

Result (type 4, 461 leaves):

$$-\frac{1}{105\, b\, c^2 e^3\sqrt{x\,(b+c x)}\,\sqrt{d+e x}}$$

$$2\left(b e x\,(b+c x)\,(d+e x)\,\left(-7 A c e\,(b e+c\,(d+3 e x))+B\left(4 b^2 e^2-b c e\,(2 d+3 e x)+c^2\left(4 d^2-3 d e x-15 e^2 x^2\right)\right)\right)+\right.$$

$$\sqrt{\frac{b}{c}}\left(\sqrt{\frac{b}{c}}\,\left(14 A c e\left(c^2 d^2-b c d e+b^2 e^2\right)+B\left(-8 c^3 d^3+5 b c^2 d^2 e+5 b^2 c d e^2-8 b^3 e^3\right)\right)(b+c x)\,(d+e x)+\right.$$

$$i\, b e\left(14 A c e\left(c^2 d^2-b c d e+b^2 e^2\right)+B\left(-8 c^3 d^3+5 b c^2 d^2 e+5 b^2 c d e^2-8 b^3 e^3\right)\right)\sqrt{1+\frac{b}{c x}}\,\sqrt{1+\frac{d}{e x}}\;x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\right.$$

$$\left.\left.\left.\frac{c d}{b e}\right]-i\, b e\,(c d-b e)\,\left(7 A c e\,(c d-2 b e)-B\left(4 c^2 d^2+b c d e-8 b^2 e^2\right)\right)\sqrt{1+\frac{b}{c x}}\,\sqrt{1+\frac{d}{e x}}\;x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]\right)\right)$$

Problem 1256: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A+B x) \sqrt{b x+c x^2}}{\sqrt{d+e x}} \, dx$$

Optimal (type 4, 318 leaves, 8 steps):

$$-\frac{2 \sqrt{d+e x} \,(4 B c d-b B e-5 A c e-3 B c e x) \sqrt{b x+c x^2}}{15 c e^2}-\left(2 \sqrt{-b} \left(5 A c e (2 c d-b e)-B \left(8 c^2 d^2-3 b c d e-2 b^2 e^2\right)\right) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{d+e x} \,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/ \left(15 c^{3/2} e^3 \sqrt{1+\frac{e x}{d}} \sqrt{b x+c x^2}\right)-\frac{2 \sqrt{-b}\, d (c d-b e) (8 B c d+b B e-10 A c e) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{1+\frac{e x}{d}} \,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]}{15 c^{3/2} e^3 \sqrt{d+e x} \sqrt{b x+c x^2}}$$

Result (type 4, 344 leaves):

$$-\frac{1}{15 b c e^3 \sqrt{x (b+c x)} \sqrt{d+e x}} 2 \left(-b e x (b+c x) (d+e x) (5 A c e+B (-4 c d+b e+3 c e x))-\sqrt{\frac{b}{c}} \left(\sqrt{\frac{b}{c}} \left(5 A c e (-2 c d+b e)+B \left(8 c^2 d^2-3 b c d e-2 b^2 e^2\right)\right) (b+c x) (d+e x)-i b e \left(5 A c e (2 c d-b e)+B \left(-8 c^2 d^2+3 b c d e+2 b^2 e^2\right)\right) \sqrt{1+\frac{b}{c x}} \sqrt{1+\frac{d}{e x}}\, x^{3/2} \,\text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right]+ i b e (c d-b e) (5 A c e-2 B (2 c d+b e)) \sqrt{1+\frac{b}{c x}} \sqrt{1+\frac{d}{e x}}\, x^{3/2} \,\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right]\right)\right)$$

Problem 1257: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A+B x) \sqrt{b x+c x^2}}{(d+e x)^{3/2}} \, dx$$

Optimal (type 4, 283 leaves, 8 steps):

$$\frac{2\,(4\,B\,d-3\,A\,e+B\,e\,x)\,\sqrt{b\,x+c\,x^2}}{3\,e^2\,\sqrt{d+e\,x}}-\frac{2\,\sqrt{-b}\,\,(8\,B\,c\,d-b\,B\,e-6\,A\,c\,e)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{3\,\sqrt{c}\,\,e^3\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}}+$$

$$\frac{2\,\sqrt{-b}\,\,(B\,d\,(8\,c\,d-5\,b\,e)-3\,A\,e\,(2\,c\,d-b\,e))\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{3\,\sqrt{c}\,\,e^3\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}$$

Result (type 4, 269 leaves) :

$$\left(2\left(b\,e\,x\,(b+c\,x)\,(4\,B\,d-3\,A\,e+B\,e\,x)+\right.\right.$$

$$\sqrt{\frac{b}{c}}\left(\sqrt{\frac{b}{c}}\,(-8\,B\,c\,d+b\,B\,e+6\,A\,c\,e)\,(b+c\,x)\,(d+e\,x)-i\,b\,e\,(8\,B\,c\,d-b\,B\,e-6\,A\,c\,e)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\right.\right.$$

$$\left.\left.\left.\left.\frac{c\,d}{b\,e}\right]+i\,b\,e\,(4\,B\,c\,d-b\,B\,e-3\,A\,c\,e)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\right)\right)\Big/\left(3\,b\,e^3\,\sqrt{x\,(b+c\,x)}\,\sqrt{d+e\,x}\right)$$

- **Problem 1258: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+B\,x)\,\sqrt{b\,x+c\,x^2}}{(d+e\,x)^{5/2}}\,dx$$

Optimal (type 4, 346 leaves, 8 steps) :

$$-\frac{2\left(d^2\,(4\,B\,c\,d-3\,b\,B\,e-A\,c\,e)+e\,(B\,d\,(5\,c\,d-4\,b\,e)-A\,e\,(2\,c\,d-b\,e))\,x\right)\sqrt{b\,x+c\,x^2}}{3\,d\,e^2\,(c\,d-b\,e)\,(d+e\,x)^{3/2}}+$$

$$\left(2\sqrt{-b}\,\sqrt{c}\,(B\,d\,(8\,c\,d-7\,b\,e)-A\,e\,(2\,c\,d-b\,e))\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]\right)\Big/$$

$$\left(3\,d\,e^3\,(c\,d-b\,e)\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}\right)-\frac{2\sqrt{-b}\,(8\,B\,c\,d-3\,b\,B\,e-2\,A\,c\,e)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{3\sqrt{c}\,e^3\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}$$

Result (type 4, 346 leaves) :

$$\frac{1}{3\sqrt{\frac{b}{c}}\,d\,e^3\,(c\,d-b\,e)\,\sqrt{x\,(b+c\,x)}\,(d+e\,x)^{3/2}}\,2\left(\sqrt{\frac{b}{c}}\,e\,x\,(b+c\,x)\left(A\,e\left(-b\,e^2\,x+c\,d\,(d+2\,e\,x)\right)+B\,d\,(b\,e\,(3\,d+4\,e\,x)-c\,d\,(4\,d+5\,e\,x))\right)+\right.$$

$$(d+e\,x)\left(\sqrt{\frac{b}{c}}\,(B\,d\,(8\,c\,d-7\,b\,e)+A\,e\,(-2\,c\,d+b\,e))\,(b+c\,x)\,(d+e\,x)-i\,b\,e\,(A\,e\,(2\,c\,d-b\,e)+B\,d\,(-8\,c\,d+7\,b\,e))\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\right.$$

$$\left.\left.\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]-i\,b\,e\,(4\,B\,d-A\,e)\,(c\,d-b\,e)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\right)$$

- **Problem 1259: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+B\,x)\,\sqrt{b\,x+c\,x^2}}{(d+e\,x)^{7/2}}\,dx$$

Optimal (type 4, 494 leaves, 9 steps) :

$$\frac{2\left(2\,A\,e\,\left(c^2\,d^2-b\,c\,d\,e+b^2\,e^2\right)+B\,d\,\left(8\,c^2\,d^2-13\,b\,c\,d\,e+3\,b^2\,e^2\right)\right)\sqrt{b\,x+c\,x^2}}{15\,d^2\,e^2\,(c\,d-b\,e)^2\,\sqrt{d+e\,x}}-$$

$$\frac{2\,(d\,(B\,d\,(4\,c\,d-3\,b\,e)+A\,e\,(c\,d-2\,b\,e))+e\,(B\,d\,(7\,c\,d-6\,b\,e)-A\,e\,(2\,c\,d-b\,e))\,x)\,\sqrt{b\,x+c\,x^2}}{15\,d\,e^2\,(c\,d-b\,e)\,(d+e\,x)^{5/2}}-$$

$$\left(2\,\sqrt{-b}\,\sqrt{c}\,\left(2\,A\,e\,\left(c^2\,d^2-b\,c\,d\,e+b^2\,e^2\right)+B\,d\,\left(8\,c^2\,d^2-13\,b\,c\,d\,e+3\,b^2\,e^2\right)\right)\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]\right)\Big/$$

$$\left(15\,d^2\,e^3\,(c\,d-b\,e)^2\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}\right)+$$

$$\left(2\,\sqrt{-b}\,\sqrt{c}\,(B\,d\,(8\,c\,d-9\,b\,e)+A\,e\,(2\,c\,d-b\,e))\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]\right)\Big/$$

$$\left(15\,d\,e^3\,(c\,d-b\,e)\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}\right)$$

Result (type 4, 491 leaves) :

$$\frac{1}{15\, b\, d^2\, e^3\, (c\, d - b\, e)^2 \sqrt{x\, (b + c\, x)}\, (d + e\, x)^{5/2}} 2 \left(b\, e\, x\, (b + c\, x) \left(3\, d^2\, (B\, d - A\, e)\, (c\, d - b\, e)^2 - d\, (c\, d - b\, e)\, (B\, d\, (7\, c\, d - 6\, b\, e) + A\, e\, (-2\, c\, d + b\, e))\, (d + e\, x) + \left(2\, A\, e \left(c^2\, d^2 - b\, c\, d\, e + b^2\, e^2\right) + B\, d \left(8\, c^2\, d^2 - 13\, b\, c\, d\, e + 3\, b^2\, e^2\right)\right) (d + e\, x)^2\right) - \sqrt{\frac{b}{c}}\, c\, (d + e\, x)^2 \left(\sqrt{\frac{b}{c}} \left(2\, A\, e \left(c^2\, d^2 - b\, c\, d\, e + b^2\, e^2\right) + B\, d \left(8\, c^2\, d^2 - 13\, b\, c\, d\, e + 3\, b^2\, e^2\right)\right) (b + c\, x)\, (d + e\, x) + i\, b\, e \left(2\, A\, e \left(c^2\, d^2 - b\, c\, d\, e + b^2\, e^2\right) + B\, d \left(8\, c^2\, d^2 - 13\, b\, c\, d\, e + 3\, b^2\, e^2\right)\right) \sqrt{1 + \frac{b}{c\, x}} \sqrt{1 + \frac{d}{e\, x}}\, x^{3/2}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c\, d}{b\, e}\right] - i\, b\, e\, (c\, d - b\, e)\, (B\, d\, (4\, c\, d - 3\, b\, e) + A\, e\, (c\, d - 2\, b\, e)) \sqrt{1 + \frac{b}{c\, x}} \sqrt{1 + \frac{d}{e\, x}}\, x^{3/2}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c\, d}{b\, e}\right] \right) \right)$$

- **Problem 1260: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B\, x) \left(b\, x + c\, x^2\right)^{3/2}}{\sqrt{d + e\, x}}\, dx$$

Optimal (type 4, 574 leaves, 9 steps) :

$$\frac{1}{315 c^2 e^4} 2 \sqrt{d+e x} \left(9 A c e \left(8 c^2 d^2 - 11 b c d e + b^2 e^2\right) - 2 B \left(32 c^3 d^3 - 42 b c^2 d^2 e + 3 b^2 c d e^2 + 2 b^3 e^3\right) - 3 c e \left(9 A c e (2 c d - b e) - B \left(16 c^2 d^2 - 7 b c d e - 4 b^2 e^2\right)\right) x\right) \sqrt{b x + c x^2} - \frac{2 \sqrt{d+e x} (8 B c d - 3 b B e - 9 A c e - 7 B c e x) \left(b x + c x^2\right)^{3/2}}{63 c e^2} - \left(2 \sqrt{-b} \left(5 b c d e (2 c d - b e) (8 B c d - 3 b B e - 9 A c e) + \left(8 c^2 d^2 - 3 b c d e - 2 b^2 e^2\right) \left(9 A c e (2 c d - b e) - B \left(16 c^2 d^2 - 7 b c d e - 4 b^2 e^2\right)\right)\right) \sqrt{x} \sqrt{1 + \frac{c x}{b}} \sqrt{d+e x} \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/ \left(315 c^{5/2} e^5 \sqrt{1 + \frac{e x}{d}} \sqrt{b x + c x^2}\right) + \left(2 \sqrt{-b} d (c d - b e) \left(9 A c e \left(16 c^2 d^2 - 16 b c d e - b^2 e^2\right) - B \left(128 c^3 d^3 - 120 b c^2 d^2 e - 9 b^2 c d e^2 - 4 b^3 e^3\right)\right) \sqrt{x} \sqrt{1 + \frac{c x}{b}} \sqrt{1 + \frac{e x}{d}} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/ \left(315 c^{5/2} e^5 \sqrt{d+e x} \sqrt{b x + c x^2}\right)$$

Result (type 4, 630 leaves) :

$$-\frac{1}{315 b c^2 e^5 x^2 (b + c x)^2 \sqrt{d+e x}} 2 (x (b + c x))^{3/2} \Bigg(b e x (b + c x) (d + e x) \left(-9 A c e \left(b^2 e^2 + b c e (-11 d + 8 e x) + c^2 \left(8 d^2 - 6 d e x + 5 e^2 x^2\right)\right) + B \left(4 b^3 e^3 - 3 b^2 c e^2 (-2 d + e x) + b c^2 e \left(-84 d^2 + 61 d e x - 50 e^2 x^2\right) + c^3 \left(64 d^3 - 48 d^2 e x + 40 d e^2 x^2 - 35 e^3 x^3\right)\right)\right) + \sqrt{\frac{b}{c}} \Bigg(\sqrt{\frac{b}{c}} \left(18 A c e \left(8 c^3 d^3 - 12 b c^2 d^2 e + 2 b^2 c d e^2 + b^3 e^3\right) - B \left(128 c^4 d^4 - 184 b c^3 d^3 e + 27 b^2 c^2 d^2 e^2 + 11 b^3 c d e^3 + 8 b^4 e^4\right)\right) (b + c x) (d + e x) + i b e \left(18 A c e \left(8 c^3 d^3 - 12 b c^2 d^2 e + 2 b^2 c d e^2 + b^3 e^3\right) - B \left(128 c^4 d^4 - 184 b c^3 d^3 e + 27 b^2 c^2 d^2 e^2 + 11 b^3 c d e^3 + 8 b^4 e^4\right)\right) \sqrt{1 + \frac{b}{c x}} \sqrt{1 + \frac{d}{e x}} x^{3/2} \, \text{EllipticE}\left[i \, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right] - i b e (c d - b e) \left(9 A c e \left(8 c^2 d^2 - 5 b c d e - 2 b^2 e^2\right) + B \left(-64 c^3 d^3 + 36 b c^2 d^2 e + 15 b^2 c d e^2 + 8 b^3 e^3\right)\right) \sqrt{1 + \frac{b}{c x}} \sqrt{1 + \frac{d}{e x}} x^{3/2} \, \text{EllipticF}\left[i \, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right]\Bigg)\Bigg)$$

Problem 1261: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A+B x)\left(b x+c x^2\right)^{3/2}}{(d+e x)^{3/2}}\,dx$$

Optimal (type 4, 449 leaves, 9 steps):

$$-\frac{1}{35 c e^4}2\sqrt{d+e x}\left(7 A c e (8 c d-7 b e)-B\left(64 c^2 d^2-60 b c d e+b^2 e^2\right)+3 c e (16 B c d-b B e-14 A c e) x\right)\sqrt{b x+c x^2}+\frac{2 (8 B d-7 A e+B e x)\left(b x+c x^2\right)^{3/2}}{7 e^2\sqrt{d+e x}}+\left(2\sqrt{-b}\left(5 b c e (8 B d-7 A e)(2 c d-b e)-(16 B c d-b B e-14 A c e)\left(8 c^2 d^2-3 b c d e-2 b^2 e^2\right)\right)\sqrt{x}\sqrt{1+\frac{c x}{b}}\sqrt{d+e x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]\right)\Big/\left(35 c^{3/2} e^5\sqrt{1+\frac{e x}{d}}\sqrt{b x+c x^2}\right)-\left(2\sqrt{-b}\, d (c d-b e)\left(56 A c e (2 c d-b e)-B\left(128 c^2 d^2-72 b c d e-b^2 e^2\right)\right)\sqrt{x}\sqrt{1+\frac{c x}{b}}\sqrt{1+\frac{e x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]\right)\Big/\left(35 c^{3/2} e^5\sqrt{d+e x}\sqrt{b x+c x^2}\right)$$

Result (type 4, 514 leaves):

$$\frac{1}{35\, b\, c\, e^5\, x^2\, (b+c\, x)^2\, \sqrt{d+e\, x}} \left(2\, (x\, (b+c\, x))^{3/2} \left(b\, e\, x\, (b+c\, x)\, \left(35\, c\, d\, (B\, d-A\, e)\, (c\, d-b\, e)+\left(7\, A\, c\, e\, (-3\, c\, d+2\, b\, e)+B\, \left(29\, c^2\, d^2-25\, b\, c\, d\, e+b^2\, e^2\right)\right)\, (d+e\, x)+\right.\right.\right.$$

$$\left. c\, e\, (-13\, B\, c\, d+8\, b\, B\, e+7\, A\, c\, e)\, x\, (d+e\, x)+5\, B\, c^2\, e^2\, x^2\, (d+e\, x)\right)+$$

$$\sqrt{\frac{b}{c}} \left(\sqrt{\frac{b}{c}}\, \left(7\, A\, c\, e\, \left(16\, c^2\, d^2-16\, b\, c\, d\, e+b^2\, e^2\right)-B\, \left(128\, c^3\, d^3-136\, b\, c^2\, d^2\, e+11\, b^2\, c\, d\, e^2+2\, b^3\, e^3\right)\right)\, (b+c\, x)\, (d+e\, x)+\right.$$

$$i\, b\, e\, \left(7\, A\, c\, e\, \left(16\, c^2\, d^2-16\, b\, c\, d\, e+b^2\, e^2\right)-B\, \left(128\, c^3\, d^3-136\, b\, c^2\, d^2\, e+11\, b^2\, c\, d\, e^2+2\, b^3\, e^3\right)\right)$$

$$\sqrt{1+\frac{b}{c\, x}}\, \sqrt{1+\frac{d}{e\, x}}\, x^{3/2}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\, \frac{c\, d}{b\, e}\right]-$$

$$\left.\left. i\, b\, e\, (c\, d-b\, e)\, \left(7\, A\, c\, e\, (8\, c\, d-b\, e)+2\, B\, \left(-32\, c^2\, d^2+6\, b\, c\, d\, e+b^2\, e^2\right)\right)\, \sqrt{1+\frac{b}{c\, x}}\, \sqrt{1+\frac{d}{e\, x}}\, x^{3/2}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\, \frac{c\, d}{b\, e}\right]\right)\right)$$

- **Problem 1262: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B\, x)\, \left(b\, x+c\, x^2\right)^{3/2}}{(d+e\, x)^{5/2}}\, dx$$

Optimal (type 4, 413 leaves, 9 steps) :

$$-\frac{2\,(4\,B\,d\,(16\,c\,d-9\,b\,e)-5\,A\,e\,(8\,c\,d-3\,b\,e)+e\,(16\,B\,c\,d-3\,b\,B\,e-10\,A\,c\,e)\,x)\,\sqrt{b\,x+c\,x^2}}{15\,e^4\,\sqrt{d+e\,x}}+\frac{2\,(8\,B\,d-5\,A\,e+3\,B\,e\,x)\,\left(b\,x+c\,x^2\right)^{3/2}}{15\,e^2\,(d+e\,x)^{3/2}}-$$

$$\left(2\,\sqrt{-b}\,\left(40\,A\,c\,e\,(2\,c\,d-b\,e)-B\,\left(128\,c^2\,d^2-88\,b\,c\,d\,e+3\,b^2\,e^2\right)\right)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]\right)\Big/$$

$$\left(15\,\sqrt{c}\,e^5\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}\right)+\left(2\,\sqrt{-b}\,\left(5\,A\,e\,\left(16\,c^2\,d^2-16\,b\,c\,d\,e+3\,b^2\,e^2\right)-B\,d\,\left(128\,c^2\,d^2-152\,b\,c\,d\,e+39\,b^2\,e^2\right)\right)\right.$$

$$\left.\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]\right)\Big/\left(15\,\sqrt{c}\,e^5\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}\right)$$

Result (type 4, 436 leaves) :

$$\frac{1}{15\,e^5\,x^2\,(b+c\,x)^2\,\sqrt{d+e\,x}}\,2\,(x\,(b+c\,x))^{3/2}\left(\frac{\left(40\,A\,c\,e\,(-2\,c\,d+b\,e)+B\,\left(128\,c^2\,d^2-88\,b\,c\,d\,e+3\,b^2\,e^2\right)\right)\,(b+c\,x)\,(d+e\,x)}{c}+\frac{1}{d+e\,x}\right.$$

$$e\,x\,(b+c\,x)\,\left(5\,A\,e\,\left(-b\,e\,(3\,d+4\,e\,x)+c\,\left(8\,d^2+10\,d\,e\,x+e^2\,x^2\right)\right)+B\,\left(b\,e\,\left(36\,d^2+47\,d\,e\,x+6\,e^2\,x^2\right)-c\,\left(64\,d^3+80\,d^2\,e\,x+8\,d\,e^2\,x^2-3\,e^3\,x^3\right)\right)\right)-$$

$$i\,\sqrt{\frac{b}{c}}\,e\,\left(40\,A\,c\,e\,(2\,c\,d-b\,e)+B\,\left(-128\,c^2\,d^2+88\,b\,c\,d\,e-3\,b^2\,e^2\right)\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]+$$

$$\left.i\,\sqrt{\frac{b}{c}}\,e\,\left(5\,A\,c\,e\,(8\,c\,d-5\,b\,e)+B\,\left(-64\,c^2\,d^2+52\,b\,c\,d\,e-3\,b^2\,e^2\right)\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)$$

- **Problem 1263: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+B\,x)\,\left(b\,x+c\,x^2\right)^{3/2}}{(d+e\,x)^{7/2}}\,dx$$

Optimal (type 4, 516 leaves, 9 steps) :

$$-\left(2\left(d\left(3 A c e (8 c d-7 b e)-B\left(64 c^2 d^2-76 b c d e+15 b^2 e^2\right)\right)-c e (B d (16 c d-13 b e)-3 A e (2 c d-b e)) x\right) \sqrt{b x+c x^2}\right) \Big/ \left(15 d e^4 (c d-b e) \sqrt{d+e x}\right)-\frac{2\left(d^2 (8 B c d-5 b B e-3 A c e)+e (B d (11 c d-8 b e)-3 A e (2 c d-b e)) x\right)\left(b x+c x^2\right)^{3/2}}{15 d e^2 (c d-b e)(d+e x)^{5/2}}+\left(2 \sqrt{-b} \sqrt{c}\left(3 A e\left(16 c^2 d^2-16 b c d e+b^2 e^2\right)-B d\left(128 c^2 d^2-168 b c d e+43 b^2 e^2\right)\right) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{d+e x}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/ \left(15 d e^5 (c d-b e) \sqrt{1+\frac{e x}{d}} \sqrt{b x+c x^2}\right)-\left(2 \sqrt{-b}\left(24 A c e (2 c d-b e)-B\left(128 c^2 d^2-104 b c d e+15 b^2 e^2\right)\right) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{1+\frac{e x}{d}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/ \left(15 \sqrt{c}\, e^5 \sqrt{d+e x} \sqrt{b x+c x^2}\right)$$

Result (type 4, 530 leaves) :

$$\frac{1}{15 \sqrt{\frac{b}{c}}\, d\, e^5 (c d - b e)\, x^2 (b + c x)^2 (d + e x)^{5/2}}$$

$$2 (x (b + c x))^{3/2} \left(\sqrt{\frac{b}{c}}\, e\, x (b + c x) \left(3 d^2 (B d - A e) (c d - b e)^2 - d (c d - b e) (B d (17 c d - 11 b e) + 6 A e (-2 c d + b e)) (d + e x) + \right.\right.$$

$$\left.\left(-3 A e \left(11 c^2 d^2 - 11 b c d e + b^2 e^2\right) + B d \left(73 c^2 d^2 - 93 b c d e + 23 b^2 e^2\right)\right) (d + e x)^2 + 5 B c d (c d - b e) (d + e x)^3\right) +$$

$$(d + e x)^2 \left(\sqrt{\frac{b}{c}} \left(B d \left(-128 c^2 d^2 + 168 b c d e - 43 b^2 e^2\right) + 3 A e \left(16 c^2 d^2 - 16 b c d e + b^2 e^2\right)\right) (b + c x) (d + e x) + i\, b\, e\right.$$

$$\left(B d \left(-128 c^2 d^2 + 168 b c d e - 43 b^2 e^2\right) + 3 A e \left(16 c^2 d^2 - 16 b c d e + b^2 e^2\right)\right) \sqrt{1 + \frac{b}{c x}} \sqrt{1 + \frac{d}{e x}}\, x^{3/2}\, \text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right] +$$

$$\left.\left. i\, b\, e (c d - b e) (4 B d (16 c d - 7 b e) + 3 A e (-8 c d + b e)) \sqrt{1 + \frac{b}{c x}} \sqrt{1 + \frac{d}{e x}}\, x^{3/2}\, \text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right]\right)\right)$$

- **Problem 1264: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B x) (d + e x)^{5/2}}{\sqrt{b x + c x^2}}\, dx$$

Optimal (type 4, 460 leaves, 10 steps) :

$$\frac{2\left(28 A c e (2 c d-b e)+B\left(15 c^2 d^2-43 b c d e+24 b^2 e^2\right)\right) \sqrt{d+e x} \sqrt{b x+c x^2}}{105 c^3}+$$

$$\frac{2 (5 B c d-6 b B e+7 A c e) (d+e x)^{3/2} \sqrt{b x+c x^2}}{35 c^2}+\frac{2 B (d+e x)^{5/2} \sqrt{b x+c x^2}}{7 c}+$$

$$\left(2 \sqrt{-b} \left(7 A c e \left(23 c^2 d^2-23 b c d e+8 b^2 e^2\right)+B\left(15 c^3 d^3-103 b c^2 d^2 e+128 b^2 c d e^2-48 b^3 e^3\right)\right) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{d+e x}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/ \left(105 c^{7/2} e \sqrt{1+\frac{e x}{d}} \sqrt{b x+c x^2}\right)-$$

$$\left(2 \sqrt{-b}\, d (c d-b e) \left(28 A c e (2 c d-b e)+B\left(15 c^2 d^2-43 b c d e+24 b^2 e^2\right)\right) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{1+\frac{e x}{d}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/$$

$$\left(105 c^{7/2} e \sqrt{d+e x} \sqrt{b x+c x^2}\right)$$

Result (type 4, 479 leaves) :

$$\frac{1}{105 c^3 \sqrt{x (b+c x)} \sqrt{d+e x}}$$

$$2 \sqrt{x} \left(\frac{1}{c e \sqrt{x}} \left(7 A c e \left(23 c^2 d^2-23 b c d e+8 b^2 e^2\right)+B\left(15 c^3 d^3-103 b c^2 d^2 e+128 b^2 c d e^2-48 b^3 e^3\right)\right) (b+c x) (d+e x)+\right.$$

$$\sqrt{x} (b+c x) (d+e x) \left(7 A c e (11 c d-4 b e+3 c e x)+B\left(24 b^2 e^2-b c e (61 d+18 e x)+15 c^2 \left(3 d^2+3 d e x+e^2 x^2\right)\right)\right)+$$

$$i \sqrt{\frac{b}{c}} \left(7 A c e \left(23 c^2 d^2-23 b c d e+8 b^2 e^2\right)+B\left(15 c^3 d^3-103 b c^2 d^2 e+128 b^2 c d e^2-48 b^3 e^3\right)\right)$$

$$\sqrt{1+\frac{b}{c x}} \sqrt{1+\frac{d}{e x}}\, x\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right]+\frac{1}{b} i \sqrt{\frac{b}{c}} (-c d+b e)$$

$$\left.\left(-105 A c^3 d^2+48 b^3 B e^2-8 b^2 c e (13 B d+7 A e)+b c^2 d (60 B d+133 A e)\right) \sqrt{1+\frac{b}{c x}} \sqrt{1+\frac{d}{e x}}\, x\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right]\right)$$

Problem 1265: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A+B x)\,(d+e x)^{3/2}}{\sqrt{b x+c x^2}}\,dx$$

Optimal (type 4, 339 leaves, 9 steps):

$$\frac{2\,(3 B c d-4 b B e+5 A c e)\,\sqrt{d+e x}\,\sqrt{b x+c x^2}}{15 c^2}+\frac{2 B\,(d+e x)^{3/2}\,\sqrt{b x+c x^2}}{5 c}+\left(2\sqrt{-b}\,\left(10 A c e\,(2 c d-b e)+B\left(3 c^2 d^2-13 b c d e+8 b^2 e^2\right)\right)\sqrt{x}\,\sqrt{1+\frac{c x}{b}}\,\sqrt{d+e x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]\right)\Big/\left(15 c^{5/2} e\,\sqrt{1+\frac{e x}{d}}\,\sqrt{b x+c x^2}\right)-\frac{2\sqrt{-b}\,d\,(c d-b e)\,(3 B c d-4 b B e+5 A c e)\,\sqrt{x}\,\sqrt{1+\frac{c x}{b}}\,\sqrt{1+\frac{e x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{15 c^{5/2} e\,\sqrt{d+e x}\,\sqrt{b x+c x^2}}$$

Result (type 4, 356 leaves):

$$\frac{1}{15 c^2\,\sqrt{x\,(b+c x)}\,\sqrt{d+e x}}\;2\sqrt{x}\left(\frac{\left(10 A c e\,(2 c d-b e)+B\left(3 c^2 d^2-13 b c d e+8 b^2 e^2\right)\right)(b+c x)\,(d+e x)}{c e\sqrt{x}}+\sqrt{x}\,(b+c x)\,(d+e x)\,(5 A c e+B\,(6 c d-4 b e+3 c e x))+\right.$$
$$i\sqrt{\frac{b}{c}}\left(10 A c e\,(2 c d-b e)+B\left(3 c^2 d^2-13 b c d e+8 b^2 e^2\right)\right)\sqrt{1+\frac{b}{c x}}\,\sqrt{1+\frac{d}{e x}}\,x\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]-$$
$$\left.\frac{1}{b}i\sqrt{\frac{b}{c}}\,(-c d+b e)\left(15 A c^2 d+8 b^2 B e-b c\,(9 B d+10 A e)\right)\sqrt{1+\frac{b}{c x}}\,\sqrt{1+\frac{d}{e x}}\,x\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]\right)$$

Problem 1266: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A+B x)\,\sqrt{d+e x}}{\sqrt{b x+c x^2}}\,dx$$

Optimal (type 4, 254 leaves, 8 steps):

$$\frac{2 B \sqrt{d+e x} \sqrt{b x+c x^2}}{3 c}+\frac{2 \sqrt{-b} (B c d-2 b B e+3 A c e) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{d+e x} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{3 c^{3/2} e \sqrt{1+\frac{e x}{d}} \sqrt{b x+c x^2}}-$$

$$\frac{2 \sqrt{-b} B d (c d-b e) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{1+\frac{e x}{d}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{3 c^{3/2} e \sqrt{d+e x} \sqrt{b x+c x^2}}$$

Result (type 4, 263 leaves) :

$$\frac{1}{3 c \sqrt{x (b+c x)} \sqrt{d+e x}}$$

$$2 x \left(B (b+c x) (d+e x)+\frac{(B c d-2 b B e+3 A c e) (b+c x) (d+e x)}{c e}+i \sqrt{\frac{b}{c}} (B c d-2 b B e+3 A c e) \sqrt{1+\frac{b}{c x}} \sqrt{1+\frac{d}{e x}} \sqrt{x}\right.$$

$$\left.\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]+\frac{i \sqrt{\frac{b}{c}} (2 b B-3 A c) (-c d+b e) \sqrt{1+\frac{b}{c x}} \sqrt{1+\frac{d}{e x}} \sqrt{x} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]}{b}\right)$$

Problem 1267: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x}{\sqrt{d+e x} \sqrt{b x+c x^2}} \, dx$$

Optimal (type 4, 204 leaves, 7 steps) :

$$\frac{2 \sqrt{-b} B \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{d+e x} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{\sqrt{c} e \sqrt{1+\frac{e x}{d}} \sqrt{b x+c x^2}}-\frac{2 \sqrt{-b} (B d-A e) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{1+\frac{e x}{d}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{\sqrt{c} e \sqrt{d+e x} \sqrt{b x+c x^2}}$$

Result (type 4, 209 leaves) :

$$\frac{1}{b\,e\,\sqrt{x}\,(b+c\,x)\,\sqrt{d+e\,x}}\left(\frac{2\,b\,B\,(b+c\,x)\,(d+e\,x)}{c}+2\,i\,b\,B\,\sqrt{\frac{b}{c}}\,e\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]-\right.$$

$$\left.2\,i\,\sqrt{\frac{b}{c}}\,(b\,B-A\,c)\,e\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)$$

- **Problem 1268: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B\,x}{(d+e\,x)^{3/2}\,\sqrt{b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 262 leaves, 8 steps):

$$\frac{2\,(B\,d-A\,e)\,\sqrt{b\,x+c\,x^2}}{d\,(c\,d-b\,e)\,\sqrt{d+e\,x}}-\frac{2\,\sqrt{-b}\,\sqrt{c}\,(B\,d-A\,e)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{d\,e\,(c\,d-b\,e)\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}}+$$

$$\frac{2\,\sqrt{-b}\,B\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{\sqrt{c}\,e\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}$$

Result (type 4, 226 leaves):

$$\left(-2\,\sqrt{\frac{b}{c}}\,d\,(B\,d-A\,e)\,(b+c\,x)+2\,i\,b\,e\,(-B\,d+A\,e)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]+\right.$$

$$\left.2\,i\,A\,e\,(c\,d-b\,e)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\Big/\left(\sqrt{\frac{b}{c}}\,d\,e\,(c\,d-b\,e)\,\sqrt{x\,(b+c\,x)}\,\sqrt{d+e\,x}\right)$$

- **Problem 1269: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+B\,x}{(d+e\,x)^{5/2}\,\sqrt{b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 369 leaves, 9 steps):

$$\frac{2\,(B\,d-A\,e)\,\sqrt{b\,x+c\,x^2}}{3\,d\,(c\,d-b\,e)\,(d+e\,x)^{3/2}}-\frac{2\,(2\,A\,e\,(2\,c\,d-b\,e)-B\,d\,(c\,d+b\,e))\,\sqrt{b\,x+c\,x^2}}{3\,d^2\,(c\,d-b\,e)^2\,\sqrt{d+e\,x}}+$$

$$\frac{2\,\sqrt{-b}\,\sqrt{c}\,(2\,A\,e\,(2\,c\,d-b\,e)-B\,d\,(c\,d+b\,e))\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{3\,d^2\,e\,(c\,d-b\,e)^2\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}}+$$

$$\frac{2\,\sqrt{-b}\,\sqrt{c}\,(B\,d-A\,e)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{3\,d\,e\,(c\,d-b\,e)\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}$$

Result (type 4, 347 leaves):

$$\frac{1}{3\,b\,d^2\,e\,(c\,d-b\,e)^2\,\sqrt{x\,(b+c\,x)}\,(d+e\,x)^{3/2}}$$

$$2\left(b\,e\,x\,(b+c\,x)\,\left(B\,d\,\left(b\,e^2\,x+c\,d\,(2\,d+e\,x)\right)+A\,e\,(b\,e\,(3\,d+2\,e\,x)-c\,d\,(5\,d+4\,e\,x))\right)-\sqrt{\frac{b}{c}}\,c\,(d+e\,x)\left(\sqrt{\frac{b}{c}}\,(2\,A\,e\,(-2\,c\,d+b\,e)+B\,d\,(c\,d+b\,e))\right.\right.$$

$$(b+c\,x)\,(d+e\,x)+i\,b\,e\,(2\,A\,e\,(-2\,c\,d+b\,e)+B\,d\,(c\,d+b\,e))\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]-$$

$$\left.\left.i\,e\,(c\,d-b\,e)\,(3\,A\,c\,d-b\,(B\,d+2\,A\,e))\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\right)$$

Problem 1270: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x}{(d+e x)^{7/2} \sqrt{b x+c x^2}} \, dx$$

Optimal (type 4, 510 leaves, 10 steps):

$$\frac{2 (B d-A e) \sqrt{b x+c x^2}}{5 d (c d-b e) (d+e x)^{5/2}}-\frac{2 (4 A e (2 c d-b e)-B d (3 c d+b e)) \sqrt{b x+c x^2}}{15 d^2 (c d-b e)^2 (d+e x)^{3/2}}+$$

$$\frac{2 \left(B d \left(3 c^2 d^2+7 b c d e-2 b^2 e^2\right)-A e \left(23 c^2 d^2-23 b c d e+8 b^2 e^2\right)\right) \sqrt{b x+c x^2}}{15 d^3 (c d-b e)^3 \sqrt{d+e x}}-$$

$$\left(2 \sqrt{-b} \sqrt{c} \left(B d \left(3 c^2 d^2+7 b c d e-2 b^2 e^2\right)-A e \left(23 c^2 d^2-23 b c d e+8 b^2 e^2\right)\right) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{d+e x} \text{ EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/$$

$$\left(15 d^3 e (c d-b e)^3 \sqrt{1+\frac{e x}{d}} \sqrt{b x+c x^2}\right)-$$

$$\left(2 \sqrt{-b} \sqrt{c} (4 A e (2 c d-b e)-B d (3 c d+b e)) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{1+\frac{e x}{d}} \text{ EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/$$

$$\left(15 d^2 e (c d-b e)^2 \sqrt{d+e x} \sqrt{b x+c x^2}\right)$$

Result (type 4, 506 leaves):

$$\frac{1}{15\, b\, d^3\, e\, (c\, d - b\, e)^3 \sqrt{x\, (b + c\, x)}\, (d + e\, x)^{5/2}}$$

$$2 \Bigg(b\, e\, x\, (b + c\, x) \left(3\, d^2\, (B\, d - A\, e)\, (c\, d - b\, e)^2 + d\, (c\, d - b\, e)\, (4\, A\, e\, (-2\, c\, d + b\, e) + B\, d\, (3\, c\, d + b\, e))\, (d + e\, x) + \left(A\, e\, \left(-23\, c^2\, d^2 + 23\, b\, c\, d\, e - 8\, b^2\, e^2\right) + B\, d\, \left(3\, c^2\, d^2 + 7\, b\, c\, d\, e - 2\, b^2\, e^2\right)\right) (d + e\, x)^2\right) -$$

$$\sqrt{\frac{b}{c}}\, c\, (d + e\, x)^2 \Bigg(\sqrt{\frac{b}{c}} \left(A\, e\, \left(-23\, c^2\, d^2 + 23\, b\, c\, d\, e - 8\, b^2\, e^2\right) + B\, d\, \left(3\, c^2\, d^2 + 7\, b\, c\, d\, e - 2\, b^2\, e^2\right)\right) (b + c\, x)\, (d + e\, x) -$$

$$i\, b\, e \left(B\, d \left(-3\, c^2\, d^2 - 7\, b\, c\, d\, e + 2\, b^2\, e^2\right) + A\, e \left(23\, c^2\, d^2 - 23\, b\, c\, d\, e + 8\, b^2\, e^2\right)\right) \sqrt{1 + \frac{b}{c\, x}} \sqrt{1 + \frac{d}{e\, x}}\, x^{3/2}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c\, d}{b\, e}\right] -$$

$$i\, e\, (c\, d - b\, e) \left(15\, A\, c^2\, d^2 + 2\, b^2\, e\, (B\, d + 4\, A\, e) - b\, c\, d\, (6\, B\, d + 19\, A\, e)\right) \sqrt{1 + \frac{b}{c\, x}} \sqrt{1 + \frac{d}{e\, x}}\, x^{3/2}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c\, d}{b\, e}\right] \Bigg) \Bigg)$$

- **Problem 1271: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B\, x)\, (d + e\, x)^{7/2}}{\left(b\, x + c\, x^2\right)^{3/2}}\, dx$$

Optimal (type 4, 527 leaves, 10 steps) :

$$-\frac{2 (d+e x)^{5/2} \left(A b c d+\left(2 A c^2 d+b^2 B e-b c (B d+A e)\right) x\right)}{b^2 c \sqrt{b x+c x^2}}+$$

$$\frac{2 e \left(30 A c^3 d^2-24 b^3 B e^2-15 b c^2 d (B d+2 A e)+b^2 c e (43 B d+20 A e)\right) \sqrt{d+e x} \sqrt{b x+c x^2}}{15 b^2 c^3}+$$

$$\frac{2 e \left(10 A c^2 d+6 b^2 B e-5 b c (B d+A e)\right) (d+e x)^{3/2} \sqrt{b x+c x^2}}{5 b^2 c^2}+$$

$$\left(2 \left(30 A c^4 d^3+48 b^4 B e^3-15 b c^3 d^2 (B d+3 A e)-8 b^3 c e^2 (16 B d+5 A e)+b^2 c^2 d e (103 B d+95 A e)\right)\right.$$

$$\left.\sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{d+e x} \text{ EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \bigg/ \left(15 (-b)^{3/2} c^{7/2} \sqrt{1+\frac{e x}{d}} \sqrt{b x+c x^2}\right)-$$

$$\left(2 d (c d-b e) \left(30 A c^3 d^2-24 b^3 B e^2-15 b c^2 d (B d+2 A e)+b^2 c e (43 B d+20 A e)\right) \sqrt{x} \sqrt{1+\frac{c x}{b}} \sqrt{1+\frac{e x}{d}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \bigg/ \left(15 (-b)^{3/2} c^{7/2} \sqrt{d+e x} \sqrt{b x+c x^2}\right)$$

Result (type 4, 493 leaves) :

$$\frac{1}{15\, b^3 c^3 \sqrt{x\,(b+c\,x)}\,\sqrt{d+e\,x}}$$

$$2\left(b\,(d+e\,x)\,\left(15\,(b\,B-A\,c)\,(c\,d-b\,e)^3\,x-15\,A\,c^3\,d^3\,(b+c\,x)+b^2\,e^2\,(16\,B\,c\,d-9\,b\,B\,e+5\,A\,c\,e)\,x\,(b+c\,x)+3\,b^2\,B\,c\,e^3\,x^2\,(b+c\,x)\right)+\right.$$

$$\sqrt{\frac{b}{c}}\left(\sqrt{\frac{b}{c}}\,\left(30\,A\,c^4\,d^3+48\,b^4\,B\,e^3-15\,b\,c^3\,d^2\,(B\,d+3\,A\,e)-8\,b^3\,c\,e^2\,(16\,B\,d+5\,A\,e)+b^2\,c^2\,d\,e\,(103\,B\,d+95\,A\,e)\right)\,(b+c\,x)\,(d+e\,x)+\right.$$

$$\mathrm{i}\,b\,e\,\left(30\,A\,c^4\,d^3+48\,b^4\,B\,e^3-15\,b\,c^3\,d^2\,(B\,d+3\,A\,e)-8\,b^3\,c\,e^2\,(16\,B\,d+5\,A\,e)+b^2\,c^2\,d\,e\,(103\,B\,d+95\,A\,e)\right)$$

$$\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]-\mathrm{i}\,b\,e\,(c\,d-b\,e)$$

$$\left.\left.\left(15\,A\,c^3\,d^2-48\,b^3\,B\,e^2-15\,b\,c^2\,d\,(4\,B\,d+5\,A\,e)+8\,b^2\,c\,e\,(13\,B\,d+5\,A\,e)\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\right)$$

- **Problem 1272: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B\,x)\,(d+e\,x)^{5/2}}{\left(b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 399 leaves, 9 steps) :

$$-\frac{2\,(d+e\,x)^{3/2}\,\left(A\,b\,c\,d+\left(2\,A\,c^2\,d+b^2\,B\,e-b\,c\,(B\,d+A\,e)\right)\,x\right)}{b^2\,c\,\sqrt{b\,x+c\,x^2}}+\frac{2\,e\,\left(6\,A\,c^2\,d+4\,b^2\,B\,e-3\,b\,c\,(B\,d+A\,e)\right)\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}{3\,b^2\,c^2}+$$

$$\left(2\,\left(6\,A\,c^3\,d^2-8\,b^3\,B\,e^2-3\,b\,c^2\,d\,(B\,d+2\,A\,e)+b^2\,c\,e\,(13\,B\,d+6\,A\,e)\right)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]\right)\Big/$$

$$\left(3\,(-b)^{3/2}\,c^{5/2}\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}\right)-$$

$$\left(2\,d\,(c\,d-b\,e)\,\left(6\,A\,c^2\,d+4\,b^2\,B\,e-3\,b\,c\,(B\,d+A\,e)\right)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]\right)\Big/$$

$$\left(3\,(-b)^{3/2}\,c^{5/2}\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}\right)$$

Result (type 4, 391 leaves) :

$$\frac{1}{3\,b^3\,c^2\,\sqrt{x\,(b+c\,x)}\,\sqrt{d+e\,x}}\,2\,\Bigg(b\,(d+e\,x)\,\left(3\,(b\,B-A\,c)\,(c\,d-b\,e)^2\,x-3\,A\,c^2\,d^2\,(b+c\,x)+b^2\,B\,e^2\,x\,(b+c\,x)\right)+$$

$$\sqrt{\frac{b}{c}}\,\Bigg(\sqrt{\frac{b}{c}}\,\left(6\,A\,c^3\,d^2-8\,b^3\,B\,e^2-3\,b\,c^2\,d\,(B\,d+2\,A\,e)+b^2\,c\,e\,(13\,B\,d+6\,A\,e)\right)\,(b+c\,x)\,(d+e\,x)+$$

$$i\,b\,e\,\left(6\,A\,c^3\,d^2-8\,b^3\,B\,e^2-3\,b\,c^2\,d\,(B\,d+2\,A\,e)+b^2\,c\,e\,(13\,B\,d+6\,A\,e)\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]-$$

$$i\,b\,e\,(c\,d-b\,e)\,\left(3\,A\,c^2\,d+8\,b^2\,B\,e-3\,b\,c\,(3\,B\,d+2\,A\,e)\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\Bigg)\Bigg)$$

- **Problem 1273: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+B\,x)\,(d+e\,x)^{3/2}}{\left(b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 295 leaves, 8 steps) :

$$-\frac{2\sqrt{d+e x}\,\left(A b c d+\left(2 A c^2 d+b^2 B e-b c\,(B d+A e)\right) x\right)}{b^2 c\sqrt{b x+c x^2}}+$$

$$\frac{2\left(2 A c^2 d+2 b^2 B e-b c\,(B d+A e)\right)\sqrt{x}\,\sqrt{1+\frac{c x}{b}}\,\sqrt{d+e x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{(-b)^{3/2} c^{3/2}\sqrt{1+\frac{e x}{d}}\,\sqrt{b x+c x^2}}+$$

$$\frac{2\,(b B-2 A c)\,d\,(c d-b e)\sqrt{x}\,\sqrt{1+\frac{c x}{b}}\,\sqrt{1+\frac{e x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{(-b)^{3/2} c^{3/2}\sqrt{d+e x}\,\sqrt{b x+c x^2}}$$

Result (type 4, 302 leaves) :

$$\frac{1}{b^3 c\sqrt{x\,(b+c x)}\,\sqrt{d+e x}}\,2\left(b\,(d+e x)\,\left((b B-A c)\,(c d-b e)\,x-A c d\,(b+c x)\right)+\right.$$

$$\sqrt{\frac{b}{c}}\left(\sqrt{\frac{b}{c}}\left(2 A c^2 d+2 b^2 B e-b c\,(B d+A e)\right)(b+c x)\,(d+e x)+i\,b\,e\left(2 A c^2 d+2 b^2 B e-b c\,(B d+A e)\right)\sqrt{1+\frac{b}{c x}}\,\sqrt{1+\frac{d}{e x}}\,x^{3/2}\right.$$

$$\left.\left.\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]-i\,b\,(-2 b B+A c)\,e\,(c d-b e)\sqrt{1+\frac{b}{c x}}\,\sqrt{1+\frac{d}{e x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]\right)\right)$$

- **Problem 1274: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+B x)\sqrt{d+e x}}{\left(b x+c x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 253 leaves, 8 steps) :

$$-\frac{2\,(A\,b-(b\,B-2\,A\,c)\,x)\,\sqrt{d+e\,x}}{b^2\,\sqrt{b\,x+c\,x^2}}-\frac{2\,(b\,B-2\,A\,c)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{(-b)^{3/2}\,\sqrt{c}\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}}+$$

$$\frac{2\,(b\,B\,d-2\,A\,c\,d+A\,b\,e)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{(-b)^{3/2}\,\sqrt{c}\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}$$

Result (type 4, 210 leaves) :

$$\left(-2\,i\,\sqrt{\frac{b}{c}}\,c\,(b\,B-2\,A\,c)\,e\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]-\right.$$

$$\left.2\,(b\,B-A\,c)\,\left(b\,(d+e\,x)-i\,\sqrt{\frac{b}{c}}\,c\,e\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\right)\Big/\left(b^2\,c\,\sqrt{x\,(b+c\,x)}\,\sqrt{d+e\,x}\right)$$

- **Problem 1275: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B\,x}{\sqrt{d+e\,x}\,\left(b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 295 leaves, 8 steps) :

$$-\frac{2\,\sqrt{d+e\,x}\,(A\,b\,(c\,d-b\,e)+c\,(2\,A\,c\,d-b\,(B\,d+A\,e))\,x)}{b^2\,d\,(c\,d-b\,e)\,\sqrt{b\,x+c\,x^2}}-\frac{2\,\sqrt{c}\,(b\,B\,d-2\,A\,c\,d+A\,b\,e)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{(-b)^{3/2}\,d\,(c\,d-b\,e)\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}}+$$

$$\frac{2\,(b\,B-2\,A\,c)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]}{(-b)^{3/2}\,\sqrt{c}\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}}$$

Result (type 4, 233 leaves) :

$$\left(2\sqrt{\frac{b}{c}}\,(b B-A c)\,d\,(d+e x)-2\,i\,e\,(2 A c d-b\,(B d+A e))\,\sqrt{1+\frac{b}{c x}}\,\sqrt{1+\frac{d}{e x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]+\right.$$

$$\left.2\,i\,A e\,(c d-b e)\,\sqrt{1+\frac{b}{c x}}\,\sqrt{1+\frac{d}{e x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]\right)\Big/\left(b\sqrt{\frac{b}{c}}\,d\,(-c d+b e)\,\sqrt{x\,(b+c x)}\,\sqrt{d+e x}\right)$$

- **Problem 1276: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B x}{(d+e x)^{3/2}\left(b x+c x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 415 leaves, 9 steps):

$$-\frac{2\,(A b\,(c d-b e)+c\,(2 A c d-b\,(B d+A e))\,x)}{b^2 d\,(c d-b e)\,\sqrt{d+e x}\,\sqrt{b x+c x^2}}-\frac{2 e\left(2 A c^2 d^2-b^2 e\,(B d-2 A e)-b c d\,(B d+2 A e)\right)\sqrt{b x+c x^2}}{b^2 d^2\,(c d-b e)^2\,\sqrt{d+e x}}+$$

$$\left(2\sqrt{c}\,\left(2 A c^2 d^2-b^2 e\,(B d-2 A e)-b c d\,(B d+2 A e)\right)\sqrt{x}\,\sqrt{1+\frac{c x}{b}}\,\sqrt{d+e x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]\right)\Big/$$

$$\left((-b)^{3/2} d^2\,(c d-b e)^2\,\sqrt{1+\frac{e x}{d}}\,\sqrt{b x+c x^2}\right)+\frac{2\sqrt{c}\,(b B d-2 A c d+A b e)\,\sqrt{x}\,\sqrt{1+\frac{c x}{b}}\,\sqrt{1+\frac{e x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{(-b)^{3/2} d\,(c d-b e)\,\sqrt{d+e x}\,\sqrt{b x+c x^2}}$$

Result (type 4, 367 leaves):

$$\frac{1}{b^2 d^2 (c d - b e)^2 \sqrt{x (b + c x)} \sqrt{d + e x}} 2 \Bigg(b^2 e^2 (B d - A e) x (b + c x) + c^2 (b B - A c) d^2 x (d + e x) - A (c d - b e)^2 (b + c x) (d + e x) + \left(2 A c^2 d^2 + b^2 e (-B d + 2 A e) - b c d (B d + 2 A e)\right)$$

$$(b + c x) (d + e x) + i \sqrt{\frac{b}{c}} c e \left(2 A c^2 d^2 + b^2 e (-B d + 2 A e) - b c d (B d + 2 A e)\right) \sqrt{1 + \frac{b}{c x}} \sqrt{1 + \frac{d}{e x}} x^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right] -$$

$$i \sqrt{\frac{b}{c}} c e (c d - b e) (b B d + A c d - 2 A b e) \sqrt{1 + \frac{b}{c x}} \sqrt{1 + \frac{d}{e x}} x^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right] \Bigg)$$

Problem 1277: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x}{(d + e x)^{5/2} \left(b x + c x^2\right)^{3/2}} \, dx$$

Optimal (type 4, 570 leaves, 10 steps):

$$-\frac{2 (A b (c d - b e) + c (2 A c d - b (B d + A e)) x)}{b^2 d (c d - b e) (d + e x)^{3/2} \sqrt{b x + c x^2}} - \frac{2 e \left(6 A c^2 d^2 - b^2 e (B d - 4 A e) - 3 b c d (B d + 2 A e)\right) \sqrt{b x + c x^2}}{3 b^2 d^2 (c d - b e)^2 (d + e x)^{3/2}} -$$

$$\frac{2 e \left(6 A c^3 d^3 - b^2 c d e (7 B d - 19 A e) + 2 b^3 e^2 (B d - 4 A e) - 3 b c^2 d^2 (B d + 3 A e)\right) \sqrt{b x + c x^2}}{3 b^2 d^3 (c d - b e)^3 \sqrt{d + e x}} +$$

$$\Bigg(2 \sqrt{c} \left(6 A c^3 d^3 - b^2 c d e (7 B d - 19 A e) + 2 b^3 e^2 (B d - 4 A e) - 3 b c^2 d^2 (B d + 3 A e)\right) \sqrt{x}$$

$$\sqrt{1 + \frac{c x}{b}} \sqrt{d + e x} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\Bigg) \Big/ \left(3 (-b)^{3/2} d^3 (c d - b e)^3 \sqrt{1 + \frac{e x}{d}} \sqrt{b x + c x^2}\right) -$$

$$\left(2 \sqrt{c} \left(6 A c^2 d^2 - b^2 e (B d - 4 A e) - 3 b c d (B d + 2 A e)\right) \sqrt{x} \sqrt{1 + \frac{c x}{b}} \sqrt{1 + \frac{e x}{d}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/$$

$$\left(3 (-b)^{3/2} d^2 (c d - b e)^2 \sqrt{d + e x} \sqrt{b x + c x^2}\right)$$

Result (type 4, 506 leaves) :

$$\frac{1}{3\,b^3\,d^3\,(c\,d-b\,e)^3\,\sqrt{x\,(b+c\,x)}\,(d+e\,x)^{3/2}}$$

$$2\left(b\left(b^2\,d\,e^2\,(B\,d-A\,e)\,(c\,d-b\,e)\,x\,(b+c\,x)+b^2\,e^2\,(B\,d\,(7\,c\,d-2\,b\,e)+5\,A\,e\,(-2\,c\,d+b\,e))\,x\,(b+c\,x)\,(d+e\,x)+\right.\right.$$

$$\left.3\,c^3\,(b\,B-A\,c)\,d^3\,x\,(d+e\,x)^2-3\,A\,(c\,d-b\,e)^3\,(b+c\,x)\,(d+e\,x)^2\right)+$$

$$\sqrt{\frac{b}{c}}\,c\,(d+e\,x)\left(\sqrt{\frac{b}{c}}\,\left(6\,A\,c^3\,d^3+2\,b^3\,e^2\,(B\,d-4\,A\,e)-3\,b\,c^2\,d^2\,(B\,d+3\,A\,e)+b^2\,c\,d\,e\,(-7\,B\,d+19\,A\,e)\right)\,(b+c\,x)\,(d+e\,x)+i\,b\,e\left(6\,A\,c^3\,d^3+\right.\right.$$

$$\left.2\,b^3\,e^2\,(B\,d-4\,A\,e)-3\,b\,c^2\,d^2\,(B\,d+3\,A\,e)+b^2\,c\,d\,e\,(-7\,B\,d+19\,A\,e)\right)\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]-$$

$$\left.\left.i\,b\,e\,(c\,d-b\,e)\,\left(3\,A\,c^2\,d^2+3\,b\,c\,d\,(2\,B\,d-5\,A\,e)+2\,b^2\,e\,(-B\,d+4\,A\,e)\right)\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\right)$$

- **Problem 1278: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B\,x)\,(d+e\,x)^{7/2}}{\left(b\,x+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 524 leaves, 9 steps) :

$$-\frac{2\,(d+e\,x)^{5/2}\,\left(A\,b\,c\,d+\left(2\,A\,c^2\,d+b^2\,B\,e-b\,c\,(B\,d+A\,e)\right)\,x\right)}{3\,b^2\,c\,\left(b\,x+c\,x^2\right)^{3/2}}+\frac{1}{3\,b^4\,c^2\,\sqrt{b\,x+c\,x^2}}2\,\sqrt{d+e\,x}$$

$$\left(b\,c\,d^2\,\left(8\,A\,c^2\,d+b^2\,B\,e-b\,c\,(4\,B\,d+9\,A\,e)\right)+\left(16\,A\,c^4\,d^3-4\,b^4\,B\,e^3+b^3\,c\,e^2\,(4\,B\,d+A\,e)-8\,b\,c^3\,d^2\,(B\,d+3\,A\,e)+b^2\,c^2\,d\,e\,(5\,B\,d+6\,A\,e)\right)\,x\right)-$$

$$\left(2\,\left(16\,A\,c^4\,d^3-8\,b^4\,B\,e^3+b^3\,c\,e^2\,(5\,B\,d+2\,A\,e)-8\,b\,c^3\,d^2\,(B\,d+3\,A\,e)+b^2\,c^2\,d\,e\,(5\,B\,d+4\,A\,e)\right)\,\sqrt{x}\right.$$

$$\left.\sqrt{1+\frac{c\,x}{b}}\,\sqrt{d+e\,x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]\right)\bigg/\left(3\,(-b)^{7/2}\,c^{5/2}\,\sqrt{1+\frac{e\,x}{d}}\,\sqrt{b\,x+c\,x^2}\right)+$$

$$\left(2\,d\,(c\,d-b\,e)\,\left(16\,A\,c^3\,d^2+4\,b^3\,B\,e^2+b^2\,c\,e\,(B\,d-A\,e)-8\,b\,c^2\,d\,(B\,d+2\,A\,e)\right)\,\sqrt{x}\,\sqrt{1+\frac{c\,x}{b}}\,\sqrt{1+\frac{e\,x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b\,e}{c\,d}\right]\right)\bigg/$$

$$\left(3\,(-b)^{7/2}\,c^{5/2}\,\sqrt{d+e\,x}\,\sqrt{b\,x+c\,x^2}\right)$$

Result (type 4, 530 leaves) :

$$-\frac{1}{3\,b^5\,c^2\,(x\,(b+c\,x))^{3/2}\,\sqrt{d+e\,x}}$$

$$2\left(b\,(d+e\,x)\,\left(b\,(b\,B-A\,c)\,(c\,d-b\,e)^3\,x^2+(c\,d-b\,e)^2\,\left(-8\,A\,c^2\,d+5\,b^2\,B\,e+b\,c\,(5\,B\,d-2\,A\,e)\right)\,x^2\,(b+c\,x)+A\,b\,c^2\,d^3\,(b+c\,x)^2+\right.\right.$$

$$\left.c^2\,d^2\,(3\,b\,B\,d-8\,A\,c\,d+10\,A\,b\,e)\,x\,(b+c\,x)^2\right)+$$

$$\sqrt{\frac{b}{c}}\,x\,(b+c\,x)\,\left(\sqrt{\frac{b}{c}}\,\left(16\,A\,c^4\,d^3-8\,b^4\,B\,e^3+b^3\,c\,e^2\,(5\,B\,d+2\,A\,e)-8\,b\,c^3\,d^2\,(B\,d+3\,A\,e)+b^2\,c^2\,d\,e\,(5\,B\,d+4\,A\,e)\right)\,(b+c\,x)\,(d+e\,x)+\right.$$

$$i\,b\,e\,\left(16\,A\,c^4\,d^3-8\,b^4\,B\,e^3+b^3\,c\,e^2\,(5\,B\,d+2\,A\,e)-8\,b\,c^3\,d^2\,(B\,d+3\,A\,e)+b^2\,c^2\,d\,e\,(5\,B\,d+4\,A\,e)\right)$$

$$\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]-i\,b\,e\,(c\,d-b\,e)$$

$$\left.\left.\left(8\,A\,c^3\,d^2+8\,b^3\,B\,e^2-b^2\,c\,e\,(B\,d+2\,A\,e)-b\,c^2\,d\,(4\,B\,d+5\,A\,e)\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\right)$$

■ Problem 1279: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A + B x) (d + e x)^{5/2}}{\left(b x + c x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 454 leaves, 9 steps):

$$-\frac{2 (d + e x)^{3/2} \left(A b c d + \left(2 A c^2 d + b^2 B e - b c (B d + A e)\right) x\right)}{3 b^2 c \left(b x + c x^2\right)^{3/2}} + \frac{1}{3 b^4 c \sqrt{b x + c x^2}}$$

$$2 \sqrt{d + e x} \left(b d \left(8 A c^2 d + b^2 B e - b c (4 B d + 7 A e)\right) + \left(16 A c^3 d^2 + 2 b^3 B e^2 + b^2 c e (3 B d + A e) - 8 b c^2 d (B d + 2 A e)\right) x\right) -$$

$$\left(2 \left(16 A c^3 d^2 + 2 b^3 B e^2 + b^2 c e (3 B d + A e) - 8 b c^2 d (B d + 2 A e)\right) \sqrt{x} \sqrt{1 + \frac{c x}{b}} \sqrt{d + e x} \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/$$

$$\left(3 (-b)^{7/2} c^{3/2} \sqrt{1 + \frac{e x}{d}} \sqrt{b x + c x^2}\right) +$$

$$\left(2 d (c d - b e) \left(16 A c^2 d - b^2 B e - 8 b c (B d + A e)\right) \sqrt{x} \sqrt{1 + \frac{c x}{b}} \sqrt{1 + \frac{e x}{d}} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/$$

$$\left(3 (-b)^{7/2} c^{3/2} \sqrt{d + e x} \sqrt{b x + c x^2}\right)$$

Result (type 4, 452 leaves):

$$-\frac{1}{3 b^5 c (x (b+c x))^{3/2} \sqrt{d+e x}} 2 \left(b (d+e x) \left(b (b B-A c) (c d-b e)^2 x^2+(c d-b e) \left(-8 A c^2 d+2 b^2 B e+b c (5 B d+A e)\right) x^2 (b+c x)+A b c d^2 (b+c x)^2+\right.\right.$$
$$\left. c d (3 b B d-8 A c d+7 A b e) x (b+c x)^2\right)+$$
$$\sqrt{\frac{b}{c}} x (b+c x) \left(\sqrt{\frac{b}{c}} \left(16 A c^3 d^2+2 b^3 B e^2+b^2 c e (3 B d+A e)-8 b c^2 d (B d+2 A e)\right) (b+c x) (d+e x)+\right.$$
$$i b e \left(16 A c^3 d^2+2 b^3 B e^2+b^2 c e (3 B d+A e)-8 b c^2 d (B d+2 A e)\right) \sqrt{1+\frac{b}{c x}} \sqrt{1+\frac{d}{e x}} x^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right]-$$
$$\left.\left. i b e (c d-b e) \left(8 A c^2 d-2 b^2 B e-b c (4 B d+A e)\right) \sqrt{1+\frac{b}{c x}} \sqrt{1+\frac{d}{e x}} x^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], \frac{c d}{b e}\right]\right)\right)$$

- **Problem 1280: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B x) (d+e x)^{3/2}}{\left(b x+c x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 410 leaves, 9 steps) :

$$-\frac{2\sqrt{d+e x}\,\left(A b c d+\left(2 A c^2 d+b^2 B e-b c\,(B d+A e)\right) x\right)}{3 b^2 c\,\left(b x+c x^2\right)^{3/2}}+$$

$$\frac{2\sqrt{d+e x}\,\left(b\,\left(8 A c^2 d+b^2 B e-b c\,(4 B d+5 A e)\right)+c\,\left(16 A c^2 d+b^2 B e-8 b c\,(B d+A e)\right) x\right)}{3 b^4 c\,\sqrt{b x+c x^2}}-$$

$$\frac{2\,\left(16 A c^2 d+b^2 B e-8 b c\,(B d+A e)\right)\sqrt{x}\,\sqrt{1+\frac{c x}{b}}\,\sqrt{d+e x}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{3\,(-b)^{7/2}\sqrt{c}\,\sqrt{1+\frac{e x}{d}}\,\sqrt{b x+c x^2}}+$$

$$\left(2\,\left(16 A c^2 d^2-8 b c d\,(B d+2 A e)+b^2 e\,(5 B d+3 A e)\right)\sqrt{x}\,\sqrt{1+\frac{c x}{b}}\,\sqrt{1+\frac{e x}{d}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]\right)\Big/$$
$$\left(3\,(-b)^{7/2}\sqrt{c}\,\sqrt{d+e x}\,\sqrt{b x+c x^2}\right)$$

Result (type 4, 378 leaves) :

$$-\frac{1}{3 b^5\,(x\,(b+c x))^{3/2}\sqrt{d+e x}}$$

$$2\left(b\,(d+e x)\,\left(b B x\,\left(8 c^2 d x^2+b^2\,(3 d-2 e x)+b c x\,(12 d-e x)\right)+A\,\left(-16 c^3 d x^3+8 b c^2 x^2\,(-3 d+e x)+b^3\,(d+4 e x)+b^2 c x\,(-6 d+13 e x)\right)\right)+\right.$$

$$\sqrt{\frac{b}{c}}\,x\,(b+c x)\left(\sqrt{\frac{b}{c}}\,\left(16 A c^2 d+b^2 B e-8 b c\,(B d+A e)\right)(b+c x)\,(d+e x)+\right.$$

$$i\,b e\,\left(16 A c^2 d+b^2 B e-8 b c\,(B d+A e)\right)\sqrt{1+\frac{b}{c x}}\,\sqrt{1+\frac{d}{e x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]-$$

$$\left.\left.i\,b e\,\left(8 A c^2 d+b^2 B e-b c\,(4 B d+5 A e)\right)\sqrt{1+\frac{b}{c x}}\,\sqrt{1+\frac{d}{e x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]\right)\right)$$

■ Problem 1281: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A+B x)\sqrt{d+e x}}{\left(b x+c x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 420 leaves, 9 steps):

$$-\frac{2\,(A b-(b B-2 A c)\,x)\,\sqrt{d+e x}}{3 b^2\left(b x+c x^2\right)^{3/2}}-\frac{2\sqrt{d+e x}\,\left(b\,(c d-b e)\,(4 b B d-8 A c d+A b e)-c\left(16 A c^2 d^2+b^2 e\,(7 B d+A e)-8 b c d\,(B d+2 A e)\right)x\right)}{3 b^4 d\,(c d-b e)\sqrt{b x+c x^2}}-$$

$$\left(2\sqrt{c}\,\left(16 A c^2 d^2+b^2 e\,(7 B d+A e)-8 b c d\,(B d+2 A e)\right)\sqrt{x}\,\sqrt{1+\frac{c x}{b}}\,\sqrt{d+e x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]\right)\Big/$$

$$\left(3\,(-b)^{7/2}\,d\,(c d-b e)\sqrt{1+\frac{e x}{d}}\,\sqrt{b x+c x^2}\right)+\frac{2\left(16 A c^2 d+3 b^2 B e-8 b c\,(B d+A e)\right)\sqrt{x}\,\sqrt{1+\frac{c x}{b}}\,\sqrt{1+\frac{e x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]}{3\,(-b)^{7/2}\sqrt{c}\,\sqrt{d+e x}\,\sqrt{b x+c x^2}}$$

Result (type 4, 441 leaves):

$$-\frac{1}{3\,b^4\,\sqrt{\frac{b}{c}}\,d\,(c\,d-b\,e)\,(x\,(b+c\,x))^{3/2}\,\sqrt{d+e\,x}}$$

$$2\left(\sqrt{\frac{b}{c}}\,(d+e\,x)\,\left(b\,c\,(b\,B-A\,c)\,d\,(c\,d-b\,e)\,x^2+c\,d\,\left(-8\,A\,c^2\,d-4\,b^2\,B\,e+b\,c\,(5\,B\,d+7\,A\,e)\right)\,x^2\,(b+c\,x)+A\,b\,d\,(c\,d-b\,e)\,(b+c\,x)^2+\right.\right.$$

$$\left.(c\,d-b\,e)\,(3\,b\,B\,d-8\,A\,c\,d+A\,b\,e)\,x\,(b+c\,x)^2\right)+x\,(b+c\,x)\,\left(\sqrt{\frac{b}{c}}\,\left(16\,A\,c^2\,d^2+b^2\,e\,(7\,B\,d+A\,e)-8\,b\,c\,d\,(B\,d+2\,A\,e)\right)\,(b+c\,x)\,(d+e\,x)+\right.$$

$$i\,b\,e\,\left(16\,A\,c^2\,d^2+b^2\,e\,(7\,B\,d+A\,e)-8\,b\,c\,d\,(B\,d+2\,A\,e)\right)\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\,\frac{c\,d}{b\,e}\right]-$$

$$\left.\left.i\,b\,e\,(c\,d-b\,e)\,(8\,A\,c\,d-b\,(4\,B\,d+A\,e))\,\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\,\frac{c\,d}{b\,e}\right]\right)\right)$$

- **Problem 1282: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B\,x}{\sqrt{d+e\,x}\,\left(b\,x+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 543 leaves, 9 steps) :

$$-\frac{2\sqrt{d+e x}\,\left(A b (c d-b e)+c (2 A c d-b (B d+A e)) x\right)}{3 b^2 d (c d-b e) \left(b x+c x^2\right)^{3/2}}+\left(2\sqrt{d+e x}\,\left(b (c d-b e) \left(8 A c^2 d^2+b^2 e (3 B d-2 A e)-b c d (4 B d+5 A e)\right)+c \left(16 A c^3 d^3-b^3 e^2 (3 B d-2 A e)-8 b c^2 d^2 (B d+3 A e)+b^2 c d e (13 B d+4 A e)\right) x\right)\right)\Big/\left(3 b^4 d^2 (c d-b e)^2 \sqrt{b x+c x^2}\right)-\left(2\sqrt{c}\,\left(16 A c^3 d^3-b^3 e^2 (3 B d-2 A e)-8 b c^2 d^2 (B d+3 A e)+b^2 c d e (13 B d+4 A e)\right)\sqrt{x}\,\sqrt{1+\frac{c x}{b}}\,\sqrt{d+e x}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]\right)\Big/\left(3 (-b)^{7/2} d^2 (c d-b e)^2 \sqrt{1+\frac{e x}{d}}\,\sqrt{b x+c x^2}\right)+\left(2\sqrt{c}\,\left(16 A c^2 d^2+b^2 e (9 B d-A e)-8 b c d (B d+2 A e)\right)\sqrt{x}\,\sqrt{1+\frac{c x}{b}}\,\sqrt{1+\frac{e x}{d}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{-b}}\right],\frac{b e}{c d}\right]\right)\Big/\left(3 (-b)^{7/2} d (c d-b e) \sqrt{d+e x}\,\sqrt{b x+c x^2}\right)$$

Result (type 4, 514 leaves):

$$-\frac{1}{3 b^5 d^2 (c d-b e)^2 (x (b+c x))^{3/2}\sqrt{d+e x}}\left(2\left(b (d+e x) \left(b c^2 (b B-A c) d^2 (c d-b e) x^2+c^2 d^2 \left(-8 A c^2 d-7 b^2 B e+5 b c (B d+2 A e)\right) x^2 (b+c x)+A b d (c d-b e)^2 (b+c x)^2+(c d-b e)^2 (3 b B d-8 A c d-2 A b e) x (b+c x)^2\right)+\sqrt{\frac{b}{c}}\,c x (b+c x)\left(\sqrt{\frac{b}{c}}\left(16 A c^3 d^3+b^3 e^2 (-3 B d+2 A e)-8 b c^2 d^2 (B d+3 A e)+b^2 c d e (13 B d+4 A e)\right)(b+c x)(d+e x)+i b e \left(16 A c^3 d^3+b^3 e^2 (-3 B d+2 A e)-8 b c^2 d^2 (B d+3 A e)+b^2 c d e (13 B d+4 A e)\right)\sqrt{1+\frac{b}{c x}}\,\sqrt{1+\frac{d}{e x}}\,x^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]-i b e (c d-b e)\left(8 A c^2 d^2+b^2 e (3 B d-2 A e)-b c d (4 B d+5 A e)\right)\sqrt{1+\frac{b}{c x}}\,\sqrt{1+\frac{d}{e x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c d}{b e}\right]\right)\right)\right)$$

■ Problem 1283: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x}{(d + e x)^{3/2} \left(b x + c x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 706 leaves, 10 steps):

$$-\frac{2 (A b (c d - b e) + c (2 A c d - b (B d + A e)) x)}{3 b^2 d (c d - b e) \sqrt{d + e x} \left(b x + c x^2\right)^{3/2}} + \left(2 \left(b (c d - b e) \left(8 A c^2 d^2 + b^2 e (3 B d - 4 A e) - b c d (4 B d + 3 A e)\right) + c \left(16 A c^3 d^3 + 15 b^2 B c d^2 e - b^3 e^2 (3 B d - 4 A e) - 8 b c^2 d^2 (B d + 3 A e)\right) x\right)\right) \Big/ \left(3 b^4 d^2 (c d - b e)^2 \sqrt{d + e x} \sqrt{b x + c x^2}\right) + \left(2 e \left(16 A c^4 d^4 - b^3 c d e^2 (9 B d - 7 A e) - 8 b c^3 d^3 (B d + 4 A e) + b^2 c^2 d^2 e (19 B d + 9 A e) + b^4 \left(6 B d e^3 - 8 A e^4\right)\right) \sqrt{b x + c x^2}\right) \Big/ \left(3 b^4 d^3 (c d - b e)^3 \sqrt{d + e x}\right) - \left(2 \sqrt{c} \left(16 A c^4 d^4 - b^3 c d e^2 (9 B d - 7 A e) + 2 b^4 e^3 (3 B d - 4 A e) - 8 b c^3 d^3 (B d + 4 A e) + b^2 c^2 d^2 e (19 B d + 9 A e)\right) \sqrt{x} \sqrt{1 + \frac{c x}{b}} \sqrt{d + e x} \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/ \left(3 (-b)^{7/2} d^3 (c d - b e)^3 \sqrt{1 + \frac{e x}{d}} \sqrt{b x + c x^2}\right) + \left(2 \sqrt{c} \left(16 A c^3 d^3 + 15 b^2 B c d^2 e - b^3 e^2 (3 B d - 4 A e) - 8 b c^2 d^2 (B d + 3 A e)\right) \sqrt{x} \sqrt{1 + \frac{c x}{b}} \sqrt{1 + \frac{e x}{d}} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{x}}{\sqrt{-b}}\right], \frac{b e}{c d}\right]\right) \Big/ \left(3 (-b)^{7/2} d^2 (c d - b e)^2 \sqrt{d + e x} \sqrt{b x + c x^2}\right)$$

Result (type 4, 628 leaves):

$$\frac{1}{3\,b^5\,d^3\,(c\,d-b\,e)^3\,(x\,(b+c\,x))^{3/2}\,\sqrt{d+e\,x}}$$

$$2\left(b\left(3\,b^4\,e^4\,(B\,d-A\,e)\,x^2\,(b+c\,x)^2+b\,c^3\,(b\,B-A\,c)\,d^3\,(-c\,d+b\,e)\,x^2\,(d+e\,x)+c^3\,d^3\left(8\,A\,c^2\,d+10\,b^2\,B\,e-b\,c\,(5\,B\,d+13\,A\,e)\right)x^2\,(b+c\,x)\,(d+e\,x)+\right.\right.$$

$$\left.A\,b\,d\,(-c\,d+b\,e)^3\,(b+c\,x)^2\,(d+e\,x)+(c\,d-b\,e)^3\,(-3\,b\,B\,d+8\,A\,c\,d+5\,A\,b\,e)\,x\,(b+c\,x)^2\,(d+e\,x)\right)-\sqrt{\frac{b}{c}}\,c\,x\,(b+c\,x)$$

$$\left(\sqrt{\frac{b}{c}}\left(16\,A\,c^4\,d^4+2\,b^4\,e^3\,(3\,B\,d-4\,A\,e)-8\,b\,c^3\,d^3\,(B\,d+4\,A\,e)+b^3\,c\,d\,e^2\,(-9\,B\,d+7\,A\,e)+b^2\,c^2\,d^2\,e\,(19\,B\,d+9\,A\,e)\right)(b+c\,x)\,(d+e\,x)+\right.$$

$$i\,b\,e\left(16\,A\,c^4\,d^4+2\,b^4\,e^3\,(3\,B\,d-4\,A\,e)-8\,b\,c^3\,d^3\,(B\,d+4\,A\,e)+b^3\,c\,d\,e^2\,(-9\,B\,d+7\,A\,e)+b^2\,c^2\,d^2\,e\,(19\,B\,d+9\,A\,e)\right)\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]-i\,b\,e\,(c\,d-b\,e)\left(8\,A\,c^3\,d^3+3\,b^2\,c\,d\,e\,(2\,B\,d-A\,e)-b\,c^2\,d^2\,(4\,B\,d+9\,A\,e)+b^3\left(-6\,B\,d\,e^2+8\,A\,e^3\right)\right)$$

$$\left.\left.\sqrt{1+\frac{b}{c\,x}}\,\sqrt{1+\frac{d}{e\,x}}\,x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right],\frac{c\,d}{b\,e}\right]\right)\right)$$

- **Problem 1284: Result more than twice size of optimal antiderivative.**

$$\int (A+B\,x)\,(d+e\,x)^5\,\left(a+c\,x^2\right)\,dx$$

Optimal (type 1, 108 leaves, 2 steps):

$$-\frac{(B\,d-A\,e)\,\left(c\,d^2+a\,e^2\right)\,(d+e\,x)^6}{6\,e^4}+\frac{\left(3\,B\,c\,d^2-2\,A\,c\,d\,e+a\,B\,e^2\right)\,(d+e\,x)^7}{7\,e^4}-\frac{c\,(3\,B\,d-A\,e)\,(d+e\,x)^8}{8\,e^4}+\frac{B\,c\,(d+e\,x)^9}{9\,e^4}$$

Result (type 1, 233 leaves):

$$a A d^5 x + \frac{1}{2} a d^4 (B d + 5 A e) x^2 + \frac{1}{3} d^3 \left(A c d^2 + 5 a B d e + 10 a A e^2\right) x^3 + \frac{1}{4} d^2 \left(B c d^3 + 5 A c d^2 e + 10 a B d e^2 + 10 a A e^3\right) x^4 + d e \left(B c d^3 + 2 A c d^2 e + 2 a B d e^2 + a A e^3\right) x^5 + \frac{1}{6} e^2 \left(10 B c d^3 + 10 A c d^2 e + 5 a B d e^2 + a A e^3\right) x^6 + \frac{1}{7} e^3 \left(10 B c d^2 + 5 A c d e + a B e^2\right) x^7 + \frac{1}{8} c e^4 (5 B d + A e) x^8 + \frac{1}{9} B c e^5 x^9$$

- **Problem 1463: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B x}{\sqrt{d + e x} \left(2 A B d - A^2 e - B^2 e x^2\right)} \, dx$$

Optimal (type 3, 155 leaves, 4 steps):

$$-\frac{\text{Log}\left[B d - A e - \sqrt{2} \sqrt{B} \sqrt{2 B d - A e} \sqrt{d + e x} + B (d + e x)\right]}{\sqrt{2} \sqrt{B} e \sqrt{2 B d - A e}} + \frac{\text{Log}\left[B d - A e + \sqrt{2} \sqrt{B} \sqrt{2 B d - A e} \sqrt{d + e x} + B (d + e x)\right]}{\sqrt{2} \sqrt{B} e \sqrt{2 B d - A e}}$$

Result (type 3, 259 leaves):

$$\left(\frac{\left(i A e + \sqrt{A} \sqrt{e} \sqrt{-2 B d + A e}\right) \text{ArcTanh}\left[\frac{\sqrt{B} \sqrt{d+e x}}{\sqrt{B d - i \sqrt{A} \sqrt{e} \sqrt{-2 B d + A e}}}\right]}{\sqrt{B d - i \sqrt{A} \sqrt{e} \sqrt{-2 B d + A e}}} + \frac{\left(-i A e + \sqrt{A} \sqrt{e} \sqrt{-2 B d + A e}\right) \text{ArcTanh}\left[\frac{\sqrt{B} \sqrt{d+e x}}{\sqrt{B d + i \sqrt{A} \sqrt{e} \sqrt{-2 B d + A e}}}\right]}{\sqrt{B d + i \sqrt{A} \sqrt{e} \sqrt{-2 B d + A e}}}\right) \Big/ \left(\sqrt{A} \sqrt{B} e^{3/2} \sqrt{-2 B d + A e}\right)$$

- **Problem 1464: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B x}{\sqrt{\frac{A^2 e - B^2 e}{2 A B} + e x} \left(1 + x^2\right)} \, dx$$

Optimal (type 3, 133 leaves, 6 steps):

$$-\frac{\sqrt{2} \sqrt{A} \sqrt{B} \text{ArcTan}\left[\frac{A}{B} - \frac{\sqrt{A} \sqrt{e \left(\frac{A}{B} - \frac{B}{A} + 2 x\right)}}{\sqrt{B} \sqrt{e}}\right]}{\sqrt{e}} + \frac{\sqrt{2} \sqrt{A} \sqrt{B} \text{ArcTan}\left[\frac{A}{B} + \frac{\sqrt{A} \sqrt{e \left(\frac{A}{B} - \frac{B}{A} + 2 x\right)}}{\sqrt{B} \sqrt{e}}\right]}{\sqrt{e}}$$

Result (type 3, 142 leaves):

$$-\frac{i \sqrt{2} \sqrt{A} \sqrt{B} \sqrt{\frac{A}{B} - \frac{B}{A} + 2 x} \left(\text{ArcTanh}\left[\frac{\sqrt{A} \sqrt{B} \sqrt{\frac{A}{B} - \frac{B}{A} + 2 x}}{A - i B}\right] - \text{ArcTanh}\left[\frac{\sqrt{A} \sqrt{B} \sqrt{\frac{A}{B} - \frac{B}{A} + 2 x}}{A + i B}\right]\right)}{\sqrt{e \left(\frac{A}{B} - \frac{B}{A} + 2 x\right)}}$$

Problem 1466: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B\,x}{\sqrt{d+e\,x}\,\left(1+x^2\right)}\,dx$$

Optimal (type 3, 440 leaves, 10 steps):

$$\frac{\left(A\,e-B\left(d-\sqrt{d^2+e^2}\right)\right)\text{ArcTanh}\left[\frac{\sqrt{d+\sqrt{d^2+e^2}}-\sqrt{2}\,\sqrt{d+e\,x}}{\sqrt{d-\sqrt{d^2+e^2}}}\right]}{\sqrt{2}\,\sqrt{d^2+e^2}\,\sqrt{d-\sqrt{d^2+e^2}}}-\frac{\left(A\,e-B\left(d-\sqrt{d^2+e^2}\right)\right)\text{ArcTanh}\left[\frac{\sqrt{d+\sqrt{d^2+e^2}}+\sqrt{2}\,\sqrt{d+e\,x}}{\sqrt{d-\sqrt{d^2+e^2}}}\right]}{\sqrt{2}\,\sqrt{d^2+e^2}\,\sqrt{d-\sqrt{d^2+e^2}}}-$$

$$\frac{\left(A\,e-B\left(d+\sqrt{d^2+e^2}\right)\right)\text{Log}\left[d+\sqrt{d^2+e^2}+e\,x-\sqrt{2}\,\sqrt{d+\sqrt{d^2+e^2}}\,\sqrt{d+e\,x}\right]}{2\,\sqrt{2}\,\sqrt{d^2+e^2}\,\sqrt{d+\sqrt{d^2+e^2}}}+$$

$$\frac{\left(A\,e-B\left(d+\sqrt{d^2+e^2}\right)\right)\text{Log}\left[d+\sqrt{d^2+e^2}+e\,x+\sqrt{2}\,\sqrt{d+\sqrt{d^2+e^2}}\,\sqrt{d+e\,x}\right]}{2\,\sqrt{2}\,\sqrt{d^2+e^2}\,\sqrt{d+\sqrt{d^2+e^2}}}$$

Result (type 3, 89 leaves):

$$-\frac{i\,(A-i\,B)\,\text{ArcTanh}\left[\frac{\sqrt{d+e\,x}}{\sqrt{d-i\,e}}\right]}{\sqrt{d-i\,e}}+\frac{i\,(A+i\,B)\,\text{ArcTanh}\left[\frac{\sqrt{d+e\,x}}{\sqrt{d+i\,e}}\right]}{\sqrt{d+i\,e}}$$

Problem 1467: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(1-x)\,\sqrt{1+x}}{1+x^2}\,dx$$

Optimal (type 3, 202 leaves, 12 steps):

$$-2\sqrt{1+x}-\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left[\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right]+\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left[\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right]-$$

$$\frac{\text{Log}\left[1+\sqrt{2}+x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right]}{2\sqrt{1+\sqrt{2}}}+\frac{\text{Log}\left[1+\sqrt{2}+x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right]}{2\sqrt{1+\sqrt{2}}}$$

Result (type 3, 60 leaves) :

$$-2\sqrt{1+x}-(-1-i)^{3/2}\,\text{ArcTan}\left[\frac{\sqrt{1+x}}{\sqrt{-1-i}}\right]-(-1+i)^{3/2}\,\text{ArcTan}\left[\frac{\sqrt{1+x}}{\sqrt{-1+i}}\right]$$

- **Problem 1468: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{3+x}{\sqrt{4+3x}\left(1+x^2\right)}\,dx$$

Optimal (type 3, 45 leaves, 6 steps) :

$$-\sqrt{2}\,\text{ArcTan}\left[3-\sqrt{2}\sqrt{4+3x}\right]+\sqrt{2}\,\text{ArcTan}\left[3+\sqrt{8+6x}\right]$$

Result (type 3, 59 leaves) :

$$\frac{(1-3i)\,\text{ArcTan}\left[\frac{\sqrt{4+3x}}{\sqrt{-4-3i}}\right]}{\sqrt{-4-3i}}+\frac{(1+3i)\,\text{ArcTan}\left[\frac{\sqrt{4+3x}}{\sqrt{-4+3i}}\right]}{\sqrt{-4+3i}}$$

- **Problem 1469: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1-3x}{\sqrt{4+3x}\left(1+x^2\right)}\,dx$$

Optimal (type 3, 53 leaves, 4 steps) :

$$-\frac{\text{Log}\left[3+x-\sqrt{2}\sqrt{4+3x}\right]}{\sqrt{2}}+\frac{\text{Log}\left[3+x+\sqrt{2}\sqrt{4+3x}\right]}{\sqrt{2}}$$

Result (type 3, 59 leaves) :

$$-\frac{(3+i)\,\text{ArcTan}\left[\frac{\sqrt{4+3x}}{\sqrt{-4-3i}}\right]}{\sqrt{-4-3i}}-\frac{(3-i)\,\text{ArcTan}\left[\frac{\sqrt{4+3x}}{\sqrt{-4+3i}}\right]}{\sqrt{-4+3i}}$$

- **Problem 1471: Result more than twice size of optimal antiderivative.**

$$\int \frac{-2+x}{\sqrt{-3+x}\,\left(-8+x^2\right)}\,dx$$

Optimal (type 3, 45 leaves, 4 steps):

$$\frac{\text{ArcTan}\left[\left(-1+\sqrt{2}\right)\sqrt{-3+x}\right]}{\sqrt{2}}+\frac{\text{ArcTan}\left[\left(1+\sqrt{2}\right)\sqrt{-3+x}\right]}{\sqrt{2}}$$

Result (type 3, 91 leaves):

$$\frac{\left(-1+\sqrt{2}\right)\text{ArcTan}\left[\frac{\sqrt{-3+x}}{\sqrt{3-2\sqrt{2}}}\right]}{\sqrt{2\left(3-2\sqrt{2}\right)}}+\frac{\left(1+\sqrt{2}\right)\text{ArcTan}\left[\frac{\sqrt{-3+x}}{\sqrt{3+2\sqrt{2}}}\right]}{\sqrt{2\left(3+2\sqrt{2}\right)}}$$

- **Problem 1472: Result unnecessarily involves imaginary or complex numbers.**

$$\int (A+B\,x)\,\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}\,dx$$

Optimal (type 4, 438 leaves, 7 steps):

$$-\frac{2\sqrt{d+e\,x}\,\left(4\,B\,c\,d^2-7\,A\,c\,d\,e+5\,a\,B\,e^2-3\,c\,e\,\left(B\,d+7\,A\,e\right)\,x\right)\sqrt{a+c\,x^2}}{105\,c\,e^2}+\frac{2\,B\sqrt{d+e\,x}\,\left(a+c\,x^2\right)^{3/2}}{7\,c}-$$

$$\left(4\sqrt{-a}\,\left(4\,B\,c\,d^3-7\,A\,c\,d^2\,e+8\,a\,B\,d\,e^2+21\,a\,A\,e^3\right)\sqrt{d+e\,x}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]\right)\Big/$$

$$\left(105\sqrt{c}\,e^3\sqrt{\frac{\sqrt{c}\,\left(d+e\,x\right)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{a+c\,x^2}\right)+$$

$$\left(4\sqrt{-a}\,\left(c\,d^2+a\,e^2\right)\left(4\,B\,c\,d^2-7\,A\,c\,d\,e+5\,a\,B\,e^2\right)\sqrt{\frac{\sqrt{c}\,\left(d+e\,x\right)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]\right)\Big/$$

$$\left(105\,c^{3/2}\,e^3\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}\right)$$

Result (type 4, 622 leaves):

$$\frac{1}{105\sqrt{a+c x^2}}\sqrt{d+e x}\left(\frac{2\left(a+c x^2\right)\left(10 a B e^2+7 A c e (d+3 e x)+B c\left(-4 d^2+3 d e x+15 e^2 x^2\right)\right)}{c e^2}+\frac{1}{c e^4\sqrt{-d-\frac{i\sqrt{a}\, e}{\sqrt{c}}}\,(d+e x)}\right.$$

$$4\left(e^2\sqrt{-d-\frac{i\sqrt{a}\, e}{\sqrt{c}}}\left(4 B c d^3-7 A c d^2 e+8 a B d e^2+21 a A e^3\right)\left(a+c x^2\right)-\sqrt{c}\left(i\sqrt{c}\, d-\sqrt{a}\, e\right)\left(4 B c d^3-7 A c d^2 e+8 a B d e^2+21 a A e^3\right)\right.$$

$$\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\, e}{\sqrt{c}}-e x}{d+e x}}\,(d+e x)^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\, d-i\sqrt{a}\, e}{\sqrt{c}\, d+i\sqrt{a}\, e}\right]+$$

$$\sqrt{a}\, e\left(\sqrt{c}\, d+i\sqrt{a}\, e\right)\left(B\left(-4 c d^2+3 i\sqrt{a}\sqrt{c}\, d e-5 a e^2\right)+7 A\left(c d e+3 i\sqrt{a}\sqrt{c}\, e^2\right)\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}$$

$$\left.\left.\sqrt{-\frac{\frac{i\sqrt{a}\, e}{\sqrt{c}}-e x}{d+e x}}\,(d+e x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\, d-i\sqrt{a}\, e}{\sqrt{c}\, d+i\sqrt{a}\, e}\right]\right)\right)$$

- **Problem 1473: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+B x)\sqrt{a+c x^2}}{\sqrt{d+e x}}\,dx$$

Optimal (type 4, 365 leaves, 6 steps) :

$$-\frac{2\sqrt{d+e x}\,(4 B d-5 A e-3 B e x)\sqrt{a+c x^2}}{15 e^2}-$$

$$\frac{4\sqrt{-a}\,(4 B c d^2-5 A c d e+3 a B e^2)\sqrt{d+e x}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\,d-a e}\right]}{15\sqrt{c}\,e^3\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{a+c x^2}}+$$

$$\left(4\sqrt{-a}\,(4 B d-5 A e)\,(c d^2+a e^2)\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\,d-a e}\right]\right)\Big/$$

$$\left(15\sqrt{c}\,e^3\sqrt{d+e x}\sqrt{a+c x^2}\right)$$

Result (type 4, 549 leaves) :

$$\frac{1}{15\sqrt{a+c x^2}}$$

$$\sqrt{d+e x}\left(\frac{2\,(-4 B d+5 A e+3 B e x)\,(a+c x^2)}{e^2}-\frac{1}{c\,e^4\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\,(d+e x)}4\left(-e^2\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\,(4 B c d^2-5 A c d e+3 a B e^2)\,(a+c x^2)+\right.\right.$$

$$\sqrt{c}\left(-i\sqrt{c}\,d+\sqrt{a}\,e\right)\left(-4 B c d^2+5 A c d e-3 a B e^2\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}\,(d+e x)^{3/2}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]+\sqrt{a}\sqrt{c}\,e\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(4 B\sqrt{c}\,d-3 i\sqrt{a}\,B e-5 A\sqrt{c}\,e\right)$$

$$\left.\left.\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}\,(d+e x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]\right)\right)$$

Problem 1474: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A+B x) \sqrt{a+c x^2}}{(d+e x)^{3/2}} \, dx$$

Optimal (type 4, 352 leaves, 6 steps):

$$\frac{2 (4 B d-3 A e+B e x) \sqrt{a+c x^2}}{3 e^2 \sqrt{d+e x}}+\frac{4 \sqrt{-a} \sqrt{c} (4 B d-3 A e) \sqrt{d+e x} \sqrt{1+\frac{c x^2}{a}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]}{3 e^3 \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{a+c x^2}}-$$

$$\left(4 \sqrt{-a} \left(4 B c d^2-3 A c d e+a B e^2\right) \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{1+\frac{c x^2}{a}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]\right) \Big/$$

$$\left(3 \sqrt{c} e^3 \sqrt{d+e x} \sqrt{a+c x^2}\right)$$

Result (type 4, 512 leaves):

$$\frac{1}{3 \sqrt{a+c x^2}}$$

$$\sqrt{d+e x} \left(\frac{2 (4 B d-3 A e+B e x) \left(a+c x^2\right)}{e^2 (d+e x)}+1 \Big/ \left(e^4 \sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}\right) 2 (d+e x) \left(\frac{2 e^2 (-4 B d+3 A e) \sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}} \left(a+c x^2\right)}{(d+e x)^2}+1 \Big/ \left(\sqrt{d+e x}\right) 2 \sqrt{c}\right.\right.$$

$$\left(-i \sqrt{c} d+\sqrt{a} e\right) (-4 B d+3 A e) \sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a} e}{\sqrt{c}}-e x}{d+e x}} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c} d-i \sqrt{a} e}{\sqrt{c} d+i \sqrt{a} e}\right]-1 \Big/ \left(\sqrt{d+e x}\right)$$

$$\left.\left.2 \sqrt{a} e \left(-4 B \sqrt{c} d-i \sqrt{a} B e+3 A \sqrt{c} e\right) \sqrt{\frac{e \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}} \sqrt{-\frac{\frac{i \sqrt{a} e}{\sqrt{c}}-e x}{d+e x}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c} d-i \sqrt{a} e}{\sqrt{c} d+i \sqrt{a} e}\right]\right)\right)$$

- **Problem 1475: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B x) \sqrt{a+c x^2}}{(d+e x)^{5/2}} \, dx$$

Optimal (type 4, 420 leaves, 6 steps) :

$$-\frac{2 \left(4 B c d^3-A c d^2 e+2 a B d e^2+a A e^3+e \left(5 B c d^2-2 A c d e+3 a B e^2\right) x\right) \sqrt{a+c x^2}}{3 e^2 \left(c d^2+a e^2\right) (d+e x)^{3/2}}-$$

$$\left(4 \sqrt{-a} \sqrt{c} \left(4 B c d^2-A c d e+3 a B e^2\right) \sqrt{d+e x} \sqrt{1+\frac{c x^2}{a}} \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]\right) \Big/$$

$$\left(3 e^3 \left(c d^2+a e^2\right) \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{a+c x^2}\right)+\frac{4 \sqrt{-a} \sqrt{c} (4 B d-A e) \sqrt{\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}} \sqrt{1+\frac{c x^2}{a}} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a} \sqrt{c} d-a e}\right]}{3 e^3 \sqrt{d+e x} \sqrt{a+c x^2}}$$

Result (type 4, 685 leaves) :

$$\sqrt{d+e x}\,\sqrt{a+c x^2}\left(-\frac{2(-B d+A e)}{3 e^2 (d+e x)^2}-\frac{2\left(5 B c d^2-2 A c d e+3 a B e^2\right)}{3 e^2 \left(c d^2+a e^2\right)(d+e x)}\right)-$$

$$\frac{1}{3 e^4 \sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}\left(c d^2+a e^2\right) \sqrt{a+\frac{c (d+e x)^2 \left(-1+\frac{d}{d+e x}\right)^2}{e^2}}}\, 4 (d+e x)^{3/2} \left(-\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}\left(4 B c d^2-A c d e+3 a B e^2\right)\left(\frac{a e^2}{(d+e x)^2}+c\left(-1+\frac{d}{d+e x}\right)^2\right)+\right.$$

$$\frac{1}{\sqrt{d+e x}}\, i \sqrt{c}\left(\sqrt{c}\, d+i \sqrt{a}\, e\right)\left(4 B c d^2-A c d e+3 a B e^2\right)\sqrt{1-\frac{d}{d+e x}-\frac{i \sqrt{a}\, e}{\sqrt{c}\,(d+e x)}}\,\sqrt{1-\frac{d}{d+e x}+\frac{i \sqrt{a}\, e}{\sqrt{c}\,(d+e x)}}$$

$$\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right], \frac{\sqrt{c}\, d-i \sqrt{a}\, e}{\sqrt{c}\, d+i \sqrt{a}\, e}\right]-\frac{1}{\sqrt{d+e x}}\sqrt{a}\,\sqrt{c}\, e\left(\sqrt{c}\, d+i \sqrt{a}\, e\right)\left(-4 B \sqrt{c}\, d+3 i \sqrt{a}\, B e+A \sqrt{c}\, e\right)$$

$$\left.\sqrt{1-\frac{d}{d+e x}-\frac{i \sqrt{a}\, e}{\sqrt{c}\,(d+e x)}}\,\sqrt{1-\frac{d}{d+e x}+\frac{i \sqrt{a}\, e}{\sqrt{c}\,(d+e x)}}\ \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right], \frac{\sqrt{c}\, d-i \sqrt{a}\, e}{\sqrt{c}\, d+i \sqrt{a}\, e}\right]\right)$$

- **Problem 1476: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B x)\left(a+c x^2\right)^{3/2}}{\sqrt{d+e x}}\, dx$$

Optimal (type 4, 498 leaves, 7 steps) :

$$-\frac{4\sqrt{d+e\,x}\,\left(32\,B\,c\,d^3-36\,A\,c\,d^2\,e+33\,a\,B\,d\,e^2-45\,a\,A\,e^3-3\,e\,\left(8\,B\,c\,d^2-9\,A\,c\,d\,e+7\,a\,B\,e^2\right)\,x\right)\,\sqrt{a+c\,x^2}}{315\,e^4}-$$

$$\frac{2\sqrt{d+e\,x}\,\left(8\,B\,d-9\,A\,e-7\,B\,e\,x\right)\,\left(a+c\,x^2\right)^{3/2}}{63\,e^2}+\left(8\sqrt{-a}\,\left(36\,A\,c\,d\,e\,\left(c\,d^2+2\,a\,e^2\right)-B\,\left(32\,c^2\,d^4+57\,a\,c\,d^2\,e^2+21\,a^2\,e^4\right)\right)\right.$$

$$\left.\sqrt{d+e\,x}\,\sqrt{1+\frac{c\,x^2}{a}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]\right)\Big/\left(315\sqrt{c}\,e^5\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{a+c\,x^2}\right)+$$

$$\left(8\sqrt{-a}\,\left(c\,d^2+a\,e^2\right)\,\left(32\,B\,c\,d^3-36\,A\,c\,d^2\,e+33\,a\,B\,d\,e^2-45\,a\,A\,e^3\right)\,\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{1+\frac{c\,x^2}{a}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]\right)\Big/\left(315\sqrt{c}\,e^5\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}\right)$$

Result (type 4, 730 leaves) :

$$\frac{1}{315\sqrt{a+c\,x^2}}\left(\sqrt{d+e\,x}\left(\frac{1}{e^4}2\left(a+c\,x^2\right)\left(135\,a\,A\,e^3+a\,B\,e^2\,(-106\,d+77\,e\,x)+9\,A\,c\,e\left(8\,d^2-6\,d\,e\,x+5\,e^2\,x^2\right)+B\,c\left(-64\,d^3+48\,d^2\,e\,x-40\,d\,e^2\,x^2+35\,e^3\,x^3\right)\right)-\right.\right.$$

$$\frac{1}{c\,e^6\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\,(d+e\,x)}8\left(e^2\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\left(36\,A\,c\,d\,e\left(c\,d^2+2\,a\,e^2\right)-B\left(32\,c^2\,d^4+57\,a\,c\,d^2\,e^2+21\,a^2\,e^4\right)\right)\left(a+c\,x^2\right)+\right.$$

$$\sqrt{c}\left(-i\sqrt{c}\,d+\sqrt{a}\,e\right)\left(36\,A\,c\,d\,e\left(c\,d^2+2\,a\,e^2\right)-B\left(32\,c^2\,d^4+57\,a\,c\,d^2\,e^2+21\,a^2\,e^4\right)\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}$$

$$\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]-$$

$$\sqrt{a}\,\sqrt{c}\,e\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(9\,A\sqrt{c}\,e\left(4\,c\,d^2-3\,i\sqrt{a}\,\sqrt{c}\,d\,e+5\,a\,e^2\right)+B\left(-32\,c^{3/2}\,d^3+24\,i\sqrt{a}\,c\,d^2\,e-33\,a\sqrt{c}\,d\,e^2+21\,i\,a^{3/2}\,e^3\right)\right)$$

$$\left.\left.\left.\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]\right)\right)\right)$$

- **Problem 1477: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+B\,x)\left(a+c\,x^2\right)^{3/2}}{(d+e\,x)^{3/2}}\,dx$$

Optimal (type 4, 448 leaves, 7 steps) :

$$\frac{4\sqrt{d+e x}\,\left(5 a B e^2+4 c d (8 B d-7 A e)-3 c e (8 B d-7 A e) x\right)\sqrt{a+c x^2}}{35 e^4}+\frac{2 (8 B d-7 A e+B e x)\left(a+c x^2\right)^{3/2}}{7 e^2\sqrt{d+e x}}+$$

$$\left(8\sqrt{-a}\,\sqrt{c}\,\left(32 B c d^3-28 A c d^2 e+29 a B d e^2-21 a A e^3\right)\sqrt{d+e x}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\,\sqrt{c}\,d-a e}\right]\right)\Big/$$

$$\left(35 e^5\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{a+c x^2}\right)-$$

$$\left(8\sqrt{-a}\,\left(c d^2+a e^2\right)\left(32 B c d^2-28 A c d e+5 a B e^2\right)\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\,\sqrt{c}\,d-a e}\right]\right)\Big/$$

$$\left(35\sqrt{c}\,e^5\sqrt{d+e x}\,\sqrt{a+c x^2}\right)$$

Result (type 4, 661 leaves) :

$$\frac{1}{35\sqrt{a+c\,x^2}}$$

$$\sqrt{d+e\,x}\left(\frac{1}{e^4\,(d+e\,x)}2\,\left(a+c\,x^2\right)\,\left(-7\,A\,e\,\left(5\,a\,e^2+c\,\left(8\,d^2+2\,d\,e\,x-e^2\,x^2\right)\right)+B\,\left(5\,a\,e^2\,(10\,d+3\,e\,x)+c\,\left(64\,d^3+16\,d^2\,e\,x-8\,d\,e^2\,x^2+5\,e^3\,x^3\right)\right)\right)+\right.$$

$$\frac{1}{e^6\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\,(d+e\,x)}8\left(e^2\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\,\left(-32\,B\,c\,d^3+28\,A\,c\,d^2\,e-29\,a\,B\,d\,e^2+21\,a\,A\,e^3\right)\,\left(a+c\,x^2\right)+\right.$$

$$\sqrt{c}\,\left(-i\sqrt{c}\,d+\sqrt{a}\,e\right)\,\left(-32\,B\,c\,d^3+28\,A\,c\,d^2\,e-29\,a\,B\,d\,e^2+21\,a\,A\,e^3\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\,\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\,\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]+\sqrt{a}\,e\,\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\,\left(32\,B\,c\,d^2-24\,i\sqrt{a}\,B\sqrt{c}\,d\,e-28\,A\,c\,d\,e+5\,a\,B\,e^2+\right.$$

$$\left.\left.\left.21\,i\sqrt{a}\,A\sqrt{c}\,e^2\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\,\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\,\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]\right)\right)$$

- **Problem 1478: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+B\,x)\,\left(a+c\,x^2\right)^{3/2}}{(d+e\,x)^{5/2}}\,dx$$

Optimal (type 4, 437 leaves, 7 steps) :

$$-\frac{4\left(9aBe^2+4cd(8Bd-5Ae)+ce(8Bd-5Ae)x\right)\sqrt{a+cx^2}}{15e^4\sqrt{d+ex}}+\frac{2(8Bd-5Ae+3Bex)\left(a+cx^2\right)^{3/2}}{15e^2(d+ex)^{3/2}}-$$

$$\left(8\sqrt{-a}\sqrt{c}\left(9aBe^2+4cd(8Bd-5Ae)\right)\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2ae}{\sqrt{-a}\sqrt{c}\,d-ae}\right]\right)\Big/$$

$$\left(15e^5\sqrt{\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{a+cx^2}\right)+\frac{1}{15e^5\sqrt{d+ex}\sqrt{a+cx^2}}$$

$$8\sqrt{-a}\sqrt{c}\left(32Bcd^3-20Acd^2e+17aBde^2-5aAe^3\right)\sqrt{\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2ae}{\sqrt{-a}\sqrt{c}\,d-ae}\right]$$

Result (type 4, 628 leaves) :

$$\frac{1}{15\sqrt{a+c x^2}}\sqrt{d+e x}\left(-\frac{1}{e^4 (d+e x)^2}2\left(a+c x^2\right)\left(5 a A e^3+5 a B e^2 (2 d+3 e x)-5 A c e\left(8 d^2+10 d e x+e^2 x^2\right)+B c\left(64 d^3+80 d^2 e x+8 d e^2 x^2-3 e^3 x^3\right)\right)-\right.$$

$$\frac{1}{e^6\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\,(d+e x)}8\left(-e^2\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\left(32 B c d^2-20 A c d e+9 a B e^2\right)\left(a+c x^2\right)+\right.$$

$$\sqrt{c}\left(-i\sqrt{c}\,d+\sqrt{a}\,e\right)\left(-32 B c d^2+20 A c d e-9 a B e^2\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}\,(d+e x)^{3/2}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]+\sqrt{a}\sqrt{c}\,e\left(B\left(32 c d^2+8 i\sqrt{a}\sqrt{c}\,d e+9 a e^2\right)-5 A\left(4 c d e+i\sqrt{a}\sqrt{c}\,e^2\right)\right)$$

$$\left.\left.\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}\,(d+e x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]\right)\right)$$

- **Problem 1479: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+B x)\left(a+c x^2\right)^{3/2}}{(d+e x)^{7/2}}\,dx$$

Optimal (type 4, 541 leaves, 7 steps) :

$$\frac{4\,c\,\left(32\,B\,c\,d^3-12\,A\,c\,d^2\,e+29\,a\,B\,d\,e^2-9\,a\,A\,e^3+e\,\left(8\,B\,c\,d^2-3\,A\,c\,d\,e+5\,a\,B\,e^2\right)\,x\right)\,\sqrt{a+c\,x^2}}{15\,e^4\,\left(c\,d^2+a\,e^2\right)\,\sqrt{d+e\,x}}-$$

$$\frac{2\,\left(2\,B\,\left(4\,c\,d^3+a\,d\,e^2\right)-3\,A\,\left(c\,d^2\,e-a\,e^3\right)+e\,\left(11\,B\,c\,d^2-6\,A\,c\,d\,e+5\,a\,B\,e^2\right)\,x\right)\,\left(a+c\,x^2\right)^{3/2}}{15\,e^2\,\left(c\,d^2+a\,e^2\right)\,\left(d+e\,x\right)^{5/2}}+$$

$$\left(8\,\sqrt{-a}\,c^{3/2}\,\left(32\,B\,c\,d^3-12\,A\,c\,d^2\,e+29\,a\,B\,d\,e^2-9\,a\,A\,e^3\right)\,\sqrt{d+e\,x}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\,-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]\right)\Bigg/$$

$$\left(15\,e^5\,\left(c\,d^2+a\,e^2\right)\,\sqrt{\frac{\sqrt{c}\,\left(d+e\,x\right)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{a+c\,x^2}\right)-\frac{1}{15\,e^5\,\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}}$$

$$8\,\sqrt{-a}\,\sqrt{c}\,\left(32\,B\,c\,d^2-12\,A\,c\,d\,e+5\,a\,B\,e^2\right)\,\sqrt{\frac{\sqrt{c}\,\left(d+e\,x\right)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\,-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]$$

Result (type 4, 789 leaves) :

$$\sqrt{d+e x}\ \sqrt{a+c x^2}\left(\frac{2 B c}{3 e^4}-\frac{2 (-B d+A e) \left(c d^2+a e^2\right)}{5 e^4 (d+e x)^3}+\frac{2 \left(-17 B c d^2+12 A c d e-5 a B e^2\right)}{15 e^4 (d+e x)^2}-\frac{2 c \left(-73 B c d^3+33 A c d^2 e-61 a B d e^2+21 a A e^3\right)}{15 e^4 \left(c d^2+a e^2\right) (d+e x)}\right)-$$

$$\frac{1}{15 e^6 \sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}} \left(c d^2+a e^2\right) \sqrt{a+\frac{c (d+e x)^2 \left(-1+\frac{d}{d+e x}\right)^2}{e^2}}}$$

$$8 c (d+e x)^{3/2} \left(\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}} \left(32 B c d^3-12 A c d^2 e+29 a B d e^2-9 a A e^3\right) \left(\frac{a e^2}{(d+e x)^2}+c \left(-1+\frac{d}{d+e x}\right)^2\right)+\right.$$

$$\frac{1}{\sqrt{d+e x}}\sqrt{c} \left(-i \sqrt{c} d+\sqrt{a} e\right) \left(32 B c d^3-12 A c d^2 e+29 a B d e^2-9 a A e^3\right) \sqrt{1-\frac{d}{d+e x}-\frac{i \sqrt{a} e}{\sqrt{c} (d+e x)}}$$

$$\sqrt{1-\frac{d}{d+e x}+\frac{i \sqrt{a} e}{\sqrt{c} (d+e x)}}\ \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right], \frac{\sqrt{c} d-i \sqrt{a} e}{\sqrt{c} d+i \sqrt{a} e}\right]+\frac{1}{\sqrt{d+e x}}$$

$$\sqrt{a} e \left(\sqrt{c} d+i \sqrt{a} e\right) \left(-32 B c d^2+24 i \sqrt{a} B \sqrt{c} d e+12 A c d e-5 a B e^2-9 i \sqrt{a} A \sqrt{c} e^2\right) \sqrt{1-\frac{d}{d+e x}-\frac{i \sqrt{a} e}{\sqrt{c} (d+e x)}}$$

$$\left.\sqrt{1-\frac{d}{d+e x}+\frac{i \sqrt{a} e}{\sqrt{c} (d+e x)}}\ \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a} e}{\sqrt{c}}}}{\sqrt{d+e x}}\right], \frac{\sqrt{c} d-i \sqrt{a} e}{\sqrt{c} d+i \sqrt{a} e}\right]\right)$$

- **Problem 1480: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B x) (d+e x)^{3/2}}{\sqrt{a+c x^2}} \, dx$$

Optimal (type 4, 388 leaves, 7 steps) :

$$\frac{2\,(3\,B\,d+5\,A\,e)\,\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}}{15\,c}+\frac{2\,B\,(d+e\,x)^{3/2}\,\sqrt{a+c\,x^2}}{5\,c}-$$

$$\frac{2\,\sqrt{-a}\,\left(3\,B\,c\,d^2+20\,A\,c\,d\,e-9\,a\,B\,e^2\right)\,\sqrt{d+e\,x}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]}{15\,c^{3/2}\,e\,\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{a+c\,x^2}}+$$

$$\left(2\,\sqrt{-a}\,(3\,B\,d+5\,A\,e)\,\left(c\,d^2+a\,e^2\right)\,\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]\right)\Big/$$

$$\left(15\,c^{3/2}\,e\,\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}\right)$$

Result (type 4, 550 leaves) :

$$\frac{1}{15\,\sqrt{a+c\,x^2}}$$

$$\sqrt{d+e\,x}\left(\frac{2\,(6\,B\,d+5\,A\,e+3\,B\,e\,x)\,\left(a+c\,x^2\right)}{c}+\frac{1}{c^2\,e^2\,\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}\,(d+e\,x)}\,2\left(e^2\,\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}\,\left(3\,B\,c\,d^2+20\,A\,c\,d\,e-9\,a\,B\,e^2\right)\,\left(a+c\,x^2\right)+\right.\right.$$

$$\sqrt{c}\,\left(-i\,\sqrt{c}\,d+\sqrt{a}\,e\right)\,\left(3\,B\,c\,d^2+20\,A\,c\,d\,e-9\,a\,B\,e^2\right)\,\sqrt{\frac{e\,\left(\frac{i\,\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\,\sqrt{-\frac{\frac{i\,\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\,\sqrt{a}\,e}{\sqrt{c}\,d+i\,\sqrt{a}\,e}\right]+i\,\sqrt{c}\,e\,\left(\sqrt{c}\,d+i\,\sqrt{a}\,e\right)\,\left(15\,A\,c\,d-9\,a\,B\,e+i\,\sqrt{a}\,\sqrt{c}\,(3\,B\,d+5\,A\,e)\right)$$

$$\left.\left.\sqrt{\frac{e\,\left(\frac{i\,\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\,\sqrt{-\frac{\frac{i\,\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\,\sqrt{a}\,e}{\sqrt{c}\,d+i\,\sqrt{a}\,e}\right]\right)\right)$$

Problem 1481: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A+Bx)\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx$$

Optimal (type 4, 331 leaves, 6 steps):

$$\frac{2B\sqrt{d+ex}\sqrt{a+cx^2}}{3c}-\frac{2\sqrt{-a}\,(Bd+3Ae)\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2ae}{\sqrt{-a}\sqrt{c}\,d-ae}\right]}{3\sqrt{c}\,e\sqrt{\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{a+cx^2}}+$$

$$\frac{2\sqrt{-a}\,B\left(cd^2+ae^2\right)\sqrt{\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2ae}{\sqrt{-a}\sqrt{c}\,d-ae}\right]}{3c^{3/2}e\sqrt{d+ex}\sqrt{a+cx^2}}$$

Result (type 4, 464 leaves):

$$\frac{1}{3c\sqrt{a+cx^2}}2\sqrt{d+ex}\left(B\left(a+cx^2\right)+\frac{(Bd+3Ae)\left(a+cx^2\right)}{d+ex}+\right.$$

$$1\Big/e^2\,\text{i}\,c\,(Bd+3Ae)\sqrt{-d-\frac{\text{i}\sqrt{a}\,e}{\sqrt{c}}}\sqrt{\frac{e\left(\frac{\text{i}\sqrt{a}}{\sqrt{c}}+x\right)}{d+ex}}\sqrt{-\frac{\frac{\text{i}\sqrt{a}\,e}{\sqrt{c}}-ex}{d+ex}}\sqrt{d+ex}\,\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{\text{i}\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+ex}}\right],\frac{\sqrt{c}\,d-\text{i}\sqrt{a}\,e}{\sqrt{c}\,d+\text{i}\sqrt{a}\,e}\right]+$$

$$1\Big/\left(e\sqrt{-d-\frac{\text{i}\sqrt{a}\,e}{\sqrt{c}}}\right)\text{i}\left(\text{i}\sqrt{a}\,B+3A\sqrt{c}\right)\left(\sqrt{c}\,d+\text{i}\sqrt{a}\,e\right)\sqrt{\frac{e\left(\frac{\text{i}\sqrt{a}}{\sqrt{c}}+x\right)}{d+ex}}$$

$$\left.\sqrt{-\frac{\frac{\text{i}\sqrt{a}\,e}{\sqrt{c}}-ex}{d+ex}}\sqrt{d+ex}\,\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{\text{i}\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+ex}}\right],\frac{\sqrt{c}\,d-\text{i}\sqrt{a}\,e}{\sqrt{c}\,d+\text{i}\sqrt{a}\,e}\right]\right)$$

Problem 1482: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x}{\sqrt{d+e x}\,\sqrt{a+c x^2}}\,dx$$

Optimal (type 4, 288 leaves, 5 steps):

$$-\frac{2\sqrt{-a}\,B\sqrt{d+e x}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\,\sqrt{c}\,d-a e}\right]}{\sqrt{c}\,e\,\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{a+c x^2}}+\frac{2\sqrt{-a}\,(B d-A e)\,\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\,\sqrt{c}\,d-a e}\right]}{\sqrt{c}\,e\,\sqrt{d+e x}\,\sqrt{a+c x^2}}$$

Result (type 4, 439 leaves):

$$-\left(2\left(-B e^2\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\,\left(a+c x^2\right)+i B\sqrt{c}\,\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\,\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}\,(d+e x)^{3/2}\right.\right.$$
$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]+\left(\sqrt{a}\,B-i A\sqrt{c}\right)\sqrt{c}\,e\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e x}}\,\sqrt{-\frac{\frac{i\sqrt{a}\,e}{\sqrt{c}}-e x}{d+e x}}$$
$$\left.\left.(d+e x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{\sqrt{c}\,d+i\sqrt{a}\,e}\right]\right)\right)\Big/\left(c e^2\sqrt{-d-\frac{i\sqrt{a}\,e}{\sqrt{c}}}\,\sqrt{d+e x}\,\sqrt{a+c x^2}\right)$$

Problem 1483: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x}{(d+e x)^{3/2}\,\sqrt{a+c x^2}}\,dx$$

Optimal (type 4, 344 leaves, 6 steps):

$$\frac{2\,(B\,d-A\,e)\,\sqrt{a+c\,x^2}}{\left(c\,d^2+a\,e^2\right)\sqrt{d+e\,x}}+\frac{2\,\sqrt{-a}\,\sqrt{c}\,(B\,d-A\,e)\,\sqrt{d+e\,x}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]}{e\,\left(c\,d^2+a\,e^2\right)\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{a+c\,x^2}}-$$

$$\frac{2\,\sqrt{-a}\,B\,\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]}{\sqrt{c}\,e\,\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 320 leaves) :

$$\left(2\,\sqrt{\frac{e\left(\frac{i\,\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\,\sqrt{-\frac{\frac{i\,\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)\left(i\,\sqrt{c}\,(B\,d-A\,e)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\,\sqrt{a}\,e}{\sqrt{c}\,d+i\,\sqrt{a}\,e}\right]+\right.\right.$$

$$\left.\left.\left(\sqrt{a}\,B+i\,A\,\sqrt{c}\right)e\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\,\sqrt{a}\,e}{\sqrt{c}\,d+i\,\sqrt{a}\,e}\right]\right)\right)\Big/\left(e^2\left(\sqrt{c}\,d-i\,\sqrt{a}\,e\right)\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}\,\sqrt{a+c\,x^2}\right)$$

■ **Problem 1484: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+B\,x)\,(d+e\,x)^{3/2}}{\left(a+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 345 leaves, 6 steps) :

$$-\frac{\sqrt{d+e\,x}\,(a\,(B\,d+A\,e)-(A\,c\,d-a\,B\,e)\,x)}{a\,c\,\sqrt{a+c\,x^2}}-\frac{(A\,c\,d-3\,a\,B\,e)\,\sqrt{d+e\,x}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]}{\sqrt{-a}\,c^{3/2}\,\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{a+c\,x^2}}+$$

$$\frac{A\left(c\,d^2+a\,e^2\right)\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]}{\sqrt{-a}\,c^{3/2}\,\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 596 leaves):

$$\frac{\sqrt{d+e x}\,(-a B d-a A e+A c d x-a B e x)}{a c \sqrt{a+c x^2}}-\left((d+e x)^{3/2}\left((A c d-3 a B e)\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}\left(\frac{a e^2}{(d+e x)^2}+c\left(-1+\frac{d}{d+e x}\right)^2\right)+1\Big/\left(\left(\sqrt{d+e x}\right)\sqrt{c}\left(-i \sqrt{c}\, d+\sqrt{a}\, e\right)(A c d-3 a B e)\sqrt{1-\frac{d}{d+e x}-\frac{i \sqrt{a}\, e}{\sqrt{c}\,(d+e x)}}\sqrt{1-\frac{d}{d+e x}+\frac{i \sqrt{a}\, e}{\sqrt{c}\,(d+e x)}}\,\text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\, d-i \sqrt{a}\, e}{\sqrt{c}\, d+i \sqrt{a}\, e}\right]-1\Big/\left(\left(\sqrt{d+e x}\right)\sqrt{a}\left(3 i \sqrt{a}\, B+A \sqrt{c}\right)\sqrt{c}\, e\left(\sqrt{c}\, d+i \sqrt{a}\, e\right)\sqrt{1-\frac{d}{d+e x}-\frac{i \sqrt{a}\, e}{\sqrt{c}\,(d+e x)}}\sqrt{1-\frac{d}{d+e x}+\frac{i \sqrt{a}\, e}{\sqrt{c}\,(d+e x)}}\,\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}}{\sqrt{d+e x}}\right],\frac{\sqrt{c}\, d-i \sqrt{a}\, e}{\sqrt{c}\, d+i \sqrt{a}\, e}\right]\right)\right)\Big/\left(a c^2 e \sqrt{-d-\frac{i \sqrt{a}\, e}{\sqrt{c}}}\sqrt{a+\frac{c (d+e x)^2\left(-1+\frac{d}{d+e x}\right)^2}{e^2}}\right)$$

- **Problem 1485: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B x)\sqrt{d+e x}}{\left(a+c x^2\right)^{3/2}}\, dx$$

Optimal (type 4, 319 leaves, 6 steps):

$$-\frac{(a B-A c x)\sqrt{d+e x}}{a c \sqrt{a+c x^2}}-\frac{A \sqrt{d+e x}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\, d-a e}\right]}{\sqrt{-a}\sqrt{c}\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}}\sqrt{a+c x^2}}+\frac{(A c d+a B e)\sqrt{\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\, d+\sqrt{-a}\, e}}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a e}{\sqrt{-a}\sqrt{c}\, d-a e}\right]}{\sqrt{-a}\, c^{3/2}\sqrt{d+e x}\sqrt{a+c x^2}}$$

Result (type 4, 431 leaves) :

$$\frac{1}{a\,c\,\sqrt{a+c\,x^2}}\sqrt{d+e\,x}\left(-a\,B+A\,c\,x-\frac{A\,e\,\left(a+c\,x^2\right)}{d+e\,x}-\right.$$

$$1\,/\,e\,\mathrm{i}\,A\,c\,\sqrt{-d-\frac{\mathrm{i}\,\sqrt{a}\,e}{\sqrt{c}}}\,\sqrt{\frac{e\left(\frac{\mathrm{i}\,\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\,\sqrt{-\frac{\frac{\mathrm{i}\,\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,\sqrt{d+e\,x}\;\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{\mathrm{i}\,\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\,\frac{\sqrt{c}\,d-\mathrm{i}\,\sqrt{a}\,e}{\sqrt{c}\,d+\mathrm{i}\,\sqrt{a}\,e}\right]+$$

$$\left.1\Big/\left(\sqrt{-d-\frac{\mathrm{i}\,\sqrt{a}\,e}{\sqrt{c}}}\right)\sqrt{a}\,\left(\mathrm{i}\,\sqrt{a}\,B+A\,\sqrt{c}\right)\sqrt{\frac{e\left(\frac{\mathrm{i}\,\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\,\sqrt{-\frac{\frac{\mathrm{i}\,\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,\sqrt{d+e\,x}\;\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{\mathrm{i}\,\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\,\frac{\sqrt{c}\,d-\mathrm{i}\,\sqrt{a}\,e}{\sqrt{c}\,d+\mathrm{i}\,\sqrt{a}\,e}\right]\right)$$

- **Problem 1486: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B\,x}{\sqrt{d+e\,x}\,\left(a+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 356 leaves, 6 steps) :

$$-\frac{\sqrt{d+e\,x}\,\left(a\,(B\,d-A\,e)-(A\,c\,d+a\,B\,e)\,x\right)}{a\,\left(c\,d^2+a\,e^2\right)\sqrt{a+c\,x^2}}-\frac{(A\,c\,d+a\,B\,e)\,\sqrt{d+e\,x}\,\sqrt{1+\frac{c\,x^2}{a}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\,-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]}{\sqrt{-a}\,\sqrt{c}\,\left(c\,d^2+a\,e^2\right)\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{a+c\,x^2}}+$$

$$\frac{A\,\sqrt{\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}}\,\sqrt{1+\frac{c\,x^2}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\,-\frac{2\,a\,e}{\sqrt{-a}\,\sqrt{c}\,d-a\,e}\right]}{\sqrt{-a}\,\sqrt{c}\,\sqrt{d+e\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 525 leaves) :

$$\frac{1}{2\,a\,(c\,d^2+a\,e^2)\,\sqrt{a+c\,x^2}}\sqrt{d+e\,x}\left(2\,(A\,c\,d\,x+a\,(-B\,d+A\,e+B\,e\,x))-\left(2\left(e^2\,(A\,c\,d+a\,B\,e)\,\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}\,(a+c\,x^2)+\sqrt{c}\,\left(-i\,\sqrt{c}\,d+\sqrt{a}\,e\right)\,(A\,c\,d+a\,B\,e)\,\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\right.\right.\right.$$

$$\sqrt{-\frac{\frac{i\,\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\,\sqrt{a}\,e}{\sqrt{c}\,d+i\,\sqrt{a}\,e}\right]+\sqrt{a}\,\left(i\,\sqrt{a}\,B-A\,\sqrt{c}\right)\,\sqrt{c}\,e\,\left(\sqrt{c}\,d+i\,\sqrt{a}\,e\right)$$

$$\left.\left.\left.\sqrt{\frac{e\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{d+e\,x}}\,\sqrt{-\frac{\frac{i\,\sqrt{a}\,e}{\sqrt{c}}-e\,x}{d+e\,x}}\,(d+e\,x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}}{\sqrt{d+e\,x}}\right],\frac{\sqrt{c}\,d-i\,\sqrt{a}\,e}{\sqrt{c}\,d+i\,\sqrt{a}\,e}\right]\right)\right)\Big/\left(c\,e\,\sqrt{-d-\frac{i\,\sqrt{a}\,e}{\sqrt{c}}}\,(d+e\,x)\right)\right)$$

- **Problem 1487: Result more than twice size of optimal antiderivative.**

$$\int (A+B\,x)\,(d+e\,x)^m\,(a+c\,x^2)^3\,dx$$

Optimal (type 3, 372 leaves, 2 steps) :

$$-\frac{(B\,d-A\,e)\,(c\,d^2+a\,e^2)^3\,(d+e\,x)^{1+m}}{e^8\,(1+m)}+\frac{(c\,d^2+a\,e^2)^2\,(7\,B\,c\,d^2-6\,A\,c\,d\,e+a\,B\,e^2)\,(d+e\,x)^{2+m}}{e^8\,(2+m)}-$$

$$\frac{3\,c\,(c\,d^2+a\,e^2)\,(7\,B\,c\,d^3-5\,A\,c\,d^2\,e+3\,a\,B\,d\,e^2-a\,A\,e^3)\,(d+e\,x)^{3+m}}{e^8\,(3+m)}-$$

$$\frac{c\,(4\,A\,c\,d\,e\,(5\,c\,d^2+3\,a\,e^2)-B\,(35\,c^2\,d^4+30\,a\,c\,d^2\,e^2+3\,a^2\,e^4))\,(d+e\,x)^{4+m}}{e^8\,(4+m)}-\frac{c^2\,(35\,B\,c\,d^3-15\,A\,c\,d^2\,e+15\,a\,B\,d\,e^2-3\,a\,A\,e^3)\,(d+e\,x)^{5+m}}{e^8\,(5+m)}+$$

$$\frac{3\,c^2\,(7\,B\,c\,d^2-2\,A\,c\,d\,e+a\,B\,e^2)\,(d+e\,x)^{6+m}}{e^8\,(6+m)}-\frac{c^3\,(7\,B\,d-A\,e)\,(d+e\,x)^{7+m}}{e^8\,(7+m)}+\frac{B\,c^3\,(d+e\,x)^{8+m}}{e^8\,(8+m)}$$

Result (type 3, 875 leaves) :

$$\frac{1}{e^8 (1+m)(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)(8+m)} (d+e x)^{1+m} \Big(A e (8+m) \big(a^3 e^6 (5040+8028 m+5104 m^2+1665 m^3+295 m^4+27 m^5+m^6)+3 a^2 c e^4 (840+638 m+179 m^2+22 m^3+m^4) (2 d^2-2 d e (1+m) x+e^2 (2+3 m+m^2) x^2)+3 a c^2 e^2 (42+13 m+m^2) (24 d^4-24 d^3 e (1+m) x+12 d^2 e^2 (2+3 m+m^2) x^2-4 d e^3 (6+11 m+6 m^2+m^3) x^3+e^4 (24+50 m+35 m^2+10 m^3+m^4) x^4)+c^3 (720 d^6-720 d^5 e (1+m) x+360 d^4 e^2 (2+3 m+m^2) x^2-120 d^3 e^3 (6+11 m+6 m^2+m^3) x^3+30 d^2 e^4 (24+50 m+35 m^2+10 m^3+m^4) x^4-6 d e^5 (120+274 m+225 m^2+85 m^3+15 m^4+m^5) x^5+e^6 (720+1764 m+1624 m^2+735 m^3+175 m^4+21 m^5+m^6) x^6)\big)-B \big(a^3 e^6 (20160+24552 m+12154 m^2+3135 m^3+445 m^4+33 m^5+m^6) (d-e (1+m) x)-3 a^2 c e^4 (1680+1066 m+251 m^2+26 m^3+m^4) (-6 d^3+6 d^2 e (1+m) x-3 d e^2 (2+3 m+m^2) x^2+e^3 (6+11 m+6 m^2+m^3) x^3)-3 a c^2 e^2 (56+15 m+m^2) (-120 d^5+120 d^4 e (1+m) x-60 d^3 e^2 (2+3 m+m^2) x^2+20 d^2 e^3 (6+11 m+6 m^2+m^3) x^3-5 d e^4 (24+50 m+35 m^2+10 m^3+m^4) x^4+e^5 (120+274 m+225 m^2+85 m^3+15 m^4+m^5) x^5)+c^3 (5040 d^7-5040 d^6 e (1+m) x+2520 d^5 e^2 (2+3 m+m^2) x^2-840 d^4 e^3 (6+11 m+6 m^2+m^3) x^3+210 d^3 e^4 (24+50 m+35 m^2+10 m^3+m^4) x^4-42 d^2 e^5 (120+274 m+225 m^2+85 m^3+15 m^4+m^5) x^5+7 d e^6 (720+1764 m+1624 m^2+735 m^3+175 m^4+21 m^5+m^6) x^6-e^7 (5040+13068 m+13132 m^2+6769 m^3+1960 m^4+322 m^5+28 m^6+m^7) x^7)\big)\Big)$$

- **Problem 1490: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B x) (d+e x)^m}{a+c x^2} \, dx$$

Optimal (type 5, 202 leaves, 4 steps):

$$-\frac{\left(a B+\sqrt{-a} A \sqrt{c}\right) (d+e x)^{1+m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{\sqrt{c} (d+e x)}{\sqrt{c} d-\sqrt{-a} e}\right]}{2 a \sqrt{c} \left(\sqrt{c} d-\sqrt{-a} e\right) (1+m)}-\frac{\left(A+\frac{\sqrt{-a} B}{\sqrt{c}}\right) (d+e x)^{1+m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}\right]}{2 \sqrt{-a} \left(\sqrt{c} d+\sqrt{-a} e\right) (1+m)}$$

Result (type 5, 241 leaves):

$$\frac{1}{2 \sqrt{a} c m} (d+e x)^m \left(\left(\sqrt{a} B-i A \sqrt{c}\right) \left(\frac{\sqrt{c} (d+e x)}{e \left(-i \sqrt{a}+\sqrt{c} x\right)}\right)^{-m} \text{Hypergeometric2F1}\left[-m, -m, 1-m, \frac{\sqrt{c} d+i \sqrt{a} e}{i \sqrt{a} e-\sqrt{c} e x}\right]+\left(\sqrt{a} B+i A \sqrt{c}\right) \left(\frac{\sqrt{c} (d+e x)}{e \left(i \sqrt{a}+\sqrt{c} x\right)}\right)^{-m} \text{Hypergeometric2F1}\left[-m, -m, 1-m, -\frac{\sqrt{c} d-i \sqrt{a} e}{i \sqrt{a} e+\sqrt{c} e x}\right]\right)$$

- **Problem 1491: Unable to integrate problem.**

$$\int \frac{(A+B x) (d+e x)^m}{\left(a+c x^2\right)^2} \, dx$$

Optimal (type 5, 361 leaves, 5 steps):

$$-\frac{(d+e x)^{1+m} (a (B d-A e)-(A c d+a B e) x)}{2 a \left(c d^2+a e^2\right) \left(a+c x^2\right)}+$$

$$\left(\left(a e (A c d+a B e) m-\sqrt{-a} \sqrt{c} \left(A \left(c d^2+a e^2 (1-m)\right)+a B d e m\right)\right) (d+e x)^{1+m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{\sqrt{c} (d+e x)}{\sqrt{c} d-\sqrt{-a} e}\right]\right) \Big/$$
$$\left(4 a^2 \sqrt{c} \left(\sqrt{c} d-\sqrt{-a} e\right) \left(c d^2+a e^2\right) (1+m)\right)+$$
$$\left(\left(a e (A c d+a B e) m+\sqrt{-a} \sqrt{c} \left(A \left(c d^2+a e^2 (1-m)\right)+a B d e m\right)\right) (d+e x)^{1+m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}\right]\right) \Big/$$
$$\left(4 a^2 \sqrt{c} \left(\sqrt{c} d+\sqrt{-a} e\right) \left(c d^2+a e^2\right) (1+m)\right)$$

Result (type 8, 24 leaves):

$$\int \frac{(A+B x) (d+e x)^m}{\left(a+c x^2\right)^2} \, dx$$

- **Problem 1492: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B x) (d+e x)^{1+m}}{a+c x^2} \, dx$$

Optimal (type 5, 202 leaves, 4 steps):

$$-\frac{\left(a B+\sqrt{-a} A \sqrt{c}\right) (d+e x)^{2+m} \text{Hypergeometric2F1}\left[1, 2+m, 3+m, \frac{\sqrt{c} (d+e x)}{\sqrt{c} d-\sqrt{-a} e}\right]}{2 a \sqrt{c} \left(\sqrt{c} d-\sqrt{-a} e\right) (2+m)}-$$
$$\frac{\left(A+\frac{\sqrt{-a} B}{\sqrt{c}}\right) (d+e x)^{2+m} \text{Hypergeometric2F1}\left[1, 2+m, 3+m, \frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}\right]}{2 \sqrt{-a} \left(\sqrt{c} d+\sqrt{-a} e\right) (2+m)}$$

Result (type 5, 303 leaves):

$$\frac{1}{2 \sqrt{a} c^{3/2} m} (d+e x)^m$$
$$\left(\frac{2 \sqrt{a} B \sqrt{c} m (d+e x)}{1+m}+\left(i \sqrt{a} B+A \sqrt{c}\right) \left(-i \sqrt{c} d+\sqrt{a} e\right) \left(\frac{\sqrt{c} (d+e x)}{e \left(-i \sqrt{a}+\sqrt{c} x\right)}\right)^{-m} \text{Hypergeometric2F1}\left[-m, -m, 1-m, \frac{\sqrt{c} d+i \sqrt{a} e}{i \sqrt{a} e-\sqrt{c} e x}\right]+\right.$$
$$\left.\left(-i \sqrt{a} B+A \sqrt{c}\right) \left(i \sqrt{c} d+\sqrt{a} e\right) \left(\frac{\sqrt{c} (d+e x)}{e \left(i \sqrt{a}+\sqrt{c} x\right)}\right)^{-m} \text{Hypergeometric2F1}\left[-m, -m, 1-m, -\frac{\sqrt{c} d-i \sqrt{a} e}{i \sqrt{a} e+\sqrt{c} e x}\right]\right)$$

- **Problem 1508: Result more than twice size of optimal antiderivative.**

$$\int (b+2\,c\,x)\,\left(a+b\,x+c\,x^2\right)^2\,dx$$

Optimal (type 1, 16 leaves, 1 step):

$$\frac{1}{3}\,\left(a+b\,x+c\,x^2\right)^3$$

Result (type 1, 36 leaves):

$$\frac{1}{3}\,x\,(b+c\,x)\,\left(3\,a^2+3\,a\,x\,(b+c\,x)+x^2\,(b+c\,x)^2\right)$$

- **Problem 1518: Result more than twice size of optimal antiderivative.**

$$\int (b+2\,c\,x)\,\left(a+b\,x+c\,x^2\right)^3\,dx$$

Optimal (type 1, 16 leaves, 1 step):

$$\frac{1}{4}\,\left(a+b\,x+c\,x^2\right)^4$$

Result (type 1, 51 leaves):

$$\frac{1}{4}\,x\,(b+c\,x)\,\left(4\,a^3+6\,a^2\,x\,(b+c\,x)+4\,a\,x^2\,(b+c\,x)^2+x^3\,(b+c\,x)^3\right)$$

- **Problem 1565: Result more than twice size of optimal antiderivative.**

$$\int (b+2\,c\,x)\,(d+e\,x)^3\,\left(a+b\,x+c\,x^2\right)^{5/2}\,dx$$

Optimal (type 3, 446 leaves, 8 steps):

$$\frac{3\,\left(b^2-4\,a\,c\right)^3\,e\,\left(40\,c^2\,d^2+11\,b^2\,e^2-4\,c\,e\,(10\,b\,d+a\,e)\right)\,(b+2\,c\,x)\,\sqrt{a+b\,x+c\,x^2}}{65\,536\,c^6}-$$
$$\frac{\left(b^2-4\,a\,c\right)^2\,e\,\left(40\,c^2\,d^2+11\,b^2\,e^2-4\,c\,e\,(10\,b\,d+a\,e)\right)\,(b+2\,c\,x)\,\left(a+b\,x+c\,x^2\right)^{3/2}}{8192\,c^5}+$$
$$\frac{\left(b^2-4\,a\,c\right)\,e\,\left(40\,c^2\,d^2+11\,b^2\,e^2-4\,c\,e\,(10\,b\,d+a\,e)\right)\,(b+2\,c\,x)\,\left(a+b\,x+c\,x^2\right)^{5/2}}{2560\,c^4}+$$
$$\frac{(2\,c\,d-b\,e)\,(d+e\,x)^2\,\left(a+b\,x+c\,x^2\right)^{7/2}}{30\,c}+\frac{1}{5}\,(d+e\,x)^3\,\left(a+b\,x+c\,x^2\right)^{7/2}+\frac{1}{6720\,c^3}$$
$$\left(128\,c^3\,d^3-99\,b^3\,e^3+4\,b\,c\,e^2\,(90\,b\,d+97\,a\,e)-8\,c^2\,d\,e\,(17\,b\,d+160\,a\,e)+14\,c\,e\,\left(8\,c^2\,d^2+11\,b^2\,e^2-4\,c\,e\,(2\,b\,d+9\,a\,e)\right)\,x\right)\,\left(a+b\,x+c\,x^2\right)^{7/2}-$$
$$\frac{3\,\left(b^2-4\,a\,c\right)^4\,e\,\left(40\,c^2\,d^2+11\,b^2\,e^2-4\,c\,e\,(10\,b\,d+a\,e)\right)\,\text{ArcTanh}\left[\frac{b+2\,c\,x}{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}\right]}{131\,072\,c^{13/2}}$$

Result (type 3, 927 leaves) :

$$\frac{1}{13\,762\,560\,c^{13/2}}\Big(2\sqrt{c}\,\sqrt{a+x\,(b+c\,x)}\,\big(3465\,b^9\,e^3-210\,b^8\,c\,e^2\,(60\,d+11\,e\,x)-640\,b^4\,c^5\,e\,x^3\,(9\,d^2+8\,d\,e\,x+2\,e^2\,x^2)+168\,b^7\,c^2\,e\,(75\,d^2+50\,d\,e\,x+11\,e^2\,x^2)+64\,b^5\,c^4\,e\,x^2\,(105\,d^2+90\,d\,e\,x+22\,e^2\,x^2)-48\,b^6\,c^3\,e\,x\,(175\,d^2+140\,d\,e\,x+33\,e^2\,x^2)+16\,384\,c^9\,x^6\,(120\,d^3+315\,d^2\,e\,x+280\,d\,e^2\,x^2+84\,e^3\,x^3)+5120\,b^3\,c^6\,x^3\,(384\,d^3+897\,d^2\,e\,x+734\,d\,e^2\,x^2+207\,e^3\,x^3)+8192\,b\,c^8\,x^5\,(720\,d^3+1845\,d^2\,e\,x+1610\,d\,e^2\,x^2+476\,e^3\,x^3)+2048\,b^2\,c^7\,x^4\,(2880\,d^3+7125\,d^2\,e\,x+6060\,d\,e^2\,x^2+1757\,e^3\,x^3)-1280\,a^4\,c^4\,e^2\,(-449\,b\,e+2\,c\,(512\,d+63\,e\,x))+1280\,a^3\,c^3\,\big(-537\,b^3\,e^3+62\,b^2\,c\,e^2\,(27\,d+4\,e\,x)-2\,b\,c^2\,e\,(837\,d^2+374\,d\,e\,x+65\,e^2\,x^2)+4\,c^3\,(384\,d^3+315\,d^2\,e\,x+128\,d\,e^2\,x^2+21\,e^3\,x^3)\big)+96\,a^2\,c^2\,\big(3003\,b^5\,e^3-10\,b^4\,c\,e^2\,(1022\,d+167\,e\,x)-40\,b^2\,c^3\,e\,x\,(141\,d^2+92\,d\,e\,x+19\,e^2\,x^2)+20\,b^3\,c^2\,e\,(511\,d^2+282\,d\,e\,x+55\,e^2\,x^2)+160\,b\,c^4\,x\,(384\,d^3+663\,d^2\,e\,x+454\,d\,e^2\,x^2+114\,e^3\,x^3)+64\,c^5\,x^2\,(960\,d^3+2065\,d^2\,e\,x+1600\,d\,e^2\,x^2+434\,e^3\,x^3)\big)+16\,a\,c\,\big(-3255\,b^7\,e^3+42\,b^6\,c\,e^2\,(275\,d+48\,e\,x)-160\,b^3\,c^4\,e\,x^2\,(33\,d^2+26\,d\,e\,x+6\,e^2\,x^2)+20\,b^4\,c^3\,e\,x\,(357\,d^2+264\,d\,e\,x+59\,e^2\,x^2)-6\,b^5\,c^2\,e\,(1925\,d^2+1190\,d\,e\,x+249\,e^2\,x^2)+960\,b^2\,c^5\,x^2\,(384\,d^3+815\,d^2\,e\,x+628\,d\,e^2\,x^2+170\,e^3\,x^3)+512\,c^7\,x^4\,(720\,d^3+1785\,d^2\,e\,x+1520\,d\,e^2\,x^2+441\,e^3\,x^3)+256\,b\,c^6\,x^3\,(2880\,d^3+6765\,d^2\,e\,x+5550\,d\,e^2\,x^2+1567\,e^3\,x^3)\big)\big)-315\,(b^2-4\,a\,c)^4\,e\,(40\,c^2\,d^2+11\,b^2\,e^2-4\,c\,e\,(10\,b\,d+a\,e))\,\mathrm{Log}\big[b+2\,c\,x+2\sqrt{c}\,\sqrt{a+x\,(b+c\,x)}\big]\Big)$$

■ Problem 1566: Result more than twice size of optimal antiderivative.

$$\int (b+2\,c\,x)\,(d+e\,x)^2\,\left(a+b\,x+c\,x^2\right)^{5/2}\,dx$$

Optimal (type 3, 289 leaves, 7 steps) :

$$\frac{5\,(b^2-4\,a\,c)^3\,e\,(2\,c\,d-b\,e)\,(b+2\,c\,x)\,\sqrt{a+b\,x+c\,x^2}}{8192\,c^5}-\frac{5\,(b^2-4\,a\,c)^2\,e\,(2\,c\,d-b\,e)\,(b+2\,c\,x)\,(a+b\,x+c\,x^2)^{3/2}}{3072\,c^4}+\frac{(b^2-4\,a\,c)\,e\,(2\,c\,d-b\,e)\,(b+2\,c\,x)\,(a+b\,x+c\,x^2)^{5/2}}{192\,c^3}+\frac{2}{9}\,(d+e\,x)^2\,(a+b\,x+c\,x^2)^{7/2}+\frac{(32\,c^2\,d^2+9\,b^2\,e^2-2\,c\,e\,(9\,b\,d+16\,a\,e)+14\,c\,e\,(2\,c\,d-b\,e)\,x)\,(a+b\,x+c\,x^2)^{7/2}}{504\,c^2}-\frac{5\,(b^2-4\,a\,c)^4\,e\,(2\,c\,d-b\,e)\,\mathrm{ArcTanh}\left[\frac{b+2\,c\,x}{2\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}\right]}{16\,384\,c^{11/2}}$$

Result (type 3, 593 leaves) :

$$\frac{1}{1032192\, c^{11/2}} \Big(2\sqrt{c}\,\sqrt{a+x\,(b+c\,x)}\,\big(-315\,b^8\,e^2-32768\,a^4\,c^4\,e^2+210\,b^7\,c\,e\,(3\,d+e\,x)-84\,b^6\,c^2\,e\,x\,(5\,d+2\,e\,x)+48\,b^5\,c^3\,e\,x^2\,(7\,d+3\,e\,x)-32\,b^4\,c^4\,e\,x^3\,(9\,d+4\,e\,x)+4096\,c^8\,x^6\,\left(36\,d^2+63\,d\,e\,x+28\,e^2\,x^2\right)+2048\,b\,c^7\,x^5\,\left(216\,d^2+369\,d\,e\,x+161\,e^2\,x^2\right)+1536\,b^2\,c^6\,x^4\,\left(288\,d^2+475\,d\,e\,x+202\,e^2\,x^2\right)+256\,b^3\,c^5\,x^3\,\left(576\,d^2+897\,d\,e\,x+367\,e^2\,x^2\right)+64\,a^3\,c^3\,\left(837\,b^2\,e^2-2\,b\,c\,e\,(837\,d+187\,e\,x)+4\,c^2\,\left(576\,d^2+315\,d\,e\,x+64\,e^2\,x^2\right)\right)+48\,a^2\,c^2\,\left(-511\,b^4\,e^2-4\,b^2\,c^2\,e\,x\,(141\,d+46\,e\,x)+2\,b^3\,c\,e\,(511\,d+141\,e\,x)+32\,c^4\,x^2\,\left(288\,d^2+413\,d\,e\,x+160\,e^2\,x^2\right)+16\,b\,c^3\,x\,\left(576\,d^2+663\,d\,e\,x+227\,e^2\,x^2\right)\right)+4\,a\,c\,\left(1155\,b^6\,e^2-32\,b^3\,c^3\,e\,x^2\,(33\,d+13\,e\,x)-42\,b^5\,c\,e\,(55\,d+17\,e\,x)+12\,b^4\,c^2\,e\,x\,(119\,d+44\,e\,x)+512\,c^6\,x^4\,\left(216\,d^2+357\,d\,e\,x+152\,e^2\,x^2\right)+768\,b\,c^5\,x^3\,\left(288\,d^2+451\,d\,e\,x+185\,e^2\,x^2\right)+192\,b^2\,c^4\,x^2\,\left(576\,d^2+815\,d\,e\,x+314\,e^2\,x^2\right)\right)\big)+315\,\left(b^2-4\,a\,c\right)^4\,e\,(-2\,c\,d+b\,e)\,\mathrm{Log}\Big[b+2\,c\,x+2\,\sqrt{c}\,\sqrt{a+x\,(b+c\,x)}\Big]\Big)$$

- **Problem 1628: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (b+2\,c\,x)\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\,dx$$

Optimal (type 4, 576 leaves, 7 steps):

$$-\frac{2\sqrt{d+e x}\left(8 c^2 d^2+b^2 e^2-c e (11 b d-10 a e)-3 c e (2 c d-b e) x\right)\sqrt{a+b x+c x^2}}{105 c e^2}+$$

$$\frac{4}{7}\sqrt{d+e x}\left(a+b x+c x^2\right)^{3/2}+\left(2\sqrt{2}\sqrt{b^2-4 a c}\,(2 c d-b e)\left(4 c^2 d^2-b^2 e^2-4 c e (b d-2 a e)\right)\sqrt{d+e x}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}\right]\right)\Bigg/\left(105 c^2 e^3\sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}}\sqrt{a+b x+c x^2}\right)-$$

$$\left(2\sqrt{2}\sqrt{b^2-4 a c}\left(c d^2-b d e+a e^2\right)\left(16 c^2 d^2-b^2 e^2-4 c e (4 b d-5 a e)\right)\sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}\right]\right)\Bigg/\left(105 c^2 e^3\sqrt{d+e x}\sqrt{a+b x+c x^2}\right)$$

Result (type 4, 5323 leaves) :

$$\sqrt{d+e x}\left(-\frac{2\left(8 c^2 d^2-11 b c d e+b^2 e^2-20 a c e^2\right)}{105 c e^2}+\frac{2 (2 c d+9 b e) x}{35 e}+\frac{4 c x^2}{7}\right)\sqrt{a+x (b+c x)}+\frac{1}{105 c e^4\sqrt{a+b x+c x^2}}$$

$$\sqrt{a+x\,(b+c\,x)}\;\Bigg(\left(4\,(2\,c\,d-b\,e)\,\left(4\,c^2\,d^2-4\,b\,c\,d\,e-b^2\,e^2+8\,a\,c\,e^2\right)\,(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)\right)\Big/$$

$$\left(c\,\sqrt{\frac{(d+e\,x)^2\left(c\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)-$$

$$\frac{1}{c\,\sqrt{\frac{(d+e\,x)^2\left(c\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\,2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\Bigg(\Bigg(4\,i\,\sqrt{2}\,c^3\,d^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\;\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Bigg)\Bigg/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(6\,\mathrm{i}\,\sqrt{2}\,b\,c^2\,d^2\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(\mathrm{i}\,\sqrt{2}\,b^2\,c\,d\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(8\, i \sqrt{2}\, a c^2 d e^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left((c d^2 - b d e + a e^2) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(i\, b^3 e^3 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2}\, (c d^2 - b d e + a e^2) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\left(4\,i\,\sqrt{2}\,a\,b\,c\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(8\,i\,\sqrt{2}\,c^3\,d^2\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(8\,i\,\sqrt{2}\,b\,c^2\,d\,e\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(\mathbb{i}\,b^2\,c\,e^2\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\ \sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(10\,\mathbb{i}\,\sqrt{2}\,a\,c^2\,e^2\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\ \sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\Bigg)$$

Problem 1629: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(b+2\,c\,x)\,\sqrt{a+b\,x+c\,x^2}}{\sqrt{d+e\,x}}\,dx$$

Optimal (type 4, 487 leaves, 6 steps):

$$-\frac{2\,\sqrt{d+e\,x}\,\left(8\,c\,d-7\,b\,e-6\,c\,e\,x\right)\,\sqrt{a+b\,x+c\,x^2}}{15\,e^2}+\left(\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(16\,c^2\,d^2+b^2\,e^2-4\,c\,e\,(4\,b\,d-3\,a\,e)\right)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/\left(15\,c\,e^3\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)-$$

$$\left(16\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,(2\,c\,d-b\,e)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/\left(15\,c\,e^3\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 3387 leaves) :

$$\left(\frac{2\,(-8\,c\,d+7\,b\,e)}{15\,e^2}+\frac{4\,c\,x}{5\,e}\right)\sqrt{d+e\,x}\,\sqrt{a+x\,(b+c\,x)}\,+$$

$$\frac{1}{15\,e^4\sqrt{a+b\,x+c\,x^2}}\sqrt{a+x\,(b+c\,x)}\left(\frac{2\left(16\,c^2\,d^2-16\,b\,c\,d\,e+b^2\,e^2+12\,a\,c\,e^2\right)(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)}{c\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}-\right.$$

$$\frac{1}{c\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\,2\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\left(\left(4\,i\sqrt{2}\,c^2\,d^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(4\,i\,\sqrt{2}\,b\,c\,d\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(i\,b^2\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(3\,i\,\sqrt{2}\,a\,c\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Bigg/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(8\,i\,\sqrt{2}\,c^2\,d\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(4\,i\,\sqrt{2}\,b\,c\,e\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)\Bigg)\Bigg)$$

- **Problem 1630: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{(b+2 c x)\sqrt{a+b x+c x^2}}{(d+e x)^{3/2}}\,dx$$

Optimal (type 4, 469 leaves, 6 steps) :

$$\frac{2\,(8\,c\,d-3\,b\,e+2\,c\,e\,x)\,\sqrt{a+b\,x+c\,x^2}}{3\,e^2\,\sqrt{d+e\,x}}-$$

$$\left(8\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,(2\,c\,d-b\,e)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/$$

$$\left(3\,e^3\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(16\,c^2\,d^2+3\,b^2\,e^2-4\,c\,e\,(4\,b\,d-a\,e)\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/\left(3\,c\,e^3\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 5706 leaves) :

$$\sqrt{d+e\,x}\,\sqrt{a+x\,(b+c\,x)}\,\left(\frac{4\,c}{3\,e^2}-\frac{2\,(-2\,c\,d+b\,e)}{e^2\,(d+e\,x)}\right)-$$

$$\frac{1}{3\,e^4\,\sqrt{a+b\,x+c\,x^2}}\,2\,\sqrt{a+x\,(b+c\,x)}\left(\frac{8\,(2\,c\,d-b\,e)\,(d+e\,x)^{3/2}\,\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)}{\sqrt{\frac{(d+e\,x)^2\,\left(c\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\,\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}-\right.$$

$$\left(4\,i\,\sqrt{2}\,c^2\,d^3\,\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}\right.$$

$$\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Bigg/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\,\sqrt{\frac{(d+e\,x)^2\,\left(c\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\,\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)+$$

$$\left(6\,i\,\sqrt{2}\,b\,c\,d^2\,e\,\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}\right.$$

$$\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)-$$

$$\left(2\,i\sqrt{2}\,b^2 d e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\,\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)-$$

$$\left(4\,\mathbb{i}\sqrt{2}\,a c d e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\,\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)+$$

$$\left(2\,\mathbb{i}\sqrt{2}\,a b e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\ \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)-$$

$$\left(8 i \sqrt{2}\, c^2 d^2 (d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\ \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)+$$

$$\left(8\, i \sqrt{2}\, b\, c\, d\, e\, (d+e x) \sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}} \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}} \sqrt{\frac{(d+e x)^2 \left(c \left(-1+\frac{d}{d+e x}\right)^2+\frac{e \left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right) -$$

$$\left(3\, i\, b^2 e^2 (d+e x) \sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}} \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}} \sqrt{\frac{(d+e x)^2 \left(c \left(-1+\frac{d}{d+e x}\right)^2+\frac{e \left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right) -$$

$$\left(2\, i \sqrt{2}\, a\, c\, e^2 (d+e x) \sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}} \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\ \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)$$

- **Problem 1631: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{(b+2 c x)\sqrt{a+b x+c x^2}}{(d+e x)^{5/2}}\,dx$$

Optimal (type 4, 548 leaves, 6 steps) :

$$-\frac{2\left(8c^2d^3+abe^3-cde(7bd-4ae)+e\left(10c^2d^2+b^2e^2-2ce(5bd-3ae)\right)x\right)\sqrt{a+bx+cx^2}}{3e^2\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}}+$$

$$\left(\sqrt{2}\sqrt{b^2-4ac}\left(16c^2d^2+b^2e^2-4ce(4bd-3ae)\right)\sqrt{d+ex}\right.$$

$$\left.\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\ e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right]\right)\Bigg/$$

$$\left(3e^3\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}\right)-\left(16\sqrt{2}\sqrt{b^2-4ac}\ (2cd-be)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right.$$

$$\left.\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\ e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right]\right)\Bigg/\left(3e^3\sqrt{d+ex}\sqrt{a+bx+cx^2}\right)$$

Result (type 4, 3463 leaves) :

$$\sqrt{d+ex}\sqrt{a+x(b+cx)}\left(-\frac{2(-2cd+be)}{3e^2(d+ex)^2}-\frac{2\left(10c^2d^2-10bcde+b^2e^2+6ace^2\right)}{3e^2\left(cd^2-bde+ae^2\right)(d+ex)}\right)-\frac{1}{3e^4\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}$$

$$2\,c\,\sqrt{a+x\,(b+c\,x)}\;\left(\frac{\left(-16\,c^2\,d^2+16\,b\,c\,d\,e-b^2\,e^2-12\,a\,c\,e^2\right)\,(d+e\,x)^{3/2}\,\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)}{c\,\sqrt{\frac{(d+e\,x)^2\,\left(c\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\,\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}+\right.$$

$$\frac{1}{c\,\sqrt{\frac{(d+e\,x)^2\,\left(c\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\,\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\left(\left(4\,i\,\sqrt{2}\,c^2\,d^2\,\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Bigg/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(4\,i\,\sqrt{2}\,b\,c\,d\,e\,\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i b^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(3 i\sqrt{2}\, a c e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\text{i ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\text{i ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(8\,\text{i}\sqrt{2}\,c^2 d\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[\text{i ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(4\,\text{i}\sqrt{2}\,b c e\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[\text{i ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left.\left.\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)\right)$$

- **Problem 1632: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{(b+2\,c\,x)\,\sqrt{a+b\,x+c\,x^2}}{(d+e\,x)^{7/2}}\,dx$$

Optimal (type 4, 691 leaves, 7 steps) :

$$\frac{4\,(2\,c\,d-b\,e)\,\left(4\,c^2\,d^2-b^2\,e^2-4\,c\,e\,(b\,d-2\,a\,e)\right)\,\sqrt{a+b\,x+c\,x^2}}{15\,e^2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,\sqrt{d+e\,x}}-$$

$$\left(2\,\left(8\,c^2\,d^3-c\,d\,e\,(5\,b\,d-4\,a\,e)-b\,e^2\,(2\,b\,d-3\,a\,e)+e\,\left(14\,c^2\,d^2+b^2\,e^2-2\,c\,e\,(7\,b\,d-5\,a\,e)\right)\,x\right)\,\sqrt{a+b\,x+c\,x^2}\right)\Big/$$

$$\left(15\,e^2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,(d+e\,x)^{5/2}\right)-\Bigg(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,(2\,c\,d-b\,e)\,\left(4\,c^2\,d^2-b^2\,e^2-4\,c\,e\,(b\,d-2\,a\,e)\right)$$

$$\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\Bigg)\Bigg/$$

$$\left(15\,e^3\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\Bigg(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(16\,c^2\,d^2-b^2\,e^2-4\,c\,e\,(4\,b\,d-5\,a\,e)\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\Bigg)\Bigg/\left(15\,e^3\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 5427 leaves) :

$$\sqrt{d+e x}\,\sqrt{a+x\,(b+c x)}\left(-\frac{2\,(-2 c d+b e)}{5 e^2\,(d+e x)^3}-\frac{2\left(14 c^2 d^2-14 b c d e+b^2 e^2+10 a c e^2\right)}{15 e^2\left(c d^2-b d e+a e^2\right)(d+e x)^2}-\frac{4\,(-2 c d+b e)\left(4 c^2 d^2-4 b c d e-b^2 e^2+8 a c e^2\right)}{15 e^2\left(c d^2-b d e+a e^2\right)^2(d+e x)}\right)+$$

$$\frac{1}{15 e^4\left(c d^2-b d e+a e^2\right)^2\sqrt{a+b x+c x^2}}$$

$$2 c\sqrt{a+x\,(b+c x)}\left(-\left(2\,(2 c d-b e)\left(4 c^2 d^2-4 b c d e-b^2 e^2+8 a c e^2\right)(d+e x)^{3/2}\left(c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)\right)\Big/\right.$$

$$\left(c\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)+$$

$$\frac{1}{c\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}\left(c d^2-b d e+a e^2\right)(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\left(\left(4\,i\sqrt{2}\,c^3 d^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(6 i \sqrt{2}\, b c^2 d^2 e \left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(i \sqrt{2}\, b^2 c d e^2 \left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(8\,i\sqrt{2}\,a c^2 d e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(i\,b^3 e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(4\,\mathbb{i}\,\sqrt{2}\,a\,b\,c\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}$$

$$\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\ \Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(8\,\mathbb{i}\,\sqrt{2}\,c^3\,d^2\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(8\,\mathrm{i}\,\sqrt{2}\,b\,c^2\,d\,e\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\;\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(\mathrm{i}\,b^2\,c\,e^2\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\;\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(10\,\mathrm{i}\,\sqrt{2}\,a\,c^2\,e^2\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\;\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right) \Bigg/$$

$$\left(\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)$$

- **Problem 1633: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(b+2\,c\,x)\,\left(a+b\,x+c\,x^2\right)^{3/2}}{\sqrt{d+e\,x}}\,dx$$

Optimal (type 4, 688 leaves, 7 steps) :

$$-\frac{1}{315\,c\,e^4}\,2\sqrt{d+e\,x}\,\left(128\,c^3\,d^3-b^3\,e^3+3\,b\,c\,e^2\,(37\,b\,d-36\,a\,e)-12\,c^2\,d\,e\,(20\,b\,d-11\,a\,e)-3\,c\,e\,\left(32\,c^2\,d^2+b^2\,e^2-4\,c\,e\,(8\,b\,d-7\,a\,e)\right)\,x\right)\sqrt{a+b\,x+c\,x^2}\,-$$

$$\frac{2\sqrt{d+e\,x}\,(16\,c\,d-15\,b\,e-14\,c\,e\,x)\,\left(a+b\,x+c\,x^2\right)^{3/2}}{63\,e^2}+$$

$$\left(2\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(128\,c^4\,d^4-b^4\,e^4-4\,c^3\,d^2\,e\,(64\,b\,d-57\,a\,e)-b^2\,c\,e^3\,(7\,b\,d-15\,a\,e)+3\,c^2\,e^2\,\left(45\,b^2\,d^2-76\,a\,b\,d\,e+28\,a^2\,e^2\right)\right)\right.$$

$$\left.\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/$$

$$\left(315\,c^2\,e^5\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)-$$

$$\left(2\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,(2\,c\,d-b\,e)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\left(128\,c^2\,d^2-b^2\,e^2-4\,c\,e\,(32\,b\,d-33\,a\,e)\right)\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\right.$$

$$\left.\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/\left(315\,c^2\,e^5\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 7917 leaves) :

$$\frac{1}{a+b\,x+c\,x^2}\sqrt{d+e\,x}\left(-\frac{2\left(128\,c^3\,d^3-240\,b\,c^2\,d^2\,e+111\,b^2\,c\,d\,e^2+212\,a\,c^2\,d\,e^2-b^3\,e^3-183\,a\,b\,c\,e^3\right)}{315\,c\,e^4}+\right.$$

$$\left.\frac{4\left(48\,c^2\,d^2-88\,b\,c\,d\,e+39\,b^2\,e^2+77\,a\,c\,e^2\right)x}{315\,e^3}-\frac{2\,c\,(16\,c\,d-29\,b\,e)\,x^2}{63\,e^2}+\frac{4\,c^2\,x^3}{9\,e}\right)(a+x\,(b+c\,x))^{3/2}-\frac{1}{315\,c\,e^6\left(a+b\,x+c\,x^2\right)^{3/2}}$$

$$2\,(a+x\,(b+c\,x))^{3/2}\left(-\left(2\left(128\,c^4\,d^4-256\,b\,c^3\,d^3\,e+135\,b^2\,c^2\,d^2\,e^2+228\,a\,c^3\,d^2\,e^2-7\,b^3\,c\,d\,e^3-228\,a\,b\,c^2\,d\,e^3-b^4\,e^4+15\,a\,b^2\,c\,e^4+84\,a^2\,c^2\,e^4\right)\right.\right.$$

$$(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)\Bigg)\Bigg/\left(c\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)+$$

$$\frac{1}{c\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\left(\left(64\,\mathbb{i}\,\sqrt{2}\,c^4\,d^4\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(128\,\mathbb{i}\,\sqrt{2}\,b\,c^3\,d^3\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(135\,\mathbb{i}\,b^2\,c^2\,d^2\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(114\,i\sqrt{2}\,a c^3 d^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(7\,i\,b^3 c d e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(114\,\mathrm{i}\,\sqrt{2}\,a b c^2 d e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\Bigg(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(\mathrm{i}\,b^4 e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(15\,i\,a b^2 c e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(42\,i\sqrt{2}\,a^2 c^2 e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(128\,\mathbb{i}\,\sqrt{2}\,c^4 d^3\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(192\,\mathbb{i}\,\sqrt{2}\,b c^3 d^2 e\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(63\,i\,\sqrt{2}\,b^2 c^2 d e^2 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(132\,i\,\sqrt{2}\,a c^3 d e^2 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i\,b^3 c e^3 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(66\,\mathbb{i}\,\sqrt{2}\,a\,b\,c^2\,e^3\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left.\left.\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)\right)$$

- **Problem 1634: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{(b+2\,c\,x)\,\left(a+b\,x+c\,x^2\right)^{3/2}}{(d+e\,x)^{3/2}}\,\mathbb{d}x$$

Optimal (type 4, 592 leaves, 7 steps) :

$$\frac{2\sqrt{d+e x}\,\left(128 c^2 d^2+51 b^2 e^2-4 c e\,(44 b d-5 a e)-48 c e\,(2 c d-b e)\,x\right)\sqrt{a+b x+c x^2}}{35 e^4}+\frac{2\,(16 c d-7 b e+2 c e x)\,\left(a+b x+c x^2\right)^{3/2}}{7 e^2\sqrt{d+e x}}-$$

$$\left(\sqrt{2}\,\sqrt{b^2-4 a c}\,(2 c d-b e)\,\left(128 c^2 d^2+3 b^2 e^2-4 c e\,(32 b d-29 a e)\right)\sqrt{d+e x}\,\sqrt{-\frac{c\,\left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\;e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}\right]\right)\Bigg/\left(35 c e^5\sqrt{\frac{c\,(d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}}\;\sqrt{a+b x+c x^2}\right)+$$

$$\left(4\sqrt{2}\,\sqrt{b^2-4 a c}\,\left(c d^2-b d e+a e^2\right)\left(128 c^2 d^2+27 b^2 e^2-4 c e\,(32 b d-5 a e)\right)\sqrt{\frac{c\,(d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}}\;\sqrt{-\frac{c\,\left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\;e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}\right]\right)\Bigg/\left(35 c e^5\sqrt{d+e x}\;\sqrt{a+b x+c x^2}\right)$$

Result (type 4, 5373 leaves) :

$$\frac{1}{a+b x+c x^2}$$

$$\sqrt{d+e x}\;(a+x\,(b+c x))^{3/2}\left(\frac{2\left(58 c^2 d^2-71 b c d e+16 b^2 e^2+30 a c e^2\right)}{35 e^4}-\frac{2 c\,(26 c d-23 b e)\,x}{35 e^3}+\frac{4 c^2 x^2}{7 e^2}-\frac{2\,(-2 c d+b e)\left(c d^2-b d e+a e^2\right)}{e^4\,(d+e x)}\right)-$$

$$\frac{1}{35 e^6\left(a+b x+c x^2\right)^{3/2}}\,2\,(a+x\,(b+c x))^{3/2}$$

$$\left(\left((2 c d-b e)\left(128 c^2 d^2-128 b c d e+3 b^2 e^2+116 a c e^2\right)(d+e x)^{3/2}\left(c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)\right)\Big/\right.$$

$$\left(c\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)-$$

$$\frac{1}{c\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}\left(c d^2-b d e+a e^2\right)(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\left(\left(64\, i \sqrt{2}\, c^3 d^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\right/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(96\,\mathrm{i}\,\sqrt{2}\,b\,c^2\,d^2\,e\,\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(67\,\mathrm{i}\,b^2\,c\,d\,e^2\,\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(58\, i \sqrt{2}\, a c^2 d e^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Bigg/$$

$$\left((c d^2 - b d e + a e^2) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\left(3\, i\, b^3 e^3 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Bigg/$$

$$\left(2 \sqrt{2}\, (c d^2 - b d e + a e^2) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\left(29\,\mathrm{i}\,\sqrt{2}\,a\,b\,c\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(128\,\mathrm{i}\,\sqrt{2}\,c^3\,d^2\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(128\,\mathrm{i}\,\sqrt{2}\,b\,c^2\,d\,e\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(27\,i\sqrt{2}\,b^2 c e^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(20\,i\sqrt{2}\,a c^2 e^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\Bigg)\Bigg)$$

- **Problem 1635: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(b+2\,c\,x)\,\left(a+b\,x+c\,x^2\right)^{3/2}}{(d+e\,x)^{5/2}}\,dx$$

Optimal (type 4, 573 leaves, 7 steps) :

$$-\frac{2\left(128 c^2 d^2+15 b^2 e^2-4 c e (28 b d-9 a e)+16 c e (2 c d-b e) x\right)\sqrt{a+b x+c x^2}}{15 e^4\sqrt{d+e x}}+$$

$$\frac{2 (16 c d-5 b e+6 c e x)\left(a+b x+c x^2\right)^{3/2}}{15 e^2 (d+e x)^{3/2}}+\left(2\sqrt{2}\sqrt{b^2-4 a c}\left(128 c^2 d^2+23 b^2 e^2-4 c e (32 b d-9 a e)\right)\sqrt{d+e x}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Bigg/\left(15 e^5\sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\sqrt{a+b x+c x^2}\right)-$$

$$\left(2\sqrt{2}\sqrt{b^2-4 a c}\,(2 c d-b e)\left(128 c^2 d^2+15 b^2 e^2-4 c e (32 b d-17 a e)\right)\sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Bigg/\left(15 c e^5\sqrt{d+e x}\sqrt{a+b x+c x^2}\right)$$

Result (type 4, 8929 leaves) :

$$\frac{1}{a+b x+c x^2}\sqrt{d+e x}\,(a+x (b+c x))^{3/2}$$

$$\left(-\frac{2 c (28 c d-17 b e)}{15 e^4}+\frac{4 c^2 x}{5 e^3}-\frac{2 (-2 c d+b e)\left(c d^2-b d e+a e^2\right)}{3 e^4 (d+e x)^2}-\frac{4\left(11 c^2 d^2-11 b c d e+2 b^2 e^2+3 a c e^2\right)}{3 e^4 (d+e x)}\right)-\frac{1}{15 e^6\left(a+b x+c x^2\right)^{3/2}}$$

$$2\,(a+x\,(b+c\,x))^{3/2}\left(-\left(2\,\left(128\,c^2\,d^2-128\,b\,c\,d\,e+23\,b^2\,e^2+36\,a\,c\,e^2\right)\,(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)\right)\Bigg/\right.$$

$$\left(\sqrt{\frac{(d+e\,x)^2\left(c\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)+\left(64\,\mathrm{i}\,\sqrt{2}\,c^3\,d^4\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)\right.$$

$$\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}\;\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\left(\mathrm{EllipticE}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Bigg/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\;\sqrt{\frac{(d+e\,x)^2\left(c\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)-$$

$$\left(128\,\mathrm{i}\,\sqrt{2}\,b\,c^2\,d^3\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\ \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)+$$

$$\left(151\, i\, b^2 c d^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/\Bigg(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)$$

$$\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\Bigg)+$$

$$\Bigg(82\,\mathbb{i}\,\sqrt{2}\,a\,c^2\,d^2\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\Bigg(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\Bigg)-$$

$$\Bigg(23\,i\,b^3 d e^3 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right)(d + e x)\sqrt{c + \frac{c d^2}{(d + e x)^2} - \frac{b d e}{(d + e x)^2} + \frac{a e^2}{(d + e x)^2} - \frac{2 c d}{d + e x} + \frac{b e}{d + e x}}$$

$$\sqrt{1 - \frac{2\left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right)(d + e x)}}\;\sqrt{1 - \frac{2\left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right)(d + e x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] -$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\Bigg)\Bigg) \Big/ \Bigg(\sqrt{2}\left(c d^2 - b d e + a e^2\right)$$

$$\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}\;\sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\;\sqrt{\frac{(d + e x)^2\left(c\left(-1 + \frac{d}{d + e x}\right)^2 + \frac{e\left(b - \frac{b d}{d + e x} + \frac{a e}{d + e x}\right)}{d + e x}\right)}{e^2}}\Bigg) -$$

$$\Bigg(82\,i\sqrt{2}\,a b c d e^3\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right)(d + e x)\sqrt{c + \frac{c d^2}{(d + e x)^2} - \frac{b d e}{(d + e x)^2} + \frac{a e^2}{(d + e x)^2} - \frac{2 c d}{d + e x} + \frac{b e}{d + e x}}$$

$$\sqrt{1 - \frac{2\left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right)(d + e x)}}\;\sqrt{1 - \frac{2\left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right)(d + e x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)+$$

$$\left(23\,i\,a b^2 e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\right.$$

$$\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\ \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\Bigg)+$$

$$\Bigg(18\, i \sqrt{2}\, a^2 c e^4 \left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\Big[i\, \text{ArcSinh}\Big[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\Big], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\Big]-$$

$$\text{EllipticF}\Big[i\, \text{ArcSinh}\Big[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\Big], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\Big]\Bigg)\Bigg)\Big/$$

$$\Bigg(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\ \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\Bigg)+$$

$$\Bigg(128\, i \sqrt{2}\, c^3 d^3 (d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\ \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)-$$

$$\Bigg(192\,i\sqrt{2}\,b c^2 d^2 e\,(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\ \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)+$$

$$\Bigg(79\,i\sqrt{2}\,b^2 c d e^2\,(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\ \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)+$$

$$\left(68\, i \sqrt{2}\, a c^2 d e^2 (d+e x) \sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \text{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\ \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)-$$

$$\left(15\, i\, b^3 e^3 (d+e x) \sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \text{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\ \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)-$$

$$\left(34\,\mathrm{i}\,\sqrt{2}\,a\,b\,c\,e^3\,(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\;\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)$$

- **Problem 1636: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{(b+2\,c\,x)\,\left(a+b\,x+c\,x^2\right)^{3/2}}{(d+e\,x)^{7/2}}\,dx$$

Optimal (type 4, 701 leaves, 7 steps) :

$$\left(2 c \left(128 c^2 d^3-4 c d e (44 b d-29 a e)+3 b e^2 (17 b d-16 a e)+e \left(32 c^2 d^2+3 b^2 e^2-4 c e (8 b d-5 a e)\right) x\right) \sqrt{a+b x+c x^2}\right) \Big/$$

$$\left(15 e^4 \left(c d^2-b d e+a e^2\right) \sqrt{d+e x}\right)-\left(2 \left(16 c^2 d^3+3 a b e^3-c d e (13 b d-4 a e)+e \left(22 c^2 d^2+3 b^2 e^2-2 c e (11 b d-5 a e)\right) x\right) \left(a+b x+c x^2\right)^{3/2}\right) \Big/$$

$$\left(15 e^2 \left(c d^2-b d e+a e^2\right) (d+e x)^{5/2}\right)-\left(\sqrt{2} \sqrt{b^2-4 a c} (2 c d-b e) \left(128 c^2 d^2+3 b^2 e^2-4 c e (32 b d-29 a e)\right)\right.$$

$$\left.\sqrt{d+e x} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c} e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right) \Big/$$

$$\left(15 e^5 \left(c d^2-b d e+a e^2\right) \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{a+b x+c x^2}\right)+$$

$$\left(4 \sqrt{2} \sqrt{b^2-4 a c} \left(128 c^2 d^2+27 b^2 e^2-4 c e (32 b d-5 a e)\right) \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c} e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right) \Big/ \left(15 e^5 \sqrt{d+e x} \sqrt{a+b x+c x^2}\right)$$

Result (type 4, 5450 leaves) :

$$\frac{1}{a+b x+c x^2}\sqrt{d+e x} (a+x (b+c x))^{3/2}$$

$$\left(\frac{4 c^2}{3 e^4}-\frac{2 (-2 c d+b e) \left(c d^2-b d e+a e^2\right)}{5 e^4 (d+e x)^3}-\frac{4 \left(17 c^2 d^2-17 b c d e+3 b^2 e^2+5 a c e^2\right)}{15 e^4 (d+e x)^2}-\frac{2 (-2 c d+b e) \left(73 c^2 d^2-73 b c d e+3 b^2 e^2+61 a c e^2\right)}{15 e^4 \left(c d^2-b d e+a e^2\right) (d+e x)}\right)-$$

$$\frac{1}{15 e^6 \left(c d^2-b d e+a e^2\right) \left(a+b x+c x^2\right)^{3/2}} 2 c (a+x (b+c x))^{3/2}$$

$$\left(\left((2 c d-b e) \left(128 c^2 d^2-128 b c d e+3 b^2 e^2+116 a c e^2\right) (d+e x)^{3/2} \left(c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)\right)\Big/\right.$$

$$\left(c \sqrt{\frac{(d+e x)^2 \left(c \left(-1+\frac{d}{d+e x}\right)^2+\frac{e \left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)-$$

$$\frac{1}{c \sqrt{\frac{(d+e x)^2 \left(c \left(-1+\frac{d}{d+e x}\right)^2+\frac{e \left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}} \left(c d^2-b d e+a e^2\right) (d+e x) \sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\left(\left(64 i \sqrt{2} c^3 d^3 \left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}\right.\right.$$

$$\sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(96\,i\sqrt{2}\,b c^2 d^2 e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(67\,i\,b^2 c d e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(58\,i\,\sqrt{2}\,a c^2 d e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\;\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(3\,i\,b^3 e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(29\,i\sqrt{2}\,a b c e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(128\,i\sqrt{2}\,c^3 d^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(128 i \sqrt{2} b c^2 d e \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(27 i \sqrt{2} b^2 c e^2 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(20 i \sqrt{2} a c^2 e^2 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)\right)$$

- **Problem 1637: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(b+2 c x)\,(d+e x)^{5/2}}{\sqrt{a+b x+c x^2}}\,dx$$

Optimal (type 4, 600 leaves, 8 steps) :

$$\frac{4\left(3c^2d^2+2b^2e^2-ce\,(3bd+5ae)\right)\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}{21c^2}+\frac{2\,(2cd-be)\,(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{7c}+$$

$$\frac{4}{7}(d+ex)^{5/2}\sqrt{a+bx+cx^2}+\left(\sqrt{2}\,\sqrt{b^2-4ac}\,(2cd-be)\left(3c^2d^2+8b^2e^2-ce\,(3bd+29ae)\right)\sqrt{d+ex}\,\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right]\right)\Bigg/\left(21c^3e\sqrt{\frac{c\,(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\,\sqrt{a+bx+cx^2}\right)-$$

$$\left(4\sqrt{2}\,\sqrt{b^2-4ac}\,\left(cd^2-bde+ae^2\right)\left(3c^2d^2+2b^2e^2-ce\,(3bd+5ae)\right)\sqrt{\frac{c\,(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\,\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right]\right)\Bigg/\left(21c^3e\sqrt{d+ex}\,\sqrt{a+bx+cx^2}\right)$$

Result (type 4, 5339 leaves) :

$$\frac{\sqrt{d+ex}\,\left(a+bx+cx^2\right)\left(\frac{2\left(18c^2d^2-9bcde+4b^2e^2-10ace^2\right)}{21c^2}+\frac{2e\,(6cd-be)\,x}{7c}+\frac{4e^2x^2}{7}\right)}{\sqrt{a+x\,(b+cx)}}-\frac{1}{21c^2e^2\sqrt{a+x\,(b+cx)}}$$

$$2\sqrt{a+b x+c x^2}\left(-\left((2 c d-b e)\left(3 c^2 d^2-3 b c d e+8 b^2 e^2-29 a c e^2\right)(d+e x)^{3/2}\left(c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)\right)\Big/\right.$$

$$\left(c\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)+$$

$$\frac{1}{c\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}\left(c d^2-b d e+a e^2\right)(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\left(\left(3\, i\, c^3 d^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/\right.$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(9\,i\,b\,c^2\,d^2\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(19\,i\,b^2\,c\,d\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(29\,i\,a\,c^2\,d\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(2\,i\sqrt{2}\,b^3\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(29\, i\, a\, b\, c\, e^3 \left(2\, c\, d - b\, e + \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}\right) \sqrt{1 - \frac{2\left(c\, d^2 - b\, d\, e + a\, e^2\right)}{\left(2\, c\, d - b\, e - \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}\right) (d + e\, x)}}\right.$$

$$\sqrt{1 - \frac{2\left(c\, d^2 - b\, d\, e + a\, e^2\right)}{\left(2\, c\, d - b\, e + \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}\right) (d + e\, x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\, \sqrt{-\frac{c\, d^2 - b\, d\, e + a\, e^2}{2\, c\, d - b\, e - \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}}}{\sqrt{d + e\, x}}\right], \frac{2\, c\, d - b\, e - \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}{2\, c\, d - b\, e + \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\, \sqrt{-\frac{c\, d^2 - b\, d\, e + a\, e^2}{2\, c\, d - b\, e - \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}}}{\sqrt{d + e\, x}}\right], \frac{2\, c\, d - b\, e - \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}{2\, c\, d - b\, e + \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}\right]\right)\right) \Big/$$

$$\left(2\sqrt{2}\, \left(c\, d^2 - b\, d\, e + a\, e^2\right) \sqrt{-\frac{c\, d^2 - b\, d\, e + a\, e^2}{2\, c\, d - b\, e - \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}}\, \sqrt{c + \frac{c\, d^2 - b\, d\, e + a\, e^2}{(d + e\, x)^2} + \frac{-2\, c\, d + b\, e}{d + e\, x}}\right) +$$

$$\left(3\, i \sqrt{2}\, c^3\, d^2 \sqrt{1 - \frac{2\left(c\, d^2 - b\, d\, e + a\, e^2\right)}{\left(2\, c\, d - b\, e - \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}\right) (d + e\, x)}} \sqrt{1 - \frac{2\left(c\, d^2 - b\, d\, e + a\, e^2\right)}{\left(2\, c\, d - b\, e + \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}\right) (d + e\, x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\, \sqrt{-\frac{c\, d^2 - b\, d\, e + a\, e^2}{2\, c\, d - b\, e - \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}}}{\sqrt{d + e\, x}}\right], \frac{2\, c\, d - b\, e - \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}{2\, c\, d - b\, e + \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c\, d^2 - b\, d\, e + a\, e^2}{2\, c\, d - b\, e - \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}}\, \sqrt{c + \frac{c\, d^2 - b\, d\, e + a\, e^2}{(d + e\, x)^2} + \frac{-2\, c\, d + b\, e}{d + e\, x}}\right) -$$

$$\left(3\, i \sqrt{2}\, b\, c^2\, d\, e \sqrt{1 - \frac{2\left(c\, d^2 - b\, d\, e + a\, e^2\right)}{\left(2\, c\, d - b\, e - \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}\right) (d + e\, x)}} \sqrt{1 - \frac{2\left(c\, d^2 - b\, d\, e + a\, e^2\right)}{\left(2\, c\, d - b\, e + \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}\right) (d + e\, x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(2\,i\sqrt{2}\,b^2 c e^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(5\,i\sqrt{2}\,a c^2 e^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Big/$$

$$\left.\left.\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)\right)$$

- **Problem 1638: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{(b+2\,c\,x)\,(d+e\,x)^{3/2}}{\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 507 leaves, 7 steps) :

$$\frac{2\,(2\,c\,d-b\,e)\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}}{5\,c}+\frac{4}{5}\,(d+e\,x)^{3/2}\,\sqrt{a+b\,x+c\,x^2}+$$

$$\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(c^2\,d^2+b^2\,e^2-c\,e\,(b\,d+3\,a\,e)\right)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/\left(5\,c^2\,e\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)-$$

$$\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,(2\,c\,d-b\,e)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/\left(5\,c^2\,e\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 3394 leaves) :

$$\frac{\left(\frac{2\,(4\,c\,d-b\,e)}{5\,c}+\frac{4\,e\,x}{5}\right)\,\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)}{\sqrt{a+x\,(b+c\,x)}}+$$

$$\frac{1}{5\,c\,e^2\,\sqrt{a+x\,(b+c\,x)}}\,2\,\sqrt{a+b\,x+c\,x^2}\,\left(\frac{2\,(c^2\,d^2-b\,c\,d\,e+b^2\,e^2-3\,a\,c\,e^2)\,(d+e\,x)^{3/2}\,\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)}{c\,\sqrt{\frac{(d+e\,x)^2\,\left(c\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\,\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}-\right.$$

$$\frac{1}{c\,\sqrt{\frac{(d+e\,x)^2\,\left(c\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\,\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\,(c\,d^2-b\,d\,e+a\,e^2)\,(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\left(\left(\mathbb{i}\,c^2\,d^2\,\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\right/$$

$$\left(\sqrt{2}\,(c\,d^2-b\,d\,e+a\,e^2)\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(\mathbb{i}\,b\,c\,d\,e\,\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(\mathbb{i}\,b^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(3\,\mathbb{i}\,a c e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(i\,\sqrt{2}\,c^2\,d\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(i\,b\,c\,e\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left.\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)$$

Problem 1639: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(b+2\,c\,x)\,\sqrt{d+e\,x}}{\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 441 leaves, 6 steps):

$$\frac{4}{3}\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}+$$

$$\left(\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,(2\,c\,d-b\,e)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/$$

$$\left(3\,c\,e\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)-\left(4\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\right.$$

$$\left.\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/\left(3\,c\,e\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 581 leaves):

$$\frac{1}{6\sqrt{a+x\,(b+c\,x)}}\left(\frac{4\,(2\,c\,d-b\,e)\,(a+x\,(b+c\,x))}{c\sqrt{d+e\,x}}+8\sqrt{d+e\,x}\,(a+x\,(b+c\,x))-\right.$$

$$\frac{1}{c\,e^2\sqrt{\frac{c\,d^2+e\,(-b\,d+a\,e)}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}}}\,\mathbb{i}\,(d+e\,x)\sqrt{1-\frac{2\left(c\,d^2+e\,(-b\,d+a\,e)\right)}{\left(2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}\right)\,(d+e\,x)}}\ \sqrt{2+\frac{4\left(c\,d^2+e\,(-b\,d+a\,e)\right)}{\left(-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}\right)\,(d+e\,x)}}$$

$$\left(-(-2\,c\,d+b\,e)\left(2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}\right)\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c\,d^2-b\,d\,e+a\,e^2}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}}}{\sqrt{d+e\,x}}\right],-\frac{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}{2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}\right]+\right.$$

$$\left(-b^2\,e^2+4\,a\,c\,e^2-2\,c\,d\sqrt{\left(b^2-4\,a\,c\right)\,e^2}+b\,e\sqrt{\left(b^2-4\,a\,c\right)\,e^2}\right)$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c\,d^2-b\,d\,e+a\,e^2}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}}}{\sqrt{d+e\,x}}\right],-\frac{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}{2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}\right]\right)\right)$$

- **Problem 1640: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{b+2\,c\,x}{\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 391 leaves, 5 steps) :

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right]}{e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}-$$

$$\left(2\sqrt{2}\sqrt{b^2-4ac}\,(2cd-be)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right]\right)\Big/\left(ce\sqrt{d+ex}\sqrt{a+bx+cx^2}\right)$$

Result (type 4, 793 leaves) :

$$\frac{1}{e^2\sqrt{\frac{c\,d^2+e\,(-b\,d+a\,e)}{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}}\,\sqrt{a+x\,(b+c\,x)}}\,(d+e\,x)^{3/2}\left(\frac{4\,e^2\sqrt{\frac{c\,d^2+e\,(-b\,d+a\,e)}{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}}\,(a+x\,(b+c\,x))}{(d+e\,x)^2}-\frac{1}{\sqrt{d+e\,x}}\,i\,\sqrt{2}\,\left(2\,c\,d-b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}\right)\right.$$

$$\sqrt{\frac{-2\,a\,e^2+2\,c\,d\,e\,x+b\,e\,(d-e\,x)+\sqrt{(b^2-4\,a\,c)\,e^2}\,(d+e\,x)}{\left(2\,c\,d-b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}\right)(d+e\,x)}}\,\sqrt{\frac{2\,a\,e^2-2\,c\,d\,e\,x+b\,e\,(-d+e\,x)+\sqrt{(b^2-4\,a\,c)\,e^2}\,(d+e\,x)}{\left(-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}\right)(d+e\,x)}}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{\frac{c\,d^2-b\,d\,e+a\,e^2}{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}}}{\sqrt{d+e\,x}}\right],-\frac{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}{2\,c\,d-b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}\right]+\frac{1}{\sqrt{d+e\,x}}\,i\,\sqrt{2}\,\sqrt{(b^2-4\,a\,c)\,e^2}$$

$$\sqrt{\frac{-2\,a\,e^2+2\,c\,d\,e\,x+b\,e\,(d-e\,x)+\sqrt{(b^2-4\,a\,c)\,e^2}\,(d+e\,x)}{\left(2\,c\,d-b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}\right)(d+e\,x)}}\,\sqrt{\frac{2\,a\,e^2-2\,c\,d\,e\,x+b\,e\,(-d+e\,x)+\sqrt{(b^2-4\,a\,c)\,e^2}\,(d+e\,x)}{\left(-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}\right)(d+e\,x)}}$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{\frac{c\,d^2-b\,d\,e+a\,e^2}{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}}}{\sqrt{d+e\,x}}\right],-\frac{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}{2\,c\,d-b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}\right]\right)$$

- **Problem 1641: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{b+2\,c\,x}{(d+e\,x)^{3/2}\,\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 458 leaves, 6 steps) :

$$\frac{2\,(2\,c\,d-b\,e)\,\sqrt{a+b\,x+c\,x^2}}{\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{d+e\,x}}-$$

$$\left(\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\,(2\,c\,d-b\,e)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/$$

$$\left(e\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)+\frac{1}{e\,\sqrt{d+e\,x}\,\,\sqrt{a+b\,x+c\,x^2}}$$

$$4\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]$$

Result (type 4, 541 leaves) :

$$\frac{1}{2\,e^2\left(c\,d^2+e\,(-b\,d+a\,e)\right)\sqrt{\frac{c\,d^2+e\,(-b\,d+a\,e)}{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}}\,\sqrt{a+x\,(b+c\,x)}}$$

$$i\,(d+e\,x)\sqrt{1-\frac{2\left(c\,d^2+e\,(-b\,d+a\,e)\right)}{\left(2\,c\,d-b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}\right)(d+e\,x)}}\,\sqrt{2+\frac{4\left(c\,d^2+e\,(-b\,d+a\,e)\right)}{\left(-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}\right)(d+e\,x)}}$$

$$\left(-(-2\,c\,d+b\,e)\left(2\,c\,d-b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{\frac{c\,d^2-b\,d\,e+a\,e^2}{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}}}{\sqrt{d+e\,x}}\right],-\frac{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}{2\,c\,d-b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}\right]+\right.$$

$$\left(-b^2\,e^2+4\,a\,c\,e^2-2\,c\,d\,\sqrt{(b^2-4\,a\,c)\,e^2}+b\,e\,\sqrt{(b^2-4\,a\,c)\,e^2}\right)$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{\frac{c\,d^2-b\,d\,e+a\,e^2}{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}}}{\sqrt{d+e\,x}}\right],-\frac{-2\,c\,d+b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}{2\,c\,d-b\,e+\sqrt{(b^2-4\,a\,c)\,e^2}}\right]\right)$$

- **Problem 1642: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{b+2\,c\,x}{(d+e\,x)^{5/2}\,\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 581 leaves, 7 steps) :

$$\frac{2\,(2\,c\,d-b\,e)\,\sqrt{a+b\,x+c\,x^2}}{3\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,(d+e\,x)^{3/2}}+\frac{4\,\left(c^2\,d^2+b^2\,e^2-c\,e\,(b\,d+3\,a\,e)\right)\,\sqrt{a+b\,x+c\,x^2}}{3\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,\sqrt{d+e\,x}}-$$

$$\Bigg(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(c^2\,d^2+b^2\,e^2-c\,e\,(b\,d+3\,a\,e)\right)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\Bigg)\Big/$$

$$\left(3\,e\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)+\Bigg(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\,(2\,c\,d-b\,e)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}$$

$$\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\Bigg)\Big/\left(3\,e\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 3483 leaves) :

$$\frac{\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)\,\left(\frac{2\,(-2\,c\,d+b\,e)}{3\,\left(-c\,d^2+b\,d\,e-a\,e^2\right)\,(d+e\,x)^2}+\frac{4\,\left(c^2\,d^2-b\,c\,d\,e+b^2\,e^2-3\,a\,c\,e^2\right)}{3\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,(d+e\,x)}\right)}{\sqrt{a+x\,(b+c\,x)}}-\frac{1}{3\,e^2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,\sqrt{a+x\,(b+c\,x)}}$$

$$2\,c\,\sqrt{a+b\,x+c\,x^2}\;\left(\frac{2\left(c^2 d^2-b\,c\,d\,e+b^2 e^2-3\,a\,c\,e^2\right)(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)}{c\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}-\right.$$

$$\frac{1}{c\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\left(\left(\mathbb{i}\,c^2 d^2\left(2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\;\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}}\right]\right)\right)\right/$$

$$\left(\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(\mathbb{i}\,b\,c\,d\,e\left(2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\;\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i\,b^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(3\,i\,a c e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i\sqrt{2}\,c^2 d\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(i\,b\,c\,e\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left.\left.\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)\right)$$

- **Problem 1643: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(b+2\,c\,x)\,(d+e\,x)^{7/2}}{\left(a+b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 540 leaves, 8 steps) :

$$-\frac{2\,(d+e\,x)^{7/2}}{\sqrt{a+b\,x+c\,x^2}}+\frac{56\,e^2\,(2\,c\,d-b\,e)\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}}{15\,c^2}+\frac{14\,e^2\,(d+e\,x)^{3/2}\,\sqrt{a+b\,x+c\,x^2}}{5\,c}+$$

$$\left(7\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\;e\,\left(23\,c^2\,d^2+8\,b^2\,e^2-c\,e\,(23\,b\,d+9\,a\,e)\right)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\;e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/\left(15\,c^3\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)-$$

$$\left(56\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\;e\,(2\,c\,d-b\,e)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\;e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/\left(15\,c^3\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 943 leaves) :

$$\frac{\sqrt{d+e x}\,\left(a+b x+c x^2\right)^2\left(\frac{2 e^2 (32 c d-13 b e)}{15 c^2}+\frac{4 e^3 x}{5 c}-\frac{2\left(c^2 d^3-3 a c d e^2+a b e^3+3 c^2 d^2 e x-3 b c d e^2 x+b^2 e^3 x-a c e^3 x\right)}{c^2\left(a+b x+c x^2\right)}\right)}{(a+x (b+c x))^{3/2}}+$$

$$\frac{1}{15 c^3 (a+x (b+c x))^{3/2}\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}$$

$$14 (d+e x)^{3/2}\left(a+b x+c x^2\right)^{3/2}\left(\left(23 c^2 d^2+8 b^2 e^2-c e (23 b d+9 a e)\right)\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)-\right.$$

$$\frac{1}{2\sqrt{2}\sqrt{\frac{c d^2+e(-b d+a e)}{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}}\sqrt{d+e x}}\,\mathrm{i}\sqrt{1-\frac{2\left(c d^2+e(-b d+a e)\right)}{\left(2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}\right)(d+e x)}}$$

$$\sqrt{1+\frac{2\left(c d^2+e(-b d+a e)\right)}{\left(-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}\right)(d+e x)}}\left(\left(2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}\right)\left(23 c^2 d^2+8 b^2 e^2-c e (23 b d+9 a e)\right)\right.$$

$$\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c d^2-b d e+a e^2}{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}}}{\sqrt{d+e x}}\right],-\frac{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}{2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}}\right]+\left(-30 c^3 d^3+8 b^2 e^2\left(b e-\sqrt{\left(b^2-4 a c\right) e^2}\right)-\right.$$

$$c^2 d\left(-45 b d e-34 a e^2+23 d\sqrt{\left(b^2-4 a c\right) e^2}\right)+c e\left(-31 b^2 d e-17 a b e^2+23 b d\sqrt{\left(b^2-4 a c\right) e^2}+9 a e\sqrt{\left(b^2-4 a c\right) e^2}\right)\Big)$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c d^2-b d e+a e^2}{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}}}{\sqrt{d+e x}}\right],-\frac{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}{2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}}\right]\right)\right)$$

- **Problem 1644: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(b+2 c x)(d+e x)^{5/2}}{\left(a+b x+c x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 468 leaves, 7 steps) :

$$-\frac{2\,(d+e\,x)^{5/2}}{\sqrt{a+b\,x+c\,x^2}}+\frac{10\,e^2\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}}{3\,c}+$$

$$\left(10\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\,e\,(2\,c\,d-b\,e)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}\,+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\,\right)\,e}\right]\right)\Big/$$

$$\left(3\,c^2\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\,\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)-\left(10\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\,e\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\,\right)\,e}}\right.$$

$$\left.\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}\,+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\,\right)\,e}\right]\right)\Big/\left(3\,c^2\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 780 leaves) :

$$\frac{\sqrt{d+e x}\,\left(a+b x+c x^2\right)^2\left(\frac{4 e^2}{3 c}-\frac{2\left(c d^2-a e^2+2 c d e x-b e^2 x\right)}{c\left(a+b x+c x^2\right)}\right)}{\left(a+x\,(b+c x)\right)^{3/2}}+$$

$$\frac{1}{3 c^2\,\left(a+x\,(b+c x)\right)^{3/2}\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}\;10\,(d+e x)^{3/2}\left(a+b x+c x^2\right)^{3/2}\left(2\,(2 c d-b e)\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)+\right.$$

$$\frac{1}{\sqrt{2}\sqrt{\frac{c d^2+e\,(-b d+a e)}{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}}\;\sqrt{d+e x}}\;i\;\sqrt{1-\frac{2\left(c d^2+e\,(-b d+a e)\right)}{\left(2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}\right)(d+e x)}}\;\sqrt{1+\frac{2\left(c d^2+e\,(-b d+a e)\right)}{\left(-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}\right)(d+e x)}}$$

$$\left((-2 c d+b e)\left(2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c d^2-b d e+a e^2}{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}}}{\sqrt{d+e x}}\right],-\frac{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}{2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}}\right]+\right.$$

$$\left(3 c^2 d^2+b e\left(b e-\sqrt{\left(b^2-4 a c\right) e^2}\right)+c\left(-3 b d e-a e^2+2 d\sqrt{\left(b^2-4 a c\right) e^2}\right)\right)$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c d^2-b d e+a e^2}{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}}}{\sqrt{d+e x}}\right],-\frac{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}{2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}}\right]\right)\right)$$

- **Problem 1645: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(b+2 c x)\,(d+e x)^{3/2}}{\left(a+b x+c x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 216 leaves, 3 steps):

$$-\frac{2\,(d+e x)^{3/2}}{\sqrt{a+b x+c x^2}}+\frac{3\sqrt{2}\,\sqrt{b^2-4 a c}\;e\,\sqrt{d+e x}\,\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\;e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]}{c\,\sqrt{\frac{c\,(d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\;\sqrt{a+b x+c x^2}}$$

Result (type 4, 378 leaves) :

$$\frac{1}{2\sqrt{a+x\,(b+c\,x)}}\left(-4\,(d+e\,x)^{3/2}+1\Big/\left(c\,\sqrt{\frac{c}{-2\,c\,d+\left(b+\sqrt{b^2-4\,a\,c}\right)e}}\right)3\,\mathrm{i}\,\sqrt{2}\,\left(2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)e\right)\sqrt{\frac{e\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}{-2\,c\,d+\left(b+\sqrt{b^2-4\,a\,c}\right)e}}\right.$$

$$\sqrt{1-\frac{2\,c\,(d+e\,x)}{2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)e}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{-2\,c\,d+\left(b+\sqrt{b^2-4\,a\,c}\right)e}}\,\sqrt{d+e\,x}\right],\frac{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)e}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{-2\,c\,d+\left(b+\sqrt{b^2-4\,a\,c}\right)e}}\,\sqrt{d+e\,x}\right],\frac{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)e}\right]\right)\right)$$

- **Problem 1646: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(b+2\,c\,x)\,\sqrt{d+e\,x}}{\left(a+b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 216 leaves, 3 steps) :

$$-\frac{2\,\sqrt{d+e\,x}}{\sqrt{a+b\,x+c\,x^2}}+\frac{1}{c\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}}$$

$$2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,e\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\right]$$

Result (type 4, 318 leaves) :

$$\frac{1}{\sqrt{a+x\,(b+c\,x)}}\left(-2\sqrt{d+e\,x}+1\Bigg/\left(\sqrt{\frac{c\,d^2+e\,(-b\,d+a\,e)}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}}\ \mathrm{i}\ (d+e\,x)\sqrt{2-\frac{4\left(c\,d^2+e\,(-b\,d+a\,e)\right)}{\left(2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}\right)\,(d+e\,x)}}\right.\right.$$

$$\left.\sqrt{1+\frac{2\left(c\,d^2+e\,(-b\,d+a\,e)\right)}{\left(-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}\right)\,(d+e\,x)}}\ \text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c\,d^2-b\,d\,e+a\,e^2}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}}}{\sqrt{d+e\,x}}\right],-\frac{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}{2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)\,e^2}}\right]\right)$$

- **Problem 1647: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{b+2\,c\,x}{\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 290 leaves, 4 steps):

$$-\frac{2\sqrt{d+e\,x}\,\left(\left(b^2-4\,a\,c\right)\,(c\,d-b\,e)-c\,\left(b^2-4\,a\,c\right)\,e\,x\right)}{\left(b^2-4\,a\,c\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{a+b\,x+c\,x^2}}-$$

$$\frac{\sqrt{2}\sqrt{b^2-4\,a\,c}\ e\sqrt{d+e\,x}\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4\,a\,c}\ e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]}{\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\ \sqrt{a+b\,x+c\,x^2}}$$

Result (type 4, 405 leaves):

$$\left(4\sqrt{d+e\,x}\,(-c\,d+b\,e+c\,e\,x)-\right.$$

$$1\Big/\left(\sqrt{\frac{c}{-2\,c\,d+\left(b+\sqrt{b^2-4\,a\,c}\right)e}}\;\mathrm{i}\sqrt{2}\,\left(2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)e\right)\sqrt{\frac{e\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}{-2\,c\,d+\left(b+\sqrt{b^2-4\,a\,c}\right)e}}\;\sqrt{1-\frac{2\,c\,(d+e\,x)}{2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)e}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{-2\,c\,d+\left(b+\sqrt{b^2-4\,a\,c}\right)e}}\,\sqrt{d+e\,x}\right],\frac{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)e}\right]-\text{EllipticF}\left[\right.\right.$$

$$\left.\left.\left.\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{-2\,c\,d+\left(b+\sqrt{b^2-4\,a\,c}\right)e}}\,\sqrt{d+e\,x}\right],\frac{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}{2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)e}\right]\right)\right)\Big/\left(2\left(c\,d^2+e\,(-b\,d+a\,e)\right)\sqrt{a+x\,(b+c\,x)}\right)$$

- **Problem 1648: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{b+2\,c\,x}{(d+e\,x)^{3/2}\left(a+b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 559 leaves, 7 steps) :

$$-\frac{2\left(\left(b^2-4ac\right)(cd-be)-c\left(b^2-4ac\right)ex\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}}+\frac{4e^2(2cd-be)\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}}-$$

$$\left(2\sqrt{2}\sqrt{b^2-4ac}\,e\,(2cd-be)\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right]\right)\Bigg/$$

$$\left(\left(cd^2-bde+ae^2\right)^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}\right)+$$

$$\left(2\sqrt{2}\sqrt{b^2-4ac}\,e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right]\right)\Bigg/$$

$$\left(\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}\right)$$

Result (type 4, 859 leaves) :

$$\frac{\sqrt{d+e x}\,\left(a+b x+c x^2\right)^2\left(\frac{2\left(2 c d e^2-b e^3\right)}{\left(c d^2-b d e+a e^2\right)^2 (d+e x)}+\frac{2\left(-c^2 d^2+2 b c d e-b^2 e^2+a c e^2+2 c^2 d e x-b c e^2 x\right)}{\left(c d^2-b d e+a e^2\right)^2 \left(a+b x+c x^2\right)}\right)}{(a+x (b+c x))^{3/2}}-$$

$$\left(2 (d+e x)^{3/2}\left(a+b x+c x^2\right)^{3/2}\left(2 (2 c d-b e)\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)+\right.\right.$$

$$\frac{1}{\sqrt{2}\sqrt{\frac{c d^2+e (-b d+a e)}{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}}\sqrt{d+e x}}\,\mathrm{i}\sqrt{1-\frac{2\left(c d^2+e (-b d+a e)\right)}{\left(2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}\right)(d+e x)}}\sqrt{1+\frac{2\left(c d^2+e (-b d+a e)\right)}{\left(-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}\right)(d+e x)}}$$

$$\left((-2 c d+b e)\left(2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}\right)\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c d^2-b d e+a e^2}{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}}}{\sqrt{d+e x}}\right],-\frac{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}{2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}}\right]+\right.$$

$$\left(3 c^2 d^2+b e\left(b e-\sqrt{\left(b^2-4 a c\right) e^2}\right)+c\left(-3 b d e-a e^2+2 d\sqrt{\left(b^2-4 a c\right) e^2}\right)\right)$$

$$\left.\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c d^2-b d e+a e^2}{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}}}{\sqrt{d+e x}}\right],-\frac{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}{2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}}\right]\right)\right)\right)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)^2 (a+x (b+c x))^{3/2}\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)$$

- **Problem 1649: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{(b+2 c x)\,(d+e x)^{7/2}}{\left(a+b x+c x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 573 leaves, 8 steps) :

$$-\frac{2\,(d+e\,x)^{7/2}}{3\,\left(a+b\,x+c\,x^2\right)^{3/2}}-\frac{14\,e\,(d+e\,x)^{3/2}\,(b\,d-2\,a\,e+(2\,c\,d-b\,e)\,x)}{3\,\left(b^2-4\,a\,c\right)\,\sqrt{a+b\,x+c\,x^2}}+\frac{14\,e^2\,(2\,c\,d-b\,e)\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}}{3\,c\,\left(b^2-4\,a\,c\right)}+$$

$$\left(14\,\sqrt{2}\,e\,\left(c^2\,d^2+b^2\,e^2-c\,e\,(b\,d+3\,a\,e)\right)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/$$

$$\left(3\,c^2\,\sqrt{b^2-4\,a\,c}\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)-$$

$$\left(14\,\sqrt{2}\,e\,(2\,c\,d-b\,e)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/\left(3\,c^2\,\sqrt{b^2-4\,a\,c}\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 3578 leaves) :

$$\frac{1}{(a+x\,(b+c\,x))^{5/2}}\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)^3\left(-\frac{2\,\left(c^2\,d^3-3\,a\,c\,d\,e^2+a\,b\,e^3+3\,c^2\,d^2\,e\,x-3\,b\,c\,d\,e^2\,x+b^2\,e^3\,x-a\,c\,e^3\,x\right)}{3\,c^2\,\left(a+b\,x+c\,x^2\right)^2}+\right.$$

$$\left(2\,\left(7\,b\,c^2\,d^2\,e+3\,b^2\,c\,d\,e^2-40\,a\,c^2\,d\,e^2-b^3\,e^3+11\,a\,b\,c\,e^3+14\,c^3\,d^2\,e\,x-14\,b\,c^2\,d\,e^2\,x+8\,b^2\,c\,e^3\,x-18\,a\,c^2\,e^3\,x\right)\right)\Big/$$

$$\left.\left(3\,c^2\,\left(-b^2+4\,a\,c\right)\,\left(a+b\,x+c\,x^2\right)\right)\right)-\frac{1}{3\,c\,\left(-b^2+4\,a\,c\right)\,(a+x\,(b+c\,x))^{5/2}}$$

$$14\left(a+b x+c x^2\right)^{5/2}\left(\frac{2\left(c^2 d^2-b c d e+b^2 e^2-3 a c e^2\right)(d+e x)^{3/2}\left(c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)}{c\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}-\right.$$

$$\frac{1}{c\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}\left(c d^2-b d e+a e^2\right)(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\left(\left(\mathrm{i}\, c^2 d^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(\mathrm{i}\, b c d e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(i\,b^2\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(3\,i\,a\,c\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i\sqrt{2}\,c^2 d\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(i\,b\,c\,e\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\Bigg)\Bigg)$$

■ Problem 1650: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{(b+2\,c\,x)\,(d+e\,x)^{5/2}}{\left(a+b\,x+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 494 leaves, 7 steps):

$$-\frac{2\,(d+e\,x)^{5/2}}{3\,\left(a+b\,x+c\,x^2\right)^{3/2}}-\frac{10\,e\,\sqrt{d+e\,x}\,\left(b\,d-2\,a\,e+(2\,c\,d-b\,e)\,x\right)}{3\,\left(b^2-4\,a\,c\right)\,\sqrt{a+b\,x+c\,x^2}}+$$

$$\frac{5\,\sqrt{2}\,e\,(2\,c\,d-b\,e)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]}{3\,c\,\sqrt{b^2-4\,a\,c}\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}}-$$

$$\left(20\,\sqrt{2}\,e\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/\left(3\,c\,\sqrt{b^2-4\,a\,c}\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 1973 leaves):

$$\frac{\sqrt{d+e\,x}\;\left(a+b\,x+c\,x^2\right)^3\left(-\frac{2\left(c\,d^2-a\,e^2+2\,c\,d\,e\,x-b\,e^2\,x\right)}{3\,c\,\left(a+b\,x+c\,x^2\right)^2}+\frac{2\left(5\,b\,c\,d\,e+b^2\,e^2-14\,a\,c\,e^2+10\,c^2\,d\,e\,x-5\,b\,c\,e^2\,x\right)}{3\,c\,\left(-b^2+4\,a\,c\right)\left(a+b\,x+c\,x^2\right)}\right)}{\left(a+x\,(b+c\,x)\right)^{5/2}}+$$

$$\frac{1}{3\left(b^2-4\,a\,c\right)\left(a+x\,(b+c\,x)\right)^{5/2}}\,5\left(a+b\,x+c\,x^2\right)^{5/2}\;\Bigg(\frac{2\,(2\,c\,d-b\,e)\,(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)}{c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}-$$

$$\frac{1}{c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\,2\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\Bigg(\Bigg(i\,c\,d\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\;\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Big],\;\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Big]-$$

$$\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Big],\;\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Big]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(\mathrm{i}\, b\, e \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(2 \sqrt{2} \left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(\mathrm{i} \sqrt{2}\, c \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right)\Bigg)$$

- **Problem 1651: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(b+2\,c\,x)\,(d+e\,x)^{3/2}}{\left(a+b\,x+c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 456 leaves, 7 steps) :

$$-\frac{2\,(d+e\,x)^{3/2}}{3\,\left(a+b\,x+c\,x^2\right)^{3/2}}-\frac{2\,e\,(b+2\,c\,x)\,\sqrt{d+e\,x}}{\left(b^2-4\,a\,c\right)\,\sqrt{a+b\,x+c\,x^2}}+\frac{2\,\sqrt{2}\,e\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\;e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]}{\sqrt{b^2-4\,a\,c}\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}}-$$

$$\left(2\,\sqrt{2}\,e\,(2\,c\,d-b\,e)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\;e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/$$

$$\left(c\,\sqrt{b^2-4\,a\,c}\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 1031 leaves) :

$$\frac{\sqrt{d+e x}\,\left(a+b x+c x^2\right)^3\left(-\frac{2(d+e x)}{3\left(a+b x+c x^2\right)^2}-\frac{2(b e+2 c e x)}{\left(b^2-4 a c\right)\left(a+b x+c x^2\right)}\right)}{(a+x(b+c x))^{5/2}}+\left((d+e x)^{3/2}\left(a+b x+c x^2\right)^{5/2}\right.$$

$$\left(4\sqrt{\frac{c d^2+e(-b d+a e)}{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}}\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)-\frac{1}{\sqrt{d+e x}}\,i\sqrt{2}\left(2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}\right)\right.$$

$$\sqrt{\frac{\sqrt{\left(b^2-4 a c\right) e^2}-\frac{2 a e^2}{d+e x}-2 c d\left(-1+\frac{d}{d+e x}\right)+b e\left(-1+\frac{2 d}{d+e x}\right)}{2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}}}\sqrt{\frac{\sqrt{\left(b^2-4 a c\right) e^2}+\frac{2 a e^2}{d+e x}+2 c d\left(-1+\frac{d}{d+e x}\right)+b\left(e-\frac{2 d e}{d+e x}\right)}{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c d^2-b d e+a e^2}{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}}}{\sqrt{d+e x}}\right],-\frac{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}{2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}}\right]+\frac{1}{\sqrt{d+e x}}\,i\sqrt{2}\sqrt{\left(b^2-4 a c\right) e^2}$$

$$\sqrt{\frac{\sqrt{\left(b^2-4 a c\right) e^2}-\frac{2 a e^2}{d+e x}-2 c d\left(-1+\frac{d}{d+e x}\right)+b e\left(-1+\frac{2 d}{d+e x}\right)}{2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}}}\sqrt{\frac{\sqrt{\left(b^2-4 a c\right) e^2}+\frac{2 a e^2}{d+e x}+2 c d\left(-1+\frac{d}{d+e x}\right)+b\left(e-\frac{2 d e}{d+e x}\right)}{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}}$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c d^2-b d e+a e^2}{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}}}{\sqrt{d+e x}}\right],-\frac{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}{2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}}\right]\right)\right)\Big/$$

$$\left(\left(b^2-4 a c\right)\sqrt{\frac{c d^2+e(-b d+a e)}{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}}\,(a+x(b+c x))^{5/2}\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)$$

- **Problem 1652: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{(b+2 c x)\sqrt{d+e x}}{\left(a+b x+c x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 517 leaves, 7 steps) :

$$-\frac{2\sqrt{d+e x}}{3\left(a+b x+c x^2\right)^{3/2}}-\frac{2 e\sqrt{d+e x}\left(b c d-b^2 e+2 a c e+c\left(2 c d-b e\right) x\right)}{3\left(b^2-4 a c\right)\left(c d^2-b d e+a e^2\right)\sqrt{a+b x+c x^2}}+$$

$$\frac{\sqrt{2}\, e\left(2 c d-b e\right)\sqrt{d+e x}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]}{3\sqrt{b^2-4 a c}\left(c d^2-b d e+a e^2\right)\sqrt{\frac{c\left(d+e x\right)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\sqrt{a+b x+c x^2}}-$$

$$\frac{4\sqrt{2}\, e\sqrt{\frac{c\left(d+e x\right)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]}{3\sqrt{b^2-4 a c}\sqrt{d+e x}\sqrt{a+b x+c x^2}}$$

Result (type 4, 2000 leaves) :

$$\frac{\sqrt{d+e x}\left(a+b x+c x^2\right)^3\left(-\frac{2}{3\left(a+b x+c x^2\right)^2}+\frac{2\left(b c d e-b^2 e^2+2 a c e^2+2 c^2 d e x-b c e^2 x\right)}{3\left(-b^2 c d^2+4 a c^2 d^2+b^3 d e-4 a b c d e-a b^2 e^2+4 a^2 c e^2\right)\left(a+b x+c x^2\right)}\right)}{\left(a+x\left(b+c x\right)\right)^{5/2}}+$$

$$\frac{1}{3\left(b^2-4 a c\right)\left(c d^2-b d e+a e^2\right)\left(a+x\left(b+c x\right)\right)^{5/2}}\,2 c\left(a+b x+c x^2\right)^{5/2}\left(\frac{\left(2 c d-b e\right)\left(d+e x\right)^{3/2}\left(c+\frac{c d^2}{\left(d+e x\right)^2}-\frac{b d e}{\left(d+e x\right)^2}+\frac{a e^2}{\left(d+e x\right)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)}{c\sqrt{\frac{\left(d+e x\right)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}-\right.$$

$$\frac{1}{c\sqrt{\frac{\left(d+e x\right)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}\left(c d^2-b d e+a e^2\right)\left(d+e x\right)\sqrt{c+\frac{c d^2}{\left(d+e x\right)^2}-\frac{b d e}{\left(d+e x\right)^2}+\frac{a e^2}{\left(d+e x\right)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\Bigg(\Bigg(i\,c\,d\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Bigg)\Big/$$

$$\left(\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(i\,b\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Bigg)\Big/$$

$$\left(2\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(i \sqrt{2}\, c \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left.\left.\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)\right)\right)$$

- **Problem 1653: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{b+2 c x}{\sqrt{d+e x}\left(a+b x+c x^2\right)^{5/2}}\, dx$$

Optimal (type 4, 665 leaves, 7 steps) :

$$-\frac{2\sqrt{d+e\,x}\,\left(\left(b^2-4\,a\,c\right)\,(c\,d-b\,e)-c\,\left(b^2-4\,a\,c\right)\,e\,x\right)}{3\,\left(b^2-4\,a\,c\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\left(a+b\,x+c\,x^2\right)^{3/2}}-$$

$$\frac{2\,e\,\sqrt{d+e\,x}\,\left(3\,b^2\,c\,d\,e-8\,a\,c^2\,d\,e-2\,b^3\,e^2-b\,c\,\left(c\,d^2-7\,a\,e^2\right)-2\,c\,\left(c^2\,d^2+b^2\,e^2-c\,e\,(b\,d+3\,a\,e)\right)\,x\right)}{3\,\left(b^2-4\,a\,c\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,\sqrt{a+b\,x+c\,x^2}}-$$

$$\left(2\,\sqrt{2}\,e\,\left(c^2\,d^2+b^2\,e^2-c\,e\,(b\,d+3\,a\,e)\right)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/$$

$$\left(3\,\sqrt{b^2-4\,a\,c}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\left(2\,\sqrt{2}\,e\,(2\,c\,d-b\,e)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/$$

$$\left(3\,\sqrt{b^2-4\,a\,c}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 3575 leaves) :

$$\frac{1}{(a+x\,(b+c\,x))^{5/2}}\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)^3\,\left(\frac{2\,(-c\,d+b\,e+c\,e\,x)}{3\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\left(a+b\,x+c\,x^2\right)^2}+\right.$$

$$\left(2\,\left(b\,c^2\,d^2\,e-3\,b^2\,c\,d\,e^2+8\,a\,c^2\,d\,e^2+2\,b^3\,e^3-7\,a\,b\,c\,e^3+2\,c^3\,d^2\,e\,x-2\,b\,c^2\,d\,e^2\,x+2\,b^2\,c\,e^3\,x-6\,a\,c^2\,e^3\,x\right)\right)\Big/$$

$$\left.\left(3\,\left(b^2-4\,a\,c\right)\,\left(-c\,d^2+b\,d\,e-a\,e^2\right)^2\,\left(a+b\,x+c\,x^2\right)\right)\right)-\frac{1}{3\,\left(b^2-4\,a\,c\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,(a+x\,(b+c\,x))^{5/2}}$$

$$2\,c\,\left(a+b\,x+c\,x^2\right)^{5/2}\,\Bigg(\frac{2\,\left(c^2\,d^2-b\,c\,d\,e+b^2\,e^2-3\,a\,c\,e^2\right)\,(d+e\,x)^{3/2}\,\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)}{c\,\sqrt{\frac{(d+e\,x)^2\,\left(c\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\,\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}-$$

$$\frac{1}{c\,\sqrt{\frac{(d+e\,x)^2\,\left(c\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\,\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\Bigg(\Bigg(\mathbb{i}\,c^2\,d^2\,\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\Big[\mathbb{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Big],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Big]-$$

$$\text{EllipticF}\Big[\mathbb{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Big],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Big]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(\mathbb{i}\,b\,c\,d\,e\,\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i\,b^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(3\,i\,a c e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i\sqrt{2}\,c^2 d\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(i\,b\,c\,e\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)\Bigg)\Bigg|$$

Problem 1654: Result more than twice size of optimal antiderivative.

$$\int (b+2 c x)(d+e x)^m \left(a+b x+c x^2\right)^3 \, dx$$

Optimal (type 3, 449 leaves, 2 steps):

$$-\frac{(2 c d-b e)\left(c d^2-b d e+a e^2\right)^3 (d+e x)^{1+m}}{e^8 (1+m)}+\frac{\left(c d^2-b d e+a e^2\right)^2 \left(14 c^2 d^2+3 b^2 e^2-2 c e (7 b d-a e)\right)(d+e x)^{2+m}}{e^8 (2+m)}-$$
$$\frac{3 (2 c d-b e)\left(c d^2-b d e+a e^2\right)\left(7 c^2 d^2+b^2 e^2-c e (7 b d-3 a e)\right)(d+e x)^{3+m}}{e^8 (3+m)}+\frac{1}{e^8 (4+m)}$$
$$\left(70 c^4 d^4+b^4 e^4-4 b^2 c e^3 (5 b d-3 a e)-20 c^3 d^2 e (7 b d-3 a e)+6 c^2 e^2 \left(15 b^2 d^2-10 a b d e+a^2 e^2\right)\right)(d+e x)^{4+m}-$$
$$\frac{5 c (2 c d-b e)\left(7 c^2 d^2+b^2 e^2-c e (7 b d-3 a e)\right)(d+e x)^{5+m}}{e^8 (5+m)}+$$
$$\frac{3 c^2 \left(14 c^2 d^2+3 b^2 e^2-2 c e (7 b d-a e)\right)(d+e x)^{6+m}}{e^8 (6+m)}-\frac{7 c^3 (2 c d-b e)(d+e x)^{7+m}}{e^8 (7+m)}+\frac{2 c^4 (d+e x)^{8+m}}{e^8 (8+m)}$$

Result (type 3, 1259 leaves):

$$\begin{aligned}
&\frac{1}{e^8 (1+m)(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)(8+m)} \\
&(d+e x)^{1+m} (-2 c^4 (5040 d^7 - 5040 d^6 e (1+m) x + 2520 d^5 e^2 (2+3 m+m^2) x^2 - 840 d^4 e^3 (6+11 m+6 m^2+m^3) x^3 + \\
&\quad 210 d^3 e^4 (24+50 m+35 m^2+10 m^3+m^4) x^4 - 42 d^2 e^5 (120+274 m+225 m^2+85 m^3+15 m^4+m^5) x^5 + \\
&\quad 7 d e^6 (720+1764 m+1624 m^2+735 m^3+175 m^4+21 m^5+m^6) x^6 - e^7 (5040+13068 m+13132 m^2+6769 m^3+1960 m^4+322 m^5+28 m^6+m^7) x^7) + \\
&\quad b e^4 (1680+1066 m+251 m^2+26 m^3+m^4) (a^3 e^3 (24+26 m+9 m^2+m^3) + 3 a^2 b e^2 (12+7 m+m^2) (-d+e (1+m) x) + \\
&\quad 3 a b^2 e (4+m) (2 d^2 - 2 d e (1+m) x + e^2 (2+3 m+m^2) x^2) + b^3 (-6 d^3 + 6 d^2 e (1+m) x - 3 d e^2 (2+3 m+m^2) x^2 + e^3 (6+11 m+6 m^2+m^3) x^3)) + \\
&\quad c e^3 (336+146 m+21 m^2+m^3) (2 a^3 e^3 (60+47 m+12 m^2+m^3) (-d+e (1+m) x) + 9 a^2 b e^2 (20+9 m+m^2) \\
&\quad (2 d^2 - 2 d e (1+m) x + e^2 (2+3 m+m^2) x^2) + 12 a b^2 e (5+m) (-6 d^3 + 6 d^2 e (1+m) x - 3 d e^2 (2+3 m+m^2) x^2 + e^3 (6+11 m+6 m^2+m^3) x^3) + \\
&\quad 5 b^3 (24 d^4 - 24 d^3 e (1+m) x + 12 d^2 e^2 (2+3 m+m^2) x^2 - 4 d e^3 (6+11 m+6 m^2+m^3) x^3 + e^4 (24+50 m+35 m^2+10 m^3+m^4) x^4)) + \\
&\quad 3 c^2 e^2 (56+15 m+m^2) (2 a^2 e^2 (30+11 m+m^2) (-6 d^3 + 6 d^2 e (1+m) x - 3 d e^2 (2+3 m+m^2) x^2 + e^3 (6+11 m+6 m^2+m^3) x^3) + \\
&\quad 5 a b e (6+m) (24 d^4 - 24 d^3 e (1+m) x + 12 d^2 e^2 (2+3 m+m^2) x^2 - 4 d e^3 (6+11 m+6 m^2+m^3) x^3 + e^4 (24+50 m+35 m^2+10 m^3+m^4) x^4) + \\
&\quad 3 b^2 (-120 d^5 + 120 d^4 e (1+m) x - 60 d^3 e^2 (2+3 m+m^2) x^2 + 20 d^2 e^3 (6+11 m+6 m^2+m^3) x^3 - \\
&\quad 5 d e^4 (24+50 m+35 m^2+10 m^3+m^4) x^4 + e^5 (120+274 m+225 m^2+85 m^3+15 m^4+m^5) x^5)) + \\
&\quad c^3 e (8+m) (6 a e (7+m) (-120 d^5 + 120 d^4 e (1+m) x - 60 d^3 e^2 (2+3 m+m^2) x^2 + 20 d^2 e^3 (6+11 m+6 m^2+m^3) x^3 - \\
&\quad 5 d e^4 (24+50 m+35 m^2+10 m^3+m^4) x^4 + e^5 (120+274 m+225 m^2+85 m^3+15 m^4+m^5) x^5) + \\
&\quad 7 b (720 d^6 - 720 d^5 e (1+m) x + 360 d^4 e^2 (2+3 m+m^2) x^2 - 120 d^3 e^3 (6+11 m+6 m^2+m^3) x^3 + 30 d^2 e^4 (24+50 m+35 m^2+10 m^3+m^4) x^4 - \\
&\quad 6 d e^5 (120+274 m+225 m^2+85 m^3+15 m^4+m^5) x^5 + e^6 (720+1764 m+1624 m^2+735 m^3+175 m^4+21 m^5+m^6) x^6)))
\end{aligned}$$

- **Problem 1655: Result more than twice size of optimal antiderivative.**

$$\int (b+2 c x) (d+e x)^m (a+b x+c x^2)^2 \, dx$$

Optimal (type 3, 270 leaves, 2 steps):

$$-\frac{(2 c d-b e) (c d^2-b d e+a e^2)^2 (d+e x)^{1+m}}{e^6 (1+m)} + \frac{2 (c d^2-b d e+a e^2) (5 c^2 d^2+b^2 e^2-c e (5 b d-a e)) (d+e x)^{2+m}}{e^6 (2+m)} - \frac{(2 c d-b e) (10 c^2 d^2+b^2 e^2-2 c e (5 b d-3 a e)) (d+e x)^{3+m}}{e^6 (3+m)} + \frac{4 c (5 c^2 d^2+b^2 e^2-c e (5 b d-a e)) (d+e x)^{4+m}}{e^6 (4+m)} - \frac{5 c^2 (2 c d-b e) (d+e x)^{5+m}}{e^6 (5+m)} + \frac{2 c^3 (d+e x)^{6+m}}{e^6 (6+m)}$$

Result (type 3, 541 leaves):

$$\frac{1}{e^6 (1+m)(2+m)(3+m)(4+m)(5+m)(6+m)}$$
$$(d+e x)^{1+m} \Big(-2 c^3 \left(120 d^5-120 d^4 e (1+m) x+60 d^3 e^2 \left(2+3 m+m^2\right) x^2-20 d^2 e^3 \left(6+11 m+6 m^2+m^3\right) x^3+ 5 d e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4-e^5 \left(120+274 m+225 m^2+85 m^3+15 m^4+m^5\right) x^5\right)+$$
$$b e^3 \left(120+74 m+15 m^2+m^3\right) \left(a^2 e^2 \left(6+5 m+m^2\right)+2 a b e (3+m) (-d+e (1+m) x)+b^2 \left(2 d^2-2 d e (1+m) x+e^2 \left(2+3 m+m^2\right) x^2\right)\right)+$$
$$2 c e^2 \left(30+11 m+m^2\right) \Big(a^2 e^2 \left(12+7 m+m^2\right) (-d+e (1+m) x)+3 a b e (4+m) \left(2 d^2-2 d e (1+m) x+e^2 \left(2+3 m+m^2\right) x^2\right)- 2 b^2 \left(6 d^3-6 d^2 e (1+m) x+3 d e^2 \left(2+3 m+m^2\right) x^2-e^3 \left(6+11 m+6 m^2+m^3\right) x^3\right)\Big)+$$
$$c^2 e (6+m) \Big(4 a e (5+m) \left(-6 d^3+6 d^2 e (1+m) x-3 d e^2 \left(2+3 m+m^2\right) x^2+e^3 \left(6+11 m+6 m^2+m^3\right) x^3\right)+ 5 b \left(24 d^4-24 d^3 e (1+m) x+12 d^2 e^2 \left(2+3 m+m^2\right) x^2-4 d e^3 \left(6+11 m+6 m^2+m^3\right) x^3+e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4\right)\Big)\Big)$$

- **Problem 1658: Unable to integrate problem.**

$$\int \frac{(b+2 c x)(d+e x)^m}{\left(a+b x+c x^2\right)^2} \, dx$$

Optimal (type 5, 358 leaves, 5 steps) :

$$-\frac{(d+e x)^{1+m} \left(\left(b^2-4 a c\right)(c d-b e)-c \left(b^2-4 a c\right) e x\right)}{\left(b^2-4 a c\right)\left(c d^2-b d e+a e^2\right)\left(a+b x+c x^2\right)}-$$
$$\frac{c e \left(2 c d-\left(b+\sqrt{b^2-4 a c}\right) e\right) m (d+e x)^{1+m} \text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\right]}{\sqrt{b^2-4 a c}\left(2 c d-\left(b-\sqrt{b^2-4 a c}\right) e\right)\left(c d^2-b d e+a e^2\right)(1+m)}+$$
$$\frac{c e \left(2 c d-\left(b-\sqrt{b^2-4 a c}\right) e\right) m (d+e x)^{1+m} \text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]}{\sqrt{b^2-4 a c}\left(2 c d-\left(b+\sqrt{b^2-4 a c}\right) e\right)\left(c d^2-b d e+a e^2\right)(1+m)}$$

Result (type 8, 28 leaves) :

$$\int \frac{(b+2 c x)(d+e x)^m}{\left(a+b x+c x^2\right)^2} \, dx$$

- **Problem 1659: Result more than twice size of optimal antiderivative.**

$$\int (A+B x)(d+e x)^5 \left(a^2+2 a b x+b^2 x^2\right) dx$$

Optimal (type 1, 120 leaves, 3 steps) :

$$-\frac{(b d-a e)^2 (B d-A e)(d+e x)^6}{6 e^4}+\frac{(b d-a e)(3 b B d-2 A b e-a B e)(d+e x)^7}{7 e^4}-\frac{b (3 b B d-A b e-2 a B e)(d+e x)^8}{8 e^4}+\frac{b^2 B (d+e x)^9}{9 e^4}$$

Result (type 1, 330 leaves) :

$$a^2 A d^5 x+\frac{1}{2} a d^4 (2 A b d+a B d+5 a A e) x^2+\frac{1}{3} d^3 \left(a B d (2 b d+5 a e)+A \left(b^2 d^2+10 a b d e+10 a^2 e^2\right)\right) x^3+$$
$$\frac{1}{4} d^2 \left(10 a^2 e^2 (B d+A e)+10 a b d e (B d+2 A e)+b^2 d^2 (B d+5 A e)\right) x^4+$$
$$d e \left(4 a b d e (B d+A e)+a^2 e^2 (2 B d+A e)+b^2 d^2 (B d+2 A e)\right) x^5+\frac{1}{6} e^2 \left(10 b^2 d^2 (B d+A e)+10 a b d e (2 B d+A e)+a^2 e^2 (5 B d+A e)\right) x^6+$$
$$\frac{1}{7} e^3 \left(a^2 B e^2+5 b^2 d (2 B d+A e)+2 a b e (5 B d+A e)\right) x^7+\frac{1}{8} b e^4 (5 b B d+A b e+2 a B e) x^8+\frac{1}{9} b^2 B e^5 x^9$$

- **Problem 1660: Result more than twice size of optimal antiderivative.**

$$\int (A+B x) (d+e x)^4 \left(a^2+2 a b x+b^2 x^2\right) dx$$

Optimal (type 1, 120 leaves, 3 steps):

$$-\frac{(b d-a e)^2 (B d-A e) (d+e x)^5}{5 e^4}+\frac{(b d-a e) (3 b B d-2 A b e-a B e) (d+e x)^6}{6 e^4}-\frac{b (3 b B d-A b e-2 a B e) (d+e x)^7}{7 e^4}+\frac{b^2 B (d+e x)^8}{8 e^4}$$

Result (type 1, 283 leaves):

$$a^2 A d^4 x+\frac{1}{2} a d^3 (2 A b d+a B d+4 a A e) x^2+\frac{1}{3} d^2 \left(2 a B d (b d+2 a e)+A \left(b^2 d^2+8 a b d e+6 a^2 e^2\right)\right) x^3+$$
$$\frac{1}{4} d \left(2 a^2 e^2 (3 B d+2 A e)+4 a b d e (2 B d+3 A e)+b^2 d^2 (B d+4 A e)\right) x^4+$$
$$\frac{1}{5} e \left(a^2 e^2 (4 B d+A e)+4 a b d e (3 B d+2 A e)+2 b^2 d^2 (2 B d+3 A e)\right) x^5+$$
$$\frac{1}{6} e^2 \left(a^2 B e^2+2 a b e (4 B d+A e)+2 b^2 d (3 B d+2 A e)\right) x^6+\frac{1}{7} b e^3 (4 b B d+A b e+2 a B e) x^7+\frac{1}{8} b^2 B e^4 x^8$$

- **Problem 1673: Result more than twice size of optimal antiderivative.**

$$\int (A+B x) (d+e x)^7 \left(a^2+2 a b x+b^2 x^2\right)^2 dx$$

Optimal (type 1, 206 leaves, 3 steps):

$$-\frac{(b d-a e)^4 (B d-A e) (d+e x)^8}{8 e^6}+\frac{(b d-a e)^3 (5 b B d-4 A b e-a B e) (d+e x)^9}{9 e^6}-\frac{b (b d-a e)^2 (5 b B d-3 A b e-2 a B e) (d+e x)^{10}}{5 e^6}+$$
$$\frac{2 b^2 (b d-a e) (5 b B d-2 A b e-3 a B e) (d+e x)^{11}}{11 e^6}-\frac{b^3 (5 b B d-A b e-4 a B e) (d+e x)^{12}}{12 e^6}+\frac{b^4 B (d+e x)^{13}}{13 e^6}$$

Result (type 1, 823 leaves):

$$a^4 A d^7 x + \frac{1}{2} a^3 d^6 (4 A b d + a B d + 7 a A e) x^2 + \frac{1}{3} a^2 d^5 \left(a B d (4 b d + 7 a e) + A \left(6 b^2 d^2 + 28 a b d e + 21 a^2 e^2\right)\right) x^3 +$$
$$\frac{1}{4} a d^4 \left(a B d \left(6 b^2 d^2 + 28 a b d e + 21 a^2 e^2\right) + A \left(4 b^3 d^3 + 42 a b^2 d^2 e + 84 a^2 b d e^2 + 35 a^3 e^3\right)\right) x^4 +$$
$$\frac{1}{5} d^3 \left(a B d \left(4 b^3 d^3 + 42 a b^2 d^2 e + 84 a^2 b d e^2 + 35 a^3 e^3\right) + A \left(b^4 d^4 + 28 a b^3 d^3 e + 126 a^2 b^2 d^2 e^2 + 140 a^3 b d e^3 + 35 a^4 e^4\right)\right) x^5 +$$
$$\frac{1}{6} d^2 \left(140 a^3 b d e^3 (B d + A e) + 28 a b^3 d^3 e (B d + 3 A e) + 7 a^4 e^4 (5 B d + 3 A e) + 42 a^2 b^2 d^2 e^2 (3 B d + 5 A e) + b^4 d^4 (B d + 7 A e)\right) x^6 +$$
$$d e \left(30 a^2 b^2 d^2 e^2 (B d + A e) + a^4 e^4 (3 B d + A e) + b^4 d^4 (B d + 3 A e) + 4 a^3 b d e^3 (5 B d + 3 A e) + 4 a b^3 d^3 e (3 B d + 5 A e)\right) x^7 +$$
$$\frac{1}{8} e^2 \left(140 a b^3 d^3 e (B d + A e) + 28 a^3 b d e^3 (3 B d + A e) + a^4 e^4 (7 B d + A e) + 42 a^2 b^2 d^2 e^2 (5 B d + 3 A e) + 7 b^4 d^4 (3 B d + 5 A e)\right) x^8 +$$
$$\frac{1}{9} e^3 \left(a^4 B e^4 + 35 b^4 d^3 (B d + A e) + 42 a^2 b^2 d e^2 (3 B d + A e) + 4 a^3 b e^3 (7 B d + A e) + 28 a b^3 d^2 e (5 B d + 3 A e)\right) x^9 +$$
$$\frac{1}{10} b e^4 \left(4 a^3 B e^3 + 28 a b^2 d e (3 B d + A e) + 6 a^2 b e^2 (7 B d + A e) + 7 b^3 d^2 (5 B d + 3 A e)\right) x^{10} +$$
$$\frac{1}{11} b^2 e^5 \left(6 a^2 B e^2 + 7 b^2 d (3 B d + A e) + 4 a b e (7 B d + A e)\right) x^{11} + \frac{1}{12} b^3 e^6 (7 b B d + A b e + 4 a B e) x^{12} + \frac{1}{13} b^4 B e^7 x^{13}$$

- **Problem 1674: Result more than twice size of optimal antiderivative.**

$$\int (A + B x) (d + e x)^6 \left(a^2 + 2 a b x + b^2 x^2\right)^2 \, dx$$

Optimal (type 1, 206 leaves, 3 steps) :

$$-\frac{(b d - a e)^4 (B d - A e) (d + e x)^7}{7 e^6} + \frac{(b d - a e)^3 (5 b B d - 4 A b e - a B e) (d + e x)^8}{8 e^6} - \frac{2 b (b d - a e)^2 (5 b B d - 3 A b e - 2 a B e) (d + e x)^9}{9 e^6} +$$
$$\frac{b^2 (b d - a e) (5 b B d - 2 A b e - 3 a B e) (d + e x)^{10}}{5 e^6} - \frac{b^3 (5 b B d - A b e - 4 a B e) (d + e x)^{11}}{11 e^6} + \frac{b^4 B (d + e x)^{12}}{12 e^6}$$

Result (type 1, 737 leaves) :

$$a^4 A d^6 x + \frac{1}{2} a^3 d^5 (4 A b d + a B d + 6 a A e) x^2 + \frac{1}{3} a^2 d^4 \left(2 a B d (2 b d + 3 a e) + 3 A \left(2 b^2 d^2 + 8 a b d e + 5 a^2 e^2\right)\right) x^3 +$$
$$\frac{1}{4} a d^3 \left(3 a B d \left(2 b^2 d^2 + 8 a b d e + 5 a^2 e^2\right) + 4 A \left(b^3 d^3 + 9 a b^2 d^2 e + 15 a^2 b d e^2 + 5 a^3 e^3\right)\right) x^4 +$$
$$\frac{1}{5} d^2 \left(4 a B d \left(b^3 d^3 + 9 a b^2 d^2 e + 15 a^2 b d e^2 + 5 a^3 e^3\right) + A \left(b^4 d^4 + 24 a b^3 d^3 e + 90 a^2 b^2 d^2 e^2 + 80 a^3 b d e^3 + 15 a^4 e^4\right)\right) x^5 +$$
$$\frac{1}{6} d \left(3 a^4 e^4 (5 B d + 2 A e) + 20 a^3 b d e^3 (4 B d + 3 A e) + 30 a^2 b^2 d^2 e^2 (3 B d + 4 A e) + 12 a b^3 d^3 e (2 B d + 5 A e) + b^4 d^4 (B d + 6 A e)\right) x^6 +$$
$$\frac{1}{7} e \left(a^4 e^4 (6 B d + A e) + 12 a^3 b d e^3 (5 B d + 2 A e) + 30 a^2 b^2 d^2 e^2 (4 B d + 3 A e) + 20 a b^3 d^3 e (3 B d + 4 A e) + 3 b^4 d^4 (2 B d + 5 A e)\right) x^7 +$$
$$\frac{1}{8} e^2 \left(a^4 B e^4 + 4 a^3 b e^3 (6 B d + A e) + 18 a^2 b^2 d e^2 (5 B d + 2 A e) + 20 a b^3 d^2 e (4 B d + 3 A e) + 5 b^4 d^3 (3 B d + 4 A e)\right) x^8 +$$
$$\frac{1}{9} b e^3 \left(4 a^3 B e^3 + 6 a^2 b e^2 (6 B d + A e) + 12 a b^2 d e (5 B d + 2 A e) + 5 b^3 d^2 (4 B d + 3 A e)\right) x^9 +$$
$$\frac{1}{10} b^2 e^4 \left(6 a^2 B e^2 + 4 a b e (6 B d + A e) + 3 b^2 d (5 B d + 2 A e)\right) x^{10} + \frac{1}{11} b^3 e^5 (6 b B d + A b e + 4 a B e) x^{11} + \frac{1}{12} b^4 B e^6 x^{12}$$

- **Problem 1675: Result more than twice size of optimal antiderivative.**

$$\int (A + B x) (d + e x)^5 \left(a^2 + 2 a b x + b^2 x^2\right)^2 dx$$

Optimal (type 1, 206 leaves, 3 steps):

$$-\frac{(b d - a e)^4 (B d - A e) (d + e x)^6}{6 e^6} + \frac{(b d - a e)^3 (5 b B d - 4 A b e - a B e) (d + e x)^7}{7 e^6} - \frac{b (b d - a e)^2 (5 b B d - 3 A b e - 2 a B e) (d + e x)^8}{4 e^6} +$$
$$\frac{2 b^2 (b d - a e) (5 b B d - 2 A b e - 3 a B e) (d + e x)^9}{9 e^6} - \frac{b^3 (5 b B d - A b e - 4 a B e) (d + e x)^{10}}{10 e^6} + \frac{b^4 B (d + e x)^{11}}{11 e^6}$$

Result (type 1, 615 leaves):

$$a^4 A d^5 x + \frac{1}{2} a^3 d^4 (4 A b d + a B d + 5 a A e) x^2 + \frac{1}{3} a^2 d^3 \left(a B d (4 b d + 5 a e) + 2 A \left(3 b^2 d^2 + 10 a b d e + 5 a^2 e^2\right)\right) x^3 +$$
$$\frac{1}{2} a d^2 \left(a B d \left(3 b^2 d^2 + 10 a b d e + 5 a^2 e^2\right) + A \left(2 b^3 d^3 + 15 a b^2 d^2 e + 20 a^2 b d e^2 + 5 a^3 e^3\right)\right) x^4 +$$
$$\frac{1}{5} d \left(2 a B d \left(2 b^3 d^3 + 15 a b^2 d^2 e + 20 a^2 b d e^2 + 5 a^3 e^3\right) + A \left(b^4 d^4 + 20 a b^3 d^3 e + 60 a^2 b^2 d^2 e^2 + 40 a^3 b d e^3 + 5 a^4 e^4\right)\right) x^5 +$$
$$\frac{1}{6} \left(60 a^2 b^2 d^2 e^2 (B d + A e) + 20 a^3 b d e^3 (2 B d + A e) + a^4 e^4 (5 B d + A e) + 20 a b^3 d^3 e (B d + 2 A e) + b^4 d^4 (B d + 5 A e)\right) x^6 +$$
$$\frac{1}{7} e \left(a^4 B e^4 + 40 a b^3 d^2 e (B d + A e) + 30 a^2 b^2 d e^2 (2 B d + A e) + 4 a^3 b e^3 (5 B d + A e) + 5 b^4 d^3 (B d + 2 A e)\right) x^7 +$$
$$\frac{1}{4} b e^2 \left(2 a^3 B e^3 + 5 b^3 d^2 (B d + A e) + 10 a b^2 d e (2 B d + A e) + 3 a^2 b e^2 (5 B d + A e)\right) x^8 +$$
$$\frac{1}{9} b^2 e^3 \left(6 a^2 B e^2 + 5 b^2 d (2 B d + A e) + 4 a b e (5 B d + A e)\right) x^9 + \frac{1}{10} b^3 e^4 (5 b B d + A b e + 4 a B e) x^{10} + \frac{1}{11} b^4 B e^5 x^{11}$$

- **Problem 1676: Result more than twice size of optimal antiderivative.**

$$\int (A + B x) (d + e x)^4 \left(a^2 + 2 a b x + b^2 x^2\right)^2 dx$$

Optimal (type 1, 204 leaves, 3 steps):

$$\frac{(A b - a B) (b d - a e)^4 (a + b x)^5}{5 b^6} + \frac{(b d - a e)^3 (b B d + 4 A b e - 5 a B e) (a + b x)^6}{6 b^6} + \frac{2 e (b d - a e)^2 (2 b B d + 3 A b e - 5 a B e) (a + b x)^7}{7 b^6} +$$
$$\frac{e^2 (b d - a e) (3 b B d + 2 A b e - 5 a B e) (a + b x)^8}{4 b^6} + \frac{e^3 (4 b B d + A b e - 5 a B e) (a + b x)^9}{9 b^6} + \frac{B e^4 (a + b x)^{10}}{10 b^6}$$

Result (type 1, 512 leaves):

$$a^4 A d^4 x + \frac{1}{2} a^3 d^3 (a B d + 4 A (b d + a e)) x^2 + \frac{2}{3} a^2 d^2 \left(2 a B d (b d + a e) + A \left(3 b^2 d^2 + 8 a b d e + 3 a^2 e^2\right)\right) x^3 +$$
$$\frac{1}{2} a d \left(a B d \left(3 b^2 d^2 + 8 a b d e + 3 a^2 e^2\right) + 2 A \left(b^3 d^3 + 6 a b^2 d^2 e + 6 a^2 b d e^2 + a^3 e^3\right)\right) x^4 +$$
$$\frac{1}{5} \left(4 a B d \left(b^3 d^3 + 6 a b^2 d^2 e + 6 a^2 b d e^2 + a^3 e^3\right) + A \left(b^4 d^4 + 16 a b^3 d^3 e + 36 a^2 b^2 d^2 e^2 + 16 a^3 b d e^3 + a^4 e^4\right)\right) x^5 +$$
$$\frac{1}{6} \left(a^4 B e^4 + 4 a^3 b e^3 (4 B d + A e) + 12 a^2 b^2 d e^2 (3 B d + 2 A e) + 8 a b^3 d^2 e (2 B d + 3 A e) + b^4 d^3 (B d + 4 A e)\right) x^6 +$$
$$\frac{2}{7} b e \left(2 a^3 B e^3 + 3 a^2 b e^2 (4 B d + A e) + 4 a b^2 d e (3 B d + 2 A e) + b^3 d^2 (2 B d + 3 A e)\right) x^7 +$$
$$\frac{1}{4} b^2 e^2 \left(3 a^2 B e^2 + 2 a b e (4 B d + A e) + b^2 d (3 B d + 2 A e)\right) x^8 + \frac{1}{9} b^3 e^3 (4 b B d + A b e + 4 a B e) x^9 + \frac{1}{10} b^4 B e^4 x^{10}$$

Problem 1677: Result more than twice size of optimal antiderivative.

$$\int (A+B x) (d+e x)^3 \left(a^2+2 a b x+b^2 x^2\right)^2 \, dx$$

Optimal (type 1, 159 leaves, 3 steps):

$$\frac{(A b-a B) (b d-a e)^3 (a+b x)^5}{5 b^5}+\frac{(b d-a e)^2 (b B d+3 A b e-4 a B e) (a+b x)^6}{6 b^5}+\frac{3 e (b d-a e) (b B d+A b e-2 a B e) (a+b x)^7}{7 b^5}+\frac{e^2 (3 b B d+A b e-4 a B e) (a+b x)^8}{8 b^5}+\frac{B e^3 (a+b x)^9}{9 b^5}$$

Result (type 1, 402 leaves):

$$a^4 A d^3 x+\frac{1}{2} a^3 d^2 (4 A b d+a B d+3 a A e) x^2+\frac{1}{3} a^2 d \left(a B d (4 b d+3 a e)+3 A \left(2 b^2 d^2+4 a b d e+a^2 e^2\right)\right) x^3+\frac{1}{4} a \left(3 a B d \left(2 b^2 d^2+4 a b d e+a^2 e^2\right)+A \left(4 b^3 d^3+18 a b^2 d^2 e+12 a^2 b d e^2+a^3 e^3\right)\right) x^4+\frac{1}{5} \left(a B \left(4 b^3 d^3+18 a b^2 d^2 e+12 a^2 b d e^2+a^3 e^3\right)+A b \left(b^3 d^3+12 a b^2 d^2 e+18 a^2 b d e^2+4 a^3 e^3\right)\right) x^5+\frac{1}{6} b \left(4 a^3 B e^3+12 a b^2 d e (B d+A e)+6 a^2 b e^2 (3 B d+A e)+b^3 d^2 (B d+3 A e)\right) x^6+\frac{1}{7} b^2 e \left(6 a^2 B e^2+3 b^2 d (B d+A e)+4 a b e (3 B d+A e)\right) x^7+\frac{1}{8} b^3 e^2 (3 b B d+A b e+4 a B e) x^8+\frac{1}{9} b^4 B e^3 x^9$$

Problem 1678: Result more than twice size of optimal antiderivative.

$$\int (A+B x) (d+e x)^2 \left(a^2+2 a b x+b^2 x^2\right)^2 \, dx$$

Optimal (type 1, 118 leaves, 3 steps):

$$\frac{(A b-a B) (b d-a e)^2 (a+b x)^5}{5 b^4}+\frac{(b d-a e) (b B d+2 A b e-3 a B e) (a+b x)^6}{6 b^4}+\frac{e (2 b B d+A b e-3 a B e) (a+b x)^7}{7 b^4}+\frac{B e^2 (a+b x)^8}{8 b^4}$$

Result (type 1, 288 leaves):

$$a^4 A d^2 x+\frac{1}{2} a^3 d (4 A b d+a B d+2 a A e) x^2+\frac{1}{3} a^2 \left(2 a B d (2 b d+a e)+A \left(6 b^2 d^2+8 a b d e+a^2 e^2\right)\right) x^3+\frac{1}{4} a \left(4 A b \left(b^2 d^2+3 a b d e+a^2 e^2\right)+a B \left(6 b^2 d^2+8 a b d e+a^2 e^2\right)\right) x^4+\frac{1}{5} b \left(4 a B \left(b^2 d^2+3 a b d e+a^2 e^2\right)+A b \left(b^2 d^2+8 a b d e+6 a^2 e^2\right)\right) x^5+\frac{1}{6} b^2 \left(6 a^2 B e^2+4 a b e (2 B d+A e)+b^2 d (B d+2 A e)\right) x^6+\frac{1}{7} b^3 e (2 b B d+A b e+4 a B e) x^7+\frac{1}{8} b^4 B e^2 x^8$$

Problem 1679: Result more than twice size of optimal antiderivative.

$$\int (A+B x) (d+e x) \left(a^2+2 a b x+b^2 x^2\right)^2 \, dx$$

Optimal (type 1, 75 leaves, 3 steps):

$$\frac{(A b-a B) (b d-a e) (a+b x)^5}{5 b^3}+\frac{(b B d+A b e-2 a B e) (a+b x)^6}{6 b^3}+\frac{B e (a+b x)^7}{7 b^3}$$

Result (type 1, 172 leaves):

$$a^4 A d x+\frac{1}{2} a^3 (4 A b d+a B d+a A e) x^2+\frac{1}{3} a^2 (a B (4 b d+a e)+2 A b (3 b d+2 a e)) x^3+\frac{1}{2} a b (a B (3 b d+2 a e)+A b (2 b d+3 a e)) x^4+\frac{1}{5} b^2 (2 a B (2 b d+3 a e)+A b (b d+4 a e)) x^5+\frac{1}{6} b^3 (b B d+A b e+4 a B e) x^6+\frac{1}{7} b^4 B e x^7$$

Problem 1680: Result more than twice size of optimal antiderivative.

$$\int (A+B x) \left(a^2+2 a b x+b^2 x^2\right)^2 \, dx$$

Optimal (type 1, 38 leaves, 3 steps):

$$\frac{(A b-a B) (a+b x)^5}{5 b^2}+\frac{B (a+b x)^6}{6 b^2}$$

Result (type 1, 84 leaves):

$$\frac{1}{30} x \left(15 a^4 (2 A+B x)+20 a^3 b x (3 A+2 B x)+15 a^2 b^2 x^2 (4 A+3 B x)+6 a b^3 x^3 (5 A+4 B x)+b^4 x^4 (6 A+5 B x)\right)$$

Problem 1686: Result more than twice size of optimal antiderivative.

$$\int \frac{(A+B x) \left(a^2+2 a b x+b^2 x^2\right)^2}{(d+e x)^6} \, dx$$

Optimal (type 3, 155 leaves, 4 steps):

$$-\frac{(B d-A e) (a+b x)^5}{5 e (b d-a e) (d+e x)^5}-\frac{B (b d-a e)^4}{4 e^6 (d+e x)^4}+\frac{4 b B (b d-a e)^3}{3 e^6 (d+e x)^3}-\frac{3 b^2 B (b d-a e)^2}{e^6 (d+e x)^2}+\frac{4 b^3 B (b d-a e)}{e^6 (d+e x)}+\frac{b^4 B \text{Log}[d+e x]}{e^6}$$

Result (type 3, 332 leaves):

$$\frac{1}{60 e^6 (d+e x)^5} \left(-3 a^4 e^4 (4 A e+B (d+5 e x))-4 a^3 b e^3 \left(3 A e (d+5 e x)+2 B \left(d^2+5 d e x+10 e^2 x^2\right)\right)-6 a^2 b^2 e^2 \left(2 A e \left(d^2+5 d e x+10 e^2 x^2\right)+3 B \left(d^3+5 d^2 e x+10 d e^2 x^2+10 e^3 x^3\right)\right)-12 a b^3 e \left(A e \left(d^3+5 d^2 e x+10 d e^2 x^2+10 e^3 x^3\right)+4 B \left(d^4+5 d^3 e x+10 d^2 e^2 x^2+10 d e^3 x^3+5 e^4 x^4\right)\right)+b^4 \left(-12 A e \left(d^4+5 d^3 e x+10 d^2 e^2 x^2+10 d e^3 x^3+5 e^4 x^4\right)+B d \left(137 d^4+625 d^3 e x+1100 d^2 e^2 x^2+900 d e^3 x^3+300 e^4 x^4\right)\right)+60 b^4 B (d+e x)^5 \text{Log}[d+e x]\right)$$

Problem 1687: Result more than twice size of optimal antiderivative.

$$\int \frac{(A+B x) \left(a^2+2 a b x+b^2 x^2\right)^2}{(d+e x)^7} \, dx$$

Optimal (type 1, 86 leaves, 3 steps):

$$-\frac{(B d-A e) (a+b x)^5}{6 e (b d-a e) (d+e x)^6}+\frac{(5 b B d+A b e-6 a B e) (a+b x)^5}{30 e (b d-a e)^2 (d+e x)^5}$$

Result (type 1, 317 leaves):

$$-\frac{1}{30 e^6 (d+e x)^6}\left(a^4 e^4 (5 A e+B (d+6 e x))+2 a^3 b e^3 \left(2 A e (d+6 e x)+B \left(d^2+6 d e x+15 e^2 x^2\right)\right)+3 a^2 b^2 e^2 \left(A e \left(d^2+6 d e x+15 e^2 x^2\right)+B \left(d^3+6 d^2 e x+15 d e^2 x^2+20 e^3 x^3\right)\right)+2 a b^3 e \left(A e \left(d^3+6 d^2 e x+15 d e^2 x^2+20 e^3 x^3\right)+2 B \left(d^4+6 d^3 e x+15 d^2 e^2 x^2+20 d e^3 x^3+15 e^4 x^4\right)\right)+b^4 \left(A e \left(d^4+6 d^3 e x+15 d^2 e^2 x^2+20 d e^3 x^3+15 e^4 x^4\right)+5 B \left(d^5+6 d^4 e x+15 d^3 e^2 x^2+20 d^2 e^3 x^3+15 d e^4 x^4+6 e^5 x^5\right)\right)\right)$$

- **Problem 1688: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+B x) \left(a^2+2 a b x+b^2 x^2\right)^2}{(d+e x)^8} \, dx$$

Optimal (type 1, 135 leaves, 4 steps):

$$-\frac{(B d-A e) (a+b x)^5}{7 e (b d-a e) (d+e x)^7}+\frac{(5 b B d+2 A b e-7 a B e) (a+b x)^5}{42 e (b d-a e)^2 (d+e x)^6}+\frac{b (5 b B d+2 A b e-7 a B e) (a+b x)^5}{210 e (b d-a e)^3 (d+e x)^5}$$

Result (type 1, 323 leaves):

$$-\frac{1}{210 e^6 (d+e x)^7}\left(5 a^4 e^4 (6 A e+B (d+7 e x))+4 a^3 b e^3 \left(5 A e (d+7 e x)+2 B \left(d^2+7 d e x+21 e^2 x^2\right)\right)+3 a^2 b^2 e^2 \left(4 A e \left(d^2+7 d e x+21 e^2 x^2\right)+3 B \left(d^3+7 d^2 e x+21 d e^2 x^2+35 e^3 x^3\right)\right)+2 a b^3 e \left(3 A e \left(d^3+7 d^2 e x+21 d e^2 x^2+35 e^3 x^3\right)+4 B \left(d^4+7 d^3 e x+21 d^2 e^2 x^2+35 d e^3 x^3+35 e^4 x^4\right)\right)+b^4 \left(2 A e \left(d^4+7 d^3 e x+21 d^2 e^2 x^2+35 d e^3 x^3+35 e^4 x^4\right)+5 B \left(d^5+7 d^4 e x+21 d^3 e^2 x^2+35 d^2 e^3 x^3+35 d e^4 x^4+21 e^5 x^5\right)\right)\right)$$

- **Problem 1731: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+B x) \left(a^2+2 a b x+b^2 x^2\right)^{3/2}}{(d+e x)^6} \, dx$$

Optimal (type 2, 106 leaves, 3 steps):

$$\frac{(A b-a B) (a+b x)^3 \sqrt{a^2+2 a b x+b^2 x^2}}{4 (b d-a e)^2 (d+e x)^4}+\frac{(B d-A e) \left(a^2+2 a b x+b^2 x^2\right)^{5/2}}{5 (b d-a e)^2 (d+e x)^5}$$

Result (type 2, 229 leaves):

$$-\frac{1}{20 e^5 (a+b x) (d+e x)^5}\sqrt{(a+b x)^2}\left(a^3 e^3 (4 A e+B (d+5 e x))+a^2 b e^2 \left(3 A e (d+5 e x)+2 B \left(d^2+5 d e x+10 e^2 x^2\right)\right)+a b^2 e \left(2 A e \left(d^2+5 d e x+10 e^2 x^2\right)+3 B \left(d^3+5 d^2 e x+10 d e^2 x^2+10 e^3 x^3\right)\right)+b^3 \left(A e \left(d^3+5 d^2 e x+10 d e^2 x^2+10 e^3 x^3\right)+4 B \left(d^4+5 d^3 e x+10 d^2 e^2 x^2+10 d e^3 x^3+5 e^4 x^4\right)\right)\right)$$

Problem 1738: Result more than twice size of optimal antiderivative.

$$\int (A+B x) (d+e x)^6 \left(a^2+2 a b x+b^2 x^2\right)^{5/2} \, dx$$

Optimal (type 2, 436 leaves, 3 steps):

$$\frac{(b d-a e)^5 (B d-A e) (d+e x)^7 \sqrt{a^2+2 a b x+b^2 x^2}}{7 e^7 (a+b x)}-\frac{(b d-a e)^4 (6 b B d-5 A b e-a B e) (d+e x)^8 \sqrt{a^2+2 a b x+b^2 x^2}}{8 e^7 (a+b x)}+$$
$$\frac{5 b (b d-a e)^3 (3 b B d-2 A b e-a B e) (d+e x)^9 \sqrt{a^2+2 a b x+b^2 x^2}}{9 e^7 (a+b x)}-\frac{b^2 (b d-a e)^2 (2 b B d-A b e-a B e) (d+e x)^{10} \sqrt{a^2+2 a b x+b^2 x^2}}{e^7 (a+b x)}+$$
$$\frac{5 b^3 (b d-a e) (3 b B d-A b e-2 a B e) (d+e x)^{11} \sqrt{a^2+2 a b x+b^2 x^2}}{11 e^7 (a+b x)}-$$
$$\frac{b^4 (6 b B d-A b e-5 a B e) (d+e x)^{12} \sqrt{a^2+2 a b x+b^2 x^2}}{12 e^7 (a+b x)}+\frac{b^5 B (d+e x)^{13} \sqrt{a^2+2 a b x+b^2 x^2}}{13 e^7 (a+b x)}$$

Result (type 2, 876 leaves):

$$\frac{1}{72072 (a+b x)} x \sqrt{(a+b x)^2} \Big(1287 a^5 \big(8 A \left(7 d^6+21 d^5 e x+35 d^4 e^2 x^2+35 d^3 e^3 x^3+21 d^2 e^4 x^4+7 d e^5 x^5+e^6 x^6\right)+$$
$$B x \left(28 d^6+112 d^5 e x+210 d^4 e^2 x^2+224 d^3 e^3 x^3+140 d^2 e^4 x^4+48 d e^5 x^5+7 e^6 x^6\right)\big)+$$
$$715 a^4 b x \big(9 A \left(28 d^6+112 d^5 e x+210 d^4 e^2 x^2+224 d^3 e^3 x^3+140 d^2 e^4 x^4+48 d e^5 x^5+7 e^6 x^6\right)+$$
$$2 B x \left(84 d^6+378 d^5 e x+756 d^4 e^2 x^2+840 d^3 e^3 x^3+540 d^2 e^4 x^4+189 d e^5 x^5+28 e^6 x^6\right)\big)+$$
$$286 a^3 b^2 x^2 \big(10 A \left(84 d^6+378 d^5 e x+756 d^4 e^2 x^2+840 d^3 e^3 x^3+540 d^2 e^4 x^4+189 d e^5 x^5+28 e^6 x^6\right)+$$
$$3 B x \left(210 d^6+1008 d^5 e x+2100 d^4 e^2 x^2+2400 d^3 e^3 x^3+1575 d^2 e^4 x^4+560 d e^5 x^5+84 e^6 x^6\right)\big)+$$
$$78 a^2 b^3 x^3 \big(11 A \left(210 d^6+1008 d^5 e x+2100 d^4 e^2 x^2+2400 d^3 e^3 x^3+1575 d^2 e^4 x^4+560 d e^5 x^5+84 e^6 x^6\right)+$$
$$4 B x \left(462 d^6+2310 d^5 e x+4950 d^4 e^2 x^2+5775 d^3 e^3 x^3+3850 d^2 e^4 x^4+1386 d e^5 x^5+210 e^6 x^6\right)\big)+$$
$$13 a b^4 x^4 \big(12 A \left(462 d^6+2310 d^5 e x+4950 d^4 e^2 x^2+5775 d^3 e^3 x^3+3850 d^2 e^4 x^4+1386 d e^5 x^5+210 e^6 x^6\right)+$$
$$5 B x \left(924 d^6+4752 d^5 e x+10395 d^4 e^2 x^2+12320 d^3 e^3 x^3+8316 d^2 e^4 x^4+3024 d e^5 x^5+462 e^6 x^6\right)\big)+$$
$$b^5 x^5 \big(13 A \left(924 d^6+4752 d^5 e x+10395 d^4 e^2 x^2+12320 d^3 e^3 x^3+8316 d^2 e^4 x^4+3024 d e^5 x^5+462 e^6 x^6\right)+$$
$$6 B x \left(1716 d^6+9009 d^5 e x+20020 d^4 e^2 x^2+24024 d^3 e^3 x^3+16380 d^2 e^4 x^4+6006 d e^5 x^5+924 e^6 x^6\right)\big)\Big)$$

Problem 1752: Result more than twice size of optimal antiderivative.

$$\int \frac{(A+B x) \left(a^2+2 a b x+b^2 x^2\right)^{5/2}}{(d+e x)^8} \, dx$$

Optimal (type 2, 106 leaves, 3 steps):

$$\frac{(A b-a B) (a+b x)^5 \sqrt{a^2+2 a b x+b^2 x^2}}{6 (b d-a e)^2 (d+e x)^6}+\frac{(B d-A e) \left(a^2+2 a b x+b^2 x^2\right)^{7/2}}{7 (b d-a e)^2 (d+e x)^7}$$

Result (type 2, 465 leaves):

$$-\frac{1}{42 e^7 (a+b x) (d+e x)^7}\sqrt{(a+b x)^2}\left(a^5 e^5 (6 A e+B (d+7 e x))+a^4 b e^4 \left(5 A e (d+7 e x)+2 B \left(d^2+7 d e x+21 e^2 x^2\right)\right)+a^3 b^2 e^3 \left(4 A e \left(d^2+7 d e x+21 e^2 x^2\right)+3 B \left(d^3+7 d^2 e x+21 d e^2 x^2+35 e^3 x^3\right)\right)+a^2 b^3 e^2 \left(3 A e \left(d^3+7 d^2 e x+21 d e^2 x^2+35 e^3 x^3\right)+4 B \left(d^4+7 d^3 e x+21 d^2 e^2 x^2+35 d e^3 x^3+35 e^4 x^4\right)\right)+a b^4 e \left(2 A e \left(d^4+7 d^3 e x+21 d^2 e^2 x^2+35 d e^3 x^3+35 e^4 x^4\right)+5 B \left(d^5+7 d^4 e x+21 d^3 e^2 x^2+35 d^2 e^3 x^3+35 d e^4 x^4+21 e^5 x^5\right)\right)+b^5 \left(A e \left(d^5+7 d^4 e x+21 d^3 e^2 x^2+35 d^2 e^3 x^3+35 d e^4 x^4+21 e^5 x^5\right)+6 B \left(d^6+7 d^5 e x+21 d^4 e^2 x^2+35 d^3 e^3 x^3+35 d^2 e^4 x^4+21 d e^5 x^5+7 e^6 x^6\right)\right)\right)$$

- **Problem 1753: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+B x) \left(a^2+2 a b x+b^2 x^2\right)^{5/2}}{(d+e x)^9} \, dx$$

Optimal (type 2, 193 leaves, 4 steps):

$$-\frac{(B d-A e) (a+b x)^5 \sqrt{a^2+2 a b x+b^2 x^2}}{8 e (b d-a e) (d+e x)^8}+\frac{(3 b B d+A b e-4 a B e) (a+b x)^5 \sqrt{a^2+2 a b x+b^2 x^2}}{28 e (b d-a e)^2 (d+e x)^7}+\frac{b (3 b B d+A b e-4 a B e) (a+b x)^5 \sqrt{a^2+2 a b x+b^2 x^2}}{168 e (b d-a e)^3 (d+e x)^6}$$

Result (type 2, 466 leaves):

$$-\frac{1}{168 e^7 (a+b x) (d+e x)^8}\sqrt{(a+b x)^2}\left(3 a^5 e^5 (7 A e+B (d+8 e x))+5 a^4 b e^4 \left(3 A e (d+8 e x)+B \left(d^2+8 d e x+28 e^2 x^2\right)\right)+2 a^3 b^2 e^3 \left(5 A e \left(d^2+8 d e x+28 e^2 x^2\right)+3 B \left(d^3+8 d^2 e x+28 d e^2 x^2+56 e^3 x^3\right)\right)+6 a^2 b^3 e^2 \left(A e \left(d^3+8 d^2 e x+28 d e^2 x^2+56 e^3 x^3\right)+B \left(d^4+8 d^3 e x+28 d^2 e^2 x^2+56 d e^3 x^3+70 e^4 x^4\right)\right)+a b^4 e \left(3 A e \left(d^4+8 d^3 e x+28 d^2 e^2 x^2+56 d e^3 x^3+70 e^4 x^4\right)+5 B \left(d^5+8 d^4 e x+28 d^3 e^2 x^2+56 d^2 e^3 x^3+70 d e^4 x^4+56 e^5 x^5\right)\right)+b^5 \left(A e \left(d^5+8 d^4 e x+28 d^3 e^2 x^2+56 d^2 e^3 x^3+70 d e^4 x^4+56 e^5 x^5\right)+3 B \left(d^6+8 d^5 e x+28 d^4 e^2 x^2+56 d^3 e^3 x^3+70 d^2 e^4 x^4+56 d e^5 x^5+28 e^6 x^6\right)\right)\right)$$

- **Problem 1800: Result more than twice size of optimal antiderivative.**

$$\int (A+B x) (d+e x)^{7/2} \left(a^2+2 a b x+b^2 x^2\right)^3 \, dx$$

Optimal (type 2, 308 leaves, 3 steps):

$$-\frac{2 (b d-a e)^6 (B d-A e) (d+e x)^{9/2}}{9 e^8}+\frac{2 (b d-a e)^5 (7 b B d-6 A b e-a B e) (d+e x)^{11/2}}{11 e^8}-\frac{6 b (b d-a e)^4 (7 b B d-5 A b e-2 a B e) (d+e x)^{13/2}}{13 e^8}+\frac{2 b^2 (b d-a e)^3 (7 b B d-4 A b e-3 a B e) (d+e x)^{15/2}}{3 e^8}-\frac{10 b^3 (b d-a e)^2 (7 b B d-3 A b e-4 a B e) (d+e x)^{17/2}}{17 e^8}+\frac{6 b^4 (b d-a e) (7 b B d-2 A b e-5 a B e) (d+e x)^{19/2}}{19 e^8}-\frac{2 b^5 (7 b B d-A b e-6 a B e) (d+e x)^{21/2}}{21 e^8}+\frac{2 b^6 B (d+e x)^{23/2}}{23 e^8}$$

Result (type 2, 628 leaves):

$$\frac{1}{66\,927\,861\,e^8} 2\,(d+e\,x)^{9/2}\,\Big(676\,039\,a^6\,e^6\,(-2\,B\,d+11\,A\,e+9\,B\,e\,x)+312\,018\,a^5\,b\,e^5\,\big(13\,A\,e\,(-2\,d+9\,e\,x)+B\,(8\,d^2-36\,d\,e\,x+99\,e^2\,x^2)\big)-156\,009\,a^4\,b^2\,e^4\,\big(-5\,A\,e\,(8\,d^2-36\,d\,e\,x+99\,e^2\,x^2)+B\,(16\,d^3-72\,d^2\,e\,x+198\,d\,e^2\,x^2-429\,e^3\,x^3)\big)+12\,236\,a^3\,b^3\,e^3\,\big(17\,A\,e\,(-16\,d^3+72\,d^2\,e\,x-198\,d\,e^2\,x^2+429\,e^3\,x^3)+B\,(128\,d^4-576\,d^3\,e\,x+1584\,d^2\,e^2\,x^2-3432\,d\,e^3\,x^3+6435\,e^4\,x^4)\big)-483\,a^2\,b^4\,e^2\,\big(-19\,A\,e\,(128\,d^4-576\,d^3\,e\,x+1584\,d^2\,e^2\,x^2-3432\,d\,e^3\,x^3+6435\,e^4\,x^4)+5\,B\,(256\,d^5-1152\,d^4\,e\,x+3168\,d^3\,e^2\,x^2-6864\,d^2\,e^3\,x^3+12\,870\,d\,e^4\,x^4-21\,879\,e^5\,x^5)\big)+138\,a\,b^5\,e\,\big(7\,A\,e\,(-256\,d^5+1152\,d^4\,e\,x-3168\,d^3\,e^2\,x^2+6864\,d^2\,e^3\,x^3-12\,870\,d\,e^4\,x^4+21\,879\,e^5\,x^5)+B\,(1024\,d^6-4608\,d^5\,e\,x+12\,672\,d^4\,e^2\,x^2-27\,456\,d^3\,e^3\,x^3+51\,480\,d^2\,e^4\,x^4-87\,516\,d\,e^5\,x^5+138\,567\,e^6\,x^6)\big)+b^6\,\big(23\,A\,e\,(1024\,d^6-4608\,d^5\,e\,x+12\,672\,d^4\,e^2\,x^2-27\,456\,d^3\,e^3\,x^3+51\,480\,d^2\,e^4\,x^4-87\,516\,d\,e^5\,x^5+138\,567\,e^6\,x^6)-7\,B\,(2048\,d^7-9216\,d^6\,e\,x+25\,344\,d^5\,e^2\,x^2-54\,912\,d^4\,e^3\,x^3+102\,960\,d^3\,e^4\,x^4-175\,032\,d^2\,e^5\,x^5+277\,134\,d\,e^6\,x^6-415\,701\,e^7\,x^7)\big)\Big)$$

Problem 1801: Result more than twice size of optimal antiderivative.

$$\int (A+B\,x)\,(d+e\,x)^{5/2}\,(a^2+2\,a\,b\,x+b^2\,x^2)^3\,dx$$

Optimal (type 2, 308 leaves, 3 steps):

$$-\frac{2\,(b\,d-a\,e)^6\,(B\,d-A\,e)\,(d+e\,x)^{7/2}}{7\,e^8}+\frac{2\,(b\,d-a\,e)^5\,(7\,b\,B\,d-6\,A\,b\,e-a\,B\,e)\,(d+e\,x)^{9/2}}{9\,e^8}-\frac{6\,b\,(b\,d-a\,e)^4\,(7\,b\,B\,d-5\,A\,b\,e-2\,a\,B\,e)\,(d+e\,x)^{11/2}}{11\,e^8}+\frac{10\,b^2\,(b\,d-a\,e)^3\,(7\,b\,B\,d-4\,A\,b\,e-3\,a\,B\,e)\,(d+e\,x)^{13/2}}{13\,e^8}-\frac{2\,b^3\,(b\,d-a\,e)^2\,(7\,b\,B\,d-3\,A\,b\,e-4\,a\,B\,e)\,(d+e\,x)^{15/2}}{3\,e^8}+\frac{6\,b^4\,(b\,d-a\,e)\,(7\,b\,B\,d-2\,A\,b\,e-5\,a\,B\,e)\,(d+e\,x)^{17/2}}{17\,e^8}-\frac{2\,b^5\,(7\,b\,B\,d-A\,b\,e-6\,a\,B\,e)\,(d+e\,x)^{19/2}}{19\,e^8}+\frac{2\,b^6\,B\,(d+e\,x)^{21/2}}{21\,e^8}$$

Result (type 2, 629 leaves):

$$\frac{1}{2\,909\,907\,e^8} 2\,(d+e\,x)^{7/2}\,\Big(46\,189\,a^6\,e^6\,(-2\,B\,d+9\,A\,e+7\,B\,e\,x)+25\,194\,a^5\,b\,e^5\,\big(11\,A\,e\,(-2\,d+7\,e\,x)+B\,(8\,d^2-28\,d\,e\,x+63\,e^2\,x^2)\big)-4845\,a^4\,b^2\,e^4\,\big(-13\,A\,e\,(8\,d^2-28\,d\,e\,x+63\,e^2\,x^2)+3\,B\,(16\,d^3-56\,d^2\,e\,x+126\,d\,e^2\,x^2-231\,e^3\,x^3)\big)+1292\,a^3\,b^3\,e^3\,\big(15\,A\,e\,(-16\,d^3+56\,d^2\,e\,x-126\,d\,e^2\,x^2+231\,e^3\,x^3)+B\,(128\,d^4-448\,d^3\,e\,x+1008\,d^2\,e^2\,x^2-1848\,d\,e^3\,x^3+3003\,e^4\,x^4)\big)-57\,a^2\,b^4\,e^2\,\big(-17\,A\,e\,(128\,d^4-448\,d^3\,e\,x+1008\,d^2\,e^2\,x^2-1848\,d\,e^3\,x^3+3003\,e^4\,x^4)+5\,B\,(256\,d^5-896\,d^4\,e\,x+2016\,d^3\,e^2\,x^2-3696\,d^2\,e^3\,x^3+6006\,d\,e^4\,x^4-9009\,e^5\,x^5)\big)+6\,a\,b^5\,e\,\big(19\,A\,e\,(-256\,d^5+896\,d^4\,e\,x-2016\,d^3\,e^2\,x^2+3696\,d^2\,e^3\,x^3-6006\,d\,e^4\,x^4+9009\,e^5\,x^5)+3\,B\,(1024\,d^6-3584\,d^5\,e\,x+8064\,d^4\,e^2\,x^2-14\,784\,d^3\,e^3\,x^3+24\,024\,d^2\,e^4\,x^4-36\,036\,d\,e^5\,x^5+51\,051\,e^6\,x^6)\big)+b^6\,\big(3\,A\,e\,(1024\,d^6-3584\,d^5\,e\,x+8064\,d^4\,e^2\,x^2-14\,784\,d^3\,e^3\,x^3+24\,024\,d^2\,e^4\,x^4-36\,036\,d\,e^5\,x^5+51\,051\,e^6\,x^6)+B\,(-2048\,d^7+7168\,d^6\,e\,x-16\,128\,d^5\,e^2\,x^2+29\,568\,d^4\,e^3\,x^3-48\,048\,d^3\,e^4\,x^4+72\,072\,d^2\,e^5\,x^5-102\,102\,d\,e^6\,x^6+138\,567\,e^7\,x^7)\big)\Big)$$

Problem 1802: Result more than twice size of optimal antiderivative.

$$\int (A+B\,x)\,(d+e\,x)^{3/2}\,(a^2+2\,a\,b\,x+b^2\,x^2)^3\,dx$$

Optimal (type 2, 308 leaves, 3 steps):

$$-\frac{2(bd-ae)^6(Bd-Ae)(d+ex)^{5/2}}{5e^8}+\frac{2(bd-ae)^5(7bBd-6Abe-aBe)(d+ex)^{7/2}}{7e^8}-\frac{2b(bd-ae)^4(7bBd-5Abe-2aBe)(d+ex)^{9/2}}{3e^8}+\frac{10b^2(bd-ae)^3(7bBd-4Abe-3aBe)(d+ex)^{11/2}}{11e^8}-\frac{10b^3(bd-ae)^2(7bBd-3Abe-4aBe)(d+ex)^{13/2}}{13e^8}+\frac{2b^4(bd-ae)(7bBd-2Abe-5aBe)(d+ex)^{15/2}}{5e^8}-\frac{2b^5(7bBd-Abe-6aBe)(d+ex)^{17/2}}{17e^8}+\frac{2b^6B(d+ex)^{19/2}}{19e^8}$$

Result (type 2, 629 leaves) :

$$\frac{1}{4849845e^8}2(d+ex)^{5/2}\Big(138567a^6e^6(-2Bd+7Ae+5Bex)+92378a^5be^5\left(9Ae(-2d+5ex)+B\left(8d^2-20dex+35e^2x^2\right)\right)-20995a^4b^2e^4\left(-11Ae\left(8d^2-20dex+35e^2x^2\right)+3B\left(16d^3-40d^2ex+70de^2x^2-105e^3x^3\right)\right)+6460a^3b^3e^3\left(13Ae\left(-16d^3+40d^2ex-70de^2x^2+105e^3x^3\right)+B\left(128d^4-320d^3ex+560d^2e^2x^2-840de^3x^3+1155e^4x^4\right)\right)-1615a^2b^4e^2\left(-3Ae\left(128d^4-320d^3ex+560d^2e^2x^2-840de^3x^3+1155e^4x^4\right)+B\left(256d^5-640d^4ex+1120d^3e^2x^2-1680d^2e^3x^3+2310de^4x^4-3003e^5x^5\right)\right)+38ab^5e\left(17Ae\left(-256d^5+640d^4ex-1120d^3e^2x^2+1680d^2e^3x^3-2310de^4x^4+3003e^5x^5\right)+3B\left(1024d^6-2560d^5ex+4480d^4e^2x^2-6720d^3e^3x^3+9240d^2e^4x^4-12012de^5x^5+15015e^6x^6\right)\right)+b^6\left(19Ae\left(1024d^6-2560d^5ex+4480d^4e^2x^2-6720d^3e^3x^3+9240d^2e^4x^4-12012de^5x^5+15015e^6x^6\right)-7B\left(2048d^7-5120d^6ex+8960d^5e^2x^2-13440d^4e^3x^3+18480d^3e^4x^4-24024d^2e^5x^5+30030de^6x^6-36465e^7x^7\right)\right)\Big)$$

Problem 1803: Result more than twice size of optimal antiderivative.

$$\int (A+Bx)\sqrt{d+ex}\left(a^2+2abx+b^2x^2\right)^3\,dx$$

Optimal (type 2, 308 leaves, 3 steps) :

$$-\frac{2(bd-ae)^6(Bd-Ae)(d+ex)^{3/2}}{3e^8}+\frac{2(bd-ae)^5(7bBd-6Abe-aBe)(d+ex)^{5/2}}{5e^8}-\frac{6b(bd-ae)^4(7bBd-5Abe-2aBe)(d+ex)^{7/2}}{7e^8}+\frac{10b^2(bd-ae)^3(7bBd-4Abe-3aBe)(d+ex)^{9/2}}{9e^8}-\frac{10b^3(bd-ae)^2(7bBd-3Abe-4aBe)(d+ex)^{11/2}}{11e^8}+\frac{6b^4(bd-ae)(7bBd-2Abe-5aBe)(d+ex)^{13/2}}{13e^8}-\frac{2b^5(7bBd-Abe-6aBe)(d+ex)^{15/2}}{15e^8}+\frac{2b^6B(d+ex)^{17/2}}{17e^8}$$

Result (type 2, 628 leaves) :

$$\frac{1}{765765 e^8} 2 (d+e x)^{3/2} \Big(51051 a^6 e^6 (-2 B d+5 A e+3 B e x)+43758 a^5 b e^5 \left(7 A e (-2 d+3 e x)+B \left(8 d^2-12 d e x+15 e^2 x^2\right)\right)-36465 a^4 b^2 e^4 \left(-3 A e \left(8 d^2-12 d e x+15 e^2 x^2\right)+B \left(16 d^3-24 d^2 e x+30 d e^2 x^2-35 e^3 x^3\right)\right)+4420 a^3 b^3 e^3 \left(11 A e \left(-16 d^3+24 d^2 e x-30 d e^2 x^2+35 e^3 x^3\right)+B \left(128 d^4-192 d^3 e x+240 d^2 e^2 x^2-280 d e^3 x^3+315 e^4 x^4\right)\right)-255 a^2 b^4 e^2 \left(-13 A e \left(128 d^4-192 d^3 e x+240 d^2 e^2 x^2-280 d e^3 x^3+315 e^4 x^4\right)+5 B \left(256 d^5-384 d^4 e x+480 d^3 e^2 x^2-560 d^2 e^3 x^3+630 d e^4 x^4-693 e^5 x^5\right)\right)+102 a b^5 e \left(5 A e \left(-256 d^5+384 d^4 e x-480 d^3 e^2 x^2+560 d^2 e^3 x^3-630 d e^4 x^4+693 e^5 x^5\right)+B \left(1024 d^6-1536 d^5 e x+1920 d^4 e^2 x^2-2240 d^3 e^3 x^3+2520 d^2 e^4 x^4-2772 d e^5 x^5+3003 e^6 x^6\right)\right)+b^6 \left(17 A e \left(1024 d^6-1536 d^5 e x+1920 d^4 e^2 x^2-2240 d^3 e^3 x^3+2520 d^2 e^4 x^4-2772 d e^5 x^5+3003 e^6 x^6\right)-7 B \left(2048 d^7-3072 d^6 e x+3840 d^5 e^2 x^2-4480 d^4 e^3 x^3+5040 d^3 e^4 x^4-5544 d^2 e^5 x^5+6006 d e^6 x^6-6435 e^7 x^7\right)\right)\Big)$$

Problem 1804: Result more than twice size of optimal antiderivative.

$$\int \frac{(A+B x) \left(a^2+2 a b x+b^2 x^2\right)^3}{\sqrt{d+e x}} \, dx$$

Optimal (type 2, 306 leaves, 3 steps):

$$-\frac{2 (b d-a e)^6 (B d-A e) \sqrt{d+e x}}{e^8}+\frac{2 (b d-a e)^5 (7 b B d-6 A b e-a B e) (d+e x)^{3/2}}{3 e^8}-\frac{6 b (b d-a e)^4 (7 b B d-5 A b e-2 a B e) (d+e x)^{5/2}}{5 e^8}+\frac{10 b^2 (b d-a e)^3 (7 b B d-4 A b e-3 a B e) (d+e x)^{7/2}}{7 e^8}-\frac{10 b^3 (b d-a e)^2 (7 b B d-3 A b e-4 a B e) (d+e x)^{9/2}}{9 e^8}+\frac{6 b^4 (b d-a e) (7 b B d-2 A b e-5 a B e) (d+e x)^{11/2}}{11 e^8}-\frac{2 b^5 (7 b B d-A b e-6 a B e) (d+e x)^{13/2}}{13 e^8}+\frac{2 b^6 B (d+e x)^{15/2}}{15 e^8}$$

Result (type 2, 628 leaves):

$$\frac{1}{45045 e^8} 2 \sqrt{d+e x} \Big(15015 a^6 e^6 (-2 B d+3 A e+B e x)+18018 a^5 b e^5 \left(5 A e (-2 d+e x)+B \left(8 d^2-4 d e x+3 e^2 x^2\right)\right)-6435 a^4 b^2 e^4 \left(-7 A e \left(8 d^2-4 d e x+3 e^2 x^2\right)+3 B \left(16 d^3-8 d^2 e x+6 d e^2 x^2-5 e^3 x^3\right)\right)+2860 a^3 b^3 e^3 \left(9 A e \left(-16 d^3+8 d^2 e x-6 d e^2 x^2+5 e^3 x^3\right)+B \left(128 d^4-64 d^3 e x+48 d^2 e^2 x^2-40 d e^3 x^3+35 e^4 x^4\right)\right)-195 a^2 b^4 e^2 \left(-11 A e \left(128 d^4-64 d^3 e x+48 d^2 e^2 x^2-40 d e^3 x^3+35 e^4 x^4\right)+5 B \left(256 d^5-128 d^4 e x+96 d^3 e^2 x^2-80 d^2 e^3 x^3+70 d e^4 x^4-63 e^5 x^5\right)\right)+30 a b^5 e \left(13 A e \left(-256 d^5+128 d^4 e x-96 d^3 e^2 x^2+80 d^2 e^3 x^3-70 d e^4 x^4+63 e^5 x^5\right)+3 B \left(1024 d^6-512 d^5 e x+384 d^4 e^2 x^2-320 d^3 e^3 x^3+280 d^2 e^4 x^4-252 d e^5 x^5+231 e^6 x^6\right)\right)+b^6 \left(15 A e \left(1024 d^6-512 d^5 e x+384 d^4 e^2 x^2-320 d^3 e^3 x^3+280 d^2 e^4 x^4-252 d e^5 x^5+231 e^6 x^6\right)-7 B \left(2048 d^7-1024 d^6 e x+768 d^5 e^2 x^2-640 d^4 e^3 x^3+560 d^3 e^4 x^4-504 d^2 e^5 x^5+462 d e^6 x^6-429 e^7 x^7\right)\right)\Big)$$

Problem 1805: Result more than twice size of optimal antiderivative.

$$\int \frac{(A+B x) \left(a^2+2 a b x+b^2 x^2\right)^3}{(d+e x)^{3/2}} \, dx$$

Optimal (type 2, 300 leaves, 3 steps):

$$\frac{2(bd-ae)^6(Bd-Ae)}{e^8\sqrt{d+ex}}+\frac{2(bd-ae)^5(7bBd-6Abe-aBe)\sqrt{d+ex}}{e^8}-\frac{2b(bd-ae)^4(7bBd-5Abe-2aBe)(d+ex)^{3/2}}{e^8}+\frac{2b^2(bd-ae)^3(7bBd-4Abe-3aBe)(d+ex)^{5/2}}{e^8}-\frac{10b^3(bd-ae)^2(7bBd-3Abe-4aBe)(d+ex)^{7/2}}{7e^8}+\frac{2b^4(bd-ae)(7bBd-2Abe-5aBe)(d+ex)^{9/2}}{3e^8}-\frac{2b^5(7bBd-Abe-6aBe)(d+ex)^{11/2}}{11e^8}+\frac{2b^6B(d+ex)^{13/2}}{13e^8}$$

Result (type 2, 624 leaves):

$$\frac{1}{3003e^8\sqrt{d+ex}}2\Big(3003a^6e^6(2Bd-Ae+Bex)+6006a^5be^5\left(3Ae(2d+ex)+B\left(-8d^2-4dex+e^2x^2\right)\right)+3003a^4b^2e^4\left(5Ae\left(-8d^2-4dex+e^2x^2\right)+3B\left(16d^3+8d^2ex-2de^2x^2+e^3x^3\right)\right)-1716a^3b^3e^3\left(-7Ae\left(16d^3+8d^2ex-2de^2x^2+e^3x^3\right)+B\left(128d^4+64d^3ex-16d^2e^2x^2+8de^3x^3-5e^4x^4\right)\right)+143a^2b^4e^2\left(9Ae\left(-128d^4-64d^3ex+16d^2e^2x^2-8de^3x^3+5e^4x^4\right)+5B\left(256d^5+128d^4ex-32d^3e^2x^2+16d^2e^3x^3-10de^4x^4+7e^5x^5\right)\right)-26ab^5e\left(-11Ae\left(256d^5+128d^4ex-32d^3e^2x^2+16d^2e^3x^3-10de^4x^4+7e^5x^5\right)+3B\left(1024d^6+512d^5ex-128d^4e^2x^2+64d^3e^3x^3-40d^2e^4x^4+28de^5x^5-21e^6x^6\right)\right)+b^6\left(13Ae\left(-1024d^6-512d^5ex+128d^4e^2x^2-64d^3e^3x^3+40d^2e^4x^4-28de^5x^5+21e^6x^6\right)+7B\left(2048d^7+1024d^6ex-256d^5e^2x^2+128d^4e^3x^3-80d^3e^4x^4+56d^2e^5x^5-42de^6x^6+33e^7x^7\right)\right)\Big)$$

- **Problem 1806: Result more than twice size of optimal antiderivative.**

$$\int\frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{(d+ex)^{5/2}}\,dx$$

Optimal (type 2, 302 leaves, 3 steps):

$$\frac{2(bd-ae)^6(Bd-Ae)}{3e^8(d+ex)^{3/2}}-\frac{2(bd-ae)^5(7bBd-6Abe-aBe)}{e^8\sqrt{d+ex}}-\frac{6b(bd-ae)^4(7bBd-5Abe-2aBe)\sqrt{d+ex}}{e^8}+\frac{10b^2(bd-ae)^3(7bBd-4Abe-3aBe)(d+ex)^{3/2}}{3e^8}-\frac{2b^3(bd-ae)^2(7bBd-3Abe-4aBe)(d+ex)^{5/2}}{e^8}+\frac{6b^4(bd-ae)(7bBd-2Abe-5aBe)(d+ex)^{7/2}}{7e^8}-\frac{2b^5(7bBd-Abe-6aBe)(d+ex)^{9/2}}{9e^8}+\frac{2b^6B(d+ex)^{11/2}}{11e^8}$$

Result (type 2, 624 leaves):

$$\frac{1}{693 e^8 (d+e x)^{3/2}} 2 \Big(-231 a^6 e^6 (2 B d+A e+3 B e x)+1386 a^5 b e^5 \left(-A e (2 d+3 e x)+B \left(8 d^2+12 d e x+3 e^2 x^2\right)\right)+$$
$$3465 a^4 b^2 e^4 \left(A e \left(8 d^2+12 d e x+3 e^2 x^2\right)+B \left(-16 d^3-24 d^2 e x-6 d e^2 x^2+e^3 x^3\right)\right)+$$
$$924 a^3 b^3 e^3 \left(5 A e \left(-16 d^3-24 d^2 e x-6 d e^2 x^2+e^3 x^3\right)+B \left(128 d^4+192 d^3 e x+48 d^2 e^2 x^2-8 d e^3 x^3+3 e^4 x^4\right)\right)-$$
$$99 a^2 b^4 e^2 \left(-7 A e \left(128 d^4+192 d^3 e x+48 d^2 e^2 x^2-8 d e^3 x^3+3 e^4 x^4\right)+5 B \left(256 d^5+384 d^4 e x+96 d^3 e^2 x^2-16 d^2 e^3 x^3+6 d e^4 x^4-3 e^5 x^5\right)\right)+$$
$$66 a b^5 e \Big(-3 A e \left(256 d^5+384 d^4 e x+96 d^3 e^2 x^2-16 d^2 e^3 x^3+6 d e^4 x^4-3 e^5 x^5\right)+$$
$$B \left(1024 d^6+1536 d^5 e x+384 d^4 e^2 x^2-64 d^3 e^3 x^3+24 d^2 e^4 x^4-12 d e^5 x^5+7 e^6 x^6\right)\Big)+$$
$$b^6 \Big(11 A e \left(1024 d^6+1536 d^5 e x+384 d^4 e^2 x^2-64 d^3 e^3 x^3+24 d^2 e^4 x^4-12 d e^5 x^5+7 e^6 x^6\right)-$$
$$7 B \left(2048 d^7+3072 d^6 e x+768 d^5 e^2 x^2-128 d^4 e^3 x^3+48 d^3 e^4 x^4-24 d^2 e^5 x^5+14 d e^6 x^6-9 e^7 x^7\right)\Big)\Big)$$

Problem 1807: Result more than twice size of optimal antiderivative.

$$\int \frac{(A+B x) \left(a^2+2 a b x+b^2 x^2\right)^3}{(d+e x)^{7/2}} \, dx$$

Optimal (type 2, 304 leaves, 3 steps):

$$\frac{2 (b d-a e)^6 (B d-A e)}{5 e^8 (d+e x)^{5/2}}-\frac{2 (b d-a e)^5 (7 b B d-6 A b e-a B e)}{3 e^8 (d+e x)^{3/2}}+\frac{6 b (b d-a e)^4 (7 b B d-5 A b e-2 a B e)}{e^8 \sqrt{d+e x}}+$$
$$\frac{10 b^2 (b d-a e)^3 (7 b B d-4 A b e-3 a B e) \sqrt{d+e x}}{e^8}-\frac{10 b^3 (b d-a e)^2 (7 b B d-3 A b e-4 a B e) (d+e x)^{3/2}}{3 e^8}+$$
$$\frac{6 b^4 (b d-a e) (7 b B d-2 A b e-5 a B e) (d+e x)^{5/2}}{5 e^8}-\frac{2 b^5 (7 b B d-A b e-6 a B e) (d+e x)^{7/2}}{7 e^8}+\frac{2 b^6 B (d+e x)^{9/2}}{9 e^8}$$

Result (type 2, 627 leaves):

$$-\frac{1}{315 e^8 (d+e x)^{5/2}} 2 \Big(21 a^6 e^6 (2 B d+3 A e+5 B e x)+126 a^5 b e^5 \left(A e (2 d+5 e x)+B \left(8 d^2+20 d e x+15 e^2 x^2\right)\right)-$$
$$315 a^4 b^2 e^4 \left(-A e \left(8 d^2+20 d e x+15 e^2 x^2\right)+3 B \left(16 d^3+40 d^2 e x+30 d e^2 x^2+5 e^3 x^3\right)\right)+$$
$$420 a^3 b^3 e^3 \left(-3 A e \left(16 d^3+40 d^2 e x+30 d e^2 x^2+5 e^3 x^3\right)+B \left(128 d^4+320 d^3 e x+240 d^2 e^2 x^2+40 d e^3 x^3-5 e^4 x^4\right)\right)-$$
$$315 a^2 b^4 e^2 \left(A e \left(-128 d^4-320 d^3 e x-240 d^2 e^2 x^2-40 d e^3 x^3+5 e^4 x^4\right)+B \left(256 d^5+640 d^4 e x+480 d^3 e^2 x^2+80 d^2 e^3 x^3-10 d e^4 x^4+3 e^5 x^5\right)\right)+$$
$$18 a b^5 e \Big(-7 A e \left(256 d^5+640 d^4 e x+480 d^3 e^2 x^2+80 d^2 e^3 x^3-10 d e^4 x^4+3 e^5 x^5\right)+$$
$$3 B \left(1024 d^6+2560 d^5 e x+1920 d^4 e^2 x^2+320 d^3 e^3 x^3-40 d^2 e^4 x^4+12 d e^5 x^5-5 e^6 x^6\right)\Big)+$$
$$b^6 \Big(9 A e \left(1024 d^6+2560 d^5 e x+1920 d^4 e^2 x^2+320 d^3 e^3 x^3-40 d^2 e^4 x^4+12 d e^5 x^5-5 e^6 x^6\right)-$$
$$7 B \left(2048 d^7+5120 d^6 e x+3840 d^5 e^2 x^2+640 d^4 e^3 x^3-80 d^3 e^4 x^4+24 d^2 e^5 x^5-10 d e^6 x^6+5 e^7 x^7\right)\Big)\Big)$$

Problem 1884: Result more than twice size of optimal antiderivative.

$$\int (A+B x) (d+e x)^m \left(a^2+2 a b x+b^2 x^2\right)^2 \, dx$$

Optimal (type 3, 234 leaves, 3 steps):

$$-\frac{(b d-a e)^4 (B d-A e) (d+e x)^{1+m}}{e^6 (1+m)}+\frac{(b d-a e)^3 (5 b B d-4 A b e-a B e) (d+e x)^{2+m}}{e^6 (2+m)}-\frac{2 b (b d-a e)^2 (5 b B d-3 A b e-2 a B e) (d+e x)^{3+m}}{e^6 (3+m)}+\frac{2 b^2 (b d-a e) (5 b B d-2 A b e-3 a B e) (d+e x)^{4+m}}{e^6 (4+m)}-\frac{b^3 (5 b B d-A b e-4 a B e) (d+e x)^{5+m}}{e^6 (5+m)}+\frac{b^4 B (d+e x)^{6+m}}{e^6 (6+m)}$$

Result (type 3, 635 leaves) :

$$\frac{1}{e^6 (1+m) (2+m) (3+m) (4+m) (5+m) (6+m)} (d+e x)^{1+m} \Big(a^4 e^4 \left(360+342 m+119 m^2+18 m^3+m^4\right) (-B d+A e (2+m)+B e (1+m) x)+ 4 a^3 b e^3 \left(120+74 m+15 m^2+m^3\right) \left(A e (3+m) (-d+e (1+m) x)+B \left(2 d^2-2 d e (1+m) x+e^2 \left(2+3 m+m^2\right) x^2\right)\right)+6 a^2 b^2 e^2 \left(30+11 m+m^2\right) \left(A e (4+m) \left(2 d^2-2 d e (1+m) x+e^2 \left(2+3 m+m^2\right) x^2\right)+B \left(-6 d^3+6 d^2 e (1+m) x-3 d e^2 \left(2+3 m+m^2\right) x^2+e^3 \left(6+11 m+6 m^2+m^3\right) x^3\right)\right)+ 4 a b^3 e (6+m) \left(A e (5+m) \left(-6 d^3+6 d^2 e (1+m) x-3 d e^2 \left(2+3 m+m^2\right) x^2+e^3 \left(6+11 m+6 m^2+m^3\right) x^3\right)+ B \left(24 d^4-24 d^3 e (1+m) x+12 d^2 e^2 \left(2+3 m+m^2\right) x^2-4 d e^3 \left(6+11 m+6 m^2+m^3\right) x^3+e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4\right)\right)- b^4 \Big(-A e (6+m) \left(24 d^4-24 d^3 e (1+m) x+12 d^2 e^2 \left(2+3 m+m^2\right) x^2-4 d e^3 \left(6+11 m+6 m^2+m^3\right) x^3+e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4\right)+ B \big(120 d^5-120 d^4 e (1+m) x+60 d^3 e^2 \left(2+3 m+m^2\right) x^2-20 d^2 e^3 \left(6+11 m+6 m^2+m^3\right) x^3+ 5 d e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4-e^5 \left(120+274 m+225 m^2+85 m^3+15 m^4+m^5\right) x^5\big)\Big)\Big)$$

- **Problem 1886: Unable to integrate problem.**

$$\int \frac{(A+B x) (d+e x)^m}{a^2+2 a b x+b^2 x^2} \, dx$$

Optimal (type 5, 112 leaves, 3 steps) :

$$-\frac{(A b-a B) (d+e x)^{1+m}}{b (b d-a e) (a+b x)}+\frac{(a B e (1+m)-b (B d+A e m)) (d+e x)^{1+m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{b (d+e x)}{b d-a e}\right]}{b (b d-a e)^2 (1+m)}$$

Result (type 8, 33 leaves) :

$$\int \frac{(A+B x) (d+e x)^m}{a^2+2 a b x+b^2 x^2} \, dx$$

- **Problem 1887: Unable to integrate problem.**

$$\int \frac{(A+B x) (d+e x)^m}{\left(a^2+2 a b x+b^2 x^2\right)^2} \, dx$$

Optimal (type 5, 126 leaves, 3 steps) :

$$-\frac{(A b-a B) (d+e x)^{1+m}}{3 b (b d-a e) (a+b x)^3}-\frac{e^2 (b (3 B d-A e (2-m))-a B e (1+m)) (d+e x)^{1+m} \text{Hypergeometric2F1}\left[3, 1+m, 2+m, \frac{b (d+e x)}{b d-a e}\right]}{3 b (b d-a e)^4 (1+m)}$$

Result (type 8, 33 leaves) :

$$\int \frac{(A+B x) (d+e x)^m}{\left(a^2+2 a b x+b^2 x^2\right)^2} \, dx$$

Problem 1888: Result more than twice size of optimal antiderivative.

$$\int (A+B x) (d+e x)^m \left(a^2+2 a b x+b^2 x^2\right)^{5/2} \, dx$$

Optimal (type 3, 471 leaves, 3 steps):

$$\frac{(b d-a e)^5 (B d-A e) (d+e x)^{1+m} \sqrt{a^2+2 a b x+b^2 x^2}}{e^7 (1+m) (a+b x)}-\frac{(b d-a e)^4 (6 b B d-5 A b e-a B e) (d+e x)^{2+m} \sqrt{a^2+2 a b x+b^2 x^2}}{e^7 (2+m) (a+b x)}+\frac{5 b (b d-a e)^3 (3 b B d-2 A b e-a B e) (d+e x)^{3+m} \sqrt{a^2+2 a b x+b^2 x^2}}{e^7 (3+m) (a+b x)}-\frac{10 b^2 (b d-a e)^2 (2 b B d-A b e-a B e) (d+e x)^{4+m} \sqrt{a^2+2 a b x+b^2 x^2}}{e^7 (4+m) (a+b x)}+\frac{5 b^3 (b d-a e) (3 b B d-A b e-2 a B e) (d+e x)^{5+m} \sqrt{a^2+2 a b x+b^2 x^2}}{e^7 (5+m) (a+b x)}-\frac{b^4 (6 b B d-A b e-5 a B e) (d+e x)^{6+m} \sqrt{a^2+2 a b x+b^2 x^2}}{e^7 (6+m) (a+b x)}+\frac{b^5 B (d+e x)^{7+m} \sqrt{a^2+2 a b x+b^2 x^2}}{e^7 (7+m) (a+b x)}$$

Result (type 3, 969 leaves):

$$\frac{1}{e^7 (1+m) (2+m) (3+m) (4+m) (5+m) (6+m) (7+m) (a+b x)} \sqrt{(a+b x)^2} (d+e x)^{1+m} \Big(a^5 e^5 \left(2520+2754 m+1175 m^2+245 m^3+25 m^4+m^5\right) (-B d+A e (2+m)+B e (1+m) x)+5 a^4 b e^4 \left(840+638 m+179 m^2+22 m^3+m^4\right) \left(A e (3+m) (-d+e (1+m) x)+B \left(2 d^2-2 d e (1+m) x+e^2 \left(2+3 m+m^2\right) x^2\right)\right)+10 a^3 b^2 e^3 \left(210+107 m+18 m^2+m^3\right) \left(A e (4+m) \left(2 d^2-2 d e (1+m) x+e^2 \left(2+3 m+m^2\right) x^2\right)+B \left(-6 d^3+6 d^2 e (1+m) x-3 d e^2 \left(2+3 m+m^2\right) x^2+e^3 \left(6+11 m+6 m^2+m^3\right) x^3\right)\right)+10 a^2 b^3 e^2 \left(42+13 m+m^2\right) \left(A e (5+m) \left(-6 d^3+6 d^2 e (1+m) x-3 d e^2 \left(2+3 m+m^2\right) x^2+e^3 \left(6+11 m+6 m^2+m^3\right) x^3\right)+B \left(24 d^4-24 d^3 e (1+m) x+12 d^2 e^2 \left(2+3 m+m^2\right) x^2-4 d e^3 \left(6+11 m+6 m^2+m^3\right) x^3+e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4\right)\right)+5 a b^4 e (7+m) \left(A e (6+m) \left(24 d^4-24 d^3 e (1+m) x+12 d^2 e^2 \left(2+3 m+m^2\right) x^2-4 d e^3 \left(6+11 m+6 m^2+m^3\right) x^3+e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4\right)+B \left(-120 d^5+120 d^4 e (1+m) x-60 d^3 e^2 \left(2+3 m+m^2\right) x^2+20 d^2 e^3 \left(6+11 m+6 m^2+m^3\right) x^3-5 d e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4+e^5 \left(120+274 m+225 m^2+85 m^3+15 m^4+m^5\right) x^5\right)\right)+b^5 \Big(A e (7+m) \left(-120 d^5+120 d^4 e (1+m) x-60 d^3 e^2 \left(2+3 m+m^2\right) x^2+20 d^2 e^3 \left(6+11 m+6 m^2+m^3\right) x^3-5 d e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4+e^5 \left(120+274 m+225 m^2+85 m^3+15 m^4+m^5\right) x^5\right)+B \left(720 d^6-720 d^5 e (1+m) x+360 d^4 e^2 \left(2+3 m+m^2\right) x^2-120 d^3 e^3 \left(6+11 m+6 m^2+m^3\right) x^3+30 d^2 e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4-6 d e^5 \left(120+274 m+225 m^2+85 m^3+15 m^4+m^5\right) x^5+e^6 \left(720+1764 m+1624 m^2+735 m^3+175 m^4+21 m^5+m^6\right) x^6\right)\Big)\Big)$$

Problem 1892: Unable to integrate problem.

$$\int \frac{(A+B x) (d+e x)^m}{\left(a^2+2 a b x+b^2 x^2\right)^{3/2}} \, dx$$

Optimal (type 5, 169 leaves, 3 steps):

$$-\frac{(A b-a B)(d+e x)^{1+m}}{2 b (b d-a e)(a+b x)\sqrt{a^2+2 a b x+b^2 x^2}}+\frac{e(b(2 B d-A e(1-m))-a B e(1+m))(a+b x)(d+e x)^{1+m}\,\text{Hypergeometric2F1}\left[2,1+m,2+m,\frac{b(d+e x)}{b d-a e}\right]}{2 b (b d-a e)^3(1+m)\sqrt{a^2+2 a b x+b^2 x^2}}$$

Result (type 8, 35 leaves):

$$\int\frac{(A+B x)(d+e x)^m}{\left(a^2+2 a b x+b^2 x^2\right)^{3/2}}\,dx$$

- **Problem 1893: Result unnecessarily involves higher level functions.**

$$\int(A+B x)(d+e x)^m\left(a^2+2 a b x+b^2 x^2\right)^p\,dx$$

Optimal (type 5, 174 leaves, 4 steps):

$$\frac{B(a+b x)(d+e x)^{1+m}\left(a^2+2 a b x+b^2 x^2\right)^p}{b e(2+m+2 p)}+\frac{1}{b e^2(1+m)(2+m+2 p)}(A b e(2+m+2 p)-B(a e(1+m)+b(d+2 d p)))$$
$$\left(-\frac{e(a+b x)}{b d-a e}\right)^{-2 p}(d+e x)^{1+m}\left(a^2+2 a b x+b^2 x^2\right)^p\,\text{Hypergeometric2F1}\left[1+m,-2 p,2+m,\frac{b(d+e x)}{b d-a e}\right]$$

Result (type 6, 204 leaves):

$$\left((a+b x)^2\right)^p(d+e x)^m\left(\left(3 a B d x^2\,\text{AppellF1}\left[2,-2 p,-m,3,-\frac{b x}{a},-\frac{e x}{d}\right]\right)\Big/\right.$$
$$\left(6 a d\,\text{AppellF1}\left[2,-2 p,-m,3,-\frac{b x}{a},-\frac{e x}{d}\right]+4 b d p x\,\text{AppellF1}\left[3,1-2 p,-m,4,-\frac{b x}{a},-\frac{e x}{d}\right]+\right.$$
$$\left.\left.2 a e m x\,\text{AppellF1}\left[3,-2 p,1-m,4,-\frac{b x}{a},-\frac{e x}{d}\right]\right)+\frac{A\left(\frac{e(a+b x)}{-b d+a e}\right)^{-2 p}(d+e x)\,\text{Hypergeometric2F1}\left[1+m,-2 p,2+m,\frac{b(d+e x)}{b d-a e}\right]}{e(1+m)}\right)$$

- **Problem 1895: Result more than twice size of optimal antiderivative.**

$$\int(a+b x)(d+e x)^5\left(a^2+2 a b x+b^2 x^2\right)\,dx$$

Optimal (type 1, 92 leaves, 3 steps):

$$-\frac{(b d-a e)^3(d+e x)^6}{6 e^4}+\frac{3 b(b d-a e)^2(d+e x)^7}{7 e^4}-\frac{3 b^2(b d-a e)(d+e x)^8}{8 e^4}+\frac{b^3(d+e x)^9}{9 e^4}$$

Result (type 1, 267 leaves):

$$a^3 d^5 x + \frac{1}{2} a^2 d^4 (3 b d + 5 a e) x^2 + \frac{1}{3} a d^3 \left(3 b^2 d^2 + 15 a b d e + 10 a^2 e^2\right) x^3 + \frac{1}{4} d^2 \left(b^3 d^3 + 15 a b^2 d^2 e + 30 a^2 b d e^2 + 10 a^3 e^3\right) x^4 + d e \left(b^3 d^3 + 6 a b^2 d^2 e + 6 a^2 b d e^2 + a^3 e^3\right) x^5 + \frac{1}{6} e^2 \left(10 b^3 d^3 + 30 a b^2 d^2 e + 15 a^2 b d e^2 + a^3 e^3\right) x^6 + \frac{1}{7} b e^3 \left(10 b^2 d^2 + 15 a b d e + 3 a^2 e^2\right) x^7 + \frac{1}{8} b^2 e^4 (5 b d + 3 a e) x^8 + \frac{1}{9} b^3 e^5 x^9$$

- **Problem 1896: Result more than twice size of optimal antiderivative.**

$$\int (a + b x) (d + e x)^4 \left(a^2 + 2 a b x + b^2 x^2\right) dx$$

Optimal (type 1, 92 leaves, 3 steps):

$$-\frac{(b d - a e)^3 (d + e x)^5}{5 e^4} + \frac{b (b d - a e)^2 (d + e x)^6}{2 e^4} - \frac{3 b^2 (b d - a e) (d + e x)^7}{7 e^4} + \frac{b^3 (d + e x)^8}{8 e^4}$$

Result (type 1, 217 leaves):

$$a^3 d^4 x + \frac{1}{2} a^2 d^3 (3 b d + 4 a e) x^2 + a d^2 \left(b^2 d^2 + 4 a b d e + 2 a^2 e^2\right) x^3 + \frac{1}{4} d \left(b^3 d^3 + 12 a b^2 d^2 e + 18 a^2 b d e^2 + 4 a^3 e^3\right) x^4 + \frac{1}{5} e \left(4 b^3 d^3 + 18 a b^2 d^2 e + 12 a^2 b d e^2 + a^3 e^3\right) x^5 + \frac{1}{2} b e^2 \left(2 b^2 d^2 + 4 a b d e + a^2 e^2\right) x^6 + \frac{1}{7} b^2 e^3 (4 b d + 3 a e) x^7 + \frac{1}{8} b^3 e^4 x^8$$

- **Problem 1905: Result more than twice size of optimal antiderivative.**

$$\int (a + b x) (d + e x)^6 \left(a^2 + 2 a b x + b^2 x^2\right)^2 dx$$

Optimal (type 1, 143 leaves, 3 steps):

$$-\frac{(b d - a e)^5 (d + e x)^7}{7 e^6} + \frac{5 b (b d - a e)^4 (d + e x)^8}{8 e^6} - \frac{10 b^2 (b d - a e)^3 (d + e x)^9}{9 e^6} + \frac{b^3 (b d - a e)^2 (d + e x)^{10}}{e^6} - \frac{5 b^4 (b d - a e) (d + e x)^{11}}{11 e^6} + \frac{b^5 (d + e x)^{12}}{12 e^6}$$

Result (type 1, 501 leaves):

$$a^5 d^6 x + \frac{1}{2} a^4 d^5 (5 b d + 6 a e) x^2 + \frac{5}{3} a^3 d^4 \left(2 b^2 d^2 + 6 a b d e + 3 a^2 e^2\right) x^3 + \frac{5}{4} a^2 d^3 \left(2 b^3 d^3 + 12 a b^2 d^2 e + 15 a^2 b d e^2 + 4 a^3 e^3\right) x^4 + a d^2 \left(b^4 d^4 + 12 a b^3 d^3 e + 30 a^2 b^2 d^2 e^2 + 20 a^3 b d e^3 + 3 a^4 e^4\right) x^5 + \frac{1}{6} d \left(b^5 d^5 + 30 a b^4 d^4 e + 150 a^2 b^3 d^3 e^2 + 200 a^3 b^2 d^2 e^3 + 75 a^4 b d e^4 + 6 a^5 e^5\right) x^6 + \frac{1}{7} e \left(6 b^5 d^5 + 75 a b^4 d^4 e + 200 a^2 b^3 d^3 e^2 + 150 a^3 b^2 d^2 e^3 + 30 a^4 b d e^4 + a^5 e^5\right) x^7 + \frac{5}{8} b e^2 \left(3 b^4 d^4 + 20 a b^3 d^3 e + 30 a^2 b^2 d^2 e^2 + 12 a^3 b d e^3 + a^4 e^4\right) x^8 + \frac{5}{9} b^2 e^3 \left(4 b^3 d^3 + 15 a b^2 d^2 e + 12 a^2 b d e^2 + 2 a^3 e^3\right) x^9 + \frac{1}{2} b^3 e^4 \left(3 b^2 d^2 + 6 a b d e + 2 a^2 e^2\right) x^{10} + \frac{1}{11} b^4 e^5 (6 b d + 5 a e) x^{11} + \frac{1}{12} b^5 e^6 x^{12}$$

Problem 1906: Result more than twice size of optimal antiderivative.

$$\int (a+b x) (d+e x)^5 \left(a^2+2 a b x+b^2 x^2\right)^2 \, dx$$

Optimal (type 1, 146 leaves, 3 steps):

$$\frac{(b d-a e)^5 (a+b x)^6}{6 b^6}+\frac{5 e (b d-a e)^4 (a+b x)^7}{7 b^6}+\frac{5 e^2 (b d-a e)^3 (a+b x)^8}{4 b^6}+\frac{10 e^3 (b d-a e)^2 (a+b x)^9}{9 b^6}+\frac{e^4 (b d-a e) (a+b x)^{10}}{2 b^6}+\frac{e^5 (a+b x)^{11}}{11 b^6}$$

Result (type 1, 413 leaves):

$$a^5 d^5 x+\frac{5}{2} a^4 d^4 (b d+a e) x^2+\frac{5}{3} a^3 d^3 \left(2 b^2 d^2+5 a b d e+2 a^2 e^2\right) x^3+\frac{5}{2} a^2 d^2 \left(b^3 d^3+5 a b^2 d^2 e+5 a^2 b d e^2+a^3 e^3\right) x^4+a d \left(b^4 d^4+10 a b^3 d^3 e+20 a^2 b^2 d^2 e^2+10 a^3 b d e^3+a^4 e^4\right) x^5+\frac{1}{6} \left(b^5 d^5+25 a b^4 d^4 e+100 a^2 b^3 d^3 e^2+100 a^3 b^2 d^2 e^3+25 a^4 b d e^4+a^5 e^5\right) x^6+\frac{5}{7} b e \left(b^4 d^4+10 a b^3 d^3 e+20 a^2 b^2 d^2 e^2+10 a^3 b d e^3+a^4 e^4\right) x^7+\frac{5}{4} b^2 e^2 \left(b^3 d^3+5 a b^2 d^2 e+5 a^2 b d e^2+a^3 e^3\right) x^8+\frac{5}{9} b^3 e^3 \left(2 b^2 d^2+5 a b d e+2 a^2 e^2\right) x^9+\frac{1}{2} b^4 e^4 (b d+a e) x^{10}+\frac{1}{11} b^5 e^5 x^{11}$$

Problem 1907: Result more than twice size of optimal antiderivative.

$$\int (a+b x) (d+e x)^4 \left(a^2+2 a b x+b^2 x^2\right)^2 \, dx$$

Optimal (type 1, 119 leaves, 3 steps):

$$\frac{(b d-a e)^4 (a+b x)^6}{6 b^5}+\frac{4 e (b d-a e)^3 (a+b x)^7}{7 b^5}+\frac{3 e^2 (b d-a e)^2 (a+b x)^8}{4 b^5}+\frac{4 e^3 (b d-a e) (a+b x)^9}{9 b^5}+\frac{e^4 (a+b x)^{10}}{10 b^5}$$

Result (type 1, 301 leaves):

$$\frac{1}{1260} x \left(252 a^5 \left(5 d^4+10 d^3 e x+10 d^2 e^2 x^2+5 d e^3 x^3+e^4 x^4\right)+210 a^4 b x \left(15 d^4+40 d^3 e x+45 d^2 e^2 x^2+24 d e^3 x^3+5 e^4 x^4\right)+120 a^3 b^2 x^2 \left(35 d^4+105 d^3 e x+126 d^2 e^2 x^2+70 d e^3 x^3+15 e^4 x^4\right)+45 a^2 b^3 x^3 \left(70 d^4+224 d^3 e x+280 d^2 e^2 x^2+160 d e^3 x^3+35 e^4 x^4\right)+10 a b^4 x^4 \left(126 d^4+420 d^3 e x+540 d^2 e^2 x^2+315 d e^3 x^3+70 e^4 x^4\right)+b^5 x^5 \left(210 d^4+720 d^3 e x+945 d^2 e^2 x^2+560 d e^3 x^3+126 e^4 x^4\right)\right)$$

Problem 1908: Result more than twice size of optimal antiderivative.

$$\int (a+b x) (d+e x)^3 \left(a^2+2 a b x+b^2 x^2\right)^2 \, dx$$

Optimal (type 1, 92 leaves, 3 steps):

$$\frac{(b d-a e)^3 (a+b x)^6}{6 b^4}+\frac{3 e (b d-a e)^2 (a+b x)^7}{7 b^4}+\frac{3 e^2 (b d-a e) (a+b x)^8}{8 b^4}+\frac{e^3 (a+b x)^9}{9 b^4}$$

Result (type 1, 235 leaves):

$$\frac{1}{504} x \left(126 a^5 \left(4 d^3+6 d^2 e x+4 d e^2 x^2+e^3 x^3\right)+126 a^4 b x \left(10 d^3+20 d^2 e x+15 d e^2 x^2+4 e^3 x^3\right)+84 a^3 b^2 x^2 \left(20 d^3+45 d^2 e x+36 d e^2 x^2+10 e^3 x^3\right)+36 a^2 b^3 x^3 \left(35 d^3+84 d^2 e x+70 d e^2 x^2+20 e^3 x^3\right)+9 a b^4 x^4 \left(56 d^3+140 d^2 e x+120 d e^2 x^2+35 e^3 x^3\right)+b^5 x^5 \left(84 d^3+216 d^2 e x+189 d e^2 x^2+56 e^3 x^3\right)\right)$$

- **Problem 1909: Result more than twice size of optimal antiderivative.**

$$\int (a+b x) (d+e x)^2 \left(a^2+2 a b x+b^2 x^2\right)^2 \, dx$$

Optimal (type 1, 65 leaves, 3 steps):

$$\frac{(b d-a e)^2 (a+b x)^6}{6 b^3}+\frac{2 e (b d-a e) (a+b x)^7}{7 b^3}+\frac{e^2 (a+b x)^8}{8 b^3}$$

Result (type 1, 189 leaves):

$$a^5 d^2 x+\frac{1}{2} a^4 d (5 b d+2 a e) x^2+\frac{1}{3} a^3 \left(10 b^2 d^2+10 a b d e+a^2 e^2\right) x^3+\frac{5}{4} a^2 b \left(2 b^2 d^2+4 a b d e+a^2 e^2\right) x^4+a b^2 \left(b^2 d^2+4 a b d e+2 a^2 e^2\right) x^5+\frac{1}{6} b^3 \left(b^2 d^2+10 a b d e+10 a^2 e^2\right) x^6+\frac{1}{7} b^4 e (2 b d+5 a e) x^7+\frac{1}{8} b^5 e^2 x^8$$

- **Problem 1910: Result more than twice size of optimal antiderivative.**

$$\int (a+b x) (d+e x) \left(a^2+2 a b x+b^2 x^2\right)^2 \, dx$$

Optimal (type 1, 38 leaves, 3 steps):

$$\frac{(b d-a e) (a+b x)^6}{6 b^2}+\frac{e (a+b x)^7}{7 b^2}$$

Result (type 1, 109 leaves):

$$a^5 d x+\frac{1}{2} a^4 (5 b d+a e) x^2+\frac{5}{3} a^3 b (2 b d+a e) x^3+\frac{5}{2} a^2 b^2 (b d+a e) x^4+a b^3 (b d+2 a e) x^5+\frac{1}{6} b^4 (b d+5 a e) x^6+\frac{1}{7} b^5 e x^7$$

- **Problem 1916: Result more than twice size of optimal antiderivative.**

$$\int (a+b x) (d+e x)^6 \left(a^2+2 a b x+b^2 x^2\right)^3 \, dx$$

Optimal (type 1, 173 leaves, 3 steps):

$$\frac{(b d-a e)^6 (a+b x)^8}{8 b^7}+\frac{2 e (b d-a e)^5 (a+b x)^9}{3 b^7}+\frac{3 e^2 (b d-a e)^4 (a+b x)^{10}}{2 b^7}+\frac{20 e^3 (b d-a e)^3 (a+b x)^{11}}{11 b^7}+\frac{5 e^4 (b d-a e)^2 (a+b x)^{12}}{4 b^7}+\frac{6 e^5 (b d-a e) (a+b x)^{13}}{13 b^7}+\frac{e^6 (a+b x)^{14}}{14 b^7}$$

Result (type 1, 581 leaves):

$$\frac{1}{24\,024} x \left(3432 a^7 \left(7 d^6+21 d^5 e x+35 d^4 e^2 x^2+35 d^3 e^3 x^3+21 d^2 e^4 x^4+7 d e^5 x^5+e^6 x^6\right)+\right.$$
$$3003 a^6 b x \left(28 d^6+112 d^5 e x+210 d^4 e^2 x^2+224 d^3 e^3 x^3+140 d^2 e^4 x^4+48 d e^5 x^5+7 e^6 x^6\right)+$$
$$2002 a^5 b^2 x^2 \left(84 d^6+378 d^5 e x+756 d^4 e^2 x^2+840 d^3 e^3 x^3+540 d^2 e^4 x^4+189 d e^5 x^5+28 e^6 x^6\right)+$$
$$1001 a^4 b^3 x^3 \left(210 d^6+1008 d^5 e x+2100 d^4 e^2 x^2+2400 d^3 e^3 x^3+1575 d^2 e^4 x^4+560 d e^5 x^5+84 e^6 x^6\right)+$$
$$364 a^3 b^4 x^4 \left(462 d^6+2310 d^5 e x+4950 d^4 e^2 x^2+5775 d^3 e^3 x^3+3850 d^2 e^4 x^4+1386 d e^5 x^5+210 e^6 x^6\right)+$$
$$91 a^2 b^5 x^5 \left(924 d^6+4752 d^5 e x+10\,395 d^4 e^2 x^2+12\,320 d^3 e^3 x^3+8316 d^2 e^4 x^4+3024 d e^5 x^5+462 e^6 x^6\right)+$$
$$14 a b^6 x^6 \left(1716 d^6+9009 d^5 e x+20\,020 d^4 e^2 x^2+24\,024 d^3 e^3 x^3+16\,380 d^2 e^4 x^4+6006 d e^5 x^5+924 e^6 x^6\right)+$$
$$\left. b^7 x^7 \left(3003 d^6+16\,016 d^5 e x+36\,036 d^4 e^2 x^2+43\,680 d^3 e^3 x^3+30\,030 d^2 e^4 x^4+11\,088 d e^5 x^5+1716 e^6 x^6\right)\right)$$

- **Problem 1917: Result more than twice size of optimal antiderivative.**

$$\int (a+b x) (d+e x)^5 \left(a^2+2 a b x+b^2 x^2\right)^3 dx$$

Optimal (type 1, 143 leaves, 3 steps):

$$\frac{(b d-a e)^5 (a+b x)^8}{8 b^6}+\frac{5 e (b d-a e)^4 (a+b x)^9}{9 b^6}+\frac{e^2 (b d-a e)^3 (a+b x)^{10}}{b^6}+\frac{10 e^3 (b d-a e)^2 (a+b x)^{11}}{11 b^6}+\frac{5 e^4 (b d-a e) (a+b x)^{12}}{12 b^6}+\frac{e^5 (a+b x)^{13}}{13 b^6}$$

Result (type 1, 493 leaves):

$$\frac{1}{10\,296} x \left(1716 a^7 \left(6 d^5+15 d^4 e x+20 d^3 e^2 x^2+15 d^2 e^3 x^3+6 d e^4 x^4+e^5 x^5\right)+1716 a^6 b x \left(21 d^5+70 d^4 e x+105 d^3 e^2 x^2+84 d^2 e^3 x^3+35 d e^4 x^4+6 e^5 x^5\right)+\right.$$
$$1287 a^5 b^2 x^2 \left(56 d^5+210 d^4 e x+336 d^3 e^2 x^2+280 d^2 e^3 x^3+120 d e^4 x^4+21 e^5 x^5\right)+$$
$$715 a^4 b^3 x^3 \left(126 d^5+504 d^4 e x+840 d^3 e^2 x^2+720 d^2 e^3 x^3+315 d e^4 x^4+56 e^5 x^5\right)+$$
$$286 a^3 b^4 x^4 \left(252 d^5+1050 d^4 e x+1800 d^3 e^2 x^2+1575 d^2 e^3 x^3+700 d e^4 x^4+126 e^5 x^5\right)+$$
$$78 a^2 b^5 x^5 \left(462 d^5+1980 d^4 e x+3465 d^3 e^2 x^2+3080 d^2 e^3 x^3+1386 d e^4 x^4+252 e^5 x^5\right)+$$
$$13 a b^6 x^6 \left(792 d^5+3465 d^4 e x+6160 d^3 e^2 x^2+5544 d^2 e^3 x^3+2520 d e^4 x^4+462 e^5 x^5\right)+$$
$$\left. b^7 x^7 \left(1287 d^5+5720 d^4 e x+10\,296 d^3 e^2 x^2+9360 d^2 e^3 x^3+4290 d e^4 x^4+792 e^5 x^5\right)\right)$$

- **Problem 1918: Result more than twice size of optimal antiderivative.**

$$\int (a+b x) (d+e x)^4 \left(a^2+2 a b x+b^2 x^2\right)^3 dx$$

Optimal (type 1, 119 leaves, 3 steps):

$$\frac{(b d-a e)^4 (a+b x)^8}{8 b^5}+\frac{4 e (b d-a e)^3 (a+b x)^9}{9 b^5}+\frac{3 e^2 (b d-a e)^2 (a+b x)^{10}}{5 b^5}+\frac{4 e^3 (b d-a e) (a+b x)^{11}}{11 b^5}+\frac{e^4 (a+b x)^{12}}{12 b^5}$$

Result (type 1, 405 leaves):

$$\frac{1}{3960} x \left(792 a^7 \left(5 d^4+10 d^3 e x+10 d^2 e^2 x^2+5 d e^3 x^3+e^4 x^4\right)+924 a^6 b x \left(15 d^4+40 d^3 e x+45 d^2 e^2 x^2+24 d e^3 x^3+5 e^4 x^4\right)+792 a^5 b^2 x^2 \left(35 d^4+105 d^3 e x+126 d^2 e^2 x^2+70 d e^3 x^3+15 e^4 x^4\right)+495 a^4 b^3 x^3 \left(70 d^4+224 d^3 e x+280 d^2 e^2 x^2+160 d e^3 x^3+35 e^4 x^4\right)+220 a^3 b^4 x^4 \left(126 d^4+420 d^3 e x+540 d^2 e^2 x^2+315 d e^3 x^3+70 e^4 x^4\right)+66 a^2 b^5 x^5 \left(210 d^4+720 d^3 e x+945 d^2 e^2 x^2+560 d e^3 x^3+126 e^4 x^4\right)+12 a b^6 x^6 \left(330 d^4+1155 d^3 e x+1540 d^2 e^2 x^2+924 d e^3 x^3+210 e^4 x^4\right)+b^7 x^7 \left(495 d^4+1760 d^3 e x+2376 d^2 e^2 x^2+1440 d e^3 x^3+330 e^4 x^4\right)\right)$$

Problem 1919: Result more than twice size of optimal antiderivative.

$$\int (a+b x) (d+e x)^3 \left(a^2+2 a b x+b^2 x^2\right)^3 dx$$

Optimal (type 1, 92 leaves, 3 steps):

$$\frac{(b d-a e)^3 (a+b x)^8}{8 b^4}+\frac{e (b d-a e)^2 (a+b x)^9}{3 b^4}+\frac{3 e^2 (b d-a e) (a+b x)^{10}}{10 b^4}+\frac{e^3 (a+b x)^{11}}{11 b^4}$$

Result (type 1, 360 leaves):

$$a^7 d^3 x+\frac{1}{2} a^6 d^2 (7 b d+3 a e) x^2+a^5 d \left(7 b^2 d^2+7 a b d e+a^2 e^2\right) x^3+\frac{1}{4} a^4 \left(35 b^3 d^3+63 a b^2 d^2 e+21 a^2 b d e^2+a^3 e^3\right) x^4+\frac{7}{5} a^3 b \left(5 b^3 d^3+15 a b^2 d^2 e+9 a^2 b d e^2+a^3 e^3\right) x^5+\frac{7}{2} a^2 b^2 \left(b^3 d^3+5 a b^2 d^2 e+5 a^2 b d e^2+a^3 e^3\right) x^6+a b^3 \left(b^3 d^3+9 a b^2 d^2 e+15 a^2 b d e^2+5 a^3 e^3\right) x^7+\frac{1}{8} b^4 \left(b^3 d^3+21 a b^2 d^2 e+63 a^2 b d e^2+35 a^3 e^3\right) x^8+\frac{1}{3} b^5 e \left(b^2 d^2+7 a b d e+7 a^2 e^2\right) x^9+\frac{1}{10} b^6 e^2 (3 b d+7 a e) x^{10}+\frac{1}{11} b^7 e^3 x^{11}$$

Problem 1920: Result more than twice size of optimal antiderivative.

$$\int (a+b x) (d+e x)^2 \left(a^2+2 a b x+b^2 x^2\right)^3 dx$$

Optimal (type 1, 65 leaves, 3 steps):

$$\frac{(b d-a e)^2 (a+b x)^8}{8 b^3}+\frac{2 e (b d-a e) (a+b x)^9}{9 b^3}+\frac{e^2 (a+b x)^{10}}{10 b^3}$$

Result (type 1, 229 leaves):

$$\frac{1}{360} x \left(120 a^7 \left(3 d^2+3 d e x+e^2 x^2\right)+210 a^6 b x \left(6 d^2+8 d e x+3 e^2 x^2\right)+252 a^5 b^2 x^2 \left(10 d^2+15 d e x+6 e^2 x^2\right)+210 a^4 b^3 x^3 \left(15 d^2+24 d e x+10 e^2 x^2\right)+120 a^3 b^4 x^4 \left(21 d^2+35 d e x+15 e^2 x^2\right)+45 a^2 b^5 x^5 \left(28 d^2+48 d e x+21 e^2 x^2\right)+10 a b^6 x^6 \left(36 d^2+63 d e x+28 e^2 x^2\right)+b^7 x^7 \left(45 d^2+80 d e x+36 e^2 x^2\right)\right)$$

Problem 1921: Result more than twice size of optimal antiderivative.

$$\int (a+b x) (d+e x) \left(a^2+2 a b x+b^2 x^2\right)^3 dx$$

Optimal (type 1, 38 leaves, 3 steps):

$$\frac{(b\,d-a\,e)\,(a+b\,x)^8}{8\,b^2}+\frac{e\,(a+b\,x)^9}{9\,b^2}$$

Result (type 1, 151 leaves):

$$a^7\,d\,x+\frac{1}{2}\,a^6\,(7\,b\,d+a\,e)\,x^2+\frac{7}{3}\,a^5\,b\,(3\,b\,d+a\,e)\,x^3+\frac{7}{4}\,a^4\,b^2\,(5\,b\,d+3\,a\,e)\,x^4+7\,a^3\,b^3\,(b\,d+a\,e)\,x^5+\frac{7}{6}\,a^2\,b^4\,(3\,b\,d+5\,a\,e)\,x^6+a\,b^5\,(b\,d+3\,a\,e)\,x^7+\frac{1}{8}\,b^6\,(b\,d+7\,a\,e)\,x^8+\frac{1}{9}\,b^7\,e\,x^9$$

- **Problem 1924: Result more than twice size of optimal antiderivative.**

$$\int\frac{(a+b\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^3}{(d+e\,x)^2}\,dx$$

Optimal (type 3, 186 leaves, 3 steps):

$$-\frac{21\,b^2\,(b\,d-a\,e)^5\,x}{e^7}+\frac{(b\,d-a\,e)^7}{e^8\,(d+e\,x)}+\frac{35\,b^3\,(b\,d-a\,e)^4\,(d+e\,x)^2}{2\,e^8}-\frac{35\,b^4\,(b\,d-a\,e)^3\,(d+e\,x)^3}{3\,e^8}+\frac{21\,b^5\,(b\,d-a\,e)^2\,(d+e\,x)^4}{4\,e^8}-\frac{7\,b^6\,(b\,d-a\,e)\,(d+e\,x)^5}{5\,e^8}+\frac{b^7\,(d+e\,x)^6}{6\,e^8}+\frac{7\,b\,(b\,d-a\,e)^6\,\mathrm{Log}[d+e\,x]}{e^8}$$

Result (type 3, 387 leaves):

$$\frac{1}{60\,e^8\,(d+e\,x)}\left(420\,a^6\,b\,d\,e^6-60\,a^7\,e^7+1260\,a^5\,b^2\,e^5\,\left(-d^2+d\,e\,x+e^2\,x^2\right)+1050\,a^4\,b^3\,e^4\,\left(2\,d^3-4\,d^2\,e\,x-3\,d\,e^2\,x^2+e^3\,x^3\right)+700\,a^3\,b^4\,e^3\,\left(-3\,d^4+9\,d^3\,e\,x+6\,d^2\,e^2\,x^2-2\,d\,e^3\,x^3+e^4\,x^4\right)+105\,a^2\,b^5\,e^2\,\left(12\,d^5-48\,d^4\,e\,x-30\,d^3\,e^2\,x^2+10\,d^2\,e^3\,x^3-5\,d\,e^4\,x^4+3\,e^5\,x^5\right)+42\,a\,b^6\,e\,\left(-10\,d^6+50\,d^5\,e\,x+30\,d^4\,e^2\,x^2-10\,d^3\,e^3\,x^3+5\,d^2\,e^4\,x^4-3\,d\,e^5\,x^5+2\,e^6\,x^6\right)+b^7\,\left(60\,d^7-360\,d^6\,e\,x-210\,d^5\,e^2\,x^2+70\,d^4\,e^3\,x^3-35\,d^3\,e^4\,x^4+21\,d^2\,e^5\,x^5-14\,d\,e^6\,x^6+10\,e^7\,x^7\right)+420\,b\,(b\,d-a\,e)^6\,(d+e\,x)\,\mathrm{Log}[d+e\,x]\right)$$

- **Problem 1925: Result more than twice size of optimal antiderivative.**

$$\int\frac{(a+b\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^3}{(d+e\,x)^3}\,dx$$

Optimal (type 3, 185 leaves, 3 steps):

$$\frac{35\,b^3\,(b\,d-a\,e)^4\,x}{e^7}+\frac{(b\,d-a\,e)^7}{2\,e^8\,(d+e\,x)^2}-\frac{7\,b\,(b\,d-a\,e)^6}{e^8\,(d+e\,x)}-\frac{35\,b^4\,(b\,d-a\,e)^3\,(d+e\,x)^2}{2\,e^8}+\frac{7\,b^5\,(b\,d-a\,e)^2\,(d+e\,x)^3}{e^8}-\frac{7\,b^6\,(b\,d-a\,e)\,(d+e\,x)^4}{4\,e^8}+\frac{b^7\,(d+e\,x)^5}{5\,e^8}-\frac{21\,b^2\,(b\,d-a\,e)^5\,\mathrm{Log}[d+e\,x]}{e^8}$$

Result (type 3, 388 leaves):

$$\frac{1}{20\,e^8\,(d+e\,x)^2}\Big(-10\,a^7\,e^7-70\,a^6\,b\,e^6\,(d+2\,e\,x)+210\,a^5\,b^2\,d\,e^5\,(3\,d+4\,e\,x)+350\,a^4\,b^3\,e^4\left(-5\,d^3-4\,d^2\,e\,x+4\,d\,e^2\,x^2+2\,e^3\,x^3\right)+350\,a^3\,b^4\,e^3\left(7\,d^4+2\,d^3\,e\,x-11\,d^2\,e^2\,x^2-4\,d\,e^3\,x^3+e^4\,x^4\right)+70\,a^2\,b^5\,e^2\left(-27\,d^5+6\,d^4\,e\,x+63\,d^3\,e^2\,x^2+20\,d^2\,e^3\,x^3-5\,d\,e^4\,x^4+2\,e^5\,x^5\right)+35\,a\,b^6\,e\left(22\,d^6-16\,d^5\,e\,x-68\,d^4\,e^2\,x^2-20\,d^3\,e^3\,x^3+5\,d^2\,e^4\,x^4-2\,d\,e^5\,x^5+e^6\,x^6\right)+b^7\left(-130\,d^7+160\,d^6\,e\,x+500\,d^5\,e^2\,x^2+140\,d^4\,e^3\,x^3-35\,d^3\,e^4\,x^4+14\,d^2\,e^5\,x^5-7\,d\,e^6\,x^6+4\,e^7\,x^7\right)-420\,b^2\,(b\,d-a\,e)^5\,(d+e\,x)^2\,\mathrm{Log}[d+e\,x]\Big)$$

- **Problem 1946: Result more than twice size of optimal antiderivative.**

$$\int\frac{(a+b\,x)\,(d+e\,x)^3}{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^3}\,dx$$

Optimal (type 1, 28 leaves, 2 steps):

$$-\frac{(d+e\,x)^4}{4\,(b\,d-a\,e)\,(a+b\,x)^4}$$

Result (type 1, 91 leaves):

$$-\frac{a^3\,e^3+a^2\,b\,e^2\,(d+4\,e\,x)+a\,b^2\,e\left(d^2+4\,d\,e\,x+6\,e^2\,x^2\right)+b^3\left(d^3+4\,d^2\,e\,x+6\,d\,e^2\,x^2+4\,e^3\,x^3\right)}{4\,b^4\,(a+b\,x)^4}$$

- **Problem 1981: Result more than twice size of optimal antiderivative.**

$$\int\frac{(a+b\,x)\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}{(d+e\,x)^6}\,dx$$

Optimal (type 2, 41 leaves, 1 step):

$$\frac{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}}{5\,(b\,d-a\,e)\,(d+e\,x)^5}$$

Result (type 2, 158 leaves):

$$-\frac{1}{5\,e^5\,(a+b\,x)\,(d+e\,x)^5}\sqrt{(a+b\,x)^2}\,\Big(a^4\,e^4+a^3\,b\,e^3\,(d+5\,e\,x)+a^2\,b^2\,e^2\left(d^2+5\,d\,e\,x+10\,e^2\,x^2\right)+a\,b^3\,e\left(d^3+5\,d^2\,e\,x+10\,d\,e^2\,x^2+10\,e^3\,x^3\right)+b^4\left(d^4+5\,d^3\,e\,x+10\,d^2\,e^2\,x^2+10\,d\,e^3\,x^3+5\,e^4\,x^4\right)\Big)$$

- **Problem 1988: Result more than twice size of optimal antiderivative.**

$$\int(a+b\,x)\,(d+e\,x)^9\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}\,dx$$

Optimal (type 2, 362 leaves, 4 steps):

$$\frac{(b d-a e)^6 (d+e x)^{10} \sqrt{a^2+2 a b x+b^2 x^2}}{10 e^7 (a+b x)}-\frac{6 b (b d-a e)^5 (d+e x)^{11} \sqrt{a^2+2 a b x+b^2 x^2}}{11 e^7 (a+b x)}+\frac{5 b^2 (b d-a e)^4 (d+e x)^{12} \sqrt{a^2+2 a b x+b^2 x^2}}{4 e^7 (a+b x)}-\frac{20 b^3 (b d-a e)^3 (d+e x)^{13} \sqrt{a^2+2 a b x+b^2 x^2}}{13 e^7 (a+b x)}+\frac{15 b^4 (b d-a e)^2 (d+e x)^{14} \sqrt{a^2+2 a b x+b^2 x^2}}{14 e^7 (a+b x)}-\frac{2 b^5 (b d-a e) (d+e x)^{15} \sqrt{a^2+2 a b x+b^2 x^2}}{5 e^7 (a+b x)}+\frac{b^6 (d+e x)^{16} \sqrt{a^2+2 a b x+b^2 x^2}}{16 e^7 (a+b x)}$$

Result (type 2, 756 leaves):

$$\frac{1}{80080 (a+b x)} x \sqrt{(a+b x)^2} \left(8008 a^6 \left(10 d^9+45 d^8 e x+120 d^7 e^2 x^2+210 d^6 e^3 x^3+252 d^5 e^4 x^4+210 d^4 e^5 x^5+120 d^3 e^6 x^6+45 d^2 e^7 x^7+10 d e^8 x^8+e^9 x^9\right)+4368 a^5 b x \left(55 d^9+330 d^8 e x+990 d^7 e^2 x^2+1848 d^6 e^3 x^3+2310 d^5 e^4 x^4+1980 d^4 e^5 x^5+1155 d^3 e^6 x^6+440 d^2 e^7 x^7+99 d e^8 x^8+10 e^9 x^9\right)+1820 a^4 b^2 x^2 \left(220 d^9+1485 d^8 e x+4752 d^7 e^2 x^2+9240 d^6 e^3 x^3+11880 d^5 e^4 x^4+10395 d^4 e^5 x^5+6160 d^3 e^6 x^6+2376 d^2 e^7 x^7+540 d e^8 x^8+55 e^9 x^9\right)+560 a^3 b^3 x^3 \left(715 d^9+5148 d^8 e x+17160 d^7 e^2 x^2+34320 d^6 e^3 x^3+45045 d^5 e^4 x^4+40040 d^4 e^5 x^5+24024 d^3 e^6 x^6+9360 d^2 e^7 x^7+2145 d e^8 x^8+220 e^9 x^9\right)+120 a^2 b^4 x^4 \left(2002 d^9+15015 d^8 e x+51480 d^7 e^2 x^2+105105 d^6 e^3 x^3+140140 d^5 e^4 x^4+126126 d^4 e^5 x^5+76440 d^3 e^6 x^6+30030 d^2 e^7 x^7+6930 d e^8 x^8+715 e^9 x^9\right)+16 a b^5 x^5 \left(5005 d^9+38610 d^8 e x+135135 d^7 e^2 x^2+280280 d^6 e^3 x^3+378378 d^5 e^4 x^4+343980 d^4 e^5 x^5+210210 d^3 e^6 x^6+83160 d^2 e^7 x^7+19305 d e^8 x^8+2002 e^9 x^9\right)+b^6 x^6 \left(11440 d^9+90090 d^8 e x+320320 d^7 e^2 x^2+672672 d^6 e^3 x^3+917280 d^5 e^4 x^4+840840 d^4 e^5 x^5+517440 d^3 e^6 x^6+205920 d^2 e^7 x^7+48048 d e^8 x^8+5005 e^9 x^9\right)\right)$$

■ Problem 2005: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b x) \left(a^2+2 a b x+b^2 x^2\right)^{5/2}}{(d+e x)^8} \, dx$$

Optimal (type 2, 41 leaves, 1 step):

$$\frac{\left(a^2+2 a b x+b^2 x^2\right)^{7/2}}{7 (b d-a e) (d+e x)^7}$$

Result (type 2, 289 leaves):

$$-\frac{1}{7 e^7 (a+b x) (d+e x)^7} \sqrt{(a+b x)^2} \left(a^6 e^6+a^5 b e^5 (d+7 e x)+a^4 b^2 e^4 \left(d^2+7 d e x+21 e^2 x^2\right)+a^3 b^3 e^3 \left(d^3+7 d^2 e x+21 d e^2 x^2+35 e^3 x^3\right)+a^2 b^4 e^2 \left(d^4+7 d^3 e x+21 d^2 e^2 x^2+35 d e^3 x^3+35 e^4 x^4\right)+a b^5 e \left(d^5+7 d^4 e x+21 d^3 e^2 x^2+35 d^2 e^3 x^3+35 d e^4 x^4+21 e^5 x^5\right)+b^6 \left(d^6+7 d^5 e x+21 d^4 e^2 x^2+35 d^3 e^3 x^3+35 d^2 e^4 x^4+21 d e^5 x^5+7 e^6 x^6\right)\right)$$

■ Problem 2006: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b x) \left(a^2+2 a b x+b^2 x^2\right)^{5/2}}{(d+e x)^9} \, dx$$

Optimal (type 2, 98 leaves, 4 steps):

$$\frac{(a+b x)^6 \sqrt{a^2+2 a b x+b^2 x^2}}{8 (b d-a e) (d+e x)^8}+\frac{b (a+b x)^6 \sqrt{a^2+2 a b x+b^2 x^2}}{56 (b d-a e)^2 (d+e x)^7}$$

Result (type 2, 295 leaves):

$$-\frac{1}{56 e^7 (a+b x) (d+e x)^8}\sqrt{(a+b x)^2} \left(7 a^6 e^6+6 a^5 b e^5 (d+8 e x)+5 a^4 b^2 e^4 \left(d^2+8 d e x+28 e^2 x^2\right)+4 a^3 b^3 e^3 \left(d^3+8 d^2 e x+28 d e^2 x^2+56 e^3 x^3\right)+3 a^2 b^4 e^2 \left(d^4+8 d^3 e x+28 d^2 e^2 x^2+56 d e^3 x^3+70 e^4 x^4\right)+2 a b^5 e \left(d^5+8 d^4 e x+28 d^3 e^2 x^2+56 d^2 e^3 x^3+70 d e^4 x^4+56 e^5 x^5\right)+b^6 \left(d^6+8 d^5 e x+28 d^4 e^2 x^2+56 d^3 e^3 x^3+70 d^2 e^4 x^4+56 d e^5 x^5+28 e^6 x^6\right)\right)$$

Problem 2146: Result more than twice size of optimal antiderivative.

$$\int (a+b x) (d+e x)^m \left(a^2+2 a b x+b^2 x^2\right)^3 \, dx$$

Optimal (type 3, 239 leaves, 3 steps):

$$-\frac{(b d-a e)^7 (d+e x)^{1+m}}{e^8 (1+m)}+\frac{7 b (b d-a e)^6 (d+e x)^{2+m}}{e^8 (2+m)}-\frac{21 b^2 (b d-a e)^5 (d+e x)^{3+m}}{e^8 (3+m)}+\frac{35 b^3 (b d-a e)^4 (d+e x)^{4+m}}{e^8 (4+m)}-\frac{35 b^4 (b d-a e)^3 (d+e x)^{5+m}}{e^8 (5+m)}+\frac{21 b^5 (b d-a e)^2 (d+e x)^{6+m}}{e^8 (6+m)}-\frac{7 b^6 (b d-a e) (d+e x)^{7+m}}{e^8 (7+m)}+\frac{b^7 (d+e x)^{8+m}}{e^8 (8+m)}$$

Result (type 3, 896 leaves):

$$\frac{1}{e^8 (1+m) (2+m) (3+m) (4+m) (5+m) (6+m) (7+m) (8+m)}(d+e x)^{1+m} \left(a^7 e^7 \left(40320+69264 m+48860 m^2+18424 m^3+4025 m^4+511 m^5+35 m^6+m^7\right)-7 a^6 b e^6 \left(20160+24552 m+12154 m^2+3135 m^3+445 m^4+33 m^5+m^6\right) (d-e (1+m) x)+21 a^5 b^2 e^5 \left(6720+5944 m+2070 m^2+355 m^3+30 m^4+m^5\right) \left(2 d^2-2 d e (1+m) x+e^2 \left(2+3 m+m^2\right) x^2\right)+35 a^4 b^3 e^4 \left(1680+1066 m+251 m^2+26 m^3+m^4\right) \left(-6 d^3+6 d^2 e (1+m) x-3 d e^2 \left(2+3 m+m^2\right) x^2+e^3 \left(6+11 m+6 m^2+m^3\right) x^3\right)+35 a^3 b^4 e^3 \left(336+146 m+21 m^2+m^3\right) \left(24 d^4-24 d^3 e (1+m) x+12 d^2 e^2 \left(2+3 m+m^2\right) x^2-4 d e^3 \left(6+11 m+6 m^2+m^3\right) x^3+e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4\right)+21 a^2 b^5 e^2 \left(56+15 m+m^2\right) \left(-120 d^5+120 d^4 e (1+m) x-60 d^3 e^2 \left(2+3 m+m^2\right) x^2+20 d^2 e^3 \left(6+11 m+6 m^2+m^3\right) x^3-5 d e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4+e^5 \left(120+274 m+225 m^2+85 m^3+15 m^4+m^5\right) x^5\right)+7 a b^6 e (8+m) \left(720 d^6-720 d^5 e (1+m) x+360 d^4 e^2 \left(2+3 m+m^2\right) x^2-120 d^3 e^3 \left(6+11 m+6 m^2+m^3\right) x^3+30 d^2 e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4-6 d e^5 \left(120+274 m+225 m^2+85 m^3+15 m^4+m^5\right) x^5+e^6 \left(720+1764 m+1624 m^2+735 m^3+175 m^4+21 m^5+m^6\right) x^6\right)-b^7 \left(5040 d^7-5040 d^6 e (1+m) x+2520 d^5 e^2 \left(2+3 m+m^2\right) x^2-840 d^4 e^3 \left(6+11 m+6 m^2+m^3\right) x^3+210 d^3 e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4-42 d^2 e^5 \left(120+274 m+225 m^2+85 m^3+15 m^4+m^5\right) x^5+7 d e^6 \left(720+1764 m+1624 m^2+735 m^3+175 m^4+21 m^5+m^6\right) x^6-e^7 \left(5040+13068 m+13132 m^2+6769 m^3+1960 m^4+322 m^5+28 m^6+m^7\right) x^7\right)\right)$$

Problem 2147: Result more than twice size of optimal antiderivative.

$$\int (a+b x) (d+e x)^m \left(a^2+2 a b x+b^2 x^2\right)^2 \, dx$$

Optimal (type 3, 175 leaves, 3 steps):

$$-\frac{(b d-a e)^5 (d+e x)^{1+m}}{e^6 (1+m)}+\frac{5 b (b d-a e)^4 (d+e x)^{2+m}}{e^6 (2+m)}-\frac{10 b^2 (b d-a e)^3 (d+e x)^{3+m}}{e^6 (3+m)}+\frac{10 b^3 (b d-a e)^2 (d+e x)^{4+m}}{e^6 (4+m)}-\frac{5 b^4 (b d-a e) (d+e x)^{5+m}}{e^6 (5+m)}+\frac{b^5 (d+e x)^{6+m}}{e^6 (6+m)}$$

Result (type 3, 449 leaves):

$$\frac{1}{e^6 (1+m) (2+m) (3+m) (4+m) (5+m) (6+m)} (d+e x)^{1+m} \left(a^5 e^5 \left(720+1044 m+580 m^2+155 m^3+20 m^4+m^5\right)-5 a^4 b e^4 \left(360+342 m+119 m^2+18 m^3+m^4\right) (d-e (1+m) x)+10 a^3 b^2 e^3 \left(120+74 m+15 m^2+m^3\right) \left(2 d^2-2 d e (1+m) x+e^2 \left(2+3 m+m^2\right) x^2\right)+10 a^2 b^3 e^2 \left(30+11 m+m^2\right) \left(-6 d^3+6 d^2 e (1+m) x-3 d e^2 \left(2+3 m+m^2\right) x^2+e^3 \left(6+11 m+6 m^2+m^3\right) x^3\right)+5 a b^4 e (6+m) \left(24 d^4-24 d^3 e (1+m) x+12 d^2 e^2 \left(2+3 m+m^2\right) x^2-4 d e^3 \left(6+11 m+6 m^2+m^3\right) x^3+e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4\right)-b^5 \left(120 d^5-120 d^4 e (1+m) x+60 d^3 e^2 \left(2+3 m+m^2\right) x^2-20 d^2 e^3 \left(6+11 m+6 m^2+m^3\right) x^3+5 d e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4-e^5 \left(120+274 m+225 m^2+85 m^3+15 m^4+m^5\right) x^5\right)\right)$$

- **Problem 2150: Unable to integrate problem.**

$$\int \frac{(a+b x) (d+e x)^m}{\left(a^2+2 a b x+b^2 x^2\right)^2} \, dx$$

Optimal (type 5, 54 leaves, 2 steps):

$$-\frac{e^2 (d+e x)^{1+m} \text{Hypergeometric2F1}\left[3, 1+m, 2+m, \frac{b (d+e x)}{b d-a e}\right]}{(b d-a e)^3 (1+m)}$$

Result (type 8, 33 leaves):

$$\int \frac{(a+b x) (d+e x)^m}{\left(a^2+2 a b x+b^2 x^2\right)^2} \, dx$$

- **Problem 2155: Unable to integrate problem.**

$$\int \frac{(a+b x) (d+e x)^m}{\left(a^2+2 a b x+b^2 x^2\right)^{3/2}} \, dx$$

Optimal (type 5, 76 leaves, 3 steps):

$$\frac{e (a+b x) (d+e x)^{1+m} \text{Hypergeometric2F1}\left[2, 1+m, 2+m, \frac{b (d+e x)}{b d-a e}\right]}{(b d-a e)^2 (1+m) \sqrt{a^2+2 a b x+b^2 x^2}}$$

Result (type 8, 35 leaves):

$$\int \frac{(a+b x)(d+e x)^m}{\left(a^2+2 a b x+b^2 x^2\right)^{3/2}}\,dx$$

- **Problem 2156: Unable to integrate problem.**

$$\int \frac{(a+b x)(d+e x)^m}{\left(a^2+2 a b x+b^2 x^2\right)^{5/2}}\,dx$$

Optimal (type 5, 78 leaves, 3 steps):

$$\frac{e^3 (a+b x)(d+e x)^{1+m}\,\text{Hypergeometric2F1}\left[4,\,1+m,\,2+m,\,\frac{b (d+e x)}{b d-a e}\right]}{(b d-a e)^4 (1+m)\sqrt{a^2+2 a b x+b^2 x^2}}$$

Result (type 8, 35 leaves):

$$\int \frac{(a+b x)(d+e x)^m}{\left(a^2+2 a b x+b^2 x^2\right)^{5/2}}\,dx$$

- **Problem 2172: Result unnecessarily involves imaginary or complex numbers.**

$$\int (d+e x)^3 (f+g x)\sqrt{c d^2-b d e-b e^2 x-c e^2 x^2}\,dx$$

Optimal (type 3, 414 leaves, 7 steps):

$$\frac{7 (2 c d-b e)^3 (4 c e f+2 c d g-3 b e g)(b+2 c x)\sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}{512 c^5 e}-\frac{7 (2 c d-b e)^2 (4 c e f+2 c d g-3 b e g)\left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{192 c^4 e^2}-\frac{7 (2 c d-b e)(4 c e f+2 c d g-3 b e g)(d+e x)\left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{160 c^3 e^2}-\frac{(4 c e f+2 c d g-3 b e g)(d+e x)^2\left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{20 c^2 e^2}-\frac{g (d+e x)^3\left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{6 c e^2}+\frac{7 (2 c d-b e)^5 (4 c e f+2 c d g-3 b e g)\,\text{ArcTan}\left[\frac{e (b+2 c x)}{2\sqrt{c}\sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}\right]}{1024 c^{11/2} e^2}$$

Result (type 3, 500 leaves):

$$\frac{1}{15360}\sqrt{(d+e x)(-b e+c(d-e x))}\left(\frac{1}{c^5 e^2}2\left(315 b^5 e^5 g-420 b^4 c e^4 (e f+7 d g)-512 c^5 d^4 (17 e f+11 d g)+56 b^3 c^2 d e^3 (65 e f+193 d g)+16 b c^4 d^3 e (1118 e f+1047 d g)-16 b^2 c^3 d^2 e^2 (749 e f+1213 d g)\right)+\frac{1}{c^4 e}4\left(-105 b^4 e^4 g-240 c^4 d^3 (-2 e f+7 d g)+28 b^3 c e^3 (5 e f+31 d g)+16 b c^3 d^2 e (179 e f+227 d g)-8 b^2 c^2 d e^2 (133 e f+335 d g)\right) x+\frac{16\left(21 b^3 e^3 g+128 c^3 d^2 (7 e f+d g)-4 b^2 c e^2 (7 e f+38 d g)+4 b c^2 d e (46 e f+95 d g)\right) x^2}{c^3}+\frac{32 e\left(-9 b^2 e^2 g+4 b c e (3 e f+14 d g)+20 c^2 d (18 e f+17 d g)\right) x^3}{c^2}+\frac{256 e^2 (b e g+12 c (e f+3 d g)) x^4}{c}+2560 e^3 g x^5-\frac{105\, i\, (-2 c d+b e)^5 (4 c e f+2 c d g-3 b e g) \operatorname{Log}\left[-\frac{i e (b+2 c x)}{\sqrt{c}}+2\sqrt{d+e x}\sqrt{-b e+c(d-e x)}\right]}{c^{11/2} e^2 \sqrt{d+e x}\sqrt{-b e+c(d-e x)}}\right)$$

- **Problem 2173: Result unnecessarily involves imaginary or complex numbers.**

$$\int (d+e x)^2 (f+g x)\sqrt{c d^2-b d e-b e^2 x-c e^2 x^2}\, dx$$

Optimal (type 3, 339 leaves, 7 steps):

$$\frac{(2 c d-b e)^2 (10 c e f+4 c d g-7 b e g)(b+2 c x)\sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}{128 c^4 e}-\frac{(2 c d-b e)(10 c e f+4 c d g-7 b e g)\left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{48 c^3 e^2}-\frac{(10 c e f+4 c d g-7 b e g)(d+e x)\left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{40 c^2 e^2}-\frac{g (d+e x)^2\left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{5 c e^2}+\frac{(2 c d-b e)^4 (10 c e f+4 c d g-7 b e g)\operatorname{ArcTan}\left[\frac{e (b+2 c x)}{2\sqrt{c}\sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}\right]}{256 c^{9/2} e^2}$$

Result (type 3, 376 leaves):

$$\frac{1}{3840}\sqrt{(d+e x)\,(-b e+c\,(d-e x))}\left(-\frac{210\,b^4 e^2 g}{c^4}-\frac{256\,d^3\,(10 e f+7 d g)}{e^2}+\frac{20\,b^3 e\,(15 e f+76 d g)}{c^3}+\frac{16\,b\,d^2\,(285 e f+274 d g)}{c e}-\frac{8\,b^2 d\,(250 e f+499 d g)}{c^2}+\frac{4\left(35\,b^3 e^3 g-120\,c^3 d^2\,(-3 e f+2 d g)-2\,b^2 c\,e^2\,(25 e f+108 d g)+4\,b\,c^2 d\,e\,(70 e f+109 d g)\right)x}{c^3 e}+\frac{16\left(-7\,b^2 e^2 g+32\,c^2 d\,(5 e f+2 d g)+2\,b\,c\,e\,(5 e f+18 d g)\right)x^2}{c^2}+\frac{96\,e\,(b e g+10\,c\,(e f+2 d g))\,x^3}{c}+768\,e^2 g\,x^4+\frac{15\,i\,(-2 c d+b e)^4\,(10 c e f+4 c d g-7 b e g)\,\text{Log}\left[-\frac{i e\,(b+2 c x)}{\sqrt{c}}+2\sqrt{d+e x}\,\sqrt{-b e+c\,(d-e x)}\right]}{c^{9/2}\,e^2\,\sqrt{d+e x}\,\sqrt{-b e+c\,(d-e x)}}\right)$$

- **Problem 2174: Result unnecessarily involves imaginary or complex numbers.**

$$\int (d+e x)\,(f+g x)\,\sqrt{c d^2-b d e-b e^2 x-c e^2 x^2}\,dx$$

Optimal (type 3, 223 leaves, 4 steps):

$$\frac{(2 c d-b e)\,(8 c e f+2 c d g-5 b e g)\,(b+2 c x)\,\sqrt{d\,(c d-b e)-b e^2 x-c e^2 x^2}}{64\,c^3 e}+\frac{(5 b e g-8 c\,(e f+d g)-6 c e g x)\,\left(d\,(c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{24\,c^2 e^2}+\frac{(2 c d-b e)^3\,(8 c e f+2 c d g-5 b e g)\,\text{ArcTan}\left[\frac{e\,(b+2 c x)}{2\sqrt{c}\,\sqrt{d\,(c d-b e)-b e^2 x-c e^2 x^2}}\right]}{128\,c^{7/2}\,e^2}$$

Result (type 3, 270 leaves):

$$\frac{1}{384}\sqrt{(d+e x)\,(-b e+c\,(d-e x))}\left(\frac{30\,b^3 e g}{c^3}-\frac{128\,d^2\,(e f+d g)}{e^2}-\frac{8\,b^2\,(6 e f+19 d g)}{c^2}+\frac{8\,b\,d\,(28 e f+29 d g)}{c e}+4\left(-\frac{12\,d^2 g}{e}+\frac{b e\,(8 c f-5 b g)}{c^2}+d\left(48 f+\frac{20\,b g}{c}\right)\right)x+\frac{16\,(b e g+8 c\,(e f+d g))\,x^2}{c}+96\,e g\,x^3-\frac{3\,i\,(-2 c d+b e)^3\,(8 c e f+2 c d g-5 b e g)\,\text{Log}\left[-\frac{i e\,(b+2 c x)}{\sqrt{c}}+2\sqrt{d+e x}\,\sqrt{-b e+c\,(d-e x)}\right]}{c^{7/2}\,e^2\,\sqrt{d+e x}\,\sqrt{-b e+c\,(d-e x)}}\right)$$

- **Problem 2175: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(f+g x)\,\sqrt{c d^2-b d e-b e^2 x-c e^2 x^2}}{d+e x}\,dx$$

Optimal (type 3, 192 leaves, 4 steps):

$$\frac{(4 c e f-2 c d g-b e g) \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}{4 c e^2}-\frac{g \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{2 c e^2 (d+e x)}+\frac{(2 c d-b e) (4 c e f-2 c d g-b e g) \text{ArcTan}\left[\frac{e (b+2 c x)}{2 \sqrt{c} \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}\right]}{8 c^{3/2} e^2}$$

Result (type 3, 157 leaves) :

$$\frac{1}{8 e^2}\sqrt{(d+e x) (-b e+c (d-e x))} \left(8 e f-8 d g+\frac{2 b e g}{c}+4 e g x-\frac{i (2 c d-b e) (-4 c e f+2 c d g+b e g) \text{Log}\left[-\frac{i e (b+2 c x)}{\sqrt{c}}+2 \sqrt{d+e x} \sqrt{-b e+c (d-e x)}\right]}{c^{3/2} \sqrt{d+e x} \sqrt{-b e+c (d-e x)}}\right)$$

Problem 2176: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(f+g x) \sqrt{c d^2-b d e-b e^2 x-c e^2 x^2}}{(d+e x)^2} \, dx$$

Optimal (type 3, 200 leaves, 4 steps) :

$$-\frac{(2 c e f-4 c d g+b e g) \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}{e^2 (2 c d-b e)}-\frac{2 (e f-d g) \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{e^2 (2 c d-b e) (d+e x)^2}-\frac{(2 c e f-4 c d g+b e g) \text{ArcTan}\left[\frac{e (b+2 c x)}{2 \sqrt{c} \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}\right]}{2 \sqrt{c} e^2}$$

Result (type 3, 147 leaves) :

$$\frac{\sqrt{(d+e x) (-b e+c (d-e x))} \left(2 g+\frac{4 (-e f+d g)}{d+e x}-\frac{i (2 c e f-4 c d g+b e g) \text{Log}\left[-\frac{i e (b+2 c x)}{\sqrt{c}}+2 \sqrt{d+e x} \sqrt{-b e+c (d-e x)}\right]}{\sqrt{c} \sqrt{d+e x} \sqrt{-b e+c (d-e x)}}\right)}{2 e^2}$$

Problem 2177: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(f+g x) \sqrt{c d^2-b d e-b e^2 x-c e^2 x^2}}{(d+e x)^3} \, dx$$

Optimal (type 3, 168 leaves, 4 steps) :

$$-\frac{2 g \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}{e^2 (d+e x)}-\frac{2 (e f-d g) \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{3 e^2 (2 c d-b e) (d+e x)^3}-\frac{\sqrt{c} g \text{ArcTan}\left[\frac{e (b+2 c x)}{2 \sqrt{c} \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}\right]}{e^2}$$

Result (type 3, 164 leaves) :

$$\frac{\sqrt{(d+e x)(-b e+c(d-e x))}\left(\frac{2(-e f+d g)}{(d+e x)^2}-\frac{2(c e f-7 c d g+3 b e g)}{(-2 c d+b e)(d+e x)}-\frac{3 i \sqrt{c}\, g \,\text{Log}\left[-\frac{i e (b+2 c x)}{\sqrt{c}}+2\sqrt{d+e x}\sqrt{-b e+c(d-e x)}\right]}{\sqrt{d+e x}\sqrt{-b e+c(d-e x)}}\right)}{3 e^2}$$

■ **Problem 2183: Result unnecessarily involves imaginary or complex numbers.**

$$\int (d+e x)^3 (f+g x)\left(c d^2-b d e-b e^2 x-c e^2 x^2\right)^{3/2} dx$$

Optimal (type 3, 488 leaves, 8 steps) :

$$\frac{9(2 c d-b e)^5(16 c e f+6 c d g-11 b e g)(b+2 c x)\sqrt{d(c d-b e)-b e^2 x-c e^2 x^2}}{16384 c^6 e}+$$
$$\frac{3(2 c d-b e)^3(16 c e f+6 c d g-11 b e g)(b+2 c x)\left(d(c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{2048 c^5 e}-$$
$$\frac{3(2 c d-b e)^2(16 c e f+6 c d g-11 b e g)\left(d(c d-b e)-b e^2 x-c e^2 x^2\right)^{5/2}}{640 c^4 e^2}-$$
$$\frac{3(2 c d-b e)(16 c e f+6 c d g-11 b e g)(d+e x)\left(d(c d-b e)-b e^2 x-c e^2 x^2\right)^{5/2}}{448 c^3 e^2}-$$
$$\frac{(16 c e f+6 c d g-11 b e g)(d+e x)^2\left(d(c d-b e)-b e^2 x-c e^2 x^2\right)^{5/2}}{112 c^2 e^2}-\frac{g(d+e x)^3\left(d(c d-b e)-b e^2 x-c e^2 x^2\right)^{5/2}}{8 c e^2}+$$
$$\frac{9(2 c d-b e)^7(16 c e f+6 c d g-11 b e g)\,\text{ArcTan}\left[\frac{e(b+2 c x)}{2\sqrt{c}\sqrt{d(c d-b e)-b e^2 x-c e^2 x^2}}\right]}{32768 c^{13/2} e^2}$$

Result (type 3, 739 leaves) :

$$\frac{1}{32768}\left((d+e x)\left(-b e+c (d-e x)\right)\right)^{3/2}\left(\frac{1}{35 c^6 e^2 (d+e x) (-c d+b e+c e x)} 2 \left(-3465 b^7 e^7 g+210 b^6 c e^6 (24 e f+218 d g+11 e g x)-84 b^5 c^2 e^5 \left(3057 d^2 g+2 e^2 x (20 f+11 g x)+d e (760 f+334 g x)\right)+128 c^7 \left(1664 d^7 g+320 d e^6 x^5 (7 f+6 g x)+80 e^7 x^6 (8 f+7 g x)-16 d^3 e^4 x^3 (175 f+136 g x)+8 d^2 e^5 x^4 (208 f+175 g x)-8 d^5 e^2 x (245 f+176 g x)+d^6 e (2944 f+945 g x)-2 d^4 e^3 x^2 (2624 f+1925 g x)\right)+24 b^4 c^3 e^4 \left(32924 d^3 g+2 e^3 x^2 (56 f+33 g x)+8 d e^2 x (203 f+107 g x)+3 d^2 e (4704 f+1963 g x)\right)+64 b c^6 e \left(-13647 d^6 g+80 e^6 x^5 (20 f+17 g x)+6 d^4 e^2 x (-116 f+123 g x)+48 d e^5 x^4 (164 f+135 g x)+8 d^3 e^3 x^2 (1574 f+1187 g x)+8 d^2 e^4 x^3 (1882 f+1483 g x)-2 d^5 e (9812 f+3263 g x)\right)-16 b^3 c^4 e^3 \left(89587 d^4 g+8 e^4 x^3 (18 f+11 g x)+8 d e^3 x^2 (222 f+125 g x)+12 d^2 e^2 x (960 f+479 g x)+4 d^3 e (15072 f+5887 g x)\right)+32 b^2 c^5 e^2 \left(47490 d^5 g+8 e^5 x^4 (8 f+5 g x)+16 d e^4 x^3 (43 f+25 g x)+12 d^2 e^3 x^2 (308 f+163 g x)+8 d^3 e^2 x (1748 f+809 g x)+d^4 e (48712 f+17401 g x)\right)\right)+\frac{9 i (2 c d-b e)^7 (16 c e f+6 c d g-11 b e g) \text{Log}\left[-\frac{i e (b+2 c x)}{\sqrt{c}}+2 \sqrt{d+e x} \sqrt{-b e+c (d-e x)}\right]}{c^{13/2} e^2 (d+e x)^{3/2} (-b e+c (d-e x))^{3/2}}\right)$$

- **Problem 2184: Result unnecessarily involves imaginary or complex numbers.**

$$\int (d+e x)^2 (f+g x) \left(c d^2-b d e-b e^2 x-c e^2 x^2\right)^{3/2} dx$$

Optimal (type 3, 413 leaves, 8 steps):

$$\frac{(2 c d-b e)^4 (14 c e f+4 c d g-9 b e g) (b+2 c x) \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}{1024 c^5 e}+\frac{(2 c d-b e)^2 (14 c e f+4 c d g-9 b e g) (b+2 c x) \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{384 c^4 e}-\frac{(2 c d-b e) (14 c e f+4 c d g-9 b e g) \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{5/2}}{120 c^3 e^2}-\frac{(14 c e f+4 c d g-9 b e g) (d+e x) \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{5/2}}{84 c^2 e^2}-\frac{g (d+e x)^2 \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{5/2}}{7 c e^2}+\frac{(2 c d-b e)^6 (14 c e f+4 c d g-9 b e g) \text{ArcTan}\left[\frac{e (b+2 c x)}{2 \sqrt{c} \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}\right]}{2048 c^{11/2} e^2}$$

Result (type 3, 599 leaves):

$$\frac{1}{2048}\left((d+e x)(-b e+c(d-e x))\right)^{3/2}\left(\frac{1}{105 c^5 e^2 (d+e x)(-c d+b e+c e x)} 2 \left(945 b^6 e^6 g-210 b^5 c e^5 (7 e f+50 d g+3 e g x)+64 c^6 \left(432 d^6 g+112 d e^5 x^4 (6 f+5 g x)+42 d^5 e (16 f+5 g x)+40 e^6 x^5 (7 f+6 g x)-2 d^2 e^4 x^3 (35 f+24 g x)-28 d^3 e^3 x^2 (48 f+35 g x)-3 d^4 e^2 x (315 f+208 g x)\right)+28 b^4 c^2 e^4 \left(1708 d^2 g+e^2 x (35 f+18 g x)+d e (560 f+226 g x)\right)+48 b^2 c^4 e^2 \left(3037 d^4 g+2 e^4 x^3 (7 f+4 g x)+4 d e^3 x^2 (35 f+18 g x)+14 d^2 e^2 x (52 f+23 g x)+4 d^3 e (763 f+255 g x)\right)-16 b^3 c^3 e^3 \left(7090 d^3 g+e^3 x^2 (49 f+27 g x)+4 d e^2 x (147 f+71 g x)+2 d^2 e (2107 f+786 g x)\right)+32 b c^5 e \left(-3054 d^5 g-123 d^4 e (35 f+11 g x)+12 d^3 e^2 x (91 f+75 g x)+8 e^5 x^4 (91 f+75 g x)+4 d e^4 x^3 (707 f+556 g x)+2 d^2 e^3 x^2 (1911 f+1409 g x)\right)\right)+\frac{i (-2 c d+b e)^6 (14 c e f+4 c d g-9 b e g) \operatorname{Log}\left[-\frac{i e (b+2 c x)}{\sqrt{c}}+2 \sqrt{d+e x} \sqrt{-b e+c (d-e x)}\right]}{c^{11/2} e^2 (d+e x)^{3/2} (-b e+c (d-e x))^{3/2}}\right)$$

- **Problem 2185: Result unnecessarily involves imaginary or complex numbers.**

$$\int (d+e x)(f+g x)\left(c d^2-b d e-b e^2 x-c e^2 x^2\right)^{3/2} dx$$

Optimal (type 3, 297 leaves, 5 steps):

$$\frac{(2 c d-b e)^3 (12 c e f+2 c d g-7 b e g)(b+2 c x)\sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}{512 c^4 e}+\frac{(2 c d-b e)(12 c e f+2 c d g-7 b e g)(b+2 c x)\left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{192 c^3 e}+\frac{(7 b e g-12 c (e f+d g)-10 c e g x)\left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{5/2}}{60 c^2 e^2}+\frac{(2 c d-b e)^5 (12 c e f+2 c d g-7 b e g) \operatorname{ArcTan}\left[\frac{e (b+2 c x)}{2 \sqrt{c} \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}\right]}{1024 c^{9/2} e^2}$$

Result (type 3, 475 leaves):

$$\frac{1}{15360\, c^{9/2}\, e^2} \left((d+e x)\,(-b e+c\,(d-e x))\right)^{3/2} \left(\frac{1}{(d+e x)\,(-c d+b e+c e x)} \sqrt{c}\, \left(-210\, b^5 e^5 g+20\, b^4 c\, e^4\,(18 e f+94 d g+7 e g x)-16\, b^3 c^2 e^3 \left(407\, d^2 g+e^2 x\,(15 f+7 g x)+3 d e\,(65 f+23 g x)\right)+96\, b^2 c^3 e^2 \left(111\, d^3 g+e^3 x^2\,(2 f+g x)+d e^2 x\,(19 f+8 g x)+d^2 e\,(107 f+33 g x)\right)+64\, c^5 \left(48\, d^5 g+12\, d e^4 x^3\,(5 f+4 g x)+8\, e^5 x^4\,(6 f+5 g x)+3\, d^4 e\,(16 f+5 g x)-6\, d^3 e^2 x\,(25 f+16 g x)-2\, d^2 e^3 x^2\,(48 f+35 g x)\right)+32\, b\, c^4 e \left(-273\, d^4 g-6\, d^3 e\,(57 f+17 g x)+4\, e^4 x^3\,(33 f+26 g x)+6\, d^2 e^2 x\,(43 f+29 g x)+4\, d e^3 x^2\,(93 f+68 g x)\right)\right)+\frac{15\, i\,(2 c d-b e)^5\,(-7 b e g+2 c\,(6 e f+d g))\, \mathrm{Log}\left[-\frac{i e\,(b+2 c x)}{\sqrt{c}}+2 \sqrt{d+e x}\, \sqrt{-b e+c\,(d-e x)}\right]}{(d+e x)^{3/2}\,(-b e+c\,(d-e x))^{3/2}}\right)$$

Problem 2186: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(f+g x)\left(c d^2-b d e-b e^2 x-c e^2 x^2\right)^{3/2}}{d+e x}\, dx$$

Optimal (type 3, 266 leaves, 5 steps):

$$\frac{(2 c d-b e)\,(8 c e f-2 c d g-3 b e g)\,(b+2 c x)\sqrt{d\,(c d-b e)-b e^2 x-c e^2 x^2}}{64\, c^2 e}+\frac{(8 c e f-2 c d g-3 b e g)\left(d\,(c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{24\, c\, e^2}-\frac{g\left(d\,(c d-b e)-b e^2 x-c e^2 x^2\right)^{5/2}}{4\, c\, e^2\,(d+e x)}+\frac{(2 c d-b e)^3\,(8 c e f-2 c d g-3 b e g)\, \mathrm{ArcTan}\left[\frac{e\,(b+2 c x)}{2 \sqrt{c}\, \sqrt{d\,(c d-b e)-b e^2 x-c e^2 x^2}}\right]}{128\, c^{5/2}\, e^2}$$

Result (type 3, 296 leaves):

$$\frac{1}{384\, c^{5/2}\, e^2} \left((d+e x)\,(-b e+c\,(d-e x))\right)^{3/2} \left(-\left(2 \sqrt{c}\, \left(-9\, b^3 e^3 g+6\, b^2 c\, e^2\,(4 e f+6 d g+e g x)+8\, c^3 \left(8\, d^3 g-4\, d e^2 x\,(3 f+2 g x)+2\, e^3 x^2\,(4 f+3 g x)-d^2 e\,(8 f+3 g x)\right)+4\, b\, c^2 e \left(-19\, d^2 g+2 d e\,(2 f+g x)+2 e^2 x\,(14 f+9 g x)\right)\right)\right) \Big/ \left((d+e x)\,(-b e+c\,(d-e x))\right)-\frac{3\, i\,(2 c d-b e)^3\,(-8 c e f+2 c d g+3 b e g)\, \mathrm{Log}\left[-\frac{i e\,(b+2 c x)}{\sqrt{c}}+2 \sqrt{d+e x}\, \sqrt{-b e+c\,(d-e x)}\right]}{(d+e x)^{3/2}\,(-b e+c\,(d-e x))^{3/2}}\right)$$

Problem 2187: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(f+g x)\left(c d^2-b d e-b e^2 x-c e^2 x^2\right)^{3/2}}{(d+e x)^2}\, dx$$

Optimal (type 3, 278 leaves, 5 steps):

$$\frac{(6 c e f-4 c d g-b e g) (b+2 c x) \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}{8 c e}+\frac{(6 c e f-4 c d g-b e g) \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{3 e^2 (2 c d-b e)}+\frac{2 (e f-d g) \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{5/2}}{e^2 (2 c d-b e) (d+e x)^2}+\frac{(2 c d-b e)^2 (6 c e f-4 c d g-b e g) \operatorname{ArcTan}\left[\frac{e (b+2 c x)}{2 \sqrt{c} \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}\right]}{16 c^{3/2} e^2}$$

Result (type 3, 231 leaves):

$$\frac{1}{48 c^{3/2} e^2}((d+e x) (-b e+c (d-e x)))^{3/2} \left(-\frac{2 \sqrt{c} \left(3 b^2 e^2 g+2 b c e (15 e f-14 d g+7 e g x)+4 c^2 \left(10 d^2 g-6 d e (2 f+g x)+e^2 x (3 f+2 g x)\right)\right)}{(d+e x) (-b e+c (d-e x))}-\frac{3 i (-2 c d+b e)^2 (-6 c e f+4 c d g+b e g) \operatorname{Log}\left[-\frac{i e (b+2 c x)}{\sqrt{c}}+2 \sqrt{d+e x} \sqrt{-b e+c (d-e x)}\right]}{(d+e x)^{3/2} (-b e+c (d-e x))^{3/2}}\right)$$

- **Problem 2188: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(f+g x) \left(c d^2-b d e-b e^2 x-c e^2 x^2\right)^{3/2}}{(d+e x)^3} \, dx$$

Optimal (type 3, 271 leaves, 5 steps):

$$-\frac{3 (4 c e f-6 c d g+b e g) \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}{4 e^2}-\frac{(4 c e f-6 c d g+b e g) \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{2 e^2 (2 c d-b e) (d+e x)}-\frac{2 (e f-d g) \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{5/2}}{e^2 (2 c d-b e) (d+e x)^3}-\frac{3 (2 c d-b e) (4 c e f-6 c d g+b e g) \operatorname{ArcTan}\left[\frac{e (b+2 c x)}{2 \sqrt{c} \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}\right]}{8 \sqrt{c} e^2}$$

Result (type 3, 214 leaves):

$$\frac{1}{8 e^2}((d+e x) (-b e+c (d-e x)))^{3/2} \left(-\frac{2 (8 (2 c d-b e) (e f-d g)+(5 b e g+4 c (e f-3 d g)) (d+e x)+2 c e g x (d+e x))}{(d+e x)^2 (-b e+c (d-e x))}-\frac{3 i (2 c d-b e) (4 c e f-6 c d g+b e g) \operatorname{Log}\left[-\frac{i e (b+2 c x)}{\sqrt{c}}+2 \sqrt{d+e x} \sqrt{-b e+c (d-e x)}\right]}{\sqrt{c} (d+e x)^{3/2} (-b e+c (d-e x))^{3/2}}\right)$$

Problem 2189: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(f+g x)\left(c d^2-b d e-b e^2 x-c e^2 x^2\right)^{3/2}}{(d+e x)^4}\,dx$$

Optimal (type 3, 276 leaves, 5 steps):

$$\frac{c\,(2 c e f-8 c d g+3 b e g)\sqrt{d\,(c d-b e)-b e^2 x-c e^2 x^2}}{e^2\,(2 c d-b e)}+\frac{2\,(2 c e f-8 c d g+3 b e g)\left(d\,(c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{3 e^2\,(2 c d-b e)\,(d+e x)^2}-\frac{2\,(e f-d g)\left(d\,(c d-b e)-b e^2 x-c e^2 x^2\right)^{5/2}}{3 e^2\,(2 c d-b e)\,(d+e x)^4}+\frac{\sqrt{c}\,(2 c e f-8 c d g+3 b e g)\,\mathrm{ArcTan}\left[\frac{e\,(b+2 c x)}{2\sqrt{c}\sqrt{d\,(c d-b e)-b e^2 x-c e^2 x^2}}\right]}{2 e^2}$$

Result (type 3, 207 leaves):

$$\left(i\,((d+e x)\,(-b e+c\,(d-e x)))^{3/2}\left(2\,i\sqrt{-b e+c\,(d-e x)}\left(-2 b e\,(2 d g+e\,(f+3 g x))+c\left(19 d^2 g+e^2 x\,(-8 f+3 g x)+d e\,(-4 f+26 g x)\right)\right)+3\sqrt{c}\,(2 c e f-8 c d g+3 b e g)\,(d+e x)^{3/2}\,\mathrm{Log}\left[-\frac{i e\,(b+2 c x)}{\sqrt{c}}+2\sqrt{d+e x}\sqrt{-b e+c\,(d-e x)}\right]\right)\right)\Big/\left(6 e^2\,(d+e x)^3\,(-b e+c\,(d-e x))^{3/2}\right)$$

Problem 2190: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(f+g x)\left(c d^2-b d e-b e^2 x-c e^2 x^2\right)^{3/2}}{(d+e x)^5}\,dx$$

Optimal (type 3, 214 leaves, 5 steps):

$$\frac{2 c g\sqrt{d\,(c d-b e)-b e^2 x-c e^2 x^2}}{e^2\,(d+e x)}-\frac{2 g\left(d\,(c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{3 e^2\,(d+e x)^3}-\frac{2\,(e f-d g)\left(d\,(c d-b e)-b e^2 x-c e^2 x^2\right)^{5/2}}{5 e^2\,(2 c d-b e)\,(d+e x)^5}+\frac{c^{3/2}\,g\,\mathrm{ArcTan}\left[\frac{e\,(b+2 c x)}{2\sqrt{c}\sqrt{d\,(c d-b e)-b e^2 x-c e^2 x^2}}\right]}{e^2}$$

Result (type 3, 225 leaves):

$$\frac{1}{15 e^2}((d+e x)(-b e+c(d-e x)))^{3/2}$$

$$\left(-\left(2\left(3(-2 c d+b e)^2(e f-d g)+(2 c d-b e)(-6 c e f+16 c d g-5 b e g)(d+e x)+c(3 c e f-43 c d g+20 b e g)(d+e x)^2\right)\right)\Big/\right.$$

$$\left.\left((2 c d-b e)(d+e x)^4(-b e+c(d-e x))\right)+\frac{15 i c^{3/2} g \operatorname{Log}\left[-\frac{i e(b+2 c x)}{\sqrt{c}}+2\sqrt{d+e x}\sqrt{-b e+c(d-e x)}\right]}{(d+e x)^{3/2}(-b e+c(d-e x))^{3/2}}\right)$$

- **Problem 2195: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (d+e x)^3 (f+g x)\left(c d^2-b d e-b e^2 x-c e^2 x^2\right)^{5/2} dx$$

Optimal (type 3, 562 leaves, 9 steps):

$$\frac{11(2 c d-b e)^7(20 c e f+6 c d g-13 b e g)(b+2 c x)\sqrt{d(c d-b e)-b e^2 x-c e^2 x^2}}{131072 c^7 e}+$$
$$\frac{11(2 c d-b e)^5(20 c e f+6 c d g-13 b e g)(b+2 c x)\left(d(c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{49152 c^6 e}+$$
$$\frac{11(2 c d-b e)^3(20 c e f+6 c d g-13 b e g)(b+2 c x)\left(d(c d-b e)-b e^2 x-c e^2 x^2\right)^{5/2}}{15360 c^5 e}-$$
$$\frac{11(2 c d-b e)^2(20 c e f+6 c d g-13 b e g)\left(d(c d-b e)-b e^2 x-c e^2 x^2\right)^{7/2}}{4480 c^4 e^2}-$$
$$\frac{11(2 c d-b e)(20 c e f+6 c d g-13 b e g)(d+e x)\left(d(c d-b e)-b e^2 x-c e^2 x^2\right)^{7/2}}{2880 c^3 e^2}-$$
$$\frac{(20 c e f+6 c d g-13 b e g)(d+e x)^2\left(d(c d-b e)-b e^2 x-c e^2 x^2\right)^{7/2}}{180 c^2 e^2}-\frac{g(d+e x)^3\left(d(c d-b e)-b e^2 x-c e^2 x^2\right)^{7/2}}{10 c e^2}+$$
$$\frac{11(2 c d-b e)^9(20 c e f+6 c d g-13 b e g)\operatorname{ArcTan}\left[\frac{e(b+2 c x)}{2\sqrt{c}\sqrt{d(c d-b e)-b e^2 x-c e^2 x^2}}\right]}{262144 c^{15/2} e^2}$$

Result (type 3, 1491 leaves):

$$\frac{1}{(d+e x)^2 \, (c d-b e-c e x)^2}$$

$$\left(\frac{1}{41\,287\,680\,c^7 e^2} \left(-19\,005\,440\,c^9 d^8 e f+87\,795\,200\,b\,c^8 d^7 e^2 f-161\,137\,920\,b^2 c^7 d^6 e^3 f+157\,489\,280\,b^3 c^6 d^5 e^4 f-93\,114\,560\,b^4 c^5 d^4 e^5 f+\right.\right.$$

$$35\,402\,400\,b^5 c^4 d^3 e^6 f-8\,445\,360\,b^6 c^3 d^2 e^7 f+1\,155\,000\,b^7 c^2 d\,e^8 f-69\,300\,b^8 c\,e^9 f-9\,830\,400\,c^9 d^9 g+$$

$$51\,078\,400\,b\,c^8 d^8 e\,g-117\,794\,560\,b^2 c^7 d^7 e^2 g+156\,115\,200\,b^3 c^6 d^6 e^3 g-130\,302\,400\,b^4 c^5 d^5 e^4 g+$$

$$\left.71\,145\,184\,b^5 c^4 d^4 e^5 g-25\,545\,168\,b^6 c^3 d^3 e^6 g+5\,835\,984\,b^7 c^2 d^2 e^7 g-771\,540\,b^8 c\,d\,e^8 g+45\,045\,b^9 e^9 g\right)+$$

$$\frac{1}{20\,643\,840\,c^6 e} \left(11\,773\,440\,c^8 d^7 e f-14\,992\,640\,b\,c^7 d^6 e^2 f-10\,945\,920\,b^2 c^6 d^5 e^3 f+21\,264\,960\,b^3 c^5 d^4 e^4 f-9\,217\,120\,b^4 c^4 d^3 e^5 f+\right.$$

$$2\,431\,440\,b^5 c^3 d^2 e^6 f-360\,360\,b^6 c^2 d\,e^7 f+23\,100\,b^7 c\,e^8 f-2\,661\,120\,c^8 d^8 g+12\,622\,080\,b\,c^7 d^7 e\,g-24\,504\,320\,b^2 c^6 d^6 e^2 g+$$

$$\left.25\,880\,640\,b^3 c^5 d^5 e^3 g-16\,587\,360\,b^4 c^4 d^4 e^4 g+6\,720\,560\,b^5 c^3 d^3 e^5 g-1\,688\,544\,b^6 c^2 d^2 e^6 g+241\,164\,b^7 c\,d\,e^7 g-15\,015\,b^8 e^8 g\right) x+$$

$$\frac{1}{5\,160\,960\,c^5} \left(6\,553\,600\,c^7 d^6 e f-16\,717\,440\,b\,c^6 d^5 e^2 f+9\,107\,520\,b^2 c^5 d^4 e^3 f+1\,415\,360\,b^3 c^4 d^3 e^4 f-417\,120\,b^4 c^3 d^2 e^5 f+\right.$$

$$67\,320\,b^5 c^2 d\,e^6 f-4620\,b^6 c\,e^7 f+1\,966\,080\,c^7 d^7 g-3\,081\,920\,b\,c^6 d^6 e\,g-336\,000\,b^2 c^5 d^5 e^2 g+$$

$$\left.2\,246\,160\,b^3 c^4 d^4 e^3 g-1\,045\,120\,b^4 c^3 d^3 e^4 g+291\,324\,b^5 c^2 d^2 e^5 g-45\,144\,b^6 c\,d\,e^6 g+3003\,b^7 e^7 g\right) x^2+\frac{1}{2\,580\,480\,c^4}$$

$$e\left(981\,120\,c^6 d^5 e f-7\,859\,520\,b\,c^5 d^4 e^2 f+7\,487\,040\,b^2 c^4 d^3 e^3 f+151\,520\,b^3 c^3 d^2 e^4 f-26\,840\,b^4 c^2 d\,e^5 f+1980\,b^5 c\,e^6 f+2\,358\,720\,c^6 d^6 g-\right.$$

$$\left.6\,092\,160\,b\,c^5 d^5 e\,g+3\,484\,080\,b^2 c^4 d^4 e^2 g+339\,840\,b^3 c^3 d^3 e^3 g-106\,540\,b^4 c^2 d^2 e^4 g+18\,040\,b^5 c\,d\,e^5 g-1287\,b^6 e^6 g\right) x^3-$$

$$\frac{1}{322\,560\,c^3} e^2 \left(337\,920\,c^5 d^4 e f+46\,560\,b\,c^4 d^3 e^2 f-730\,320\,b^2 c^3 d^2 e^3 f-2760\,b^3 c^2 d\,e^4 f+220\,b^4 c\,e^5 f-92\,160\,c^5 d^5 g+\right.$$

$$\left.762\,000\,b\,c^4 d^4 e\,g-733\,200\,b^2 c^3 d^3 e^2 g-9960\,b^3 c^2 d^2 e^3 g+1860\,b^4 c\,d\,e^4 g-143\,b^5 e^5 g\right) x^4+$$

$$\frac{1}{161\,280\,c^2} e^3 \left(-144\,480\,c^4 d^3 e f+278\,160\,b\,c^3 d^2 e^2 f+147\,720\,b^2 c^2 d\,e^3 f+100\,b^3 c\,e^4 f-140\,112\,c^4 d^4 g-\right.$$

$$\left.16\,176\,b\,c^3 d^3 e\,g+298\,968\,b^2 c^2 d^2 e^2 g+780\,b^3 c\,d\,e^3 g-65\,b^4 e^4 g\right) x^5+\frac{1}{40\,320\,c}$$

$$e^4 \left(5120\,c^3 d^2 e f+47\,800\,b\,c^2 d\,e^2 f+6180\,b^2 c\,e^3 f-30\,720\,c^3 d^3 g+59\,396\,b\,c^2 d^2 e\,g+31\,264\,b^2 c\,d\,e^2 g+15\,b^3 e^3 g\right) x^6+$$

$$\frac{e^5 \left(1080\,c^2 d\,e f+740\,b\,c\,e^2 f+324\,c^2 d^2 g+2976\,b\,c\,d\,e\,g+383\,b^2 e^2 g\right) x^7}{2880}+$$

$$\left.\frac{1}{180} c\,e^6 (20\,c\,e f+60\,c\,d\,g+41\,b\,e\,g)\,x^8+\frac{1}{10} c^2 e^7 g\,x^9\right) ((d+e x)\,(-b e+c\,(d-e x)))^{5/2}-$$

$$\left(11\,\mathrm{i}\,(-2\,c\,d+b\,e)^9\,(20\,c\,e f+6\,c\,d\,g-13\,b\,e\,g)\,((d+e x)\,(-b e+c\,(d-e x)))^{5/2}\right.$$

$$\left.\mathrm{Log}\left[-\frac{\mathrm{i}\,e\,(b+2\,c\,x)}{\sqrt{c}}+2\,\sqrt{d+e x}\,\sqrt{c\,d-b\,e-c\,e x}\right]\right) \Big/$$

$$\left(262\,144\,c^{15/2} e^2\,(d+e x)^{5/2}\,(c\,d-b\,e-c\,e x)^{5/2}\right)$$

Problem 2196: Result unnecessarily involves imaginary or complex numbers.

$$\int (d+e x)^2 (f+g x) \left(c d^2-b d e-b e^2 x-c e^2 x^2\right)^{5/2} dx$$

Optimal (type 3, 487 leaves, 9 steps):

$$\frac{5 (2 c d-b e)^6 (18 c e f+4 c d g-11 b e g) (b+2 c x) \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}{32768 c^6 e}+\frac{5 (2 c d-b e)^4 (18 c e f+4 c d g-11 b e g) (b+2 c x) \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{12288 c^5 e}+\frac{(2 c d-b e)^2 (18 c e f+4 c d g-11 b e g) (b+2 c x) \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{5/2}}{768 c^4 e}-\frac{(2 c d-b e) (18 c e f+4 c d g-11 b e g) \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{7/2}}{224 c^3 e^2}-\frac{(18 c e f+4 c d g-11 b e g) (d+e x) \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{7/2}}{144 c^2 e^2}-\frac{g (d+e x)^2 \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{7/2}}{9 c e^2}+\frac{5 (2 c d-b e)^8 (18 c e f+4 c d g-11 b e g) \text{ArcTan}\left[\frac{e (b+2 c x)}{2 \sqrt{c} \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}\right]}{65536 c^{13/2} e^2}$$

Result (type 3, 895 leaves):

$$\frac{1}{65536} ((d+e x) (-b e+c (d-e x)))^{5/2} \left(\frac{1}{63 c^6 e^2 (d+e x)^2 (-c d+b e+c e x)^2} 2 \left(-3465 b^8 e^8 g+210 b^7 c e^7 (27 e f+248 d g+11 e g x)-84 b^6 c^2 e^6 \left(4037 d^2 g+e^2 x (45 f+22 g x)+6 d e (165 f+64 g x)\right)+72 b^5 c^3 e^5 \left(17298 d^3 g+2 e^3 x^2 (21 f+11 g x)+2 d e^2 x (357 f+166 g x)+d^2 e (7287 f+2663 g x)\right)-256 c^8 \left(1408 d^8 g-288 d e^7 x^6 (8 f+7 g x)-112 e^8 x^7 (9 f+8 g x)+18 d^7 e (128 f+35 g x)+48 d^3 e^5 x^4 (144 f+119 g x)+8 d^2 e^6 x^5 (189 f+160 g x)+6 d^4 e^4 x^3 (315 f+256 g x)-12 d^5 e^3 x^2 (576 f+413 g x)-d^6 e^2 x (5229 f+3328 g x)\right)+192 b^2 c^6 e^2 \left(-17681 d^6 g-38 d^5 e (639 f+182 g x)+8 e^6 x^5 (243 f+206 g x)+16 d e^5 x^4 (603 f+494 g x)+8 d^3 e^3 x^2 (2097 f+1546 g x)+d^4 e^2 x (1215 f+2198 g x)+4 d^2 e^4 x^3 (4707 f+3674 g x)\right)+128 b c^7 e \left(12938 d^7 g-78 d^5 e^2 x (225 f+154 g x)+16 e^7 x^6 (297 f+259 g x)+24 d^2 e^5 x^4 (549 f+457 g x)-24 d^3 e^4 x^3 (837 f+646 g x)+16 d e^6 x^5 (1053 f+898 g x)-18 d^4 e^3 x^2 (2235 f+1613 g x)+d^6 e (21357 f+5837 g x)\right)+32 b^3 c^5 e^3 \left(123452 d^5 g+8 e^5 x^4 (9 f+5 g x)+24 d e^4 x^3 (39 f+20 g x)+4 d^2 e^3 x^2 (1539 f+713 g x)+4 d^3 e^2 x (7173 f+2884 g x)+3 d^4 e (40875 f+12587 g x)\right)-16 b^4 c^4 e^4 \left(175531 d^4 g+2 e^4 x^3 (81 f+44 g x)+4 d e^3 x^2 (594 f+295 g x)+3 d^2 e^2 x (6147 f+2684 g x)+2 d^3 e (57726 f+19583 g x)\right)\right)+\frac{5 i (-2 c d+b e)^8 (18 c e f+4 c d g-11 b e g) \text{Log}\left[-\frac{i e (b+2 c x)}{\sqrt{c}}+2 \sqrt{d+e x} \sqrt{-b e+c (d-e x)}\right]}{c^{13/2} e^2 (d+e x)^{5/2} (-b e+c (d-e x))^{5/2}}\right)$$

Problem 2197: Result unnecessarily involves imaginary or complex numbers.

$$\int (d+e x) (f+g x) \left(c d^2-b d e-b e^2 x-c e^2 x^2\right)^{5/2} dx$$

Optimal (type 3, 371 leaves, 6 steps):

$$\frac{5 (2 c d - b e)^5 (16 c e f + 2 c d g - 9 b e g) (b + 2 c x) \sqrt{d (c d - b e) - b e^2 x - c e^2 x^2}}{16384 c^5 e} + \frac{5 (2 c d - b e)^3 (16 c e f + 2 c d g - 9 b e g) (b + 2 c x) \left(d (c d - b e) - b e^2 x - c e^2 x^2\right)^{3/2}}{6144 c^4 e} + \frac{(2 c d - b e) (16 c e f + 2 c d g - 9 b e g) (b + 2 c x) \left(d (c d - b e) - b e^2 x - c e^2 x^2\right)^{5/2}}{384 c^3 e} + \frac{(9 b e g - 16 c (e f + d g) - 14 c e g x) \left(d (c d - b e) - b e^2 x - c e^2 x^2\right)^{7/2}}{112 c^2 e^2} + \frac{5 (2 c d - b e)^7 (16 c e f + 2 c d g - 9 b e g) \text{ArcTan}\left[\frac{e (b+2 c x)}{2 \sqrt{c} \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}\right]}{32768 c^{11/2} e^2}$$

Result (type 3, 741 leaves):

$$\frac{1}{32768} ((d + e x) (-b e + c (d - e x)))^{5/2} \left(\frac{1}{21 c^5 e^2 (d + e x)^2 (-c d + b e + c e x)^2} 2 \left(945 b^7 e^7 g - 210 b^6 c e^6 (8 e f + 58 d g + 3 e g x) + 28 b^5 c^2 e^5 \left(2363 d^2 g + 38 d e (20 f + 7 g x) + 2 e^2 x (20 f + 9 g x)\right) - 128 c^7 \left(384 d^7 g - 64 d e^6 x^5 (7 f + 6 g x) - 48 e^7 x^6 (8 f + 7 g x) + 3 d^6 e (128 f + 35 g x) - 24 d^5 e^2 x (77 f + 48 g x) + 16 d^3 e^4 x^3 (91 f + 72 g x) + 8 d^2 e^5 x^4 (144 f + 119 g x) - 2 d^4 e^3 x^2 (576 f + 413 g x)\right) + 64 b c^6 e \left(2967 d^6 g - 8 d^2 e^4 x^3 (30 f + 19 g x) + 16 e^6 x^5 (116 f + 99 g x) + 6 d^5 e (692 f + 181 g x) + 16 d e^5 x^4 (284 f + 235 g x) - 24 d^3 e^3 x^2 (374 f + 269 g x) - 6 d^4 e^2 x (1156 f + 739 g x)\right) + 16 b^3 c^4 e^3 \left(20779 d^4 g + 24 e^4 x^3 (2 f + g x) + 8 d e^3 x^2 (74 f + 33 g x) + 20 d^2 e^2 x (192 f + 73 g x) + 4 d^3 e (5024 f + 1431 g x)\right) + 32 b^2 c^5 e^2 \left(-10434 d^5 g + 1224 d^3 e^2 x (4 f + 3 g x) + 8 e^5 x^4 (296 f + 243 g x) + 16 d e^4 x^3 (583 f + 455 g x) - 3 d^4 e (4616 f + 1227 g x) + 4 d^2 e^3 x^2 (3276 f + 2375 g x)\right) - 8 b^4 c^3 e^4 \left(24372 d^3 g + 2 e^3 x^2 (56 f + 27 g x) + 8 d e^2 x (203 f + 85 g x) + d^2 e (14112 f + 4523 g x)\right)\right) + \frac{5 i (2 c d - b e)^7 (-9 b e g + 2 c (8 e f + d g)) \text{Log}\left[-\frac{i e (b+2 c x)}{\sqrt{c}} + 2 \sqrt{d + e x} \sqrt{-b e + c (d - e x)}\right]}{c^{11/2} e^2 (d + e x)^{5/2} (-b e + c (d - e x))^{5/2}}\right)$$

- **Problem 2198: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(f + g x) \left(c d^2 - b d e - b e^2 x - c e^2 x^2\right)^{5/2}}{d + e x} \, dx$$

Optimal (type 3, 346 leaves, 6 steps):

$$-\frac{(2cd-be)^3 (5beg-2c(6ef-dg))(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{512c^3e}-\frac{(2cd-be)(5beg-2c(6ef-dg))(b+2cx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{192c^2e}+\frac{(12cef-2cdg-5beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{60ce^2}-\frac{g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{6ce^2(d+ex)}-\frac{(2cd-be)^5(5beg-2c(6ef-dg))\,\text{ArcTan}\left[\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right]}{1024c^{7/2}e^2}$$

Result (type 3, 476 leaves) :

$$\frac{1}{15360c^{7/2}e^2}((d+ex)(-be+c(d-ex)))^{5/2}\left(\frac{1}{(d+ex)^2(-cd+be+cex)^2}\sqrt{c}\left(150b^5e^5g-20b^4ce^4(18ef+62dg+5egx)+80b^3c^2e^3\left(47d^2g+e^2x(3f+gx)+de(39f+9gx)\right)-64c^5\left(48d^5g+12de^4x^3(5f+4gx)-8e^5x^4(6f+5gx)-3d^4e(16f+5gx)-6d^3e^2x(25f+16gx)+2d^2e^3x^2(48f+35gx)\right)+32bc^4e\left(207d^4g+4de^3x^2(3f+2gx)-6d^3e(7f+3gx)+4e^4x^3(63f+50gx)-6d^2e^2x(107f+67gx)\right)-96b^2c^3e^2\left(67d^3g+d^2e(43f+9gx)-e^3x^2(62f+45gx)-de^2x(109f+68gx)\right)\right)-\frac{15i(2cd-be)^5(5beg+2c(-6ef+dg))\,\text{Log}\left[-\frac{ie(b+2cx)}{\sqrt{c}}+2\sqrt{d+ex}\sqrt{-be+c(d-ex)}\right]}{(d+ex)^{5/2}(-be+c(d-ex))^{5/2}}\right)$$

Problem 2199: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$$

Optimal (type 3, 354 leaves, 6 steps) :

$$\frac{(2cd-be)^2(10cef-4cdg-3beg)(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{128c^2e}+\frac{(10cef-4cdg-3beg)(b+2cx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{48ce}+\frac{(10cef-4cdg-3beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{15e^2(2cd-be)}+\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{3e^2(2cd-be)(d+ex)^2}+\frac{(2cd-be)^4(10cef-4cdg-3beg)\,\text{ArcTan}\left[\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right]}{256c^{5/2}e^2}$$

Result (type 3, 381 leaves) :

$$\frac{1}{3840\,c^{5/2}\,e^2}\,(-c\,d+b\,e+c\,e\,x)^2\,\sqrt{(d+e\,x)\,(-b\,e+c\,(d-e\,x))}\,\left(\frac{1}{(-c\,d+b\,e+c\,e\,x)^2}\,\sqrt{c}\,\left(-90\,b^4\,e^4\,g+60\,b^3\,c\,e^3\,(5\,e\,f+8\,d\,g+e\,g\,x)-32\,c^4\,\left(56\,d^4\,g+20\,d\,e^3\,x^2\,(4\,f+3\,g\,x)-10\,d^3\,e\,(8\,f+3\,g\,x)-6\,e^4\,x^3\,(5\,f+4\,g\,x)-d^2\,e^2\,x\,(45\,f+32\,g\,x)\right)+16\,b\,c^3\,e\,\left(174\,d^3\,g+2\,e^3\,x^2\,(85\,f+63\,g\,x)-d^2\,e\,(195\,f+71\,g\,x)-2\,d\,e^2\,x\,(125\,f+82\,g\,x)\right)+8\,b^2\,c^2\,e^2\,\left(-199\,d^2\,g+d\,e\,(70\,f+32\,g\,x)+e^2\,x\,(295\,f+186\,g\,x)\right)\right)+\frac{15\,i\,(-2\,c\,d+b\,e)^4\,(10\,c\,e\,f-4\,c\,d\,g-3\,b\,e\,g)\,\mathrm{Log}\left[-\frac{i\,e\,(b+2\,c\,x)}{\sqrt{c}}+2\,\sqrt{d+e\,x}\,\sqrt{-b\,e+c\,(d-e\,x)}\right]}{\sqrt{d+e\,x}\,(-b\,e+c\,(d-e\,x))^{5/2}}\right)$$

- **Problem 2200: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-b\,e^2\,x-c\,e^2\,x^2\right)^{5/2}}{(d+e\,x)^3}\,dx$$

Optimal (type 3, 354 leaves, 7 steps):

$$\frac{5\,(2\,c\,d-b\,e)\,(8\,c\,e\,f-6\,c\,d\,g-b\,e\,g)\,(b+2\,c\,x)\,\sqrt{d\,(c\,d-b\,e)-b\,e^2\,x-c\,e^2\,x^2}}{64\,c\,e}+\frac{5\,(8\,c\,e\,f-6\,c\,d\,g-b\,e\,g)\,\left(d\,(c\,d-b\,e)-b\,e^2\,x-c\,e^2\,x^2\right)^{3/2}}{24\,e^2}+\frac{(8\,c\,e\,f-6\,c\,d\,g-b\,e\,g)\,(c\,d-b\,e-c\,e\,x)\,\left(d\,(c\,d-b\,e)-b\,e^2\,x-c\,e^2\,x^2\right)^{3/2}}{4\,e^2\,(2\,c\,d-b\,e)}+\frac{2\,(e\,f-d\,g)\,\left(d\,(c\,d-b\,e)-b\,e^2\,x-c\,e^2\,x^2\right)^{7/2}}{e^2\,(2\,c\,d-b\,e)\,(d+e\,x)^3}+\frac{5\,(2\,c\,d-b\,e)^3\,(8\,c\,e\,f-6\,c\,d\,g-b\,e\,g)\,\mathrm{ArcTan}\left[\frac{e\,(b+2\,c\,x)}{2\,\sqrt{c}\,\sqrt{d\,(c\,d-b\,e)-b\,e^2\,x-c\,e^2\,x^2}}\right]}{128\,c^{3/2}\,e^2}$$

Result (type 3, 295 leaves):

$$\frac{1}{384\,c^{3/2}\,e^2}\,((d+e\,x)\,(-b\,e+c\,(d-e\,x)))^{5/2}\,\left(\left(\sqrt{c}\,\left(30\,b^3\,e^3\,g+4\,b^2\,c\,e^2\,(132\,e\,f-118\,d\,g+59\,e\,g\,x)-16\,c^3\,\left(72\,d^3\,g+12\,d\,e^2\,x\,(3\,f+2\,g\,x)-2\,e^3\,x^2\,(4\,f+3\,g\,x)-d^2\,e\,(88\,f+45\,g\,x)\right)+8\,b\,c^2\,e\,\left(173\,d^2\,g+2\,e^2\,x\,(26\,f+17\,g\,x)-2\,d\,e\,(106\,f+51\,g\,x)\right)\right)\right)\Big/\left((d+e\,x)^2\,(-c\,d+b\,e+c\,e\,x)^2\right)-\frac{15\,i\,(2\,c\,d-b\,e)^3\,(-8\,c\,e\,f+6\,c\,d\,g+b\,e\,g)\,\mathrm{Log}\left[-\frac{i\,e\,(b+2\,c\,x)}{\sqrt{c}}+2\,\sqrt{d+e\,x}\,\sqrt{-b\,e+c\,(d-e\,x)}\right]}{(d+e\,x)^{5/2}\,(-b\,e+c\,(d-e\,x))^{5/2}}\right)$$

- **Problem 2201: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(f+g x)\left(c d^2-b d e-b e^2 x-c e^2 x^2\right)^{5/2}}{(d+e x)^4}\,dx$$

Optimal (type 3, 342 leaves, 6 steps):

$$-\frac{5(6 c e f-8 c d g+b e g)(b+2 c x)\sqrt{d(c d-b e)-b e^2 x-c e^2 x^2}}{8 e}-\frac{5 c(6 c e f-8 c d g+b e g)\left(d(c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{3 e^2(2 c d-b e)}-\frac{2(6 c e f-8 c d g+b e g)\left(d(c d-b e)-b e^2 x-c e^2 x^2\right)^{5/2}}{e^2(2 c d-b e)(d+e x)^2}-\frac{2(e f-d g)\left(d(c d-b e)-b e^2 x-c e^2 x^2\right)^{7/2}}{e^2(2 c d-b e)(d+e x)^4}-\frac{5(2 c d-b e)^2(6 c e f-8 c d g+b e g)\,\text{ArcTan}\left[\frac{e(b+2 c x)}{2\sqrt{c}\sqrt{d(c d-b e)-b e^2 x-c e^2 x^2}}\right]}{16\sqrt{c}\,e^2}$$

Result (type 3, 273 leaves):

$$\frac{1}{48 e^2}\left((d+e x)(-b e+c(d-e x))\right)^{5/2}\left(\left(6 b^2 e^2(-16 e f+27 d g+11 e g x)+4 b c e\left(-176 d^2 g+d e(123 f-67 g x)+e^2 x(27 f+13 g x)\right)+8 c^2\left(94 d^3 g+e^3 x^2(3 f+2 g x)-d e^2 x(21 f+10 g x)+d^2 e(-72 f+34 g x)\right)\right)\Big/\left((d+e x)^3(-c d+b e+c e x)^2\right)-\frac{15\,i\,(-2 c d+b e)^2(6 c e f-8 c d g+b e g)\,\text{Log}\left[-\frac{i e(b+2 c x)}{\sqrt{c}}+2\sqrt{d+e x}\sqrt{-b e+c(d-e x)}\right]}{\sqrt{c}\,(d+e x)^{5/2}(-b e+c(d-e x))^{5/2}}\right)$$

- **Problem 2202: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(f+g x)\left(c d^2-b d e-b e^2 x-c e^2 x^2\right)^{5/2}}{(d+e x)^5}\,dx$$

Optimal (type 3, 350 leaves, 6 steps):

$$\frac{5 c(4 c e f-10 c d g+3 b e g)\sqrt{d(c d-b e)-b e^2 x-c e^2 x^2}}{4 e^2}+\frac{5 c(4 c e f-10 c d g+3 b e g)\left(d(c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{6 e^2(2 c d-b e)(d+e x)}+\frac{2(4 c e f-10 c d g+3 b e g)\left(d(c d-b e)-b e^2 x-c e^2 x^2\right)^{5/2}}{3 e^2(2 c d-b e)(d+e x)^3}-\frac{2(e f-d g)\left(d(c d-b e)-b e^2 x-c e^2 x^2\right)^{7/2}}{3 e^2(2 c d-b e)(d+e x)^5}+\frac{5\sqrt{c}(2 c d-b e)(4 c e f-10 c d g+3 b e g)\,\text{ArcTan}\left[\frac{e(b+2 c x)}{2\sqrt{c}\sqrt{d(c d-b e)-b e^2 x-c e^2 x^2}}\right]}{8 e^2}$$

Result (type 3, 260 leaves):

$$\frac{1}{24 e^2 (d+e x)^4 (-b e+c (d-e x))^{5/2}} i ((d+e x) (-b e+c (d-e x)))^{5/2} \left(2 i \sqrt{-b e+c (d-e x)} \left(8 (-2 c d+b e)^2 (e f-d g)+8 (2 c d-b e) (-7 c e f+13 c d g-3 b e g) (d+e x)-3 c (9 b e g+4 c (e f-5 d g)) (d+e x)^2-6 c^2 e g x (d+e x)^2\right)+15 \sqrt{c} (2 c d-b e) (4 c e f-10 c d g+3 b e g) (d+e x)^{3/2} \text{Log}\left[-\frac{i e (b+2 c x)}{\sqrt{c}}+2 \sqrt{d+e x} \sqrt{-b e+c (d-e x)}\right]\right)$$

- **Problem 2203: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(f+g x) \left(c d^2-b d e-b e^2 x-c e^2 x^2\right)^{5/2}}{(d+e x)^6} \, dx$$

Optimal (type 3, 352 leaves, 6 steps):

$$-\frac{c^2 (2 c e f-12 c d g+5 b e g) \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}{e^2 (2 c d-b e)}-\frac{2 c (2 c e f-12 c d g+5 b e g) \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{3 e^2 (2 c d-b e) (d+e x)^2}+\frac{2 (2 c e f-12 c d g+5 b e g) \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{5/2}}{15 e^2 (2 c d-b e) (d+e x)^4}-\frac{2 (e f-d g) \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{7/2}}{5 e^2 (2 c d-b e) (d+e x)^6}-\frac{c^{3/2} (2 c e f-12 c d g+5 b e g) \text{ArcTan}\left[\frac{e (b+2 c x)}{2 \sqrt{c} \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}\right]}{2 e^2}$$

Result (type 3, 244 leaves):

$$-\frac{1}{30 e^2} ((d+e x) (-b e+c (d-e x)))^{5/2} \left(\left(2 \left(6 (-2 c d+b e)^2 (e f-d g)+2 (2 c d-b e) (-11 c e f+21 c d g-5 b e g) (d+e x)+2 c (23 c e f-93 c d g+35 b e g) (d+e x)^2-15 c^2 g (d+e x)^3\right)\right) \Big/ \left((d+e x)^5 (-c d+b e+c e x)^2\right)+\frac{15 i c^{3/2} (5 b e g+2 c (e f-6 d g)) \text{Log}\left[-\frac{i e (b+2 c x)}{\sqrt{c}}+2 \sqrt{d+e x} \sqrt{-b e+c (d-e x)}\right]}{(d+e x)^{5/2} (-b e+c (d-e x))^{5/2}}\right)$$

- **Problem 2204: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(f+g x) \left(c d^2-b d e-b e^2 x-c e^2 x^2\right)^{5/2}}{(d+e x)^7} \, dx$$

Optimal (type 3, 264 leaves, 6 steps):

$$-\frac{2 c^2 g \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}{e^2 (d+e x)}+\frac{2 c g \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}{3 e^2 (d+e x)^3}-$$

$$\frac{2 g \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{5/2}}{5 e^2 (d+e x)^5}-\frac{2 (e f-d g) \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{7/2}}{7 e^2 (2 c d-b e) (d+e x)^7}-\frac{c^{5/2} g \,\text{ArcTan}\left[\frac{e (b+2 c x)}{2 \sqrt{c} \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}\right]}{e^2}$$

Result (type 3, 266 leaves) :

$$((d+e x) (-b e+c (d-e x)))^{5/2}$$

$$\left(-\left(2 \left(15 (2 c d-b e)^3 (e f-d g)+3 (-2 c d+b e)^2 (-15 c e f+29 c d g-7 b e g) (d+e x)+c (2 c d-b e) (45 c e f-199 c d g+77 b e g) (d+e x)^2-c^2 (15 c e f-337 c d g+161 b e g) (d+e x)^3\right)\right) \Big/ \left(105 e^2 (2 c d-b e) (d+e x)^6 (-c d+b e+c e x)^2\right)-\frac{i c^{5/2} g \,\text{Log}\left[-\frac{i e (b+2 c x)}{\sqrt{c}}+2 \sqrt{d+e x} \sqrt{-b e+c (d-e x)}\right]}{e^2 (d+e x)^{5/2} (-b e+c (d-e x))^{5/2}}\right)$$

Problem 2209: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d+e x)^3 (f+g x)}{\sqrt{c d^2-b d e-b e^2 x-c e^2 x^2}} \, dx$$

Optimal (type 3, 340 leaves, 6 steps) :

$$-\frac{5 (2 c d-b e)^2 (8 c e f+6 c d g-7 b e g) \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}{64 c^4 e^2}-$$

$$\frac{5 (2 c d-b e) (8 c e f+6 c d g-7 b e g) (d+e x) \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}{96 c^3 e^2}-$$

$$\frac{(8 c e f+6 c d g-7 b e g) (d+e x)^2 \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}{24 c^2 e^2}-\frac{g (d+e x)^3 \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}{4 c e^2}+$$

$$\frac{5 (2 c d-b e)^3 (8 c e f+6 c d g-7 b e g) \,\text{ArcTan}\left[\frac{e (b+2 c x)}{2 \sqrt{c} \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}\right]}{128 c^{9/2} e^2}$$

Result (type 3, 293 leaves) :

$$\frac{1}{384\,c^{9/2}\,e^2\,\sqrt{(d+e\,x)\,(-b\,e+c\,(d-e\,x))}}\Bigg(-2\,\sqrt{c}\,(d+e\,x)\,(-b\,e+c\,(d-e\,x))\,\Big(-105\,b^3\,e^3\,g+10\,b^2\,c\,e^2\,(12\,e\,f+58\,d\,g+7\,e\,g\,x)-4\,b\,c^2\,e\,\big(259\,d^2\,g+2\,e^2\,x\,(10\,f+7\,g\,x)+2\,d\,e\,(70\,f+39\,g\,x)\big)+8\,c^3\,\big(72\,d^3\,g+12\,d\,e^2\,x\,(3\,f+2\,g\,x)+2\,e^3\,x^2\,(4\,f+3\,g\,x)+d^2\,e\,(88\,f+45\,g\,x)\big)\Big)+15\,i\,(2\,c\,d-b\,e)^3\,(8\,c\,e\,f+6\,c\,d\,g-7\,b\,e\,g)\,\sqrt{d+e\,x}\,\sqrt{-b\,e+c\,(d-e\,x)}\,\text{Log}\Big[-\frac{i\,e\,(b+2\,c\,x)}{\sqrt{c}}+2\,\sqrt{d+e\,x}\,\sqrt{-b\,e+c\,(d-e\,x)}\Big]\Bigg)$$

- **Problem 2210: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e\,x)^2\,(f+g\,x)}{\sqrt{c\,d^2-b\,d\,e-b\,e^2\,x-c\,e^2\,x^2}}\,dx$$

Optimal (type 3, 265 leaves, 6 steps):

$$-\frac{(2\,c\,d-b\,e)\,(6\,c\,e\,f+4\,c\,d\,g-5\,b\,e\,g)\,\sqrt{d\,(c\,d-b\,e)-b\,e^2\,x-c\,e^2\,x^2}}{8\,c^3\,e^2}-\frac{(6\,c\,e\,f+4\,c\,d\,g-5\,b\,e\,g)\,(d+e\,x)\,\sqrt{d\,(c\,d-b\,e)-b\,e^2\,x-c\,e^2\,x^2}}{12\,c^2\,e^2}-\frac{g\,(d+e\,x)^2\,\sqrt{d\,(c\,d-b\,e)-b\,e^2\,x-c\,e^2\,x^2}}{3\,c\,e^2}+\frac{(2\,c\,d-b\,e)^2\,(6\,c\,e\,f+4\,c\,d\,g-5\,b\,e\,g)\,\text{ArcTan}\Big[\frac{e\,(b+2\,c\,x)}{2\,\sqrt{c}\,\sqrt{d\,(c\,d-b\,e)-b\,e^2\,x-c\,e^2\,x^2}}\Big]}{16\,c^{7/2}\,e^2}$$

Result (type 3, 228 leaves):

$$\Bigg(-2\,\sqrt{c}\,(d+e\,x)\,(-b\,e+c\,(d-e\,x))\,\Big(15\,b^2\,e^2\,g-2\,b\,c\,e\,(9\,e\,f+26\,d\,g+5\,e\,g\,x)+4\,c^2\,\big(10\,d^2\,g+6\,d\,e\,(2\,f+g\,x)+e^2\,x\,(3\,f+2\,g\,x)\big)\Big)+3\,i\,(-2\,c\,d+b\,e)^2\,(6\,c\,e\,f+4\,c\,d\,g-5\,b\,e\,g)\,\sqrt{d+e\,x}\,\sqrt{-b\,e+c\,(d-e\,x)}\,\text{Log}\Big[-\frac{i\,e\,(b+2\,c\,x)}{\sqrt{c}}+2\,\sqrt{d+e\,x}\,\sqrt{-b\,e+c\,(d-e\,x)}\Big]\Bigg)\Big/\Big(48\,c^{7/2}\,e^2\,\sqrt{(d+e\,x)\,(-b\,e+c\,(d-e\,x))}\Big)$$

- **Problem 2211: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e\,x)\,(f+g\,x)}{\sqrt{c\,d^2-b\,d\,e-b\,e^2\,x-c\,e^2\,x^2}}\,dx$$

Optimal (type 3, 149 leaves, 3 steps):

$$\frac{(3\,b\,e\,g-4\,c\,(e\,f+d\,g)-2\,c\,e\,g\,x)\,\sqrt{d\,(c\,d-b\,e)-b\,e^2\,x-c\,e^2\,x^2}}{4\,c^2\,e^2}+\frac{(2\,c\,d-b\,e)\,(4\,c\,e\,f+2\,c\,d\,g-3\,b\,e\,g)\,\text{ArcTan}\Big[\frac{e\,(b+2\,c\,x)}{2\,\sqrt{c}\,\sqrt{d\,(c\,d-b\,e)-b\,e^2\,x-c\,e^2\,x^2}}\Big]}{8\,c^{5/2}\,e^2}$$

Result (type 3, 184 leaves):

$$\left(-2\sqrt{c}\,(d+e x)\,(-b e+c\,(d-e x))\,(-3 b e g+2 c\,(2 e f+2 d g+e g x))+i\,(2 c d-b e)\,(4 c e f+2 c d g-3 b e g)\,\sqrt{d+e x}\,\sqrt{-b e+c\,(d-e x)}\,\text{Log}\left[-\frac{i e\,(b+2 c x)}{\sqrt{c}}+2\sqrt{d+e x}\,\sqrt{-b e+c\,(d-e x)}\right]\right)\Big/\left(8 c^{5/2} e^2\sqrt{(d+e x)\,(-b e+c\,(d-e x))}\right)$$

Problem 2212: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{f+g x}{(d+e x)\sqrt{c d^2-b d e-b e^2 x-c e^2 x^2}}\,dx$$

Optimal (type 3, 121 leaves, 3 steps):

$$-\frac{2\,(e f-d g)\sqrt{d\,(c d-b e)-b e^2 x-c e^2 x^2}}{e^2\,(2 c d-b e)\,(d+e x)}+\frac{g\,\text{ArcTan}\left[\frac{e\,(b+2 c x)}{2\sqrt{c}\,\sqrt{d\,(c d-b e)-b e^2 x-c e^2 x^2}}\right]}{\sqrt{c}\,e^2}$$

Result (type 3, 156 leaves):

$$\left(-2\sqrt{c}\,(e f-d g)\,(-c d+b e+c e x)-i\,(2 c d-b e)\,g\sqrt{d+e x}\,\sqrt{-b e+c\,(d-e x)}\,\text{Log}\left[-\frac{i e\,(b+2 c x)}{\sqrt{c}}+2\sqrt{d+e x}\,\sqrt{-b e+c\,(d-e x)}\right]\right)\Big/\left(\sqrt{c}\,e^2\,(-2 c d+b e)\sqrt{(d+e x)\,(-b e+c\,(d-e x))}\right)$$

Problem 2217: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(d+e x)^3\,(f+g x)}{\left(c d^2-b d e-b e^2 x-c e^2 x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 287 leaves, 5 steps):

$$\frac{2\,(c e f+c d g-b e g)\,(d+e x)^3}{c e^2\,(2 c d-b e)\sqrt{d\,(c d-b e)-b e^2 x-c e^2 x^2}}+\frac{3\,(4 c e f+6 c d g-5 b e g)\sqrt{d\,(c d-b e)-b e^2 x-c e^2 x^2}}{4 c^3 e^2}+$$
$$\frac{(4 c e f+6 c d g-5 b e g)\,(d+e x)\sqrt{d\,(c d-b e)-b e^2 x-c e^2 x^2}}{2 c^2 e^2\,(2 c d-b e)}-\frac{3\,(2 c d-b e)\,(4 c e f+6 c d g-5 b e g)\,\text{ArcTan}\left[\frac{e\,(b+2 c x)}{2\sqrt{c}\,\sqrt{d\,(c d-b e)-b e^2 x-c e^2 x^2}}\right]}{8 c^{7/2} e^2}$$

Result (type 3, 229 leaves):

$$\left(2\sqrt{c}\,(d+e x)^2\,(-b e+c\,(d-e x))\,\left(15 b^2 e^2 g+b c e\,(-12 e f-43 d g+5 e g x)+2 c^2\left(14 d^2 g+5 d e\,(2 f-g x)-e^2 x\,(2 f+g x)\right)\right)-3 i\,(2 c d-b e)\,(4 c e f+6 c d g-5 b e g)\,(d+e x)^{3/2}\,(-b e+c\,(d-e x))^{3/2}\,\text{Log}\left[-\frac{i e\,(b+2 c x)}{\sqrt{c}}+2\sqrt{d+e x}\,\sqrt{-b e+c\,(d-e x)}\right]\right)\Big/\left(8 c^{7/2} e^2\,((d+e x)\,(-b e+c\,(d-e x)))^{3/2}\right)$$

Problem 2218: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d+e x)^2 (f+g x)}{\left(c d^2-b d e-b e^2 x-c e^2 x^2\right)^{3/2}} \, dx$$

Optimal (type 3, 213 leaves, 4 steps):

$$\frac{2 (c e f+c d g-b e g) (d+e x)^2}{c e^2 (2 c d-b e) \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}+\frac{(2 c e f+4 c d g-3 b e g) \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}{c^2 e^2 (2 c d-b e)}-\frac{(2 c e f+4 c d g-3 b e g) \text{ArcTan}\left[\frac{e (b+2 c x)}{2 \sqrt{c} \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}\right]}{2 c^{5/2} e^2}$$

Result (type 3, 162 leaves):

$$\left(2 \sqrt{c} (d+e x) (-3 b e g+c (2 e f+3 d g-e g x))-i (2 c e f+4 c d g-3 b e g) \sqrt{d+e x} \sqrt{-b e+c (d-e x)} \text{Log}\left[-\frac{i e (b+2 c x)}{\sqrt{c}}+2 \sqrt{d+e x} \sqrt{-b e+c (d-e x)}\right]\right) \Big/ \left(2 c^{5/2} e^2 \sqrt{(d+e x) (-b e+c (d-e x))}\right)$$

Problem 2219: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d+e x) (f+g x)}{\left(c d^2-b d e-b e^2 x-c e^2 x^2\right)^{3/2}} \, dx$$

Optimal (type 3, 129 leaves, 3 steps):

$$\frac{2 (c e f+c d g-b e g) (d+e x)}{c e^2 (2 c d-b e) \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}-\frac{g \text{ArcTan}\left[\frac{e (b+2 c x)}{2 \sqrt{c} \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}\right]}{c^{3/2} e^2}$$

Result (type 3, 155 leaves):

$$\left(-2 \sqrt{c} (c e f+c d g-b e g) (d+e x)+i (2 c d-b e) g \sqrt{d+e x} \sqrt{-b e+c (d-e x)} \text{Log}\left[-\frac{i e (b+2 c x)}{\sqrt{c}}+2 \sqrt{d+e x} \sqrt{-b e+c (d-e x)}\right]\right) \Big/ \left(c^{3/2} e^2 (-2 c d+b e) \sqrt{(d+e x) (-b e+c (d-e x))}\right)$$

Problem 2223: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d+e x)^5 (f+g x)}{\left(c d^2-b d e-b e^2 x-c e^2 x^2\right)^{5/2}} \, dx$$

Optimal (type 3, 364 leaves, 6 steps):

$$\frac{2\,(c\,e\,f+c\,d\,g-b\,e\,g)\,(d+e\,x)^5}{3\,c\,e^2\,(2\,c\,d-b\,e)\,\left(d\,(c\,d-b\,e)-b\,e^2\,x-c\,e^2\,x^2\right)^{3/2}}-\frac{2\,(4\,c\,e\,f+10\,c\,d\,g-7\,b\,e\,g)\,(d+e\,x)^3}{3\,c^2\,e^2\,(2\,c\,d-b\,e)\,\sqrt{d\,(c\,d-b\,e)-b\,e^2\,x-c\,e^2\,x^2}}-$$
$$\frac{5\,(4\,c\,e\,f+10\,c\,d\,g-7\,b\,e\,g)\,\sqrt{d\,(c\,d-b\,e)-b\,e^2\,x-c\,e^2\,x^2}}{4\,c^4\,e^2}-\frac{5\,(4\,c\,e\,f+10\,c\,d\,g-7\,b\,e\,g)\,(d+e\,x)\,\sqrt{d\,(c\,d-b\,e)-b\,e^2\,x-c\,e^2\,x^2}}{6\,c^3\,e^2\,(2\,c\,d-b\,e)}+$$
$$\frac{5\,(2\,c\,d-b\,e)\,(4\,c\,e\,f+10\,c\,d\,g-7\,b\,e\,g)\,\mathrm{ArcTan}\left[\frac{e\,(b+2\,c\,x)}{2\,\sqrt{c}\,\sqrt{d\,(c\,d-b\,e)-b\,e^2\,x-c\,e^2\,x^2}}\right]}{8\,c^{9/2}\,e^2}$$

Result (type 3, 291 leaves) :

$$\Bigg(1\Big/\left(3\,c^4\,e^2\right)2\,(d+e\,x)^3\,(-c\,d+b\,e+c\,e\,x)$$
$$\Big(-105\,b^3\,e^3\,g+10\,b^2\,c\,e^2\,(6\,e\,f+43\,d\,g-14\,e\,g\,x)+2\,c^3\,\left(118\,d^3\,g+23\,d^2\,e\,(2\,f-7\,g\,x)+3\,e^3\,x^2\,(2\,f+g\,x)+4\,d\,e^2\,x\,(-17\,f+6\,g\,x)\right)+$$
$$b\,c^2\,e\,\left(-561\,d^2\,g+e^2\,x\,(80\,f-21\,g\,x)+d\,e\,(-160\,f+438\,g\,x)\right)\Big)-1\Big/\left(c^{9/2}\,e^2\right)5\,i\,(-2\,c\,d+b\,e)\,(4\,c\,e\,f+10\,c\,d\,g-7\,b\,e\,g)$$
$$(d+e\,x)^{5/2}\,(-b\,e+c\,(d-e\,x))^{5/2}\,\mathrm{Log}\left[-\frac{i\,e\,(b+2\,c\,x)}{\sqrt{c}}+2\,\sqrt{d+e\,x}\,\sqrt{-b\,e+c\,(d-e\,x)}\right]\Bigg)\Big/\left(8\,((d+e\,x)\,(-b\,e+c\,(d-e\,x)))^{5/2}\right)$$

Problem 2224: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(d+e\,x)^4\,(f+g\,x)}{\left(c\,d^2-b\,d\,e-b\,e^2\,x-c\,e^2\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 291 leaves, 5 steps) :

$$\frac{2\,(c\,e\,f+c\,d\,g-b\,e\,g)\,(d+e\,x)^4}{3\,c\,e^2\,(2\,c\,d-b\,e)\,\left(d\,(c\,d-b\,e)-b\,e^2\,x-c\,e^2\,x^2\right)^{3/2}}-\frac{2\,(2\,c\,e\,f+8\,c\,d\,g-5\,b\,e\,g)\,(d+e\,x)^2}{3\,c^2\,e^2\,(2\,c\,d-b\,e)\,\sqrt{d\,(c\,d-b\,e)-b\,e^2\,x-c\,e^2\,x^2}}-$$
$$\frac{(2\,c\,e\,f+8\,c\,d\,g-5\,b\,e\,g)\,\sqrt{d\,(c\,d-b\,e)-b\,e^2\,x-c\,e^2\,x^2}}{c^3\,e^2\,(2\,c\,d-b\,e)}+\frac{(2\,c\,e\,f+8\,c\,d\,g-5\,b\,e\,g)\,\mathrm{ArcTan}\left[\frac{e\,(b+2\,c\,x)}{2\,\sqrt{c}\,\sqrt{d\,(c\,d-b\,e)-b\,e^2\,x-c\,e^2\,x^2}}\right]}{2\,c^{7/2}\,e^2}$$

Result (type 3, 219 leaves) :

$$\Bigg(2\,\sqrt{c}\,(d+e\,x)^3\,(-b\,e+c\,(d-e\,x))\,\left(-15\,b^2\,e^2\,g+2\,b\,c\,e\,(3\,e\,f+17\,d\,g-10\,e\,g\,x)+c^2\,\left(-19\,d^2\,g+e^2\,x\,(8\,f-3\,g\,x)+d\,e\,(-4\,f+26\,g\,x)\right)\right)+$$
$$3\,i\,(2\,c\,e\,f+8\,c\,d\,g-5\,b\,e\,g)\,(d+e\,x)^{5/2}\,(-b\,e+c\,(d-e\,x))^{5/2}\,\mathrm{Log}\left[-\frac{i\,e\,(b+2\,c\,x)}{\sqrt{c}}+2\,\sqrt{d+e\,x}\,\sqrt{-b\,e+c\,(d-e\,x)}\right]\Bigg)\Big/$$
$$\left(6\,c^{7/2}\,e^2\,((d+e\,x)\,(-b\,e+c\,(d-e\,x)))^{5/2}\right)$$

Problem 2225: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d+e x)^3 (f+g x)}{\left(c d^2-b d e-b e^2 x-c e^2 x^2\right)^{5/2}} \, dx$$

Optimal (type 3, 177 leaves, 4 steps):

$$\frac{2 (c e f+c d g-b e g) (d+e x)^3}{3 c e^2 (2 c d-b e) \left(d (c d-b e)-b e^2 x-c e^2 x^2\right)^{3/2}}-\frac{2 g (d+e x)}{c^2 e^2 \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}+\frac{g \operatorname{ArcTan}\left[\frac{e (b+2 c x)}{2 \sqrt{c} \sqrt{d (c d-b e)-b e^2 x-c e^2 x^2}}\right]}{c^{5/2} e^2}$$

Result (type 3, 202 leaves):

$$\left(-1 / (2 c d-b e) 2 \sqrt{c} (d+e x)^3 (-b e+c (d-e x)) \left(3 b^2 e^2 g+4 b c e g (-2 d+e x)+c^2 \left(5 d^2 g-e^2 f x-d e (f+7 g x)\right)\right)+3 i g (d+e x)^{5/2} (-b e+c (d-e x))^{5/2} \operatorname{Log}\left[-\frac{i e (b+2 c x)}{\sqrt{c}}+2 \sqrt{d+e x} \sqrt{-b e+c (d-e x)}\right]\right) \Big/ \left(3 c^{5/2} e^2 ((d+e x) (-b e+c (d-e x)))^{5/2}\right)$$

Problem 2292: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (d+e x)^{-3-2 p} (f+g x) \left(d (e f+d g+d g p)+e (e f+3 d g+2 d g p) x+e^2 g (2+p) x^2\right)^p \, dx$$

Optimal (type 3, 64 leaves, 1 step):

$$-\frac{(d+e x)^{-3-2 p} \left(d (e f+d g (1+p))+e (e f+d g (3+2 p)) x+e^2 g (2+p) x^2\right)^{1+p}}{e^2 (2+p)}$$

Result (type 5, 139 leaves):

$$-\frac{1}{e^2 (e f-d g)^2 (1+p)} g (d+e x)^{-2 (1+p)} ((d+e x) (d g (1+p)+e (f+g (2+p) x)))^{1+p} \left(e f-d g+g (2+p)^2 (d+e x) \left(\frac{g (2+p) (d+e x)}{-e f+d g}\right)^p \operatorname{Hypergeometric2F1}\left[1+p, 3+p, 2+p, \frac{d g (1+p)+e (f+g (2+p) x)}{e f-d g}\right]\right)$$

Problem 2301: Result unnecessarily involves imaginary or complex numbers.

$$\int (1+x)^{3/2} (a+b x) \left(1-x+x^2\right)^{3/2} \, dx$$

Optimal (type 4, 365 leaves, 6 steps):

$$\frac{54\, b\, \sqrt{1+x}\, \sqrt{1-x+x^2}}{91\left(1+\sqrt{3}+x\right)} + \frac{18\, \sqrt{1+x}\, \sqrt{1-x+x^2}\, \left(91\, a\, x + 55\, b\, x^2\right)}{5005} + \frac{2}{143}\, \sqrt{1+x}\, \sqrt{1-x+x^2}\, \left(13\, a\, x + 11\, b\, x^2\right)\, \left(1+x^3\right) -$$

$$\frac{27\times 3^{1/4}\, \sqrt{2-\sqrt{3}}\, b\, (1+x)^{3/2}\, \sqrt{1-x+x^2}\, \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{91\, \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\, \left(1+x^3\right)} +$$

$$\left(18\times 3^{3/4}\, \sqrt{2+\sqrt{3}}\, \left(91\, a - 55\left(1-\sqrt{3}\right) b\right)\, (1+x)^{3/2}\, \sqrt{1-x+x^2}\, \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]\right) \Big/$$

$$\left(5005\, \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\, \left(1+x^3\right)\right)$$

Result (type 4, 437 leaves) :

$$\frac{2 x \sqrt{1+x} \sqrt{1-x+x^2} \left(91 a \left(14+5 x^3\right)+55 b x \left(16+7 x^3\right)\right)}{5005} -$$

$$\left(9 (1+x)^{3/2} \left(-\frac{660 \sqrt{-\frac{i}{3 i+\sqrt{3}}}\, b \left(1-x+x^2\right)}{(1+x)^2}+\frac{165 i \sqrt{2} \left(i+\sqrt{3}\right) b \sqrt{\frac{3 i+\sqrt{3}-\frac{6 i}{1+x}}{3 i+\sqrt{3}}} \sqrt{\frac{-3 i+\sqrt{3}+\frac{6 i}{1+x}}{-3 i+\sqrt{3}}}\, \text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6 i}{3 i+\sqrt{3}}}}{\sqrt{1+x}}\right], \frac{3 i+\sqrt{3}}{3 i-\sqrt{3}}\right]}{\sqrt{1+x}}+\right.\right.$$

$$1 \Big/ \left(\sqrt{1+x}\right) \sqrt{2} \left(-182 i \sqrt{3}\, a+55 \left(3-i \sqrt{3}\right) b\right) \sqrt{\frac{3 i+\sqrt{3}-\frac{6 i}{1+x}}{3 i+\sqrt{3}}} \sqrt{\frac{-3 i+\sqrt{3}+\frac{6 i}{1+x}}{-3 i+\sqrt{3}}}$$

$$\left.\left. \text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6 i}{3 i+\sqrt{3}}}}{\sqrt{1+x}}\right], \frac{3 i+\sqrt{3}}{3 i-\sqrt{3}}\right]\right)\right) \Big/ \left(10010 \sqrt{-\frac{i}{3 i+\sqrt{3}}} \sqrt{1-x+x^2}\right)$$

- **Problem 2302: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{1+x}\, (a+b x) \sqrt{1-x+x^2}\, dx$$

Optimal (type 4, 326 leaves, 5 steps):

$$\frac{6 b \sqrt{1+x} \sqrt{1-x+x^2}}{7 \left(1+\sqrt{3}+x\right)}+\frac{2}{35} \sqrt{1+x} \sqrt{1-x+x^2} \left(7 a x+5 b x^2\right)-$$

$$\frac{3 \times 3^{1/4} \sqrt{2-\sqrt{3}}\, b\, (1+x)^{3/2} \sqrt{1-x+x^2} \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]}{7 \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \left(1+x^3\right)}+\frac{1}{35 \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \left(1+x^3\right)}$$

$$2 \times 3^{3/4} \sqrt{2+\sqrt{3}} \left(7 a-5 \left(1-\sqrt{3}\right) b\right) (1+x)^{3/2} \sqrt{1-x+x^2} \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]$$

Result (type 4, 423 leaves) :

$$\frac{2}{35} x \sqrt{1+x} (7 a+5 b x) \sqrt{1-x+x^2}-\frac{1}{70 \sqrt{-\frac{i}{3 i+\sqrt{3}}} \sqrt{1-x+x^2}}$$

$$(1+x)^{3/2} \left(-\frac{60 \sqrt{-\frac{i}{3 i+\sqrt{3}}} b (1-x+x^2)}{(1+x)^2}+\frac{15 i \sqrt{2} (i+\sqrt{3}) b \sqrt{\frac{3 i+\sqrt{3}-\frac{6 i}{1+x}}{3 i+\sqrt{3}}} \sqrt{\frac{-3 i+\sqrt{3}+\frac{6 i}{1+x}}{-3 i+\sqrt{3}}} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{6 i}{3 i+\sqrt{3}}}}{\sqrt{1+x}}\right], \frac{3 i+\sqrt{3}}{3 i-\sqrt{3}}\right]}{\sqrt{1+x}}+\right.$$

$$\left.1 \Big/ \left(\sqrt{1+x}\right) \sqrt{2} \left(-14 i \sqrt{3} a+5 \left(3-i \sqrt{3}\right) b\right) \sqrt{\frac{3 i+\sqrt{3}-\frac{6 i}{1+x}}{3 i+\sqrt{3}}} \sqrt{\frac{-3 i+\sqrt{3}+\frac{6 i}{1+x}}{-3 i+\sqrt{3}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{6 i}{3 i+\sqrt{3}}}}{\sqrt{1+x}}\right], \frac{3 i+\sqrt{3}}{3 i-\sqrt{3}}\right]\right)$$

Problem 2303: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b x}{\sqrt{1+x} \sqrt{1-x+x^2}} \, dx$$

Optimal (type 4, 275 leaves, 4 steps) :

$$\frac{2 b (1+x^3)}{\sqrt{1+x} \left(1+\sqrt{3}+x\right) \sqrt{1-x+x^2}}-\frac{3^{1/4} \sqrt{2-\sqrt{3}} b \sqrt{1+x} \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{1-x+x^2}}+$$

$$\frac{2 \sqrt{2+\sqrt{3}} \left(a-\left(1-\sqrt{3}\right) b\right) \sqrt{1+x} \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]}{3^{1/4} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{1-x+x^2}}$$

Result (type 4, 389 leaves) :

$$-\frac{1}{6\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

$$(1+x)^{3/2}\left(-\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}\,b\,(1-x+x^2)}{(1+x)^2}+\frac{3\,i\sqrt{2}\,\left(i+\sqrt{3}\right)b\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]}{\sqrt{1+x}}+\right.$$

$$\left.1\Big/\left(\left(\sqrt{1+x}\right)\sqrt{2}\left(-2\,i\sqrt{3}\,a+\left(3-i\sqrt{3}\right)b\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]\right)\right)$$

- **Problem 2304: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{a+b\,x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 304 leaves, 5 steps):

$$\frac{2x\,(a+b\,x)}{3\sqrt{1+x}\sqrt{1-x+x^2}}-\frac{2b\left(1+x^3\right)}{3\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}}+\frac{\sqrt{2-\sqrt{3}}\,b\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}+$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(a+b-\sqrt{3}\,b\right)\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{3\times 3^{1/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Result (type 4, 417 leaves):

$$\left(12\sqrt{-\frac{i}{3i+\sqrt{3}}}\,x\,(a+b\,x)-12\sqrt{-\frac{i}{3i+\sqrt{3}}}\,b\left(1-x+x^2\right)+\right.$$

$$3i\sqrt{2}\left(i+\sqrt{3}\right)b\,(1+x)^{3/2}\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{-\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{3i-\sqrt{3}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]+$$

$$\left.\sqrt{2}\left(2i\sqrt{3}\,a+\left(3-i\sqrt{3}\right)b\right)(1+x)^{3/2}\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{-\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{3i-\sqrt{3}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]\right)\Bigg/$$

$$\left(18\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1+x}\sqrt{1-x+x^2}\right)$$

- **Problem 2305: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{a+b\,x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 351 leaves, 6 steps):

$$\frac{2x\,(7a+5b\,x)}{27\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{2x\,(a+b\,x)}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)}-\frac{10b\left(1+x^3\right)}{27\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}}+$$

$$\frac{5\sqrt{2-\sqrt{3}}\,b\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{9\times3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}+$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(7a+5\left(1-\sqrt{3}\right)b\right)\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{27\times3^{1/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Result (type 4, 435 leaves):

$$\frac{2 x \left(b x \left(8+5 x^3\right)+a \left(10+7 x^3\right)\right)}{27 (1+x)^{3/2} \left(1-x+x^2\right)^{3/2}}+$$

$$\left((1+x)^{3/2} \left(-\frac{60 \sqrt{-\frac{i}{3 i+\sqrt{3}}}\, b \left(1-x+x^2\right)}{(1+x)^2}+\frac{15 i \sqrt{2} \left(i+\sqrt{3}\right) b \sqrt{\frac{3 i+\sqrt{3}-\frac{6 i}{1+x}}{3 i+\sqrt{3}}} \sqrt{\frac{-3 i+\sqrt{3}+\frac{6 i}{1+x}}{-3 i+\sqrt{3}}}\, \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{6 i}{3 i+\sqrt{3}}}}{\sqrt{1+x}}\right], \frac{3 i+\sqrt{3}}{3 i-\sqrt{3}}\right]}{\sqrt{1+x}}+\right.\right.$$

$$1 \Big/ \left(\sqrt{1+x}\right) \sqrt{2} \left(14 i \sqrt{3}\, a+5 \left(3-i \sqrt{3}\right) b\right) \sqrt{\frac{3 i+\sqrt{3}-\frac{6 i}{1+x}}{3 i+\sqrt{3}}} \sqrt{\frac{-3 i+\sqrt{3}+\frac{6 i}{1+x}}{-3 i+\sqrt{3}}}$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{6 i}{3 i+\sqrt{3}}}}{\sqrt{1+x}}\right], \frac{3 i+\sqrt{3}}{3 i-\sqrt{3}}\right]\right)\right) \Big/ \left(162 \sqrt{-\frac{i}{3 i+\sqrt{3}}} \sqrt{1-x+x^2}\right)$$

- **Problem 2318: Result more than twice size of optimal antiderivative.**

$$\int (A+B x) (d+e x)^5 \left(a+b x+c x^2\right)^2 \, dx$$

Optimal (type 1, 304 leaves, 2 steps):

$$-\frac{(B d-A e) \left(c d^2-b d e+a e^2\right)^2 (d+e x)^6}{6 e^6}-\frac{\left(c d^2-b d e+a e^2\right) \left(2 A e (2 c d-b e)-B \left(5 c d^2-e (3 b d-a e)\right)\right) (d+e x)^7}{7 e^6}-$$

$$\frac{\left(B \left(10 c^2 d^3+b e^2 (3 b d-2 a e)-6 c d e (2 b d-a e)\right)-A e \left(6 c^2 d^2+b^2 e^2-2 c e (3 b d-a e)\right)\right) (d+e x)^8}{8 e^6}-$$

$$\frac{\left(2 A c e (2 c d-b e)-B \left(10 c^2 d^2+b^2 e^2-2 c e (4 b d-a e)\right)\right) (d+e x)^9}{9 e^6}-\frac{c (5 B c d-2 b B e-A c e) (d+e x)^{10}}{10 e^6}+\frac{B c^2 (d+e x)^{11}}{11 e^6}$$

Result (type 1, 665 leaves):

$$a^2 A d^5 x + \frac{1}{2} a d^4 (2 A b d + a B d + 5 a A e) x^2 + \frac{1}{3} d^3 \left(a B d (2 b d + 5 a e) + A \left(b^2 d^2 + 10 a b d e + 2 a \left(c d^2 + 5 a e^2\right)\right)\right) x^3 +$$
$$\frac{1}{4} d^2 \left(b^2 d^2 (B d + 5 A e) + 2 b d \left(A c d^2 + 5 a B d e + 10 a A e^2\right) + 2 a \left(B c d^3 + 5 A c d^2 e + 5 a B d e^2 + 5 a A e^3\right)\right) x^4 +$$
$$\frac{1}{5} d \left(5 b^2 d^2 e (B d + 2 A e) + 10 a B d e \left(c d^2 + a e^2\right) + 2 b d \left(B c d^3 + 5 A c d^2 e + 10 a B d e^2 + 10 a A e^3\right) + A \left(c^2 d^4 + 20 a c d^2 e^2 + 5 a^2 e^4\right)\right) x^5 +$$
$$\frac{1}{6} \left(B \left(c^2 d^5 + 10 c d^3 e (b d + 2 a e) + 5 d e^2 \left(2 b^2 d^2 + 4 a b d e + a^2 e^2\right)\right) + A e \left(5 c^2 d^4 + 20 c d^2 e (b d + a e) + e^2 \left(10 b^2 d^2 + 10 a b d e + a^2 e^2\right)\right)\right) x^6 +$$
$$\frac{1}{7} e \left(A e \left(10 c^2 d^3 + 10 c d e (2 b d + a e) + b e^2 (5 b d + 2 a e)\right) + B \left(5 c^2 d^4 + 20 c d^2 e (b d + a e) + e^2 \left(10 b^2 d^2 + 10 a b d e + a^2 e^2\right)\right)\right) x^7 +$$
$$\frac{1}{8} e^2 \left(A e \left(10 c^2 d^2 + b^2 e^2 + 2 c e (5 b d + a e)\right) + B \left(10 c^2 d^3 + 10 c d e (2 b d + a e) + b e^2 (5 b d + 2 a e)\right)\right) x^8 +$$
$$\frac{1}{9} e^3 \left(A c e (5 c d + 2 b e) + B \left(10 c^2 d^2 + b^2 e^2 + 2 c e (5 b d + a e)\right)\right) x^9 + \frac{1}{10} c e^4 (5 B c d + 2 b B e + A c e) x^{10} + \frac{1}{11} B c^2 e^5 x^{11}$$

- **Problem 2333: Result more than twice size of optimal antiderivative.**

$$\int (A + B x) (d + e x)^5 \left(a + b x + c x^2\right)^3 \, dx$$

Optimal (type 1, 555 leaves, 2 steps) :

$$-\frac{(B d - A e) \left(c d^2 - b d e + a e^2\right)^3 (d + e x)^6}{6 e^8} - \frac{\left(c d^2 - b d e + a e^2\right)^2 \left(3 A e (2 c d - b e) - B \left(7 c d^2 - e (4 b d - a e)\right)\right) (d + e x)^7}{7 e^8} -$$
$$\frac{1}{8 e^8} 3 \left(c d^2 - b d e + a e^2\right) \left(B \left(7 c^2 d^3 - c d e (8 b d - 3 a e) + b e^2 (2 b d - a e)\right) - A e \left(5 c^2 d^2 + b^2 e^2 - c e (5 b d - a e)\right)\right) (d + e x)^8 -$$
$$\frac{1}{9 e^8} \left(A e (2 c d - b e) \left(10 c^2 d^2 + b^2 e^2 - 2 c e (5 b d - 3 a e)\right) -\right.$$
$$\left. B \left(35 c^3 d^4 - b^2 e^3 (4 b d - 3 a e) - 30 c^2 d^2 e (2 b d - a e) + 3 c e^2 \left(10 b^2 d^2 - 8 a b d e + a^2 e^2\right)\right)\right) (d + e x)^9 - \frac{1}{10 e^8}$$
$$\left(B \left(35 c^3 d^3 - b^3 e^3 + 3 b c e^2 (5 b d - 2 a e) - 15 c^2 d e (3 b d - a e)\right) - 3 A c e \left(5 c^2 d^2 + b^2 e^2 - c e (5 b d - a e)\right)\right) (d + e x)^{10} -$$
$$\frac{3 c \left(A c e (2 c d - b e) - B \left(7 c^2 d^2 + b^2 e^2 - c e (6 b d - a e)\right)\right) (d + e x)^{11}}{11 e^8} - \frac{c^2 (7 B c d - 3 b B e - A c e) (d + e x)^{12}}{12 e^8} + \frac{B c^3 (d + e x)^{13}}{13 e^8}$$

Result (type 1, 1178 leaves) :

$$a^3 A d^5 x + \frac{1}{2} a^2 d^4 (3 A b d + a B d + 5 a A e) x^2 + \frac{1}{3} a d^3 \left(a B d (3 b d + 5 a e) + A \left(3 b^2 d^2 + 15 a b d e + a \left(3 c d^2 + 10 a e^2\right)\right)\right) x^3 +$$
$$\frac{1}{4} d^2 \left(A \left(b^3 d^3 + 15 a b^2 d^2 e + 5 a^2 e \left(3 c d^2 + 2 a e^2\right) + 6 a b d \left(c d^2 + 5 a e^2\right)\right) + a B d \left(3 b^2 d^2 + 15 a b d e + a \left(3 c d^2 + 10 a e^2\right)\right)\right) x^4 +$$
$$\frac{1}{5} d \left(b^3 d^3 (B d + 5 A e) + 3 b^2 d^2 \left(A c d^2 + 5 a B d e + 10 a A e^2\right) + \right.$$
$$\left. 6 a b d \left(B c d^3 + 5 A c d^2 e + 5 a B d e^2 + 5 a A e^3\right) + a \left(5 a B d e \left(3 c d^2 + 2 a e^2\right) + A \left(3 c^2 d^4 + 30 a c d^2 e^2 + 5 a^2 e^4\right)\right)\right) x^5 +$$
$$\frac{1}{6} \left(5 b^3 d^3 e (B d + 2 A e) + 3 b^2 d^2 \left(B c d^3 + 5 A c d^2 e + 10 a B d e^2 + 10 a A e^3\right) + 3 b d \left(10 a B d e \left(c d^2 + a e^2\right) + A \left(c^2 d^4 + 20 a c d^2 e^2 + 5 a^2 e^4\right)\right) + \right.$$
$$\left. a \left(A e \left(15 c^2 d^4 + 30 a c d^2 e^2 + a^2 e^4\right) + B \left(3 c^2 d^5 + 30 a c d^3 e^2 + 5 a^2 d e^4\right)\right)\right) x^6 +$$
$$\frac{1}{7} \left(10 b^3 d^2 e^2 (B d + A e) + 15 b^2 d e \left(B c d^3 + 2 A c d^2 e + 2 a B d e^2 + a A e^3\right) + a B e \left(15 c^2 d^4 + 30 a c d^2 e^2 + a^2 e^4\right) + \right.$$
$$\left. A c d \left(c^2 d^4 + 30 a c d^2 e^2 + 15 a^2 e^4\right) + 3 b \left(A e \left(5 c^2 d^4 + 20 a c d^2 e^2 + a^2 e^4\right) + B \left(c^2 d^5 + 20 a c d^3 e^2 + 5 a^2 d e^4\right)\right)\right) x^7 +$$
$$\frac{1}{8} \left(A e \left(5 c^3 d^4 + 30 c^2 d^2 e (b d + a e) + b^2 e^3 (5 b d + 3 a e) + 3 c e^2 \left(10 b^2 d^2 + 10 a b d e + a^2 e^2\right)\right) + \right.$$
$$\left. B \left(c^3 d^5 + 15 c^2 d^3 e (b d + 2 a e) + 15 c d e^2 \left(2 b^2 d^2 + 4 a b d e + a^2 e^2\right) + b e^3 \left(10 b^2 d^2 + 15 a b d e + 3 a^2 e^2\right)\right)\right) x^8 +$$
$$\frac{1}{9} e \left(A e \left(10 c^3 d^3 + b^3 e^3 + 15 c^2 d e (2 b d + a e) + 3 b c e^2 (5 b d + 2 a e)\right) + \right.$$
$$\left. B \left(5 c^3 d^4 + 30 c^2 d^2 e (b d + a e) + b^2 e^3 (5 b d + 3 a e) + 3 c e^2 \left(10 b^2 d^2 + 10 a b d e + a^2 e^2\right)\right)\right) x^9 +$$
$$\frac{1}{10} e^2 \left(A c e \left(10 c^2 d^2 + 3 b^2 e^2 + 3 c e (5 b d + a e)\right) + B \left(10 c^3 d^3 + b^3 e^3 + 15 c^2 d e (2 b d + a e) + 3 b c e^2 (5 b d + 2 a e)\right)\right) x^{10} +$$
$$\frac{1}{11} c e^3 \left(A c e (5 c d + 3 b e) + B \left(10 c^2 d^2 + 3 b^2 e^2 + 3 c e (5 b d + a e)\right)\right) x^{11} +$$
$$\frac{1}{12} c^2 e^4 (5 B c d + 3 b B e + A c e) x^{12} + \frac{1}{13} B c^3 e^5 x^{13}$$

- **Problem 2372: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d + e x)^3 (f + g x)}{\left(a + b x + c x^2\right)^3} \, dx$$

Optimal (type 3, 195 leaves, 4 steps):

$$-\frac{(d + e x)^3 (b f - 2 a g + (2 c f - b g) x)}{2 \left(b^2 - 4 a c\right) \left(a + b x + c x^2\right)^2} + \frac{3 (2 c d f - b e f - b d g + 2 a e g) (d + e x) (b d - 2 a e + (2 c d - b e) x)}{2 \left(b^2 - 4 a c\right)^2 \left(a + b x + c x^2\right)} -$$
$$\frac{6 \left(c d^2 - b d e + a e^2\right) (2 c d f - b e f - b d g + 2 a e g) \operatorname{ArcTanh}\left[\frac{b+2 c x}{\sqrt{b^2-4 a c}}\right]}{\left(b^2 - 4 a c\right)^{5/2}}$$

Result (type 3, 550 leaves):

$$\frac{1}{2}\left(\frac{1}{c^3 \left(b^2-4 a c\right)^2 (a+x (b+c x))}\left(b^5 e^3 g+b^3 c e \left(-8 a e^2 g+3 c d (e f+d g)\right)-b^4 c e^2 (3 d g+e (f+2 g x))-4 c^3 \left(-3 c^2 d^3 f x-3 a c d e (e f+d g) x+a^2 e^2 (4 e f+12 d g+5 e g x)\right)+b^2 c^2 \left(a e^2 (5 e f+15 d g+16 e g x)-3 c d \left(3 d e f+d^2 g-2 e^2 f x-2 d e g x\right)\right)+2 b c^2 \left(11 a^2 e^3 g+3 a c e \left(d^2 g-e^2 f x+d e (f-3 g x)\right)+3 c^2 d^2 (-3 e f x+d (f-g x))\right)\right)+\left(b^4 e^3 g x+b^3 e^2 (a e g-c (e f+3 d g) x)-b^2 c e (-3 c d (e f+d g) x+a e (e f+3 d g+4 e g x))+2 c^2 \left(c^2 d^3 f x+a^2 e^2 (3 d g+e (f+g x))-a c d \left(d^2 g+3 e^2 f x+3 d e (f+g x)\right)\right)+b c \left(-3 a^2 e^3 g+c^2 d^2 (-3 e f x+d (f-g x))+3 a c e \left(d^2 g+e^2 f x+d e (f+3 g x)\right)\right)\right) \Big/ \left(c^3 \left(-b^2+4 a c\right) (a+x (b+c x))^2\right)+\frac{12 \left(c d^2+e (-b d+a e)\right) (2 c d f+2 a e g-b (e f+d g)) \operatorname{ArcTan}\left[\frac{b+2 c x}{\sqrt{-b^2+4 a c}}\right]}{\left(-b^2+4 a c\right)^{5/2}}\right)$$

Problem 2482: Result more than twice size of optimal antiderivative.

$$\int \frac{(A+B x) (d+e x)^2}{\left(a+b x+c x^2\right)^{5/2}}\, dx$$

Optimal (type 2, 121 leaves, 2 steps):

$$-\frac{2 (A b-2 a B-(b B-2 A c) x) (d+e x)^2}{3 \left(b^2-4 a c\right) \left(a+b x+c x^2\right)^{3/2}}-\frac{8 (b B d-2 A c d+A b e-2 a B e) (b d-2 a e+(2 c d-b e) x)}{3 \left(b^2-4 a c\right)^2 \sqrt{a+b x+c x^2}}$$

Result (type 2, 314 leaves):

$$\frac{1}{3 \left(b^2-4 a c\right)^2 (a+x (b+c x))^{3/2}}\left(2 A \left(-b^3 \left(d^2+6 d e x-3 e^2 x^2\right)+4 b \left(2 a^2 e^2+2 c^2 d x^2 (3 d-2 e x)+3 a c (d-e x)^2\right)+8 c \left(-2 a^2 d e+2 c^2 d^2 x^3+a c x \left(3 d^2+e^2 x^2\right)\right)+b^2 \left(-4 a e (d-3 e x)+2 c x \left(3 d^2-12 d e x+e^2 x^2\right)\right)\right)-2 B \left(16 a^3 e^2+b x \left(8 c^2 d^2 x^2+4 b c d x (3 d-e x)+b^2 \left(3 d^2-6 d e x-e^2 x^2\right)\right)+8 a^2 \left(b e (-2 d+3 e x)+c \left(d^2+3 e^2 x^2\right)\right)+2 a \left(-8 c^2 d e x^3+6 b c x (d-e x)^2+b^2 \left(d^2-12 d e x+3 e^2 x^2\right)\right)\right)\right)$$

Problem 2488: Result more than twice size of optimal antiderivative.

$$\int \frac{(A+B x) (d+e x)^5}{\left(a+b x+c x^2\right)^{7/2}}\, dx$$

Optimal (type 3, 942 leaves, 5 steps):

$$\frac{2\,(d+e\,x)^4\,\left(2\,a\,c\,(B\,d+A\,e)-b\,(A\,c\,d+a\,B\,e)-\left(b^2\,B\,e-b\,c\,(B\,d+A\,e)+2\,c\,(A\,c\,d-a\,B\,e)\right)\,x\right)}{5\,c\,\left(b^2-4\,a\,c\right)\,\left(a+b\,x+c\,x^2\right)^{5/2}}+$$

$$\frac{1}{15\,c^2\,\left(b^2-4\,a\,c\right)^2\,\left(a+b\,x+c\,x^2\right)^{3/2}}\,2\,(d+e\,x)^2\,\Big(b^3\,B\,e\,\left(3\,c\,d^2-5\,a\,e^2\right)-4\,b^2\,c\,d\,\left(2\,B\,c\,d^2+4\,A\,c\,d\,e+a\,B\,e^2\right)-16\,a\,c^2\,e\,\left(5\,a\,B\,d\,e+2\,A\,\left(c\,d^2+a\,e^2\right)\right)+$$
$$4\,b\,c\,\left(9\,a\,B\,e\,\left(c\,d^2+a\,e^2\right)+4\,A\,c\,d\,\left(c\,d^2+3\,a\,e^2\right)\right)+\Big(2\,b^3\,B\,c\,d\,e^2-5\,b^4\,B\,e^3+2\,b^2\,c\,e\,\left(7\,B\,c\,d^2+8\,A\,c\,d\,e+19\,a\,B\,e^2\right)-$$
$$8\,b\,c^2\,\left(2\,B\,c\,d^3+6\,A\,c\,d^2\,e+7\,a\,B\,d\,e^2+2\,a\,A\,e^3\right)+8\,c^2\,\left(5\,a\,B\,e\,\left(c\,d^2-a\,e^2\right)+4\,A\,c\,d\,\left(c\,d^2+a\,e^2\right)\right)\Big)\,x\Big)+$$

$$\frac{1}{15\,c^3\,\left(b^2-4\,a\,c\right)^3\,\sqrt{a+b\,x+c\,x^2}}\,2\,\Big(4\,b^4\,B\,c^2\,d^3\,e^2+5\,b^5\,B\,e^3\,\left(c\,d^2-3\,a\,e^2\right)+32\,b^2\,c^3\,d^2\,\left(2\,B\,c\,d^3+8\,A\,c\,d^2\,e+17\,a\,B\,d\,e^2+16\,a\,A\,e^3\right)+$$
$$64\,a\,c^3\,e\,\left(4\,A\,\left(c\,d^2+a\,e^2\right)^2+5\,a\,B\,d\,e\,\left(c\,d^2+4\,a\,e^2\right)\right)-8\,b^3\,c\,e\,\left(16\,A\,c^2\,d^3\,e+B\,\left(11\,c^2\,d^4+7\,a\,c\,d^2\,e^2-20\,a^2\,e^4\right)\right)-$$
$$16\,b\,c^2\,\left(8\,A\,c\,d\,\left(c^2\,d^4+6\,a\,c\,d^2\,e^2+5\,a^2\,e^4\right)+a\,B\,e\,\left(18\,c^2\,d^4+71\,a\,c\,d^2\,e^2+33\,a^2\,e^4\right)\right)+$$
$$\Big(10\,b^5\,B\,c\,d\,e^4-15\,b^6\,B\,e^5+2\,b^4\,B\,c\,e^3\,\left(3\,c\,d^2+85\,a\,e^2\right)+16\,b^3\,c^2\,d\,e^2\,\left(6\,B\,c\,d^2+8\,A\,c\,d\,e-7\,a\,B\,e^2\right)-32\,c^3$$
$$\left(8\,A\,c\,d\,\left(c\,d^2+a\,e^2\right)^2+5\,a\,B\,e\,\left(2\,c^2\,d^4+5\,a\,c\,d^2\,e^2-3\,a^2\,e^4\right)\right)-16\,b^2\,c^2\,e\,\left(16\,A\,c\,d\,e\,\left(2\,c\,d^2+a\,e^2\right)+B\,\left(15\,c^2\,d^4+29\,a\,c\,d^2\,e^2+39\,a^2\,e^4\right)\right)+$$
$$32\,b\,c^3\,\left(4\,A\,e\,\left(5\,c^2\,d^4+6\,a\,c\,d^2\,e^2+a^2\,e^4\right)+B\,\left(4\,c^2\,d^5+28\,a\,c\,d^3\,e^2+29\,a^2\,d\,e^4\right)\right)\Big)\,x\Big)+\frac{B\,e^5\,\text{ArcTanh}\left[\frac{b+2\,c\,x}{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}\right]}{c^{7/2}}$$

Result (type 3, 2431 leaves) :

$$\frac{1}{(a+x\,(b+c\,x))^{7/2}}$$

$$\left(a+b\,x+c\,x^2\right)^4\left(\frac{1}{5\,c^5\,\left(-b^2+4\,a\,c\right)\,\left(a+b\,x+c\,x^2\right)^3}\,2\,\left(A\,b\,c^5\,d^5-2\,a\,B\,c^5\,d^5+5\,a\,b\,B\,c^4\,d^4\,e-10\,a\,A\,c^5\,d^4\,e-10\,a\,b^2\,B\,c^3\,d^3\,e^2+10\,a\,A\,b\,c^4\,d^3\,e^2+\right.\right.$$

$$20\,a^2\,B\,c^4\,d^3\,e^2+10\,a\,b^3\,B\,c^2\,d^2\,e^3-10\,a\,A\,b^2\,c^3\,d^2\,e^3-30\,a^2\,b\,B\,c^3\,d^2\,e^3+20\,a^2\,A\,c^4\,d^2\,e^3-5\,a\,b^4\,B\,c\,d\,e^4+5\,a\,A\,b^3\,c^2\,d\,e^4+$$
$$20\,a^2\,b^2\,B\,c^2\,d\,e^4-15\,a^2\,A\,b\,c^3\,d\,e^4-10\,a^3\,B\,c^3\,d\,e^4+a\,b^5\,B\,e^5-a\,A\,b^4\,c\,e^5-5\,a^2\,b^3\,B\,c\,e^5+4\,a^2\,A\,b^2\,c^2\,e^5+5\,a^3\,b\,B\,c^2\,e^5-$$
$$2\,a^3\,A\,c^3\,e^5-b\,B\,c^5\,d^5\,x+2\,A\,c^6\,d^5\,x+5\,b^2\,B\,c^4\,d^4\,e\,x-5\,A\,b\,c^5\,d^4\,e\,x-10\,a\,B\,c^5\,d^4\,e\,x-10\,b^3\,B\,c^3\,d^3\,e^2\,x+10\,A\,b^2\,c^4\,d^3\,e^2\,x+$$
$$30\,a\,b\,B\,c^4\,d^3\,e^2\,x-20\,a\,A\,c^5\,d^3\,e^2\,x+10\,b^4\,B\,c^2\,d^2\,e^3\,x-10\,A\,b^3\,c^3\,d^2\,e^3\,x-40\,a\,b^2\,B\,c^3\,d^2\,e^3\,x+30\,a\,A\,b\,c^4\,d^2\,e^3\,x+$$
$$20\,a^2\,B\,c^4\,d^2\,e^3\,x-5\,b^5\,B\,c\,d\,e^4\,x+5\,A\,b^4\,c^2\,d\,e^4\,x+25\,a\,b^3\,B\,c^2\,d\,e^4\,x-20\,a\,A\,b^2\,c^3\,d\,e^4\,x-25\,a^2\,b\,B\,c^3\,d\,e^4\,x+$$
$$\left.10\,a^2\,A\,c^4\,d\,e^4\,x+b^6\,B\,e^5\,x-A\,b^5\,c\,e^5\,x-6\,a\,b^4\,B\,c\,e^5\,x+5\,a\,A\,b^3\,c^2\,e^5\,x+9\,a^2\,b^2\,B\,c^2\,e^5\,x-5\,a^2\,A\,b\,c^3\,e^5\,x-2\,a^3\,B\,c^3\,e^5\,x\right)+$$

$$\frac{1}{15\,c^5\,\left(-b^2+4\,a\,c\right)^2\,\left(a+b\,x+c\,x^2\right)^2}\,2\,\left(-8\,b^2\,B\,c^5\,d^5+16\,A\,b\,c^6\,d^5+15\,b^3\,B\,c^4\,d^4\,e-40\,A\,b^2\,c^5\,d^4\,e+20\,a\,b\,B\,c^5\,d^4\,e-30\,b^4\,B\,c^3\,d^3\,e^2+\right.$$

$$30\,A\,b^3\,c^4\,d^3\,e^2+140\,a\,b^2\,B\,c^4\,d^3\,e^2+40\,a\,A\,b\,c^5\,d^3\,e^2-400\,a^2\,B\,c^5\,d^3\,e^2+30\,b^5\,B\,c^2\,d^2\,e^3-30\,A\,b^4\,c^3\,d^2\,e^3-220\,a\,b^3\,B\,c^3\,d^2\,e^3+$$
$$140\,a\,A\,b^2\,c^4\,d^2\,e^3+560\,a^2\,b\,B\,c^4\,d^2\,e^3-400\,a^2\,A\,c^5\,d^2\,e^3-15\,b^6\,B\,c\,d\,e^4+15\,A\,b^5\,c^2\,d\,e^4+150\,a\,b^4\,B\,c^2\,d\,e^4-110\,a\,A\,b^3\,c^3\,d\,e^4-$$
$$500\,a^2\,b^2\,B\,c^3\,d\,e^4+280\,a^2\,A\,b\,c^4\,d\,e^4+400\,a^3\,B\,c^4\,d\,e^4+3\,b^7\,B\,e^5-3\,A\,b^6\,c\,e^5-38\,a\,b^5\,B\,c\,e^5+30\,a\,A\,b^4\,c^2\,e^5+157\,a^2\,b^3\,B\,c^2\,e^5-$$
$$100\,a^2\,A\,b^2\,c^3\,e^5-196\,a^3\,b\,B\,c^3\,e^5+80\,a^3\,A\,c^4\,e^5-16\,b\,B\,c^6\,d^5\,x+32\,A\,c^7\,d^5\,x+30\,b^2\,B\,c^5\,d^4\,e\,x-80\,A\,b\,c^6\,d^4\,e\,x+40\,a\,B\,c^6\,d^4\,e\,x-$$
$$10\,b^3\,B\,c^4\,d^3\,e^2\,x+60\,A\,b^2\,c^5\,d^3\,e^2\,x-120\,a\,b\,B\,c^5\,d^3\,e^2\,x+80\,a\,A\,c^6\,d^3\,e^2\,x-40\,b^4\,B\,c^3\,d^2\,e^3\,x-10\,A\,b^3\,c^4\,d^2\,e^3\,x+360\,a\,b^2\,B\,c^4\,d^2\,e^3\,x-$$
$$120\,a\,A\,b\,c^5\,d^2\,e^3\,x-480\,a^2\,B\,c^5\,d^2\,e^3\,x+45\,b^5\,B\,c^2\,d\,e^4\,x-20\,A\,b^4\,c^3\,d\,e^4\,x-350\,a\,b^3\,B\,c^3\,d\,e^4\,x+180\,a\,A\,b^2\,c^4\,d\,e^4\,x+600\,a^2\,b\,B\,c^4\,d\,e^4\,x-$$
$$\left.240\,a^2\,A\,c^5\,d\,e^4\,x-14\,b^6\,B\,c\,e^5\,x+9\,A\,b^5\,c^2\,e^5\,x+114\,a\,b^4\,B\,c^2\,e^5\,x-70\,a\,A\,b^3\,c^3\,e^5\,x-246\,a^2\,b^2\,B\,c^3\,e^5\,x+120\,a^2\,A\,b\,c^4\,e^5\,x+88\,a^3\,B\,c^4\,e^5\,x\right)+$$

$$\frac{1}{15\,c^4\,\left(-b^2+4\,a\,c\right)^3\,\left(a+b\,x+c\,x^2\right)}\,2\,\left(-64\,b^2\,B\,c^5\,d^5+128\,A\,b\,c^6\,d^5+120\,b^3\,B\,c^4\,d^4\,e-320\,A\,b^2\,c^5\,d^4\,e+160\,a\,b\,B\,c^5\,d^4\,e-\right.$$

$$40\,b^4\,B\,c^3\,d^3\,e^2+240\,A\,b^3\,c^4\,d^3\,e^2-480\,a\,b^2\,B\,c^4\,d^3\,e^2+320\,a\,A\,b\,c^5\,d^3\,e^2-10\,b^5\,B\,c^2\,d^2\,e^3-40\,A\,b^4\,c^3\,d^2\,e^3+240\,a\,b^3\,B\,c^3\,d^2\,e^3-$$
$$480\,a\,A\,b^2\,c^4\,d^2\,e^3+480\,a^2\,b\,B\,c^4\,d^2\,e^3+30\,b^6\,B\,c\,d\,e^4-5\,A\,b^5\,c^2\,d\,e^4-350\,a\,b^4\,B\,c^2\,d\,e^4+120\,a\,A\,b^3\,c^3\,d\,e^4+1200\,a^2\,b^2\,B\,c^3\,d\,e^4+$$
$$240\,a^2\,A\,b\,c^4\,d\,e^4-2400\,a^3\,B\,c^4\,d\,e^4-11\,b^7\,B\,e^5+6\,A\,b^6\,c\,e^5+141\,a\,b^5\,B\,c\,e^5-70\,a\,A\,b^4\,c^2\,e^5-624\,a^2\,b^3\,B\,c^2\,e^5+240\,a^2\,A\,b^2\,c^3\,e^5+$$
$$1072\,a^3\,b\,B\,c^3\,e^5-480\,a^3\,A\,c^4\,e^5-128\,b\,B\,c^6\,d^5\,x+256\,A\,c^7\,d^5\,x+240\,b^2\,B\,c^5\,d^4\,e\,x-640\,A\,b\,c^6\,d^4\,e\,x+320\,a\,B\,c^6\,d^4\,e\,x-80\,b^3\,B\,c^4\,d^3\,e^2\,x+$$
$$480\,A\,b^2\,c^5\,d^3\,e^2\,x-960\,a\,b\,B\,c^5\,d^3\,e^2\,x+640\,a\,A\,c^6\,d^3\,e^2\,x-20\,b^4\,B\,c^3\,d^2\,e^3\,x-80\,A\,b^3\,c^4\,d^2\,e^3\,x+480\,a\,b^2\,B\,c^4\,d^2\,e^3\,x-960\,a\,A\,b\,c^5\,d^2\,e^3\,x+$$
$$960\,a^2\,B\,c^5\,d^2\,e^3\,x-15\,b^5\,B\,c^2\,d\,e^4\,x-10\,A\,b^4\,c^3\,d\,e^4\,x+200\,a\,b^3\,B\,c^3\,d\,e^4\,x+240\,a\,A\,b^2\,c^4\,d\,e^4\,x-1200\,a^2\,b\,B\,c^4\,d\,e^4\,x+480\,a^2\,A\,c^5\,d\,e^4\,x+$$
$$\left.\left.23\,b^6\,B\,c\,e^5\,x-3\,A\,b^5\,c^2\,e^5\,x-258\,a\,b^4\,B\,c^2\,e^5\,x+40\,a\,A\,b^3\,c^3\,e^5\,x+912\,a^2\,b^2\,B\,c^3\,e^5\,x-240\,a^2\,A\,b\,c^4\,e^5\,x-736\,a^3\,B\,c^4\,e^5\,x\right)\right)+$$

$$\frac{B\,e^5\,\left(a+b\,x+c\,x^2\right)^{7/2}\,\mathrm{Log}\left[b+2\,c\,x+2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}\,\right]}{c^{7/2}\,(a+x\,(b+c\,x))^{7/2}}$$

- **Problem 2489: Result more than twice size of optimal antiderivative.**

$$\int\frac{(A+B\,x)\,(d+e\,x)^4}{\left(a+b\,x+c\,x^2\right)^{7/2}}\,dx$$

Optimal (type 2, 210 leaves, 3 steps) :

$$-\frac{2\,(A\,b-2\,a\,B-(b\,B-2\,A\,c)\,x)\,(d+e\,x)^4}{5\,(b^2-4\,a\,c)\,(a+b\,x+c\,x^2)^{5/2}}-\frac{16\,(b\,B\,d-2\,A\,c\,d+A\,b\,e-2\,a\,B\,e)\,(d+e\,x)^2\,(b\,d-2\,a\,e+(2\,c\,d-b\,e)\,x)}{15\,(b^2-4\,a\,c)^2\,(a+b\,x+c\,x^2)^{3/2}}+$$
$$\frac{128\,(b\,B\,d-2\,A\,c\,d+A\,b\,e-2\,a\,B\,e)\,(c\,d^2-b\,d\,e+a\,e^2)\,(b\,d-2\,a\,e+(2\,c\,d-b\,e)\,x)}{15\,(b^2-4\,a\,c)^3\,\sqrt{a+b\,x+c\,x^2}}$$

Result (type 2, 1196 leaves):

$$\frac{1}{15\,(b^2-4\,a\,c)^3\,(a+x\,(b+c\,x))^{5/2}}$$
$$\Big(-2\,A\,\Big(b^5\,(3\,d^4+20\,d^3\,e\,x+90\,d^2\,e^2\,x^2-60\,d\,e^3\,x^3-5\,e^4\,x^4)+16\,b\,\big(8\,a^4\,e^4+8\,c^4\,d^3\,x^4\,(5\,d-4\,e\,x)+15\,a^2\,c^2\,(d-e\,x)^4+$$
$$4\,a^3\,c\,e^2\,(9\,d^2-10\,d\,e\,x+5\,e^2\,x^2)+4\,a\,c^3\,d\,x^2\,(15\,d^3-20\,d^2\,e\,x+15\,d\,e^2\,x^2-6\,e^3\,x^3)\big)+$$
$$8\,b^3\,\big(-5\,a\,c\,(d-e\,x)^2\,(d^2+14\,d\,e\,x-3\,e^2\,x^2)+6\,a^2\,e^2\,(d^2-10\,d\,e\,x+5\,e^2\,x^2)+2\,c^2\,d\,x^2\,(5\,d^3-60\,d^2\,e\,x+45\,d\,e^2\,x^2-2\,e^3\,x^3)\big)+$$
$$32\,c\,\big(-8\,a^4\,d\,e^3+8\,c^4\,d^4\,x^5+4\,a\,c^3\,d^2\,x^3\,(5\,d^2+3\,e^2\,x^2)-4\,a^3\,c\,d\,e\,(3\,d^2+5\,e^2\,x^2)+3\,a^2\,c^2\,x\,(5\,d^4+10\,d^2\,e^2\,x^2+e^4\,x^4)\big)+$$
$$16\,b^2\,\big(4\,a^3\,e^3\,(-3\,d+5\,e\,x)+2\,c^3\,d^2\,x^3\,(15\,d^2-40\,d\,e\,x+9\,e^2\,x^2)+$$
$$6\,a^2\,c\,e\,(-2\,d^3+15\,d^2\,e\,x-10\,d\,e^2\,x^2+5\,e^3\,x^3)+3\,a\,c^2\,x\,(5\,d^4-40\,d^3\,e\,x+30\,d^2\,e^2\,x^2-20\,d\,e^3\,x^3+e^4\,x^4)\big)+$$
$$2\,b^4\,\big(4\,a\,e\,(d^3+15\,d^2\,e\,x-45\,d\,e^2\,x^2+5\,e^3\,x^3)-c\,x\,(5\,d^4+80\,d^3\,e\,x-270\,d^2\,e^2\,x^2+40\,d\,e^3\,x^3+e^4\,x^4)\big)\Big)+$$
$$2\,B\,\Big(256\,a^5\,e^4+128\,a^4\,e^2\,\big(b\,e\,(-4\,d+5\,e\,x)+c\,(3\,d^2+5\,e^2\,x^2)\big)+b\,x\,\big(128\,c^4\,d^4\,x^4+64\,b\,c^3\,d^3\,x^3\,(5\,d-3\,e\,x)+48\,b^2\,c^2\,d^2\,x^2\,(5\,d^2-10\,d\,e\,x+e^2\,x^2)+$$
$$8\,b^3\,c\,d\,x\,(5\,d^3-45\,d^2\,e\,x+15\,d\,e^2\,x^2+e^3\,x^3)+b^4\,(-5\,d^4-60\,d^3\,e\,x+90\,d^2\,e^2\,x^2+20\,d\,e^3\,x^3+3\,e^4\,x^4)\big)+$$
$$32\,a^3\,\big(b^2\,e^2\,(9\,d^2-40\,d\,e\,x+15\,e^2\,x^2)+2\,b\,c\,e\,(-6\,d^3+15\,d^2\,e\,x-20\,d\,e^2\,x^2+15\,e^3\,x^3)+3\,c^2\,(d^4+10\,d^2\,e^2\,x^2+5\,e^4\,x^4)\big)-$$
$$16\,a^2\,\big(-15\,b\,c^2\,x\,(d-e\,x)^4+8\,c^3\,d\,e\,x^3\,(5\,d^2+3\,e^2\,x^2)+$$
$$b^3\,e\,(2\,d^3-45\,d^2\,e\,x+60\,d\,e^2\,x^2-5\,e^3\,x^3)-3\,b^2\,c\,(d^4-20\,d^3\,e\,x+30\,d^2\,e^2\,x^2-40\,d\,e^3\,x^3+5\,e^4\,x^4)\big)-$$
$$2\,a\,\big(128\,c^4\,d^3\,e\,x^5+20\,b^3\,c\,x\,(d-e\,x)^2\,(-3\,d^2+14\,d\,e\,x+e^2\,x^2)-32\,b\,c^3\,d^2\,x^3\,(5\,d^2-10\,d\,e\,x+9\,e^2\,x^2)+$$
$$48\,b^2\,c^2\,d\,x^2\,(-5\,d^3+10\,d^2\,e\,x-15\,d\,e^2\,x^2+2\,e^3\,x^3)+b^4\,(d^4+40\,d^3\,e\,x-270\,d^2\,e^2\,x^2+80\,d\,e^3\,x^3+5\,e^4\,x^4)\big)\Big)\Big)$$

- **Problem 2490: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+B\,x)\,(d+e\,x)^3}{(a+b\,x+c\,x^2)^{7/2}}\,dx$$

Optimal (type 2, 264 leaves, 3 steps):

$$-\frac{2\,(A\,b-2\,a\,B-(b\,B-2\,A\,c)\,x)\,(d+e\,x)^3}{5\,(b^2-4\,a\,c)\,(a+b\,x+c\,x^2)^{5/2}}-$$
$$\Big(4\,(d+e\,x)^2\,\big(4\,a\,A\,c\,e+b^2\,(4\,B\,d+3\,A\,e)-8\,b\,(A\,c\,d+a\,B\,e)-(b^2\,B\,e-8\,b\,c\,(B\,d+A\,e)+4\,c\,(4\,A\,c\,d+3\,a\,B\,e))\,x\big)\Big)\Big/$$
$$\Big(15\,(b^2-4\,a\,c)^2\,(a+b\,x+c\,x^2)^{3/2}\Big)-$$
$$\Big(16\,\big(b^2\,e\,(5\,B\,d+3\,A\,e)+4\,c\,(4\,A\,c\,d^2+3\,a\,B\,d\,e+a\,A\,e^2)-8\,b\,(B\,c\,d^2+2\,A\,c\,d\,e+a\,B\,e^2)\big)\,(b\,d-2\,a\,e+(2\,c\,d-b\,e)\,x)\Big)\Big/$$
$$\Big(15\,(b^2-4\,a\,c)^3\,\sqrt{a+b\,x+c\,x^2}\Big)$$

Result (type 2, 965 leaves):

$$
\begin{aligned}
&-\frac{1}{15\left(b^2-4ac\right)^3(a+x(b+cx))^{5/2}}\\
&2\left(A\left(3b^5\left(d^3+5d^2ex+15de^2x^2-5e^3x^3\right)+32c\left(-2a^4e^3+8c^4d^3x^5+15a^2c^2dx\left(d^2+e^2x^2\right)+2ac^3dx^3\left(10d^2+3e^2x^2\right)-a^3ce\left(9d^2+\right.\right.\right.\right.\\
&\left.\left.5e^2x^2\right)\right)+16bc\left(2a^3e^2(9d-5ex)+8c^3d^2x^4(5d-3ex)+15a^2c(d-ex)^3-6ac^2x^2\left(-10d^3+10d^2ex-5de^2x^2+e^3x^3\right)\right)-\\
&48b^2\left(a^3e^3+c^3dx^3\left(-10d^2+20dex-3e^2x^2\right)+a^2ce\left(3d^2-15dex+5e^2x^2\right)+5ac^2x\left(-d^3+6d^2ex-3de^2x^2+e^3x^3\right)\right)+\\
&2b^4\left(3ae\left(d^2+10dex-15e^2x^2\right)-5cx\left(d^3+12d^2ex-27de^2x^2+2e^3x^3\right)\right)+\\
&\left.8b^3\left(3a^2e^2(d-5ex)+c^2x^2\left(10d^3-90d^2ex+45de^2x^2-e^3x^3\right)-5ac\left(d^3+9d^2ex-15de^2x^2+5e^3x^3\right)\right)\right)+\\
&B\left(64a^4e^2(-3cd+2be)-16a^3\left(b^2e^2(9d-20ex)-2bce\left(9d^2-15dex+10e^2x^2\right)+6c^2\left(d^3+5de^2x^2\right)\right)+\right.\\
&24a^2\left(10bc^2x(-d+ex)^3+4c^3ex^3\left(5d^2+e^2x^2\right)+b^3e\left(d^2-15dex+10e^2x^2\right)-2b^2c\left(d^3-15d^2ex+15de^2x^2-10e^3x^3\right)\right)-\\
&bx\left(128c^4d^3x^4+16bc^3d^2x^3(20d-9ex)+24b^2c^2dx^2\left(10d^2-15dex+e^2x^2\right)-5b^4\left(d^3+9d^2ex-9de^2x^2-e^3x^3\right)+\right.\\
&\left.2b^3cx\left(20d^3-135d^2ex+30de^2x^2+e^3x^3\right)\right)+2a\left(96c^4d^2ex^5-16bc^3dx^3\left(10d^2-15dex+9e^2x^2\right)+24b^2c^2x^2\right.\\
&\left.\left.\left.\left(-10d^3+15d^2ex-15de^2x^2+e^3x^3\right)+60b^3cx\left(-d^3+5d^2ex-5de^2x^2+e^3x^3\right)+b^4\left(d^3+30d^2ex-135de^2x^2+20e^3x^3\right)\right)\right)\right)
\end{aligned}
$$

- **Problem 2575: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{5+\sqrt{35}+10x}{\sqrt{1+2x}\left(2+3x+5x^2\right)}\,dx$$

Optimal (type 3, 105 leaves, 6 steps):

$$-2\sqrt{\frac{10}{-2+\sqrt{35}}}\,\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{35}}-\sqrt{10+20x}}{\sqrt{-2+\sqrt{35}}}\right]+2\sqrt{\frac{10}{-2+\sqrt{35}}}\,\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{35}}+\sqrt{10+20x}}{\sqrt{-2+\sqrt{35}}}\right]$$

Result (type 3, 130 leaves):

$$2\sqrt{\frac{5}{31}}\left(\frac{\left(-2i+\sqrt{31}-i\sqrt{35}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2-i\sqrt{31}}}\right]}{\sqrt{-2-i\sqrt{31}}}+\frac{\left(2i+\sqrt{31}+i\sqrt{35}\right)\text{ArcTan}\left[\frac{\sqrt{5+10x}}{\sqrt{-2+i\sqrt{31}}}\right]}{\sqrt{-2+i\sqrt{31}}}\right)$$

- **Problem 2630: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{(A+Bx)(d+ex)^{3/2}}{\sqrt{a+bx+cx^2}}\,dx$$

Optimal (type 4, 545 leaves, 7 steps):

$$\frac{2\,(3\,B\,c\,d-4\,b\,B\,e+5\,A\,c\,e)\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}}{15\,c^2}+\frac{2\,B\,(d+e\,x)^{3/2}\,\sqrt{a+b\,x+c\,x^2}}{5\,c}+$$

$$\left(\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(10\,A\,c\,e\,(2\,c\,d-b\,e)+B\,\left(3\,c^2\,d^2+8\,b^2\,e^2-c\,e\,(13\,b\,d+9\,a\,e)\right)\right)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/\left(15\,c^3\,e\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)-$$

$$\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\,(3\,B\,c\,d-4\,b\,B\,e+5\,A\,c\,e)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/\left(15\,c^3\,e\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 4932 leaves) :

$$\frac{\sqrt{d+e\,x}\,\left(\frac{2\,(6\,B\,c\,d-4\,b\,B\,e+5\,A\,c\,e)}{15\,c^2}+\frac{2\,B\,e\,x}{5\,c}\right)\,\left(a+b\,x+c\,x^2\right)}{\sqrt{a+x\,(b+c\,x)}}+$$

$$\frac{1}{15\,c^2\,e^2\,\sqrt{a+x\,(b+c\,x)}}\,2\,\sqrt{a+b\,x+c\,x^2}\,\Bigg(\Big(\left(3\,B\,c^2\,d^2-13\,b\,B\,c\,d\,e+20\,A\,c^2\,d\,e+8\,b^2\,B\,e^2-10\,A\,b\,c\,e^2-9\,a\,B\,c\,e^2\right)\,(d+e\,x)^{3/2}$$

$$\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)\Big)\Big/\left(c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)-$$

$$\frac{1}{c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\left(c\,d^2-b\,d\,e+a\,e^2\right)\,(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\Bigg(\Bigg(3\,i\,B\,c^2\,d^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Bigg)\Bigg/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(13\,i\,b\,B\,c\,d\,e\left(2cd-be+\sqrt{b^2e^2-4ace^2}\right)\sqrt{1-\frac{2\left(cd^2-bde+ae^2\right)}{\left(2cd-be-\sqrt{b^2e^2-4ace^2}\right)(d+ex)}}\right.$$

$$\sqrt{1-\frac{2\left(cd^2-bde+ae^2\right)}{\left(2cd-be+\sqrt{b^2e^2-4ace^2}\right)(d+ex)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{cd^2-bde+ae^2}{2cd-be-\sqrt{b^2e^2-4ace^2}}}}{\sqrt{d+ex}}\right],\frac{2cd-be-\sqrt{b^2e^2-4ace^2}}{2cd-be+\sqrt{b^2e^2-4ace^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{cd^2-bde+ae^2}{2cd-be-\sqrt{b^2e^2-4ace^2}}}}{\sqrt{d+ex}}\right],\frac{2cd-be-\sqrt{b^2e^2-4ace^2}}{2cd-be+\sqrt{b^2e^2-4ace^2}}\right]\right)\right)\Bigg/$$

$$\left(2\sqrt{2}\left(cd^2-bde+ae^2\right)\sqrt{-\frac{cd^2-bde+ae^2}{2cd-be-\sqrt{b^2e^2-4ace^2}}}\sqrt{c+\frac{cd^2-bde+ae^2}{(d+ex)^2}+\frac{-2cd+be}{d+ex}}\right)+$$

$$\left(5\,i\sqrt{2}\,A\,c^2\,d\,e\left(2cd-be+\sqrt{b^2e^2-4ace^2}\right)\sqrt{1-\frac{2\left(cd^2-bde+ae^2\right)}{\left(2cd-be-\sqrt{b^2e^2-4ace^2}\right)(d+ex)}}\right.$$

$$\sqrt{1-\frac{2\left(cd^2-bde+ae^2\right)}{\left(2cd-be+\sqrt{b^2e^2-4ace^2}\right)(d+ex)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{cd^2-bde+ae^2}{2cd-be-\sqrt{b^2e^2-4ace^2}}}}{\sqrt{d+ex}}\right],\frac{2cd-be-\sqrt{b^2e^2-4ace^2}}{2cd-be+\sqrt{b^2e^2-4ace^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{cd^2-bde+ae^2}{2cd-be-\sqrt{b^2e^2-4ace^2}}}}{\sqrt{d+ex}}\right],\frac{2cd-be-\sqrt{b^2e^2-4ace^2}}{2cd-be+\sqrt{b^2e^2-4ace^2}}\right]\right)\right)\Bigg/$$

$$\left(\left(cd^2-bde+ae^2\right)\sqrt{-\frac{cd^2-bde+ae^2}{2cd-be-\sqrt{b^2e^2-4ace^2}}}\sqrt{c+\frac{cd^2-bde+ae^2}{(d+ex)^2}+\frac{-2cd+be}{d+ex}}\right)+$$

$$\left(2\,\mathrm{i}\,\sqrt{2}\,b^2\,B\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\mathrm{EllipticE}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(5\,\mathrm{i}\,A\,b\,c\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\mathrm{EllipticE}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(9\,\mathbb{i}\,a\,B\,c\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(3\,\mathbb{i}\,B\,c^2\,d\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(2\,\mathbb{i}\sqrt{2}\,b\,B\,c\,e\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\text{EllipticF}\Big[\text{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\Big],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\Big]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(5\,\text{i}\,A c^2 e\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\text{EllipticF}\Big[\text{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\Big],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\Big]\Bigg)\Bigg/$$

$$\left.\left(\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)\right)$$

- **Problem 2631: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(A+B x)\sqrt{d+e x}}{\sqrt{a+b x+c x^2}}\,dx$$

Optimal (type 4, 452 leaves, 6 steps) :

$$\frac{2 B \sqrt{d+e x} \sqrt{a+b x+c x^2}}{3 c}+$$

$$\left(\sqrt{2} \sqrt{b^2-4 a c} \,(B c d-2 b B e+3 A c e) \sqrt{d+e x} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right) \Big/$$

$$\left(3 c^2 e \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{a+b x+c x^2}\right)-\left(2 \sqrt{2} B \sqrt{b^2-4 a c} \left(c d^2-b d e+a e^2\right) \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\right.$$

$$\left.\sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right) \Big/ \left(3 c^2 e \sqrt{d+e x} \sqrt{a+b x+c x^2}\right)$$

Result (type 4, 781 leaves) :

$$\frac{2\,B\,\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)}{3\,c\,\sqrt{a+x\,(b+c\,x)}}+\frac{1}{3\,c^2\,e^2\,\sqrt{a+x\,(b+c\,x)}\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}$$

$$2\,(d+e\,x)^{3/2}\,\sqrt{a+b\,x+c\,x^2}\,\Bigg((B\,c\,d-2\,b\,B\,e+3\,A\,c\,e)\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)+\frac{1}{2\,\sqrt{2}\,\sqrt{\frac{c\,d^2+e\,(-b\,d+a\,e)}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}}}\,\sqrt{d+e\,x}}$$

$$i\,\sqrt{1-\frac{2\left(c\,d^2+e\,(-b\,d+a\,e)\right)}{\left(2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}\right)(d+e\,x)}}\,\sqrt{1+\frac{2\left(c\,d^2+e\,(-b\,d+a\,e)\right)}{\left(-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}\right)(d+e\,x)}}\,\Bigg((-B\,c\,d+2\,b\,B\,e-3\,A\,c\,e)$$

$$\left(2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{\frac{c\,d^2-b\,d\,e+a\,e^2}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}}}}{\sqrt{d+e\,x}}\right],-\frac{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}}{2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}}\right]+$$

$$\left(2\,b^2\,B\,e^2-b\,e\left(3\,B\,c\,d+3\,A\,c\,e+2\,B\,\sqrt{\left(b^2-4\,a\,c\right)e^2}\right)+c\left(-2\,a\,B\,e^2+B\,d\,\sqrt{\left(b^2-4\,a\,c\right)e^2}+3\,A\,e\left(2\,c\,d+\sqrt{\left(b^2-4\,a\,c\right)e^2}\right)\right)\right)$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{\frac{c\,d^2-b\,d\,e+a\,e^2}{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}}}}{\sqrt{d+e\,x}}\right],-\frac{-2\,c\,d+b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}}{2\,c\,d-b\,e+\sqrt{\left(b^2-4\,a\,c\right)e^2}}\right]\Bigg)\Bigg)$$

- **Problem 2632: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{A+B\,x}{\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 393 leaves, 5 steps) :

$$\frac{\sqrt{2}\,B\sqrt{b^2-4ac}\,\sqrt{d+ex}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right]}{c\,e\,\sqrt{\frac{c\,(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\,\sqrt{a+bx+cx^2}}-$$

$$\left(2\sqrt{2}\,\sqrt{b^2-4ac}\,(B\,d-A\,e)\,\sqrt{\frac{c\,(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right]\right)\Big/\left(c\,e\,\sqrt{d+ex}\,\sqrt{a+bx+cx^2}\right)$$

Result (type 4, 2732 leaves) :

$$-\frac{1}{e^2\sqrt{a+x\,(b+cx)}}2\sqrt{a+bx+cx^2}\left(-\frac{B\,(d+ex)^{3/2}\left(c+\frac{cd^2}{(d+ex)^2}-\frac{bde}{(d+ex)^2}+\frac{ae^2}{(d+ex)^2}-\frac{2cd}{d+ex}+\frac{be}{d+ex}\right)}{c\sqrt{\frac{(d+ex)^2\left(c\left(-1+\frac{d}{d+ex}\right)^2+\frac{e\left(b-\frac{bd}{d+ex}+\frac{ae}{d+ex}\right)}{d+ex}\right)}{e^2}}}+\right.$$

$$\frac{1}{c\sqrt{\frac{(d+ex)^2\left(c\left(-1+\frac{d}{d+ex}\right)^2+\frac{e\left(b-\frac{bd}{d+ex}+\frac{ae}{d+ex}\right)}{d+ex}\right)}{e^2}}}(d+ex)\sqrt{c+\frac{cd^2}{(d+ex)^2}-\frac{bde}{(d+ex)^2}+\frac{ae^2}{(d+ex)^2}-\frac{2cd}{d+ex}+\frac{be}{d+ex}}$$

$$\left(\left(i\,B\,c\,d^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(i\,b\,B\,d\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(i\, a\, B\, e^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2\left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right)(d + e x)}} \sqrt{1 - \frac{2\left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right)(d + e x)}}\right.$$

$$\left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(2\sqrt{2}\left(c d^2 - b d e + a e^2\right)\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(i\, B\, c\, d \sqrt{1 - \frac{2\left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right)(d + e x)}} \sqrt{1 - \frac{2\left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right)(d + e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\left(i\, A\, c\, e \sqrt{1 - \frac{2\left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right)(d + e x)}} \sqrt{1 - \frac{2\left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right)(d + e x)}}\right.$$

$$\text{EllipticF}\left[\dot{\imath}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\Bigg)\Bigg)$$

- **Problem 2633: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B\,x}{(d+e\,x)^{3/2}\,\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 460 leaves, 6 steps) :

$$\frac{2\,(B\,d-A\,e)\,\sqrt{a+b\,x+c\,x^2}}{\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{d+e\,x}}-\left(\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\,(B\,d-A\,e)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/$$

$$\left(e\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\left(2\,\sqrt{2}\,B\,\sqrt{b^2-4\,a\,c}\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/$$

$$\left(c\,e\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 550 leaves) :

$$-\frac{1}{\sqrt{2}\, e^2 \left(c d^2+e (-b d+a e)\right) \sqrt{\frac{c d^2+e (-b d+a e)}{-2 c d+b e+\sqrt{(b^2-4 a c) e^2}}}\, \sqrt{a+x (b+c x)}}$$

$$i (d+e x) \sqrt{1-\frac{2 \left(c d^2+e (-b d+a e)\right)}{\left(2 c d-b e+\sqrt{(b^2-4 a c) e^2}\right) (d+e x)}}\, \sqrt{1+\frac{2 \left(c d^2+e (-b d+a e)\right)}{\left(-2 c d+b e+\sqrt{(b^2-4 a c) e^2}\right) (d+e x)}}$$

$$\left((-B d+A e) \left(2 c d-b e+\sqrt{(b^2-4 a c) e^2}\right) \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{\frac{c d^2-b d e+a e^2}{-2 c d+b e+\sqrt{(b^2-4 a c) e^2}}}}{\sqrt{d+e x}}\right], -\frac{-2 c d+b e+\sqrt{(b^2-4 a c) e^2}}{2 c d-b e+\sqrt{(b^2-4 a c) e^2}}\right]+\right.$$

$$\left(-2 a B e^2+B d \sqrt{(b^2-4 a c) e^2}+b e (B d+A e)-A e \left(2 c d+\sqrt{(b^2-4 a c) e^2}\right)\right)$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{\frac{c d^2-b d e+a e^2}{-2 c d+b e+\sqrt{(b^2-4 a c) e^2}}}}{\sqrt{d+e x}}\right], -\frac{-2 c d+b e+\sqrt{(b^2-4 a c) e^2}}{2 c d-b e+\sqrt{(b^2-4 a c) e^2}}\right]\right)$$

- **Problem 2634: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A+B x}{(d+e x)^{5/2} \sqrt{a+b x+c x^2}}\, dx$$

Optimal (type 4, 591 leaves, 7 steps) :

$$\frac{2\,(B\,d-A\,e)\,\sqrt{a+b\,x+c\,x^2}}{3\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,(d+e\,x)^{3/2}}-\frac{2\,\left(2\,A\,e\,(2\,c\,d-b\,e)-B\,\left(c\,d^2+e\,(b\,d-3\,a\,e)\right)\right)\,\sqrt{a+b\,x+c\,x^2}}{3\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,\sqrt{d+e\,x}}+$$

$$\left(\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(2\,A\,e\,(2\,c\,d-b\,e)-B\,\left(c\,d^2+e\,(b\,d-3\,a\,e)\right)\right)\,\sqrt{d+e\,x}\right.$$

$$\left.\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\;e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/$$

$$\left(3\,e\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)+\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,(B\,d-A\,e)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\right.$$

$$\left.\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\;e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/\left(3\,e\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 4053 leaves) :

$$\frac{\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)\,\left(-\frac{2\,(-B\,d+A\,e)}{3\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,(d+e\,x)^2}-\frac{2\,\left(-B\,c\,d^2-b\,B\,d\,e+4\,A\,c\,d\,e-2\,A\,b\,e^2+3\,a\,B\,e^2\right)}{3\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,(d+e\,x)}\right)}{\sqrt{a+x\,(b+c\,x)}}+\frac{1}{3\,e^2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,\sqrt{a+x\,(b+c\,x)}}$$

$$2\,c\,\sqrt{a+b\,x+c\,x^2}\left(\left(\left(-B\,c\,d^2-b\,B\,d\,e+4\,A\,c\,d\,e-2\,A\,b\,e^2+3\,a\,B\,e^2\right)(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)\right)\Big/\right.$$

$$\left(c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)+$$

$$\frac{1}{c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\left(\left(\left(i\,B\,c\,d^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\,\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(i\, b\, B\, d\, e \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(2 \sqrt{2} \left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\left(i \sqrt{2}\, A\, c\, d\, e \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(\mathbb{i}\, A\, b\, e^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2} \left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\left(3\, \mathbb{i}\, a\, B\, e^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(2 \sqrt{2} \left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(i\,B\,c\,d\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(i\,A\,c\,e\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left.\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)\right)$$

- **Problem 2635: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{(A+B\,x)\,(d+e\,x)^{5/2}}{\left(a+b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 678 leaves, 7 steps) :

$$\frac{2\,(d+e\,x)^{3/2}\,\left(2\,a\,c\,(B\,d+A\,e)-b\,(A\,c\,d+a\,B\,e)-\left(b^2\,B\,e-b\,c\,(B\,d+A\,e)+2\,c\,(A\,c\,d-a\,B\,e)\right)\,x\right)}{c\,\left(b^2-4\,a\,c\right)\,\sqrt{a+b\,x+c\,x^2}}+$$

$$\frac{2\,e\,\left(4\,b^2\,B\,e-3\,b\,c\,(B\,d+A\,e)+2\,c\,(3\,A\,c\,d-5\,a\,B\,e)\right)\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}}{3\,c^2\,\left(b^2-4\,a\,c\right)}-$$

$$\left(\sqrt{2}\,\left(8\,b^3\,B\,e^2-b^2\,c\,e\,(13\,B\,d+6\,A\,e)-2\,c^2\,\left(3\,A\,c\,d^2-20\,a\,B\,d\,e-9\,a\,A\,e^2\right)+b\,c\,\left(3\,B\,c\,d^2+6\,A\,c\,d\,e-29\,a\,B\,e^2\right)\right)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/\left(3\,c^3\,\sqrt{b^2-4\,a\,c}\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)-$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\left(4\,b^2\,B\,e-3\,b\,c\,(B\,d+A\,e)+2\,c\,(3\,A\,c\,d-5\,a\,B\,e)\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/\left(3\,c^3\,\sqrt{b^2-4\,a\,c}\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 7589 leaves) :

$$\frac{1}{(a+x\,(b+c\,x))^{3/2}}$$

$$\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)^2\,\left(\frac{2\,B\,e^2}{3\,c^2}+\left(2\,\left(A\,b\,c^2\,d^2-2\,a\,B\,c^2\,d^2+2\,a\,b\,B\,c\,d\,e-4\,a\,A\,c^2\,d\,e-a\,b^2\,B\,e^2+a\,A\,b\,c\,e^2+2\,a^2\,B\,c\,e^2-b\,B\,c^2\,d^2\,x+2\,A\,c^3\,d^2\,x+\right.\right.\right.$$

$$2 b^2 B c d e x - 2 A b c^2 d e x - 4 a B c^2 d e x - b^3 B e^2 x + A b^2 c e^2 x + 3 a b B c e^2 x - 2 a A c^2 e^2 x)) /$$

$$\left(c^2 \left(-b^2 + 4 a c\right) \left(a + b x + c x^2\right)\right)\Big) + \frac{1}{3 c^2 \left(-b^2 + 4 a c\right) e \left(a + x (b + c x)\right)^{3/2}} 2 \left(a + b x + c x^2\right)^{3/2}$$

$$\Bigg(\Big(\left(3 b B c^2 d^2 - 6 A c^3 d^2 - 13 b^2 B c d e + 6 A b c^2 d e + 40 a B c^2 d e + 8 b^3 B e^2 - 6 A b^2 c e^2 - 29 a b B c e^2 + 18 a A c^2 e^2\right) (d + e x)^{3/2}$$

$$\left(c + \frac{c d^2}{(d + e x)^2} - \frac{b d e}{(d + e x)^2} + \frac{a e^2}{(d + e x)^2} - \frac{2 c d}{d + e x} + \frac{b e}{d + e x}\right)\Big) \Big/ \left(c \sqrt{\frac{(d + e x)^2 \left(c \left(-1 + \frac{d}{d+e x}\right)^2 + \frac{e \left(b - \frac{b d}{d+e x} + \frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right) -$$

$$\frac{1}{c \sqrt{\frac{(d+e x)^2 \left(c \left(-1+\frac{d}{d+e x}\right)^2 + \frac{e \left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}} \left(c d^2 - b d e + a e^2\right) (d + e x) \sqrt{c + \frac{c d^2}{(d + e x)^2} - \frac{b d e}{(d + e x)^2} + \frac{a e^2}{(d + e x)^2} - \frac{2 c d}{d + e x} + \frac{b e}{d + e x}}$$

$$\Bigg(\Bigg(3 i b B c^2 d^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}$$

$$\left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] -\right.$$

$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\Bigg) \Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(3 i A c^3 d^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(13 i b^2 B c d e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(3\,\mathbb{i}\,A\,b\,c^2\,d\,e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(10\,\mathbb{i}\sqrt{2}\,a\,B\,c^2\,d\,e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left((c d^2 - b d e + a e^2) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(2 i \sqrt{2} b^3 B e^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left((c d^2 - b d e + a e^2) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\left(3 i A b^2 c e^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(29\,\mathbb{i}\,a\,b\,B\,c\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(9\,\mathbb{i}\,a\,A\,c^2\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(3\,i\,b B c^2 d\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(3\,i\sqrt{2}\,A c^3 d\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(2\,i\sqrt{2}\,b^2 B c e\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(3\,i\,A\,b\,c^2 e\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(5\,i\sqrt{2}\,a\,B\,c^2 e\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\Bigg)\Bigg)$$

- **Problem 2636: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{(A+B\,x)\,(d+e\,x)^{3/2}}{\left(a+b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 530 leaves, 6 steps) :

$$\frac{2\sqrt{d+e\,x}\,\left(2\,a\,c\,(B\,d+A\,e)-b\,(A\,c\,d+a\,B\,e)-\left(b^2\,B\,e-b\,c\,(B\,d+A\,e)+2\,c\,(A\,c\,d-a\,B\,e)\right)\,x\right)}{c\,\left(b^2-4\,a\,c\right)\,\sqrt{a+b\,x+c\,x^2}}+$$

$$\left(\sqrt{2}\,\left(2\,A\,c^2\,d+2\,b^2\,B\,e-c\,(b\,B\,d+A\,b\,e+6\,a\,B\,e)\right)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\bigg/\left(c^2\,\sqrt{b^2-4\,a\,c}\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\left(2\,\sqrt{2}\,(b\,B-2\,A\,c)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\bigg/\left(c^2\,\sqrt{b^2-4\,a\,c}\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 4019 leaves) :

$$\left(2\,\sqrt{d+e\,x}\,\left(A\,b\,c\,d-2\,a\,B\,c\,d+a\,b\,B\,e-2\,a\,A\,c\,e-b\,B\,c\,d\,x+2\,A\,c^2\,d\,x+b^2\,B\,e\,x-A\,b\,c\,e\,x-2\,a\,B\,c\,e\,x\right)\,\left(a+b\,x+c\,x^2\right)\right)\bigg/$$

$$\left(c\,\left(-b^2+4\,a\,c\right)\,\left(a+x\,(b+c\,x)\right)^{3/2}\right)-\frac{1}{c\,\left(-b^2+4\,a\,c\right)\,e\,\left(a+x\,(b+c\,x)\right)^{3/2}}$$

$$2\left(a+bx+cx^2\right)^{3/2}\left(\left(\left(-bBcd+2Ac^2d+2b^2Be-Abce-6aBce\right)(d+ex)^{3/2}\left(c+\frac{cd^2}{(d+ex)^2}-\frac{bde}{(d+ex)^2}+\frac{ae^2}{(d+ex)^2}-\frac{2cd}{d+ex}+\frac{be}{d+ex}\right)\right)\Big/\right.$$

$$\left(c\sqrt{\frac{(d+ex)^2\left(c\left(-1+\frac{d}{d+ex}\right)^2+\frac{e\left(b-\frac{bd}{d+ex}+\frac{ae}{d+ex}\right)}{d+ex}\right)}{e^2}}\right)-$$

$$\frac{1}{c\sqrt{\frac{(d+ex)^2\left(c\left(-1+\frac{d}{d+ex}\right)^2+\frac{e\left(b-\frac{bd}{d+ex}+\frac{ae}{d+ex}\right)}{d+ex}\right)}{e^2}}}\left(cd^2-bde+ae^2\right)(d+ex)\sqrt{c+\frac{cd^2}{(d+ex)^2}-\frac{bde}{(d+ex)^2}+\frac{ae^2}{(d+ex)^2}-\frac{2cd}{d+ex}+\frac{be}{d+ex}}$$

$$\left(-\left(i\, bBcd\left(2cd-be+\sqrt{b^2e^2-4ace^2}\right)\sqrt{1-\frac{2\left(cd^2-bde+ae^2\right)}{\left(2cd-be-\sqrt{b^2e^2-4ace^2}\right)(d+ex)}}\right.\right.$$

$$\sqrt{1-\frac{2\left(cd^2-bde+ae^2\right)}{\left(2cd-be+\sqrt{b^2e^2-4ace^2}\right)(d+ex)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{cd^2-bde+ae^2}{2cd-be-\sqrt{b^2e^2-4ace^2}}}}{\sqrt{d+ex}}\right],\right.\right.$$

$$\left.\left.\left.\frac{2cd-be-\sqrt{b^2e^2-4ace^2}}{2cd-be+\sqrt{b^2e^2-4ace^2}}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{cd^2-bde+ae^2}{2cd-be-\sqrt{b^2e^2-4ace^2}}}}{\sqrt{d+ex}}\right],\frac{2cd-be-\sqrt{b^2e^2-4ace^2}}{2cd-be+\sqrt{b^2e^2-4ace^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(cd^2-bde+ae^2\right)\sqrt{-\frac{cd^2-bde+ae^2}{2cd-be-\sqrt{b^2e^2-4ace^2}}}\sqrt{c+\frac{cd^2-bde+ae^2}{(d+ex)^2}+\frac{-2cd+be}{d+ex}}\right)+$$

$$\left(i\,A\,c^2\,d\,\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(i\,b^2\,B\,e\,\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(i\,A\,b\,c\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(3\,i\,a\,B\,c\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(i\, b\, B\, c \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i \sqrt{2}\, A\, c^2 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left.\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)\right)$$

- **Problem 2637: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A+B x) \sqrt{d+e x}}{\left(a+b x+c x^2\right)^{3/2}} \, dx$$

Optimal (type 4, 460 leaves, 6 steps) :

$$-\frac{2\left(A b-2 a B-(b B-2 A c) x\right) \sqrt{d+e x}}{\left(b^2-4 a c\right) \sqrt{a+b x+c x^2}}-\frac{\sqrt{2}\,(b B-2 A c) \sqrt{d+e x}\,\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\;e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]}{c \sqrt{b^2-4 a c}\,\sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\,\sqrt{a+b x+c x^2}}+$$

$$\left(2 \sqrt{2}\,(b B d-2 A c d+A b e-2 a B e) \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\,\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\;e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Big/\left(c \sqrt{b^2-4 a c}\,\sqrt{d+e x}\,\sqrt{a+b x+c x^2}\right)$$

Result (type 4, 5246 leaves) :

$$\frac{2 (-A b+2 a B+b B x-2 A c x) \sqrt{d+e x}\left(a+b x+c x^2\right)}{\left(b^2-4 a c\right) (a+x (b+c x))^{3/2}}+$$

$$\frac{1}{\left(-b^2+4 a c\right) e\,(a+x (b+c x))^{3/2}}\,2\left(a+b x+c x^2\right)^{3/2}\left(\frac{(b B-2 A c)\,(d+e x)^{3/2}\left(c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)}{c \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}+\right.$$

$$\frac{1}{c \sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}\,(d+e x) \sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\left(-\left(i\, b\, B\, c\, d^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.\right.$$

$$\sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right],\right.\right.$$

$$\left.\left.\left.\frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(2 \sqrt{2} \left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(i\, A\, c^2 d^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2} \left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(i\, b^2 B\, d\, e \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(2 \sqrt{2} \left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\left(i\, A\, b\, c\, d\, e \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2} \left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\left(i\, a\, b\, B\, e^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(2 \sqrt{2} \left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(i\, a\, A\, c\, e^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2} \left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\left(i\, b\, B\, c\, d \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i \sqrt{2}\, A\, c^2 d \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(i\, A\, b\, c\, e \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(i\,\sqrt{2}\,a\,B\,c\,e\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Bigg/$$

$$\left(\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)$$

- **Problem 2638: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+B\,x}{\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 528 leaves, 6 steps) :

$$\frac{2\sqrt{d+e\,x}\,\left(a\,B\,(2\,c\,d-b\,e)-A\left(b\,c\,d-b^2\,e+2\,a\,c\,e\right)+c\,(b\,B\,d-2\,A\,c\,d+A\,b\,e-2\,a\,B\,e)\,x\right)}{\left(b^2-4\,a\,c\right)\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{a+b\,x+c\,x^2}}-$$

$$\left(\sqrt{2}\,(b\,B\,d-2\,A\,c\,d+A\,b\,e-2\,a\,B\,e)\sqrt{d+e\,x}\,\sqrt{-\frac{c\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4\,a\,c}\;e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\right]\right)\Bigg/$$

$$\left(\sqrt{b^2-4\,a\,c}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}}\,\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\left(2\sqrt{2}\,(b\,B-2\,A\,c)\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}}\,\sqrt{-\frac{c\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4\,a\,c}\;e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\right]\right)\Bigg/$$

$$\left(c\sqrt{b^2-4\,a\,c}\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 893 leaves) :

$$\left(2\sqrt{d+e x}\,\left(A b c d-2 a B c d-A b^2 e+a b B e+2 a A c e-b B c d x+2 A c^2 d x-A b c e x+2 a B c e x\right)\left(a+b x+c x^2\right)\right)\Big/\left(\left(-b^2+4 a c\right)\left(c d^2-b d e+a e^2\right)\left(a+x\left(b+c x\right)\right)^{3/2}\right)-$$

$$\left(2\left(d+e x\right)^{3/2}\left(a+b x+c x^2\right)^{3/2}\left(-\left(b B d-2 A c d+A b e-2 a B e\right)\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)+\frac{1}{2\sqrt{2}\sqrt{\frac{c d^2+e\left(-b d+a e\right)}{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}}\sqrt{d+e x}}\right.\right.$$

$$\mathbb{i}\sqrt{1-\frac{2\left(c d^2+e\left(-b d+a e\right)\right)}{\left(2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}\right)\left(d+e x\right)}}\sqrt{1+\frac{2\left(c d^2+e\left(-b d+a e\right)\right)}{\left(-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}\right)\left(d+e x\right)}}\left(\left(b B d-2 A c d+A b e-2 a B e\right)\right.$$

$$\left(2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}\right)\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c d^2-b d e+a e^2}{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}}}{\sqrt{d+e x}}\right],-\frac{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}{2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}}\right]-$$

$$\left(-2 A c d\sqrt{\left(b^2-4 a c\right) e^2}+b^2 e\left(B d-A e\right)+b\sqrt{\left(b^2-4 a c\right) e^2}\left(B d+A e\right)-2 a e\left(2 B c d-2 A c e+B\sqrt{\left(b^2-4 a c\right) e^2}\right)\right)$$

$$\left.\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c d^2-b d e+a e^2}{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}}}{\sqrt{d+e x}}\right],-\frac{-2 c d+b e+\sqrt{\left(b^2-4 a c\right) e^2}}{2 c d-b e+\sqrt{\left(b^2-4 a c\right) e^2}}\right]\right)\right)\right)\Big/$$

$$\left(\left(-b^2+4 a c\right) e\left(c d^2-b d e+a e^2\right)\left(a+x\left(b+c x\right)\right)^{3/2}\sqrt{\frac{\left(d+e x\right)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)$$

- **Problem 2639: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{A+B x}{\left(d+e x\right)^{3/2}\left(a+b x+c x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 705 leaves, 7 steps) :

$$\frac{2\left(a B (2 c d-b e)-A\left(b c d-b^2 e+2 a c e\right)+c (b B d-2 A c d+A b e-2 a B e) x\right)}{\left(b^2-4 a c\right)\left(c d^2-b d e+a e^2\right) \sqrt{d+e x} \sqrt{a+b x+c x^2}}+$$

$$\frac{2 e\left(b^2 e (B d-2 A e)-2 c\left(A c d^2+4 a B d e-3 a A e^2\right)+b\left(B c d^2+2 A c d e+a B e^2\right)\right) \sqrt{a+b x+c x^2}}{\left(b^2-4 a c\right)\left(c d^2-b d e+a e^2\right)^2 \sqrt{d+e x}}-$$

$$\left(\sqrt{2}\left(b^2 e (B d-2 A e)-2 c\left(A c d^2+4 a B d e-3 a A e^2\right)+b\left(B c d^2+2 A c d e+a B e^2\right)\right)\right.$$

$$\left.\sqrt{d+e x} \sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right) /$$

$$\left(\sqrt{b^2-4 a c}\left(c d^2-b d e+a e^2\right)^2 \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{a+b x+c x^2}\right)+$$

$$\left(2 \sqrt{2} (b B d-2 A c d+A b e-2 a B e) \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right) / \left(\sqrt{b^2-4 a c}\left(c d^2-b d e+a e^2\right) \sqrt{d+e x} \sqrt{a+b x+c x^2}\right)$$

Result (type 4, 6669 leaves) :

$$\frac{1}{(a+x\,(b+c\,x))^{3/2}}$$

$$\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)^2\left(-\frac{2\,e^2\,(-B\,d+A\,e)}{\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,(d+e\,x)}+\left(2\left(A\,b\,c^2\,d^2-2\,a\,B\,c^2\,d^2-2\,A\,b^2\,c\,d\,e+2\,a\,b\,B\,c\,d\,e+4\,a\,A\,c^2\,d\,e+A\,b^3\,e^2-a\,b^2\,B\,e^2-\right.\right.\right.$$

$$\left.\left.3\,a\,A\,b\,c\,e^2+2\,a^2\,B\,c\,e^2-b\,B\,c^2\,d^2\,x+2\,A\,c^3\,d^2\,x-2\,A\,b\,c^2\,d\,e\,x+4\,a\,B\,c^2\,d\,e\,x+A\,b^2\,c\,e^2\,x-a\,b\,B\,c\,e^2\,x-2\,a\,A\,c^2\,e^2\,x\right)\right)\Big/$$

$$\left.\left(\left(-b^2+4\,a\,c\right)\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\left(a+b\,x+c\,x^2\right)\right)\right)-\frac{1}{\left(-b^2+4\,a\,c\right)e\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,(a+x\,(b+c\,x))^{3/2}}$$

$$2\,c\left(a+b\,x+c\,x^2\right)^{3/2}\left(\left(\left(-b\,B\,c\,d^2+2\,A\,c^2\,d^2-b^2\,B\,d\,e-2\,A\,b\,c\,d\,e+8\,a\,B\,c\,d\,e+2\,A\,b^2\,e^2-a\,b\,B\,e^2-6\,a\,A\,c\,e^2\right)(d+e\,x)^{3/2}\right.\right.$$

$$\left.\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)\right)\Big/\left(c\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)+$$

$$\frac{1}{c\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\left(\left(i\,b\,B\,c\,d^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(\mathbb{i}\,A\,c^2 d^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(\mathbb{i}\,b^2 B\,d\,e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(\mathbb{i}\,A\,b\,c\,d\,e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(2\,\mathbb{i}\sqrt{2}\,a\,B\,c\,d\,e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(i\,A\,b^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(i\,a\,b\,B\,e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(3\,\mathrm{i}\,a\,A\,c\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(\mathrm{i}\,b\,B\,c\,d\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(i\,\sqrt{2}\,A\,c^2\,d\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(i\,A\,b\,c\,e\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(i\,\sqrt{2}\,a\,B\,c\,e\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)\Bigg)$$

- **Problem 2640: Result more than twice size of optimal antiderivative.**

$$\int (A+B x)\,(d+e x)^m\,\left(a+b x+c x^2\right)^3\,dx$$

Optimal (type 3, 594 leaves, 2 steps):

$$-\frac{(B d-A e)\left(c d^2-b d e+a e^2\right)^3 (d+e x)^{1+m}}{e^8 (1+m)}-\frac{\left(c d^2-b d e+a e^2\right)^2\left(3 A e (2 c d-b e)-B\left(7 c d^2-e (4 b d-a e)\right)\right)(d+e x)^{2+m}}{e^8 (2+m)}-\frac{1}{e^8 (3+m)}$$
$$3\left(c d^2-b d e+a e^2\right)\left(B\left(7 c^2 d^3-c d e (8 b d-3 a e)+b e^2 (2 b d-a e)\right)-A e\left(5 c^2 d^2+b^2 e^2-c e (5 b d-a e)\right)\right)(d+e x)^{3+m}-$$
$$\frac{1}{e^8 (4+m)}\left(A e (2 c d-b e)\left(10 c^2 d^2+b^2 e^2-2 c e (5 b d-3 a e)\right)-\right.$$
$$\left.B\left(35 c^3 d^4-b^2 e^3 (4 b d-3 a e)-30 c^2 d^2 e (2 b d-a e)+3 c e^2\left(10 b^2 d^2-8 a b d e+a^2 e^2\right)\right)\right)(d+e x)^{4+m}-\frac{1}{e^8 (5+m)}$$
$$\left(B\left(35 c^3 d^3-b^3 e^3+3 b c e^2 (5 b d-2 a e)-15 c^2 d e (3 b d-a e)\right)-3 A c e\left(5 c^2 d^2+b^2 e^2-c e (5 b d-a e)\right)\right)(d+e x)^{5+m}-$$
$$\frac{3 c\left(A c e (2 c d-b e)-B\left(7 c^2 d^2+b^2 e^2-c e (6 b d-a e)\right)\right)(d+e x)^{6+m}}{e^8 (6+m)}-\frac{c^2 (7 B c d-3 b B e-A c e)(d+e x)^{7+m}}{e^8 (7+m)}+\frac{B c^3 (d+e x)^{8+m}}{e^8 (8+m)}$$

Result (type 3, 6116 leaves):

$$(d+e x)^m$$
$$\left(-\frac{1}{e^8 (1+m)(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)(8+m)}d\left(5040 B c^3 d^7-17280 b B c^2 d^6 e-5760 A c^3 d^6 e+20160 b^2 B c d^5 e^2+20160 A b\right.\right.$$
$$c^2 d^5 e^2+20160 a B c^2 d^5 e^2-8064 b^3 B d^4 e^3-24192 A b^2 c d^4 e^3-48384 a b B c d^4 e^3-24192 a A c^2 d^4 e^3+10080 A b^3 d^3 e^4+$$
$$30240 a b^2 B d^3 e^4+60480 a A b c d^3 e^4+30240 a^2 B c d^3 e^4-40320 a A b^2 d^2 e^5-40320 a^2 b B d^2 e^5-40320 a^2 A c d^2 e^5+$$
$$60480 a^2 A b d e^6+20160 a^3 B d e^6-40320 a^3 A e^7-2160 b B c^2 d^6 e m-720 A c^3 d^6 e m+5400 b^2 B c d^5 e^2 m+5400 A b c^2 d^5 e^2 m+$$
$$5400 a B c^2 d^5 e^2 m-3504 b^3 B d^4 e^3 m-10512 A b^2 c d^4 e^3 m-21024 a b B c d^4 e^3 m-10512 a A c^2 d^4 e^3 m+6396 A b^3 d^3 e^4 m+$$
$$19188 a b^2 B d^3 e^4 m+38376 a A b c d^3 e^4 m+19188 a^2 B c d^3 e^4 m-35664 a A b^2 d^2 e^5 m-35664 a^2 b B d^2 e^5 m-35664 a^2 A c d^2 e^5 m+$$
$$73656 a^2 A b d e^6 m+24552 a^3 B d e^6 m-69264 a^3 A e^7 m+360 b^2 B c d^5 e^2 m^2+360 A b c^2 d^5 e^2 m^2+360 a B c^2 d^5 e^2 m^2-504 b^3 B d^4 e^3 m^2-$$

$$1512 A b^2 c d^4 e^3 m^2 - 3024 a b B c d^4 e^3 m^2 - 1512 a A c^2 d^4 e^3 m^2 + 1506 A b^3 d^3 e^4 m^2 + 4518 a b^2 B d^3 e^4 m^2 + 9036 a A b c d^3 e^4 m^2 +$$
$$4518 a^2 B c d^3 e^4 m^2 - 12420 a A b^2 d^2 e^5 m^2 - 12420 a^2 b B d^2 e^5 m^2 - 12420 a^2 A c d^2 e^5 m^2 + 36462 a^2 A b d e^6 m^2 + 12154 a^3 B d e^6 m^2 -$$
$$48860 a^3 A e^7 m^2 - 24 b^3 B d^4 e^3 m^3 - 72 A b^2 c d^4 e^3 m^3 - 144 a b B c d^4 e^3 m^3 - 72 a A c^2 d^4 e^3 m^3 + 156 A b^3 d^3 e^4 m^3 + 468 a b^2 B d^3 e^4 m^3 +$$
$$936 a A b c d^3 e^4 m^3 + 468 a^2 B c d^3 e^4 m^3 - 2130 a A b^2 d^2 e^5 m^3 - 2130 a^2 b B d^2 e^5 m^3 - 2130 a^2 A c d^2 e^5 m^3 + 9405 a^2 A b d e^6 m^3 +$$
$$3135 a^3 B d e^6 m^3 - 18424 a^3 A e^7 m^3 + 6 A b^3 d^3 e^4 m^4 + 18 a b^2 B d^3 e^4 m^4 + 36 a A b c d^3 e^4 m^4 + 18 a^2 B c d^3 e^4 m^4 - 180 a A b^2 d^2 e^5 m^4 -$$
$$180 a^2 b B d^2 e^5 m^4 - 180 a^2 A c d^2 e^5 m^4 + 1335 a^2 A b d e^6 m^4 + 445 a^3 B d e^6 m^4 - 4025 a^3 A e^7 m^4 - 6 a A b^2 d^2 e^5 m^5 - 6 a^2 b B d^2 e^5 m^5 -$$
$$6 a^2 A c d^2 e^5 m^5 + 99 a^2 A b d e^6 m^5 + 33 a^3 B d e^6 m^5 - 511 a^3 A e^7 m^5 + 3 a^2 A b d e^6 m^6 + a^3 B d e^6 m^6 - 35 a^3 A e^7 m^6 - a^3 A e^7 m^7) +$$

$$\frac{1}{e^6 (2+m)(3+m)(4+m)(5+m)(6+m)(7+m)(8+m)(e+e m)} (40320 a^3 A e^7 + 5040 B c^3 d^7 m - 17280 b B c^2 d^6 e m - 5760 A c^3 d^6 e m +$$
$$20160 b^2 B c d^5 e^2 m + 20160 A b c^2 d^5 e^2 m + 20160 a B c^2 d^5 e^2 m - 8064 b^3 B d^4 e^3 m - 24192 A b^2 c d^4 e^3 m - 48384 a b B c d^4 e^3 m -$$
$$24192 a A c^2 d^4 e^3 m + 10080 A b^3 d^3 e^4 m + 30240 a b^2 B d^3 e^4 m + 60480 a A b c d^3 e^4 m + 30240 a^2 B c d^3 e^4 m - 40320 a A b^2 d^2 e^5 m -$$
$$40320 a^2 b B d^2 e^5 m - 40320 a^2 A c d^2 e^5 m + 60480 a^2 A b d e^6 m + 20160 a^3 B d e^6 m + 69264 a^3 A e^7 m - 2160 b B c^2 d^6 e m^2 - 720 A c^3 d^6 e m^2 +$$
$$5400 b^2 B c d^5 e^2 m^2 + 5400 A b c^2 d^5 e^2 m^2 + 5400 a B c^2 d^5 e^2 m^2 - 3504 b^3 B d^4 e^3 m^2 - 10512 A b^2 c d^4 e^3 m^2 - 21024 a b B c d^4 e^3 m^2 -$$
$$10512 a A c^2 d^4 e^3 m^2 + 6396 A b^3 d^3 e^4 m^2 + 19188 a b^2 B d^3 e^4 m^2 + 38376 a A b c d^3 e^4 m^2 + 19188 a^2 B c d^3 e^4 m^2 - 35664 a A b^2 d^2 e^5 m^2 -$$
$$35664 a^2 b B d^2 e^5 m^2 - 35664 a^2 A c d^2 e^5 m^2 + 73656 a^2 A b d e^6 m^2 + 24552 a^3 B d e^6 m^2 + 48860 a^3 A e^7 m^2 + 360 b^2 B c d^5 e^2 m^3 +$$
$$360 A b c^2 d^5 e^2 m^3 + 360 a B c^2 d^5 e^2 m^3 - 504 b^3 B d^4 e^3 m^3 - 1512 A b^2 c d^4 e^3 m^3 - 3024 a b B c d^4 e^3 m^3 - 1512 a A c^2 d^4 e^3 m^3 + 1506 A b^3 d^3 e^4 m^3 +$$
$$4518 a b^2 B d^3 e^4 m^3 + 9036 a A b c d^3 e^4 m^3 + 4518 a^2 B c d^3 e^4 m^3 - 12420 a A b^2 d^2 e^5 m^3 - 12420 a^2 b B d^2 e^5 m^3 - 12420 a^2 A c d^2 e^5 m^3 +$$
$$36462 a^2 A b d e^6 m^3 + 12154 a^3 B d e^6 m^3 + 18424 a^3 A e^7 m^3 - 24 b^3 B d^4 e^3 m^4 - 72 A b^2 c d^4 e^3 m^4 - 144 a b B c d^4 e^3 m^4 - 72 a A c^2 d^4 e^3 m^4 +$$
$$156 A b^3 d^3 e^4 m^4 + 468 a b^2 B d^3 e^4 m^4 + 936 a A b c d^3 e^4 m^4 + 468 a^2 B c d^3 e^4 m^4 - 2130 a A b^2 d^2 e^5 m^4 - 2130 a^2 b B d^2 e^5 m^4 - 2130 a^2 A c d^2 e^5 m^4 +$$
$$9405 a^2 A b d e^6 m^4 + 3135 a^3 B d e^6 m^4 + 4025 a^3 A e^7 m^4 + 6 A b^3 d^3 e^4 m^5 + 18 a b^2 B d^3 e^4 m^5 + 36 a A b c d^3 e^4 m^5 + 18 a^2 B c d^3 e^4 m^5 -$$
$$180 a A b^2 d^2 e^5 m^5 - 180 a^2 b B d^2 e^5 m^5 - 180 a^2 A c d^2 e^5 m^5 + 1335 a^2 A b d e^6 m^5 + 445 a^3 B d e^6 m^5 + 511 a^3 A e^7 m^5 - 6 a A b^2 d^2 e^5 m^6 -$$
$$6 a^2 b B d^2 e^5 m^6 - 6 a^2 A c d^2 e^5 m^6 + 99 a^2 A b d e^6 m^6 + 33 a^3 B d e^6 m^6 + 35 a^3 A e^7 m^6 + 3 a^2 A b d e^6 m^7 + a^3 B d e^6 m^7 + a^3 A e^7 m^7) x +$$

$$\frac{1}{e^5 (3+m)(4+m)(5+m)(6+m)(7+m)(8+m)(2 e+e m)} (60480 a^2 A b e^6 + 20160 a^3 B e^6 - 2520 B c^3 d^6 m + 8640 b B c^2 d^5 e m +$$
$$2880 A c^3 d^5 e m - 10080 b^2 B c d^4 e^2 m - 10080 A b c^2 d^4 e^2 m - 10080 a B c^2 d^4 e^2 m + 4032 b^3 B d^3 e^3 m + 12096 A b^2 c d^3 e^3 m +$$
$$24192 a b B c d^3 e^3 m + 12096 a A c^2 d^3 e^3 m - 5040 A b^3 d^2 e^4 m - 15120 a b^2 B d^2 e^4 m - 30240 a A b c d^2 e^4 m - 15120 a^2 B c d^2 e^4 m +$$
$$20160 a A b^2 d e^5 m + 20160 a^2 b B d e^5 m + 20160 a^2 A c d e^5 m + 73656 a^2 A b e^6 m + 24552 a^3 B e^6 m + 1080 b B c^2 d^5 e m^2 + 360 A c^3 d^5 e m^2 -$$
$$2700 b^2 B c d^4 e^2 m^2 - 2700 A b c^2 d^4 e^2 m^2 - 2700 a B c^2 d^4 e^2 m^2 + 1752 b^3 B d^3 e^3 m^2 + 5256 A b^2 c d^3 e^3 m^2 + 10512 a b B c d^3 e^3 m^2 +$$
$$5256 a A c^2 d^3 e^3 m^2 - 3198 A b^3 d^2 e^4 m^2 - 9594 a b^2 B d^2 e^4 m^2 - 19188 a A b c d^2 e^4 m^2 - 9594 a^2 B c d^2 e^4 m^2 + 17832 a A b^2 d e^5 m^2 +$$
$$17832 a^2 b B d e^5 m^2 + 17832 a^2 A c d e^5 m^2 + 36462 a^2 A b e^6 m^2 + 12154 a^3 B e^6 m^2 - 180 b^2 B c d^4 e^2 m^3 - 180 A b c^2 d^4 e^2 m^3 -$$
$$180 a B c^2 d^4 e^2 m^3 + 252 b^3 B d^3 e^3 m^3 + 756 A b^2 c d^3 e^3 m^3 + 1512 a b B c d^3 e^3 m^3 + 756 a A c^2 d^3 e^3 m^3 - 753 A b^3 d^2 e^4 m^3 - 2259 a b^2 B d^2 e^4 m^3 -$$
$$4518 a A b c d^2 e^4 m^3 - 2259 a^2 B c d^2 e^4 m^3 + 6210 a A b^2 d e^5 m^3 + 6210 a^2 b B d e^5 m^3 + 6210 a^2 A c d e^5 m^3 + 9405 a^2 A b e^6 m^3 +$$
$$3135 a^3 B e^6 m^3 + 12 b^3 B d^3 e^3 m^4 + 36 A b^2 c d^3 e^3 m^4 + 72 a b B c d^3 e^3 m^4 + 36 a A c^2 d^3 e^3 m^4 - 78 A b^3 d^2 e^4 m^4 - 234 a b^2 B d^2 e^4 m^4 -$$
$$468 a A b c d^2 e^4 m^4 - 234 a^2 B c d^2 e^4 m^4 + 1065 a A b^2 d e^5 m^4 + 1065 a^2 b B d e^5 m^4 + 1065 a^2 A c d e^5 m^4 + 1335 a^2 A b e^6 m^4 +$$
$$445 a^3 B e^6 m^4 - 3 A b^3 d^2 e^4 m^5 - 9 a b^2 B d^2 e^4 m^5 - 18 a A b c d^2 e^4 m^5 - 9 a^2 B c d^2 e^4 m^5 + 90 a A b^2 d e^5 m^5 + 90 a^2 b B d e^5 m^5 +$$
$$90 a^2 A c d e^5 m^5 + 99 a^2 A b e^6 m^5 + 33 a^3 B e^6 m^5 + 3 a A b^2 d e^5 m^6 + 3 a^2 b B d e^5 m^6 + 3 a^2 A c d e^5 m^6 + 3 a^2 A b e^6 m^6 + a^3 B e^6 m^6) x^2 +$$

$$\frac{1}{e^4 (4+m)(5+m)(6+m)(7+m)(8+m)(3 e+e m)} (20160 a A b^2 e^5 + 20160 a^2 b B e^5 + 20160 a^2 A c e^5 + 840 B c^3 d^5 m - 2880 b B c^2 d^4 e m -$$
$$960 A c^3 d^4 e m + 3360 b^2 B c d^3 e^2 m + 3360 A b c^2 d^3 e^2 m + 3360 a B c^2 d^3 e^2 m - 1344 b^3 B d^2 e^3 m - 4032 A b^2 c d^2 e^3 m - 8064 a b B c d^2 e^3 m -$$
$$4032 a A c^2 d^2 e^3 m + 1680 A b^3 d e^4 m + 5040 a b^2 B d e^4 m + 10080 a A b c d e^4 m + 5040 a^2 B c d e^4 m + 17832 a A b^2 e^5 m + 17832 a^2 b B e^5 m +$$
$$17832 a^2 A c e^5 m - 360 b B c^2 d^4 e m^2 - 120 A c^3 d^4 e m^2 + 900 b^2 B c d^3 e^2 m^2 + 900 A b c^2 d^3 e^2 m^2 + 900 a B c^2 d^3 e^2 m^2 - 584 b^3 B d^2 e^3 m^2 -$$
$$1752 A b^2 c d^2 e^3 m^2 - 3504 a b B c d^2 e^3 m^2 - 1752 a A c^2 d^2 e^3 m^2 + 1066 A b^3 d e^4 m^2 + 3198 a b^2 B d e^4 m^2 + 6396 a A b c d e^4 m^2 +$$

$$3198\,a^2 B c d e^4 m^2+6210\,a A b^2 e^5 m^2+6210\,a^2 b B e^5 m^2+6210\,a^2 A c e^5 m^2+60\,b^2 B c d^3 e^2 m^3+60\,A b c^2 d^3 e^2 m^3+60\,a B c^2 d^3 e^2 m^3-$$
$$84\,b^3 B d^2 e^3 m^3-252\,A b^2 c d^2 e^3 m^3-504\,a b B c d^2 e^3 m^3-252\,a A c^2 d^2 e^3 m^3+251\,A b^3 d e^4 m^3+753\,a b^2 B d e^4 m^3+1506\,a A b c d e^4 m^3+$$
$$753\,a^2 B c d e^4 m^3+1065\,a A b^2 e^5 m^3+1065\,a^2 b B e^5 m^3+1065\,a^2 A c e^5 m^3-4\,b^3 B d^2 e^3 m^4-12\,A b^2 c d^2 e^3 m^4-24\,a b B c d^2 e^3 m^4-$$
$$12\,a A c^2 d^2 e^3 m^4+26\,A b^3 d e^4 m^4+78\,a b^2 B d e^4 m^4+156\,a A b c d e^4 m^4+78\,a^2 B c d e^4 m^4+90\,a A b^2 e^5 m^4+90\,a^2 b B e^5 m^4+$$
$$90\,a^2 A c e^5 m^4+A b^3 d e^4 m^5+3\,a b^2 B d e^4 m^5+6\,a A b c d e^4 m^5+3\,a^2 B c d e^4 m^5+3\,a A b^2 e^5 m^5+3\,a^2 b B e^5 m^5+3\,a^2 A c e^5 m^5\Big)\,x^3+$$

$$\frac{1}{e^3\,(5+m)\,(6+m)\,(7+m)\,(8+m)\,(4\,e+e\,m)}\Big(1680\,A b^3 e^4+5040\,a b^2 B e^4+10\,080\,a A b c e^4+5040\,a^2 B c e^4-210\,B c^3 d^4 m+$$
$$720\,b B c^2 d^3 e m+240\,A c^3 d^3 e m-840\,b^2 B c d^2 e^2 m-840\,A b c^2 d^2 e^2 m-840\,a B c^2 d^2 e^2 m+336\,b^3 B d e^3 m+1008\,A b^2 c d e^3 m+$$
$$2016\,a b B c d e^3 m+1008\,a A c^2 d e^3 m+1066\,A b^3 e^4 m+3198\,a b^2 B e^4 m+6396\,a A b c e^4 m+3198\,a^2 B c e^4 m+90\,b B c^2 d^3 e m^2+$$
$$30\,A c^3 d^3 e m^2-225\,b^2 B c d^2 e^2 m^2-225\,A b c^2 d^2 e^2 m^2-225\,a B c^2 d^2 e^2 m^2+146\,b^3 B d e^3 m^2+438\,A b^2 c d e^3 m^2+876\,a b B c d e^3 m^2+$$
$$438\,a A c^2 d e^3 m^2+251\,A b^3 e^4 m^2+753\,a b^2 B e^4 m^2+1506\,a A b c e^4 m^2+753\,a^2 B c e^4 m^2-15\,b^2 B c d^2 e^2 m^3-15\,A b c^2 d^2 e^2 m^3-$$
$$15\,a B c^2 d^2 e^2 m^3+21\,b^3 B d e^3 m^3+63\,A b^2 c d e^3 m^3+126\,a b B c d e^3 m^3+63\,a A c^2 d e^3 m^3+26\,A b^3 e^4 m^3+78\,a b^2 B e^4 m^3+156\,a A b c e^4 m^3+$$
$$78\,a^2 B c e^4 m^3+b^3 B d e^3 m^4+3\,A b^2 c d e^3 m^4+6\,a b B c d e^3 m^4+3\,a A c^2 d e^3 m^4+A b^3 e^4 m^4+3\,a b^2 B e^4 m^4+6\,a A b c e^4 m^4+3\,a^2 B c e^4 m^4\Big)\,x^4+$$

$$\frac{1}{e^2\,(6+m)\,(7+m)\,(8+m)\,(5\,e+e\,m)}\Big(336\,b^3 B e^3+1008\,A b^2 c e^3+2016\,a b B c e^3+1008\,a A c^2 e^3+42\,B c^3 d^3 m-144\,b B c^2 d^2 e m-$$
$$48\,A c^3 d^2 e m+168\,b^2 B c d e^2 m+168\,A b c^2 d e^2 m+168\,a B c^2 d e^2 m+146\,b^3 B e^3 m+438\,A b^2 c e^3 m+876\,a b B c e^3 m+438\,a A c^2 e^3 m-$$
$$18\,b B c^2 d^2 e m^2-6\,A c^3 d^2 e m^2+45\,b^2 B c d e^2 m^2+45\,A b c^2 d e^2 m^2+45\,a B c^2 d e^2 m^2+21\,b^3 B e^3 m^2+63\,A b^2 c e^3 m^2+126\,a b B c e^3 m^2+$$
$$63\,a A c^2 e^3 m^2+3\,b^2 B c d e^2 m^3+3\,A b c^2 d e^2 m^3+3\,a B c^2 d e^2 m^3+b^3 B e^3 m^3+3\,A b^2 c e^3 m^3+6\,a b B c e^3 m^3+3\,a A c^2 e^3 m^3\Big)\,x^5+$$
$$\Big(\Big(168\,b^2 B c e^2+168\,A b c^2 e^2+168\,a B c^2 e^2-7\,B c^3 d^2 m+24\,b B c^2 d e m+8\,A c^3 d e m+45\,b^2 B c e^2 m+45\,A b c^2 e^2 m+$$
$$45\,a B c^2 e^2 m+3\,b B c^2 d e m^2+A c^3 d e m^2+3\,b^2 B c e^2 m^2+3\,A b c^2 e^2 m^2+3\,a B c^2 e^2 m^2\Big)\,x^6\Big)\Big/$$
$$(e\,(7+m)\,(8+m)\,(6\,e+e\,m))+\frac{\left(24\,b B c^2 e+8\,A c^3 e+B c^3 d m+3\,b B c^2 e m+A c^3 e m\right)x^7}{(8+m)\,(7\,e+e\,m)}+\frac{B c^3 e x^8}{8\,e+e\,m}\Bigg)$$

- **Problem 2641: Result more than twice size of optimal antiderivative.**

$$\int (A+B x)\,(d+e x)^m \left(a+b x+c x^2\right)^2 dx$$

Optimal (type 3, 333 leaves, 2 steps):

$$-\frac{(B d-A e)\left(c d^2-b d e+a e^2\right)^2 (d+e x)^{1+m}}{e^6\,(1+m)}-\frac{\left(c d^2-b d e+a e^2\right)\left(2 A e\,(2 c d-b e)-B\left(5 c d^2-e\,(3 b d-a e)\right)\right)(d+e x)^{2+m}}{e^6\,(2+m)}-$$
$$\frac{1}{e^6\,(3+m)}\left(B\left(10 c^2 d^3+b e^2\,(3 b d-2 a e)-6 c d e\,(2 b d-a e)\right)-A e\left(6 c^2 d^2+b^2 e^2-2 c e\,(3 b d-a e)\right)\right)(d+e x)^{3+m}-$$
$$\frac{\left(2 A c e\,(2 c d-b e)-B\left(10 c^2 d^2+b^2 e^2-2 c e\,(4 b d-a e)\right)\right)(d+e x)^{4+m}}{e^6\,(4+m)}-\frac{c\,(5 B c d-2 b B e-A c e)\,(d+e x)^{5+m}}{e^6\,(5+m)}+\frac{B c^2\,(d+e x)^{6+m}}{e^6\,(6+m)}$$

Result (type 3, 722 leaves):

$$\frac{1}{e^6 (1+m) (2+m) (3+m) (4+m) (5+m) (6+m)} (d+e x)^{1+m} \Big(A e (6+m) \Big(c^2 \left(24 d^4 - 24 d^3 e (1+m) x + 12 d^2 e^2 \left(2+3 m+m^2\right) x^2 - 4 d e^3 \left(6+11 m+6 m^2+m^3\right) x^3 + e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4\right) + e^2 \left(20+9 m+m^2\right) \left(a^2 e^2 \left(6+5 m+m^2\right) + 2 a b e (3+m) (-d+e (1+m) x) + b^2 \left(2 d^2 - 2 d e (1+m) x + e^2 \left(2+3 m+m^2\right) x^2\right)\right) + 2 c e (5+m) \left(a e (4+m) \left(2 d^2 - 2 d e (1+m) x + e^2 \left(2+3 m+m^2\right) x^2\right) + b \left(-6 d^3 + 6 d^2 e (1+m) x - 3 d e^2 \left(2+3 m+m^2\right) x^2 + e^3 \left(6+11 m+6 m^2+m^3\right) x^3\right)\right)\Big) + B \Big(-c^2 \left(120 d^5 - 120 d^4 e (1+m) x + 60 d^3 e^2 \left(2+3 m+m^2\right) x^2 - 20 d^2 e^3 \left(6+11 m+6 m^2+m^3\right) x^3 + 5 d e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4 - e^5 \left(120+274 m+225 m^2+85 m^3+15 m^4+m^5\right) x^5\right) + e^2 \left(30+11 m+m^2\right) \left(a^2 e^2 \left(12+7 m+m^2\right) (-d+e (1+m) x) + 2 a b e (4+m) \left(2 d^2 - 2 d e (1+m) x + e^2 \left(2+3 m+m^2\right) x^2\right) + b^2 \left(-6 d^3 + 6 d^2 e (1+m) x - 3 d e^2 \left(2+3 m+m^2\right) x^2 + e^3 \left(6+11 m+6 m^2+m^3\right) x^3\right)\right) + 2 c e (6+m) \Big(a e (5+m) \left(-6 d^3 + 6 d^2 e (1+m) x - 3 d e^2 \left(2+3 m+m^2\right) x^2 + e^3 \left(6+11 m+6 m^2+m^3\right) x^3\right) + b \left(24 d^4 - 24 d^3 e (1-m) x + 12 d^2 e^2 \left(2+3 m+m^2\right) x^2 - 4 d e^3 \left(6+11 m+6 m^2+m^3\right) x^3 + e^4 \left(24+50 m+35 m^2+10 m^3+m^4\right) x^4\right)\Big)\Big)\Big)$$

- **Problem 2644: Unable to integrate problem.**

$$\int \frac{(A+B x) (d+e x)^m}{\left(a+b x+c x^2\right)^2} \, dx$$

Optimal (type 5, 538 leaves, 5 steps):

$$\frac{(d+e x)^{1+m} \left(a B (2 c d - b e) - A \left(b c d - b^2 e + 2 a c e\right) + c (b B d - 2 A c d + A b e - 2 a B e) x\right)}{\left(b^2-4 a c\right) \left(c d^2 - b d e + a e^2\right) \left(a+b x+c x^2\right)} +$$

$$\left(c \left(e (b B d - 2 A c d + A b e - 2 a B e) m - \frac{2 b \left(B c d^2 + 2 A c d e + a B e^2\right) - b^2 e (B d (2-m) + A e m) - 4 c \left(A \left(c d^2 + a e^2 (1-m)\right) + a B d e m\right)}{\sqrt{b^2-4 a c}}\right) (d+e x)^{1+m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{2 c (d+e x)}{2 c d - \left(b - \sqrt{b^2-4 a c}\right) e}\right]\right) \Big/ \left(\left(b^2-4 a c\right) \left(2 c d - \left(b - \sqrt{b^2-4 a c}\right) e\right) \left(c d^2 - b d e + a e^2\right) (1+m)\right) +$$

$$\left(c \left(e (b B d - 2 A c d + A b e - 2 a B e) m + \frac{2 b \left(B c d^2 + 2 A c d e + a B e^2\right) - b^2 e (B d (2-m) + A e m) - 4 c \left(A \left(c d^2 + a e^2 (1-m)\right) + a B d e m\right)}{\sqrt{b^2-4 a c}}\right) (d+e x)^{1+m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{2 c (d+e x)}{2 c d - \left(b + \sqrt{b^2-4 a c}\right) e}\right]\right) \Big/ \left(\left(b^2-4 a c\right) \left(2 c d - \left(b + \sqrt{b^2-4 a c}\right) e\right) \left(c d^2 - b d e + a e^2\right) (1+m)\right)$$

Result (type 8, 27 leaves):

$$\int \frac{(A+B x) (d+e x)^m}{\left(a+b x+c x^2\right)^2} \, dx$$

Problem 2645: Result more than twice size of optimal antiderivative.

$$\int \frac{(A+B x)\,(d+e x)^{1+m}}{a+b x+c x^2}\,dx$$

Optimal (type 5, 212 leaves, 4 steps):

$$-\frac{\left(B-\frac{b B-2 A c}{\sqrt{b^2-4 a c}}\right)(d+e x)^{2+m}\,\text{Hypergeometric2F1}\left[1,2+m,3+m,\frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\right]}{\left(2 c d-\left(b-\sqrt{b^2-4 a c}\right) e\right)(2+m)}-\frac{\left(B+\frac{b B-2 A c}{\sqrt{b^2-4 a c}}\right)(d+e x)^{2+m}\,\text{Hypergeometric2F1}\left[1,2+m,3+m,\frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]}{\left(2 c d-\left(b+\sqrt{b^2-4 a c}\right) e\right)(2+m)}$$

Result (type 5, 1358 leaves):

$$-\frac{1}{4 c^2 \sqrt{(b^2-4 a c)\, e^2}\; m\,(1+m)}(d+e x)^m \left(-2^{1-m} A c e\,(1+m)\left(\frac{c\,(d+e x)}{b e-\sqrt{(b^2-4 a c)\, e^2}+2 c e x}\right)^{-m}\left(\frac{c\,(d+e x)}{b e+\sqrt{(b^2-4 a c)\, e^2}+2 c e x}\right)^{-m}\right.$$
$$\left(\left(2 c d-b e+\sqrt{(b^2-4 a c)\, e^2}\right)\left(\frac{c\,(d+e x)}{b e+\sqrt{(b^2-4 a c)\, e^2}+2 c e x}\right)^{m}\text{Hypergeometric2F1}\left[-m,-m,1-m,\frac{2 c d-b e+\sqrt{(b^2-4 a c)\, e^2}}{-b e+\sqrt{(b^2-4 a c)\, e^2}-2 c e x}\right]+\right.$$
$$\left.\left(-2 c d+b e+\sqrt{(b^2-4 a c)\, e^2}\right)\left(\frac{c\,(d+e x)}{b e-\sqrt{(b^2-4 a c)\, e^2}+2 c e x}\right)^{m}\text{Hypergeometric2F1}\left[-m,-m,1-m,\frac{-2 c d+b e+\sqrt{(b^2-4 a c)\, e^2}}{b e+\sqrt{(b^2-4 a c)\, e^2}+2 c e x}\right]\right)+$$
$$B\left(2 c\left(2 c d-b e-\sqrt{(b^2-4 a c)\, e^2}\right) m\,(d+e x)-2 c\left(2 c d-b e+\sqrt{(b^2-4 a c)\, e^2}\right) m\,(d+e x)-2^{-m}\left(2 c d-b e+\sqrt{(b^2-4 a c)\, e^2}\right)^2\right.$$
$$\left(\frac{c\,(d+e x)}{b e-\sqrt{(b^2-4 a c)\, e^2}+2 c e x}\right)^{-m}\text{Hypergeometric2F1}\left[-m,-m,1-m,\frac{2 c d-b e+\sqrt{(b^2-4 a c)\, e^2}}{-b e+\sqrt{(b^2-4 a c)\, e^2}-2 c e x}\right]-$$
$$2^{-m}\left(2 c d-b e+\sqrt{(b^2-4 a c)\, e^2}\right)^2 m\left(\frac{c\,(d+e x)}{b e-\sqrt{(b^2-4 a c)\, e^2}+2 c e x}\right)^{-m}\text{Hypergeometric2F1}\left[-m,-m,1-m,\right.$$
$$\left.\frac{2 c d-b e+\sqrt{(b^2-4 a c)\, e^2}}{-b e+\sqrt{(b^2-4 a c)\, e^2}-2 c e x}\right]+2^{-m}\left(-2 c d+b e+\sqrt{(b^2-4 a c)\, e^2}\right)^2\left(\frac{c\,(d+e x)}{b e+\sqrt{(b^2-4 a c)\, e^2}+2 c e x}\right)^{-m}$$

$$\text{Hypergeometric2F1}\left[-m,-m,1-m,\frac{-2cd+be+\sqrt{(b^2-4ac)e^2}}{be+\sqrt{(b^2-4ac)e^2}+2cex}\right]+2^{-m}\left(-2cd+be+\sqrt{(b^2-4ac)e^2}\right)^2 m$$

$$\left(\frac{c(d+ex)}{be+\sqrt{(b^2-4ac)e^2}+2cex}\right)^{-m}\text{Hypergeometric2F1}\left[-m,-m,1-m,\frac{-2cd+be+\sqrt{(b^2-4ac)e^2}}{be+\sqrt{(b^2-4ac)e^2}+2cex}\right]+2^{1-m}cd(1+m)$$

$$\left(\frac{c(d+ex)}{be-\sqrt{(b^2-4ac)e^2}+2cex}\right)^{-m}\left(\frac{c(d+ex)}{be+\sqrt{(b^2-4ac)e^2}+2cex}\right)^{-m}\left(\left(2cd-be+\sqrt{(b^2-4ac)e^2}\right)\left(\frac{c(d+ex)}{be+\sqrt{(b^2-4ac)e^2}+2cex}\right)^{m}\right.$$

$$\text{Hypergeometric2F1}\left[-m,-m,1-m,\frac{2cd-be+\sqrt{(b^2-4ac)e^2}}{-be+\sqrt{(b^2-4ac)e^2}-2cex}\right]+\left(-2cd+be+\sqrt{(b^2-4ac)e^2}\right)$$

$$\left.\left.\left.\left(\frac{c(d+ex)}{be-\sqrt{(b^2-4ac)e^2}+2cex}\right)^{m}\text{Hypergeometric2F1}\left[-m,-m,1-m,\frac{-2cd+be+\sqrt{(b^2-4ac)e^2}}{be+\sqrt{(b^2-4ac)e^2}+2cex}\right]\right)\right)\right)$$

Problem 2646: Result more than twice size of optimal antiderivative.

$$\int(A+Bx)(d+ex)^{-3-2p}\left(a+bx+cx^2\right)^p\,dx$$

Optimal (type 5, 349 leaves, 2 steps):

$$\frac{(Bd-Ae)(d+ex)^{-2(1+p)}\left(a+bx+cx^2\right)^{1+p}}{2\left(cd^2-bde+ae^2\right)(1+p)}-$$

$$\left((bBd-2Acd+Abe-2aBe)\left(b-\sqrt{b^2-4ac}+2cx\right)\left(\frac{\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\left(b-\sqrt{b^2-4ac}+2cx\right)}\right)^{-p}(d+ex)^{-1-2p}\right.$$

$$\left.\left(a+bx+cx^2\right)^p\text{Hypergeometric2F1}\left[-1-2p,-p,-2p,-\frac{4c\sqrt{b^2-4ac}\,(d+ex)}{\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\left(b-\sqrt{b^2-4ac}+2cx\right)}\right]\right)\Big/$$

$$\left(2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\left(cd^2-bde+ae^2\right)(1+2p)\right)$$

Result (type 5, 1806 leaves):

$$\left(2^{-2p-2(1+p)}\,B\,d\left(-\frac{-b-\sqrt{b^2-4ac}}{2c}+x\right)^{-p}\left(-\frac{-b+\sqrt{b^2-4ac}}{2c}+x\right)^{-p}\left(\frac{b-\sqrt{b^2-4ac}+2cx}{c}\right)^{p}\right.$$

$$\left(\frac{b+\sqrt{b^2-4ac}+2cx}{c}\right)^{p}(d+ex)^{-2-2p}\left(\frac{-be-\sqrt{b^2-4ac}\,e-2cex}{2cd-be-\sqrt{b^2-4ac}\,e}\right)^{-p}\left(\frac{-be+\sqrt{b^2-4ac}\,e-2cex}{2cd-be+\sqrt{b^2-4ac}\,e}\right)^{-p}$$

$$\left(a+bx+cx^2\right)^{p}\left(1-\frac{2c(d+ex)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)^{1+p}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{p}\text{Gamma}\left[-\frac{1}{2}-p\right]$$

$$\left(\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\left(4cd(1+p)+\left(-b+\sqrt{b^2-4ac}\right)e(1+2p)+2cex\right)\text{Gamma}[1-2p]\,\text{Gamma}[-p]\,\text{Hypergeometric2F1}\left[1,-p,\right.\right.$$

$$\left.-2p,\frac{4c\sqrt{b^2-4ac}\,(d+ex)}{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\left(b+\sqrt{b^2-4ac}+2cx\right)}\right]+\frac{1}{b+\sqrt{b^2-4ac}+2cx}4ce\left(-b^2+4ac+b\sqrt{b^2-4ac}+2c\sqrt{b^2-4ac}\,x\right)$$

$$\left.\left.(d+ex)\,\text{Gamma}[1-p]\,\text{Gamma}[-2p]\,\text{Hypergeometric2F1}\left[2,1-p,1-2p,\frac{4c\sqrt{b^2-4ac}\,(d+ex)}{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\left(b+\sqrt{b^2-4ac}+2cx\right)}\right]\right)\right)\Big/$$

$$\left(e^2\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)^2(-2-2p)\sqrt{\pi}\,\text{Gamma}[1-2p]\,\text{Gamma}[-2p]\right)-$$

$$\left(2^{-2p-2(1+p)}\,A\left(-\frac{-b-\sqrt{b^2-4ac}}{2c}+x\right)^{-p}\left(-\frac{-b+\sqrt{b^2-4ac}}{2c}+x\right)^{-p}\left(\frac{b-\sqrt{b^2-4ac}+2cx}{c}\right)^{p}\right.$$

$$\left(\frac{b+\sqrt{b^2-4ac}+2cx}{c}\right)^{p}(d+ex)^{-2-2p}\left(\frac{-be-\sqrt{b^2-4ac}\,e-2cex}{2cd-be-\sqrt{b^2-4ac}\,e}\right)^{-p}\left(\frac{-be+\sqrt{b^2-4ac}\,e-2cex}{2cd-be+\sqrt{b^2-4ac}\,e}\right)^{-p}$$

$$\left(a+bx+cx^2\right)^{p}\left(1-\frac{2c(d+ex)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)^{1+p}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{p}\text{Gamma}\left[-\frac{1}{2}-p\right]$$

$$\left(\left(2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)e\right)\left(4\,c\,d\,(1+p)+\left(-b+\sqrt{b^2-4\,a\,c}\right)e\,(1+2\,p)+2\,c\,e\,x\right)\text{Gamma}[1-2\,p]\,\text{Gamma}[-p]\,\text{Hypergeometric2F1}\left[1,-p,\right.\right.$$

$$\left.-2\,p,\frac{4\,c\sqrt{b^2-4\,a\,c}\,(d+e\,x)}{\left(2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)e\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}\right]+\frac{1}{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}4\,c\,e\left(-b^2+4\,a\,c+b\sqrt{b^2-4\,a\,c}+2\,c\sqrt{b^2-4\,a\,c}\,x\right)$$

$$\left.\left.(d+e\,x)\,\text{Gamma}[1-p]\,\text{Gamma}[-2\,p]\,\text{Hypergeometric2F1}\left[2,1-p,1-2\,p,\frac{4\,c\sqrt{b^2-4\,a\,c}\,(d+e\,x)}{\left(2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)e\right)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}\right]\right)\right)\Big/$$

$$\left(e\left(2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)e\right)^2(-2-2\,p)\sqrt{\pi}\,\text{Gamma}[1-2\,p]\,\text{Gamma}[-2\,p]\right)+\frac{1}{e^2\,(-1-2\,p)}2^{-2\,p}$$

B

$$\left(-\frac{-b-\sqrt{b^2-4\,a\,c}}{2\,c}+x\right)^{-p}$$

$$\left(-\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c}+x\right)^{-p}$$

$$\left(\frac{b-\sqrt{b^2-4\,a\,c}+2\,c\,x}{c}\right)^{p}$$

$$\left(\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{c}\right)^{p}(d+e\,x)^{-1-2\,p}$$

$$\left(\frac{-b\,e-\sqrt{b^2-4\,a\,c}\,e-2\,c\,e\,x}{2\,c\,d-b\,e-\sqrt{b^2-4\,a\,c}\,e}\right)^{-p}$$

$$\left(\frac{-b\,e+\sqrt{b^2-4\,a\,c}\,e-2\,c\,e\,x}{2\,c\,d-b\,e+\sqrt{b^2-4\,a\,c}\,e}\right)^{-p}\left(a+b\,x+c\,x^2\right)^{p}$$

$$\left(1-\frac{2\,c\,(d+e\,x)}{2\,c\,d+\left(-b+\sqrt{b^2-4\,a\,c}\right)e}\right)^{1+2\,p}$$

$$\text{Hypergeometric2F1}\left[-1-2p,-p,-2p,\frac{-\frac{2c(d+ex)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}+\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}{1-\frac{2c(d+ex)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}}\right]$$

Test results for the 958 problems in "1.2.1.4 (d+e x)^m (f+g x)^n (a+b x+c x^2)^p.m"

- **Problem 221: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx$$

Optimal (type 3, 35 leaves, 4 steps):

$$\sqrt{x}\sqrt{1-ax}+\frac{\text{ArcSin}\left[\sqrt{a}\sqrt{x}\right]}{\sqrt{a}}$$

Result (type 3, 80 leaves):

$$\frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}+\frac{i\,\text{Log}\left[-2i\sqrt{a}\sqrt{x}+\frac{2\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\right]}{\sqrt{a}}$$

- **Problem 223: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx$$

Optimal (type 3, 34 leaves, 4 steps):

$$\sqrt{x}\sqrt{1+ax}+\frac{\text{ArcSinh}\left[\sqrt{a}\sqrt{x}\right]}{\sqrt{a}}$$

Result (type 3, 100 leaves):

$$\frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1-ax}}+\frac{\text{Log}[1-ax]}{\sqrt{a}}-\frac{\text{Log}\left[-a\sqrt{x}+a^2x^{3/2}+\sqrt{a}\sqrt{1-ax}\sqrt{1-a^2x^2}\right]}{\sqrt{a}}$$

- **Problem 225: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx$$

Optimal (type 3, 63 leaves, 5 steps):

$$-\frac{\sqrt{x}\,\sqrt{1-a x}}{4 a}+\frac{1}{2} x^{3/2} \sqrt{1-a x}+\frac{\text{ArcSin}\left[\sqrt{a}\,\sqrt{x}\right]}{4 a^{3/2}}$$

Result (type 3, 94 leaves):

$$\frac{\sqrt{x}\,(-1+2 a x) \sqrt{1-a^2 x^2}}{4 a \sqrt{1+a x}}+\frac{i \,\text{Log}\left[-2 i \sqrt{a}\,\sqrt{x}+\frac{2 \sqrt{1-a^2 x^2}}{\sqrt{1+a x}}\right]}{4 a^{3/2}}$$

- **Problem 229: Result more than twice size of optimal antiderivative.**

$$\int (g x)^m \left(d^2-e^2 x^2\right)^{5/2} dx$$

Optimal (type 5, 80 leaves, 2 steps):

$$\frac{d^4 (g x)^{1+m} \sqrt{d^2-e^2 x^2}\ \text{Hypergeometric2F1}\left[-\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2 x^2}{d^2}\right]}{g (1+m) \sqrt{1-\frac{e^2 x^2}{d^2}}}$$

Result (type 5, 183 leaves):

$$\left(x (g x)^m \sqrt{d^2-e^2 x^2}\left(d^4 \left(15+8 m+m^2\right) \text{Hypergeometric2F1}\left[-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2 x^2}{d^2}\right]-e^2 (1+m) x^2 \left(2 d^2 (5+m) \text{Hypergeometric2F1}\left[-\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},\frac{e^2 x^2}{d^2}\right]-e^2 (3+m) x^2 \text{Hypergeometric2F1}\left[-\frac{1}{2},\frac{5+m}{2},\frac{7+m}{2},\frac{e^2 x^2}{d^2}\right]\right)\right)\right)\Big/\left((1+m) (3+m) (5+m) \sqrt{1-\frac{e^2 x^2}{d^2}}\right)$$

- **Problem 232: Result unnecessarily involves higher level functions.**

$$\int \frac{(g x)^m \left(d^2-e^2 x^2\right)^{5/2}}{(d+e x)^3} dx$$

Optimal (type 5, 250 leaves, 8 steps):

$$-\frac{3\,d\,(g\,x)^{1+m}\sqrt{d^2-e^2\,x^2}}{g\,(2+m)}+\frac{e\,(g\,x)^{2+m}\sqrt{d^2-e^2\,x^2}}{g^2\,(3+m)}+\frac{d^3\,(5+4\,m)\,(g\,x)^{1+m}\sqrt{1-\frac{e^2\,x^2}{d^2}}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2\,x^2}{d^2}\right]}{g\,(1+m)\,(2+m)\sqrt{d^2-e^2\,x^2}}-\frac{d^2\,e\,(11+4\,m)\,(g\,x)^{2+m}\sqrt{1-\frac{e^2\,x^2}{d^2}}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2\,x^2}{d^2}\right]}{g^2\,(2+m)\,(3+m)\sqrt{d^2-e^2\,x^2}}$$

Result (type 6, 272 leaves) :

$$\frac{1}{(1+m)\,(2+m)}x\,(g\,x)^m\left(1\Big/\left(\sqrt{1-\frac{e^2\,x^2}{d^2}}\right)\sqrt{d^2-e^2\,x^2}\right.$$
$$\left(e\,(1+m)\,x\,\text{Hypergeometric2F1}\left[-\frac{1}{2},1+\frac{m}{2},2+\frac{m}{2},\frac{e^2\,x^2}{d^2}\right]-3\,d\,(2+m)\,\text{Hypergeometric2F1}\left[-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2\,x^2}{d^2}\right]\right)+$$
$$\left(8\,d^3\,(2+m)^2\sqrt{d-e\,x}\,\text{AppellF1}\left[1+m,-\frac{1}{2},\frac{1}{2},2+m,\frac{e\,x}{d},-\frac{e\,x}{d}\right]\right)\Big/\left(\sqrt{d+e\,x}\left(2\,d\,(2+m)\,\text{AppellF1}\left[1+m,-\frac{1}{2},\frac{1}{2},2+m,\frac{e\,x}{d},-\frac{e\,x}{d}\right]-\right.\right.$$
$$\left.\left.\left.e\,x\left(\text{AppellF1}\left[2+m,-\frac{1}{2},\frac{3}{2},3+m,\frac{e\,x}{d},-\frac{e\,x}{d}\right]+\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},1+\frac{m}{2}\right\},\left\{2+\frac{m}{2}\right\},\frac{e^2\,x^2}{d^2}\right]\right)\right)\right)\right)$$

- **Problem 237: Result unnecessarily involves higher level functions.**

$$\int\frac{(g\,x)^m}{(d+e\,x)\,\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Optimal (type 5, 163 leaves, 8 steps) :

$$\frac{(g\,x)^{1+m}\sqrt{1-\frac{e^2\,x^2}{d^2}}\,\text{Hypergeometric2F1}\left[\frac{9}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2\,x^2}{d^2}\right]}{d^7\,g\,(1+m)\sqrt{d^2-e^2\,x^2}}-\frac{e\,(g\,x)^{2+m}\sqrt{1-\frac{e^2\,x^2}{d^2}}\,\text{Hypergeometric2F1}\left[\frac{9}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2\,x^2}{d^2}\right]}{d^8\,g^2\,(2+m)\sqrt{d^2-e^2\,x^2}}$$

Result (type 6, 161 leaves) :

$$\left(2\,d\,(2+m)\,x\,(g\,x)^m\,\text{AppellF1}\left[1+m,\frac{7}{2},\frac{9}{2},2+m,\frac{e\,x}{d},-\frac{e\,x}{d}\right]\right)\Big/$$
$$\left((1+m)\,(d-e\,x)^{7/2}\,(d+e\,x)^{9/2}\left(2\,d\,(2+m)\,\text{AppellF1}\left[1+m,\frac{7}{2},\frac{9}{2},2+m,\frac{e\,x}{d},-\frac{e\,x}{d}\right]+\right.\right.$$
$$\left.\left.e\,x\left(-9\,\text{AppellF1}\left[2+m,\frac{7}{2},\frac{11}{2},3+m,\frac{e\,x}{d},-\frac{e\,x}{d}\right]+7\,\text{HypergeometricPFQ}\left[\left\{\frac{9}{2},1+\frac{m}{2}\right\},\left\{2+\frac{m}{2}\right\},\frac{e^2\,x^2}{d^2}\right]\right)\right)\right)$$

Problem 238: Result unnecessarily involves higher level functions.

$$\int \frac{(g x)^m}{(d+e x)^2 \left(d^2-e^2 x^2\right)^{7/2}} \, dx$$

Optimal (type 5, 217 leaves, 7 steps):

$$\frac{2 (g x)^{1+m} (d-e x)}{9 d g \left(d^2-e^2 x^2\right)^{9/2}}+\frac{(7-2 m) (g x)^{1+m} \sqrt{1-\frac{e^2 x^2}{d^2}} \text{Hypergeometric2F1}\left[\frac{9}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2 x^2}{d^2}\right]}{9 d^8 g (1+m) \sqrt{d^2-e^2 x^2}}-$$

$$\frac{2 e (7-m) (g x)^{2+m} \sqrt{1-\frac{e^2 x^2}{d^2}} \text{Hypergeometric2F1}\left[\frac{9}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2 x^2}{d^2}\right]}{9 d^9 g^2 (2+m) \sqrt{d^2-e^2 x^2}}$$

Result (type 6, 157 leaves):

$$\left(2 d (2+m) x (g x)^m \text{AppellF1}\left[1+m,\frac{7}{2},\frac{11}{2},2+m,\frac{e x}{d},-\frac{e x}{d}\right]\right) \Big/$$

$$\left((1+m) (d-e x)^{7/2} (d+e x)^{11/2} \left(2 d (2+m) \text{AppellF1}\left[1+m,\frac{7}{2},\frac{11}{2},2+m,\frac{e x}{d},-\frac{e x}{d}\right]+\right.\right.$$

$$\left.\left.e x \left(-11 \text{AppellF1}\left[2+m,\frac{7}{2},\frac{13}{2},3+m,\frac{e x}{d},-\frac{e x}{d}\right]+7 \text{AppellF1}\left[2+m,\frac{9}{2},\frac{11}{2},3+m,\frac{e x}{d},-\frac{e x}{d}\right]\right)\right)\right)$$

Problem 239: Result unnecessarily involves higher level functions.

$$\int \frac{(g x)^m}{(d+e x)^3 \left(d^2-e^2 x^2\right)^{7/2}} \, dx$$

Optimal (type 5, 214 leaves, 7 steps):

$$\frac{4 (g x)^{1+m} (d-e x)}{11 g \left(d^2-e^2 x^2\right)^{11/2}}+\frac{(7-4 m) (g x)^{1+m} \sqrt{1-\frac{e^2 x^2}{d^2}} \text{Hypergeometric2F1}\left[\frac{11}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2 x^2}{d^2}\right]}{11 d^9 g (1+m) \sqrt{d^2-e^2 x^2}}-$$

$$\frac{e (25-4 m) (g x)^{2+m} \sqrt{1-\frac{e^2 x^2}{d^2}} \text{Hypergeometric2F1}\left[\frac{11}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2 x^2}{d^2}\right]}{11 d^{10} g^2 (2+m) \sqrt{d^2-e^2 x^2}}$$

Result (type 6, 157 leaves):

$$\left(2 d (2+m) x (g x)^m \text{AppellF1}\left[1+m, \frac{7}{2}, \frac{13}{2}, 2+m, \frac{e x}{d}, -\frac{e x}{d}\right]\right) \Big/ \left((1+m) (d-e x)^{7/2} (d+e x)^{13/2} \left(2 d (2+m) \text{AppellF1}\left[1+m, \frac{7}{2}, \frac{13}{2}, 2+m, \frac{e x}{d}, -\frac{e x}{d}\right] + e x \left(-13 \text{AppellF1}\left[2+m, \frac{7}{2}, \frac{15}{2}, 3+m, \frac{e x}{d}, -\frac{e x}{d}\right] + 7 \text{AppellF1}\left[2+m, \frac{9}{2}, \frac{13}{2}, 3+m, \frac{e x}{d}, -\frac{e x}{d}\right]\right)\right)\right)$$

- **Problem 254: Result more than twice size of optimal antiderivative.**

$$\int (d+e x)^2 \left(d^2-e^2 x^2\right)^p \, dx$$

Optimal (type 5, 71 leaves, 2 steps):

$$-\frac{2^{2+p} d \left(1+\frac{e x}{d}\right)^{-1-p} \left(d^2-e^2 x^2\right)^{1+p} \text{Hypergeometric2F1}\left[-2-p, 1+p, 2+p, \frac{d-e x}{2 d}\right]}{e (1+p)}$$

Result (type 5, 150 leaves):

$$\frac{1}{3 e (1+p)} \left(d^2-e^2 x^2\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \left(3 d e^2 x^2 \left(1-\frac{e^2 x^2}{d^2}\right)^p - 3 d^3 \left(-1+\left(1-\frac{e^2 x^2}{d^2}\right)^p\right) + 3 d^2 e (1+p) x \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, \frac{e^2 x^2}{d^2}\right] + e^3 (1+p) x^3 \text{Hypergeometric2F1}\left[\frac{3}{2}, -p, \frac{5}{2}, \frac{e^2 x^2}{d^2}\right]\right)$$

- **Problem 263: Result more than twice size of optimal antiderivative.**

$$\int (d+e x)^3 \left(d^2-e^2 x^2\right)^p \, dx$$

Optimal (type 5, 73 leaves, 2 steps):

$$-\frac{2^{3+p} d^2 \left(1+\frac{e x}{d}\right)^{-1-p} \left(d^2-e^2 x^2\right)^{1+p} \text{Hypergeometric2F1}\left[-3-p, 1+p, 2+p, \frac{d-e x}{2 d}\right]}{e (1+p)}$$

Result (type 5, 271 leaves):

$$\frac{1}{2 e (1+p) (2+p)} \left(d^2-e^2 x^2\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \left(7 d^4 + 3 d^4 p - 7 d^4 \left(1-\frac{e^2 x^2}{d^2}\right)^p - 3 d^4 p \left(1-\frac{e^2 x^2}{d^2}\right)^p + 6 d^2 e^2 x^2 \left(1-\frac{e^2 x^2}{d^2}\right)^p + 2 d^2 e^2 p x^2 \left(1-\frac{e^2 x^2}{d^2}\right)^p + e^4 x^4 \left(1-\frac{e^2 x^2}{d^2}\right)^p + e^4 p x^4 \left(1-\frac{e^2 x^2}{d^2}\right)^p + 2 d^3 e \left(2+3 p+p^2\right) x \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, \frac{e^2 x^2}{d^2}\right] + 2 d e^3 \left(2+3 p+p^2\right) x^3 \text{Hypergeometric2F1}\left[\frac{3}{2}, -p, \frac{5}{2}, \frac{e^2 x^2}{d^2}\right]\right)$$

- **Problem 267: Result unnecessarily involves higher level functions.**

$$\int \frac{x^4 \left(d^2-e^2 x^2\right)^p}{d+e x} \, dx$$

Optimal (type 5, 148 leaves, 7 steps):

$$\frac{d^4 \left(d^2-e^2 x^2\right)^p}{2 e^5 p}-\frac{d^2 \left(d^2-e^2 x^2\right)^{1+p}}{e^5 (1+p)}+\frac{\left(d^2-e^2 x^2\right)^{2+p}}{2 e^5 (2+p)}+\frac{x^5 \left(d^2-e^2 x^2\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[\frac{5}{2},1-p,\frac{7}{2},\frac{e^2 x^2}{d^2}\right]}{5 d}$$

Result (type 6, 225 leaves):

$$\left(6 d e^5 (1+p) (2+p) x^{10} (d-e x)^p (d+e x)^{-1+p} \text{AppellF1}\left[5,-p,1-p,6,\frac{e x}{d},-\frac{e x}{d}\right]\right) \Big/ \left(5 \left(6 d^4 e^2 p x^2 \left(1-\frac{e^2 x^2}{d^2}\right)^p+3 d^2 e^4 p (1+p) x^4 \left(1-\frac{e^2 x^2}{d^2}\right)^p+6 d^6 \left(-1+\left(1-\frac{e^2 x^2}{d^2}\right)^p\right)+6 d e^5 \left(2+3 p+p^2\right) x^5 \text{AppellF1}\left[5,-p,1-p,6,\frac{e x}{d},-\frac{e x}{d}\right]+e^6 \left(-2-p+2 p^2+p^3\right) x^6 \text{AppellF1}\left[6,-p,2-p,7,\frac{e x}{d},-\frac{e x}{d}\right]\right)\right)$$

Problem 268: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3 \left(d^2-e^2 x^2\right)^p}{d+e x} \, dx$$

Optimal (type 5, 121 leaves, 7 steps):

$$-\frac{d^3 \left(d^2-e^2 x^2\right)^p}{2 e^4 p}+\frac{d \left(d^2-e^2 x^2\right)^{1+p}}{2 e^4 (1+p)}-\frac{e x^5 \left(d^2-e^2 x^2\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[\frac{5}{2},1-p,\frac{7}{2},\frac{e^2 x^2}{d^2}\right]}{5 d^2}$$

Result (type 5, 245 leaves):

$$\frac{1}{6 e^4 (1+p)} \left(1+\frac{e x}{d}\right)^{-p} \left(d^2-e^2 x^2\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \left(6 d^2 e (1+p) x \left(1+\frac{e x}{d}\right)^p \text{Hypergeometric2F1}\left[\frac{1}{2},-p,\frac{3}{2},\frac{e^2 x^2}{d^2}\right]+2 e^3 (1+p) x^3 \left(1+\frac{e x}{d}\right)^p \text{Hypergeometric2F1}\left[\frac{3}{2},-p,\frac{5}{2},\frac{e^2 x^2}{d^2}\right]+3 d \left(\left(1+\frac{e x}{d}\right)^p \left(-e^2 x^2 \left(1-\frac{e^2 x^2}{d^2}\right)^p+d^2 \left(-1+\left(1-\frac{e^2 x^2}{d^2}\right)^p\right)\right)+d (d-e x) \left(2-\frac{2 e^2 x^2}{d^2}\right)^p \text{Hypergeometric2F1}\left[1-p,1+p,2+p,\frac{d-e x}{2 d}\right]\right)\right)$$

Problem 274: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(d^2-e^2 x^2\right)^p}{x^3 (d+e x)} \, dx$$

Optimal (type 5, 108 leaves, 6 steps):

$$\frac{e \left(d^2-e^2 x^2\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[-\frac{1}{2},1-p,\frac{1}{2},\frac{e^2 x^2}{d^2}\right]}{d^2 x}-\frac{e^2 \left(d^2-e^2 x^2\right)^p \text{Hypergeometric2F1}\left[2,p,1+p,1-\frac{e^2 x^2}{d^2}\right]}{2 d^3 p}$$

Result (type 5, 219 leaves):

$$\frac{1}{2 d^4}\left(d^2-e^2 x^2\right)^p \left(\frac{2 d^2 e \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[-\frac{1}{2},-p,\frac{1}{2},\frac{e^2 x^2}{d^2}\right]}{x}+\left(1-\frac{d^2}{e^2 x^2}\right)^{-p} \left(\frac{d^3 \text{Hypergeometric2F1}\left[1-p,-p,2-p,\frac{d^2}{e^2 x^2}\right]}{(-1+p) x^2}+ e^2 \left(\frac{\left(2-\frac{2 d^2}{e^2 x^2}\right)^p (d-e x) \left(1+\frac{e x}{d}\right)^{-p} \text{Hypergeometric2F1}\left[1-p,1+p,2+p,\frac{d-e x}{2 d}\right]}{1+p}+\frac{d \text{Hypergeometric2F1}\left[-p,-p,1-p,\frac{d^2}{e^2 x^2}\right]}{p}\right)\right)\right)$$

Problem 275: Result unnecessarily involves higher level functions.

$$\int \frac{x^5 \left(d^2-e^2 x^2\right)^p}{(d+e x)^2} \, dx$$

Optimal (type 5, 179 leaves, 8 steps) :

$$\frac{d^6 \left(d^2-e^2 x^2\right)^{-1+p}}{e^6 (1-p)}+\frac{5 d^4 \left(d^2-e^2 x^2\right)^p}{2 e^6 p}-\frac{2 d^2 \left(d^2-e^2 x^2\right)^{1+p}}{e^6 (1+p)}+\frac{\left(d^2-e^2 x^2\right)^{2+p}}{2 e^6 (2+p)}-\frac{2 e x^7 \left(d^2-e^2 x^2\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[\frac{7}{2},2-p,\frac{9}{2},\frac{e^2 x^2}{d^2}\right]}{7 d^3}$$

Result (type 6, 140 leaves) :

$$-\left(7 d x^6 (d-e x)^p (d+e x)^{-2+p} \text{AppellF1}\left[6,-p,2-p,7,\frac{e x}{d},-\frac{e x}{d}\right]\right) \Big/ \left(6 \left(-7 d \text{AppellF1}\left[6,-p,2-p,7,\frac{e x}{d},-\frac{e x}{d}\right]+ e x \left(p \text{AppellF1}\left[7,1-p,2-p,8,\frac{e x}{d},-\frac{e x}{d}\right]-(-2+p) \text{AppellF1}\left[7,-p,3-p,8,\frac{e x}{d},-\frac{e x}{d}\right]\right)\right)\right)$$

Problem 276: Result unnecessarily involves higher level functions.

$$\int \frac{x^4 \left(d^2-e^2 x^2\right)^p}{(d+e x)^2} \, dx$$

Optimal (type 5, 184 leaves, 9 steps) :

$$-\frac{d^5 \left(d^2-e^2 x^2\right)^{-1+p}}{e^5 (1-p)}-\frac{x^5 \left(d^2-e^2 x^2\right)^{-1+p}}{3+2 p}-\frac{2 d^3 \left(d^2-e^2 x^2\right)^p}{e^5 p}+ \frac{d \left(d^2-e^2 x^2\right)^{1+p}}{e^5 (1+p)}+\frac{2 (4+p) x^5 \left(d^2-e^2 x^2\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[\frac{5}{2},2-p,\frac{7}{2},\frac{e^2 x^2}{d^2}\right]}{5 d^2 (3+2 p)}$$

Result (type 6, 140 leaves) :

$$-\left(6 d x^5 (d-e x)^p (d+e x)^{-2+p} \text{AppellF1}\left[5,-p,2-p,6,\frac{e x}{d},-\frac{e x}{d}\right]\right) \Big/ \left(5 \left(-6 d \text{AppellF1}\left[5,-p,2-p,6,\frac{e x}{d},-\frac{e x}{d}\right]+ e x \left(p \text{AppellF1}\left[6,1-p,2-p,7,\frac{e x}{d},-\frac{e x}{d}\right]-(-2+p) \text{AppellF1}\left[6,-p,3-p,7,\frac{e x}{d},-\frac{e x}{d}\right]\right)\right)\right)$$

Problem 277: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3 \left(d^2-e^2 x^2\right)^p}{(d+e x)^2} \, dx$$

Optimal (type 5, 150 leaves, 8 steps):

$$\frac{d^4 \left(d^2-e^2 x^2\right)^{-1+p}}{e^4 (1-p)}+\frac{3 d^2 \left(d^2-e^2 x^2\right)^p}{2 e^4 p}-\frac{\left(d^2-e^2 x^2\right)^{1+p}}{2 e^4 (1+p)}-\frac{2 e x^5 \left(d^2-e^2 x^2\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[\frac{5}{2},2-p,\frac{7}{2},\frac{e^2 x^2}{d^2}\right]}{5 d^3}$$

Result (type 5, 332 leaves):

$$\frac{1}{e^4 (1+p)} 2^{-2+p} \left(1+\frac{e x}{d}\right)^{-p} \left(d^2-e^2 x^2\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^{-p}$$
$$\left(2 d^2 \left(\frac{1}{2}+\frac{e x}{2 d}\right)^p-2 d^2 \left(\frac{1}{2}+\frac{e x}{2 d}\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^p+2 e^2 x^2 \left(\frac{1}{2}+\frac{e x}{2 d}\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^p-8 d e (1+p) x \left(\frac{1}{2}+\frac{e x}{2 d}\right)^p \text{Hypergeometric2F1}\left[\frac{1}{2},-p,\frac{3}{2},\frac{e^2 x^2}{d^2}\right]-\right.$$
$$6 d (d-e x) \left(1-\frac{e^2 x^2}{d^2}\right)^p \text{Hypergeometric2F1}\left[1-p,1+p,2+p,\frac{d-e x}{2 d}\right]+d^2 \left(1-\frac{e^2 x^2}{d^2}\right)^p \text{Hypergeometric2F1}\left[2-p,1+p,2+p,\frac{d-e x}{2 d}\right]-$$
$$\left.d e x \left(1-\frac{e^2 x^2}{d^2}\right)^p \text{Hypergeometric2F1}\left[2-p,1+p,2+p,\frac{d-e x}{2 d}\right]\right)$$

Problem 282: Result unnecessarily involves higher level functions.

$$\int \frac{\left(d^2-e^2 x^2\right)^p}{x^2 (d+e x)^2} \, dx$$

Optimal (type 5, 137 leaves, 7 steps):

$$-\frac{\left(d^2-e^2 x^2\right)^{-1+p}}{x}+\frac{2 e^2 (2-p) x \left(d^2-e^2 x^2\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[\frac{1}{2},2-p,\frac{3}{2},\frac{e^2 x^2}{d^2}\right]}{d^4}-$$
$$\frac{e \left(d^2-e^2 x^2\right)^{-1+p} \text{Hypergeometric2F1}\left[1,-1+p,p,1-\frac{e^2 x^2}{d^2}\right]}{d (1-p)}$$

Result (type 6, 195 leaves):

$$\left(2 e (-2+p) (d-e x)^p (d+e x)^{-2+p} \text{AppellF1}\left[3-2 p,-p,2-p,4-2 p,\frac{d}{e x},-\frac{d}{e x}\right]\right) \Big/$$
$$\left((-3+2 p) \left(2 e (-2+p) x \text{AppellF1}\left[3-2 p,-p,2-p,4-2 p,\frac{d}{e x},-\frac{d}{e x}\right]+\right.\right.$$
$$\left.\left.d p \text{AppellF1}\left[4-2 p,1-p,2-p,5-2 p,\frac{d}{e x},-\frac{d}{e x}\right]-d (-2+p) \text{AppellF1}\left[4-2 p,-p,3-p,5-2 p,\frac{d}{e x},-\frac{d}{e x}\right]\right)\right)$$

Problem 284: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(d^2-e^2 x^2\right)^p}{x^4 (d+e x)^2} \, dx$$

Optimal (type 5, 145 leaves, 7 steps):

$$-\frac{\left(d^2-e^2 x^2\right)^{-1+p}}{3 x^3}-\frac{2 e^2 (4-p) \left(d^2-e^2 x^2\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[-\frac{1}{2},2-p,\frac{1}{2},\frac{e^2 x^2}{d^2}\right]}{3 d^4 x}-\frac{e^3 \left(d^2-e^2 x^2\right)^{-1+p} \text{Hypergeometric2F1}\left[2,-1+p,p,1-\frac{e^2 x^2}{d^2}\right]}{d^3 (1-p)}$$

Result (type 5, 334 leaves):

$$\frac{1}{12 d^6}\left(d^2-e^2 x^2\right)^p \left(-\frac{4 d^4 \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[-\frac{3}{2},-p,-\frac{1}{2},\frac{e^2 x^2}{d^2}\right]}{x^3}-\frac{36 d^2 e^2 \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[-\frac{1}{2},-p,\frac{1}{2},\frac{e^2 x^2}{d^2}\right]}{x}-\frac{12 d^3 e \left(1-\frac{d^2}{e^2 x^2}\right)^{-p} \text{Hypergeometric2F1}\left[1-p,-p,2-p,\frac{d^2}{e^2 x^2}\right]}{(-1+p) x^2}+\frac{3\times 2^{3+p} e^3 (-d+e x) \left(1+\frac{e x}{d}\right)^{-p} \text{Hypergeometric2F1}\left[1-p,1+p,2+p,\frac{d-e x}{2 d}\right]}{1+p}+\frac{3\times 2^p e^3 (-d+e x) \left(1+\frac{e x}{d}\right)^{-p} \text{Hypergeometric2F1}\left[2-p,1+p,2+p,\frac{d-e x}{2 d}\right]}{1+p}-\frac{24 d e^3 \left(1-\frac{d^2}{e^2 x^2}\right)^{-p} \text{Hypergeometric2F1}\left[-p,-p,1-p,\frac{d^2}{e^2 x^2}\right]}{p}\right)$$

Problem 285: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(d^2-e^2 x^2\right)^p}{x^5 (d+e x)^2} \, dx$$

Optimal (type 5, 145 leaves, 7 steps):

$$-\frac{\left(d^2-e^2 x^2\right)^{-1+p}}{4 x^4}+\frac{2 e \left(d^2-e^2 x^2\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[-\frac{3}{2},2-p,-\frac{1}{2},\frac{e^2 x^2}{d^2}\right]}{3 d^3 x^3}+\frac{e^4 (5-p) \left(d^2-e^2 x^2\right)^{-1+p} \text{Hypergeometric2F1}\left[2,-1+p,p,1-\frac{e^2 x^2}{d^2}\right]}{4 d^4 (1-p)}$$

Result (type 5, 389 leaves):

$$\frac{1}{12 d^7}\left(d^2-e^2 x^2\right)^p \left(\frac{8 d^4 e \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[-\frac{3}{2},-p,-\frac{1}{2},\frac{e^2 x^2}{d^2}\right]}{x^3}+\right.$$

$$\frac{48 d^2 e^3 \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[-\frac{1}{2},-p,\frac{1}{2},\frac{e^2 x^2}{d^2}\right]}{x}+\frac{18 d^3 e^2 \left(1-\frac{d^2}{e^2 x^2}\right)^{-p} \text{Hypergeometric2F1}\left[1-p,-p,2-p,\frac{d^2}{e^2 x^2}\right]}{(-1+p)\, x^2}+$$

$$\frac{15\times 2^{1+p} e^4 (d-e x) \left(1+\frac{e x}{d}\right)^{-p} \text{Hypergeometric2F1}\left[1-p,1+p,2+p,\frac{d-e x}{2 d}\right]}{1+p}+\frac{6 d^5 \left(1-\frac{d^2}{e^2 x^2}\right)^{-p} \text{Hypergeometric2F1}\left[2-p,-p,3-p,\frac{d^2}{e^2 x^2}\right]}{(-2+p)\, x^4}+$$

$$\left.\frac{3\times 2^{p} e^4 (d-e x) \left(1+\frac{e x}{d}\right)^{-p} \text{Hypergeometric2F1}\left[2-p,1+p,2+p,\frac{d-e x}{2 d}\right]}{1+p}+\frac{30 d e^4 \left(1-\frac{d^2}{e^2 x^2}\right)^{-p} \text{Hypergeometric2F1}\left[-p,-p,1-p,\frac{d^2}{e^2 x^2}\right]}{p}\right)$$

Problem 286: Result unnecessarily involves higher level functions.

$$\int \frac{x^4 \left(d^2-e^2 x^2\right)^p}{(d+e x)^3}\, dx$$

Optimal (type 5, 220 leaves, 8 steps):

$$-\frac{2 d^6 \left(d^2-e^2 x^2\right)^{-2+p}}{e^5 (2-p)}-\frac{3 d x^5 \left(d^2-e^2 x^2\right)^{-2+p}}{1+2 p}+\frac{9 d^4 \left(d^2-e^2 x^2\right)^{-1+p}}{2 e^5 (1-p)}+\frac{3 d^2 \left(d^2-e^2 x^2\right)^{p}}{e^5 p}-$$

$$\frac{\left(d^2-e^2 x^2\right)^{1+p}}{2 e^5 (1+p)}+\frac{2 (8+p) x^5 \left(d^2-e^2 x^2\right)^{p} \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[\frac{5}{2},3-p,\frac{7}{2},\frac{e^2 x^2}{d^2}\right]}{5 d^3 (1+2 p)}$$

Result (type 6, 140 leaves):

$$-\left(6 d x^5 (d-e x)^p (d+e x)^{-3+p} \text{AppellF1}\left[5,-p,3-p,6,\frac{e x}{d},-\frac{e x}{d}\right]\right) \Big/ \left(5 \left(-6 d \,\text{AppellF1}\left[5,-p,3-p,6,\frac{e x}{d},-\frac{e x}{d}\right]+\right.\right.$$

$$\left.\left. e x \left(p \,\text{AppellF1}\left[6,1-p,3-p,7,\frac{e x}{d},-\frac{e x}{d}\right]-(-3+p) \,\text{AppellF1}\left[6,-p,4-p,7,\frac{e x}{d},-\frac{e x}{d}\right]\right)\right)\right)$$

Problem 287: Result unnecessarily involves higher level functions.

$$\int \frac{x^3 \left(d^2-e^2 x^2\right)^p}{(d+e x)^3}\, dx$$

Optimal (type 5, 194 leaves, 8 steps):

$$\frac{2 d^5 \left(d^2-e^2 x^2\right)^{-2+p}}{e^4 (2-p)}+\frac{e x^5 \left(d^2-e^2 x^2\right)^{-2+p}}{1+2 p}-\frac{7 d^3 \left(d^2-e^2 x^2\right)^{-1+p}}{2 e^4 (1-p)}-$$

$$\frac{3 d \left(d^2-e^2 x^2\right)^{p}}{2 e^4 p}-\frac{2 e (4+3 p) x^5 \left(d^2-e^2 x^2\right)^{p} \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[\frac{5}{2},3-p,\frac{7}{2},\frac{e^2 x^2}{d^2}\right]}{5 d^4 (1+2 p)}$$

Result (type 6, 140 leaves) :

$$-\left(5\,d\,x^4\,(d-e\,x)^p\,(d+e\,x)^{-3+p}\,\text{AppellF1}\left[4,-p,3-p,5,\frac{e\,x}{d},-\frac{e\,x}{d}\right]\right)\Big/\left(4\left(-5\,d\,\text{AppellF1}\left[4,-p,3-p,5,\frac{e\,x}{d},-\frac{e\,x}{d}\right]+e\,x\left(p\,\text{AppellF1}\left[5,1-p,3-p,6,\frac{e\,x}{d},-\frac{e\,x}{d}\right]-(-3+p)\,\text{AppellF1}\left[5,-p,4-p,6,\frac{e\,x}{d},-\frac{e\,x}{d}\right]\right)\right)\right)$$

- **Problem 291: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(d^2-e^2\,x^2\right)^p}{x\,(d+e\,x)^3}\,dx$$

Optimal (type 5, 175 leaves, 8 steps) :

$$\frac{2\,d\left(d^2-e^2\,x^2\right)^{-2+p}}{2-p}-\frac{e\,x\left(d^2-e^2\,x^2\right)^{-2+p}}{3-2\,p}-\frac{2\,e\,(4-3\,p)\,x\left(d^2-e^2\,x^2\right)^p\left(1-\frac{e^2\,x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left[\frac{1}{2},3-p,\frac{3}{2},\frac{e^2\,x^2}{d^2}\right]}{d^4\,(3-2\,p)}+\frac{\left(d^2-e^2\,x^2\right)^{-1+p}\text{Hypergeometric2F1}\left[1,-1+p,p,1-\frac{e^2\,x^2}{d^2}\right]}{2\,d\,(1-p)}$$

Result (type 6, 196 leaves) :

$$\left(2\,e\,(-2+p)\,x\,(d-e\,x)^p\,(d+e\,x)^{-3+p}\,\text{AppellF1}\left[3-2\,p,-p,3-p,4-2\,p,\frac{d}{e\,x},-\frac{d}{e\,x}\right]\right)\Big/\left((-3+2\,p)\left(2\,e\,(-2+p)\,x\,\text{AppellF1}\left[3-2\,p,-p,3-p,4-2\,p,\frac{d}{e\,x},-\frac{d}{e\,x}\right]+d\,p\,\text{AppellF1}\left[4-2\,p,1-p,3-p,5-2\,p,\frac{d}{e\,x},-\frac{d}{e\,x}\right]-d\,(-3+p)\,\text{AppellF1}\left[4-2\,p,-p,4-p,5-2\,p,\frac{d}{e\,x},-\frac{d}{e\,x}\right]\right)\right)$$

- **Problem 292: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(d^2-e^2\,x^2\right)^p}{x^2\,(d+e\,x)^3}\,dx$$

Optimal (type 5, 166 leaves, 9 steps) :

$$-\frac{2\,e\left(d^2-e^2\,x^2\right)^{-2+p}}{2-p}-\frac{d\left(d^2-e^2\,x^2\right)^{-2+p}}{x}+\frac{2\,e^2\,(4-p)\,x\left(d^2-e^2\,x^2\right)^p\left(1-\frac{e^2\,x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left[\frac{1}{2},3-p,\frac{3}{2},\frac{e^2\,x^2}{d^2}\right]}{d^5}-\frac{3\,e\left(d^2-e^2\,x^2\right)^{-1+p}\text{Hypergeometric2F1}\left[1,-1+p,p,1-\frac{e^2\,x^2}{d^2}\right]}{2\,d^2\,(1-p)}$$

Result (type 6, 198 leaves) :

$$\left(e\,(-5+2p)\,(d-e\,x)^{p}\,(d+e\,x)^{-3+p}\,\text{AppellF1}\left[4-2p,-p,3-p,5-2p,\frac{d}{e\,x},-\frac{d}{e\,x}\right]\right)\Big/$$
$$\left(2\,(-2+p)\,\left(e\,(-5+2p)\,x\,\text{AppellF1}\left[4-2p,-p,3-p,5-2p,\frac{d}{e\,x},-\frac{d}{e\,x}\right]+\right.\right.$$
$$\left.\left.d\,p\,\text{AppellF1}\left[5-2p,1-p,3-p,6-2p,\frac{d}{e\,x},-\frac{d}{e\,x}\right]-d\,(-3+p)\,\text{AppellF1}\left[5-2p,-p,4-p,6-2p,\frac{d}{e\,x},-\frac{d}{e\,x}\right]\right)\right)$$

- **Problem 294: Result more than twice size of optimal antiderivative.**

$$\int\frac{\left(d^2-e^2x^2\right)^p}{x^4\,(d+e\,x)^3}\,dx$$

Optimal (type 5, 179 leaves, 8 steps):

$$-\frac{d\,\left(d^2-e^2x^2\right)^{-2+p}}{3\,x^3}+\frac{3\,e\,\left(d^2-e^2x^2\right)^{-2+p}}{2\,x^2}-\frac{2\,e^2\,(8-p)\,\left(d^2-e^2x^2\right)^{p}\,\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,\text{Hypergeometric2F1}\left[-\frac{1}{2},3-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right]}{3\,d^5\,x}-$$
$$\frac{e^3\,(10-3p)\,\left(d^2-e^2x^2\right)^{-2+p}\,\text{Hypergeometric2F1}\left[1,-2+p,-1+p,1-\frac{e^2x^2}{d^2}\right]}{2\,d^2\,(2-p)}$$

Result (type 5, 393 leaves):

$$\frac{1}{24\,d^7}\left(d^2-e^2x^2\right)^p\left(-\frac{8\,d^4\,\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,\text{Hypergeometric2F1}\left[-\frac{3}{2},-p,-\frac{1}{2},\frac{e^2x^2}{d^2}\right]}{x^3}-\right.$$
$$\frac{144\,d^2\,e^2\,\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,\text{Hypergeometric2F1}\left[-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right]}{x}-\frac{36\,d^3\,e\,\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\,\text{Hypergeometric2F1}\left[1-p,-p,2-p,\frac{d^2}{e^2x^2}\right]}{(-1+p)\,x^2}+$$
$$\frac{15\times 2^{3+p}\,e^3\,(-d+e\,x)\,\left(1+\frac{e\,x}{d}\right)^{-p}\,\text{Hypergeometric2F1}\left[1-p,1+p,2+p,\frac{d-e\,x}{2\,d}\right]}{1+p}+$$
$$\frac{3\times 2^{3+p}\,e^3\,(-d+e\,x)\,\left(1+\frac{e\,x}{d}\right)^{-p}\,\text{Hypergeometric2F1}\left[2-p,1+p,2+p,\frac{d-e\,x}{2\,d}\right]}{1+p}+$$
$$\left.\frac{3\times 2^{p}\,e^3\,(-d+e\,x)\,\left(1+\frac{e\,x}{d}\right)^{-p}\,\text{Hypergeometric2F1}\left[3-p,1+p,2+p,\frac{d-e\,x}{2\,d}\right]}{1+p}-\frac{120\,d\,e^3\,\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\,\text{Hypergeometric2F1}\left[-p,-p,1-p,\frac{d^2}{e^2x^2}\right]}{p}\right)$$

- **Problem 295: Result more than twice size of optimal antiderivative.**

$$\int\frac{\left(d^2-e^2x^2\right)^p}{x^5\,(d+e\,x)^3}\,dx$$

Optimal (type 5, 174 leaves, 8 steps):

$$-\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{4x^4}+\frac{e\left(d^2-e^2x^2\right)^{-2+p}}{x^3}+\frac{2e^3(4-p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left[-\frac{1}{2},3-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right]}{d^6x}+\frac{e^4(10-p)\left(d^2-e^2x^2\right)^{-2+p}\text{Hypergeometric2F1}\left[2,-2+p,-1+p,1-\frac{e^2x^2}{d^2}\right]}{4d^3(2-p)}$$

Result (type 5, 446 leaves) :

$$\frac{1}{8d^8}\left(d^2-e^2x^2\right)^p\left(\frac{8d^4e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left[-\frac{3}{2},-p,-\frac{1}{2},\frac{e^2x^2}{d^2}\right]}{x^3}+\frac{80d^2e^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left[-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right]}{x}+\frac{24d^3e^2\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\text{Hypergeometric2F1}\left[1-p,-p,2-p,\frac{d^2}{e^2x^2}\right]}{(-1+p)x^2}+\frac{15\times2^{2+p}e^4(d-ex)\left(1+\frac{ex}{d}\right)^{-p}\text{Hypergeometric2F1}\left[1-p,1+p,2+p,\frac{d-ex}{2d}\right]}{1+p}+\frac{4d^5\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\text{Hypergeometric2F1}\left[2-p,-p,3-p,\frac{d^2}{e^2x^2}\right]}{(-2+p)x^4}+\frac{5\times2^{1+p}e^4(d-ex)\left(1+\frac{ex}{d}\right)^{-p}\text{Hypergeometric2F1}\left[2-p,1+p,2+p,\frac{d-ex}{2d}\right]}{1+p}+\frac{2^pe^4(d-ex)\left(1+\frac{ex}{d}\right)^{-p}\text{Hypergeometric2F1}\left[3-p,1+p,2+p,\frac{d-ex}{2d}\right]}{1+p}+\frac{60de^4\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\text{Hypergeometric2F1}\left[-p,-p,1-p,\frac{d^2}{e^2x^2}\right]}{p}\right)$$

- **Problem 296: Result unnecessarily involves higher level functions.**

$$\int\frac{x^4\left(d^2-e^2x^2\right)^p}{(d+ex)^4}\,dx$$

Optimal (type 5, 265 leaves, 9 steps) :

$$-\frac{4d^7\left(d^2-e^2x^2\right)^{-3+p}}{e^5(3-p)}+\frac{d^2(13+12p)x^5\left(d^2-e^2x^2\right)^{-3+p}}{1-4p^2}-\frac{e^2x^7\left(d^2-e^2x^2\right)^{-3+p}}{1+2p}+\frac{10d^5\left(d^2-e^2x^2\right)^{-2+p}}{e^5(2-p)}-\frac{8d^3\left(d^2-e^2x^2\right)^{-1+p}}{e^5(1-p)}-\frac{2d\left(d^2-e^2x^2\right)^p}{e^5p}-\frac{4\left(16+15p+p^2\right)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left[\frac{5}{2},4-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right]}{5d^4\left(1-4p^2\right)}$$

Result (type 6, 140 leaves) :

$$-\left(6dx^5(d-ex)^p(d+ex)^{-4+p}\text{AppellF1}\left[5,-p,4-p,6,\frac{ex}{d},-\frac{ex}{d}\right]\right)\Big/\left(5\left(-6d\,\text{AppellF1}\left[5,-p,4-p,6,\frac{ex}{d},-\frac{ex}{d}\right]+ex\left(p\,\text{AppellF1}\left[6,1-p,4-p,7,\frac{ex}{d},-\frac{ex}{d}\right]-(-4+p)\text{AppellF1}\left[6,-p,5-p,7,\frac{ex}{d},-\frac{ex}{d}\right]\right)\right)\right)$$

- **Problem 297: Result unnecessarily involves higher level functions.**

$$\int\frac{x^3\left(d^2-e^2x^2\right)^p}{(d+ex)^4}\,dx$$

Optimal (type 5, 211 leaves, 5 steps):

$$\frac{d^2 \left(d^2-e^2 x^2\right)^{1+p}}{2 e^4 (3-p) (d+e x)^4}-\frac{d (1+2 p) \left(d^2-e^2 x^2\right)^{1+p}}{e^4 (1-2 p) p (d+e x)^3}-\frac{\left(d^2-e^2 x^2\right)^{1+p}}{2 e^4 p (d+e x)^2}+\frac{3\times 2^{-2+p} (2+p) \left(1+\frac{e x}{d}\right)^{-1-p} \left(d^2-e^2 x^2\right)^{1+p} \text{Hypergeometric2F1}\left[3-p,1+p,2+p,\frac{d-e x}{2 d}\right]}{d^2 e^4 (1-2 p) (3-p) p (1+p)}$$

Result (type 6, 140 leaves):

$$-\left(5 d x^4 (d-e x)^p (d+e x)^{-4+p} \text{AppellF1}\left[4,-p,4-p,5,\frac{e x}{d},-\frac{e x}{d}\right]\right)\Big/\left(4 \left(-5 d \text{AppellF1}\left[4,-p,4-p,5,\frac{e x}{d},-\frac{e x}{d}\right]+e x \left(p \text{AppellF1}\left[5,1-p,4-p,6,\frac{e x}{d},-\frac{e x}{d}\right]-(-4+p) \text{AppellF1}\left[5,-p,5-p,6,\frac{e x}{d},-\frac{e x}{d}\right]\right)\right)\right)$$

■ Problem 301: Result unnecessarily involves higher level functions.

$$\int \frac{\left(d^2-e^2 x^2\right)^p}{x (d+e x)^4} \, dx$$

Optimal (type 5, 204 leaves, 9 steps):

$$\frac{4 d^2 \left(d^2-e^2 x^2\right)^{-3+p}}{3-p}-\frac{4 d e x \left(d^2-e^2 x^2\right)^{-3+p}}{5-2 p}-\frac{\left(d^2-e^2 x^2\right)^{-2+p}}{2 (2-p)}-\frac{8 e (2-p) x \left(d^2-e^2 x^2\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[\frac{1}{2},4-p,\frac{3}{2},\frac{e^2 x^2}{d^2}\right]}{d^5 (5-2 p)}+\frac{\left(d^2-e^2 x^2\right)^{-2+p} \text{Hypergeometric2F1}\left[1,-2+p,-1+p,1-\frac{e^2 x^2}{d^2}\right]}{2 (2-p)}$$

Result (type 6, 199 leaves):

$$\left(e (-5+2 p) x (d-e x)^p (d+e x)^{-4+p} \text{AppellF1}\left[4-2 p,-p,4-p,5-2 p,\frac{d}{e x},-\frac{d}{e x}\right]\right)\Big/\left(2 (-2+p) \left(e (-5+2 p) x \text{AppellF1}\left[4-2 p,-p,4-p,5-2 p,\frac{d}{e x},-\frac{d}{e x}\right]+d p \text{AppellF1}\left[5-2 p,1-p,4-p,6-2 p,\frac{d}{e x},-\frac{d}{e x}\right]-d (-4+p) \text{AppellF1}\left[5-2 p,-p,5-p,6-2 p,\frac{d}{e x},-\frac{d}{e x}\right]\right)\right)$$

■ Problem 302: Result unnecessarily involves higher level functions.

$$\int \frac{\left(d^2-e^2 x^2\right)^p}{x^2 (d+e x)^4} \, dx$$

Optimal (type 5, 207 leaves, 9 steps):

$$-\frac{4\,d\,e\,\left(d^2-e^2\,x^2\right)^{-3+p}}{3-p}-\frac{d^2\,\left(d^2-e^2\,x^2\right)^{-3+p}}{x}+\frac{e^2\,x\,\left(d^2-e^2\,x^2\right)^{-3+p}}{5-2\,p}+$$

$$\frac{4\,e^2\,\left(16-9\,p+p^2\right)\,x\,\left(d^2-e^2\,x^2\right)^p\,\left(1-\frac{e^2\,x^2}{d^2}\right)^{-p}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,4-p,\,\frac{3}{2},\,\frac{e^2\,x^2}{d^2}\right]}{d^6\,(5-2\,p)}-$$

$$\frac{2\,e\,\left(d^2-e^2\,x^2\right)^{-2+p}\,\text{Hypergeometric2F1}\left[1,\,-2+p,\,-1+p,\,1-\frac{e^2\,x^2}{d^2}\right]}{d\,(2-p)}$$

Result (type 6, 195 leaves) :

$$\left(2\,e\,(-3+p)\,(d-e\,x)^p\,(d+e\,x)^{-4+p}\,\text{AppellF1}\left[5-2\,p,\,-p,\,4-p,\,6-2\,p,\,\frac{d}{e\,x},\,-\frac{d}{e\,x}\right]\right)\Big/$$
$$\left((-5+2\,p)\,\left(2\,e\,(-3+p)\,x\,\text{AppellF1}\left[5-2\,p,\,-p,\,4-p,\,6-2\,p,\,\frac{d}{e\,x},\,-\frac{d}{e\,x}\right]+\right.\right.$$
$$\left.\left.d\,p\,\text{AppellF1}\left[6-2\,p,\,1-p,\,4-p,\,7-2\,p,\,\frac{d}{e\,x},\,-\frac{d}{e\,x}\right]-d\,(-4+p)\,\text{AppellF1}\left[6-2\,p,\,-p,\,5-p,\,7-2\,p,\,\frac{d}{e\,x},\,-\frac{d}{e\,x}\right]\right)\right)$$

- **Problem 304: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(d^2-e^2\,x^2\right)^p}{x^4\,(d+e\,x)^4}\,dx$$

Optimal (type 5, 210 leaves, 9 steps) :

$$-\frac{d^2\,\left(d^2-e^2\,x^2\right)^{-3+p}}{3\,x^3}+\frac{2\,d\,e\,\left(d^2-e^2\,x^2\right)^{-3+p}}{x^2}-\frac{e^2\,(27-2\,p)\,\left(d^2-e^2\,x^2\right)^{-3+p}}{3\,x}+$$

$$\frac{4\,e^4\,\left(48-17\,p+p^2\right)\,x\,\left(d^2-e^2\,x^2\right)^p\,\left(1-\frac{e^2\,x^2}{d^2}\right)^{-p}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,4-p,\,\frac{3}{2},\,\frac{e^2\,x^2}{d^2}\right]}{3\,d^8}-$$

$$\frac{2\,e^3\,(5-p)\,\left(d^2-e^2\,x^2\right)^{-3+p}\,\text{Hypergeometric2F1}\left[1,\,-3+p,\,-2+p,\,1-\frac{e^2\,x^2}{d^2}\right]}{d\,(3-p)}$$

Result (type 5, 452 leaves) :

$$\frac{1}{48\,d^8}\left(d^2-e^2x^2\right)^p\left(-\frac{16\,d^4\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left[-\frac{3}{2},-p,-\frac{1}{2},\frac{e^2x^2}{d^2}\right]}{x^3}-\right.$$

$$\frac{480\,d^2e^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left[-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right]}{x}-\frac{96\,d^3e\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\text{Hypergeometric2F1}\left[1-p,-p,2-p,\frac{d^2}{e^2x^2}\right]}{(-1+p)\,x^2}+$$

$$\frac{15\times 2^{5+p}e^3\,(-d+e\,x)\left(1+\frac{e\,x}{d}\right)^{-p}\text{Hypergeometric2F1}\left[1-p,1+p,2+p,\frac{d-e\,x}{2\,d}\right]}{1+p}+$$

$$\frac{15\times 2^{3+p}e^3\,(-d+e\,x)\left(1+\frac{e\,x}{d}\right)^{-p}\text{Hypergeometric2F1}\left[2-p,1+p,2+p,\frac{d-e\,x}{2\,d}\right]}{1+p}+$$

$$\frac{3\times 2^{3+p}e^3\,(-d+e\,x)\left(1+\frac{e\,x}{d}\right)^{-p}\text{Hypergeometric2F1}\left[3-p,1+p,2+p,\frac{d-e\,x}{2\,d}\right]}{1+p}+$$

$$\left.\frac{3\times 2^{p}e^3\,(-d+e\,x)\left(1+\frac{e\,x}{d}\right)^{-p}\text{Hypergeometric2F1}\left[4-p,1+p,2+p,\frac{d-e\,x}{2\,d}\right]}{1+p}-\frac{480\,d\,e^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\text{Hypergeometric2F1}\left[-p,-p,1-p,\frac{d^2}{e^2x^2}\right]}{p}\right)$$

- **Problem 305: Result more than twice size of optimal antiderivative.**

$$\int\frac{\left(d^2-e^2x^2\right)^p}{x^5\,(d+e\,x)^4}\,dx$$

Optimal (type 5, 216 leaves, 9 steps):

$$-\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{4\,x^4}+\frac{4\,d\,e\left(d^2-e^2x^2\right)^{-3+p}}{3\,x^3}-\frac{e^2\,(17-p)\left(d^2-e^2x^2\right)^{-3+p}}{4\,x^2}+$$

$$\frac{8\,e^3\,(6-p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left[-\frac{1}{2},4-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right]}{3\,d^7\,x}+$$

$$\frac{e^4\left(70-21\,p+p^2\right)\left(d^2-e^2x^2\right)^{-3+p}\text{Hypergeometric2F1}\left[1,-3+p,-2+p,1-\frac{e^2x^2}{d^2}\right]}{4\,d^2\,(3-p)}$$

Result (type 5, 505 leaves):

$$\frac{1}{48 d^9}\left(d^2-e^2 x^2\right)^p \left(\frac{64 d^4 e \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[-\frac{3}{2},-p,-\frac{1}{2},\frac{e^2 x^2}{d^2}\right]}{x^3}+\right.$$

$$\frac{960 d^2 e^3 \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[-\frac{1}{2},-p,\frac{1}{2},\frac{e^2 x^2}{d^2}\right]}{x}+\frac{240 d^3 e^2 \left(1-\frac{d^2}{e^2 x^2}\right)^{-p} \text{Hypergeometric2F1}\left[1-p,-p,2-p,\frac{d^2}{e^2 x^2}\right]}{(-1+p)\, x^2}+$$

$$\frac{105\times 2^{3+p} e^4 (d-e x) \left(1+\frac{e x}{d}\right)^{-p} \text{Hypergeometric2F1}\left[1-p,1+p,2+p,\frac{d-e x}{2 d}\right]}{1+p}+$$

$$\frac{24 d^5 \left(1-\frac{d^2}{e^2 x^2}\right)^{-p} \text{Hypergeometric2F1}\left[2-p,-p,3-p,\frac{d^2}{e^2 x^2}\right]}{(-2+p)\, x^4}+\frac{45\times 2^{2+p} e^4 (d-e x) \left(1+\frac{e x}{d}\right)^{-p} \text{Hypergeometric2F1}\left[2-p,1+p,2+p,\frac{d-e x}{2 d}\right]}{1+p}+$$

$$\frac{15\times 2^{1+p} e^4 (d-e x) \left(1+\frac{e x}{d}\right)^{-p} \text{Hypergeometric2F1}\left[3-p,1+p,2+p,\frac{d-e x}{2 d}\right]}{1+p}+$$

$$\left.\frac{3\times 2^{p} e^4 (d-e x) \left(1+\frac{e x}{d}\right)^{-p} \text{Hypergeometric2F1}\left[4-p,1+p,2+p,\frac{d-e x}{2 d}\right]}{1+p}+\frac{840 d e^4 \left(1-\frac{d^2}{e^2 x^2}\right)^{-p} \text{Hypergeometric2F1}\left[-p,-p,1-p,\frac{d^2}{e^2 x^2}\right]}{p}\right)$$

Problem 310: Result unnecessarily involves higher level functions.

$$\int \frac{(g x)^m \left(d^2-e^2 x^2\right)^p}{d+e x}\, dx$$

Optimal (type 5, 163 leaves, 8 steps):

$$\frac{(g x)^{1+m} \left(d^2-e^2 x^2\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[\frac{1+m}{2},1-p,\frac{3+m}{2},\frac{e^2 x^2}{d^2}\right]}{d g (1+m)}-$$

$$\frac{e (g x)^{2+m} \left(d^2-e^2 x^2\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[\frac{2+m}{2},1-p,\frac{4+m}{2},\frac{e^2 x^2}{d^2}\right]}{d^2 g^2 (2+m)}$$

Result (type 6, 168 leaves):

$$\left(d (2+m) x (g x)^m (d-e x)^p (d+e x)^{-1+p} \text{AppellF1}\left[1+m,-p,1-p,2+m,\frac{e x}{d},-\frac{e x}{d}\right]\right) \Big/$$

$$\left((1+m) \left(d (2+m) \text{AppellF1}\left[1+m,-p,1-p,2+m,\frac{e x}{d},-\frac{e x}{d}\right]+\right.\right.$$

$$\left.\left.e x \left((-1+p) \text{AppellF1}\left[2+m,-p,2-p,3+m,\frac{e x}{d},-\frac{e x}{d}\right]-p\, \text{HypergeometricPFQ}\left[\left\{1+\frac{m}{2},1-p\right\},\left\{2+\frac{m}{2}\right\},\frac{e^2 x^2}{d^2}\right]\right)\right)\right)$$

Problem 311: Result unnecessarily involves higher level functions.

$$\int \frac{(g x)^m \left(d^2-e^2 x^2\right)^p}{(d+e x)^2} \, dx$$

Optimal (type 5, 214 leaves, 7 steps):

$$\frac{(g x)^{1+m} \left(d^2-e^2 x^2\right)^{-1+p}}{g (1-m-2 p)}-\frac{2 (m+p) (g x)^{1+m} \left(d^2-e^2 x^2\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[\frac{1+m}{2},2-p,\frac{3+m}{2},\frac{e^2 x^2}{d^2}\right]}{d^2 g (1+m) (1-m-2 p)}-\frac{2 e (g x)^{2+m} \left(d^2-e^2 x^2\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[\frac{2+m}{2},2-p,\frac{4+m}{2},\frac{e^2 x^2}{d^2}\right]}{d^3 g^2 (2+m)}$$

Result (type 6, 166 leaves):

$$\left(d (2+m) x (g x)^m (d-e x)^p (d+e x)^{-2+p} \text{AppellF1}\left[1+m,-p,2-p,2+m,\frac{e x}{d},-\frac{e x}{d}\right]\right) \Big/ \left((1+m) \left(d (2+m) \text{AppellF1}\left[1+m,-p,2-p,2+m,\frac{e x}{d},-\frac{e x}{d}\right]+e x \left(-p \text{AppellF1}\left[2+m,1-p,2-p,3+m,\frac{e x}{d},-\frac{e x}{d}\right]+(-2+p) \text{AppellF1}\left[2+m,-p,3-p,3+m,\frac{e x}{d},-\frac{e x}{d}\right]\right)\right)\right)$$

Problem 312: Result unnecessarily involves higher level functions.

$$\int \frac{(g x)^m \left(d^2-e^2 x^2\right)^p}{(d+e x)^3} \, dx$$

Optimal (type 5, 275 leaves, 8 steps):

$$\frac{3 d (g x)^{1+m} \left(d^2-e^2 x^2\right)^{-2+p}}{g (3-m-2 p)}-\frac{e (g x)^{2+m} \left(d^2-e^2 x^2\right)^{-2+p}}{g^2 (2-m-2 p)}-\frac{2 (2 m+p) (g x)^{1+m} \left(d^2-e^2 x^2\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[\frac{1+m}{2},3-p,\frac{3+m}{2},\frac{e^2 x^2}{d^2}\right]}{d^3 g (1+m) (3-m-2 p)}-\frac{2 e (2-2 m-3 p) (g x)^{2+m} \left(d^2-e^2 x^2\right)^p \left(1-\frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left[\frac{2+m}{2},3-p,\frac{4+m}{2},\frac{e^2 x^2}{d^2}\right]}{d^4 g^2 (2+m) (2-m-2 p)}$$

Result (type 6, 166 leaves):

$$\left(d (2+m) x (g x)^m (d-e x)^p (d+e x)^{-3+p} \text{AppellF1}\left[1+m,-p,3-p,2+m,\frac{e x}{d},-\frac{e x}{d}\right]\right) \Big/ \left((1+m) \left(d (2+m) \text{AppellF1}\left[1+m,-p,3-p,2+m,\frac{e x}{d},-\frac{e x}{d}\right]+e x \left(-p \text{AppellF1}\left[2+m,1-p,3-p,3+m,\frac{e x}{d},-\frac{e x}{d}\right]+(-3+p) \text{AppellF1}\left[2+m,-p,4-p,3+m,\frac{e x}{d},-\frac{e x}{d}\right]\right)\right)\right)$$

- **Problem 313: Result unnecessarily involves higher level functions.**

$$\int \frac{(g x)^m \left(1-a^2 x^2\right)^p}{1+a x} \, dx$$

Optimal (type 5, 89 leaves, 6 steps):

$$\frac{(g x)^{1+m} \text{Hypergeometric2F1}\left[\frac{1+m}{2}, 1-p, \frac{3+m}{2}, a^2 x^2\right]}{g (1+m)} - \frac{a (g x)^{2+m} \text{Hypergeometric2F1}\left[\frac{2+m}{2}, 1-p, \frac{4+m}{2}, a^2 x^2\right]}{g^2 (2+m)}$$

Result (type 6, 145 leaves):

$$\Big((2+m)\, x\, (g x)^m (1-a x)^p (1+a x)^{-1+p} \text{AppellF1}[1+m, -p, 1-p, 2+m, a x, -a x]\Big) \Big/ \Big((1+m) \Big((2+m) \text{AppellF1}[1+m, -p, 1-p, 2+m, a x, -a x] + a x \Big((-1+p) \text{AppellF1}[2+m, -p, 2-p, 3+m, a x, -a x] - p\, \text{HypergeometricPFQ}\Big[\Big\{1+\frac{m}{2}, 1-p\Big\}, \Big\{2+\frac{m}{2}\Big\}, a^2 x^2\Big]\Big)\Big)\Big)$$

- **Problem 315: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x \sqrt{1+x}}{1+x^2} \, dx$$

Optimal (type 3, 214 leaves, 11 steps):

$$2 \sqrt{1+x} + \frac{\text{ArcTan}\left[\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right]}{\sqrt{2\left(1+\sqrt{2}\right)}} - \frac{\text{ArcTan}\left[\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right]}{\sqrt{2\left(1+\sqrt{2}\right)}} + \frac{1}{2} \sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)} \text{Log}\left[1+\sqrt{2}+x-\sqrt{2\left(1+\sqrt{2}\right)} \sqrt{1+x}\right] - \frac{1}{2} \sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)} \text{Log}\left[1+\sqrt{2}+x+\sqrt{2\left(1+\sqrt{2}\right)} \sqrt{1+x}\right]$$

Result (type 3, 60 leaves):

$$2 \sqrt{1+x} - \sqrt{1-i}\, \text{ArcTanh}\left[\frac{\sqrt{1+x}}{\sqrt{1-i}}\right] - \sqrt{1+i}\, \text{ArcTanh}\left[\frac{\sqrt{1+x}}{\sqrt{1+i}}\right]$$

- **Problem 359: Result more than twice size of optimal antiderivative.**

$$\int x^2 (a+b x)^n \left(c+d x^2\right)^3 \, dx$$

Optimal (type 3, 343 leaves, 2 steps):

$$\frac{a^2 (b^2 c+a^2 d)^3 (a+b x)^{1+n}}{b^9 (1+n)}-\frac{2 a (b^2 c+a^2 d)^2 (b^2 c+4 a^2 d) (a+b x)^{2+n}}{b^9 (2+n)}+\frac{(b^2 c+a^2 d) (b^4 c^2+17 a^2 b^2 c d+28 a^4 d^2) (a+b x)^{3+n}}{b^9 (3+n)}-\frac{4 a d (3 b^4 c^2+15 a^2 b^2 c d+14 a^4 d^2) (a+b x)^{4+n}}{b^9 (4+n)}+\frac{d (3 b^4 c^2+45 a^2 b^2 c d+70 a^4 d^2) (a+b x)^{5+n}}{b^9 (5+n)}-\frac{2 a d^2 (9 b^2 c+28 a^2 d) (a+b x)^{6+n}}{b^9 (6+n)}+\frac{d^2 (3 b^2 c+28 a^2 d) (a+b x)^{7+n}}{b^9 (7+n)}-\frac{8 a d^3 (a+b x)^{8+n}}{b^9 (8+n)}+\frac{d^3 (a+b x)^{9+n}}{b^9 (9+n)}$$

Result (type 3, 746 leaves) :

$$\frac{1}{b^9 (1+n) (2+n) (3+n) (4+n) (5+n) (6+n) (7+n) (8+n) (9+n)} (a+b x)^{1+n} \Big(40320 a^8 d^3-40320 a^7 b d^3 (1+n) x+720 a^6 b^2 d^2 \left(3 c \left(72+17 n+n^2\right)+28 d \left(2+3 n+n^2\right) x^2\right)- 240 a^5 b^3 d^2 (1+n) x \left(9 c \left(72+17 n+n^2\right)+28 d \left(6+5 n+n^2\right) x^2\right)+ 24 a^4 b^4 d \left(3 c^2 \left(3024+1650 n+335 n^2+30 n^3+n^4\right)+45 c d \left(144+250 n+125 n^2+20 n^3+n^4\right) x^2+70 d^2 \left(24+50 n+35 n^2+10 n^3+n^4\right) x^4\right)-24 a^3 b^5 d (1+n) x \left(3 c^2 \left(3024+1650 n+335 n^2+30 n^3+n^4\right)+15 c d \left(432+462 n+163 n^2+22 n^3+n^4\right) x^2+14 d^2 \left(120+154 n+71 n^2+14 n^3+n^4\right) x^4\right)+ b^8 \left(384+784 n+540 n^2+160 n^3+21 n^4+n^5\right) x^2 \left(c^3 \left(315+143 n+21 n^2+n^3\right)+3 c^2 d \left(189+111 n+19 n^2+n^3\right) x^2+3 c d^2 \left(135+87 n+17 n^2+n^3\right) x^4+d^3 \left(105+71 n+15 n^2+n^3\right) x^6\right)+ 2 a^2 b^6 \left(c^3 \left(60480+60216 n+24574 n^2+5265 n^3+625 n^4+39 n^5+n^6\right)+18 c^2 d \left(6048+12372 n+8644 n^2+2715 n^3+427 n^4+33 n^5+n^6\right) x^2+ 45 c d^2 \left(1728+4008 n+3394 n^2+1365 n^3+277 n^4+27 n^5+n^6\right) x^4+28 d^3 \left(720+1764 n+1624 n^2+735 n^3+175 n^4+21 n^5+n^6\right) x^6\right)-2 a b^7 (1+n) x \left(c^3 \left(60480+60216 n+24574 n^2+5265 n^3+625 n^4+39 n^5+n^6\right)+6 c^2 d \left(18144+25020 n+13284 n^2+3505 n^3+491 n^4+35 n^5+n^6\right) x^2+ 9 c d^2 \left(8640+13128 n+7850 n^2+2369 n^3+381 n^4+31 n^5+n^6\right) x^4+4 d^3 \left(5040+8028 n+5104 n^2+1665 n^3+295 n^4+27 n^5+n^6\right) x^6\right)\Big)$$

- **Problem 360: Result more than twice size of optimal antiderivative.**

$$\int x (a+b x)^n (c+d x^2)^3 \, dx$$

Optimal (type 3, 282 leaves, 2 steps) :

$$-\frac{a (b^2 c+a^2 d)^3 (a+b x)^{1+n}}{b^8 (1+n)}+\frac{(b^2 c+a^2 d)^2 (b^2 c+7 a^2 d) (a+b x)^{2+n}}{b^8 (2+n)}-\frac{3 a d (b^2 c+a^2 d) (3 b^2 c+7 a^2 d) (a+b x)^{3+n}}{b^8 (3+n)}+\frac{d (3 b^4 c^2+30 a^2 b^2 c d+35 a^4 d^2) (a+b x)^{4+n}}{b^8 (4+n)}-\frac{5 a d^2 (3 b^2 c+7 a^2 d) (a+b x)^{5+n}}{b^8 (5+n)}+\frac{3 d^2 (b^2 c+7 a^2 d) (a+b x)^{6+n}}{b^8 (6+n)}-\frac{7 a d^3 (a+b x)^{7+n}}{b^8 (7+n)}+\frac{d^3 (a+b x)^{8+n}}{b^8 (8+n)}$$

Result (type 3, 578 leaves) :

$$\frac{1}{b^8 (1+n) (2+n) (3+n) (4+n) (5+n) (6+n) (7+n) (8+n)}$$

$$(a+b x)^{1+n} \Big(-5040 a^7 d^3 + 5040 a^6 b d^3 (1+n) x - 360 a^5 b^2 d^2 \left(c \left(56+15 n+n^2\right) + 7 d \left(2+3 n+n^2\right) x^2\right) + 120 a^4 b^3 d^2 (1+n) x \left(3 c \left(56+15 n+n^2\right) + 7 d \left(6+5 n+n^2\right) x^2\right) - 6 a^3 b^4 d \left(3 c^2 \left(1680+1066 n+251 n^2+26 n^3+n^4\right) + 30 c d \left(112+198 n+103 n^2+18 n^3+n^4\right) x^2 + 35 d^2 \left(24+50 n+35 n^2+10 n^3+n^4\right) x^4\right) + 6 a^2 b^5 d (1+n) x \left(3 c^2 \left(1680+1066 n+251 n^2+26 n^3+n^4\right) + 10 c d \left(336+370 n+137 n^2+20 n^3+n^4\right) x^2 + 7 d^2 \left(120+154 n+71 n^2+14 n^3+n^4\right) x^4\right) + b^7 \left(105+176 n+86 n^2+16 n^3+n^4\right) x \left(c^3 \left(192+104 n+18 n^2+n^3\right) + 3 c^2 d \left(96+76 n+16 n^2+n^3\right) x^2 + 3 c d^2 \left(64+56 n+14 n^2+n^3\right) x^4 + d^3 \left(48+44 n+12 n^2+n^3\right) x^6\right) - a b^6 \left(c^3 \left(20160+24552 n+12154 n^2+3135 n^3+445 n^4+33 n^5+n^6\right) + 9 c^2 d \left(3360+7172 n+5380 n^2+1871 n^3+331 n^4+29 n^5+n^6\right) x^2 + 15 c d^2 \left(1344+3160 n+2734 n^2+1135 n^3+241 n^4+25 n^5+n^6\right) x^4 + 7 d^3 \left(720+1764 n+1624 n^2+735 n^3+175 n^4+21 n^5+n^6\right) x^6\right)\Big)$$

- **Problem 362: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b x)^n \left(c+d x^2\right)^3}{x} \, dx$$

Optimal (type 5, 246 leaves, 4 steps):

$$-\frac{a d \left(3 b^4 c^2+3 a^2 b^2 c d+a^4 d^2\right) (a+b x)^{1+n}}{b^6 (1+n)} + \frac{d \left(3 b^4 c^2+9 a^2 b^2 c d+5 a^4 d^2\right) (a+b x)^{2+n}}{b^6 (2+n)} - \frac{a d^2 \left(9 b^2 c+10 a^2 d\right) (a+b x)^{3+n}}{b^6 (3+n)} + \frac{d^2 \left(3 b^2 c+10 a^2 d\right) (a+b x)^{4+n}}{b^6 (4+n)} - \frac{5 a d^3 (a+b x)^{5+n}}{b^6 (5+n)} + \frac{d^3 (a+b x)^{6+n}}{b^6 (6+n)} - \frac{c^3 (a+b x)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, 1+\frac{b x}{a}\right]}{a (1+n)}$$

Result (type 5, 546 leaves):

$$(a+b x)^n \Bigg(\frac{3 c^2 d \left(1+\frac{b x}{a}\right)^{-n} \left(a b n x \left(1+\frac{b x}{a}\right)^n + b^2 (1+n) x^2 \left(1+\frac{b x}{a}\right)^n - a^2 \left(-1+\left(1+\frac{b x}{a}\right)^n\right)\right)}{b^2 (1+n) (2+n)} + \left(3 c d^2 \left(1+\frac{b x}{a}\right)^{-n} \left(6 a^3 b n x \left(1+\frac{b x}{a}\right)^n - 3 a^2 b^2 n (1+n) x^2 \left(1+\frac{b x}{a}\right)^n + a b^3 n \left(2+3 n+n^2\right) x^3 \left(1+\frac{b x}{a}\right)^n + b^4 \left(6+11 n+6 n^2+n^3\right) x^4 \left(1+\frac{b x}{a}\right)^n - 6 a^4 \left(-1+\left(1+\frac{b x}{a}\right)^n\right)\right)\right) \Big/ \left(b^4 (1+n) (2+n) (3+n) (4+n)\right) + \left(d^3 \left(1+\frac{b x}{a}\right)^{-n} \left(120 a^5 b n x \left(1+\frac{b x}{a}\right)^n - 60 a^4 b^2 n (1+n) x^2 \left(1+\frac{b x}{a}\right)^n + 20 a^3 b^3 n \left(2+3 n+n^2\right) x^3 \left(1+\frac{b x}{a}\right)^n - 5 a^2 b^4 n \left(6+11 n+6 n^2+n^3\right) x^4 \left(1+\frac{b x}{a}\right)^n + a b^5 n \left(24+50 n+35 n^2+10 n^3+n^4\right) x^5 \left(1+\frac{b x}{a}\right)^n + b^6 \left(120+274 n+225 n^2+85 n^3+15 n^4+n^5\right) x^6 \left(1+\frac{b x}{a}\right)^n - 120 a^6 \left(-1+\left(1+\frac{b x}{a}\right)^n\right)\right)\right) \Big/ \left(b^6 (1+n) (2+n) (3+n) (4+n) (5+n) (6+n)\right) + \frac{c^3 \left(1+\frac{a}{b x}\right)^{-n} \text{Hypergeometric2F1}\left[-n, -n, 1-n, -\frac{a}{b x}\right]}{n}\Bigg)$$

Problem 363: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4 (d+e x)^n}{a+c x^2} \, dx$$

Optimal (type 5, 250 leaves, 6 steps):

$$\frac{\left(c d^2-a e^2\right) (d+e x)^{1+n}}{c^2 e^3 (1+n)}-\frac{2 d (d+e x)^{2+n}}{c e^3 (2+n)}+\frac{(d+e x)^{3+n}}{c e^3 (3+n)}+$$
$$\frac{(-a)^{3/2} (d+e x)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{\sqrt{c} (d+e x)}{\sqrt{c} d-\sqrt{-a} e}\right]}{2 c^2 \left(\sqrt{c} d-\sqrt{-a} e\right) (1+n)}-\frac{(-a)^{3/2} (d+e x)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}\right]}{2 c^2 \left(\sqrt{c} d+\sqrt{-a} e\right) (1+n)}$$

Result (type 5, 354 leaves):

$$\frac{1}{2 c^3 e^3} (d+e x)^n \left(-\frac{2 a c e^2 (d+e x)}{1+n}+1/((1+n) (2+n) (3+n)) 2 c^2 \left(1+\frac{e x}{d}\right)^{-n}\right.$$
$$\left(-2 d^2 e n x \left(1+\frac{e x}{d}\right)^n+d e^2 n (1+n) x^2 \left(1+\frac{e x}{d}\right)^n+e^3 \left(2+3 n+n^2\right) x^3 \left(1+\frac{e x}{d}\right)^n+2 d^3 \left(-1+\left(1+\frac{e x}{d}\right)^n\right)\right)+$$
$$1/n \, i \, a^{3/2} \sqrt{c} e^3 \left(-\left(\frac{\sqrt{c} (d+e x)}{e \left(-i \sqrt{a}+\sqrt{c} x\right)}\right)^{-n} \text{Hypergeometric2F1}\left[-n,-n,1-n,\frac{\sqrt{c} d+i \sqrt{a} e}{i \sqrt{a} e-\sqrt{c} e x}\right]+\right.$$
$$\left.\left.\left(\frac{\sqrt{c} (d+e x)}{e \left(i \sqrt{a}+\sqrt{c} x\right)}\right)^{-n} \text{Hypergeometric2F1}\left[-n,-n,1-n,-\frac{\sqrt{c} d-i \sqrt{a} e}{i \sqrt{a} e+\sqrt{c} e x}\right]\right)\right)$$

Problem 364: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3 (d+e x)^n}{a+c x^2} \, dx$$

Optimal (type 5, 209 leaves, 6 steps):

$$-\frac{d (d+e x)^{1+n}}{c e^2 (1+n)}+\frac{(d+e x)^{2+n}}{c e^2 (2+n)}+\frac{a (d+e x)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{\sqrt{c} (d+e x)}{\sqrt{c} d-\sqrt{-a} e}\right]}{2 c^{3/2} \left(\sqrt{c} d-\sqrt{-a} e\right) (1+n)}+$$
$$\frac{a (d+e x)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}\right]}{2 c^{3/2} \left(\sqrt{c} d+\sqrt{-a} e\right) (1+n)}$$

Result (type 5, 275 leaves):

$$\frac{1}{2 c^2 n (1+n) (2+n)} (d+e x)^n \left(\frac{2 c d n^2 x}{e} + 2 c n (1+n) x^2 + \frac{2 c d^2 n \left(-1+\left(1+\frac{e x}{d}\right)^{-n}\right)}{e^2} - \right.$$

$$a \left(2+3 n+n^2\right) \left(\frac{\sqrt{c} (d+e x)}{e \left(-i \sqrt{a}+\sqrt{c} x\right)}\right)^{-n} \text{Hypergeometric2F1}\left[-n, -n, 1-n, \frac{\sqrt{c} d+i \sqrt{a} e}{i \sqrt{a} e-\sqrt{c} e x}\right] -$$

$$\left. a \left(2+3 n+n^2\right) \left(\frac{\sqrt{c} (d+e x)}{e \left(i \sqrt{a}+\sqrt{c} x\right)}\right)^{-n} \text{Hypergeometric2F1}\left[-n, -n, 1-n, -\frac{\sqrt{c} d-i \sqrt{a} e}{i \sqrt{a} e+\sqrt{c} e x}\right]\right)$$

- **Problem 365: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^2 (d+e x)^n}{a+c x^2} \, dx$$

Optimal (type 5, 194 leaves, 6 steps):

$$\frac{(d+e x)^{1+n}}{c e (1+n)} + \frac{\sqrt{-a} (d+e x)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{\sqrt{c} (d+e x)}{\sqrt{c} d-\sqrt{-a} e}\right]}{2 c \left(\sqrt{c} d-\sqrt{-a} e\right) (1+n)} - \frac{\sqrt{-a} (d+e x)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}\right]}{2 c \left(\sqrt{c} d+\sqrt{-a} e\right) (1+n)}$$

Result (type 5, 233 leaves):

$$\frac{1}{2 c^2 e}$$

$$(d+e x)^n \left(\frac{2 c (d+e x)}{1+n} - 1/n\, i \sqrt{a} \sqrt{c} e \left(-\left(\frac{\sqrt{c} (d+e x)}{e \left(-i \sqrt{a}+\sqrt{c} x\right)}\right)^{-n} \text{Hypergeometric2F1}\left[-n, -n, 1-n, \frac{\sqrt{c} d+i \sqrt{a} e}{i \sqrt{a} e-\sqrt{c} e x}\right] + \left(\frac{\sqrt{c} (d+e x)}{e \left(i \sqrt{a}+\sqrt{c} x\right)}\right)^{-n} \right.\right.$$

$$\left.\left. \text{Hypergeometric2F1}\left[-n, -n, 1-n, -\frac{\sqrt{c} d-i \sqrt{a} e}{i \sqrt{a} e+\sqrt{c} e x}\right]\right)\right)$$

- **Problem 366: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x (d+e x)^n}{a+c x^2} \, dx$$

Optimal (type 5, 163 leaves, 4 steps):

$$-\frac{(d+e x)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{\sqrt{c} (d+e x)}{\sqrt{c} d-\sqrt{-a} e}\right]}{2 \sqrt{c} \left(\sqrt{c} d-\sqrt{-a} e\right) (1+n)} - \frac{(d+e x)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}\right]}{2 \sqrt{c} \left(\sqrt{c} d+\sqrt{-a} e\right) (1+n)}$$

Result (type 5, 200 leaves):

$$\frac{1}{2\,c\,n}(d+e\,x)^n\left(\left(\frac{\sqrt{c}\,(d+e\,x)}{e\left(-i\,\sqrt{a}+\sqrt{c}\,x\right)}\right)^{-n}\text{Hypergeometric2F1}\left[-n,-n,1-n,\frac{\sqrt{c}\,d+i\,\sqrt{a}\,e}{i\,\sqrt{a}\,e-\sqrt{c}\,e\,x}\right]+\left(\frac{\sqrt{c}\,(d+e\,x)}{e\left(i\,\sqrt{a}+\sqrt{c}\,x\right)}\right)^{-n}\text{Hypergeometric2F1}\left[-n,-n,1-n,-\frac{\sqrt{c}\,d-i\,\sqrt{a}\,e}{i\,\sqrt{a}\,e+\sqrt{c}\,e\,x}\right]\right)$$

- **Problem 367: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(d+e\,x)^n}{a+c\,x^2}\,dx$$

Optimal (type 5, 167 leaves, 4 steps):

$$\frac{(d+e\,x)^{1+n}\,\text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right]}{2\,\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)}-\frac{(d+e\,x)^{1+n}\,\text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right]}{2\,\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(1+n)}$$

Result (type 5, 210 leaves):

$$-\frac{1}{2\,\sqrt{a}\,\sqrt{c}\,n}i\,(d+e\,x)^n\left(\left(\frac{\sqrt{c}\,(d+e\,x)}{e\left(-i\,\sqrt{a}+\sqrt{c}\,x\right)}\right)^{-n}\text{Hypergeometric2F1}\left[-n,-n,1-n,\frac{\sqrt{c}\,d+i\,\sqrt{a}\,e}{i\,\sqrt{a}\,e-\sqrt{c}\,e\,x}\right]-\left(\frac{\sqrt{c}\,(d+e\,x)}{e\left(i\,\sqrt{a}+\sqrt{c}\,x\right)}\right)^{-n}\text{Hypergeometric2F1}\left[-n,-n,1-n,-\frac{\sqrt{c}\,d-i\,\sqrt{a}\,e}{i\,\sqrt{a}\,e+\sqrt{c}\,e\,x}\right]\right)$$

- **Problem 368: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(d+e\,x)^n}{x\left(a+c\,x^2\right)}\,dx$$

Optimal (type 5, 207 leaves, 7 steps):

$$\frac{\sqrt{c}\,(d+e\,x)^{1+n}\,\text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right]}{2\,a\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)}+\frac{\sqrt{c}\,(d+e\,x)^{1+n}\,\text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right]}{2\,a\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(1+n)}-\frac{(d+e\,x)^{1+n}\,\text{Hypergeometric2F1}\left[1,1+n,2+n,1+\frac{e\,x}{d}\right]}{a\,d\,(1+n)}$$

Result (type 5, 239 leaves):

$$\frac{1}{2\,a\,n}(d+e\,x)^n\left(2\left(1+\frac{d}{e\,x}\right)^{-n}\text{Hypergeometric2F1}\left[-n,\,-n,\,1-n,\,-\frac{d}{e\,x}\right]-\left(\frac{\sqrt{c}\,(d+e\,x)}{e\left(-i\sqrt{a}+\sqrt{c}\,x\right)}\right)^{-n}\text{Hypergeometric2F1}\left[-n,\,-n,\,1-n,\,\frac{\sqrt{c}\,d+i\sqrt{a}\,e}{i\sqrt{a}\,e-\sqrt{c}\,e\,x}\right]-\left(\frac{\sqrt{c}\,(d+e\,x)}{e\left(i\sqrt{a}+\sqrt{c}\,x\right)}\right)^{-n}\text{Hypergeometric2F1}\left[-n,\,-n,\,1-n,\,-\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{i\sqrt{a}\,e+\sqrt{c}\,e\,x}\right]\right)$$

- **Problem 369: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(d+e\,x)^n}{x^2\left(a+c\,x^2\right)}\,dx$$

Optimal (type 5, 207 leaves, 7 steps):

$$\frac{c\,(d+e\,x)^{1+n}\,\text{Hypergeometric2F1}\left[1,\,1+n,\,2+n,\,\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right]}{2\,(-a)^{3/2}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)}-\frac{c\,(d+e\,x)^{1+n}\,\text{Hypergeometric2F1}\left[1,\,1+n,\,2+n,\,\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right]}{2\,(-a)^{3/2}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(1+n)}+\frac{e\,(d+e\,x)^{1+n}\,\text{Hypergeometric2F1}\left[2,\,1+n,\,2+n,\,1+\frac{e\,x}{d}\right]}{a\,d^2\,(1+n)}$$

Result (type 5, 263 leaves):

$$\frac{1}{2\,a}(d+e\,x)^n\left(\frac{2\left(1+\frac{d}{e\,x}\right)^{-n}\text{Hypergeometric2F1}\left[1-n,\,-n,\,2-n,\,-\frac{d}{e\,x}\right]}{(-1+n)\,x}+1\Big/\left(\sqrt{a}\,n\right)i\sqrt{c}\left(\left(\frac{\sqrt{c}\,(d+e\,x)}{e\left(-i\sqrt{a}+\sqrt{c}\,x\right)}\right)^{-n}\text{Hypergeometric2F1}\left[-n,\,-n,\,1-n,\,\frac{\sqrt{c}\,d+i\sqrt{a}\,e}{i\sqrt{a}\,e-\sqrt{c}\,e\,x}\right]-\left(\frac{\sqrt{c}\,(d+e\,x)}{e\left(i\sqrt{a}+\sqrt{c}\,x\right)}\right)^{-n}\text{Hypergeometric2F1}\left[-n,\,-n,\,1-n,\,-\frac{\sqrt{c}\,d-i\sqrt{a}\,e}{i\sqrt{a}\,e+\sqrt{c}\,e\,x}\right]\right)\right)$$

- **Problem 370: Unable to integrate problem.**

$$\int\frac{x^4\,(d+e\,x)^n}{\left(a+c\,x^2\right)^2}\,dx$$

Optimal (type 5, 332 leaves, 5 steps):

$$\frac{(d+e x)^{1+n}}{c^2 e (1+n)}+\frac{a (a e+c d x) (d+e x)^{1+n}}{2 c^2 \left(c d^2+a e^2\right) \left(a+c x^2\right)}+$$
$$\left(\left(3 \sqrt{-a} c d^2+a \sqrt{c} d e n+\sqrt{-a} a e^2 (3+n)\right) (d+e x)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{\sqrt{c} (d+e x)}{\sqrt{c} d-\sqrt{-a} e}\right]\right)\Big/$$
$$\left(4 c^2 \left(\sqrt{c} d-\sqrt{-a} e\right) \left(c d^2+a e^2\right) (1+n)\right)-$$
$$\left(\left(3 \sqrt{-a} c d^2-a \sqrt{c} d e n+\sqrt{-a} a e^2 (3+n)\right) (d+e x)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}\right]\right)\Big/$$
$$\left(4 c^2 \left(\sqrt{c} d+\sqrt{-a} e\right) \left(c d^2+a e^2\right) (1+n)\right)$$

Result (type 8, 22 leaves) :

$$\int \frac{x^4 (d+e x)^n}{\left(a+c x^2\right)^2} \, dx$$

- **Problem 371: Unable to integrate problem.**

$$\int \frac{x^3 (d+e x)^n}{\left(a+c x^2\right)^2} \, dx$$

Optimal (type 5, 297 leaves, 5 steps) :

$$\frac{a (d-e x) (d+e x)^{1+n}}{2 c \left(c d^2+a e^2\right) \left(a+c x^2\right)}+\frac{\left(\sqrt{-a} d e n-\frac{2 c d^2+a e^2 (2+n)}{\sqrt{c}}\right) (d+e x)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{\sqrt{c} (d+e x)}{\sqrt{c} d-\sqrt{-a} e}\right]}{4 c \left(\sqrt{c} d-\sqrt{-a} e\right) \left(c d^2+a e^2\right) (1+n)}-$$
$$\frac{\left(2 c d^2+\sqrt{-a} \sqrt{c} d e n+a e^2 (2+n)\right) (d+e x)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}\right]}{4 c^{3/2} \left(\sqrt{c} d+\sqrt{-a} e\right) \left(c d^2+a e^2\right) (1+n)}$$

Result (type 8, 22 leaves) :

$$\int \frac{x^3 (d+e x)^n}{\left(a+c x^2\right)^2} \, dx$$

- **Problem 372: Unable to integrate problem.**

$$\int \frac{x^2 (d+e x)^n}{\left(a+c x^2\right)^2} \, dx$$

Optimal (type 5, 306 leaves, 5 steps) :

$$-\frac{(a e+c d x)\,(d+e x)^{1+n}}{2 c\left(c d^2+a e^2\right)\left(a+c x^2\right)}+\frac{\left(c d^2-\sqrt{-a}\,\sqrt{c}\,d e n+a e^2\,(1+n)\right)\,(d+e x)^{1+n}\,\text{Hypergeometric2F1}\left[1,\,1+n,\,2+n,\,\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right]}{4\sqrt{-a}\,c\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(c d^2+a e^2\right)(1+n)}-\frac{\left(c d^2+\sqrt{-a}\,\sqrt{c}\,d e n+a e^2\,(1+n)\right)\,(d+e x)^{1+n}\,\text{Hypergeometric2F1}\left[1,\,1+n,\,2+n,\,\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right]}{4\sqrt{-a}\,c\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)\left(c d^2+a e^2\right)(1+n)}$$

Result (type 8, 22 leaves):

$$\int\frac{x^2\,(d+e x)^n}{\left(a+c x^2\right)^2}\,dx$$

Problem 373: Unable to integrate problem.

$$\int\frac{x\,(d+e x)^n}{\left(a+c x^2\right)^2}\,dx$$

Optimal (type 5, 279 leaves, 5 steps):

$$-\frac{(d-e x)\,(d+e x)^{1+n}}{2\left(c d^2+a e^2\right)\left(a+c x^2\right)}+\frac{e\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)n\,(d+e x)^{1+n}\,\text{Hypergeometric2F1}\left[1,\,1+n,\,2+n,\,\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right]}{4\sqrt{-a}\,\sqrt{c}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(c d^2+a e^2\right)(1+n)}+\frac{e\left(\sqrt{-a}\,\sqrt{c}\,d+a e\right)n\,(d+e x)^{1+n}\,\text{Hypergeometric2F1}\left[1,\,1+n,\,2+n,\,\frac{\sqrt{c}\,(d+e x)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right]}{4 a\sqrt{c}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)\left(c d^2+a e^2\right)(1+n)}$$

Result (type 8, 20 leaves):

$$\int\frac{x\,(d+e x)^n}{\left(a+c x^2\right)^2}\,dx$$

Problem 374: Unable to integrate problem.

$$\int\frac{(d+e x)^n}{\left(a+c x^2\right)^2}\,dx$$

Optimal (type 5, 304 leaves, 5 steps):

$$\frac{(a e+c d x) (d+e x)^{1+n}}{2 a \left(c d^2+a e^2\right) \left(a+c x^2\right)}-\frac{\left(c d^2+a e^2 (1-n)+\sqrt{-a} \sqrt{c} d e n\right) (d+e x)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{\sqrt{c} (d+e x)}{\sqrt{c} d-\sqrt{-a} e}\right]}{4 (-a)^{3/2} \left(\sqrt{c} d-\sqrt{-a} e\right) \left(c d^2+a e^2\right) (1+n)}+$$
$$\frac{\left(c d^2+a e^2 (1-n)-\sqrt{-a} \sqrt{c} d e n\right) (d+e x)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}\right]}{4 (-a)^{3/2} \left(\sqrt{c} d+\sqrt{-a} e\right) \left(c d^2+a e^2\right) (1+n)}$$

Result (type 8, 19 leaves):

$$\int \frac{(d+e x)^n}{\left(a+c x^2\right)^2} \, dx$$

Problem 375: Unable to integrate problem.

$$\int \frac{(d+e x)^n}{x \left(a+c x^2\right)^2} \, dx$$

Optimal (type 5, 489 leaves, 12 steps):

$$\frac{c (d-e x) (d+e x)^{1+n}}{2 a \left(c d^2+a e^2\right) \left(a+c x^2\right)}+\frac{\sqrt{c} (d+e x)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{\sqrt{c} (d+e x)}{\sqrt{c} d-\sqrt{-a} e}\right]}{2 a^2 \left(\sqrt{c} d-\sqrt{-a} e\right) (1+n)}+$$
$$\frac{\sqrt{c} e \left(\sqrt{c} d+\sqrt{-a} e\right) n (d+e x)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{\sqrt{c} (d+e x)}{\sqrt{c} d-\sqrt{-a} e}\right]}{4 (-a)^{3/2} \left(\sqrt{c} d-\sqrt{-a} e\right) \left(c d^2+a e^2\right) (1+n)}+$$
$$\frac{\sqrt{c} (d+e x)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}\right]}{2 a^2 \left(\sqrt{c} d+\sqrt{-a} e\right) (1+n)}-$$
$$\frac{\sqrt{c} e \left(\sqrt{-a} \sqrt{c} d+a e\right) n (d+e x)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}\right]}{4 a^2 \left(\sqrt{c} d+\sqrt{-a} e\right) \left(c d^2+a e^2\right) (1+n)}-\frac{(d+e x)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,1+\frac{e x}{d}\right]}{a^2 d (1+n)}$$

Result (type 8, 22 leaves):

$$\int \frac{(d+e x)^n}{x \left(a+c x^2\right)^2} \, dx$$

Problem 376: Unable to integrate problem.

$$\int \frac{(d+e x)^n}{x^2 \left(a+c x^2\right)^2} \, dx$$

Optimal (type 5, 513 leaves, 12 steps):

$$-\frac{c (a e+c d x) (d+e x)^{1+n}}{2 a^2 \left(c d^2+a e^2\right) \left(a+c x^2\right)}-\frac{c (d+e x)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{\sqrt{c} (d+e x)}{\sqrt{c} d-\sqrt{-a} e}\right]}{2 (-a)^{5/2} \left(\sqrt{c} d-\sqrt{-a} e\right) (1+n)}-$$
$$\frac{c \left(c d^2+a e^2 (1-n)+\sqrt{-a} \sqrt{c} d e n\right) (d+e x)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{\sqrt{c} (d+e x)}{\sqrt{c} d-\sqrt{-a} e}\right]}{4 (-a)^{5/2} \left(\sqrt{c} d-\sqrt{-a} e\right) \left(c d^2+a e^2\right) (1+n)}+$$
$$\frac{c (d+e x)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}\right]}{2 (-a)^{5/2} \left(\sqrt{c} d+\sqrt{-a} e\right) (1+n)}+$$
$$\frac{c \left(c d^2+a e^2 (1-n)-\sqrt{-a} \sqrt{c} d e n\right) (d+e x)^{1+n} \text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{\sqrt{c} (d+e x)}{\sqrt{c} d+\sqrt{-a} e}\right]}{4 (-a)^{5/2} \left(\sqrt{c} d+\sqrt{-a} e\right) \left(c d^2+a e^2\right) (1+n)}+$$
$$\frac{e (d+e x)^{1+n} \text{Hypergeometric2F1}\left[2,1+n,2+n,1+\frac{e x}{d}\right]}{a^2 d^2 (1+n)}$$

Result (type 8, 22 leaves):

$$\int \frac{(d+e x)^n}{x^2 \left(a+c x^2\right)^2} \, dx$$

Problem 379: Unable to integrate problem.

$$\int \frac{(g x)^m (d+e x)^n}{a+c x^2} \, dx$$

Optimal (type 6, 148 leaves, 6 steps):

$$\frac{(g x)^{1+m} (d+e x)^n \left(1+\frac{e x}{d}\right)^{-n} \text{AppellF1}\left[1+m,-n,1,2+m,-\frac{e x}{d},-\frac{\sqrt{c} x}{\sqrt{-a}}\right]}{2 a g (1+m)}+\frac{(g x)^{1+m} (d+e x)^n \left(1+\frac{e x}{d}\right)^{-n} \text{AppellF1}\left[1+m,-n,1,2+m,-\frac{e x}{d},\frac{\sqrt{c} x}{\sqrt{-a}}\right]}{2 a g (1+m)}$$

Result (type 8, 24 leaves):

$$\int \frac{(g x)^m (d+e x)^n}{a+c x^2} \, dx$$

Problem 380: Unable to integrate problem.

$$\int \frac{(g x)^m (d+e x)^n}{\left(a+c x^2\right)^2} \, dx$$

Optimal (type 6, 295 leaves, 12 steps):

$$\frac{(g x)^{1+m} (d+e x)^n \left(1+\frac{e x}{d}\right)^{-n} \text{AppellF1}\left[1+m,-n,1,2+m,-\frac{e x}{d},-\frac{\sqrt{c} x}{\sqrt{-a}}\right]}{4 a^2 g (1+m)}+\frac{(g x)^{1+m} (d+e x)^n \left(1+\frac{e x}{d}\right)^{-n} \text{AppellF1}\left[1+m,-n,1,2+m,-\frac{e x}{d},\frac{\sqrt{c} x}{\sqrt{-a}}\right]}{4 a^2 g (1+m)}+\frac{(g x)^{1+m} (d+e x)^n \left(1+\frac{e x}{d}\right)^{-n} \text{AppellF1}\left[1+m,-n,2,2+m,-\frac{e x}{d},-\frac{\sqrt{c} x}{\sqrt{-a}}\right]}{4 a^2 g (1+m)}+\frac{(g x)^{1+m} (d+e x)^n \left(1+\frac{e x}{d}\right)^{-n} \text{AppellF1}\left[1+m,-n,2,2+m,-\frac{e x}{d},\frac{\sqrt{c} x}{\sqrt{-a}}\right]}{4 a^2 g (1+m)}$$

Result (type 8, 24 leaves):

$$\int \frac{(g x)^m (d+e x)^n}{\left(a+c x^2\right)^2} \, dx$$

Problem 408: Unable to integrate problem.

$$\int \frac{x^4 \left(a+b x^2\right)^p}{d+e x} \, dx$$

Optimal (type 6, 199 leaves, 7 steps):

$$\frac{\left(b d^2-a e^2\right) \left(a+b x^2\right)^{1+p}}{2 b^2 e^3 (1+p)}+\frac{\left(a+b x^2\right)^{2+p}}{2 b^2 e (2+p)}+\frac{x^5 \left(a+b x^2\right)^p \left(1+\frac{b x^2}{a}\right)^{-p} \text{AppellF1}\left[\frac{5}{2},-p,1,\frac{7}{2},-\frac{b x^2}{a},\frac{e^2 x^2}{d^2}\right]}{5 d}-\frac{d^4 \left(a+b x^2\right)^{1+p} \text{Hypergeometric2F1}\left[1,1+p,2+p,\frac{e^2 \left(a+b x^2\right)}{b d^2+a e^2}\right]}{2 e^3 \left(b d^2+a e^2\right) (1+p)}$$

Result (type 8, 22 leaves):

$$\int \frac{x^4 \left(a+b x^2\right)^p}{d+e x} \, dx$$

Problem 416: Unable to integrate problem.

$$\int \frac{x^4 \left(a+b x^2\right)^p}{(d+e x)^2} \, dx$$

Optimal (type 6, 392 leaves, 12 steps):

$$-\frac{d\,(4+3p)\,\left(a+b x^2\right)^{1+p}}{b e^3\,(1+p)\,(3+2p)}-\frac{d^4\,\left(a+b x^2\right)^{1+p}}{e^3\,\left(b d^2+a e^2\right)\,(d+e x)}+\frac{(d+e x)\,\left(a+b x^2\right)^{1+p}}{b e^3\,(3+2p)}-$$

$$\frac{2 d^2\,\left(2 a e^2+b d^2\,(2+p)\right)\,x\,\left(a+b x^2\right)^p\,\left(1+\frac{b x^2}{a}\right)^{-p}\,\text{AppellF1}\left[\frac{1}{2},\,-p,\,1,\,\frac{3}{2},\,-\frac{b x^2}{a},\,\frac{e^2 x^2}{d^2}\right]}{e^4\,\left(b d^2+a e^2\right)}-\frac{1}{b e^4\,\left(b d^2+a e^2\right)\,(3+2p)}$$

$$\left(a^2 e^4-2 a b d^2 e^2\,(4+3p)-2 b^2 d^4\,\left(6+7p+2p^2\right)\right)\,x\,\left(a+b x^2\right)^p\,\left(1+\frac{b x^2}{a}\right)^{-p}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,-p,\,\frac{3}{2},\,-\frac{b x^2}{a}\right]+$$

$$\frac{d^3\,\left(2 a e^2+b d^2\,(2+p)\right)\,\left(a+b x^2\right)^{1+p}\,\text{Hypergeometric2F1}\left[1,\,1+p,\,2+p,\,\frac{e^2\,\left(a+b x^2\right)}{b d^2+a e^2}\right]}{e^3\,\left(b d^2+a e^2\right)^2\,(1+p)}$$

Result (type 8, 22 leaves) :

$$\int\frac{x^4\,\left(a+b x^2\right)^p}{(d+e x)^2}\,dx$$

Problem 422: Unable to integrate problem.

$$\int\frac{\left(a+b x^2\right)^p}{x^2\,(d+e x)^2}\,dx$$

Optimal (type 6, 421 leaves, 20 steps) :

$$\frac{2 e^2 x\,\left(a+b x^2\right)^p\,\left(1+\frac{b x^2}{a}\right)^{-p}\,\text{AppellF1}\left[\frac{1}{2},\,-p,\,1,\,\frac{3}{2},\,-\frac{b x^2}{a},\,\frac{e^2 x^2}{d^2}\right]}{d^4}+$$

$$\frac{e^2 x\,\left(a+b x^2\right)^p\,\left(1+\frac{b x^2}{a}\right)^{-p}\,\text{AppellF1}\left[\frac{1}{2},\,-p,\,2,\,\frac{3}{2},\,-\frac{b x^2}{a},\,\frac{e^2 x^2}{d^2}\right]}{d^4}+\frac{e^4 x^3\,\left(a+b x^2\right)^p\,\left(1+\frac{b x^2}{a}\right)^{-p}\,\text{AppellF1}\left[\frac{3}{2},\,-p,\,2,\,\frac{5}{2},\,-\frac{b x^2}{a},\,\frac{e^2 x^2}{d^2}\right]}{3 d^6}-$$

$$\frac{\left(a+b x^2\right)^p\,\left(1+\frac{b x^2}{a}\right)^{-p}\,\text{Hypergeometric2F1}\left[-\frac{1}{2},\,-p,\,\frac{1}{2},\,-\frac{b x^2}{a}\right]}{d^2 x}-\frac{e^3\,\left(a+b x^2\right)^{1+p}\,\text{Hypergeometric2F1}\left[1,\,1+p,\,2+p,\,\frac{e^2\,\left(a+b x^2\right)}{b d^2+a e^2}\right]}{d^3\,\left(b d^2+a e^2\right)\,(1+p)}+$$

$$\frac{e\,\left(a+b x^2\right)^{1+p}\,\text{Hypergeometric2F1}\left[1,\,1+p,\,2+p,\,1+\frac{b x^2}{a}\right]}{a d^3\,(1+p)}-\frac{b e^3\,\left(a+b x^2\right)^{1+p}\,\text{Hypergeometric2F1}\left[2,\,1+p,\,2+p,\,\frac{e^2\,\left(a+b x^2\right)}{b d^2+a e^2}\right]}{d\,\left(b d^2+a e^2\right)^2\,(1+p)}$$

Result (type 8, 22 leaves) :

$$\int\frac{\left(a+b x^2\right)^p}{x^2\,(d+e x)^2}\,dx$$

Problem 423: Unable to integrate problem.

$$\int \frac{x^4 \left(a+b x^2\right)^p}{(d+e x)^3} \, dx$$

Optimal (type 6, 449 leaves, 12 steps):

$$\frac{\left(a+b x^2\right)^{1+p}}{2 b e^3 (1+p)}-\frac{d^4 \left(a+b x^2\right)^{1+p}}{2 e^3 \left(b d^2+a e^2\right) (d+e x)^2}+\frac{d^3 \left(4 a e^2+b d^2 (3+p)\right) \left(a+b x^2\right)^{1+p}}{e^3 \left(b d^2+a e^2\right)^2 (d+e x)}+\frac{1}{e^4 \left(b d^2+a e^2\right)^2}$$
$$d \left(6 a^2 e^4+3 a b d^2 e^2 (4+3 p)+b^2 d^4 \left(6+7 p+2 p^2\right)\right) x \left(a+b x^2\right)^p \left(1+\frac{b x^2}{a}\right)^{-p} \text{AppellF1}\left[\frac{1}{2},-p,1,\frac{3}{2},-\frac{b x^2}{a},\frac{e^2 x^2}{d^2}\right]-$$
$$\frac{1}{e^4 \left(b d^2+a e^2\right)^2}d \left(3 a^2 e^4+2 a b d^2 e^2 (5+4 p)+b^2 d^4 \left(6+7 p+2 p^2\right)\right) x \left(a+b x^2\right)^p \left(1+\frac{b x^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left[\frac{1}{2},-p,\frac{3}{2},-\frac{b x^2}{a}\right]-$$
$$\frac{1}{2 e^3 \left(b d^2+a e^2\right)^3 (1+p)}d^2 \left(6 a^2 e^4+3 a b d^2 e^2 (4+3 p)+b^2 d^4 \left(6+7 p+2 p^2\right)\right) \left(a+b x^2\right)^{1+p} \text{Hypergeometric2F1}\left[1,1+p,2+p,\frac{e^2 \left(a+b x^2\right)}{b d^2+a e^2}\right]$$

Result (type 8, 22 leaves):

$$\int \frac{x^4 \left(a+b x^2\right)^p}{(d+e x)^3} \, dx$$

Problem 424: Unable to integrate problem.

$$\int \frac{x^3 \left(a+b x^2\right)^p}{(d+e x)^3} \, dx$$

Optimal (type 6, 416 leaves, 11 steps):

$$\frac{d^3 \left(a+b x^2\right)^{1+p}}{2 e^2 \left(b d^2+a e^2\right) (d+e x)^2}-\frac{d^2 \left(3 a e^2+b d^2 (2+p)\right) \left(a+b x^2\right)^{1+p}}{e^2 \left(b d^2+a e^2\right)^2 (d+e x)}-\frac{1}{e^3 \left(b d^2+a e^2\right)^2}$$
$$\left(3 a^2 e^4+a b d^2 e^2 (6+7 p)+b^2 d^4 \left(3+5 p+2 p^2\right)\right) x \left(a+b x^2\right)^p \left(1+\frac{b x^2}{a}\right)^{-p} \text{AppellF1}\left[\frac{1}{2},-p,1,\frac{3}{2},-\frac{b x^2}{a},\frac{e^2 x^2}{d^2}\right]+$$
$$\frac{1}{e^3 \left(b d^2+a e^2\right)^2}\left(a^2 e^4+a b d^2 e^2 (5+6 p)+b^2 d^4 \left(3+5 p+2 p^2\right)\right) x \left(a+b x^2\right)^p \left(1+\frac{b x^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left[\frac{1}{2},-p,\frac{3}{2},-\frac{b x^2}{a}\right]+$$
$$\frac{1}{2 e^2 \left(b d^2+a e^2\right)^3 (1+p)}d \left(3 a^2 e^4+a b d^2 e^2 (6+7 p)+b^2 d^4 \left(3+5 p+2 p^2\right)\right) \left(a+b x^2\right)^{1+p} \text{Hypergeometric2F1}\left[1,1+p,2+p,\frac{e^2 \left(a+b x^2\right)}{b d^2+a e^2}\right]$$

Result (type 8, 22 leaves):

$$\int \frac{x^3 \left(a+b x^2\right)^p}{(d+e x)^3} \, dx$$

Problem 428: Unable to integrate problem.

$$\int \frac{\left(a+b x^2\right)^p}{x (d+e x)^3} \, dx$$

Optimal (type 6, 700 leaves, 29 steps):

$$\frac{d e^2 \left(a+b x^2\right)^{1+p}}{4 \left(b d^2+a e^2\right) \left(d^2-e^2 x^2\right)^2}-\frac{e x \left(a+b x^2\right)^p \left(1+\frac{b x^2}{a}\right)^{-p} \text{AppellF1}\left[\frac{1}{2},-p,1,\frac{3}{2},-\frac{b x^2}{a},\frac{e^2 x^2}{d^2}\right]}{d^4}-$$
$$\frac{e x \left(a+b x^2\right)^p \left(1+\frac{b x^2}{a}\right)^{-p} \text{AppellF1}\left[\frac{1}{2},-p,2,\frac{3}{2},-\frac{b x^2}{a},\frac{e^2 x^2}{d^2}\right]}{d^4}-\frac{e x \left(a+b x^2\right)^p \left(1+\frac{b x^2}{a}\right)^{-p} \text{AppellF1}\left[\frac{1}{2},-p,3,\frac{3}{2},-\frac{b x^2}{a},\frac{e^2 x^2}{d^2}\right]}{d^4}-$$
$$\frac{e^3 x^3 \left(a+b x^2\right)^p \left(1+\frac{b x^2}{a}\right)^{-p} \text{AppellF1}\left[\frac{3}{2},-p,2,\frac{5}{2},-\frac{b x^2}{a},\frac{e^2 x^2}{d^2}\right]}{3 d^6}-$$
$$\frac{e^3 x^3 \left(a+b x^2\right)^p \left(1+\frac{b x^2}{a}\right)^{-p} \text{AppellF1}\left[\frac{3}{2},-p,3,\frac{5}{2},-\frac{b x^2}{a},\frac{e^2 x^2}{d^2}\right]}{d^6}+\frac{e^2 \left(a+b x^2\right)^{1+p} \text{Hypergeometric2F1}\left[1,1+p,2+p,\frac{e^2 \left(a+b x^2\right)}{b d^2+a e^2}\right]}{2 d^3 \left(b d^2+a e^2\right) (1+p)}-$$
$$\frac{\left(a+b x^2\right)^{1+p} \text{Hypergeometric2F1}\left[1,1+p,2+p,1+\frac{b x^2}{a}\right]}{2 a d^3 (1+p)}+\frac{b e^2 \left(a+b x^2\right)^{1+p} \text{Hypergeometric2F1}\left[2,1+p,2+p,\frac{e^2 \left(a+b x^2\right)}{b d^2+a e^2}\right]}{d \left(b d^2+a e^2\right)^2 (1+p)}-$$
$$\frac{b e^2 \left(2 a e^2+b d^2 (1+p)\right) \left(a+b x^2\right)^{1+p} \text{Hypergeometric2F1}\left[2,1+p,2+p,\frac{e^2 \left(a+b x^2\right)}{b d^2+a e^2}\right]}{4 d \left(b d^2+a e^2\right)^3 (1+p)}+$$
$$\frac{3 b^2 d e^2 \left(a+b x^2\right)^{1+p} \text{Hypergeometric2F1}\left[3,1+p,2+p,\frac{e^2 \left(a+b x^2\right)}{b d^2+a e^2}\right]}{2 \left(b d^2+a e^2\right)^3 (1+p)}$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a+b x^2\right)^p}{x (d+e x)^3} \, dx$$

Problem 429: Unable to integrate problem.

$$\int \frac{\left(a+b x^2\right)^p}{x^2 (d+e x)^3} \, dx$$

Optimal (type 6, 754 leaves, 31 steps):

$$-\frac{e^3\,(a+b\,x^2)^{1+p}}{4\,(b\,d^2+a\,e^2)\,(d^2-e^2\,x^2)^2}+\frac{3\,e^2\,x\,(a+b\,x^2)^p\,\left(1+\frac{b\,x^2}{a}\right)^{-p}\,\text{AppellF1}\left[\frac{1}{2},-p,1,\frac{3}{2},-\frac{b\,x^2}{a},\frac{e^2\,x^2}{d^2}\right]}{d^5}+$$

$$\frac{2\,e^2\,x\,(a+b\,x^2)^p\,\left(1+\frac{b\,x^2}{a}\right)^{-p}\,\text{AppellF1}\left[\frac{1}{2},-p,2,\frac{3}{2},-\frac{b\,x^2}{a},\frac{e^2\,x^2}{d^2}\right]}{d^5}+\frac{e^2\,x\,(a+b\,x^2)^p\,\left(1+\frac{b\,x^2}{a}\right)^{-p}\,\text{AppellF1}\left[\frac{1}{2},-p,3,\frac{3}{2},-\frac{b\,x^2}{a},\frac{e^2\,x^2}{d^2}\right]}{d^5}+$$

$$\frac{2\,e^4\,x^3\,(a+b\,x^2)^p\,\left(1+\frac{b\,x^2}{a}\right)^{-p}\,\text{AppellF1}\left[\frac{3}{2},-p,2,\frac{5}{2},-\frac{b\,x^2}{a},\frac{e^2\,x^2}{d^2}\right]}{3\,d^7}+\frac{e^4\,x^3\,(a+b\,x^2)^p\,\left(1+\frac{b\,x^2}{a}\right)^{-p}\,\text{AppellF1}\left[\frac{3}{2},-p,3,\frac{5}{2},-\frac{b\,x^2}{a},\frac{e^2\,x^2}{d^2}\right]}{d^7}-$$

$$\frac{(a+b\,x^2)^p\,\left(1+\frac{b\,x^2}{a}\right)^{-p}\,\text{Hypergeometric2F1}\left[-\frac{1}{2},-p,\frac{1}{2},-\frac{b\,x^2}{a}\right]}{d^3\,x}-\frac{3\,e^3\,(a+b\,x^2)^{1+p}\,\text{Hypergeometric2F1}\left[1,1+p,2+p,\frac{e^2\,(a+b\,x^2)}{b\,d^2+a\,e^2}\right]}{2\,d^4\,(b\,d^2+a\,e^2)\,(1+p)}+$$

$$\frac{3\,e\,(a+b\,x^2)^{1+p}\,\text{Hypergeometric2F1}\left[1,1+p,2+p,1+\frac{b\,x^2}{a}\right]}{2\,a\,d^4\,(1+p)}-\frac{2\,b\,e^3\,(a+b\,x^2)^{1+p}\,\text{Hypergeometric2F1}\left[2,1+p,2+p,\frac{e^2\,(a+b\,x^2)}{b\,d^2+a\,e^2}\right]}{d^2\,(b\,d^2+a\,e^2)^2\,(1+p)}+$$

$$\frac{b\,e^3\,(2\,a\,e^2+b\,d^2\,(1+p))\,(a+b\,x^2)^{1+p}\,\text{Hypergeometric2F1}\left[2,1+p,2+p,\frac{e^2\,(a+b\,x^2)}{b\,d^2+a\,e^2}\right]}{4\,d^2\,(b\,d^2+a\,e^2)^3\,(1+p)}-$$

$$\frac{3\,b^2\,e^3\,(a+b\,x^2)^{1+p}\,\text{Hypergeometric2F1}\left[3,1+p,2+p,\frac{e^2\,(a+b\,x^2)}{b\,d^2+a\,e^2}\right]}{2\,(b\,d^2+a\,e^2)^3\,(1+p)}$$

Result (type 8, 22 leaves) :

$$\int\frac{(a+b\,x^2)^p}{x^2\,(d+e\,x)^3}\,dx$$

- **Problem 434: Unable to integrate problem.**

$$\int\frac{(g\,x)^m\,(a+c\,x^2)^p}{d+e\,x}\,dx$$

Optimal (type 6, 157 leaves, 5 steps) :

$$\frac{x\,(g\,x)^m\,(a+c\,x^2)^p\,\left(1+\frac{c\,x^2}{a}\right)^{-p}\,\text{AppellF1}\left[\frac{1+m}{2},-p,1,\frac{3+m}{2},-\frac{c\,x^2}{a},\frac{e^2\,x^2}{d^2}\right]}{d\,(1+m)}-\frac{e\,x^2\,(g\,x)^m\,(a+c\,x^2)^p\,\left(1+\frac{c\,x^2}{a}\right)^{-p}\,\text{AppellF1}\left[\frac{2+m}{2},-p,1,\frac{4+m}{2},-\frac{c\,x^2}{a},\frac{e^2\,x^2}{d^2}\right]}{d^2\,(2+m)}$$

Result (type 8, 24 leaves) :

$$\int\frac{(g\,x)^m\,(a+c\,x^2)^p}{d+e\,x}\,dx$$

Problem 435: Unable to integrate problem.

$$\int \frac{(g x)^m \left(a+c x^2\right)^p}{(d+e x)^2} \, dx$$

Optimal (type 6, 238 leaves, 8 steps) :

$$\frac{x (g x)^m \left(a+c x^2\right)^p \left(1+\frac{c x^2}{a}\right)^{-p} \text{AppellF1}\left[\frac{1+m}{2},-p,2,\frac{3+m}{2},-\frac{c x^2}{a},\frac{e^2 x^2}{d^2}\right]}{d^2 (1+m)}-\frac{2 e x^2 (g x)^m \left(a+c x^2\right)^p \left(1+\frac{c x^2}{a}\right)^{-p} \text{AppellF1}\left[\frac{2+m}{2},-p,2,\frac{4+m}{2},-\frac{c x^2}{a},\frac{e^2 x^2}{d^2}\right]}{d^3 (2+m)}+\frac{e^2 x^3 (g x)^m \left(a+c x^2\right)^p \left(1+\frac{c x^2}{a}\right)^{-p} \text{AppellF1}\left[\frac{3+m}{2},-p,2,\frac{5+m}{2},-\frac{c x^2}{a},\frac{e^2 x^2}{d^2}\right]}{d^4 (3+m)}$$

Result (type 8, 24 leaves) :

$$\int \frac{(g x)^m \left(a+c x^2\right)^p}{(d+e x)^2} \, dx$$

Problem 436: Unable to integrate problem.

$$\int \frac{(g x)^m \left(a+c x^2\right)^p}{(d+e x)^3} \, dx$$

Optimal (type 6, 321 leaves, 10 steps) :

$$\frac{x (g x)^m \left(a+c x^2\right)^p \left(1+\frac{c x^2}{a}\right)^{-p} \text{AppellF1}\left[\frac{1+m}{2},-p,3,\frac{3+m}{2},-\frac{c x^2}{a},\frac{e^2 x^2}{d^2}\right]}{d^3 (1+m)}-\frac{3 e x^2 (g x)^m \left(a+c x^2\right)^p \left(1+\frac{c x^2}{a}\right)^{-p} \text{AppellF1}\left[\frac{2+m}{2},-p,3,\frac{4+m}{2},-\frac{c x^2}{a},\frac{e^2 x^2}{d^2}\right]}{d^4 (2+m)}+\frac{3 e^2 x^3 (g x)^m \left(a+c x^2\right)^p \left(1+\frac{c x^2}{a}\right)^{-p} \text{AppellF1}\left[\frac{3+m}{2},-p,3,\frac{5+m}{2},-\frac{c x^2}{a},\frac{e^2 x^2}{d^2}\right]}{d^5 (3+m)}-\frac{e^3 x^4 (g x)^m \left(a+c x^2\right)^p \left(1+\frac{c x^2}{a}\right)^{-p} \text{AppellF1}\left[\frac{4+m}{2},-p,3,\frac{6+m}{2},-\frac{c x^2}{a},\frac{e^2 x^2}{d^2}\right]}{d^6 (4+m)}$$

Result (type 8, 24 leaves) :

$$\int \frac{(g x)^m \left(a+c x^2\right)^p}{(d+e x)^3} \, dx$$

- **Problem 489: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^3 \sqrt{1+x} \sqrt{1-x+x^2} \, dx$$

Optimal (type 4, 170 leaves, 4 steps):

$$\frac{6}{55} x \sqrt{1+x} \sqrt{1-x+x^2} + \frac{2}{11} x^4 \sqrt{1+x} \sqrt{1-x+x^2} - \frac{4 \times 3^{3/4} \sqrt{2+\sqrt{3}} (1+x)^{3/2} \sqrt{1-x+x^2} \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{55 \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \left(1+x^3\right)}$$

Result (type 4, 221 leaves):

$$\frac{1}{55 \sqrt{1-x+x^2}} 2 \left(x \sqrt{1+x} \left(3-3x+3x^2+5x^3-5x^4+5x^5\right) + \sqrt{-\frac{6 i}{3 i+\sqrt{3}}} \left(3 i+\sqrt{3}\right) (1+x) \sqrt{\frac{3 i+\sqrt{3}+\left(-3 i+\sqrt{3}\right) x}{\left(-3 i+\sqrt{3}\right) (1+x)}} \sqrt{\frac{-3 i+\sqrt{3}+\left(3 i+\sqrt{3}\right) x}{\left(3 i+\sqrt{3}\right) (1+x)}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{6 i}{3 i+\sqrt{3}}}}{\sqrt{1+x}}\right], \frac{3 i+\sqrt{3}}{3 i-\sqrt{3}}\right]\right)$$

- **Problem 491: Result unnecessarily involves imaginary or complex numbers.**

$$\int x \sqrt{1+x} \sqrt{1-x+x^2} \, dx$$

Optimal (type 4, 294 leaves, 5 steps):

$$\frac{2}{7} x^2 \sqrt{1+x} \sqrt{1-x+x^2} + \frac{6 \sqrt{1+x} \sqrt{1-x+x^2}}{7 \left(1+\sqrt{3}+x\right)} - \frac{3 \times 3^{1/4} \sqrt{2-\sqrt{3}} (1+x)^{3/2} \sqrt{1-x+x^2} \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{7 \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \left(1+x^3\right)} + \frac{2 \sqrt{2}\, 3^{3/4} (1+x)^{3/2} \sqrt{1-x+x^2} \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{7 \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \left(1+x^3\right)}$$

Result (type 4, 347 leaves):

$$\frac{1}{14\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}\sqrt{1+x}\left(4x^2\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\left(1-x+x^2\right)-3\sqrt{2}\left(-3i+\sqrt{3}\right)\sqrt{\frac{i+\sqrt{3}-2ix}{3i+\sqrt{3}}}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]+3\sqrt{2}\left(-i+\sqrt{3}\right)\sqrt{\frac{i+\sqrt{3}-2ix}{3i+\sqrt{3}}}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]\right)$$

- **Problem 492: Result unnecessarily involves imaginary or complex numbers.**

$$\int\sqrt{1+x}\,\sqrt{1-x+x^2}\,dx$$

Optimal (type 4, 144 leaves, 3 steps):

$$\frac{2}{5}x\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2\times 3^{3/4}\sqrt{2+\sqrt{3}}\,(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{5\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

Result (type 4, 169 leaves):

$$\frac{2x\sqrt{1+x}\left(1-x+x^2\right)+\frac{i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{5\sqrt{1-x+x^2}}$$

- **Problem 493: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{x}\,dx$$

Optimal (type 3, 66 leaves, 5 steps):

$$\frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}-\frac{2\sqrt{1+x}\sqrt{1-x+x^2}\,\text{ArcTanh}\left[\sqrt{1+x^3}\right]}{3\sqrt{1+x^3}}$$

Result (type 4, 197 leaves) :

$$\frac{\sqrt{1+x}\left(2\left(1-x+x^2\right)+\frac{3\,i\,\sqrt{2}\,\sqrt{\frac{i+\sqrt{3}-2\,i\,x}{3\,i+\sqrt{3}}}\,\sqrt{\frac{-i+\sqrt{3}+2\,i\,x}{-3\,i+\sqrt{3}}}\,\text{EllipticPi}\left[\frac{3}{2}-\frac{i\,\sqrt{3}}{2},\,i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{i\,(1+x)}{3\,i+\sqrt{3}}}\right],\,\frac{3\,i+\sqrt{3}}{3\,i-\sqrt{3}}\right]}{\sqrt{-\frac{i\,(1+x)}{3\,i+\sqrt{3}}}}\right)}{3\,\sqrt{1-x+x^2}}$$

Problem 494: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{x^2}\,dx$$

Optimal (type 4, 287 leaves, 5 steps) :

$$-\frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{x}+\frac{3\,\sqrt{1+x}\,\sqrt{1-x+x^2}}{1+\sqrt{3}+x}-\frac{3\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,(1+x)^{3/2}\,\sqrt{1-x+x^2}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],\,-7-4\,\sqrt{3}\right]}{2\,\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\left(1+x^3\right)}+$$

$$\frac{\sqrt{2}\,3^{3/4}\,(1+x)^{3/2}\,\sqrt{1-x+x^2}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],\,-7-4\,\sqrt{3}\right]}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\left(1+x^3\right)}$$

Result (type 4, 349 leaves) :

$$-\frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{x}+\left(3\,\sqrt{1+\frac{2\,i\,(1+x)}{-3\,i+\sqrt{3}}}\,\sqrt{1-\frac{2\,i\,(1+x)}{3\,i+\sqrt{3}}}\,\left(-\frac{\left(-3\,i+\sqrt{3}\right)\,\sqrt{-\frac{i}{3\,i+\sqrt{3}}}\,\sqrt{1+x}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{i\,(1+x)}{3\,i+\sqrt{3}}}\right],\,\frac{3\,i+\sqrt{3}}{3\,i-\sqrt{3}}\right]}{\sqrt{-\frac{i\,(1+x)}{3\,i+\sqrt{3}}}}+\right.\right.$$

$$\left.\left.\frac{\left(-i+\sqrt{3}\right)\,\sqrt{-\frac{i}{3\,i+\sqrt{3}}}\,\sqrt{1+x}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{i\,(1+x)}{3\,i+\sqrt{3}}}\right],\,\frac{3\,i+\sqrt{3}}{3\,i-\sqrt{3}}\right]}{\sqrt{-\frac{i\,(1+x)}{3\,i+\sqrt{3}}}}\right)\right)\Bigg/\left(2\,\sqrt{2}\,\sqrt{-\frac{i}{3\,i+\sqrt{3}}}\,\sqrt{3-3\,(1+x)+(1+x)^2}\right)$$

- **Problem 495: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{x^3}\,dx$$

Optimal (type 4, 146 leaves, 3 steps):

$$-\frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{2\,x^2}+\frac{3^{3/4}\,\sqrt{2+\sqrt{3}}\,(1+x)^{3/2}\,\sqrt{1-x+x^2}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\,\sqrt{3}\right]}{2\,\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\left(1+x^3\right)}$$

Result (type 4, 185 leaves):

$$\frac{\sqrt{1+x}\,\left(-\frac{2\,\left(1-x+x^2\right)}{x^2}-\frac{3\,i\,\sqrt{2}\,\sqrt{\frac{i+\sqrt{3}-2\,i\,x}{3\,i+\sqrt{3}}}\,\sqrt{\frac{-i+\sqrt{3}+2\,i\,x}{-3\,i+\sqrt{3}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{i\,(1+x)}{3\,i+\sqrt{3}}}\right],\frac{3\,i+\sqrt{3}}{3\,i-\sqrt{3}}\right]}{\sqrt{-\frac{i\,(1+x)}{3\,i+\sqrt{3}}}}\right)}{4\,\sqrt{1-x+x^2}}$$

- **Problem 496: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^3\,(1+x)^{3/2}\,\left(1-x+x^2\right)^{3/2}\,dx$$

Optimal (type 4, 201 leaves, 5 steps):

$$\frac{54}{935}\,x\,\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{18}{187}\,x^4\,\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{2}{17}\,x^4\,\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)-$$

$$\frac{36\times 3^{3/4}\,\sqrt{2+\sqrt{3}}\,(1+x)^{3/2}\,\sqrt{1-x+x^2}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\,\sqrt{3}\right]}{935\,\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\left(1+x^3\right)}$$

Result (type 4, 235 leaves):

$$\frac{1}{935\sqrt{1-x+x^2}}2\left(x\sqrt{1+x}\left(27-27x+27x^2+100x^3-100x^4+100x^5+55x^6-55x^7+55x^8\right)-\right.$$

$$\left.\frac{9\,i\sqrt{6}\,(1+x)\sqrt{\frac{3i+\sqrt{3}+\left(-3i+\sqrt{3}\right)x}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{\frac{-3i+\sqrt{3}+\left(3i+\sqrt{3}\right)x}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}\right)$$

- **Problem 498: Result unnecessarily involves imaginary or complex numbers.**

$$\int x\,(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}dx$$

Optimal (type 4, 325 leaves, 6 steps) :

$$\frac{18}{91}x^2\sqrt{1+x}\sqrt{1-x+x^2}+\frac{54\sqrt{1+x}\sqrt{1-x+x^2}}{91\left(1+\sqrt{3}+x\right)}+\frac{2}{13}x^2\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)-$$

$$\frac{27\times 3^{1/4}\sqrt{2-\sqrt{3}}\,(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{91\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}+$$

$$\frac{18\sqrt{2}\,3^{3/4}\,(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{91\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

Result (type 4, 244 leaves) :

$$\frac{1}{182\sqrt{1-x+x^2}}\sqrt{1+x}\left(4x^2\left(1-x+x^2\right)\left(16+7x^3\right)-1\Big/\left(\sqrt{-\frac{i(1+x)}{i+\sqrt{3}-2ix}}\right)27\sqrt{2}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\right.$$

$$\left.\left(\left(-3i+\sqrt{3}\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]-\left(-i+\sqrt{3}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]\right)\right)$$

- **Problem 499: Result unnecessarily involves imaginary or complex numbers.**

$$\int(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}dx$$

Optimal (type 4, 173 leaves, 4 steps) :

$$\frac{18}{55}x\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{11}x\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)+$$

$$\frac{18\times 3^{3/4}\sqrt{2+\sqrt{3}}\,(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{55\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

Result (type 4, 176 leaves) :

$$\frac{1}{55\sqrt{1-x+x^2}}\left(2x\sqrt{1+x}\left(1-x+x^2\right)\left(14+5x^3\right)+\frac{9i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}\right)$$

- **Problem 500: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}dx$$

Optimal (type 3, 94 leaves, 6 steps) :

$$\frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{9}\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)-\frac{2\sqrt{1+x}\sqrt{1-x+x^2}\,\text{ArcTanh}\left[\sqrt{1+x^3}\right]}{3\sqrt{1+x^3}}$$

Result (type 4, 201 leaves) :

$$\frac{1}{\sqrt{1-x+x^2}}\sqrt{1+x}\left(\frac{2}{9}\left(1-x+x^2\right)\left(4+x^3\right)+\frac{i\sqrt{2}\sqrt{\frac{i+\sqrt{3}-2ix}{3i+\sqrt{3}}}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\,\text{EllipticPi}\left[\frac{3}{2}-\frac{i\sqrt{3}}{2},\,i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]}{\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}}\right)$$

Problem 501: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^2}\,dx$$

Optimal (type 4, 323 leaves, 6 steps):

$$\frac{9}{7}x^2\sqrt{1+x}\sqrt{1-x+x^2}+\frac{27\sqrt{1+x}\sqrt{1-x+x^2}}{7\left(1+\sqrt{3}+x\right)}-\frac{\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)}{x}-$$

$$\frac{27\times 3^{1/4}\sqrt{2-\sqrt{3}}\,(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{14\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}+$$

$$\frac{9\sqrt{2}\,3^{3/4}\,(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{7\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

Result (type 4, 244 leaves):

$$\frac{1}{28\sqrt{1-x+x^2}}\sqrt{1+x}\left(\frac{4\left(1-x+x^2\right)\left(-7+2x^3\right)}{x}-1\middle/\left(\sqrt{-\frac{i(1+x)}{i+\sqrt{3}-2ix}}\right)27\sqrt{2}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\right.$$

$$\left.\left(\left(-3i+\sqrt{3}\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]-\left(-i+\sqrt{3}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]\right)\right)$$

Problem 502: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx$$

Optimal (type 4, 175 leaves, 4 steps) :

$$\frac{9}{10}x\sqrt{1+x}\sqrt{1-x+x^2}-\frac{\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)}{2x^2}+$$
$$\frac{9\times 3^{3/4}\sqrt{2+\sqrt{3}}\,(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{10\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

Result (type 4, 192 leaves) :

$$\frac{\sqrt{1+x}\left(\frac{2\left(1-x+x^2\right)\left(-5+4x^3\right)}{x^2}-\frac{27\,i\sqrt{2}\sqrt{\frac{i+\sqrt{3}-2ix}{3i+\sqrt{3}}}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]}{\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}}\right)}{20\sqrt{1-x+x^2}}$$

▪ Problem 503: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$$

Optimal (type 4, 142 leaves, 3 steps) :

$$\frac{2x\left(1+x^3\right)}{5\sqrt{1+x}\sqrt{1-x+x^2}}-\frac{4\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{5\times 3^{1/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Result (type 4, 169 leaves) :

$$\frac{6x\sqrt{1+x}\left(1-x+x^2\right)-\frac{2i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{15\sqrt{1-x+x^2}}$$

- **Problem 505: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x}{\sqrt{1+x}\,\sqrt{1-x+x^2}}\,dx$$

Optimal (type 4, 253 leaves, 4 steps):

$$\frac{2\left(1+x^3\right)}{\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}}-\frac{3^{1/4}\sqrt{2-\sqrt{3}}\,\sqrt{1+x}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1-x+x^2}}+$$

$$\frac{2\sqrt{2}\,\sqrt{1+x}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{3^{1/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1-x+x^2}}$$

Result (type 4, 375 leaves):

$$\frac{1}{6\sqrt{-\frac{i}{3\,i+\sqrt{3}}}\,\sqrt{1-x+x^2}}$$

$$(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3\,i+\sqrt{3}}}\left(1-x+x^2\right)}{(1+x)^2}+\frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3\,i+\sqrt{3}-\frac{6\,i}{1+x}}{3\,i+\sqrt{3}}}\,\sqrt{\frac{-3\,i+\sqrt{3}+\frac{6\,i}{1+x}}{-3\,i+\sqrt{3}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6\,i}{3\,i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3\,i+\sqrt{3}}{3\,i-\sqrt{3}}\right]}{\sqrt{1+x}}+\right.$$

$$\left.\frac{i\sqrt{2}\left(3\,i+\sqrt{3}\right)\sqrt{\frac{3\,i+\sqrt{3}-\frac{6\,i}{1+x}}{3\,i+\sqrt{3}}}\,\sqrt{\frac{-3\,i+\sqrt{3}+\frac{6\,i}{1+x}}{-3\,i+\sqrt{3}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6\,i}{3\,i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3\,i+\sqrt{3}}{3\,i-\sqrt{3}}\right]}{\sqrt{1+x}}\right)$$

Problem 506: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{1+x}\,\sqrt{1-x+x^2}}\,dx$$

Optimal (type 4, 110 leaves, 2 steps):

$$\frac{2\sqrt{2+\sqrt{3}}\,\sqrt{1+x}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{3^{1/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1-x+x^2}}$$

Result (type 4, 148 leaves):

$$\frac{i\,(1+x)\,\sqrt{1+\frac{6\,i}{\left(-3\,i+\sqrt{3}\right)(1+x)}}\,\sqrt{\frac{2}{3}-\frac{4\,i}{\left(3\,i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6\,i}{3\,i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3\,i+\sqrt{3}}{3\,i-\sqrt{3}}\right]}{\sqrt{-\frac{i}{3\,i+\sqrt{3}}}\,\sqrt{1-x+x^2}}$$

Problem 507: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x\sqrt{1+x}\,\sqrt{1-x+x^2}}\,dx$$

Optimal (type 3, 42 leaves, 4 steps):

$$-\frac{2\sqrt{1+x^3}\,\text{ArcTanh}\left[\sqrt{1+x^3}\right]}{3\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

Result (type 4, 68 leaves):

$$-\frac{2\sqrt{1+x}\,\text{EllipticPi}\left[1+(-1)^{1/3},\text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right],(-1)^{1/3}\right]}{\sqrt{3}\,\sqrt{\frac{1+x}{1+(-1)^{1/3}}}}$$

Problem 508: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2\sqrt{1+x}\,\sqrt{1-x+x^2}}\,dx$$

Optimal (type 4, 282 leaves, 5 steps):

$$-\frac{1+x^3}{x\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{1+x^3}{\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}}-$$

$$\frac{3^{1/4}\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}+\frac{\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{3^{1/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Result (type 4, 400 leaves) :

$$-\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}+\frac{1}{12\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

$$(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}\left(1-x+x^2\right)}{(1+x)^2}+\frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]}{\sqrt{1+x}}+\right.$$

$$\left.\frac{i\sqrt{2}\left(3i+\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]}{\sqrt{1+x}}\right)$$

- **Problem 509: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$$

Optimal (type 4, 144 leaves, 3 steps) :

$$-\frac{1+x^3}{2\,x^2\,\sqrt{1+x}\,\sqrt{1-x+x^2}}-\frac{\sqrt{2+\sqrt{3}}\,\sqrt{1+x}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{2\times 3^{1/4}\,\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1-x+x^2}}$$

Result (type 4, 171 leaves):

$$\frac{-\frac{6\sqrt{1+x}\,\left(1-x+x^2\right)}{x^2}-\frac{i\,(1+x)\,\sqrt{1+\frac{6\,i}{\left(-3\,i+\sqrt{3}\right)(1+x)}}\,\sqrt{6-\frac{36\,i}{\left(3\,i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6\,i}{3\,i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3\,i+\sqrt{3}}{3\,i-\sqrt{3}}\right]}{\sqrt{-\frac{i}{3\,i+\sqrt{3}}}}}{12\,\sqrt{1-x+x^2}}$$

- **Problem 510: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^3}{(1+x)^{3/2}\,\left(1-x+x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 137 leaves, 3 steps):

$$-\frac{2\,x}{3\,\sqrt{1+x}\,\sqrt{1-x+x^2}}+\frac{4\,\sqrt{2+\sqrt{3}}\,\sqrt{1+x}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{3\times 3^{1/4}\,\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1-x+x^2}}$$

Result (type 4, 161 leaves):

$$\frac{-\frac{6\,x}{\sqrt{1+x}}+\frac{2\,i\,(1+x)\,\sqrt{1+\frac{6\,i}{\left(-3\,i+\sqrt{3}\right)(1+x)}}\,\sqrt{6-\frac{36\,i}{\left(3\,i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6\,i}{3\,i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3\,i+\sqrt{3}}{3\,i-\sqrt{3}}\right]}{\sqrt{-\frac{i}{3\,i+\sqrt{3}}}}}{9\,\sqrt{1-x+x^2}}$$

- **Problem 512: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x}{(1+x)^{3/2}\,\left(1-x+x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 282 leaves, 5 steps):

$$\frac{2x^2}{3\sqrt{1+x}\sqrt{1-x+x^2}}-\frac{2\left(1+x^3\right)}{3\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}}+$$

$$\frac{\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}-\frac{2\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{3\times 3^{1/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Result (type 4, 402 leaves):

$$\frac{2x^2}{3\sqrt{1+x}\sqrt{1-x+x^2}}-\frac{1}{18\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

$$(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}\left(1-x+x^2\right)}{(1+x)^2}+\frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]}{\sqrt{1+x}}+\right.$$

$$\left.\frac{i\sqrt{2}\left(3i+\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]}{\sqrt{1+x}}\right)$$

- **Problem 513: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 137 leaves, 3 steps):

$$\frac{2x}{3\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{2\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{3\times 3^{1/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Result (type 4, 216 leaves) :

$$\sqrt{3-3(1+x)+(1+x)^2}\left(-\frac{2}{9\sqrt{1+x}}+\frac{2(1+x)^{3/2}}{9\left(3-3(1+x)+(1+x)^2\right)}\right)+$$

$$\frac{i\sqrt{\frac{2}{3}}(1+x)\sqrt{1-\frac{6}{\left(3-i\sqrt{3}\right)(1+x)}}\sqrt{1-\frac{6}{\left(3+i\sqrt{3}\right)(1+x)}}\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6}{3-i\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3-i\sqrt{3}}{3+i\sqrt{3}}\right]}{3\sqrt{-\frac{1}{3-i\sqrt{3}}}\sqrt{3-3(1+x)+(1+x)^2}}$$

- **Problem 514: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 66 leaves, 5 steps) :

$$\frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}}-\frac{2\sqrt{1+x^3}\,\text{ArcTanh}\left[\sqrt{1+x^3}\right]}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

Result (type 4, 88 leaves) :

$$\frac{2\left(\frac{1}{\sqrt{1-x+x^2}}-\frac{\sqrt{3}(1+x)\,\text{EllipticPi}\left[1+(-1)^{1/3},\text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right],(-1)^{1/3}\right]}{\sqrt{\frac{1+x}{1+(-1)^{1/3}}}}\right)}{3\sqrt{1+x}}$$

- **Problem 515: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 316 leaves, 6 steps) :

$$\frac{2}{3 x \sqrt{1+x} \sqrt{1-x+x^2}}-\frac{5 \left(1+x^3\right)}{3 x \sqrt{1+x} \sqrt{1-x+x^2}}+\frac{5 \left(1+x^3\right)}{3 \sqrt{1+x} \left(1+\sqrt{3}+x\right) \sqrt{1-x+x^2}}-$$

$$\frac{5 \sqrt{2-\sqrt{3}} \sqrt{1+x} \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4 \sqrt{3}\right]}{2\times 3^{3/4} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{1-x+x^2}}+\frac{5 \sqrt{2} \sqrt{1+x} \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4 \sqrt{3}\right]}{3\times 3^{1/4} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{1-x+x^2}}$$

Result (type 4, 409 leaves) :

$$-\frac{3+5 x^3}{3 x \sqrt{1+x} \sqrt{1-x+x^2}}+\frac{1}{36 \sqrt{-\frac{i}{3 i+\sqrt{3}}} \sqrt{1-x+x^2}}$$

$$5 (1+x)^{3/2} \left(\frac{12 \sqrt{-\frac{i}{3 i+\sqrt{3}}} \left(1-x+x^2\right)}{(1+x)^2}+\frac{3 \sqrt{2} \left(1-i \sqrt{3}\right) \sqrt{\frac{3 i+\sqrt{3}-\frac{6 i}{1+x}}{3 i+\sqrt{3}}} \sqrt{\frac{-3 i+\sqrt{3}+\frac{6 i}{1+x}}{-3 i+\sqrt{3}}} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{6 i}{3 i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3 i+\sqrt{3}}{3 i-\sqrt{3}}\right]}{\sqrt{1+x}}+\right.$$

$$\left.\frac{i \sqrt{2} \left(3 i+\sqrt{3}\right) \sqrt{\frac{3 i+\sqrt{3}-\frac{6 i}{1+x}}{3 i+\sqrt{3}}} \sqrt{\frac{-3 i+\sqrt{3}+\frac{6 i}{1+x}}{-3 i+\sqrt{3}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{6 i}{3 i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3 i+\sqrt{3}}{3 i-\sqrt{3}}\right]}{\sqrt{1+x}}\right)$$

- **Problem 516: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{x^3 (1+x)^{3/2} \left(1-x+x^2\right)^{3/2}} \, dx$$

Optimal (type 4, 170 leaves, 4 steps) :

$$\frac{2}{3x^2\sqrt{1+x}\,\sqrt{1-x+x^2}}-\frac{7\left(1+x^3\right)}{6x^2\sqrt{1+x}\,\sqrt{1-x+x^2}}-\frac{7\sqrt{2+\sqrt{3}}\,\sqrt{1+x}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{6\times 3^{1/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1-x+x^2}}$$

Result (type 4, 170 leaves) :

$$\frac{-\frac{6\left(3+7x^3\right)}{x^2\sqrt{1+x}}-\frac{7\,i\,(1+x)\sqrt{1+\frac{6\,i}{\left(-3\,i+\sqrt{3}\right)(1+x)}}\,\sqrt{6-\frac{36\,i}{\left(3\,i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6\,i}{3\,i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3\,i+\sqrt{3}}{3\,i-\sqrt{3}}\right]}{\sqrt{-\frac{i}{3\,i+\sqrt{3}}}}}{36\sqrt{1-x+x^2}}$$

- **Problem 517: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 168 leaves, 4 steps) :

$$\frac{4x}{27\sqrt{1+x}\,\sqrt{1-x+x^2}}-\frac{2x}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)}+\frac{4\sqrt{2+\sqrt{3}}\,\sqrt{1+x}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{27\times 3^{1/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1-x+x^2}}$$

Result (type 4, 178 leaves) :

$$\frac{\frac{6x\left(-1+2x^3\right)}{(1+x)^{3/2}\left(1-x+x^2\right)}+\frac{2\,i\,(1+x)\sqrt{1+\frac{6\,i}{\left(-3\,i+\sqrt{3}\right)(1+x)}}\,\sqrt{6-\frac{36\,i}{\left(3\,i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6\,i}{3\,i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3\,i+\sqrt{3}}{3\,i-\sqrt{3}}\right]}{\sqrt{-\frac{i}{3\,i+\sqrt{3}}}}}{81\sqrt{1-x+x^2}}$$

- **Problem 519: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 318 leaves, 6 steps) :

$$\frac{10 x^2}{27 \sqrt{1+x} \sqrt{1-x+x^2}}+\frac{2 x^2}{9 \sqrt{1+x} \sqrt{1-x+x^2} \left(1+x^3\right)}-\frac{10 \left(1+x^3\right)}{27 \sqrt{1+x} \left(1+\sqrt{3}+x\right) \sqrt{1-x+x^2}}+$$

$$\frac{5 \sqrt{2-\sqrt{3}} \sqrt{1+x} \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4 \sqrt{3}\right]}{9\times 3^{3/4} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{1-x+x^2}}-\frac{10 \sqrt{2} \sqrt{1+x} \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4 \sqrt{3}\right]}{27\times 3^{1/4} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{1-x+x^2}}$$

Result (type 4, 409 leaves) :

$$\frac{2 x^2 \left(8+5 x^3\right)}{27 (1+x)^{3/2} \left(1-x+x^2\right)^{3/2}}-$$

$$\left(5 (1+x)^{3/2} \left(\frac{12 \sqrt{-\frac{i}{3 i+\sqrt{3}}} \left(1-x+x^2\right)}{(1+x)^2}+\frac{3 \sqrt{2} \left(1-i \sqrt{3}\right) \sqrt{\frac{3 i+\sqrt{3}-\frac{6 i}{1+x}}{3 i+\sqrt{3}}} \sqrt{\frac{-3 i+\sqrt{3}+\frac{6 i}{1+x}}{-3 i+\sqrt{3}}} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{6 i}{3 i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3 i+\sqrt{3}}{3 i-\sqrt{3}}\right]}{\sqrt{1+x}}+\right.\right.$$

$$\left.\left.\frac{i \sqrt{2} \left(3 i+\sqrt{3}\right) \sqrt{\frac{3 i+\sqrt{3}-\frac{6 i}{1+x}}{3 i+\sqrt{3}}} \sqrt{\frac{-3 i+\sqrt{3}+\frac{6 i}{1+x}}{-3 i+\sqrt{3}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{6 i}{3 i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3 i+\sqrt{3}}{3 i-\sqrt{3}}\right]}{\sqrt{1+x}}\right)\right) \Big/ \left(162 \sqrt{-\frac{i}{3 i+\sqrt{3}}} \sqrt{1-x+x^2}\right)$$

- **Problem 520: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(1+x)^{5/2} \left(1-x+x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 168 leaves, 4 steps) :

$$\frac{14 x}{27 \sqrt{1+x} \sqrt{1-x+x^2}}+\frac{2 x}{9 \sqrt{1+x} \sqrt{1-x+x^2} \left(1+x^3\right)}+\frac{14 \sqrt{2+\sqrt{3}} \sqrt{1+x} \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4 \sqrt{3}\right]}{27\times 3^{1/4} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{1-x+x^2}}$$

Result (type 4, 178 leaves) :

$$\frac{\frac{6 x \left(10+7 x^3\right)}{(1+x)^{3/2} \left(1-x+x^2\right)}+\frac{7 i (1+x) \sqrt{1+\frac{6 i}{\left(-3 i+\sqrt{3}\right) (1+x)}} \sqrt{6-\frac{36 i}{\left(3 i+\sqrt{3}\right) (1+x)}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{6 i}{3 i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3 i+\sqrt{3}}{3 i-\sqrt{3}}\right]}{\sqrt{-\frac{i}{3 i+\sqrt{3}}}}}{81 \sqrt{1-x+x^2}}$$

Problem 521: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x (1+x)^{5/2} \left(1-x+x^2\right)^{5/2}} \, dx$$

Optimal (type 3, 96 leaves, 6 steps) :

$$\frac{2}{3 \sqrt{1+x} \sqrt{1-x+x^2}}+\frac{2}{9 \sqrt{1+x} \sqrt{1-x+x^2} \left(1+x^3\right)}-\frac{2 \sqrt{1+x^3} \text{ArcTanh}\left[\sqrt{1+x^3}\right]}{3 \sqrt{1+x} \sqrt{1-x+x^2}}$$

Result (type 4, 98 leaves) :

$$\frac{2 \left(\frac{4+3 x^3}{\left(1-x+x^2\right)^{3/2}}-\frac{3 \sqrt{3} (1+x)^2 \text{EllipticPi}\left[1+(-1)^{1/3},\text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right],(-1)^{1/3}\right]}{\sqrt{\frac{1+x}{1+(-1)^{1/3}}}}\right)}{9 (1+x)^{3/2}}$$

Problem 522: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 (1+x)^{5/2} \left(1-x+x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 349 leaves, 7 steps) :

$$\frac{22}{27 x \sqrt{1+x} \sqrt{1-x+x^2}}+\frac{2}{9 x \sqrt{1+x} \sqrt{1-x+x^2} \left(1+x^3\right)}-\frac{55 \left(1+x^3\right)}{27 x \sqrt{1+x} \sqrt{1-x+x^2}}+\frac{55 \left(1+x^3\right)}{27 \sqrt{1+x} \left(1+\sqrt{3}+x\right) \sqrt{1-x+x^2}}-$$

$$\frac{55 \sqrt{2-\sqrt{3}} \sqrt{1+x} \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4 \sqrt{3}\right]}{18 \times 3^{3/4} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{1-x+x^2}}+\frac{55 \sqrt{2} \sqrt{1+x} \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4 \sqrt{3}\right]}{27 \times 3^{1/4} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{1-x+x^2}}$$

Result (type 4, 414 leaves):

$$-\frac{27+88x^3+55x^6}{27x(1+x)^{3/2}(1-x+x^2)^{3/2}}+\left(55(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}(1-x+x^2)}{(1+x)^2}+\frac{3\sqrt{2}(1-i\sqrt{3})\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]}{\sqrt{1+x}}+\frac{i\sqrt{2}(3i+\sqrt{3})\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right]}{\sqrt{1+x}}\right)\right)\Bigg/\left(324\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\right)$$

Problem 523: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{x^3(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$$

Optimal (type 4, 203 leaves, 5 steps):

$$\frac{26}{27x^2\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{2}{9x^2\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3)}-\frac{91(1+x^3)}{54x^2\sqrt{1+x}\sqrt{1-x+x^2}}-\frac{91\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{54\times3^{1/4}\sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}\sqrt{1-x+x^2}}$$

Result (type 4, 183 leaves):

$$\frac{-\frac{6\left(27+130x^3+91x^6\right)}{x^2(1+x)^{3/2}}-\frac{91\,i\,(1+x)\left(1-x+x^2\right)\sqrt{6+\frac{36\,i}{\left(-3\,i+\sqrt{3}\right)(1+x)}}\sqrt{1-\frac{6\,i}{\left(3\,i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{6\,i}{3\,i+\sqrt{3}}}}{\sqrt{1+x}}\right],\frac{3\,i+\sqrt{3}}{3\,i-\sqrt{3}}\right]}{\sqrt{-\frac{i}{3\,i+\sqrt{3}}}}}{324\left(1-x+x^2\right)^{3/2}}$$

Problem 541: Unable to integrate problem.

$$\int\frac{x^m\,(e+f\,x)^n}{a+b\,x+c\,x^2}\,dx$$

Optimal (type 6, 201 leaves, 6 steps):

$$\frac{2\,c\,x^{1+m}\,(e+f\,x)^n\left(1+\frac{f\,x}{e}\right)^{-n}\text{AppellF1}\left[1+m,\,-n,\,1,\,2+m,\,-\frac{f\,x}{e},\,-\frac{2\,c\,x}{b-\sqrt{b^2-4\,a\,c}}\right]}{\sqrt{b^2-4\,a\,c}\left(b-\sqrt{b^2-4\,a\,c}\right)(1+m)}-$$

$$\frac{2\,c\,x^{1+m}\,(e+f\,x)^n\left(1+\frac{f\,x}{e}\right)^{-n}\text{AppellF1}\left[1+m,\,-n,\,1,\,2+m,\,-\frac{f\,x}{e},\,-\frac{2\,c\,x}{b+\sqrt{b^2-4\,a\,c}}\right]}{\sqrt{b^2-4\,a\,c}\left(b+\sqrt{b^2-4\,a\,c}\right)(1+m)}$$

Result (type 8, 25 leaves):

$$\int\frac{x^m\,(e+f\,x)^n}{a+b\,x+c\,x^2}\,dx$$

Problem 586: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(d+e\,x)^3}{(f+g\,x)^2\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Optimal (type 3, 311 leaves, 6 steps):

$$\frac{4\,d\,e\,(d+e\,x)}{5\,(e\,f+d\,g)^2\left(d^2-e^2\,x^2\right)^{5/2}}-\frac{e\,(5\,d\,(e\,f-3\,d\,g)-e\,(e\,f+21\,d\,g)\,x)}{15\,d\,(e\,f+d\,g)^3\left(d^2-e^2\,x^2\right)^{3/2}}+$$

$$\frac{e\left(45\,d^3\,g^2+e\left(2\,e^2\,f^2+14\,d\,e\,f\,g+57\,d^2\,g^2\right)x\right)}{15\,d^3\,(e\,f+d\,g)^4\sqrt{d^2-e^2\,x^2}}+\frac{g^4\sqrt{d^2-e^2\,x^2}}{(e\,f-d\,g)\,(e\,f+d\,g)^4\,(f+g\,x)}+\frac{e\,g^3\,(4\,e\,f-3\,d\,g)\,\text{ArcTan}\left[\frac{d^2\,g+e^2\,f\,x}{\sqrt{e^2\,f^2-d^2\,g^2}\,\sqrt{d^2-e^2\,x^2}}\right]}{(e\,f-d\,g)\,(e\,f+d\,g)^4\sqrt{e^2\,f^2-d^2\,g^2}}$$

Result (type 3, 308 leaves):

$$\frac{1}{15\,(e f+d g)^4}\left(\sqrt{d^2-e^2 x^2}\left(\frac{3 e\,(e f+d g)^2}{d\,(d-e x)^3}+\frac{2 e\,(e f+d g)\,(e f+6 d g)}{d^2\,(d-e x)^2}+\frac{e\left(2 e^2 f^2+14 d e f g+57 d^2 g^2\right)}{d^3\,(d-e x)}+\frac{15 g^4}{(e f-d g)\,(f+g x)}\right)-\frac{15\,\mathrm{i}\,e g^3\,(4 e f-3 d g)\,\mathrm{Log}\left[\frac{2\,(e f-d g)\,(e f+d g)^4\left(\mathrm{i}\,d^2 g+\mathrm{i}\,e^2 f x+\sqrt{e^2 f^2-d^2 g^2}\,\sqrt{d^2-e^2 x^2}\right)}{e g^2\,(4 e f-3 d g)\,\sqrt{e^2 f^2-d^2 g^2}\,(f+g x)}\right]}{(e f-d g)\,\sqrt{e^2 f^2-d^2 g^2}}\right)$$

- **Problem 587: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e x)^3}{(f+g x)^3\left(d^2-e^2 x^2\right)^{7/2}}\,dx$$

Optimal (type 3, 398 leaves, 7 steps):

$$\frac{4 d e^2\,(d+e x)}{5\,(e f+d g)^3\left(d^2-e^2 x^2\right)^{5/2}}-\frac{e^2\,(5 d\,(e f-5 d g)-e\,(e f+31 d g)\,x)}{15 d\,(e f+d g)^4\left(d^2-e^2 x^2\right)^{3/2}}+\frac{e^2\left(90 d^3 g^2+e\left(2 e^2 f^2+19 d e f g+107 d^2 g^2\right) x\right)}{15 d^3\,(e f+d g)^5\sqrt{d^2-e^2 x^2}}+$$

$$\frac{g^4\sqrt{d^2-e^2 x^2}}{2\,(e f-d g)\,(e f+d g)^4\,(f+g x)^2}+\frac{3 e g^4\,(3 e f-2 d g)\sqrt{d^2-e^2 x^2}}{2\,(e f-d g)^2\,(e f+d g)^5\,(f+g x)}+\frac{e^2 g^3\left(20 e^2 f^2-30 d e f g+13 d^2 g^2\right)\mathrm{ArcTan}\left[\frac{d^2 g+e^2 f x}{\sqrt{e^2 f^2-d^2 g^2}\,\sqrt{d^2-e^2 x^2}}\right]}{2\,(e f-d g)^2\,(e f+d g)^5\sqrt{e^2 f^2-d^2 g^2}}$$

Result (type 3, 387 leaves):

$$\frac{1}{30\,(e f+d g)^5}\left(\sqrt{d^2-e^2 x^2}\left(\frac{6 e^2\,(e f+d g)^2}{d\,(d-e x)^3}+\frac{2 e^2\,(e f+d g)\,(2 e f+17 d g)}{d^2\,(d-e x)^2}+\frac{2 e^2\left(2 e^2 f^2+19 d e f g+107 d^2 g^2\right)}{d^3\,(d-e x)}+\frac{15 g^4\,(e f+d g)}{(e f-d g)\,(f+g x)^2}+\frac{45 e g^4\,(3 e f-2 d g)}{(e f-d g)^2\,(f+g x)}\right)-\frac{15\,\mathrm{i}\,e^2 g^3\left(20 e^2 f^2-30 d e f g+13 d^2 g^2\right)\mathrm{Log}\left[\frac{4\,(e f-d g)^2\,(e f+d g)^5\left(\mathrm{i}\,d^2 g+\mathrm{i}\,e^2 f x+\sqrt{e^2 f^2-d^2 g^2}\,\sqrt{d^2-e^2 x^2}\right)}{e^2 g^2\sqrt{e^2 f^2-d^2 g^2}\left(20 e^2 f^2-30 d e f g+13 d^2 g^2\right)(f+g x)}\right]}{(e f-d g)^2\sqrt{e^2 f^2-d^2 g^2}}\right)$$

Problem 605: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d+e x)^{3/2} \sqrt{f+g x}}{a+c x^2} \, dx$$

Optimal (type 3, 411 leaves, 11 steps):

$$\frac{e \sqrt{d+e x} \sqrt{f+g x}}{c}+\frac{\sqrt{e} (e f+3 d g) \text{ArcTanh}\left[\frac{\sqrt{g} \sqrt{d+e x}}{\sqrt{e} \sqrt{f+g x}}\right]}{c \sqrt{g}}+\frac{\left(\frac{a \left(a e^2 g-c d (2 e f+d g)\right)}{\sqrt{c}}-\sqrt{-a} \left(c d^2 f-a e (e f+2 d g)\right)\right) \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c} f-\sqrt{-a} g} \sqrt{d+e x}}{\sqrt{\sqrt{c} d-\sqrt{-a} e} \sqrt{f+g x}}\right]}{a c \sqrt{\sqrt{c} d-\sqrt{-a} e} \sqrt{\sqrt{c} f-\sqrt{-a} g}}+$$

$$\frac{\left(\frac{a \left(a e^2 g-c d (2 e f+d g)\right)}{\sqrt{c}}+\sqrt{-a} \left(c d^2 f-a e (e f+2 d g)\right)\right) \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c} f+\sqrt{-a} g} \sqrt{d+e x}}{\sqrt{\sqrt{c} d+\sqrt{-a} e} \sqrt{f+g x}}\right]}{a c \sqrt{\sqrt{c} d+\sqrt{-a} e} \sqrt{\sqrt{c} f+\sqrt{-a} g}}$$

Result (type 3, 575 leaves):

$$\frac{1}{2 c^2}\Bigg(2 c e \sqrt{d+e x} \sqrt{f+g x}+$$

$$\frac{c \sqrt{e} (e f+3 d g) \text{Log}\left[e f+d g+2 e g x+2 \sqrt{e} \sqrt{g} \sqrt{d+e x} \sqrt{f+g x}\right]}{\sqrt{g}}+\frac{1}{\sqrt{a}} i \sqrt{c} \left(\sqrt{c} d+i \sqrt{a} e\right)^{3/2} \sqrt{\sqrt{c} f+i \sqrt{a} g}$$

$$\text{Log}\Big[\Big(i \sqrt{a} c^{3/2} \Big(2 \sqrt{\sqrt{c} d+i \sqrt{a} e} \sqrt{\sqrt{c} f+i \sqrt{a} g} \sqrt{d+e x} \sqrt{f+g x}+\sqrt{c} (2 d f+e f x+d g x)+i \sqrt{a} (e f+d g+2 e g x)\Big)\Big)\Big/$$

$$\left(\left(\sqrt{c} d+i \sqrt{a} e\right)^{5/2} \left(\sqrt{c} f+i \sqrt{a} g\right)^{3/2} \left(-i \sqrt{a}+\sqrt{c} x\right)\right)\Big]-\frac{1}{\sqrt{a}} i \sqrt{c} \left(\sqrt{c} d-i \sqrt{a} e\right)^{3/2} \sqrt{\sqrt{c} f-i \sqrt{a} g}$$

$$\text{Log}\Big[-\Big(\sqrt{a} c^{3/2} \Big(2 i \sqrt{\sqrt{c} d-i \sqrt{a} e} \sqrt{\sqrt{c} f-i \sqrt{a} g} \sqrt{d+e x} \sqrt{f+g x}+i \sqrt{c} (2 d f+e f x+d g x)+\sqrt{a} (e f+d g+2 e g x)\Big)\Big)\Big/$$

$$\left(\left(\sqrt{c} d-i \sqrt{a} e\right)^{5/2} \left(\sqrt{c} f-i \sqrt{a} g\right)^{3/2} \left(i \sqrt{a}+\sqrt{c} x\right)\right)\Big]\Bigg)$$

Problem 606: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{d+e x} \sqrt{f+g x}}{a+c x^2} \, dx$$

Optimal (type 3, 342 leaves, 10 steps):

$$\frac{2\sqrt{e}\sqrt{g}\,\text{ArcTanh}\left[\frac{\sqrt{g}\sqrt{d+e x}}{\sqrt{e}\sqrt{f+g x}}\right]}{c}+\frac{\left(c d f-a e g-\sqrt{-a}\sqrt{c}\,(e f+d g)\right)\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+e x}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+g x}}\right]}{\sqrt{-a}\,c\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-$$

$$\frac{\left(c d f-a e g+\sqrt{-a}\sqrt{c}\,(e f+d g)\right)\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\sqrt{d+e x}}{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{f+g x}}\right]}{\sqrt{-a}\,c\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}}$$

Result (type 3, 524 leaves) :

$$\frac{1}{2\sqrt{a}\,c}\left(2\sqrt{a}\sqrt{e}\sqrt{g}\,\text{Log}\left[e f+d g+2 e g x+2\sqrt{e}\sqrt{g}\sqrt{d+e x}\sqrt{f+g x}\right]-i\sqrt{\sqrt{c}\,d-i\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f-i\sqrt{a}\,g}\right.$$
$$\text{Log}\left[-\left(\sqrt{a}\,c\left(2 i\sqrt{\sqrt{c}\,d-i\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f-i\sqrt{a}\,g}\,\sqrt{d+e x}\sqrt{f+g x}+i\sqrt{c}\,(2 d f+e f x+d g x)+\sqrt{a}\,(e f+d g+2 e g x)\right)\right)\Big/\right.$$
$$\left.\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)^{3/2}\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)^{3/2}\left(i\sqrt{a}+\sqrt{c}\,x\right)\right)\right]+i\sqrt{\sqrt{c}\,d+i\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f+i\sqrt{a}\,g}$$
$$\text{Log}\left[\left(-a c\,(d g+e\,(f+2 g x))+i\sqrt{a}\left(2 c\sqrt{\sqrt{c}\,d+i\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f+i\sqrt{a}\,g}\,\sqrt{d+e x}\sqrt{f+g x}+c^{3/2}\,(2 d f+e f x+d g x)\right)\right)\Big/\right.$$
$$\left.\left.\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)^{3/2}\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)^{3/2}\left(-i\sqrt{a}+\sqrt{c}\,x\right)\right)\right]\right)$$

- **Problem 607: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{f+g x}}{\sqrt{d+e x}\,\left(a+c x^2\right)}\,dx$$

Optimal (type 3, 240 leaves, 6 steps) :

$$\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+e x}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+g x}}\right]}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}-\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\sqrt{d+e x}}{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{f+g x}}\right]}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}}$$

Result (type 3, 496 leaves) :

$$\frac{1}{2\sqrt{a}\,c}\,i\left(\left(\left(c f+i\sqrt{a}\sqrt{c}\,g\right)\right.\right.$$

$$\text{Log}\Big[\Big(i\sqrt{a}\sqrt{c}\left(2\sqrt{\sqrt{c}\,d+i\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f+i\sqrt{a}\,g}\,\sqrt{d+e x}\,\sqrt{f+g x}+\sqrt{c}\,(2 d f+e f x+d g x)+i\sqrt{a}\,(d g+e\,(f+2 g x))\right)\Big)\Big/$$

$$\left(\sqrt{\sqrt{c}\,d+i\sqrt{a}\,e}\,\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)^{3/2}\left(-i\sqrt{a}+\sqrt{c}\,x\right)\right)\Big]\Big)\Big/$$

$$\left(\sqrt{\sqrt{c}\,d+i\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f+i\sqrt{a}\,g}\right)-1\Big/\left(\sqrt{\sqrt{c}\,d-i\sqrt{a}\,e}\right)\sqrt{c}\,\sqrt{\sqrt{c}\,f-i\sqrt{a}\,g}$$

$$\text{Log}\Big[-\Big(\sqrt{a}\sqrt{c}\left(2 i\sqrt{\sqrt{c}\,d-i\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f-i\sqrt{a}\,g}\,\sqrt{d+e x}\,\sqrt{f+g x}+i\sqrt{c}\,(2 d f+e f x+d g x)+\sqrt{a}\,(d g+e\,(f+2 g x))\right)\Big)\Big/$$

$$\left(\sqrt{\sqrt{c}\,d-i\sqrt{a}\,e}\,\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)^{3/2}\left(i\sqrt{a}+\sqrt{c}\,x\right)\right)\Big]\Big)$$

- **Problem 608: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{f+g x}}{(d+e x)^{3/2}\left(a+c x^2\right)}\,dx$$

Optimal (type 3, 351 leaves, 8 steps) :

$$-\frac{2 e\sqrt{f+g x}}{\left(c d^2+a e^2\right)\sqrt{d+e x}}+\frac{\left(c d f+a e g+\sqrt{-a}\sqrt{c}\,(e f-d g)\right)\text{ArcTanh}\Big[\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+e x}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+g x}}\Big]}{\sqrt{-a}\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\left(c d^2+a e^2\right)\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-$$

$$\frac{\left(c d f+a e g-\sqrt{-a}\sqrt{c}\,(e f-d g)\right)\text{ArcTanh}\Big[\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\sqrt{d+e x}}{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{f+g x}}\Big]}{\sqrt{-a}\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\left(c d^2+a e^2\right)\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}}$$

Result (type 3, 531 leaves) :

$$\frac{1}{2\left(c\,d^2+a\,e^2\right)}\left(-\frac{4\,e\,\sqrt{f+g\,x}}{\sqrt{d+e\,x}}+1\Big/\left(\sqrt{a}\,\sqrt{\sqrt{c}\,d+i\,\sqrt{a}\,e}\right)\left(i\,\sqrt{c}\,d+\sqrt{a}\,e\right)\sqrt{\sqrt{c}\,f+i\,\sqrt{a}\,g}\right.$$
$$\text{Log}\left[\left(i\,\sqrt{a}\,\sqrt{\sqrt{c}\,d+i\,\sqrt{a}\,e}\,\left(2\,\sqrt{\sqrt{c}\,d+i\,\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f+i\,\sqrt{a}\,g}\,\sqrt{d+e\,x}\,\sqrt{f+g\,x}+\sqrt{c}\,\left(2\,d\,f+e\,f\,x+d\,g\,x\right)+i\,\sqrt{a}\,\left(e\,f+d\,g+2\,e\,g\,x\right)\right)\right)\Big/\right.$$
$$\left.\left(\left(\sqrt{c}\,f+i\,\sqrt{a}\,g\right)^{3/2}\left(-i\,\sqrt{a}+\sqrt{c}\,x\right)\right)\right]+1\Big/\left(\sqrt{a}\,\sqrt{\sqrt{c}\,d-i\,\sqrt{a}\,e}\right)\left(-i\,\sqrt{c}\,d+\sqrt{a}\,e\right)\sqrt{\sqrt{c}\,f-i\,\sqrt{a}\,g}\,\text{Log}\Big[$$
$$-\left(i\,\sqrt{a}\,\sqrt{\sqrt{c}\,d-i\,\sqrt{a}\,e}\,\left(2\,\sqrt{\sqrt{c}\,d-i\,\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f-i\,\sqrt{a}\,g}\,\sqrt{d+e\,x}\,\sqrt{f+g\,x}+\sqrt{c}\,\left(2\,d\,f+e\,f\,x+d\,g\,x\right)-i\,\sqrt{a}\,\left(d\,g+e\,\left(f+2\,g\,x\right)\right)\right)\right)\Big/$$
$$\left.\left(\left(\sqrt{c}\,f-i\,\sqrt{a}\,g\right)^{3/2}\left(i\,\sqrt{a}+\sqrt{c}\,x\right)\right)\Big]\right)$$

- **Problem 609: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{f+g\,x}}{\left(d+e\,x\right)^{5/2}\left(a+c\,x^2\right)}\,dx$$

Optimal (type 3, 613 leaves, 11 steps):

$$-\frac{2\,e\,\sqrt{f+g\,x}}{3\,\left(c\,d^2+a\,e^2\right)\left(d+e\,x\right)^{3/2}}+\frac{4\,e\,g\,\sqrt{f+g\,x}}{3\,\left(c\,d^2+a\,e^2\right)\left(e\,f-d\,g\right)\sqrt{d+e\,x}}+\frac{e\,\left(c\,d\,f+a\,e\,g-\sqrt{-a}\,\sqrt{c}\,\left(e\,f-d\,g\right)\right)\sqrt{f+g\,x}}{\sqrt{-a}\,\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)\left(c\,d^2+a\,e^2\right)\left(e\,f-d\,g\right)\sqrt{d+e\,x}}-$$
$$\frac{e\,\left(c\,d\,f+a\,e\,g+\sqrt{-a}\,\sqrt{c}\,\left(e\,f-d\,g\right)\right)\sqrt{f+g\,x}}{\sqrt{-a}\,\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(c\,d^2+a\,e^2\right)\left(e\,f-d\,g\right)\sqrt{d+e\,x}}+\frac{\sqrt{c}\,\left(c\,d\,f+a\,e\,g+\sqrt{-a}\,\sqrt{c}\,\left(e\,f-d\,g\right)\right)\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+e\,x}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+g\,x}}\right]}{\sqrt{-a}\,\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)^{3/2}\left(c\,d^2+a\,e^2\right)\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}+$$
$$\frac{\sqrt{c}\,\left(\sqrt{-a}\,c\,d\,f+\sqrt{-a}\,a\,e\,g+a\,\sqrt{c}\,\left(e\,f-d\,g\right)\right)\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\sqrt{d+e\,x}}{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{f+g\,x}}\right]}{a\,\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)^{3/2}\left(c\,d^2+a\,e^2\right)\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}}$$

Result (type 3, 600 leaves):

$$\frac{1}{2}\left(-\frac{4 e \sqrt{f+g x}\,\left(a e^3 (f+g x)+c d\left(-6 d^2 g+6 e^2 f x+d e (7 f-5 g x)\right)\right)}{3\left(c d^2+a e^2\right)^2 (e f-d g)\,(d+e x)^{3/2}}-\right.$$
$$\left(\mathrm{i}\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g\right)\mathrm{Log}\left[-\left(\mathrm{i}\sqrt{a}\left(\sqrt{c}\,d-\mathrm{i}\sqrt{a}\,e\right)^{3/2}\left(2\sqrt{\sqrt{c}\,d-\mathrm{i}\sqrt{a}\,e}\sqrt{\sqrt{c}\,f-\mathrm{i}\sqrt{a}\,g}\sqrt{d+e x}\sqrt{f+g x}+\sqrt{c}\,(2 d f+e f x+d g x)-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\mathrm{i}\sqrt{a}\,(e f+d g+2 e g x)\right)\right)\Big/\left(\sqrt{c}\left(\sqrt{c}\,f-\mathrm{i}\sqrt{a}\,g\right)^{3/2}\left(\mathrm{i}\sqrt{a}+\sqrt{c}\,x\right)\right)\right]\right)\Big/\left(\sqrt{a}\left(\sqrt{c}\,d-\mathrm{i}\sqrt{a}\,e\right)^{5/2}\sqrt{\sqrt{c}\,f-\mathrm{i}\sqrt{a}\,g}\right)+$$
$$\left(\mathrm{i}\left(c f+\mathrm{i}\sqrt{a}\sqrt{c}\,g\right)\mathrm{Log}\left[\left(\mathrm{i}\sqrt{a}\left(\sqrt{c}\,d+\mathrm{i}\sqrt{a}\,e\right)^{3/2}\left(2\sqrt{\sqrt{c}\,d+\mathrm{i}\sqrt{a}\,e}\sqrt{\sqrt{c}\,f+\mathrm{i}\sqrt{a}\,g}\sqrt{d+e x}\sqrt{f+g x}+\sqrt{c}\,(2 d f+e f x+d g x)+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\left.\mathrm{i}\sqrt{a}\,(e f+d g+2 e g x)\right)\right)\Big/\left(\sqrt{c}\left(\sqrt{c}\,f+\mathrm{i}\sqrt{a}\,g\right)^{3/2}\left(-\mathrm{i}\sqrt{a}+\sqrt{c}\,x\right)\right)\right]\right)\Big/\left(\sqrt{a}\left(\sqrt{c}\,d+\mathrm{i}\sqrt{a}\,e\right)^{5/2}\sqrt{\sqrt{c}\,f+\mathrm{i}\sqrt{a}\,g}\right)\right)$$

- **Problem 610: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(d+e x)^{3/2}}{\sqrt{f+g x}\,\left(a+c x^2\right)}\,dx$$

Optimal (type 3, 337 leaves, 11 steps):

$$\frac{2 e^{3/2}\,\mathrm{ArcTanh}\left[\frac{\sqrt{g}\sqrt{d+e x}}{\sqrt{e}\sqrt{f+g x}}\right]}{c\sqrt{g}}+\frac{\left(c d^2-2\sqrt{-a}\sqrt{c}\,d e-a e^2\right)\mathrm{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+e x}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+g x}}\right]}{\sqrt{-a}\,c\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-$$
$$\frac{\left(c d^2+2\sqrt{-a}\sqrt{c}\,d e-a e^2\right)\mathrm{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\sqrt{d+e x}}{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{f+g x}}\right]}{\sqrt{-a}\,c\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}}$$

Result (type 3, 527 leaves):

$$\frac{1}{2c}\left(\frac{2 e^{3/2}\,\text{Log}\left[e f+d g+2 e g x+2\sqrt{e}\sqrt{g}\sqrt{d+e x}\sqrt{f+g x}\right]}{\sqrt{g}}+1\Big/\left(\sqrt{a}\sqrt{\sqrt{c}\,f+i\sqrt{a}\,g}\right)i\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)^{3/2}\right.$$

$$\text{Log}\left[\left(i\sqrt{a}\,c\left(2\sqrt{\sqrt{c}\,d+i\sqrt{a}\,e}\sqrt{\sqrt{c}\,f+i\sqrt{a}\,g}\sqrt{d+e x}\sqrt{f+g x}+\sqrt{c}\,(2 d f+e f x+d g x)+i\sqrt{a}\,(e f+d g+2 e g x)\right)\right)\Big/\right.$$

$$\left.\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)^{5/2}\sqrt{\sqrt{c}\,f+i\sqrt{a}\,g}\left(-i\sqrt{a}+\sqrt{c}\,x\right)\right)\right]-1\Big/\left(\sqrt{a}\sqrt{\sqrt{c}\,f-i\sqrt{a}\,g}\right)i\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)^{3/2}$$

$$\text{Log}\left[-\left(\sqrt{a}\,c\left(2 i\sqrt{\sqrt{c}\,d-i\sqrt{a}\,e}\sqrt{\sqrt{c}\,f-i\sqrt{a}\,g}\sqrt{d+e x}\sqrt{f+g x}+i\sqrt{c}\,(2 d f+e f x+d g x)+\sqrt{a}\,(e f+d g+2 e g x)\right)\right)\Big/\right.$$

$$\left.\left.\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)^{5/2}\sqrt{\sqrt{c}\,f-i\sqrt{a}\,g}\left(i\sqrt{a}+\sqrt{c}\,x\right)\right)\right]\right)$$

- **Problem 611: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{d+e x}}{\sqrt{f+g x}\,\left(a+c x^2\right)}\,dx$$

Optimal (type 3, 240 leaves, 6 steps):

$$\frac{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\;\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\sqrt{d+e x}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\sqrt{f+g x}}\right]}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\;\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\sqrt{d+e x}}{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\sqrt{f+g x}}\right]}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}}$$

Result (type 3, 496 leaves):

$$\frac{1}{2\sqrt{a}\,c}i\left(\left(\left(c d+i\sqrt{a}\sqrt{c}\,e\right)\right.\right.$$

$$\text{Log}\left[\left(i\sqrt{a}\sqrt{c}\left(2\sqrt{\sqrt{c}\,d+i\sqrt{a}\,e}\sqrt{\sqrt{c}\,f+i\sqrt{a}\,g}\sqrt{d+e x}\sqrt{f+g x}+\sqrt{c}\,(2 d f+e f x+d g x)+i\sqrt{a}\,(d g+e\,(f+2 g x))\right)\right)\Big/\right.$$

$$\left.\left.\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)^{3/2}\sqrt{\sqrt{c}\,f+i\sqrt{a}\,g}\left(-i\sqrt{a}+\sqrt{c}\,x\right)\right)\right]\right)\Big/$$

$$\left(\sqrt{\sqrt{c}\,d+i\sqrt{a}\,e}\sqrt{\sqrt{c}\,f+i\sqrt{a}\,g}\right)-1\Big/\left(\sqrt{\sqrt{c}\,f-i\sqrt{a}\,g}\right)\sqrt{c}\sqrt{\sqrt{c}\,d-i\sqrt{a}\,e}$$

$$\text{Log}\left[-\left(\sqrt{a}\sqrt{c}\left(2 i\sqrt{\sqrt{c}\,d-i\sqrt{a}\,e}\sqrt{\sqrt{c}\,f-i\sqrt{a}\,g}\sqrt{d+e x}\sqrt{f+g x}+i\sqrt{c}\,(2 d f+e f x+d g x)+\sqrt{a}\,(d g+e\,(f+2 g x))\right)\right)\Big/\right.$$

$$\left.\left.\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)^{3/2}\sqrt{\sqrt{c}\,f-i\sqrt{a}\,g}\left(i\sqrt{a}+\sqrt{c}\,x\right)\right)\right]\right)$$

- **Problem 612: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{d+e x}\,\sqrt{f+g x}\,\left(a+c x^2\right)}\,dx$$

Optimal (type 3, 230 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+e x}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+g x}}\right]}{\sqrt{-a}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\sqrt{d+e x}}{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{f+g x}}\right]}{\sqrt{-a}\,\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}}$$

Result (type 3, 451 leaves):

$$-\frac{1}{2\sqrt{a}}\,i\left(\frac{\text{Log}\left[\frac{-a\,(d g+e\,(f+2 g x))-i\sqrt{a}\left(2\sqrt{\sqrt{c}\,d-i\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f-i\sqrt{a}\,g}\,\sqrt{d+e x}\,\sqrt{f+g x}+\sqrt{c}\,(2 d f+e f x+d g x)\right)}{\sqrt{\sqrt{c}\,d-i\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f-i\sqrt{a}\,g}\,\left(i\sqrt{a}+\sqrt{c}\,x\right)}\right]}{\sqrt{\sqrt{c}\,d-i\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f-i\sqrt{a}\,g}}-\frac{\text{Log}\left[\frac{-a\,(d g+e\,(f+2 g x))+i\sqrt{a}\left(2\sqrt{\sqrt{c}\,d+i\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f+i\sqrt{a}\,g}\,\sqrt{d+e x}\,\sqrt{f+g x}+\sqrt{c}\,(2 d f+e f x+d g x)\right)}{\sqrt{\sqrt{c}\,d+i\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f+i\sqrt{a}\,g}\,\left(-i\sqrt{a}+\sqrt{c}\,x\right)}\right]}{\sqrt{\sqrt{c}\,d+i\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f+i\sqrt{a}\,g}}\right)$$

- **Problem 613: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(d+e x)^{3/2}\,\sqrt{f+g x}\,\left(a+c x^2\right)}\,dx$$

Optimal (type 3, 354 leaves, 8 steps):

$$-\frac{e\sqrt{f+g x}}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(e f-d g)\sqrt{d+e x}}+\frac{e\sqrt{f+g x}}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(e f-d g)\sqrt{d+e x}}+$$

$$\frac{\sqrt{c}\,\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+e x}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+g x}}\right]}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)^{3/2}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\sqrt{c}\,\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\sqrt{d+e x}}{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{f+g x}}\right]}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)^{3/2}\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}}$$

Result (type 3, 555 leaves):

$$\begin{aligned}
&\frac{1}{2\left(c d^2+a e^2\right)}\left(-\frac{4 e^2\sqrt{f+g x}}{(e f-d g)\sqrt{d+e x}}+\left(\left(-i c d+\sqrt{a}\,\sqrt{c}\,e\right)\text{Log}\Big[\right.\right.\\
&\quad -\left(i\sqrt{a}\,\sqrt{\sqrt{c}\,d-i\sqrt{a}\,e}\,\left(2\sqrt{\sqrt{c}\,d-i\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f-i\sqrt{a}\,g}\,\sqrt{d+e x}\,\sqrt{f+g x}+\sqrt{c}\,(2 d f+e f x+d g x)-i\sqrt{a}\,(e f+d g+2 e g x)\right)\right)\Big/\\
&\quad\quad \left.\left(\sqrt{c}\,\sqrt{\sqrt{c}\,f-i\sqrt{a}\,g}\,\left(i\sqrt{a}+\sqrt{c}\,x\right)\right)\Big]\right)\Big/\left(\sqrt{a}\,\sqrt{\sqrt{c}\,d-i\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f-i\sqrt{a}\,g}\right)+\left(\left(i c d+\sqrt{a}\,\sqrt{c}\,e\right)\text{Log}\Big[\right.\\
&\quad\left(i\sqrt{a}\,\sqrt{\sqrt{c}\,d+i\sqrt{a}\,e}\,\left(2\sqrt{\sqrt{c}\,d+i\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f+i\sqrt{a}\,g}\,\sqrt{d+e x}\,\sqrt{f+g x}+\sqrt{c}\,(2 d f+e f x+d g x)+i\sqrt{a}\,(e f+d g+2 e g x)\right)\right)\Big/\\
&\quad\quad \left.\left.\left(\sqrt{c}\,\sqrt{\sqrt{c}\,f+i\sqrt{a}\,g}\,\left(-i\sqrt{a}+\sqrt{c}\,x\right)\right)\Big]\right)\Big/\left(\sqrt{a}\,\sqrt{\sqrt{c}\,d+i\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f+i\sqrt{a}\,g}\right)\right)
\end{aligned}$$

- **Problem 614: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(d+e x)^{3/2}}{(f+g x)^{3/2}\left(a+c x^2\right)}\,dx$$

Optimal (type 3, 625 leaves, 21 steps):

$$\frac{2\,(e f-d g)\,\sqrt{d+e x}}{\left(c f^2+a g^2\right)\sqrt{f+g x}}-\frac{2\,\sqrt{e}\,(e f-d g)\,\text{ArcTanh}\left[\frac{\sqrt{g}\,\sqrt{d+e x}}{\sqrt{e}\,\sqrt{f+g x}}\right]}{\sqrt{g}\,\left(c f^2+a g^2\right)}-$$

$$\frac{\sqrt{e}\,\left(c d f+a e g-\sqrt{-a}\,\sqrt{c}\,(e f-d g)\right)\text{ArcTanh}\left[\frac{\sqrt{g}\,\sqrt{d+e x}}{\sqrt{e}\,\sqrt{f+g x}}\right]}{\sqrt{-a}\,\sqrt{c}\,\sqrt{g}\,\left(c f^2+a g^2\right)}+\frac{\sqrt{e}\,\left(c d f+a e g+\sqrt{-a}\,\sqrt{c}\,(e f-d g)\right)\text{ArcTanh}\left[\frac{\sqrt{g}\,\sqrt{d+e x}}{\sqrt{e}\,\sqrt{f+g x}}\right]}{\sqrt{-a}\,\sqrt{c}\,\sqrt{g}\,\left(c f^2+a g^2\right)}+$$

$$\frac{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\left(c d f+a e g-\sqrt{-a}\,\sqrt{c}\,(e f-d g)\right)\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+e x}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+g x}}\right]}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\left(c f^2+a g^2\right)}-$$

$$\frac{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\left(c d f+a e g+\sqrt{-a}\,\sqrt{c}\,(e f-d g)\right)\text{ArcTanh}\left[\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\sqrt{d+e x}}{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{f+g x}}\right]}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\left(c f^2+a g^2\right)}$$

Result (type 3, 558 leaves) :

$$\frac{1}{2\left(c f^2+a g^2\right)}\left(\frac{4\,(e f-d g)\,\sqrt{d+e x}}{\sqrt{f+g x}}+\right.$$
$$\left(\left(\sqrt{c}\,d-i\,\sqrt{a}\,e\right)^{3/2}\left(-i\,\sqrt{c}\,f+\sqrt{a}\,g\right)\text{Log}\left[-\left(i\,\sqrt{a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,f-i\,\sqrt{a}\,g}\,\left(2\,\sqrt{\sqrt{c}\,d-i\,\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f-i\,\sqrt{a}\,g}\,\sqrt{d+e x}\,\sqrt{f+g x}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\sqrt{c}\,(2 d f+e f x+d g x)-i\,\sqrt{a}\,(e f+d g+2 e g x)\right)\right)\Big/\left(\left(\sqrt{c}\,d-i\,\sqrt{a}\,e\right)^{5/2}\left(i\,\sqrt{a}+\sqrt{c}\,x\right)\right)\right]\right)\Big/\left(\sqrt{a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,f-i\,\sqrt{a}\,g}\right)+$$
$$\left(\left(\sqrt{c}\,d+i\,\sqrt{a}\,e\right)^{3/2}\left(i\,\sqrt{c}\,f+\sqrt{a}\,g\right)\text{Log}\left[\left(i\,\sqrt{a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,f+i\,\sqrt{a}\,g}\,\left(2\,\sqrt{\sqrt{c}\,d+i\,\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f+i\,\sqrt{a}\,g}\,\sqrt{d+e x}\,\sqrt{f+g x}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\left.\sqrt{c}\,(2 d f+e f x+d g x)+i\,\sqrt{a}\,(e f+d g+2 e g x)\right)\right)\Big/\left(\left(\sqrt{c}\,d+i\,\sqrt{a}\,e\right)^{5/2}\left(-i\,\sqrt{a}+\sqrt{c}\,x\right)\right)\right]\right)\Big/\left(\sqrt{a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,f+i\,\sqrt{a}\,g}\right)\right)$$

- **Problem 615: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{d+e x}}{(f+g x)^{3/2}\left(a+c x^2\right)}\,dx$$

Optimal (type 3, 351 leaves, 8 steps) :

$$-\frac{2 g \sqrt{d+e x}}{\left(c f^2+a g^2\right) \sqrt{f+g x}}+\frac{\left(c d f+a e g-\sqrt{-a} \sqrt{c} (e f-d g)\right) \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c} f-\sqrt{-a} g} \sqrt{d+e x}}{\sqrt{\sqrt{c} d-\sqrt{-a} e} \sqrt{f+g x}}\right]}{\sqrt{-a} \sqrt{\sqrt{c} d-\sqrt{-a} e} \sqrt{\sqrt{c} f-\sqrt{-a} g} \left(c f^2+a g^2\right)}-$$

$$\frac{\left(c d f+a e g+\sqrt{-a} \sqrt{c} (e f-d g)\right) \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c} f+\sqrt{-a} g} \sqrt{d+e x}}{\sqrt{\sqrt{c} d+\sqrt{-a} e} \sqrt{f+g x}}\right]}{\sqrt{-a} \sqrt{\sqrt{c} d+\sqrt{-a} e} \sqrt{\sqrt{c} f+\sqrt{-a} g} \left(c f^2+a g^2\right)}$$

Result (type 3, 531 leaves) :

$$\frac{1}{2 \left(c f^2+a g^2\right)}\left(-\frac{4 g \sqrt{d+e x}}{\sqrt{f+g x}}+1\Big/\left(\left(\sqrt{a} \sqrt{\sqrt{c} f+i \sqrt{a} g}\right) \sqrt{\sqrt{c} d+i \sqrt{a} e} \left(i \sqrt{c} f+\sqrt{a} g\right)\right.\right.$$
$$\text{Log}\left[\left(i \sqrt{a} \sqrt{\sqrt{c} f+i \sqrt{a} g} \left(2 \sqrt{\sqrt{c} d+i \sqrt{a} e} \sqrt{\sqrt{c} f+i \sqrt{a} g} \sqrt{d+e x} \sqrt{f+g x}+\sqrt{c} (2 d f+e f x+d g x)+i \sqrt{a} (e f+d g+2 e g x)\right)\right)\Big/\right.$$
$$\left.\left.\left(\left(\sqrt{c} d+i \sqrt{a} e\right)^{3/2} \left(-i \sqrt{a}+\sqrt{c} x\right)\right)\right]+1\Big/\left(\sqrt{a} \sqrt{\sqrt{c} f-i \sqrt{a} g}\right) \sqrt{\sqrt{c} d-i \sqrt{a} e} \left(-i \sqrt{c} f+\sqrt{a} g\right) \text{Log}\Big[$$
$$-\left(i \sqrt{a} \sqrt{\sqrt{c} f-i \sqrt{a} g} \left(2 \sqrt{\sqrt{c} d-i \sqrt{a} e} \sqrt{\sqrt{c} f-i \sqrt{a} g} \sqrt{d+e x} \sqrt{f+g x}+\sqrt{c} (2 d f+e f x+d g x)-i \sqrt{a} (d g+e (f+2 g x))\right)\right)\Big/$$
$$\left.\left.\left(\left(\sqrt{c} d-i \sqrt{a} e\right)^{3/2} \left(i \sqrt{a}+\sqrt{c} x\right)\right)\right]\right)$$

- **Problem 616: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{d+e x} (f+g x)^{3/2} \left(a+c x^2\right)} \, dx$$

Optimal (type 3, 354 leaves, 8 steps) :

$$\frac{g \sqrt{d+e x}}{\sqrt{-a} \left(\sqrt{c} f-\sqrt{-a} g\right) (e f-d g) \sqrt{f+g x}}-\frac{g \sqrt{d+e x}}{\sqrt{-a} \left(\sqrt{c} f+\sqrt{-a} g\right) (e f-d g) \sqrt{f+g x}}+$$

$$\frac{\sqrt{c} \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c} f-\sqrt{-a} g} \sqrt{d+e x}}{\sqrt{\sqrt{c} d-\sqrt{-a} e} \sqrt{f+g x}}\right]}{\sqrt{-a} \sqrt{\sqrt{c} d-\sqrt{-a} e} \left(\sqrt{c} f-\sqrt{-a} g\right)^{3/2}}-\frac{\sqrt{c} \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c} f+\sqrt{-a} g} \sqrt{d+e x}}{\sqrt{\sqrt{c} d+\sqrt{-a} e} \sqrt{f+g x}}\right]}{\sqrt{-a} \sqrt{\sqrt{c} d+\sqrt{-a} e} \left(\sqrt{c} f+\sqrt{-a} g\right)^{3/2}}$$

Result (type 3, 555 leaves) :

$$\frac{1}{2\left(c f^2+a g^2\right)}\left(\frac{4 g^2 \sqrt{d+e x}}{(e f-d g) \sqrt{f+g x}}+\left(\left(i c f+\sqrt{a} \sqrt{c} g\right) \text{Log}\left[\left(i \sqrt{a} \sqrt{\sqrt{c} f+i \sqrt{a} g} \left(2 \sqrt{\sqrt{c} d+i \sqrt{a} e} \sqrt{\sqrt{c} f+i \sqrt{a} g} \sqrt{d+e x} \sqrt{f+g x}+\sqrt{c} (2 d f+e f x+d g x)+i \sqrt{a} (e f+d g+2 e g x)\right)\right)\Big/\left(\sqrt{c} \sqrt{\sqrt{c} d+i \sqrt{a} e} \left(-i \sqrt{a}+\sqrt{c} x\right)\right)\right]\right)\Big/\left(\sqrt{a} \sqrt{\sqrt{c} d+i \sqrt{a} e} \sqrt{\sqrt{c} f+i \sqrt{a} g}\right)+\left(\left(-i c f+\sqrt{a} \sqrt{c} g\right) \text{Log}\left[-\left(i \sqrt{a} \sqrt{\sqrt{c} f-i \sqrt{a} g} \left(2 \sqrt{\sqrt{c} d-i \sqrt{a} e} \sqrt{\sqrt{c} f-i \sqrt{a} g} \sqrt{d+e x} \sqrt{f+g x}+\sqrt{c} (2 d f+e f x+d g x)-i \sqrt{a} (d g+e (f+2 g x))\right)\right)\Big/\left(\sqrt{c} \sqrt{\sqrt{c} d-i \sqrt{a} e} \left(i \sqrt{a}+\sqrt{c} x\right)\right)\right]\right)\Big/\left(\sqrt{a} \sqrt{\sqrt{c} d-i \sqrt{a} e} \sqrt{\sqrt{c} f-i \sqrt{a} g}\right)\right)$$

■ Problem 617: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(d+e x)^{3/2} (f+g x)^{3/2} \left(a+c x^2\right)} \, dx$$

Optimal (type 3, 549 leaves, 12 steps):

$$-\frac{e}{\sqrt{-a} \left(\sqrt{c} d-\sqrt{-a} e\right) (e f-d g) \sqrt{d+e x} \sqrt{f+g x}}+\frac{e}{\sqrt{-a} \left(\sqrt{c} d+\sqrt{-a} e\right) (e f-d g) \sqrt{d+e x} \sqrt{f+g x}}+$$

$$\frac{g \left(2 \sqrt{-a} e g-\sqrt{c} (e f+d g)\right) \sqrt{d+e x}}{\sqrt{-a} \left(\sqrt{c} d-\sqrt{-a} e\right) \left(\sqrt{c} f-\sqrt{-a} g\right) (e f-d g)^2 \sqrt{f+g x}}+\frac{g \left(2 \sqrt{-a} e g+\sqrt{c} (e f+d g)\right) \sqrt{d+e x}}{\sqrt{-a} \left(\sqrt{c} d+\sqrt{-a} e\right) \left(\sqrt{c} f+\sqrt{-a} g\right) (e f-d g)^2 \sqrt{f+g x}}+$$

$$\frac{c \, \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c} f-\sqrt{-a} g} \sqrt{d+e x}}{\sqrt{\sqrt{c} d-\sqrt{-a} e} \sqrt{f+g x}}\right]}{\sqrt{-a} \left(\sqrt{c} d-\sqrt{-a} e\right)^{3/2} \left(\sqrt{c} f-\sqrt{-a} g\right)^{3/2}}-\frac{c \, \text{ArcTanh}\left[\frac{\sqrt{\sqrt{c} f+\sqrt{-a} g} \sqrt{d+e x}}{\sqrt{\sqrt{c} d+\sqrt{-a} e} \sqrt{f+g x}}\right]}{\sqrt{-a} \left(\sqrt{c} d+\sqrt{-a} e\right)^{3/2} \left(\sqrt{c} f+\sqrt{-a} g\right)^{3/2}}$$

Result (type 3, 548 leaves):

$$\frac{1}{2}\left(\frac{4\left(-\frac{g^3 (d+e x)}{c f^2+a g^2}-\frac{e^3 (f+g x)}{c d^2+a e^2}\right)}{(e f-d g)^2 \sqrt{d+e x} \sqrt{f+g x}}-\left(i c \operatorname{Log}\left[-1\Big/\left(i \sqrt{a} c+c^{3/2} x\right) i \sqrt{a} \sqrt{\sqrt{c} d-i \sqrt{a} e} \sqrt{\sqrt{c} f-i \sqrt{a} g} \left(2 \sqrt{\sqrt{c} d-i \sqrt{a} e} \sqrt{\sqrt{c} f-i \sqrt{a} g} \sqrt{d+e x} \sqrt{f+g x}+\sqrt{c} (2 d f+e f x+d g x)-i \sqrt{a} (e f+d g+2 e g x)\right)\right]\right)\Big/\left(\sqrt{a} \left(\sqrt{c} d-i \sqrt{a} e\right)^{3/2} \left(\sqrt{c} f-i \sqrt{a} g\right)^{3/2}\right)+\left(i c \operatorname{Log}\left[1\Big/\left(-i \sqrt{a} c+c^{3/2} x\right) i \sqrt{a} \sqrt{\sqrt{c} d+i \sqrt{a} e} \sqrt{\sqrt{c} f+i \sqrt{a} g} \left(2 \sqrt{\sqrt{c} d+i \sqrt{a} e} \sqrt{\sqrt{c} f+i \sqrt{a} g} \sqrt{d+e x} \sqrt{f+g x}+\sqrt{c} (2 d f+e f x+d g x)+i \sqrt{a} (e f+d g+2 e g x)\right)\right]\right)\Big/\left(\sqrt{a} \left(\sqrt{c} d+i \sqrt{a} e\right)^{3/2} \left(\sqrt{c} f+i \sqrt{a} g\right)^{3/2}\right)\right)$$

Problem 620: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(1-a^2 x^2\right)^{3/2}}{(1-a x)^2 (c+d x)} \, dx$$

Optimal (type 3, 107 leaves, 6 steps):

$$-\frac{\sqrt{1-a^2 x^2}}{d}-\frac{(a c-2 d) \operatorname{ArcSin}[a x]}{d^2}+\frac{(a c-d)^2 \operatorname{ArcTan}\left[\frac{d+a^2 c x}{\sqrt{a^2 c^2-d^2} \sqrt{1-a^2 x^2}}\right]}{d^2 \sqrt{a^2 c^2-d^2}}$$

Result (type 3, 148 leaves):

$$-\frac{d \sqrt{1-a^2 x^2}+(a c-2 d) \operatorname{ArcSin}[a x]+\frac{i (-a c+d)^2 \operatorname{Log}\left[\frac{2 d^3 \left(i d+i a^2 c x+\sqrt{a^2 c^2-d^2} \sqrt{1-a^2 x^2}\right)}{(-a c+d)^2 \sqrt{a^2 c^2-d^2} (c+d x)}\right]}{\sqrt{a^2 c^2-d^2}}}{d^2}$$

Problem 621: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(1+a x)^2}{(c+d x) \sqrt{1-a^2 x^2}} \, dx$$

Optimal (type 3, 107 leaves, 5 steps):

$$-\frac{\sqrt{1-a^2 x^2}}{d}-\frac{(a c-2 d) \operatorname{ArcSin}[a x]}{d^2}+\frac{(a c-d)^2 \operatorname{ArcTan}\left[\frac{d+a^2 c x}{\sqrt{a^2 c^2-d^2} \sqrt{1-a^2 x^2}}\right]}{d^2 \sqrt{a^2 c^2-d^2}}$$

Result (type 3, 148 leaves):

$$-\frac{d\sqrt{1-a^2x^2}+(a c-2 d)\,\text{ArcSin}[a x]+\frac{i\,(-a c+d)^2\,\text{Log}\left[\frac{2 d^3\left(i\,d+i\,a^2 c x+\sqrt{a^2c^2-d^2}\,\sqrt{1-a^2x^2}\right)}{(-a c+d)^2\sqrt{a^2c^2-d^2}\,(c+d x)}\right]}{\sqrt{a^2c^2-d^2}}}{d^2}$$

- **Problem 622: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (d+e x)^3\sqrt{f+g x}\,\sqrt{a+c x^2}\,dx$$

Optimal (type 4, 851 leaves, 10 steps):

$$-\frac{1}{3465 c^2 e g^4}2\left(150 a^2 e^4 g^4-6 a c e^2 g^2\left(2 e^2 f^2-33 d e f g+165 d^2 g^2\right)+c^2\left(187 e^4 f^4-732 d e^3 f^3 g+1098 d^2 e^2 f^2 g^2-798 d^3 e f g^3+315 d^4 g^4\right)\right)$$
$$\sqrt{f+g x}\,\sqrt{a+c x^2}+\frac{2 (d+e x)^4\sqrt{f+g x}\,\sqrt{a+c x^2}}{11 e}-\frac{1}{3465 c g^4}$$
$$2\left(2 a e^2 g^2(74 e f-231 d g)-c\left(233 e^3 f^3-843 d e^2 f^2 g+1107 d^2 e f g^2-567 d^3 g^3\right)\right)(f+g x)^{3/2}\sqrt{a+c x^2}+$$
$$\frac{2 e\left(18 a e^2 g^2-c\left(29 e^2 f^2-96 d e f g+81 d^2 g^2\right)\right)(f+g x)^{5/2}\sqrt{a+c x^2}}{693 c g^4}+\frac{2 e^2(e f-3 d g)(f+g x)^{7/2}\sqrt{a+c x^2}}{99 g^4}+$$
$$\left(4\sqrt{-a}\left(3 a^2 e^2 g^4(26 e f+231 d g)-c^2 f^2\left(64 e^3 f^3-264 d e^2 f^2 g+396 d^2 e f g^2-231 d^3 g^3\right)-9 a c g^2\left(6 e^3 f^3-33 d e^2 f^2 g+88 d^2 e f g^2+77 d^3 g^3\right)\right)\right.$$
$$\left.\sqrt{f+g x}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]\right)\Bigg/\left(3465 c^{3/2} g^5\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+c x^2}\right)-$$
$$\left(4\sqrt{-a}\left(c f^2+a g^2\right)\left(75 a^2 e^3 g^4-3 a c e g^2\left(2 e^2 f^2-33 d e f g+165 d^2 g^2\right)-c^2 f\left(64 e^3 f^3-264 d e^2 f^2 g+396 d^2 e f g^2-231 d^3 g^3\right)\right)\right.$$
$$\left.\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]\right)\Bigg/\left(3465 c^{5/2} g^5\sqrt{f+g x}\,\sqrt{a+c x^2}\right)$$

Result (type 4, 6884 leaves) :

$$\sqrt{f+g x}\ \sqrt{a+c x^2}\left(\frac{1}{3465 c^2 g^4}2\left(-64 c^2 e^3 f^4+264 c^2 d e^2 f^3 g-396 c^2 d^2 e f^2 g^2-46 a c e^3 f^2 g^2+231 c^2 d^3 f g^3+264 a c d e^2 f g^3+990 a c d^2 e g^4-150 a^2 e^3 g^4\right)+\frac{2\left(48 c e^3 f^3-198 c d e^2 f^2 g+297 c d^2 e f g^2+32 a e^3 f g^2+693 c d^3 g^3+462 a d e^2 g^3\right) x}{3465 c g^3}+\frac{2 e\left(-8 c e^2 f^2+33 c d e f g+297 c d^2 g^2+18 a e^2 g^2\right) x^2}{693 c g^2}+\frac{2 e^2 (e f+33 d g) x^3}{99 g}+\frac{2 e^3 x^4}{11}\right)-$$

$$\frac{1}{3465 c^2 g^6} 4\left(\frac{1}{\sqrt{a+\frac{c (f+g x)^2\left(-1+\frac{f}{f+g x}\right)^2}{g^2}}}\left(-64 c^2 e^3 f^5+264 c^2 d e^2 f^4 g-396 c^2 d^2 e f^3 g^2-54 a c e^3 f^3 g^2+231 c^2 d^3 f^2 g^3+297 a c d e^2 f^2 g^3-\right.\right.$$

$$\left.792 a c d^2 e f g^4+78 a^2 e^3 f g^4-693 a c d^3 g^5+693 a^2 d e^2 g^5\right)(f+g x)^{3/2}\left(c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}\right)+$$

$$\frac{1}{\sqrt{a+\frac{c (f+g x)^2\left(-1+\frac{f}{f+g x}\right)^2}{g^2}}}\left(c f^2+a g^2\right)(f+g x)\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}$$

$$\left(\left(64 i c^2 e^3 f^5\left(c f+i \sqrt{a} \sqrt{c} g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right)(f+g x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]-\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right)\Bigg)\Bigg)\Big/$$

$$\left(\left(c f^2+a g^2\right) \sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i} \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-\left(264 \mathbb{i} c^2 d e^2 f^4 g \left(c f+\mathbb{i} \sqrt{a} \sqrt{c} g\right)\right.$$

$$\sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathbb{i} \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathbb{i} \sqrt{a} \sqrt{c} g\right) (f+g x)}} \left(\text{EllipticE}\left[\mathbb{i} \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i} \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-\mathbb{i} \sqrt{a} \sqrt{c} g}{c f+\mathbb{i} \sqrt{a} \sqrt{c} g}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i} \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i} \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-\mathbb{i} \sqrt{a} \sqrt{c} g}{c f+\mathbb{i} \sqrt{a} \sqrt{c} g}\right]\right)\right) \Big/ \left(\left(c f^2+a g^2\right) \sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i} \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\left(396 \mathbb{i} c^2 d^2 e f^3 g^2 \left(c f+\mathbb{i} \sqrt{a} \sqrt{c} g\right) \sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathbb{i} \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathbb{i} \sqrt{a} \sqrt{c} g\right) (f+g x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[\mathbb{i} \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i} \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-\mathbb{i} \sqrt{a} \sqrt{c} g}{c f+\mathbb{i} \sqrt{a} \sqrt{c} g}\right]-\text{EllipticF}\left[\mathbb{i} \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i} \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-\mathbb{i} \sqrt{a} \sqrt{c} g}{c f+\mathbb{i} \sqrt{a} \sqrt{c} g}\right]\right)\right) \Big/$$

$$\left(\left(c f^2+a g^2\right) \sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i} \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\left(54 \mathbb{i} a c e^3 f^3 g^2 \left(c f+\mathbb{i} \sqrt{a} \sqrt{c} g\right) \sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathbb{i} \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathbb{i} \sqrt{a} \sqrt{c} g\right) (f+g x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\Bigg)\Bigg/$$

$$\left(\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-\Bigg(231\,i\,c^2 d^3 f^2 g^3\left(c f+i\sqrt{a}\sqrt{c}\,g\right)$$

$$\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\Bigg)\Bigg)\Bigg/\left(\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\Bigg(297\,i\,a c d e^2 f^2 g^3\left(c f+i\sqrt{a}\sqrt{c}\,g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\Bigg)\Bigg/$$

$$\left(\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+\Bigg(792\,i\,a c d^2 e f g^4\left(c f+i\sqrt{a}\sqrt{c}\,g\right)$$

$$\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right)\right) \Big/ \left(\left(c f^2+a g^2\right) \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right) -$$

$$\left(78 i a^2 e^3 f g^4 \left(c f+i \sqrt{a} \sqrt{c} g\right) \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right)\right) \Big/$$

$$\left(\left(c f^2+a g^2\right) \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right) + \left(693 i a c d^3 g^5 \left(c f+i \sqrt{a} \sqrt{c} g\right)\right.$$

$$\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right)\right) \Big/ \left(\left(c f^2+a g^2\right) \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right) -$$

$$\left(693\,i\,a^2\,d\,e^2\,g^5\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]\right)\right)\Big/$$

$$\left(\left(c\,f^2+a\,g^2\right)\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)+\left(64\,i\,c^2\,e^3\,f^4\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\right.$$

$$\left.\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-\left(264\,i\,c^2\,d\,e^2\,f^3\,g\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\right.$$

$$\left.\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)+\left(396\,i\,c^2\,d^2\,e\,f^2\,g^2\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\right.$$

$$\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+\Bigg(6\,i\,a\,c\,e^3 f^2 g^2\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}$$

$$\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-\Bigg(231\,i\,c^2 d^3 f g^3\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}$$

$$\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-\Bigg(99\,i\,a\,c\,d\,e^2 f g^3\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}$$

$$\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+\left(495 i a c d^2 e g^4 \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}}\right.$$

$$\left.\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-\left(75 i a^2 e^3 g^4 \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right) \Big/ \left(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\Bigg)$$

- **Problem 623: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (d+e x)^2 \sqrt{f+g x} \sqrt{a+c x^2}\, dx$$

Optimal (type 4, 635 leaves, 9 steps) :

$$-\frac{2\left(6 a e^2 g^2 (e f-10 d g)-c\left(19 e^3 f^3-57 d e^2 f^2 g+63 d^2 e f g^2-35 d^3 g^3\right)\right)\sqrt{f+g x}\,\sqrt{a+c x^2}}{315 c e g^3}+\frac{2 (d+e x)^3\sqrt{f+g x}\,\sqrt{a+c x^2}}{9 e}+$$

$$\frac{4\left(7 a e^2 g^2-c\left(8 e^2 f^2-24 d e f g+21 d^2 g^2\right)\right)(f+g x)^{3/2}\sqrt{a+c x^2}}{315 c g^3}+\frac{2 e (e f-3 d g)(f+g x)^{5/2}\sqrt{a+c x^2}}{63 g^3}+$$

$$\left(4\sqrt{-a}\left(21 a^2 e^2 g^4+3 a c g^2\left(3 e^2 f^2-16 d e f g-21 d^2 g^2\right)+c^2 f^2\left(8 e^2 f^2-24 d e f g+21 d^2 g^2\right)\right)\sqrt{f+g x}\right.$$

$$\left.\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]\right)\Bigg/\left(315 c^{3/2} g^4\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+c x^2}\right)-$$

$$\left(4\sqrt{-a}\left(c f^2+a g^2\right)\left(3 a e g^2 (e f-10 d g)+c f\left(8 e^2 f^2-24 d e f g+21 d^2 g^2\right)\right)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]\right)\Bigg/\left(315 c^{3/2} g^4\sqrt{f+g x}\,\sqrt{a+c x^2}\right)$$

Result (type 4, 4647 leaves) :

$$\sqrt{f+g x}\,\sqrt{a+c x^2}\left(\frac{2\left(8 c e^2 f^3-24 c d e f^2 g+21 c d^2 f g^2+8 a e^2 f g^2+60 a d e g^3\right)}{315 c g^3}+\right.$$

$$\left.\frac{2\left(-6 c e^2 f^2+18 c d e f g+63 c d^2 g^2+14 a e^2 g^2\right) x}{315 c g^2}+\frac{2 e (e f+18 d g) x^2}{63 g}+\frac{2 e^2 x^3}{9}\right)+$$

$$\frac{1}{315\,c\,g^5}\,2\Bigg(-\Bigg(2\left(8 c^2 e^2 f^4-24 c^2 d e f^3 g+21 c^2 d^2 f^2 g^2+9 a c e^2 f^2 g^2-48 a c d e f g^3-63 a c d^2 g^4+21 a^2 e^2 g^4\right)$$

$$(f+g x)^{3/2}\left(c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}\right)\Bigg)\Bigg/\left(c\sqrt{a+\frac{c\,(f+g x)^2\left(-1+\frac{f}{f+g x}\right)^2}{g^2}}\right)+$$

$$\frac{1}{c\sqrt{a+\frac{c\,(f+g x)^2\left(-1+\frac{f}{f+g x}\right)^2}{g^2}}}\,2\left(c f^2+a g^2\right)(f+g x)\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}$$

$$\Bigg(\Bigg(8\,i\,c^2 e^2 f^4\left(c f+i\sqrt{a}\sqrt{c}\,g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\Bigg)\Bigg/$$

$$\left(\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-\Bigg(24\,i\,c^2 d e f^3 g\left(c f+i\sqrt{a}\sqrt{c}\,g\right)$$

$$\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\Bigg)\Bigg)\Bigg/\left(\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\Bigg(21\,i\,c^2 d^2 f^2 g^2\left(c f+i\sqrt{a}\sqrt{c}\,g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\Bigg)\Bigg/$$

$$\left(\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+\Bigg(9\,i\,a\,c\,e^2 f^2 g^2\left(c f+i\sqrt{a}\sqrt{c}\,g\right)$$

$$\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\Bigg)\Bigg)\Bigg/\left(\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\Bigg(48\,i\,a\,c\,d\,e\,f\,g^3\left(c f+i\sqrt{a}\sqrt{c}\,g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\Bigg)\Bigg/$$

$$\left(\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-\left(63\,i\,a c d^2 g^4\left(c f+i\sqrt{a}\sqrt{c}\,g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\right)\Bigg/\left(\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\left(21\,i\,a^2 e^2 g^4\left(c f+i\sqrt{a}\sqrt{c}\,g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\Bigg)\Bigg/$$

$$\left(\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+\left(8\,i\,c^2 e^2 f^3\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\Bigg/ \left(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}\right.$$

$$\left.\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right) - \left(24 i c^2 d e f^2 g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right)\Bigg/ \left(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right) +$$

$$\left(21 i c^2 d^2 f g^2 \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right)\Bigg/ \left(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right) +$$

$$\left(3 i a c e^2 f g^2 \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right)\Bigg/ \left(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right) -$$

$$\left(30\, i\, a\, c\, d\, e\, g^3 \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}}\right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c}\, g}{c f+i \sqrt{a} \sqrt{c}\, g}\right]\right) \Big/ \left(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}} \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\right)$$

■ Problem 624: Result unnecessarily involves imaginary or complex numbers.

$$\int (d+e x) \sqrt{f+g x} \sqrt{a+c x^2}\, dx$$

Optimal (type 4, 434 leaves, 7 steps):

$$-\frac{2 \sqrt{f+g x} \left(5 a e g^2+c f (4 e f-7 d g)-3 c g (e f+7 d g) x\right) \sqrt{a+c x^2}}{105 c g^2}+\frac{2 e \sqrt{f+g x} \left(a+c x^2\right)^{3/2}}{7 c}-$$

$$\left(4 \sqrt{-a} \left(c f^2 (4 e f-7 d g)+a g^2 (8 e f+21 d g)\right) \sqrt{f+g x} \sqrt{1+\frac{c x^2}{a}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right], -\frac{2 a g}{\sqrt{-a} \sqrt{c}\, f-a g}\right]\right) \Big/$$

$$\left(105 \sqrt{c}\, g^3 \sqrt{\frac{\sqrt{c}\, (f+g x)}{\sqrt{c}\, f+\sqrt{-a}\, g}} \sqrt{a+c x^2}\right)+$$

$$\left(4 \sqrt{-a} \left(c f^2+a g^2\right) \left(5 a e g^2+c f (4 e f-7 d g)\right) \sqrt{\frac{\sqrt{c}\, (f+g x)}{\sqrt{c}\, f+\sqrt{-a}\, g}} \sqrt{1+\frac{c x^2}{a}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right], -\frac{2 a g}{\sqrt{-a} \sqrt{c}\, f-a g}\right]\right) \Big/$$

$$\left(105 c^{3/2} g^3 \sqrt{f+g x} \sqrt{a+c x^2}\right)$$

Result (type 4, 610 leaves):

$$\frac{1}{105\sqrt{a+c x^2}}\sqrt{f+g x}\left(\frac{2\left(a+c x^2\right)\left(10 a e g^2+7 c d g (f+3 g x)+c e\left(-4 f^2+3 f g x+15 g^2 x^2\right)\right)}{c g^2}+\right.$$

$$\frac{1}{c g^4\sqrt{-f-\frac{i\sqrt{a} g}{\sqrt{c}}}(f+g x)}4\left(g^2\sqrt{-f-\frac{i\sqrt{a} g}{\sqrt{c}}}\left(c f^2 (4 e f-7 d g)+a g^2 (8 e f+21 d g)\right)\left(a+c x^2\right)+\right.$$

$$i\sqrt{c}\left(\sqrt{c} f+i\sqrt{a} g\right)\left(c f^2 (-4 e f+7 d g)-a g^2 (8 e f+21 d g)\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+g x}}\sqrt{-\frac{\frac{i\sqrt{a} g}{\sqrt{c}}-g x}{f+g x}}(f+g x)^{3/2}\text{EllipticE}\Big[$$

$$i\,\text{ArcSinh}\Big[\frac{\sqrt{-f-\frac{i\sqrt{a} g}{\sqrt{c}}}}{\sqrt{f+g x}}\Big],\frac{\sqrt{c} f-i\sqrt{a} g}{\sqrt{c} f+i\sqrt{a} g}\Big]+\sqrt{a} g\left(i\sqrt{c} f-\sqrt{a} g\right)\left(5 i a e g^2+i c f (4 e f-7 d g)+3\sqrt{a}\sqrt{c} g (e f+7 d g)\right)$$

$$\left.\left.\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+g x}}\sqrt{-\frac{\frac{i\sqrt{a} g}{\sqrt{c}}-g x}{f+g x}}(f+g x)^{3/2}\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\frac{\sqrt{-f-\frac{i\sqrt{a} g}{\sqrt{c}}}}{\sqrt{f+g x}}\Big],\frac{\sqrt{c} f-i\sqrt{a} g}{\sqrt{c} f+i\sqrt{a} g}\Big]\right)\right)$$

- **Problem 625: Result unnecessarily involves imaginary or complex numbers.**

$$\int\sqrt{f+g x}\sqrt{a+c x^2}\,dx$$

Optimal (type 4, 362 leaves, 7 steps) :

$$-\frac{4 f \sqrt{f+g x} \sqrt{a+c x^2}}{15 g}+\frac{2 (f+g x)^{3/2} \sqrt{a+c x^2}}{5 g}+\frac{4 \sqrt{-a} \left(c f^2-3 a g^2\right) \sqrt{f+g x} \sqrt{1+\frac{c x^2}{a}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a} \sqrt{c} f-a g}\right]}{15 \sqrt{c} g^2 \sqrt{\frac{\sqrt{c} (f+g x)}{\sqrt{c} f+\sqrt{-a} g}} \sqrt{a+c x^2}}-$$

$$\frac{4 \sqrt{-a} f \left(c f^2+a g^2\right) \sqrt{\frac{\sqrt{c} (f+g x)}{\sqrt{c} f+\sqrt{-a} g}} \sqrt{1+\frac{c x^2}{a}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a} \sqrt{c} f-a g}\right]}{15 \sqrt{c} g^2 \sqrt{f+g x} \sqrt{a+c x^2}}$$

Result (type 4, 536 leaves) :

$$\frac{1}{15 \sqrt{a+c x^2}}\sqrt{f+g x}$$

$$\left(\frac{2 (f+3 g x) \left(a+c x^2\right)}{g}-\left(4 \left(g^2 \sqrt{-f-\frac{i \sqrt{a} g}{\sqrt{c}}} \left(-3 a^2 g^2+c^2 f^2 x^2+a c \left(f^2-3 g^2 x^2\right)\right)+\sqrt{c} \left(-i c^{3/2} f^3+\sqrt{a} c f^2 g+3 i a \sqrt{c} f g^2-3 a^{3/2} g^3\right)\right.\right.\right.$$

$$\sqrt{\frac{g \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{f+g x}} \sqrt{-\frac{\frac{i \sqrt{a} g}{\sqrt{c}}-g x}{f+g x}} (f+g x)^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i \sqrt{a} g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c} f-i \sqrt{a} g}{\sqrt{c} f+i \sqrt{a} g}\right]-$$

$$\sqrt{a} \sqrt{c} g \left(c f^2+4 i \sqrt{a} \sqrt{c} f g-3 a g^2\right) \sqrt{\frac{g \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{f+g x}} \sqrt{-\frac{\frac{i \sqrt{a} g}{\sqrt{c}}-g x}{f+g x}} (f+g x)^{3/2}$$

$$\left.\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i \sqrt{a} g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c} f-i \sqrt{a} g}{\sqrt{c} f+i \sqrt{a} g}\right]\right)\right)\Big/\left(c g^3 \sqrt{-f-\frac{i \sqrt{a} g}{\sqrt{c}}} (f+g x)\right)\right)$$

- **Problem 626: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{f+g x} \sqrt{a+c x^2}}{d+e x} \, dx$$

Optimal (type 4, 683 leaves, 14 steps):

$$\frac{2\sqrt{f+g x}\,\sqrt{a+c x^2}}{3 e}-\frac{2\sqrt{-a}\,\sqrt{c}\,(e f-3 d g)\sqrt{f+g x}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]}{3 e^2 g\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+c x^2}}+$$

$$\frac{2\sqrt{-a}\,\sqrt{c}\,f\,(e f-3 d g)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]}{3 e^2 g\sqrt{f+g x}\,\sqrt{a+c x^2}}-$$

$$\left(2\sqrt{-a}\,\left(2 a e^2 g-3 c d\,(e f-d g)\right)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]\right)\Big/$$

$$\left(3\sqrt{c}\,e^3\sqrt{f+g x}\,\sqrt{a+c x^2}\right)-\frac{2\left(c d^2+a e^2\right)(e f-d g)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticPi}\left[\frac{2 e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e},\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right]}{e^3\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)\sqrt{f+g x}\,\sqrt{a+c x^2}}$$

Result (type 4, 1216 leaves):

$$\frac{2\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}{3\,e}+$$

$$\frac{1}{3\,e^3\,g^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,\sqrt{a+\frac{c\,(f+g\,x)^2\left(-1+\frac{f}{f+g\,x}\right)^2}{g^2}}}(f+g\,x)^{3/2}\left(2\,c\,e^2\,f\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}-6\,c\,d\,e\,g\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}+\frac{2\,c\,e^2\,f^3\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{(f+g\,x)^2}-\right.$$

$$\frac{6\,c\,d\,e\,f^2\,g\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{(f+g\,x)^2}+\frac{2\,a\,e^2\,f\,g^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{(f+g\,x)^2}-\frac{6\,a\,d\,e\,g^3\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{(f+g\,x)^2}-\frac{4\,c\,e^2\,f^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{f+g\,x}+\frac{12\,c\,d\,e\,f\,g\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{f+g\,x}+$$

$$\frac{1}{\sqrt{f+g\,x}}2\sqrt{c}\,e\left(-i\sqrt{c}\,f+\sqrt{a}\,g\right)(e\,f-3\,d\,g)\sqrt{1-\frac{f}{f+g\,x}-\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}\,\sqrt{1-\frac{f}{f+g\,x}+\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]+\frac{1}{\sqrt{f+g\,x}}2\,e\left(3\sqrt{c}\,d-i\sqrt{a}\,e\right)g\left(-i\sqrt{c}\,f+\sqrt{a}\,g\right)\sqrt{1-\frac{f}{f+g\,x}-\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}$$

$$\sqrt{1-\frac{f}{f+g\,x}+\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]+\frac{1}{\sqrt{f+g\,x}}6\,i\,c\,d^2\,g^2\sqrt{1-\frac{f}{f+g\,x}-\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}$$

$$\sqrt{1-\frac{f}{f+g\,x}+\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}\,\text{EllipticPi}\left[\frac{\sqrt{c}\,(e\,f-d\,g)}{e\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)},i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]+\frac{1}{\sqrt{f+g\,x}}6\,i\,a\,e^2\,g^2$$

$$\left.\sqrt{1-\frac{f}{f+g\,x}-\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}\,\sqrt{1-\frac{f}{f+g\,x}+\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}\,\text{EllipticPi}\left[\frac{\sqrt{c}\,(e\,f-d\,g)}{e\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)},i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]\right)$$

- **Problem 627: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}{(d+e\,x)^2}\,dx$$

Optimal (type 4, 650 leaves, 14 steps):

$$-\frac{\sqrt{f+g x}\,\sqrt{a+c x^2}}{e\,(d+e x)}-\frac{3\sqrt{-a}\,\sqrt{c}\,\sqrt{f+g x}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]}{e^2\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+c x^2}}+$$

$$\frac{3\sqrt{-a}\,\sqrt{c}\,f\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]}{e^2\sqrt{f+g x}\,\sqrt{a+c x^2}}-$$

$$\frac{\sqrt{-a}\,\sqrt{c}\,(2 e f-3 d g)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]}{e^3\sqrt{f+g x}\,\sqrt{a+c x^2}}-$$

$$\left(\left(a e^2 g-c d\,(2 e f-3 d g)\right)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticPi}\left[\frac{2 e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e},\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right]\right)\Big/$$

$$\left(e^3\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)\sqrt{f+g x}\,\sqrt{a+c x^2}\right)$$

Result (type 4, 1446 leaves):

$$-\frac{\sqrt{f+g x}\,\sqrt{a+c x^2}}{e\,(d+e x)}-\frac{1}{e^3 g\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(e f-d g)\sqrt{a+\frac{c\,(f+g x)^2\left(-1+\frac{f}{f+g x}\right)^2}{g^2}}}$$

$$(f+g x)^{3/2}\left(-3 c e^2 f\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}+3 c d e g\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}-\frac{3 c e^2 f^3\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{(f+g x)^2}+\frac{3 c d e f^2 g\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{(f+g x)^2}-\frac{3 a e^2 f g^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{(f+g x)^2}+\right.$$

$$\frac{3\,a\,d\,e\,g^3\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{(f+g\,x)^2}+\frac{6\,c\,e^2\,f^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{f+g\,x}-\frac{6\,c\,d\,e\,f\,g\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{f+g\,x}+\frac{1}{\sqrt{f+g\,x}}3\,i\,\sqrt{c}\,e\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)(e\,f-d\,g)$$

$$\sqrt{1-\frac{f}{f+g\,x}-\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}\sqrt{1-\frac{f}{f+g\,x}+\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]+$$

$$\frac{1}{\sqrt{f+g\,x}}e\left(-i\sqrt{c}\,f+\sqrt{a}\,g\right)\left(i\sqrt{a}\,e\,g+\sqrt{c}\,(2\,e\,f-3\,d\,g)\right)\sqrt{1-\frac{f}{f+g\,x}-\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}\sqrt{1-\frac{f}{f+g\,x}+\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]+\frac{1}{\sqrt{f+g\,x}}2\,i\,c\,d\,e\,f\,g\sqrt{1-\frac{f}{f+g\,x}-\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}\sqrt{1-\frac{f}{f+g\,x}+\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}$$

$$\text{EllipticPi}\left[\frac{\sqrt{c}\,(e\,f-d\,g)}{e\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)},i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]-\frac{1}{\sqrt{f+g\,x}}3\,i\,c\,d^2\,g^2\sqrt{1-\frac{f}{f+g\,x}-\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}$$

$$\sqrt{1-\frac{f}{f+g\,x}+\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}\;\text{EllipticPi}\left[\frac{\sqrt{c}\,(e\,f-d\,g)}{e\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)},i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]-\frac{1}{\sqrt{f+g\,x}}i\,a\,e^2\,g^2$$

$$\left.\sqrt{1-\frac{f}{f+g\,x}-\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}\sqrt{1-\frac{f}{f+g\,x}+\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}\;\text{EllipticPi}\left[\frac{\sqrt{c}\,(e\,f-d\,g)}{e\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)},i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]\right)$$

- **Problem 628: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}{(d+e\,x)^3}\,dx$$

Optimal (type 4, 1205 leaves, 23 steps) :

$$-\frac{\sqrt{f+g x}\,\sqrt{a+c x^2}}{2 e (d+e x)^2}-\frac{\left(a e^2 g-c d (2 e f-3 d g)\right)\sqrt{f+g x}\,\sqrt{a+c x^2}}{4 e \left(c d^2+a e^2\right)(e f-d g)(d+e x)}-$$

$$\frac{\sqrt{-a}\,\sqrt{c}\,\left(a e^2 g-c d (2 e f-3 d g)\right)\sqrt{f+g x}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]}{4 e^2 \left(c d^2+a e^2\right)(e f-d g)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+c x^2}}-$$

$$\frac{3\sqrt{-a}\,\sqrt{c}\,g\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]}{2 e^3\sqrt{f+g x}\,\sqrt{a+c x^2}}+$$

$$\left(\sqrt{-a}\,\sqrt{c}\,f\left(a e^2 g-c d (2 e f-3 d g)\right)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]\right)\Big/$$

$$\left(4 e^2\left(c d^2+a e^2\right)(e f-d g)\sqrt{f+g x}\,\sqrt{a+c x^2}\right)-$$

$$\left(\sqrt{-a}\,\sqrt{c}\,d g\left(a e^2 g-c d (2 e f-3 d g)\right)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]\right)\Big/$$

$$\left(4 e^3\left(c d^2+a e^2\right)(e f-d g)\sqrt{f+g x}\,\sqrt{a+c x^2}\right)-\frac{c\,(e f-3 d g)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticPi}\left[\frac{2 e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e},\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right]}{e^3\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)\sqrt{f+g x}\,\sqrt{a+c x^2}}+$$

$$\left(\left(a e^2 g-c d\,(2 e f-3 d g)\right)^2 \sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\;\sqrt{1+\frac{c x^2}{a}}\;\text{EllipticPi}\left[\frac{2 e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e},\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\,\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right]\right)\Big/$$

$$\left(4 e^3\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)\left(c d^2+a e^2\right)(e f-d g)\sqrt{f+g x}\;\sqrt{a+c x^2}\right)$$

Result (type 4, 12 364 leaves) :

$$\sqrt{f+g x}\;\sqrt{a+c x^2}\left(-\frac{1}{2 e\,(d+e x)^2}+\frac{2 c d e f-3 c d^2 g-a e^2 g}{4 e\left(c d^2+a e^2\right)(e f-d g)\,(d+e x)}\right)+$$

$$\frac{1}{4 e\left(c d^2+a e^2\right) g\,(e f-d g)}\left(\frac{\left(-2 c d e f+3 c d^2 g+a e^2 g\right)(f+g x)^{3/2}\left(c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}\right)}{e\sqrt{a+\frac{c\,(f+g x)^2\left(-1+\frac{f}{f+g x}\right)^2}{g^2}}}+\frac{1}{e\sqrt{a+\frac{c\,(f+g x)^2\left(-1+\frac{f}{f+g x}\right)^2}{g^2}}}(e f-d g)\,(f+g x)\right.$$

$$\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\left(\left(2\,\mathrm{i}\,c^2 d e f^3\left(c f+\mathrm{i}\sqrt{a}\,\sqrt{c}\,g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g\right)(f+g x)}}\;\sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i}\sqrt{a}\,\sqrt{c}\,g\right)(f+g x)}}\right.\right.$$

$$\left.\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\,\frac{c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}{c f+\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}\right]-\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\,\frac{c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}{c f+\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}\right]\right)\right)\Big/$$

$$\left((e f-d g)\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}}\;\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(3\,\mathrm{i}\,c^2 d^2 f^2 g\left(c f+\mathrm{i}\sqrt{a}\,\sqrt{c}\,g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g\right)(f+g x)}}\;\sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i}\sqrt{a}\,\sqrt{c}\,g\right)(f+g x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\bigg)\bigg/$$

$$\left((e f-d g)\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\Bigg(i\,a\,c\,e^2 f^2 g\left(c f+i\sqrt{a}\sqrt{c}\,g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\Bigg)\bigg/$$

$$\left((e f-d g)\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\Bigg(2\,i\,a\,c\,d\,e\,f\,g^2\left(c f+i\sqrt{a}\sqrt{c}\,g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\Bigg)\bigg/$$

$$\left((e f-d g)\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(3\,i\,a\,c\,d^2\,g^3\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]\right)\right)\Big/$$

$$\left((e\,f-d\,g)\left(c\,f^2+a\,g^2\right)\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(i\,a^2\,e^2\,g^3\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]\right)\right)\Big/$$

$$\left((e\,f-d\,g)\left(c\,f^2+a\,g^2\right)\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(2\,i\,c^2\,d\,e^2\,f^3\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left((e\,f-d\,g)^2\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)+$$

$$\left(3\, i\, c^2 d^2 e f^2 g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c}\, g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c}\, g\right) (f+g x)}}\right.$$

$$\left.\mathrm{EllipticF}\left[i\, \mathrm{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c}\, g}{c f+i \sqrt{a} \sqrt{c}\, g}\right]\right) \Big/ \left((e f-d g)^2 \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}} \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\left(i\, a c e^3 f^2 g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c}\, g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c}\, g\right) (f+g x)}}\ \mathrm{EllipticF}\left[i\, \mathrm{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}}}{\sqrt{f+g x}}\right],\right.\right.$$

$$\left.\left.\frac{c f-i \sqrt{a} \sqrt{c}\, g}{c f+i \sqrt{a} \sqrt{c}\, g}\right]\right) \Big/ \left((e f-d g)^2 \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}} \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(2\, i\, a c d e^2 f g^2 \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c}\, g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c}\, g\right) (f+g x)}}\right.$$

$$\left.\mathrm{EllipticF}\left[i\, \mathrm{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c}\, g}{c f+i \sqrt{a} \sqrt{c}\, g}\right]\right) \Big/ \left((e f-d g)^2 \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}} \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\left(3\, i\, a c d^2 e g^3 \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c}\, g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c}\, g\right) (f+g x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\Bigg)\Bigg/\left((e f-d g)^2\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\Bigg(i a^2 e^3 g^3\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\right.$$

$$\left.\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\Bigg)\Bigg/\left((e f-d g)^2\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\Bigg(4 i c^2 d e f^2\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\right.$$

$$\left.\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\Bigg)\Bigg/\left((e f-d g)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\Bigg(4 i a c d e g^2\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\right.$$

$$\left.\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\Bigg)\Bigg/\left((e f-d g)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+\frac{1}{(e f-d g)^3}$$

$$2 c^2 d e^3 f^3 \Bigg(\Bigg(i f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right)(f+g x)}} \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\right.$$

$$\left. i \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\Bigg) \Bigg/ \left(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\Bigg(i d g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right)(f+g x)}} \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right)(e f-d g)}{e\left(c f^2+a g^2\right)}, i \text{ArcSinh}\left[\right.\right.$$

$$\left.\left.\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\Bigg) \Bigg/ \left(e \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\Bigg)-\frac{1}{(e f-d g)^3}$$

$$3 c^2 d^2 e^2 f^2 g \Bigg(\Bigg(i f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right)(f+g x)}} \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\right.$$

$$\left. i \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\Bigg) \Bigg/ \left(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\Bigg(i d g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right)(f+g x)}} \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right)(e f-d g)}{e\left(c f^2+a g^2\right)}, i \text{ArcSinh}\left[\right.\right.$$

$$\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\Big],\ \frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\Big]\Bigg)\Bigg/\left(e\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\Bigg)-\frac{1}{(e f-d g)^3}$$

$$a c e^4 f^2 g\Bigg(\Bigg(i f\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\ \text{EllipticPi}\Big[\frac{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},$$

$$i\,\text{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\Big],\ \frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\Big]\Bigg)\Bigg/\left(\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\Bigg(i d g\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\ \text{EllipticPi}\Big[\frac{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\ i\,\text{ArcSinh}\Big[$$

$$\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\Big],\ \frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\Big]\Bigg)\Bigg/\left(e\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\Bigg)+\frac{1}{(e f-d g)^3}$$

$$2 a c d e^3 f g^2\Bigg(\Bigg(i f\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\ \text{EllipticPi}\Big[\frac{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},$$

$$i\,\text{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\Big],\ \frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\Big]\Bigg)\Bigg/\left(\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(\mathrm{i}\, d\, g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(f+g x)}}\ \mathrm{EllipticPi}\left[\frac{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(e f-d g)}{e\left(c f^2+a g^2\right)}, \mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g}}}{\sqrt{f+g x}}\right], \frac{c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g}{c f+\mathrm{i}\sqrt{a}\sqrt{c}\, g}\right]\right) \Bigg/ \left(e\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\right)-\frac{1}{(e f-d g)^3}$$

$$3\, a\, c\, d^2 e^2 g^3\left(\left(\mathrm{i}\, f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(f+g x)}}\ \mathrm{EllipticPi}\left[\frac{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\right.\right.\right.$$

$$\left.\left.\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g}}}{\sqrt{f+g x}}\right], \frac{c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g}{c f+\mathrm{i}\sqrt{a}\sqrt{c}\, g}\right]\right) \Bigg/ \left(\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(\mathrm{i}\, d\, g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(f+g x)}}\ \mathrm{EllipticPi}\left[\frac{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(e f-d g)}{e\left(c f^2+a g^2\right)}, \mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g}}}{\sqrt{f+g x}}\right], \frac{c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g}{c f+\mathrm{i}\sqrt{a}\sqrt{c}\, g}\right]\right) \Bigg/ \left(e\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\right)-\frac{1}{(e f-d g)^3}$$

$$a^2 e^4 g^3\left(\left(\mathrm{i}\, f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(f+g x)}}\ \mathrm{EllipticPi}\left[\frac{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\right.\right.\right.$$

$$\mathrm{i}\,\mathrm{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\Big],\ \frac{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}{c f+\mathrm{i}\sqrt{a}\sqrt{c}\,g}\Big]\Bigg)\Bigg/\left(\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}}\ \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\Bigg(\mathrm{i}\,d\,g\sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\ \sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\ \mathrm{EllipticPi}\Big[\frac{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\ \mathrm{i}\,\mathrm{ArcSinh}\Big[$$

$$\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\Big],\ \frac{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}{c f+\mathrm{i}\sqrt{a}\sqrt{c}\,g}\Big]\Bigg)\Bigg/\left(e\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}}\ \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\Bigg)-\frac{1}{(e f-d g)^2}$$

$$4 c^2 d e^2 f^2\Bigg(\Bigg(\Bigg(\mathrm{i}\,f\sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\ \sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\ \mathrm{EllipticPi}\Big[\frac{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},$$

$$\mathrm{i}\,\mathrm{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\Big],\ \frac{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}{c f+\mathrm{i}\sqrt{a}\sqrt{c}\,g}\Big]\Bigg)\Bigg/\left(\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}}\ \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\Bigg(\mathrm{i}\,d\,g\sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\ \sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\ \mathrm{EllipticPi}\Big[\frac{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\ \mathrm{i}\,\mathrm{ArcSinh}\Big[$$

$$\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\Big],\ \frac{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}{c f+\mathrm{i}\sqrt{a}\sqrt{c}\,g}\Big]\Bigg)\Bigg/\left(e\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}}\ \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\Bigg)-\frac{1}{(e f-d g)^2}$$

$$4\,a\,c\,d\,e^2\,g^2\left(\left(\left(\mathrm{i}\,f\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\left[\frac{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\right.\right.\right.\right.$$

$$\left.\left.\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\,\frac{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left(\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(\mathrm{i}\,d\,g\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\left[\frac{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\,\mathrm{i}\right.\right.$$

$$\left.\left.\left.\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\,\frac{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left(e\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)\right)+\frac{1}{e\,f-d\,g}$$

$$2\,c^2\,d\,e\,f\left(\left(\left(\mathrm{i}\,f\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\left[\frac{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\right.\right.\right.\right.$$

$$\left.\left.\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\,\frac{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left(\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(\mathrm{i}\,d\,g\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\left[\frac{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\,\mathrm{i}\right.\right.$$

$$\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\Bigg) \Bigg/ \left(e \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\Bigg) + \frac{1}{e f-d g}$$

$$3 c^2 d^2 g \Bigg(\Bigg(i f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \text{ EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)},\right.$$

$$i \text{ ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\Bigg) \Bigg/ \left(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right) -$$

$$\Bigg(i d g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \text{ EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)}, i\right.$$

$$\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\Bigg) \Bigg/ \left(e \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\Bigg) + \frac{1}{e f-d g}$$

$$5 a c e^2 g \Bigg(\Bigg(i f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \text{ EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)},\right.$$

$$i \text{ ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\Bigg) \Bigg/ \left(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right) -$$

$$\left(i\, d\, g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right)(f+g x)}} \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right)(e f-d g)}{e\left(c f^2+a g^2\right)}, i\right.\right.$$

$$\left.\left.\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right) \Big/ \left(e \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\Bigg)\Bigg)$$

- **Problem 629: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(d+e x)^3 \sqrt{a+c x^2}}{\sqrt{f+g x}}\, dx$$

Optimal (type 4, 666 leaves, 9 steps) :

$$-\frac{4\left(9 a e^2 g^2 (2 e f-5 d g)+c\left(76 e^3 f^3-204 d e^2 f^2 g+168 d^2 e f g^2-35 d^3 g^3\right)\right)\sqrt{f+g x}\,\sqrt{a+c x^2}}{315 c g^4}+\frac{2 (d+e x)^3\sqrt{f+g x}\,\sqrt{a+c x^2}}{9 g}+$$

$$\frac{4 e\left(7 a e^2 g^2+c\left(64 e^2 f^2-111 d e f g+42 d^2 g^2\right)\right)(f+g x)^{3/2}\sqrt{a+c x^2}}{315 c g^4}-\frac{4 e^2 (4 e f-3 d g)(f+g x)^{5/2}\sqrt{a+c x^2}}{63 g^4}+$$

$$\Bigg(4\sqrt{-a}\left(21 a^2 e^3 g^4-3 a c e g^2\left(10 e^2 f^2-39 d e f g+63 d^2 g^2\right)-c^2 f\left(64 e^3 f^3-216 d e^2 f^2 g+252 d^2 e f g^2-105 d^3 g^3\right)\right)$$

$$\sqrt{f+g x}\,\sqrt{1+\frac{c x^2}{a}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\sqrt{c}\,f-a g}\right]\Bigg)\Bigg/\left(315 c^{3/2} g^5\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+c x^2}\right)-$$

$$\Bigg(4\sqrt{-a}\left(c f^2+a g^2\right)\left(9 a e^2 g^2 (2 e f-5 d g)-c\left(64 e^3 f^3-216 d e^2 f^2 g+252 d^2 e f g^2-105 d^3 g^3\right)\right)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}$$

$$\sqrt{1+\frac{c x^2}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\sqrt{c}\,f-a g}\right]\Bigg)\Bigg/\left(315 c^{3/2} g^5\sqrt{f+g x}\,\sqrt{a+c x^2}\right)$$

Result (type 4, 5379 leaves) :

$$\sqrt{f+g x}\,\sqrt{a+c x^2}\left(\frac{2\left(-64 c e^3 f^3+216 c d e^2 f^2 g-252 c d^2 e f g^2-22 a e^3 f g^2+105 c d^3 g^3+90 a d e^2 g^3\right)}{315 c g^4}+\right.$$

$$\left.\frac{2 e\left(48 c e^2 f^2-162 c d e f g+189 c d^2 g^2+14 a e^2 g^2\right)x}{315 c g^3}-\frac{2 e^2 (8 e f-27 d g)x^2}{63 g^2}+\frac{2 e^3 x^3}{9 g}\right)+$$

$$\frac{1}{315\,c\,g^6}\,4\,\Bigg(\Bigg(\left(64\,c^2\,e^3\,f^4-216\,c^2\,d\,e^2\,f^3\,g+252\,c^2\,d^2\,e\,f^2\,g^2+30\,a\,c\,e^3\,f^2\,g^2-105\,c^2\,d^3\,f\,g^3-117\,a\,c\,d\,e^2\,f\,g^3+189\,a\,c\,d^2\,e\,g^4-21\,a^2\,e^3\,g^4\right)$$

$$(f+g\,x)^{3/2}\left(c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}\right)\Bigg)\Bigg/\left(c\,\sqrt{a+\frac{c\,(f+g\,x)^2\left(-1+\frac{f}{f+g\,x}\right)^2}{g^2}}\right)-$$

$$\frac{1}{c\,\sqrt{a+\frac{c\,(f+g\,x)^2\left(-1+\frac{f}{f+g\,x}\right)^2}{g^2}}}\left(c\,f^2+a\,g^2\right)\,(f+g\,x)\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}$$

$$\Bigg(\Bigg(64\,i\,c^2\,e^3\,f^4\left(c\,f+i\,\sqrt{a}\,\sqrt{c}\,g\right)\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\,\frac{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\,\sqrt{a}\,\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\,\frac{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Bigg)\Bigg/$$

$$\left(\left(c\,f^2+a\,g^2\right)\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-\Bigg(216\,i\,c^2\,d\,e^2\,f^3\,g\left(c\,f+i\,\sqrt{a}\,\sqrt{c}\,g\right)$$

$$\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\,\frac{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\,\sqrt{a}\,\sqrt{c}\,g}\right]-$$

$$\left.\left.\mathrm{EllipticF}\left[i\,\mathrm{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right],\frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right)\right) \bigg/ \left(\left(c f^2+a g^2\right) \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\left(252\, i\, c^2 d^2 e f^2 g^2 \left(c f+i \sqrt{a} \sqrt{c} g\right) \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right)(f+g x)}}\right.$$

$$\left.\left(\mathrm{EllipticE}\left[i\,\mathrm{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right],\frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]-\mathrm{EllipticF}\left[i\,\mathrm{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right],\frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right)\right) \bigg/$$

$$\left(\left(c f^2+a g^2\right) \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\left(30\, i\, a\, c\, e^3 f^2 g^2 \left(c f+i \sqrt{a} \sqrt{c} g\right) \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right)(f+g x)}}\right.$$

$$\left.\left(\mathrm{EllipticE}\left[i\,\mathrm{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right],\frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]-\mathrm{EllipticF}\left[i\,\mathrm{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right],\frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right)\right) \bigg/$$

$$\left(\left(c f^2+a g^2\right) \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-\left(105\, i\, c^2 d^3 f g^3 \left(c f+i \sqrt{a} \sqrt{c} g\right)\right.$$

$$\sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathbb{i}\sqrt{a}\ \sqrt{c}\ g\right)\ (f+g x)}}\ \sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathbb{i}\sqrt{a}\ \sqrt{c}\ g\right)\ (f+g x)}}\ \left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\ \sqrt{c}\ g}}}{\sqrt{f+g x}}\right],\ \frac{c f-\mathbb{i}\sqrt{a}\ \sqrt{c}\ g}{c f+\mathbb{i}\sqrt{a}\ \sqrt{c}\ g}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\ \sqrt{c}\ g}}}{\sqrt{f+g x}}\right],\ \frac{c f-\mathbb{i}\sqrt{a}\ \sqrt{c}\ g}{c f+\mathbb{i}\sqrt{a}\ \sqrt{c}\ g}\right]\right)\right)\Bigg/\left(\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\ \sqrt{c}\ g}}\ \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(117\,\mathbb{i}\,a c d e^2 f g^3\left(c f+\mathbb{i}\sqrt{a}\ \sqrt{c}\ g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathbb{i}\sqrt{a}\ \sqrt{c}\ g\right)\ (f+g x)}}\ \sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathbb{i}\sqrt{a}\ \sqrt{c}\ g\right)\ (f+g x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\ \sqrt{c}\ g}}}{\sqrt{f+g x}}\right],\ \frac{c f-\mathbb{i}\sqrt{a}\ \sqrt{c}\ g}{c f+\mathbb{i}\sqrt{a}\ \sqrt{c}\ g}\right]-\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\ \sqrt{c}\ g}}}{\sqrt{f+g x}}\right],\ \frac{c f-\mathbb{i}\sqrt{a}\ \sqrt{c}\ g}{c f+\mathbb{i}\sqrt{a}\ \sqrt{c}\ g}\right]\right)\right)\Bigg/$$

$$\left(\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\ \sqrt{c}\ g}}\ \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+\left(189\,\mathbb{i}\,a c d^2 e g^4\left(c f+\mathbb{i}\sqrt{a}\ \sqrt{c}\ g\right)\right.$$

$$\sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathbb{i}\sqrt{a}\ \sqrt{c}\ g\right)\ (f+g x)}}\ \sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathbb{i}\sqrt{a}\ \sqrt{c}\ g\right)\ (f+g x)}}\ \left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\ \sqrt{c}\ g}}}{\sqrt{f+g x}}\right],\ \frac{c f-\mathbb{i}\sqrt{a}\ \sqrt{c}\ g}{c f+\mathbb{i}\sqrt{a}\ \sqrt{c}\ g}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\ \sqrt{c}\ g}}}{\sqrt{f+g x}}\right],\ \frac{c f-\mathbb{i}\sqrt{a}\ \sqrt{c}\ g}{c f+\mathbb{i}\sqrt{a}\ \sqrt{c}\ g}\right]\right)\right)\Bigg/\left(\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\ \sqrt{c}\ g}}\ \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(21\,i\,a^2 e^3 g^4\left(c f+i\sqrt{a}\sqrt{c}\,g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\right)\Big/$$

$$\left(\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+\left(64\,i\,c^2 e^3 f^3\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\right.$$

$$\left.\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\Big/\left(\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\right.$$

$$\left.\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-\left(216\,i\,c^2 d\,e^2 f^2 g\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\Big/\left(\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\left(252\,i\,c^2 d^2 e f g^2\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\Bigg/\left(\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(18\,i\,a\,c\,e^3 f g^2\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\Bigg/\left(\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(105\,i\,c^2 d^3 g^3\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\Bigg/\left(\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\left(45\,i\,a\,c\,d\,e^2 g^3\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\Bigg/\left(\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\right)\right)$$

- **Problem 630: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e x)^2 \sqrt{a+c x^2}}{\sqrt{f+g x}} \, dx$$

Optimal (type 4, 508 leaves, 8 steps):

$$\frac{4 \left(5 a e^2 g^2+c \left(21 e^2 f^2-34 d e f g+10 d^2 g^2\right)\right) \sqrt{f+g x} \sqrt{a+c x^2}}{105 c g^3}+\frac{2 (d+e x)^2 \sqrt{f+g x} \sqrt{a+c x^2}}{7 g}-\frac{4 e (3 e f-2 d g) (f+g x)^{3/2} \sqrt{a+c x^2}}{35 g^3}+$$

$$\left(4 \sqrt{-a} \left(a e g^2 (13 e f-42 d g)+c f \left(24 e^2 f^2-56 d e f g+35 d^2 g^2\right)\right) \sqrt{f+g x} \sqrt{1+\frac{c x^2}{a}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right], -\frac{2 a g}{\sqrt{-a} \sqrt{c} f-a g}\right]\right) \Big/$$

$$\left(105 \sqrt{c} g^4 \sqrt{\frac{\sqrt{c} (f+g x)}{\sqrt{c} f+\sqrt{-a} g}} \sqrt{a+c x^2}\right)+\left(4 \sqrt{-a} \left(c f^2+a g^2\right) \left(5 a e^2 g^2-c \left(24 e^2 f^2-56 d e f g+35 d^2 g^2\right)\right)\right.$$

$$\left.\sqrt{\frac{\sqrt{c} (f+g x)}{\sqrt{c} f+\sqrt{-a} g}} \sqrt{1+\frac{c x^2}{a}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c} x}{\sqrt{-a}}}}{\sqrt{2}}\right], -\frac{2 a g}{\sqrt{-a} \sqrt{c} f-a g}\right]\right) \Big/ \left(105 c^{3/2} g^4 \sqrt{f+g x} \sqrt{a+c x^2}\right)$$

Result (type 4, 712 leaves):

$$\frac{1}{105\, c\, g^5 \sqrt{a+c\, x^2}}\, 2 \sqrt{f+g\, x}$$

$$\left(g^2 \left(a+c\, x^2\right) \left(10\, a\, e^2\, g^2 + c \left(35\, d^2\, g^2 + 14\, d\, e\, g\, (-4\, f + 3\, g\, x) + 3\, e^2 \left(8\, f^2 - 6\, f\, g\, x + 5\, g^2\, x^2\right)\right)\right) - \frac{1}{\sqrt{-f-\frac{i \sqrt{a}\, g}{\sqrt{c}}}\, (f+g\, x)}\, 2 \left(g^2 \sqrt{-f-\frac{i\sqrt{a}\, g}{\sqrt{c}}}\right.\right.$$

$$\left(a^2\, e\, g^2\, (13\, e\, f - 42\, d\, g) + c^2\, f \left(24\, e^2\, f^2 - 56\, d\, e\, f\, g + 35\, d^2\, g^2\right) x^2 + a\, c \left(35\, d^2\, f\, g^2 - 14\, d\, e\, g \left(4\, f^2 + 3\, g^2\, x^2\right) + e^2 \left(24\, f^3 + 13\, f\, g^2\, x^2\right)\right)\right) -$$

$$i \sqrt{c} \left(\sqrt{c}\, f + i \sqrt{a}\, g\right) \left(a\, e\, g^2\, (13\, e\, f - 42\, d\, g) + c\, f \left(24\, e^2\, f^2 - 56\, d\, e\, f\, g + 35\, d^2\, g^2\right)\right) \sqrt{\frac{g \left(\frac{i\sqrt{a}}{\sqrt{c}} + x\right)}{f+g\, x}}$$

$$\sqrt{-\frac{\frac{i\sqrt{a}\, g}{\sqrt{c}} - g\, x}{f+g\, x}}\, (f+g\, x)^{3/2}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\, g}{\sqrt{c}}}}{\sqrt{f+g\, x}}\right], \frac{\sqrt{c}\, f - i\sqrt{a}\, g}{\sqrt{c}\, f + i\sqrt{a}\, g}\right] +$$

$$\sqrt{a}\, g \left(\sqrt{c}\, f + i\sqrt{a}\, g\right) \left(5\, a\, e^2\, g^2 + 6\, i \sqrt{a}\, \sqrt{c}\, e\, g\, (3\, e\, f - 7\, d\, g) + c \left(-24\, e^2\, f^2 + 56\, d\, e\, f\, g - 35\, d^2\, g^2\right)\right)$$

$$\left.\left.\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}} + x\right)}{f+g\, x}}\, \sqrt{-\frac{\frac{i\sqrt{a}\, g}{\sqrt{c}} - g\, x}{f+g\, x}}\, (f+g\, x)^{3/2}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\, g}{\sqrt{c}}}}{\sqrt{f+g\, x}}\right], \frac{\sqrt{c}\, f - i\sqrt{a}\, g}{\sqrt{c}\, f + i\sqrt{a}\, g}\right]\right)\right)$$

- **Problem 631: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e\, x)\sqrt{a+c\, x^2}}{\sqrt{f+g\, x}}\, dx$$

Optimal (type 4, 364 leaves, 6 steps) :

$$-\frac{2\sqrt{f+g x}\,(4 e f-5 d g-3 e g x)\sqrt{a+c x^2}}{15 g^2}-$$

$$\frac{4\sqrt{-a}\left(3 a e g^2+c f(4 e f-5 d g)\right)\sqrt{f+g x}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\sqrt{c}\,f-a g}\right]}{15\sqrt{c}\,g^3\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\sqrt{a+c x^2}}+$$

$$\left(4\sqrt{-a}\,(4 e f-5 d g)\left(c f^2+a g^2\right)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\sqrt{c}\,f-a g}\right]\right)\Big/$$

$$\left(15\sqrt{c}\,g^3\sqrt{f+g x}\sqrt{a+c x^2}\right)$$

Result (type 4, 545 leaves) :

$$\frac{1}{15\sqrt{a+c x^2}}\sqrt{f+g x}\left(\frac{2(-4 e f+5 d g+3 e g x)\left(a+c x^2\right)}{g^2}+\right.$$

$$\frac{1}{c g^4\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(f+g x)}4\left(g^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\left(3 a e g^2+c f(4 e f-5 d g)\right)\left(a+c x^2\right)-\sqrt{c}\left(i\sqrt{c}\,f-\sqrt{a}\,g\right)\left(3 a e g^2+c f(4 e f-5 d g)\right)\right.$$

$$\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+g x}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-g x}{f+g x}}\,(f+g x)^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]+\sqrt{a}\sqrt{c}\,g\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)$$

$$\left.\left.\left(3 i\sqrt{a}\,e g+\sqrt{c}\,(-4 e f+5 d g)\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+g x}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-g x}{f+g x}}\,(f+g x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]\right)\right)$$

■ Problem 632: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+c\,x^2}}{\sqrt{f+g\,x}}\,dx$$

Optimal (type 4, 322 leaves, 6 steps):

$$\frac{2\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}{3\,g}+\frac{4\sqrt{-a}\,\sqrt{c}\,f\,\sqrt{f+g\,x}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\,-\frac{2\,a\,g}{\sqrt{-a}\,\sqrt{c}\,f-a\,g}\right]}{3\,g^2\,\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+c\,x^2}}-$$

$$\frac{4\sqrt{-a}\,\left(c\,f^2+a\,g^2\right)\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\,-\frac{2\,a\,g}{\sqrt{-a}\,\sqrt{c}\,f-a\,g}\right]}{3\sqrt{c}\,g^2\,\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 456 leaves):

$$\frac{1}{3\,g^3\,\sqrt{a+c\,x^2}}$$

$$2\sqrt{f+g\,x}\left(g^2\left(a+c\,x^2\right)-1\Big/\left(\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(f+g\,x)\right)2\left(f\,g^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,\left(a+c\,x^2\right)+\sqrt{c}\,f\left(-i\sqrt{c}\,f+\sqrt{a}\,g\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+g\,x}}\,\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-g\,x}{f+g\,x}}\right.\right.$$

$$(f+g\,x)^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\,\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]-$$

$$\left.\left.\sqrt{a}\,g\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+g\,x}}\,\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-g\,x}{f+g\,x}}\,(f+g\,x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\,\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]\right)\right)$$

- **Problem 633: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+c x^2}}{(d+e x)\sqrt{f+g x}}\,dx$$

Optimal (type 4, 473 leaves, 10 steps):

$$-\frac{2\sqrt{-a}\,\sqrt{c}\,\sqrt{f+g x}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]}{e g\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+c x^2}}+$$

$$\frac{2\sqrt{-a}\,\sqrt{c}\,(e f+d g)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]}{e^2 g\sqrt{f+g x}\,\sqrt{a+c x^2}}-$$

$$\frac{2\left(c d^2+a e^2\right)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticPi}\left[\frac{2 e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e},\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right]}{e^2\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)\sqrt{f+g x}\,\sqrt{a+c x^2}}$$

Result (type 4, 1096 leaves):

$$-\frac{1}{e^2 g^2 \sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(e f-d g)\sqrt{f+g x}\,\sqrt{a+c x^2}}$$

$$2\left(-c e^2 f^3\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}+c d e f^2 g\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}-a e^2 f g^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}+a d e g^3\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}+2 c e^2 f^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(f+g x)-\right.$$

$$2 c d e f g\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(f+g x)-c e^2 f\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(f+g x)^2+c d e g\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(f+g x)^2+$$

$$\sqrt{c}\,e\left(-i\sqrt{c}\,f+\sqrt{a}\,g\right)(-e f+d g)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+g x}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-g x}{f+g x}}\,(f+g x)^{3/2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]+$$

$$e\left(i\sqrt{c}\,d+\sqrt{a}\,e\right)g\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+g x}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-g x}{f+g x}}\,(f+g x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]-$$

$$i\,c\,d^2 g^2\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+g x}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-g x}{f+g x}}\,(f+g x)^{3/2}\,\text{EllipticPi}\left[\frac{\sqrt{c}\,(e f-d g)}{e\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)},i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]-$$

$$\left.i\,a\,e^2 g^2\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+g x}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-g x}{f+g x}}\,(f+g x)^{3/2}\,\text{EllipticPi}\left[\frac{\sqrt{c}\,(e f-d g)}{e\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)},i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]\right)$$

- **Problem 634: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{a+c x^2}}{(d+e x)^2\sqrt{f+g x}}\,dx$$

Optimal (type 4, 694 leaves, 14 steps):

$$-\frac{\sqrt{f+g x}\,\sqrt{a+c x^2}}{(e f-d g)\,(d+e x)}-\frac{\sqrt{-a}\,\sqrt{c}\,\sqrt{f+g x}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]}{e\,(e f-d g)\,\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+c x^2}}+$$

$$\frac{\sqrt{-a}\,\sqrt{c}\,f\,\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]}{e\,(e f-d g)\,\sqrt{f+g x}\,\sqrt{a+c x^2}}-$$

$$\frac{\sqrt{-a}\,\sqrt{c}\,(2 e f-d g)\,\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]}{e^2\,(e f-d g)\,\sqrt{f+g x}\,\sqrt{a+c x^2}}+$$

$$\frac{\left(a e^2 g+c d\,(2 e f-d g)\right)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticPi}\left[\frac{2 e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e},\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right]}{e^2\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)(e f-d g)\,\sqrt{f+g x}\,\sqrt{a+c x^2}}$$

Result (type 4, 1456 leaves) :

$$\frac{\sqrt{f+g x}\,\sqrt{a+c x^2}}{(-e f+d g)\,(d+e x)}-\frac{1}{e^2\,g\,\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(e f-d g)\,(-e f+d g)\,\sqrt{a+\frac{c\,(f+g x)^2\left(-1+\frac{f}{f+g x}\right)^2}{g^2}}}$$

$$(f+g x)^{3/2}\left(c\,e^2\,f\,\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}-c\,d\,e\,g\,\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}+\frac{c\,e^2\,f^3\,\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{(f+g x)^2}-\frac{c\,d\,e\,f^2\,g\,\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{(f+g x)^2}+\frac{a\,e^2\,f\,g^2\,\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{(f+g x)^2}-\right.$$

$$\frac{a\,d\,e\,g^3\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{(f+g\,x)^2}-\frac{2\,c\,e^2\,f^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{f+g\,x}+\frac{2\,c\,d\,e\,f\,g\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{f+g\,x}+\frac{1}{\sqrt{f+g\,x}}\sqrt{c}\;e\left(-i\sqrt{c}\;f+\sqrt{a}\;g\right)(e\,f-d\,g)$$

$$\sqrt{1-\frac{f}{f+g\,x}-\frac{i\sqrt{a}\;g}{\sqrt{c}\;(f+g\,x)}}\;\sqrt{1-\frac{f}{f+g\,x}+\frac{i\sqrt{a}\;g}{\sqrt{c}\;(f+g\,x)}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\frac{\sqrt{c}\;f-i\sqrt{a}\;g}{\sqrt{c}\;f+i\sqrt{a}\;g}\right]+$$

$$\frac{1}{\sqrt{f+g\,x}}e\left(\sqrt{c}\;f+i\sqrt{a}\;g\right)\left(\sqrt{a}\;e\,g+i\sqrt{c}\;(2\,e\,f-d\,g)\right)\sqrt{1-\frac{f}{f+g\,x}-\frac{i\sqrt{a}\;g}{\sqrt{c}\;(f+g\,x)}}\;\sqrt{1-\frac{f}{f+g\,x}+\frac{i\sqrt{a}\;g}{\sqrt{c}\;(f+g\,x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\frac{\sqrt{c}\;f-i\sqrt{a}\;g}{\sqrt{c}\;f+i\sqrt{a}\;g}\right]-\frac{1}{\sqrt{f+g\,x}}2\,i\,c\,d\,e\,f\,g\sqrt{1-\frac{f}{f+g\,x}-\frac{i\sqrt{a}\;g}{\sqrt{c}\;(f+g\,x)}}\;\sqrt{1-\frac{f}{f+g\,x}+\frac{i\sqrt{a}\;g}{\sqrt{c}\;(f+g\,x)}}$$

$$\text{EllipticPi}\left[\frac{\sqrt{c}\;(e\,f-d\,g)}{e\left(\sqrt{c}\;f+i\sqrt{a}\;g\right)},\,i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\frac{\sqrt{c}\;f-i\sqrt{a}\;g}{\sqrt{c}\;f+i\sqrt{a}\;g}\right]+\frac{1}{\sqrt{f+g\,x}}i\,c\,d^2\,g^2\sqrt{1-\frac{f}{f+g\,x}-\frac{i\sqrt{a}\;g}{\sqrt{c}\;(f+g\,x)}}$$

$$\sqrt{1-\frac{f}{f+g\,x}+\frac{i\sqrt{a}\;g}{\sqrt{c}\;(f+g\,x)}}\;\text{EllipticPi}\left[\frac{\sqrt{c}\;(e\,f-d\,g)}{e\left(\sqrt{c}\;f+i\sqrt{a}\;g\right)},\,i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\frac{\sqrt{c}\;f-i\sqrt{a}\;g}{\sqrt{c}\;f+i\sqrt{a}\;g}\right]-\frac{1}{\sqrt{f+g\,x}}i\,a\,e^2\,g^2$$

$$\left.\sqrt{1-\frac{f}{f+g\,x}-\frac{i\sqrt{a}\;g}{\sqrt{c}\;(f+g\,x)}}\;\sqrt{1-\frac{f}{f+g\,x}+\frac{i\sqrt{a}\;g}{\sqrt{c}\;(f+g\,x)}}\;\text{EllipticPi}\left[\frac{\sqrt{c}\;(e\,f-d\,g)}{e\left(\sqrt{c}\;f+i\sqrt{a}\;g\right)},\,i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\frac{\sqrt{c}\;f-i\sqrt{a}\;g}{\sqrt{c}\;f+i\sqrt{a}\;g}\right]\right)$$

- **Problem 635: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{a+c\,x^2}}{(d+e\,x)^3\sqrt{f+g\,x}}\,dx$$

Optimal (type 4, 1241 leaves, 23 steps) :

$$-\frac{\sqrt{f+g x}\,\sqrt{a+c x^2}}{2\,(e f-d g)\,(d+e x)^2}+\frac{\left(3 a e^2 g+c d\,(2 e f+d g)\right)\sqrt{f+g x}\,\sqrt{a+c x^2}}{4\left(c d^2+a e^2\right)(e f-d g)^2\,(d+e x)}+$$

$$\frac{\sqrt{-a}\,\sqrt{c}\,\left(3 a e^2 g+c d\,(2 e f+d g)\right)\sqrt{f+g x}\,\sqrt{1+\frac{c x^2}{a}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]}{4 e\left(c d^2+a e^2\right)(e f-d g)^2\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+c x^2}}+$$

$$\frac{\sqrt{-a}\,\sqrt{c}\,g\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]}{2 e^2\,(e f-d g)\,\sqrt{f+g x}\,\sqrt{a+c x^2}}-$$

$$\left(\sqrt{-a}\,\sqrt{c}\,f\left(3 a e^2 g+c d\,(2 e f+d g)\right)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]\right)\Big/$$

$$\left(4 e\left(c d^2+a e^2\right)(e f-d g)^2\sqrt{f+g x}\,\sqrt{a+c x^2}\right)+$$

$$\left(\sqrt{-a}\,\sqrt{c}\,d g\left(3 a e^2 g+c d\,(2 e f+d g)\right)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]\right)\Big/$$

$$\left(4 e^2\left(c d^2+a e^2\right)(e f-d g)^2\sqrt{f+g x}\,\sqrt{a+c x^2}\right)-\frac{c\,(e f+d g)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\;\text{EllipticPi}\left[\frac{2 e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e},\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right]}{e^2\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)(e f-d g)\sqrt{f+g x}\,\sqrt{a+c x^2}}-$$

$$\left(\left(a e^2 g - c d (2 e f - 3 d g)\right) \left(3 a e^2 g + c d (2 e f + d g)\right) \sqrt{\frac{\sqrt{c}\, (f+g x)}{\sqrt{c}\, f+\sqrt{-a}\, g}}\, \sqrt{1+\frac{c x^2}{a}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{2 e}{\frac{\sqrt{c}\, d}{\sqrt{-a}}+e}, \text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right], \frac{2 \sqrt{-a}\, g}{\sqrt{c}\, f+\sqrt{-a}\, g}\right]\right) \Big/ \left(4 e^2 \left(\frac{\sqrt{c}\, d}{\sqrt{-a}}+e\right) \left(c d^2+a e^2\right) (e f-d g)^2 \sqrt{f+g x}\, \sqrt{a+c x^2}\right)$$

Result (type 4, 12 365 leaves) :

$$\sqrt{f+g x}\, \sqrt{a+c x^2} \left(\frac{1}{2 (-e f+d g) (d+e x)^2}+\frac{2 c d e f+c d^2 g+3 a e^2 g}{4 \left(c d^2+a e^2\right) (e f-d g)^2 (d+e x)}\right)+$$

$$\frac{1}{4 \left(c d^2+a e^2\right) g (-e f+d g)^2} \left(\frac{\left(-2 c d e f-c d^2 g-3 a e^2 g\right) (f+g x)^{3/2} \left(c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}\right)}{e \sqrt{a+\frac{c (f+g x)^2 \left(-1+\frac{f}{f+g x}\right)^2}{g^2}}}-\right.$$

$$\frac{1}{e \sqrt{a+\frac{c (f+g x)^2 \left(-1+\frac{f}{f+g x}\right)^2}{g^2}}} (e f-d g) (f+g x) \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}$$

$$\left(-\left(2 i c^2 d e f^3 \left(c f+i \sqrt{a} \sqrt{c}\, g\right) \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c}\, g\right) (f+g x)}}\, \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c}\, g\right) (f+g x)}}\right.\right.$$

$$\left.\left.\left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c}\, g}{c f+i \sqrt{a} \sqrt{c}\, g}\right]-\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c}\, g}{c f+i \sqrt{a} \sqrt{c}\, g}\right]\right)\right)\right) \Big/$$

$$\left((e f-d g)\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\, g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(i c^2 d^2 f^2 g\left(c f+i\sqrt{a}\sqrt{c}\, g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\, g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\, g\right)(f+g x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\, g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\, g}{c f+i\sqrt{a}\sqrt{c}\, g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\, g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\, g}{c f+i\sqrt{a}\sqrt{c}\, g}\right]\right)\right)\Bigg/$$

$$\left((e f-d g)\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\, g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(3 i a c e^2 f^2 g\left(c f+i\sqrt{a}\sqrt{c}\, g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\, g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\, g\right)(f+g x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\, g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\, g}{c f+i\sqrt{a}\sqrt{c}\, g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\, g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\, g}{c f+i\sqrt{a}\sqrt{c}\, g}\right]\right)\right)\Bigg/$$

$$\left((e f-d g)\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\, g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(2 i a c d e f g^2\left(c f+i\sqrt{a}\sqrt{c}\, g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\, g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\, g\right)(f+g x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\Big/$$

$$\left((e f-d g)\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(i\,a\,c\,d^2 g^3\left(c f+i\sqrt{a}\sqrt{c}\,g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\right)\Big/$$

$$\left((e f-d g)\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(3\,i\,a^2 e^2 g^3\left(c f+i\sqrt{a}\sqrt{c}\,g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\right)\Big/$$

$$\left((e f-d g)\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\left(2\,\mathrm{i}\,c^2\,d\,e^2\,f^3\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\,\frac{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left((e\,f-d\,g)^2\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)+$$

$$\left(\mathrm{i}\,c^2\,d^2\,e\,f^2\,g\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\,\frac{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left((e\,f-d\,g)^2\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)+$$

$$\left(3\,\mathrm{i}\,a\,c\,e^3\,f^2\,g\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\,\frac{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left((e\,f-d\,g)^2\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)+$$

$$\left(2\,\mathrm{i}\,a\,c\,d\,e^2\,f\,g^2\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\ \frac{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left((e\,f-d\,g)^2\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)+$$

$$\left(\mathbb{i}\,a\,c\,d^2\,e\,g^3\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\right.\right.$$

$$\left.\left.\frac{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left((e\,f-d\,g)^2\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)+$$

$$\left(3\,\mathbb{i}\,a^2\,e^3\,g^3\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\right.\right.$$

$$\left.\left.\frac{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left((e\,f-d\,g)^2\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(4\,\mathbb{i}\,c^2\,d\,e\,f^2\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\right.\right.$$

$$\left.\left.\frac{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left((e\,f-d\,g)\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\Bigg(4\, i\, a\, c\, d\, e\, g^2 \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c}\, g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c}\, g\right) (f+g x)}}\ \mathrm{EllipticF}\Big[i\, \mathrm{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}}}{\sqrt{f+g x}}\Big],$$

$$\frac{c f-i \sqrt{a} \sqrt{c}\, g}{c f+i \sqrt{a} \sqrt{c}\, g}\Big]\Bigg) \Bigg/ \Bigg((e f-d g) \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}} \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\Bigg)-\frac{1}{(e f-d g)^3}$$

$$2 c^2 d\, e^3 f^3 \Bigg(\Bigg(i\, f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c}\, g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c}\, g\right) (f+g x)}}\ \mathrm{EllipticPi}\Big[\frac{\left(c f-i \sqrt{a} \sqrt{c}\, g\right) (e f-d g)}{e \left(c f^2+a g^2\right)},$$

$$i\, \mathrm{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}}}{\sqrt{f+g x}}\Big], \frac{c f-i \sqrt{a} \sqrt{c}\, g}{c f+i \sqrt{a} \sqrt{c}\, g}\Big]\Bigg) \Bigg/ \Bigg(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\Bigg)-$$

$$\Bigg(i\, d\, g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c}\, g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c}\, g\right) (f+g x)}}\ \mathrm{EllipticPi}\Big[\frac{\left(c f-i \sqrt{a} \sqrt{c}\, g\right) (e f-d g)}{e \left(c f^2+a g^2\right)}, i\, \mathrm{ArcSinh}\Big[$$

$$\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}}}{\sqrt{f+g x}}\Big], \frac{c f-i \sqrt{a} \sqrt{c}\, g}{c f+i \sqrt{a} \sqrt{c}\, g}\Big]\Bigg) \Bigg/ \Bigg(e \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\Bigg)\Bigg)-\frac{1}{(e f-d g)^3}$$

$$c^2 d^2 e^2 f^2 g \Bigg(\Bigg(i\, f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c}\, g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c}\, g\right) (f+g x)}}\ \mathrm{EllipticPi}\Big[\frac{\left(c f-i \sqrt{a} \sqrt{c}\, g\right) (e f-d g)}{e \left(c f^2+a g^2\right)},$$

$$\left.\left.\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i} \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}},\ \frac{c f-\mathrm{i} \sqrt{a} \sqrt{c} g}{c f+\mathrm{i} \sqrt{a} \sqrt{c} g}\right]\right)\right/\left(\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i} \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(\mathrm{i}\, d g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i} \sqrt{a} \sqrt{c} g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i} \sqrt{a} \sqrt{c} g\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{\left(c f-\mathrm{i} \sqrt{a} \sqrt{c} g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\ \mathrm{i}\,\text{ArcSinh}\right[\right.$$

$$\left.\left.\left.\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i} \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}},\ \frac{c f-\mathrm{i} \sqrt{a} \sqrt{c} g}{c f+\mathrm{i} \sqrt{a} \sqrt{c} g}\right]\right)\right/\left(e \sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i} \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\right)-\frac{1}{(e f-d g)^3}$$

$$3 a c e^4 f^2 g\left(\left(\mathrm{i}\, f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i} \sqrt{a} \sqrt{c} g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i} \sqrt{a} \sqrt{c} g\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{\left(c f-\mathrm{i} \sqrt{a} \sqrt{c} g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\right.\right.\right.$$

$$\left.\left.\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i} \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}},\ \frac{c f-\mathrm{i} \sqrt{a} \sqrt{c} g}{c f+\mathrm{i} \sqrt{a} \sqrt{c} g}\right]\right)\right/\left(\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i} \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(\mathrm{i}\, d g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i} \sqrt{a} \sqrt{c} g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i} \sqrt{a} \sqrt{c} g\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{\left(c f-\mathrm{i} \sqrt{a} \sqrt{c} g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\ \mathrm{i}\,\text{ArcSinh}\right[\right.$$

$$\left.\left.\left.\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i} \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}},\ \frac{c f-\mathrm{i} \sqrt{a} \sqrt{c} g}{c f+\mathrm{i} \sqrt{a} \sqrt{c} g}\right]\right)\right/\left(e \sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i} \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\right)-\frac{1}{(e f-d g)^3}$$

$$2\,a\,c\,d\,e^3\,f\,g^2\left(\left(\left(\mathrm{i}\,f\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\Big[\frac{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\right.\right.\right.$$

$$\left.\left.\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\Big],\,\frac{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}\Big]\right)\Big/\left(\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(\mathrm{i}\,d\,g\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\Big[\frac{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\,\mathrm{i}\,\text{ArcSinh}\Big[\right.$$

$$\left.\left.\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\Big],\,\frac{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}\Big]\right)\Big/\left(e\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)\right)-\frac{1}{(e\,f-d\,g)^3}$$

$$a\,c\,d^2\,e^2\,g^3\left(\left(\left(\mathrm{i}\,f\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\Big[\frac{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\right.\right.\right.$$

$$\left.\left.\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\Big],\,\frac{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}\Big]\right)\Big/\left(\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(\mathrm{i}\,d\,g\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\Big[\frac{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\,\mathrm{i}\,\text{ArcSinh}\Big[\right.$$

$$\left.\left.\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right) \Bigg/ \left(e \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\right)-\frac{1}{(e f-d g)^3}$$

$$3 a^2 e^4 g^3 \left(\left(i f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)},\right.\right.\right.$$

$$\left.\left.i \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right) \Bigg/ \left(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(i d g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)}, i \text{ArcSinh}\left[\right.\right.\right.$$

$$\left.\left.\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right) \Bigg/ \left(e \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\right)+\frac{1}{(e f-d g)^2}$$

$$4 c^2 d e^2 f^2 \left(\left(i f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)},\right.\right.\right.$$

$$\left.\left.i \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right) \Bigg/ \left(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(\mathrm{i}\, d\, g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(e f-d g)}{e\left(c f^2+a g^2\right)}, \mathrm{i}\,\text{ArcSinh}\left[\right.\right.$$

$$\left.\left.\left.\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g}}}{\sqrt{f+g x}}\right], \frac{c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g}{c f+\mathrm{i}\sqrt{a}\sqrt{c}\, g}\right]\right) \Big/ \left(e\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\right) + \frac{1}{(e f-d g)^2}$$

$$4 a c d e^2 g^2 \left(\left(\mathrm{i}\, f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\right.\right.\right.$$

$$\left.\left.\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g}}}{\sqrt{f+g x}}\right], \frac{c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g}{c f+\mathrm{i}\sqrt{a}\sqrt{c}\, g}\right]\right) \Big/ \left(\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right) -$$

$$\left(\mathrm{i}\, d\, g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(e f-d g)}{e\left(c f^2+a g^2\right)}, \mathrm{i}\right.\right.$$

$$\left.\left.\left.\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g}}}{\sqrt{f+g x}}\right], \frac{c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g}{c f+\mathrm{i}\sqrt{a}\sqrt{c}\, g}\right]\right) \Big/ \left(e\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\right) - \frac{1}{e f-d g}$$

$$2 c^2 d e f \left(\left(\mathrm{i}\, f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\, g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\right.\right.\right.$$

$$\left.\left.\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\ \frac{c\,f-\mathbb{i}\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathbb{i}\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left(\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\sqrt{a}\,\sqrt{c}\,g}}\ \sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(\mathbb{i}\,d\,g\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathbb{i}\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\ \sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathbb{i}\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\ \text{EllipticPi}\left[\frac{\left(c\,f-\mathbb{i}\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\ \mathbb{i}\right.\right.$$

$$\left.\left.\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\ \frac{c\,f-\mathbb{i}\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathbb{i}\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left(e\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\sqrt{a}\,\sqrt{c}\,g}}\ \sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)\Bigg)+\frac{1}{e\,f-d\,g}$$

$$c^2\,d^2\,g\left(\left(\mathbb{i}\,f\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathbb{i}\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\ \sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathbb{i}\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\ \text{EllipticPi}\left[\frac{\left(c\,f-\mathbb{i}\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\right.\right.\right.$$

$$\left.\left.\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\ \frac{c\,f-\mathbb{i}\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathbb{i}\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left(\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\sqrt{a}\,\sqrt{c}\,g}}\ \sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(\mathbb{i}\,d\,g\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathbb{i}\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\ \sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathbb{i}\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\ \text{EllipticPi}\left[\frac{\left(c\,f-\mathbb{i}\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\ \mathbb{i}\right.\right.$$

$$\left.\left.\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\ \frac{c\,f-\mathbb{i}\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathbb{i}\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left(e\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\sqrt{a}\,\sqrt{c}\,g}}\ \sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)\Bigg)-\frac{1}{e\,f-d\,g}$$

$$a\,c\,e^2\,g\left(\left(\mathbb{i}\,f\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\left[\frac{\left(c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\right.\right.\right.$$

$$\left.\left.\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left(\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(\mathbb{i}\,d\,g\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\left[\frac{\left(c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\mathbb{i}\right.\right.$$

$$\left.\left.\left.\left.\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left(e\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)\right)\right)\right)$$

- **Problem 636: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(d+e\,x)^3\,\sqrt{f+g\,x}}{\sqrt{a+c\,x^2}}\,dx$$

Optimal (type 4, 531 leaves, 8 steps) :

$$-\frac{2 e \left(25 a e^2 g^2+c \left(7 e^2 f^2+12 d e f g-90 d^2 g^2\right)\right) \sqrt{f+g x} \sqrt{a+c x^2}}{105 c^2 g^2}+\frac{2 e (d+e x)^2 \sqrt{f+g x} \sqrt{a+c x^2}}{7 c}+$$

$$\frac{2 e^2 (e f+11 d g) (f+g x)^{3/2} \sqrt{a+c x^2}}{35 c g^2}+\Bigg(2 \sqrt{-a} \left(a e^2 g^2 (19 e f+189 d g)-c \left(8 e^3 f^3-42 d e^2 f^2 g+105 d^2 e f g^2+105 d^3 g^3\right)\right)$$

$$\sqrt{f+g x} \sqrt{1+\frac{c x^2}{a}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a} \sqrt{c}\, f-a g}\right]\Bigg)\Bigg/\left(105 c^{3/2} g^3 \sqrt{\frac{\sqrt{c}\, (f+g x)}{\sqrt{c}\, f+\sqrt{-a}\, g}} \sqrt{a+c x^2}\right)-$$

$$\Bigg(2 \sqrt{-a}\, e \left(c f^2+a g^2\right) \left(25 a e^2 g^2-c \left(8 e^2 f^2-42 d e f g+105 d^2 g^2\right)\right) \sqrt{\frac{\sqrt{c}\, (f+g x)}{\sqrt{c}\, f+\sqrt{-a}\, g}} \sqrt{1+\frac{c x^2}{a}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a} \sqrt{c}\, f-a g}\right]\Bigg)\Bigg/\left(105 c^{5/2} g^3 \sqrt{f+g x} \sqrt{a+c x^2}\right)$$

Result (type 4, 959 leaves) :

$$\sqrt{f+g x}\,\sqrt{a+c x^2}\left(-\frac{2 e\left(4 c e^2 f^2-21 c d e f g-105 c d^2 g^2+25 a e^2 g^2\right)}{105 c^2 g^2}+\frac{2 e^2 (e f+21 d g) x}{35 c g}+\frac{2 e^3 x^2}{7 c}\right)-$$

$$\frac{1}{105 c^2 g^4 \sqrt{-f-\frac{i \sqrt{a} g}{\sqrt{c}}}\sqrt{a+\frac{c (f+g x)^2\left(-1+\frac{f}{f+g x}\right)^2}{g^2}}}$$

$$2 (f+g x)^{3/2}\left(\sqrt{-f-\frac{i \sqrt{a} g}{\sqrt{c}}}\left(\frac{a^2 e^2 g^4 (19 e f+189 d g)}{(f+g x)^2}-c^2\left(8 e^3 f^3-42 d e^2 f^2 g+105 d^2 e f g^2+105 d^3 g^3\right)\left(-1+\frac{f}{f+g x}\right)^2+\right.\right.$$

$$\left.\left. a c g^2\left(-\frac{105 d^2 e f g^2}{(f+g x)^2}-\frac{105 d^3 g^3}{(f+g x)^2}+e^3 f\left(19+\frac{11 f^2}{(f+g x)^2}-\frac{38 f}{f+g x}\right)+21 d e^2 g\left(9+\frac{11 f^2}{(f+g x)^2}-\frac{18 f}{f+g x}\right)\right)\right)+\frac{1}{\sqrt{f+g x}}\right.$$

$$\sqrt{c}\left(-i a \sqrt{c} e^2 f g^2 (19 e f+189 d g)+a^{3/2} e^2 g^3 (19 e f+189 d g)+i c^{3/2} f\left(8 e^3 f^3-42 d e^2 f^2 g+105 d^2 e f g^2+105 d^3 g^3\right)-\right.$$

$$\left.\sqrt{a} c g\left(8 e^3 f^3-42 d e^2 f^2 g+105 d^2 e f g^2+105 d^3 g^3\right)\right)\sqrt{1-\frac{f}{f+g x}-\frac{i \sqrt{a} g}{\sqrt{c} (f+g x)}}$$

$$\sqrt{1-\frac{f}{f+g x}+\frac{i \sqrt{a} g}{\sqrt{c} (f+g x)}}\ \text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i \sqrt{a} g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c} f-i \sqrt{a} g}{\sqrt{c} f+i \sqrt{a} g}\right]+\frac{1}{\sqrt{f+g x}}$$

$$g\left(\sqrt{c} f+i \sqrt{a} g\right)\left(-105 i c^{3/2} d^3 g^2-25 a^{3/2} e^3 g^2-3 i a \sqrt{c} e^2 g (2 e f-63 d g)+\sqrt{a} c e\left(8 e^2 f^2-42 d e f g+105 d^2 g^2\right)\right)$$

$$\left.\sqrt{1-\frac{f}{f+g x}-\frac{i \sqrt{a} g}{\sqrt{c} (f+g x)}}\sqrt{1-\frac{f}{f+g x}+\frac{i \sqrt{a} g}{\sqrt{c} (f+g x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i \sqrt{a} g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c} f-i \sqrt{a} g}{\sqrt{c} f+i \sqrt{a} g}\right]\right)$$

- **Problem 637: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e x)^2 \sqrt{f+g x}}{\sqrt{a+c x^2}}\,dx$$

Optimal (type 4, 410 leaves, 7 steps):

$$\frac{2\,e\,(e\,f+7\,d\,g)\,\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}{15\,c\,g}+\frac{2\,e\,(d+e\,x)\,\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}{5\,c}+$$

$$\left(2\,\sqrt{-a}\,\left(9\,a\,e^2\,g^2+c\,\left(2\,e^2\,f^2-10\,d\,e\,f\,g-15\,d^2\,g^2\right)\right)\,\sqrt{f+g\,x}\,\sqrt{1+\frac{c\,x^2}{a}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,g}{\sqrt{-a}\,\sqrt{c}\,f-a\,g}\right]\right)\Big/$$

$$\left(15\,c^{3/2}\,g^2\,\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+c\,x^2}\right)-$$

$$\left(4\,\sqrt{-a}\,e\,(e\,f-5\,d\,g)\,\left(c\,f^2+a\,g^2\right)\,\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c\,x^2}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,g}{\sqrt{-a}\,\sqrt{c}\,f-a\,g}\right]\right)\Big/$$

$$\left(15\,c^{3/2}\,g^2\,\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}\right)$$

Result (type 4, 596 leaves) :

$$\frac{1}{15\,c^2\,g^3\,\sqrt{a+c\,x^2}}\,2\,\sqrt{f+g\,x}\,\Bigg(c\,e\,g^2\,\left(a+c\,x^2\right)\,\left(10\,d\,g+e\,\left(f+3\,g\,x\right)\right)+\frac{g^2\,\left(-9\,a^2\,e^2\,g^2+c^2\,\left(-2\,e^2\,f^2+10\,d\,e\,f\,g+15\,d^2\,g^2\right)\,x^2+a\,c\,\left(10\,d\,e\,f\,g+15\,d^2\,g^2-e^2\,\left(2\,f^2+9\,g^2\,x^2\right)\right)\right)}{f+g\,x}-$$

$$i\,c\,\sqrt{-f-\frac{i\,\sqrt{a}\,g}{\sqrt{c}}}\,\left(9\,a\,e^2\,g^2+c\,\left(2\,e^2\,f^2-10\,d\,e\,f\,g-15\,d^2\,g^2\right)\right)\,\sqrt{\frac{g\,\left(\frac{i\,\sqrt{a}}{\sqrt{c}}+x\right)}{f+g\,x}}$$

$$\sqrt{-\frac{\frac{i\,\sqrt{a}\,g}{\sqrt{c}}-g\,x}{f+g\,x}}\,\sqrt{f+g\,x}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\,\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\,\frac{\sqrt{c}\,f-i\,\sqrt{a}\,g}{\sqrt{c}\,f+i\,\sqrt{a}\,g}\right]+$$

$$\frac{1}{\sqrt{-f-\frac{i\,\sqrt{a}\,g}{\sqrt{c}}}}\sqrt{c}\,g\,\left(\sqrt{c}\,f+i\,\sqrt{a}\,g\right)\,\left(15\,i\,c\,d^2\,g-9\,i\,a\,e^2\,g+2\,\sqrt{a}\,\sqrt{c}\,e\,\left(e\,f-5\,d\,g\right)\right)\,\sqrt{\frac{g\,\left(\frac{i\,\sqrt{a}}{\sqrt{c}}+x\right)}{f+g\,x}}$$

$$\sqrt{-\frac{\frac{i\,\sqrt{a}\,g}{\sqrt{c}}-g\,x}{f+g\,x}}\,\sqrt{f+g\,x}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\,\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\,\frac{\sqrt{c}\,f-i\,\sqrt{a}\,g}{\sqrt{c}\,f+i\,\sqrt{a}\,g}\right]\Bigg)$$

- **Problem 638: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(d+e\,x\right)\,\sqrt{f+g\,x}}{\sqrt{a+c\,x^2}}\,dx$$

Optimal (type 4, 331 leaves, 6 steps) :

$$\frac{2 e \sqrt{f+g x}\, \sqrt{a+c x^2}}{3 c}-\frac{2 \sqrt{-a}\, (e f+3 d g) \sqrt{f+g x}\, \sqrt{1+\frac{c x^2}{a}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\, \sqrt{c}\, f-a g}\right]}{3 \sqrt{c}\, g \sqrt{\frac{\sqrt{c}\, (f+g x)}{\sqrt{c}\, f+\sqrt{-a}\, g}}\, \sqrt{a+c x^2}}+$$

$$\frac{2 \sqrt{-a}\, e \left(c f^2+a g^2\right) \sqrt{\frac{\sqrt{c}\, (f+g x)}{\sqrt{c}\, f+\sqrt{-a}\, g}}\, \sqrt{1+\frac{c x^2}{a}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\, \sqrt{c}\, f-a g}\right]}{3 c^{3/2} g \sqrt{f+g x}\, \sqrt{a+c x^2}}$$

Result (type 4, 464 leaves) :

$$\frac{1}{3 c \sqrt{a+c x^2}} 2 \sqrt{f+g x} \left(e \left(a+c x^2\right)+\frac{(e f+3 d g) \left(a+c x^2\right)}{f+g x}+\right.$$

$$1\Big/ g^2 i\, c \sqrt{-f-\frac{i \sqrt{a}\, g}{\sqrt{c}}}\, (e f+3 d g) \sqrt{\frac{g \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{f+g x}}\, \sqrt{-\frac{\frac{i \sqrt{a}\, g}{\sqrt{c}}-g x}{f+g x}}\, \sqrt{f+g x}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i \sqrt{a}\, g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c}\, f-i \sqrt{a}\, g}{\sqrt{c}\, f+i \sqrt{a}\, g}\right]+$$

$$1\Big/ \left(g \sqrt{-f-\frac{i \sqrt{a}\, g}{\sqrt{c}}}\right) i \left(3 \sqrt{c}\, d+i \sqrt{a}\, e\right) \left(\sqrt{c}\, f+i \sqrt{a}\, g\right) \sqrt{\frac{g \left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{f+g x}}$$

$$\left.\sqrt{-\frac{\frac{i \sqrt{a}\, g}{\sqrt{c}}-g x}{f+g x}}\, \sqrt{f+g x}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i \sqrt{a}\, g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c}\, f-i \sqrt{a}\, g}{\sqrt{c}\, f+i \sqrt{a}\, g}\right]\right)$$

- **Problem 639: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{f+g x}}{\sqrt{a+c x^2}}\, dx$$

Optimal (type 4, 136 leaves, 2 steps) :

$$-\frac{2\sqrt{-a}\,\sqrt{f+g\,x}\,\sqrt{1+\frac{c\,x^2}{a}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\,-\frac{2\,a\,g}{\sqrt{-a}\,\sqrt{c}\,f-a\,g}\right]}{\sqrt{c}\,\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 294 leaves):

$$\left(2\,\mathrm{i}\left(\sqrt{c}\,f+\mathrm{i}\sqrt{a}\,g\right)\sqrt{\frac{g\left(\sqrt{a}+\mathrm{i}\sqrt{c}\,x\right)}{-\mathrm{i}\sqrt{c}\,f+\sqrt{a}\,g}}\,\sqrt{f+g\,x}\right.$$

$$\left.\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f-\mathrm{i}\sqrt{a}\,g}}\right],\,\frac{\sqrt{c}\,f-\mathrm{i}\sqrt{a}\,g}{\sqrt{c}\,f+\mathrm{i}\sqrt{a}\,g}\right]-\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f-\mathrm{i}\sqrt{a}\,g}}\right],\,\frac{\sqrt{c}\,f-\mathrm{i}\sqrt{a}\,g}{\sqrt{c}\,f+\mathrm{i}\sqrt{a}\,g}\right]\right)\right)\Big/$$

$$\left(\sqrt{c}\,g\,\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{g\left(\mathrm{i}\sqrt{a}+\sqrt{c}\,x\right)}}\,\sqrt{a+c\,x^2}\right)$$

■ Problem 640: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{f+g\,x}}{(d+e\,x)\,\sqrt{a+c\,x^2}}\,dx$$

Optimal (type 4, 319 leaves, 7 steps):

$$-\frac{2\sqrt{-a}\,g\,\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c\,x^2}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\,-\frac{2\,a\,g}{\sqrt{-a}\,\sqrt{c}\,f-a\,g}\right]}{\sqrt{c}\,e\,\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}-$$

$$\frac{2\,(e\,f-d\,g)\,\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c\,x^2}{a}}\;\text{EllipticPi}\left[\frac{2\,e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e},\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\,\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right]}{e\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 300 leaves):

$$-\left(2\,i\sqrt{\frac{g\left(\sqrt{a}+i\sqrt{c}\,x\right)}{-i\sqrt{c}\,f+\sqrt{a}\,g}}\,\sqrt{f+g\,x}\,\left(\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f-i\sqrt{a}\,g}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]-\text{EllipticPi}\left[\frac{e\left(f-\frac{i\sqrt{a}\,g}{\sqrt{c}}\right)}{e\,f-d\,g},\,i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f-i\sqrt{a}\,g}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]\right)\right)\Big/\left(e\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{g\left(i\sqrt{a}+\sqrt{c}\,x\right)}}\,\sqrt{a+c\,x^2}\right)$$

- **Problem 641: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{f+g\,x}}{(d+e\,x)^2\sqrt{a+c\,x^2}}\,dx$$

Optimal (type 4, 698 leaves, 14 steps):

$$-\frac{e\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}{\left(c\,d^2+a\,e^2\right)(d+e\,x)}-\frac{\sqrt{-a}\,\sqrt{c}\,\sqrt{f+g\,x}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,g}{\sqrt{-a}\,\sqrt{c}\,f-a\,g}\right]}{\left(c\,d^2+a\,e^2\right)\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+c\,x^2}}+$$

$$\frac{\sqrt{-a}\,\sqrt{c}\,f\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,g}{\sqrt{-a}\,\sqrt{c}\,f-a\,g}\right]}{\left(c\,d^2+a\,e^2\right)\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}-$$

$$\frac{\sqrt{-a}\,\sqrt{c}\,d\,g\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,g}{\sqrt{-a}\,\sqrt{c}\,f-a\,g}\right]}{e\left(c\,d^2+a\,e^2\right)\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}-$$

$$\frac{\left(a\,e^2\,g+c\,d\,(2\,e\,f-d\,g)\right)\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticPi}\left[\frac{2\,e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e},\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right]}{e\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)\left(c\,d^2+a\,e^2\right)\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 1459 leaves) :

$$-\frac{e\sqrt{f+g x}\,\sqrt{a+c x^2}}{\left(c d^2+a e^2\right)(d+e x)}-\frac{1}{e\left(c d^2+a e^2\right) g\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(e f-d g)\sqrt{a+\frac{c(f+g x)^2\left(-1+\frac{f}{f+g x}\right)^2}{g^2}}}$$

$$(f+g x)^{3/2}\left(-c e^2 f\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}+c d e g\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}-\frac{c e^2 f^3\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{(f+g x)^2}+\frac{c d e f^2 g\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{(f+g x)^2}-\frac{a e^2 f g^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{(f+g x)^2}+\right.$$

$$\frac{a d e g^3\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{(f+g x)^2}+\frac{2 c e^2 f^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{f+g x}-\frac{2 c d e f g\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{f+g x}+\frac{1}{\sqrt{f+g x}}i\sqrt{c}\,e\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)(e f-d g)$$

$$\sqrt{1-\frac{f}{f+g x}-\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g x)}}\,\sqrt{1-\frac{f}{f+g x}+\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g x)}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]+$$

$$\frac{1}{\sqrt{f+g x}}e\left(i\sqrt{c}\,d+\sqrt{a}\,e\right) g\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\sqrt{1-\frac{f}{f+g x}-\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g x)}}\,\sqrt{1-\frac{f}{f+g x}+\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]-\frac{1}{\sqrt{f+g x}}2\,i\,c d e f g\sqrt{1-\frac{f}{f+g x}-\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g x)}}\,\sqrt{1-\frac{f}{f+g x}+\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g x)}}$$

$$\text{EllipticPi}\left[\frac{\sqrt{c}\,(e f-d g)}{e\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)},i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]+\frac{1}{\sqrt{f+g x}}i\,c d^2 g^2\sqrt{1-\frac{f}{f+g x}-\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g x)}}$$

$$\sqrt{1-\frac{f}{f+g x}+\frac{i\sqrt{a}\,g}{\sqrt{c}\,(f+g x)}}\,\text{EllipticPi}\left[\frac{\sqrt{c}\,(e f-d g)}{e\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)},i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]-\frac{1}{\sqrt{f+g x}}i\,a e^2 g^2$$

$$\sqrt{1-\frac{f}{f+g x}-\frac{i \sqrt{a}\, g}{\sqrt{c}\,(f+g x)}}\,\sqrt{1-\frac{f}{f+g x}+\frac{i \sqrt{a}\, g}{\sqrt{c}\,(f+g x)}}\;\text{EllipticPi}\left[\frac{\sqrt{c}\,(e f-d g)}{e\left(\sqrt{c}\, f+i \sqrt{a}\, g\right)},\, i\, \text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i \sqrt{a}\, g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\, \frac{\sqrt{c}\, f-i \sqrt{a}\, g}{\sqrt{c}\, f+i \sqrt{a}\, g}\right]\Bigg)$$

- **Problem 642: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{f+g x}}{(d+e x)^3 \sqrt{a+c x^2}}\, dx$$

Optimal (type 4, 1246 leaves, 23 steps) :

$$-\frac{e \sqrt{f+g x}\, \sqrt{a+c x^2}}{2\left(c d^2+a e^2\right)(d+e x)^2}-\frac{e\left(a e^2 g+c d\,(6 e f-5 d g)\right)\sqrt{f+g x}\, \sqrt{a+c x^2}}{4\left(c d^2+a e^2\right)^2 (e f-d g)(d+e x)}-$$

$$\frac{\sqrt{-a}\, \sqrt{c}\left(a e^2 g+c d\,(6 e f-5 d g)\right)\sqrt{f+g x}\, \sqrt{1+\frac{c x^2}{a}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],\, -\frac{2 a g}{\sqrt{-a}\, \sqrt{c}\, f-a g}\right]}{4\left(c d^2+a e^2\right)^2 (e f-d g)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\, f+\sqrt{-a}\, g}}\, \sqrt{a+c x^2}}+$$

$$\frac{\sqrt{-a}\, \sqrt{c}\, g \sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\, f+\sqrt{-a}\, g}}\, \sqrt{1+\frac{c x^2}{a}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],\, -\frac{2 a g}{\sqrt{-a}\, \sqrt{c}\, f-a g}\right]}{2 e\left(c d^2+a e^2\right)\sqrt{f+g x}\, \sqrt{a+c x^2}}+$$

$$\left(\sqrt{-a}\, \sqrt{c}\, f\left(a e^2 g+c d\,(6 e f-5 d g)\right)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\, f+\sqrt{-a}\, g}}\, \sqrt{1+\frac{c x^2}{a}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],\, -\frac{2 a g}{\sqrt{-a}\, \sqrt{c}\, f-a g}\right]\right)\Big/$$

$$\left(4\left(c d^2+a e^2\right)^2 (e f-d g)\sqrt{f+g x}\, \sqrt{a+c x^2}\right)-$$

$$\left(\sqrt{-a}\,\sqrt{c}\,d\,g\,\left(a\,e^2\,g+c\,d\,(6\,e\,f-5\,d\,g)\right)\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,g}{\sqrt{-a}\,\sqrt{c}\,f-a\,g}\right]\right)\Big/$$

$$\left(4\,e\,\left(c\,d^2+a\,e^2\right)^2\,(e\,f-d\,g)\,\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}\right)+\frac{c\,(e\,f-3\,d\,g)\,\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticPi}\left[\frac{2\,e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e},\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\frac{2\,\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right]}{e\,\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)\left(c\,d^2+a\,e^2\right)\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}+$$

$$\left(\left(a\,e^2\,g+c\,d\,(6\,e\,f-5\,d\,g)\right)\left(a\,e^2\,g-c\,d\,(2\,e\,f-3\,d\,g)\right)\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c\,x^2}{a}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{2\,e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e},\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\frac{2\,\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right]\right)\Big/\left(4\,e\,\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)\left(c\,d^2+a\,e^2\right)^2\,(e\,f-d\,g)\,\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}\right)$$

Result (type 4, 12 363 leaves) :

$$\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}\,\left(-\frac{e}{2\,\left(c\,d^2+a\,e^2\right)(d+e\,x)^2}-\frac{e\,\left(6\,c\,d\,e\,f-5\,c\,d^2\,g+a\,e^2\,g\right)}{4\,\left(c\,d^2+a\,e^2\right)^2\,(e\,f-d\,g)\,(d+e\,x)}\right)-$$

$$\frac{1}{4\,\left(c\,d^2+a\,e^2\right)^2\,g\,(-e\,f+d\,g)}\left(\frac{\left(6\,c\,d\,e\,f-5\,c\,d^2\,g+a\,e^2\,g\right)(f+g\,x)^{3/2}\left(c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}\right)}{\sqrt{a+\frac{c\,(f+g\,x)^2\left(-1+\frac{f}{f+g\,x}\right)^2}{g^2}}}+\right.$$

$$\frac{1}{\sqrt{a+\frac{c\,(f+g\,x)^2\left(-1+\frac{f}{f+g\,x}\right)^2}{g^2}}}\,(e\,f-d\,g)\,(f+g\,x)\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}$$

$$\left(-\left(6\,i\,c^2\,d\,e\,f^3\left(c\,f+i\sqrt{a}\sqrt{c}\,g\right)\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\sqrt{c}\,g\right)(f+g\,x)}}\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\sqrt{c}\,g\right)(f+g\,x)}}\right.\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\sqrt{c}\,g}{c\,f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\sqrt{c}\,g}{c\,f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\right)\Big/$$

$$\left((e\,f-d\,g)\left(c\,f^2+a\,g^2\right)\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)+$$

$$\left(5\,i\,c^2\,d^2\,f^2\,g\left(c\,f+i\sqrt{a}\sqrt{c}\,g\right)\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\sqrt{c}\,g\right)(f+g\,x)}}\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\sqrt{c}\,g\right)(f+g\,x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\sqrt{c}\,g}{c\,f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\sqrt{c}\,g}{c\,f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\right)\Big/$$

$$\left((e\,f-d\,g)\left(c\,f^2+a\,g^2\right)\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(i\,a\,c\,e^2\,f^2\,g\left(c\,f+i\sqrt{a}\sqrt{c}\,g\right)\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\sqrt{c}\,g\right)(f+g\,x)}}\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\sqrt{c}\,g\right)(f+g\,x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\sqrt{c}\,g}{c\,f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\sqrt{c}\,g}{c\,f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\right)\Big/$$

$$\left((e f-d g)\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(6 i a c d e f g^2\left(c f+i\sqrt{a}\sqrt{c}g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}g\right)(f+g x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}g}{c f+i\sqrt{a}\sqrt{c}g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}g}{c f+i\sqrt{a}\sqrt{c}g}\right]\right)\right)\Big/$$

$$\left((e f-d g)\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\left(5 i a c d^2 g^3\left(c f+i\sqrt{a}\sqrt{c}g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}g\right)(f+g x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}g}{c f+i\sqrt{a}\sqrt{c}g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}g}{c f+i\sqrt{a}\sqrt{c}g}\right]\right)\right)\Big/$$

$$\left((e f-d g)\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(i a^2 e^2 g^3\left(c f+i\sqrt{a}\sqrt{c}g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}g\right)(f+g x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\right)\Big/$$

$$\left((e f-d g)\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\left(6\,i\,c^2 d\,e^2 f^3\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\Big/\left((e f-d g)^2\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(5\,i\,c^2 d^2 e\,f^2 g\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\Big/\left((e f-d g)^2\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\left(i\,a\,c\,e^3 f^2 g\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\right.\right.$$

$$\left.\left.\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Bigg/\left((e\,f-d\,g)^2\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)+$$

$$\left(6\,i\,a\,c\,d\,e^2\,f\,g^2\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Bigg/\left((e\,f-d\,g)^2\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(5\,i\,a\,c\,d^2\,e\,g^3\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Bigg/\left((e\,f-d\,g)^2\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)+$$

$$\left(i\,a^2\,e^3\,g^3\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\right.\right.$$

$$\left.\left.\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Bigg/\left((e\,f-d\,g)^2\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(12\,i\,c^2\,d\,e\,f^2\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\right.\right.$$

$$\left.\left.\frac{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left((e\,f-d\,g)\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(12\,i\,a\,c\,d\,e\,g^2\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\right.\right.$$

$$\left.\left.\frac{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left((e\,f-d\,g)\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-\frac{1}{(e\,f-d\,g)^3}$$

$$6\,c^2\,d\,e^3\,f^3\left(\left(i\,f\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\left[\frac{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\right.\right.\right.$$

$$\left.\left.i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\,\frac{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left(\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(i\,d\,g\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\left[\frac{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\,i\,\text{ArcSinh}\left[\right.\right.\right.$$

$$\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\Big],\ \frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\Big]\Bigg)\Bigg/\left(e\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\Bigg)+\frac{1}{(e f-d g)^3}$$

$$5 c^2 d^2 e^2 f^2 g\left(\left(\left(i f\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\ \text{EllipticPi}\Big[\frac{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\right.\right.\right.$$

$$i\,\text{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\Big],\ \frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\Big]\Bigg)\Bigg/\left(\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(i d g\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\ \text{EllipticPi}\Big[\frac{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\ i\,\text{ArcSinh}\Big[\right.$$

$$\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\Big],\ \frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\Big]\Bigg)\Bigg/\left(e\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\Bigg)-\frac{1}{(e f-d g)^3}$$

$$a c e^4 f^2 g\left(\left(\left(i f\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\ \text{EllipticPi}\Big[\frac{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\right.\right.\right.$$

$$i\,\text{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\Big],\ \frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\Big]\Bigg)\Bigg/\left(\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(i\, d\, g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c}\, g\right)(e f-d g)}{e\left(c f^2+a g^2\right)}, i\, \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c}\, g}{c f+i \sqrt{a} \sqrt{c}\, g}\right]\right) \Bigg/ \left(e \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\right) - \frac{1}{(e f-d g)^3}$$

$$6\, a\, c\, d\, e^3 f g^2 \left(\left(i\, f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c}\, g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\right.\right.\right.$$

$$\left. i\, \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c}\, g}{c f+i \sqrt{a} \sqrt{c}\, g}\right]\right) \Bigg/ \left(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right) -$$

$$\left(i\, d\, g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c}\, g\right)(e f-d g)}{e\left(c f^2+a g^2\right)}, i\, \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c}\, g}{c f+i \sqrt{a} \sqrt{c}\, g}\right]\right) \Bigg/ \left(e \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\right) + \frac{1}{(e f-d g)^3}$$

$$5\, a\, c\, d^2 e^2 g^3 \left(\left(i\, f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c}\, g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\right.\right.\right.$$

$$\left.\left.\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}{c f+\mathbb{i}\sqrt{a}\sqrt{c}\,g}\right]\right)\Bigg/\left(\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(\mathbb{i}\,d g\sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathbb{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\,\text{EllipticPi}\left[\frac{\left(c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\mathbb{i}\,\text{ArcSinh}\left[\right.\right.\right.$$

$$\left.\left.\left.\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}{c f+\mathbb{i}\sqrt{a}\sqrt{c}\,g}\right]\right)\Bigg/\left(e\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\Bigg)-\frac{1}{(e f-d g)^3}$$

$$a^2 e^4 g^3\left(\left(\mathbb{i}\,f\sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathbb{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\,\text{EllipticPi}\left[\frac{\left(c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\right.\right.\right.$$

$$\left.\left.\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}{c f+\mathbb{i}\sqrt{a}\sqrt{c}\,g}\right]\right)\Bigg/\left(\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(\mathbb{i}\,d g\sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathbb{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\,\text{EllipticPi}\left[\frac{\left(c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\mathbb{i}\,\text{ArcSinh}\left[\right.\right.\right.$$

$$\left.\left.\left.\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}{c f+\mathbb{i}\sqrt{a}\sqrt{c}\,g}\right]\right)\Bigg/\left(e\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\Bigg)+\frac{1}{(e f-d g)^2}$$

$$12\,c^2\,d\,e^2\,f^2\left(\left(\left(i\,f\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\left[\frac{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\right.\right.\right.\right.$$

$$\left.\left.i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\,\frac{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left(\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(i\,d\,g\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\left[\frac{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\,i\,\text{ArcSinh}\left[\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\,\frac{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left(e\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)\right)+\frac{1}{(e\,f-d\,g)^2}$$

$$12\,a\,c\,d\,e^2\,g^2\left(\left(\left(i\,f\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\left[\frac{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\right.\right.\right.\right.$$

$$\left.\left.i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\,\frac{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left(\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(i\,d\,g\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\left[\frac{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\,i\right.\right.$$

$$\text{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c} g}}}{\sqrt{f+g x}}\Big],\ \frac{c f-i\sqrt{a}\sqrt{c} g}{c f+i\sqrt{a}\sqrt{c} g}\Big]\Bigg)\Bigg/\left(e\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c} g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-\frac{1}{e f-d g}$$

$$6 c^2 d e f\Bigg(\Bigg(i f\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c} g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c} g\right)(f+g x)}}\ \text{EllipticPi}\Big[\frac{\left(c f-i\sqrt{a}\sqrt{c} g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},$$

$$i\,\text{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c} g}}}{\sqrt{f+g x}}\Big],\ \frac{c f-i\sqrt{a}\sqrt{c} g}{c f+i\sqrt{a}\sqrt{c} g}\Big]\Bigg)\Bigg/\left(\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c} g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\Bigg(i d g\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c} g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c} g\right)(f+g x)}}\ \text{EllipticPi}\Big[\frac{\left(c f-i\sqrt{a}\sqrt{c} g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\ i$$

$$\text{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c} g}}}{\sqrt{f+g x}}\Big],\ \frac{c f-i\sqrt{a}\sqrt{c} g}{c f+i\sqrt{a}\sqrt{c} g}\Big]\Bigg)\Bigg/\left(e\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c} g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+\frac{1}{e f-d g}$$

$$3 c^2 d^2 g\Bigg(\Bigg(i f\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c} g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c} g\right)(f+g x)}}\ \text{EllipticPi}\Big[\frac{\left(c f-i\sqrt{a}\sqrt{c} g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},$$

$$i\,\text{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c} g}}}{\sqrt{f+g x}}\Big],\ \frac{c f-i\sqrt{a}\sqrt{c} g}{c f+i\sqrt{a}\sqrt{c} g}\Big]\Bigg)\Bigg/\left(\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c} g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(i\,d\,g\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\,\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\text{EllipticPi}\left[\frac{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)(e\,f-d\,g)}{e\left(c\,f^2+a\,g^2\right)},\,i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\,\frac{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Bigg/\left(e\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-\frac{1}{e\,f-d\,g}$$

$$3\,a\,c\,e^2\,g\left(\left(i\,f\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\,\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\text{EllipticPi}\left[\frac{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)(e\,f-d\,g)}{e\left(c\,f^2+a\,g^2\right)},\right.\right.\right.$$

$$\left.\left.i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\,\frac{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Bigg/\left(\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(i\,d\,g\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\,\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\text{EllipticPi}\left[\frac{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)(e\,f-d\,g)}{e\left(c\,f^2+a\,g^2\right)},\,i\right.\right.$$

$$\left.\left.\left.\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\,\frac{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Bigg/\left(e\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)\right)\right)$$

- **Problem 643: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(f+g\,x)^{5/2}}{(d+e\,x)\,\sqrt{a+c\,x^2}}\,dx$$

Optimal (type 4, 600 leaves, 16 steps):

$$\frac{2 g^2 \sqrt{f+g x}\,\sqrt{a+c x^2}}{3 c e}-\frac{2 \sqrt{-a}\, g\,(7 e f-3 d g)\,\sqrt{f+g x}\,\sqrt{1+\frac{c x^2}{a}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1+\frac{a \sqrt{c}\, x}{(-a)^{3/2}}}}{\sqrt{2}}\right],\frac{2 a g}{-\sqrt{-a}\,\sqrt{c}\, f+a g}\right]}{3 \sqrt{c}\, e^2 \sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\, f+\sqrt{-a}\, g}}\,\sqrt{a+c x^2}}+$$

$$\left(2 \sqrt{-a}\, g\left(a e^2 g^2+c\left(-2 e^2 f^2+6 d e f g-3 d^2 g^2\right)\right)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\, f+\sqrt{-a}\, g}}\,\sqrt{1+\frac{c x^2}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1+\frac{a \sqrt{c}\, x}{(-a)^{3/2}}}}{\sqrt{2}}\right],\frac{2 a g}{-\sqrt{-a}\,\sqrt{c}\, f+a g}\right]\right)\Big/$$

$$\left(3 c^{3/2} e^3 \sqrt{f+g x}\,\sqrt{a+c x^2}\right)-$$

$$\left(2\,(e f-d g)^2 \sqrt{\frac{g\left(\sqrt{-a}-\sqrt{c}\, x\right)}{\sqrt{c}\, f+\sqrt{-a}\, g}}\,\sqrt{-\frac{g\left(\sqrt{-a}+\sqrt{c}\, x\right)}{\sqrt{c}\, f-\sqrt{-a}\, g}}\;\text{EllipticPi}\left[\frac{e\left(f+\frac{\sqrt{-a}\, g}{\sqrt{c}}\right)}{e f-d g},\text{ArcSin}\left[\sqrt{\frac{c}{c f+\sqrt{-a}\,\sqrt{c}\, g}}\,\sqrt{f+g x}\right],\frac{\sqrt{c}\, f+\sqrt{-a}\, g}{\sqrt{c}\, f-\sqrt{-a}\, g}\right]\right)\Big/$$

$$\left(e^3 \sqrt{\frac{c}{c f+\sqrt{-a}\,\sqrt{c}\, g}}\,\sqrt{a+c x^2}\right)$$

Result (type 4, 1440 leaves) :

$$\frac{2 g^2 \sqrt{f+g x}\,\sqrt{a+c x^2}}{3 c e}+\frac{1}{3 c e^3 \sqrt{-f-\frac{i \sqrt{a}\, g}{\sqrt{c}}}\,\sqrt{a+\frac{c\,(f+g x)^2\left(-1+\frac{f}{f+g x}\right)^2}{g^2}}}$$

$$2\,(f+g x)^{3/2}\left(7 c e^2 f \sqrt{-f-\frac{i \sqrt{a}\, g}{\sqrt{c}}}-3 c d e g \sqrt{-f-\frac{i \sqrt{a}\, g}{\sqrt{c}}}+\frac{7 c e^2 f^3 \sqrt{-f-\frac{i \sqrt{a}\, g}{\sqrt{c}}}}{(f+g x)^2}-\frac{3 c d e f^2 g \sqrt{-f-\frac{i \sqrt{a}\, g}{\sqrt{c}}}}{(f+g x)^2}+\frac{7 a e^2 f g^2 \sqrt{-f-\frac{i \sqrt{a}\, g}{\sqrt{c}}}}{(f+g x)^2}-\right.$$

$$\frac{3 a d e g^3 \sqrt{-f-\frac{i \sqrt{a}\, g}{\sqrt{c}}}}{(f+g x)^2}-\frac{14 c e^2 f^2 \sqrt{-f-\frac{i \sqrt{a}\, g}{\sqrt{c}}}}{f+g x}+\frac{6 c d e f g \sqrt{-f-\frac{i \sqrt{a}\, g}{\sqrt{c}}}}{f+g x}+\frac{1}{\sqrt{f+g x}}\sqrt{c}\, e\left(-i \sqrt{c}\, f+\sqrt{a}\, g\right)(7 e f-3 d g)$$

$$\sqrt{1-\frac{f}{f+g x}-\frac{i \sqrt{a}\, g}{\sqrt{c}\,(f+g x)}}\,\sqrt{1-\frac{f}{f+g x}+\frac{i \sqrt{a}\, g}{\sqrt{c}\,(f+g x)}}\ \text{EllipticE}\Big[i\,\text{ArcSinh}\Big[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g x}}\Big],\ \frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\Big]+$$

$$\frac{1}{\sqrt{f+g x}}\,i\,e\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\left(i\sqrt{a}\,e\,g+\sqrt{c}\,(6 e f-3 d g)\right)\sqrt{1-\frac{f}{f+g x}-\frac{i \sqrt{a}\, g}{\sqrt{c}\,(f+g x)}}\,\sqrt{1-\frac{f}{f+g x}+\frac{i \sqrt{a}\, g}{\sqrt{c}\,(f+g x)}}$$

$$\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g x}}\Big],\ \frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\Big]+\frac{1}{\sqrt{f+g x}}3\,i\,c\,e^2 f^2\sqrt{1-\frac{f}{f+g x}-\frac{i \sqrt{a}\, g}{\sqrt{c}\,(f+g x)}}\,\sqrt{1-\frac{f}{f+g x}+\frac{i \sqrt{a}\, g}{\sqrt{c}\,(f+g x)}}$$

$$\text{EllipticPi}\Big[\frac{\sqrt{c}\,(e f-d g)}{e\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)},\ i\,\text{ArcSinh}\Big[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g x}}\Big],\ \frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\Big]-\frac{1}{\sqrt{f+g x}}6\,i\,c\,d\,e\,f\,g\sqrt{1-\frac{f}{f+g x}-\frac{i \sqrt{a}\, g}{\sqrt{c}\,(f+g x)}}$$

$$\sqrt{1-\frac{f}{f+g x}+\frac{i \sqrt{a}\, g}{\sqrt{c}\,(f+g x)}}\ \text{EllipticPi}\Big[\frac{\sqrt{c}\,(e f-d g)}{e\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)},\ i\,\text{ArcSinh}\Big[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g x}}\Big],\ \frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\Big]+\frac{1}{\sqrt{f+g x}}3\,i\,c\,d^2 g^2$$

$$\sqrt{1-\frac{f}{f+g x}-\frac{i \sqrt{a}\, g}{\sqrt{c}\,(f+g x)}}\,\sqrt{1-\frac{f}{f+g x}+\frac{i \sqrt{a}\, g}{\sqrt{c}\,(f+g x)}}\ \text{EllipticPi}\Big[\frac{\sqrt{c}\,(e f-d g)}{e\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)},\ i\,\text{ArcSinh}\Big[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g x}}\Big],\ \frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\Big]\Bigg)$$

- **Problem 644: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(f+g x)^{3/2}}{(d+e x)\sqrt{a+c x^2}}\,dx$$

Optimal (type 4, 469 leaves, 10 steps) :

$$-\frac{2\sqrt{-a}\,g\sqrt{f+g x}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]}{\sqrt{c}\,e\,\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+c x^2}}-$$

$$\frac{2\sqrt{-a}\,g\,(e f-d g)\,\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]}{\sqrt{c}\,e^2\,\sqrt{f+g x}\,\sqrt{a+c x^2}}-$$

$$\frac{2\,(e f-d g)^2\,\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticPi}\left[\frac{2 e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e},\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right]}{e^2\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)\sqrt{f+g x}\,\sqrt{a+c x^2}}$$

Result (type 4, 927 leaves) :

$$\frac{1}{\sqrt{f+g x}\ \sqrt{a+c x^2}}\, 2\sqrt{\frac{\sqrt{c}\ (f+g x)}{\sqrt{c}\ f-\mathbb{i}\sqrt{a}\ g}}$$

$$\left(\frac{2\,\mathbb{i}\sqrt{a}\ f g\sqrt{1+\frac{c x^2}{a}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\mathbb{i}\sqrt{c}\ x}{\sqrt{a}}}}{\sqrt{2}}\right],\frac{2\sqrt{a}\ g}{\mathbb{i}\sqrt{c}\ f+\sqrt{a}\ g}\right]}{\sqrt{c}\ e}-\frac{\mathbb{i}\sqrt{a}\ d g^2\sqrt{1+\frac{c x^2}{a}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\mathbb{i}\sqrt{c}\ x}{\sqrt{a}}}}{\sqrt{2}}\right],\frac{2\sqrt{a}\ g}{\mathbb{i}\sqrt{c}\ f+\sqrt{a}\ g}\right]}{\sqrt{c}\ e^2}+\right.$$

$$\frac{1}{c\, e\sqrt{\frac{g\left(\sqrt{a}-\mathbb{i}\sqrt{c}\ x\right)}{\mathbb{i}\sqrt{c}\ f+\sqrt{a}\ g}}}\, g\sqrt{\frac{g\left(\sqrt{a}+\mathbb{i}\sqrt{c}\ x\right)}{-\mathbb{i}\sqrt{c}\ f+\sqrt{a}\ g}}\ \left(\mathbb{i}\sqrt{a}+\sqrt{c}\ x\right)\left(\left(\sqrt{c}\ f+\mathbb{i}\sqrt{a}\ g\right)\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{\sqrt{c}\ (f+g x)}{\sqrt{c}\ f-\mathbb{i}\sqrt{a}\ g}}\right],\frac{\sqrt{c}\ f-\mathbb{i}\sqrt{a}\ g}{\sqrt{c}\ f+\mathbb{i}\sqrt{a}\ g}\right]-\right.$$

$$\left.\mathbb{i}\sqrt{a}\ g\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\sqrt{c}\ (f+g x)}{\sqrt{c}\ f-\mathbb{i}\sqrt{a}\ g}}\right],\frac{\sqrt{c}\ f-\mathbb{i}\sqrt{a}\ g}{\sqrt{c}\ f+\mathbb{i}\sqrt{a}\ g}\right]\right)-$$

$$\frac{\sqrt{a}\ f^2\sqrt{1+\frac{c x^2}{a}}\ \text{EllipticPi}\left[\frac{2\sqrt{a}\ e}{\mathbb{i}\sqrt{c}\ d+\sqrt{a}\ e},\text{ArcSin}\left[\frac{\sqrt{1-\frac{\mathbb{i}\sqrt{c}\ x}{\sqrt{a}}}}{\sqrt{2}}\right],\frac{2\sqrt{a}\ g}{\mathbb{i}\sqrt{c}\ f+\sqrt{a}\ g}\right]}{\mathbb{i}\sqrt{c}\ d+\sqrt{a}\ e}+$$

$$\frac{2\sqrt{a}\ d f g\sqrt{1+\frac{c x^2}{a}}\ \text{EllipticPi}\left[\frac{2\sqrt{a}\ e}{\mathbb{i}\sqrt{c}\ d+\sqrt{a}\ e},\text{ArcSin}\left[\frac{\sqrt{1-\frac{\mathbb{i}\sqrt{c}\ x}{\sqrt{a}}}}{\sqrt{2}}\right],\frac{2\sqrt{a}\ g}{\mathbb{i}\sqrt{c}\ f+\sqrt{a}\ g}\right]}{\mathbb{i}\sqrt{c}\ d e+\sqrt{a}\ e^2}-$$

$$\left.\frac{\sqrt{a}\ d^2 g^2\sqrt{1+\frac{c x^2}{a}}\ \text{EllipticPi}\left[\frac{2\sqrt{a}\ e}{\mathbb{i}\sqrt{c}\ d+\sqrt{a}\ e},\text{ArcSin}\left[\frac{\sqrt{1-\frac{\mathbb{i}\sqrt{c}\ x}{\sqrt{a}}}}{\sqrt{2}}\right],\frac{2\sqrt{a}\ g}{\mathbb{i}\sqrt{c}\ f+\sqrt{a}\ g}\right]}{e^2\left(\mathbb{i}\sqrt{c}\ d+\sqrt{a}\ e\right)}\right)$$

- **Problem 645: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d+e x)^3}{\sqrt{f+g x}\,\sqrt{a+c x^2}}\,dx$$

Optimal (type 4, 457 leaves, 7 steps):

$$-\frac{8 e^2 (e f-3 d g) \sqrt{f+g x} \sqrt{a+c x^2}}{15 c g^2}+\frac{2 e^2 (d+e x) \sqrt{f+g x} \sqrt{a+c x^2}}{5 c g}+\left(2 \sqrt{-a}\, e \left(9 a e^2 g^2-c \left(8 e^2 f^2-30 d e f g+45 d^2 g^2\right)\right) \sqrt{f+g x} \sqrt{1+\frac{c x^2}{a}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a} \sqrt{c}\, f-a g}\right]\right) \Big/ \left(15 c^{3/2} g^3 \sqrt{\frac{\sqrt{c}\, (f+g x)}{\sqrt{c}\, f+\sqrt{-a}\, g}} \sqrt{a+c x^2}\right)-\left(2 \sqrt{-a} \left(a e^2 g^2 (7 e f-15 d g)-c \left(8 e^3 f^3-30 d e^2 f^2 g+45 d^2 e f g^2-15 d^3 g^3\right)\right) \sqrt{\frac{\sqrt{c}\, (f+g x)}{\sqrt{c}\, f+\sqrt{-a}\, g}} \sqrt{1+\frac{c x^2}{a}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a} \sqrt{c}\, f-a g}\right]\right) \Big/ \left(15 c^{3/2} g^3 \sqrt{f+g x} \sqrt{a+c x^2}\right)$$

Result (type 4, 625 leaves):

$$\frac{1}{15\, c^2\, g^4\, \sqrt{a + c\, x^2}}\, 2\, \sqrt{f + g\, x}\, \Bigg(c\, e^2\, g^2\, (-4\, e\, f + 15\, d\, g + 3\, e\, g\, x)\, \left(a + c\, x^2\right) +$$

$$\frac{e\, g^2\, \left(-9\, a^2\, e^2\, g^2 + c^2\, \left(8\, e^2\, f^2 - 30\, d\, e\, f\, g + 45\, d^2\, g^2\right)\, x^2 + a\, c\, \left(-30\, d\, e\, f\, g + 45\, d^2\, g^2 + e^2\, \left(8\, f^2 - 9\, g^2\, x^2\right)\right)\right)}{f + g\, x} +$$

$$i\, c\, e\, \sqrt{-f - \frac{i\, \sqrt{a}\, g}{\sqrt{c}}}\, \left(-9\, a\, e^2\, g^2 + c\, \left(8\, e^2\, f^2 - 30\, d\, e\, f\, g + 45\, d^2\, g^2\right)\right)\, \sqrt{\frac{g\, \left(\frac{i\, \sqrt{a}}{\sqrt{c}} + x\right)}{f + g\, x}}$$

$$\sqrt{-\frac{\frac{i\, \sqrt{a}\, g}{\sqrt{c}} - g\, x}{f + g\, x}}\, \sqrt{f + g\, x}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-f - \frac{i\, \sqrt{a}\, g}{\sqrt{c}}}}{\sqrt{f + g\, x}}\right],\, \frac{\sqrt{c}\, f - i\, \sqrt{a}\, g}{\sqrt{c}\, f + i\, \sqrt{a}\, g}\right] + \frac{1}{\sqrt{-f - \frac{i\, \sqrt{a}\, g}{\sqrt{c}}}}$$

$$\sqrt{c}\, g\, \left(15\, i\, c^{3/2}\, d^3\, g^2 + 9\, a^{3/2}\, e^3\, g^2 - i\, a\, \sqrt{c}\, e^2\, g\, (2\, e\, f + 15\, d\, g) + \sqrt{a}\, c\, e\, \left(-8\, e^2\, f^2 + 30\, d\, e\, f\, g - 45\, d^2\, g^2\right)\right)$$

$$\sqrt{\frac{g\, \left(\frac{i\, \sqrt{a}}{\sqrt{c}} + x\right)}{f + g\, x}}\, \sqrt{-\frac{\frac{i\, \sqrt{a}\, g}{\sqrt{c}} - g\, x}{f + g\, x}}\, \sqrt{f + g\, x}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-f - \frac{i\, \sqrt{a}\, g}{\sqrt{c}}}}{\sqrt{f + g\, x}}\right],\, \frac{\sqrt{c}\, f - i\, \sqrt{a}\, g}{\sqrt{c}\, f + i\, \sqrt{a}\, g}\right]\Bigg)$$

- **Problem 646: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d + e\, x)^2}{\sqrt{f + g\, x}\, \sqrt{a + c\, x^2}}\, dx$$

Optimal (type 4, 356 leaves, 7 steps) :

$$\frac{2 e^2 \sqrt{f+g x}\,\sqrt{a+c x^2}}{3 c g}+\frac{4 \sqrt{-a}\, e\,(e f-3 d g) \sqrt{f+g x}\, \sqrt{1+\frac{c x^2}{a}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\, f-a g}\right]}{3 \sqrt{c}\, g^2 \sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\, f+\sqrt{-a}\, g}}\, \sqrt{a+c x^2}}-$$

$$\left(2 \sqrt{-a}\,\left(\left(3 c d^2-a e^2\right) g^2+2 c e f\,(e f-3 d g)\right) \sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\, f+\sqrt{-a}\, g}}\, \sqrt{1+\frac{c x^2}{a}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\, f-a g}\right]\right)\Big/$$

$$\left(3 c^{3/2} g^2 \sqrt{f+g x}\, \sqrt{a+c x^2}\right)$$

Result (type 4, 473 leaves) :

$$\frac{1}{3 c g^3 \sqrt{a+c x^2}}$$

$$2 \sqrt{f+g x}\, \left(e^2 g^2 \left(a+c x^2\right)-\frac{2 e g^2\,(e f-3 d g)\,\left(a+c x^2\right)}{f+g x}-2\, \mathrm{i}\, c e \sqrt{-f-\frac{\mathrm{i} \sqrt{a}\, g}{\sqrt{c}}}\,(e f-3 d g) \sqrt{\frac{g \left(\frac{\mathrm{i} \sqrt{a}}{\sqrt{c}}+x\right)}{f+g x}}\, \sqrt{-\frac{\frac{\mathrm{i} \sqrt{a}\, g}{\sqrt{c}}-g x}{f+g x}}\, \sqrt{f+g x}\right.$$

$$\text{EllipticE}\left[\mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{-f-\frac{\mathrm{i} \sqrt{a}\, g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c}\, f-\mathrm{i} \sqrt{a}\, g}{\sqrt{c}\, f+\mathrm{i} \sqrt{a}\, g}\right]+1\Big/\left(\sqrt{-f-\frac{\mathrm{i} \sqrt{a}\, g}{\sqrt{c}}}\right) g \left(3\, \mathrm{i}\, c d^2 g-\mathrm{i}\, a e^2 g+2 \sqrt{a}\, \sqrt{c}\, e\,(e f-3 d g)\right)$$

$$\left.\sqrt{\frac{g \left(\frac{\mathrm{i} \sqrt{a}}{\sqrt{c}}+x\right)}{f+g x}}\, \sqrt{-\frac{\frac{\mathrm{i} \sqrt{a}\, g}{\sqrt{c}}-g x}{f+g x}}\, \sqrt{f+g x}\, \text{EllipticF}\left[\mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{-f-\frac{\mathrm{i} \sqrt{a}\, g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c}\, f-\mathrm{i} \sqrt{a}\, g}{\sqrt{c}\, f+\mathrm{i} \sqrt{a}\, g}\right]\right)$$

- **Problem 647: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{d+e x}{\sqrt{f+g x}\, \sqrt{a+c x^2}}\, dx$$

Optimal (type 4, 288 leaves, 5 steps) :

$$-\frac{2\sqrt{-a}\,e\sqrt{f+g x}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\sqrt{c}\,f-a g}\right]}{\sqrt{c}\,g\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\sqrt{a+c x^2}}+$$

$$\frac{2\sqrt{-a}\,(e f-d g)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\sqrt{c}\,f-a g}\right]}{\sqrt{c}\,g\sqrt{f+g x}\sqrt{a+c x^2}}$$

Result (type 4, 439 leaves) :

$$-\left(2\left(-e g^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(a+c x^2)+i\sqrt{c}\,e\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+g x}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-g x}{f+g x}}\,(f+g x)^{3/2}\right.\right.$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]+\sqrt{c}\left(-i\sqrt{c}\,d+\sqrt{a}\,e\right)g\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+g x}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-g x}{f+g x}}$$

$$\left.\left.(f+g x)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]\right)\right)\Big/\left(c\,g^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\sqrt{f+g x}\sqrt{a+c x^2}\right)$$

- **Problem 648: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\sqrt{f+g x}\sqrt{a+c x^2}}\,dx$$

Optimal (type 4, 136 leaves, 2 steps) :

$$-\frac{2\sqrt{-a}\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\sqrt{c}\,f-a g}\right]}{\sqrt{c}\sqrt{f+g x}\sqrt{a+c x^2}}$$

Result (type 4, 186 leaves):

$$\frac{2\,i\,\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+g\,x}}\,\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-g\,x}{f+g\,x}}\,(f+g\,x)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]}{g\,\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,\sqrt{a+c\,x^2}}$$

- **Problem 649: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{(d+e\,x)\,\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}\,dx$$

Optimal (type 4, 167 leaves, 4 steps):

$$-\frac{2\,\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticPi}\left[\frac{2\,e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e},\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\,\frac{2\,\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right]}{\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)\,\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}$$

Result (type 4, 311 leaves):

$$-\left(2\,i\,\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+g\,x}}\,\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-g\,x}{f+g\,x}}\,(f+g\,x)\left(\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]-\right.\right.$$
$$\left.\left.\text{EllipticPi}\left[\frac{\sqrt{c}\,(e\,f-d\,g)}{e\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)},\,i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\,\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right]\right)\right)\Big/\left(\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(e\,f-d\,g)\,\sqrt{a+c\,x^2}\right)$$

- **Problem 650: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{(d+e\,x)^2\,\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}\,dx$$

Optimal (type 4, 746 leaves, 14 steps):

$$-\frac{e^2\sqrt{f+g x}\,\sqrt{a+c x^2}}{\left(c d^2+a e^2\right)(e f-d g)(d+e x)}-\frac{\sqrt{-a}\,\sqrt{c}\,e\sqrt{f+g x}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]}{\left(c d^2+a e^2\right)(e f-d g)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+c x^2}}+$$

$$\frac{\sqrt{-a}\,\sqrt{c}\,e f\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]}{\left(c d^2+a e^2\right)(e f-d g)\sqrt{f+g x}\,\sqrt{a+c x^2}}-$$

$$\frac{\sqrt{-a}\,\sqrt{c}\,d g\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]}{\left(c d^2+a e^2\right)(e f-d g)\sqrt{f+g x}\,\sqrt{a+c x^2}}+$$

$$\left(\left(a e^2 g-c d(2 e f-3 d g)\right)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticPi}\left[\frac{2 e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e},\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right]\right)\Big/$$

$$\left(\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)\left(c d^2+a e^2\right)(e f-d g)\sqrt{f+g x}\,\sqrt{a+c x^2}\right)$$

Result (type 4, 1491 leaves) :

$$-\frac{e^2\sqrt{f+g x}\,\sqrt{a+c x^2}}{\left(c d^2+a e^2\right)(e f-d g)(d+e x)}+$$

$$\left((f+g x)^{3/2}\left(-2 c e^2 f\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}+2 c d e g\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}-\frac{2 c e^2 f^3\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{(f+g x)^2}+\frac{2 c d e f^2 g\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{(f+g x)^2}-\frac{2 a e^2 f g^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{(f+g x)^2}+\right.\right.$$

$$\frac{2\,a\,d\,e\,g^3\sqrt{-f-\frac{\mathrm{i}\sqrt{a}\,g}{\sqrt{c}}}}{(f+g\,x)^2}+\frac{4\,c\,e^2\,f^2\sqrt{-f-\frac{\mathrm{i}\sqrt{a}\,g}{\sqrt{c}}}}{f+g\,x}-\frac{4\,c\,d\,e\,f\,g\sqrt{-f-\frac{\mathrm{i}\sqrt{a}\,g}{\sqrt{c}}}}{f+g\,x}+\frac{1}{\sqrt{f+g\,x}}2\,\mathrm{i}\sqrt{c}\,e\left(\sqrt{c}\,f+\mathrm{i}\sqrt{a}\,g\right)(e\,f-d\,g)$$

$$\sqrt{1-\frac{f}{f+g\,x}-\frac{\mathrm{i}\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}\sqrt{1-\frac{f}{f+g\,x}+\frac{\mathrm{i}\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}\;\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{\mathrm{i}\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\frac{\sqrt{c}\,f-\mathrm{i}\sqrt{a}\,g}{\sqrt{c}\,f+\mathrm{i}\sqrt{a}\,g}\right]+$$

$$\frac{1}{\sqrt{f+g\,x}}2\left(\sqrt{c}\,d-\mathrm{i}\sqrt{a}\,e\right)g\left(\sqrt{a}\,e\,g+\mathrm{i}\sqrt{c}\,(e\,f-2\,d\,g)\right)\sqrt{1-\frac{f}{f+g\,x}-\frac{\mathrm{i}\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}\sqrt{1-\frac{f}{f+g\,x}+\frac{\mathrm{i}\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{\mathrm{i}\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\frac{\sqrt{c}\,f-\mathrm{i}\sqrt{a}\,g}{\sqrt{c}\,f+\mathrm{i}\sqrt{a}\,g}\right]-\frac{1}{\sqrt{f+g\,x}}4\,\mathrm{i}\,c\,d\,e\,f\,g\sqrt{1-\frac{f}{f+g\,x}-\frac{\mathrm{i}\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}$$

$$\sqrt{1-\frac{f}{f+g\,x}+\frac{\mathrm{i}\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}\;\text{EllipticPi}\left[\frac{\sqrt{c}\,(e\,f-d\,g)}{e\left(\sqrt{c}\,f+\mathrm{i}\sqrt{a}\,g\right)},\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{\mathrm{i}\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\frac{\sqrt{c}\,f-\mathrm{i}\sqrt{a}\,g}{\sqrt{c}\,f+\mathrm{i}\sqrt{a}\,g}\right]+$$

$$\frac{1}{\sqrt{f+g\,x}}6\,\mathrm{i}\,c\,d^2\,g^2\sqrt{1-\frac{f}{f+g\,x}-\frac{\mathrm{i}\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}\sqrt{1-\frac{f}{f+g\,x}+\frac{\mathrm{i}\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}$$

$$\text{EllipticPi}\left[\frac{\sqrt{c}\,(e\,f-d\,g)}{e\left(\sqrt{c}\,f+\mathrm{i}\sqrt{a}\,g\right)},\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{\mathrm{i}\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\frac{\sqrt{c}\,f-\mathrm{i}\sqrt{a}\,g}{\sqrt{c}\,f+\mathrm{i}\sqrt{a}\,g}\right]+\frac{1}{\sqrt{f+g\,x}}2\,\mathrm{i}\,a\,e^2\,g^2\sqrt{1-\frac{f}{f+g\,x}-\frac{\mathrm{i}\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}$$

$$\left.\left.\sqrt{1-\frac{f}{f+g\,x}+\frac{\mathrm{i}\sqrt{a}\,g}{\sqrt{c}\,(f+g\,x)}}\;\text{EllipticPi}\left[\frac{\sqrt{c}\,(e\,f-d\,g)}{e\left(\sqrt{c}\,f+\mathrm{i}\sqrt{a}\,g\right)},\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{\mathrm{i}\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g\,x}}\right],\frac{\sqrt{c}\,f-\mathrm{i}\sqrt{a}\,g}{\sqrt{c}\,f+\mathrm{i}\sqrt{a}\,g}\right]\right)\right)\Big/$$

$$\left(2\left(c\,d^2+a\,e^2\right)g\sqrt{-f-\frac{\mathrm{i}\sqrt{a}\,g}{\sqrt{c}}}\,(e\,f-d\,g)\,(-e\,f+d\,g)\sqrt{a+\frac{c\,(f+g\,x)^2\left(-1+\frac{f}{f+g\,x}\right)^2}{g^2}}\right)$$

Problem 651: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(d+e\,x)^3\,\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}\,dx$$

Optimal (type 4, 1257 leaves, 23 steps):

$$-\frac{e^2\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}{2\left(c\,d^2+a\,e^2\right)(e\,f-d\,g)\,(d+e\,x)^2}+\frac{3\,e^2\left(a\,e^2\,g-c\,d\,(2\,e\,f-3\,d\,g)\right)\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}{4\left(c\,d^2+a\,e^2\right)^2(e\,f-d\,g)^2\,(d+e\,x)}+$$

$$\left(3\sqrt{-a}\,\sqrt{c}\,e\left(a\,e^2\,g-c\,d\,(2\,e\,f-3\,d\,g)\right)\sqrt{f+g\,x}\,\sqrt{1+\frac{c\,x^2}{a}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,g}{\sqrt{-a}\,\sqrt{c}\,f-a\,g}\right]\right)\Bigg/$$

$$\left(4\left(c\,d^2+a\,e^2\right)^2(e\,f-d\,g)^2\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+c\,x^2}\right)+\frac{\sqrt{-a}\,\sqrt{c}\,g\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c\,x^2}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,g}{\sqrt{-a}\,\sqrt{c}\,f-a\,g}\right]}{2\left(c\,d^2+a\,e^2\right)(e\,f-d\,g)\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}-$$

$$\left(3\sqrt{-a}\,\sqrt{c}\,e\,f\left(a\,e^2\,g-c\,d\,(2\,e\,f-3\,d\,g)\right)\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c\,x^2}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,g}{\sqrt{-a}\,\sqrt{c}\,f-a\,g}\right]\right)\Bigg/$$

$$\left(4\left(c\,d^2+a\,e^2\right)^2(e\,f-d\,g)^2\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}\right)+$$

$$\left(3\sqrt{-a}\,\sqrt{c}\,d\,g\left(a\,e^2\,g-c\,d\,(2\,e\,f-3\,d\,g)\right)\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c\,x^2}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2\,a\,g}{\sqrt{-a}\,\sqrt{c}\,f-a\,g}\right]\right)\Bigg/$$

$$\left(4\left(c\,d^2+a\,e^2\right)^2(e\,f-d\,g)^2\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}\right)+\frac{c\,(e\,f-3\,d\,g)\sqrt{\frac{\sqrt{c}\,(f+g\,x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c\,x^2}{a}}\;\text{EllipticPi}\left[\frac{2\,e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e},\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right]}{\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)\left(c\,d^2+a\,e^2\right)(e\,f-d\,g)\sqrt{f+g\,x}\,\sqrt{a+c\,x^2}}-$$

$$\left(3\left(a e^2 g-c d(2 e f-3 d g)\right)^2 \sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\, f+\sqrt{-a}\, g}} \sqrt{1+\frac{c x^2}{a}} \text{ EllipticPi}\left[\frac{2 e}{\frac{\sqrt{c}\, d}{\sqrt{-a}}+e}, \text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right], \frac{2 \sqrt{-a}\, g}{\sqrt{c}\, f+\sqrt{-a}\, g}\right]\right) \Big/$$

$$\left(4\left(\frac{\sqrt{c}\, d}{\sqrt{-a}}+e\right)\left(c d^2+a e^2\right)^2 (e f-d g)^2 \sqrt{f+g x} \sqrt{a+c x^2}\right)$$

Result (type 4, 15 233 leaves) :

$$\sqrt{f+g x} \sqrt{a+c x^2}\left(-\frac{e^2}{2\left(c d^2+a e^2\right)(e f-d g)(d+e x)^2}+\frac{3 e^2\left(-2 c d e f+3 c d^2 g+a e^2 g\right)}{4\left(c d^2+a e^2\right)^2 (e f-d g)^2 (d+e x)}\right)-$$

$$\frac{1}{4\left(c d^2+a e^2\right)^2 g(-e f+d g)^2}\left(\frac{3 e\left(-2 c d e f+3 c d^2 g+a e^2 g\right)(f+g x)^{3/2}\left(c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}\right)}{\sqrt{a+\frac{c (f+g x)^2\left(-1+\frac{f}{f+g x}\right)^2}{g^2}}}+\right.$$

$$\frac{1}{\sqrt{a+\frac{c (f+g x)^2\left(-1+\frac{f}{f+g x}\right)^2}{g^2}}}(e f-d g)(f+g x) \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}$$

$$\left(\left(6\, i\, c^2 d e^2 f^3\left(c f+i \sqrt{a} \sqrt{c}\, g\right) \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}}\right.\right.$$

$$\left.\left.\left(\text{EllipticE}\left[i \text{ ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c}\, g}{c f+i \sqrt{a} \sqrt{c}\, g}\right]-\text{EllipticF}\left[i \text{ ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c}\, g}{c f+i \sqrt{a} \sqrt{c}\, g}\right]\right)\right)\right) \Big/$$

$$\left((e f-d g)\left(c f^2+a g^2\right) \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}} \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(9\,i\,c^2 d^2 e f^2 g\left(c f+i\sqrt{a}\sqrt{c}\,g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\right)\Big/$$

$$\left((e f-d g)\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(3\,i\,a c e^3 f^2 g\left(c f+i\sqrt{a}\sqrt{c}\,g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\right)\Big/$$

$$\left((e f-d g)\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\left(6\,i\,a c d e^2 f g^2\left(c f+i\sqrt{a}\sqrt{c}\,g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\right)\Big/$$

$$\left((e f-d g)\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(9\,i\,a\,c\,d^2 e\,g^3\left(c f+i\sqrt{a}\sqrt{c}\,g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\right)\Big/$$

$$\left((e f-d g)\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(3\,i\,a^2 e^3 g^3\left(c f+i\sqrt{a}\sqrt{c}\,g\right)\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-i\sqrt{a}\sqrt{c}\,g}{c f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\right)\Big/$$

$$\left((e f-d g)\left(c f^2+a g^2\right)\sqrt{-\frac{c f^2+a g^2}{c f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(6\,i\,c^2 d\,e^3 f^3\sqrt{1-\frac{c f^2+a g^2}{\left(c f-i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+i\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\right.$$

$$\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g x}}\Big],\ \frac{c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}{c f+\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}\Big]\Bigg/\left((e f-d g)^2\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}}\ \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\Bigg(9\,\mathrm{i}\,c^2 d^2 e^2 f^2 g\sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g\right)(f+g x)}}\ \sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i}\sqrt{a}\,\sqrt{c}\,g\right)(f+g x)}}$$

$$\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g x}}\Big],\ \frac{c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}{c f+\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}\Big]\Bigg)\Bigg/\left((e f-d g)^2\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}}\ \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\Bigg(3\,\mathrm{i}\,a\,c\,e^4 f^2 g\sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g\right)(f+g x)}}\ \sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i}\sqrt{a}\,\sqrt{c}\,g\right)(f+g x)}}$$

$$\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g x}}\Big],\ \frac{c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}{c f+\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}\Big]\Bigg)\Bigg/\left((e f-d g)^2\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}}\ \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\Bigg(6\,\mathrm{i}\,a\,c\,d\,e^3 f g^2\sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g\right)(f+g x)}}\ \sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i}\sqrt{a}\,\sqrt{c}\,g\right)(f+g x)}}$$

$$\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g x}}\Big],\ \frac{c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}{c f+\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}\Big]\Bigg)\Bigg/\left((e f-d g)^2\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\,\sqrt{c}\,g}}\ \sqrt{c+\frac{c f^2+a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+$$

$$\left(9\,i\,a\,c\,d^2\,e^2\,g^3\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\,\frac{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\bigg/\left((e\,f-d\,g)^2\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)+$$

$$\left(3\,i\,a^2\,e^4\,g^3\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\right.\right.$$

$$\left.\left.\frac{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\bigg/\left((e\,f-d\,g)^2\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)+$$

$$\left(12\,i\,c^2\,d\,e^2\,f^2\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\right.\right.$$

$$\left.\left.\frac{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\bigg/\left((e\,f-d\,g)\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(8\,i\,c^2\,d^2\,e\,f\,g\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\right.\right.$$

$$\left.\left.\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Bigg/\left((e\,f-d\,g)\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(8\,i\,a\,c\,e^3\,f\,g\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\right.\right.$$

$$\left.\left.\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Bigg/\left((e\,f-d\,g)\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(8\,i\,c^2\,d^3\,g^2\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\right.\right.$$

$$\left.\left.\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Bigg/\left((e\,f-d\,g)\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)+$$

$$\left(4\,i\,a\,c\,d\,e^2\,g^2\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\right.\right.$$

$$\left.\left.\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Bigg/\left((e\,f-d\,g)\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)+\frac{1}{(e\,f-d\,g)^3}$$

$$6 c^2 d e^4 f^3 \left(\left(\left(i f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)},\right.\right.\right.\right.$$

$$\left.\left. i \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right) \Big/ \left(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right) -$$

$$\left(i d g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)}, i \text{ArcSinh}\left[\right.\right.\right.$$

$$\left.\left.\left.\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right) \Big/ \left(e \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\Bigg) - \frac{1}{(e f-d g)^3}$$

$$9 c^2 d^2 e^3 f^2 g \left(\left(\left(i f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)},\right.\right.\right.\right.$$

$$\left.\left. i \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right) \Big/ \left(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right) -$$

$$\left(i d g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)}, i \text{ArcSinh}\left[\right.\right.\right.$$

$$\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\Big], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\Big]\Bigg) \Big/ \Bigg(e \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\Bigg)\Bigg)-\frac{1}{(e f-d g)^3}$$

$$3 a c e^5 f^2 g \Bigg(\Bigg(i f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \text{EllipticPi}\Big[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)},$$

$$i \text{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\Big], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\Big]\Bigg) \Big/ \Bigg(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\Bigg)-$$

$$\Bigg(i d g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \text{EllipticPi}\Big[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)}, i \text{ArcSinh}\Big[$$

$$\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\Big], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\Big]\Bigg) \Big/ \Bigg(e \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\Bigg)\Bigg)+\frac{1}{(e f-d g)^3}$$

$$6 a c d e^4 f g^2 \Bigg(\Bigg(i f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \text{EllipticPi}\Big[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)},$$

$$i \text{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\Big], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\Big]\Bigg) \Big/ \Bigg(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\Bigg)-$$

$$\left(i\, d\, g\, \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}}\, \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}}\ \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)}, i\, \text{ArcSinh}\left[\right.\right.\right.$$

$$\left.\left.\left.\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right) \Bigg/ \left(e \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}\, \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\right) - \frac{1}{(e f-d g)^3}$$

$$9\, a\, c\, d^2 e^3 g^3 \left(\left(\left(i\, f\, \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}}\, \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}}\ \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)},\right.\right.\right.\right.$$

$$\left.\left.i\, \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right) \Bigg/ \left(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}\, \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right) -$$

$$\left(i\, d\, g\, \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}}\, \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}}\ \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)}, i\, \text{ArcSinh}\left[\right.\right.\right.$$

$$\left.\left.\left.\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\right]\right) \Bigg/ \left(e \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}\, \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\right) - \frac{1}{(e f-d g)^3}$$

$$3\, a^2 e^5 g^3 \left(\left(\left(i\, f\, \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}}\, \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}}\ \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)},\right.\right.\right.\right.$$

$$\left.\left.\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}},\frac{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}{c f+\mathrm{i}\sqrt{a}\sqrt{c}\,g}\right]\right)\right/\left(\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(\mathrm{i}\,d g\sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\,\mathrm{EllipticPi}\left[\frac{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\mathrm{i}\,\mathrm{ArcSinh}\left[\right.\right.\right.$$

$$\left.\left.\left.\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}{c f+\mathrm{i}\sqrt{a}\sqrt{c}\,g}\right]\right)\Bigg/\left(e\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\Bigg)-\frac{1}{(e f-d g)^2}$$

$$12 c^2 d e^3 f^2\left(\left(\mathrm{i}\,f\sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\,\mathrm{EllipticPi}\left[\frac{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\right.\right.\right.$$

$$\left.\left.\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}{c f+\mathrm{i}\sqrt{a}\sqrt{c}\,g}\right]\right)\Bigg/\left(\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(\mathrm{i}\,d g\sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathrm{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\,\mathrm{EllipticPi}\left[\frac{\left(c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\mathrm{i}\,\mathrm{ArcSinh}\left[\right.\right.\right.$$

$$\left.\left.\left.\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}{c f+\mathrm{i}\sqrt{a}\sqrt{c}\,g}\right]\right)\Bigg/\left(e\sqrt{-\frac{c f^2+a g^2}{c f-\mathrm{i}\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\Bigg)+\frac{1}{(e f-d g)^2}$$

$$8\,c^2\,d^2\,e^2\,f\,g\,\Bigg(\Bigg(\Bigg(\mathrm{i}\,f\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\Big[\frac{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},$$

$$\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\Big],\,\frac{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}\Big]\Bigg)\Bigg/\Bigg(\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\Bigg)-$$

$$\Bigg(\mathrm{i}\,d\,g\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\Big[\frac{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\,\mathrm{i}\,\text{ArcSinh}\Big[$$

$$\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\Big],\,\frac{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}\Big]\Bigg)\Bigg/\Bigg(e\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\Bigg)\Bigg)+\frac{1}{(e\,f-d\,g)^2}$$

$$8\,a\,c\,e^4\,f\,g\,\Bigg(\Bigg(\Bigg(\mathrm{i}\,f\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\Big[\frac{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},$$

$$\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\Big],\,\frac{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}\Big]\Bigg)\Bigg/\Bigg(\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\Bigg)-$$

$$\Bigg(\mathrm{i}\,d\,g\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\Big[\frac{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\,\mathrm{i}\,\text{ArcSinh}\Big[$$

$$\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\Big], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\Big]\Bigg) \Big/ \Bigg(e \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\Bigg)\Bigg)+\frac{1}{(e f-d g)^2}$$

$$8 c^2 d^3 e g^2 \Bigg(\Bigg(i f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}}\ \text{EllipticPi}\Big[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)},$$

$$i\, \text{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\Big], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\Big]\Bigg) \Big/ \Bigg(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\Bigg)-$$

$$\Bigg(i d g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}}\ \text{EllipticPi}\Big[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)}, i\, \text{ArcSinh}\Big[$$

$$\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\Big], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\Big]\Bigg) \Big/ \Bigg(e \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\Bigg)\Bigg)-\frac{1}{(e f-d g)^2}$$

$$4 a c d e^3 g^2 \Bigg(\Bigg(i f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}}\ \text{EllipticPi}\Big[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)},$$

$$i\, \text{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\Big], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\Big]\Bigg) \Big/ \Bigg(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\Bigg)-$$

$$\left(i\, d\, g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c}\, g\right)(e f-d g)}{e\left(c f^2+a g^2\right)}, i\right.\right.$$

$$\left.\left.\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c}\, g}{c f+i \sqrt{a} \sqrt{c}\, g}\right]\right) \Big/ \left(e \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)+\frac{1}{e f-d g}$$

$$6 c^2 d e^2 f\left(\left(i f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c}\, g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\right.\right.\right.$$

$$\left.\left.i\, \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c}\, g}{c f+i \sqrt{a} \sqrt{c}\, g}\right]\right) \Big/ \left(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(i\, d\, g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c}\, g\right)(e f-d g)}{e\left(c f^2+a g^2\right)}, i\right.\right.$$

$$\left.\left.\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c}\, g}{c f+i \sqrt{a} \sqrt{c}\, g}\right]\right) \Big/ \left(e \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-\frac{1}{e f-d g}$$

$$7 c^2 d^2 e\, g\left(\left(i f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c}\, g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\right.\right.\right.$$

$$\left.\left.\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}},\ \frac{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\right/\left(\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}}\ \sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(\mathbb{i}\,d\,g\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\ \sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\ \text{EllipticPi}\left[\frac{\left(c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\ \mathbb{i}\right.\right.$$

$$\left.\left.\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}},\ \frac{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\right/\left(e\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}}\ \sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)\Bigg)-\frac{1}{e\,f-d\,g}$$

$$a\,c\,e^3\,g\left(\left(\mathbb{i}\,f\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\ \sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\ \text{EllipticPi}\left[\frac{\left(c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\right.\right.\right.$$

$$\left.\left.\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}},\ \frac{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\right/\left(\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}}\ \sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(\mathbb{i}\,d\,g\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\ \sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\ \text{EllipticPi}\left[\frac{\left(c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\ \mathbb{i}\right.\right.$$

$$\left.\left.\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}},\ \frac{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}\right]\right)\right/\left(e\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathbb{i}\,\sqrt{a}\,\sqrt{c}\,g}}\ \sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)\Bigg)\Bigg)\Bigg)$$

- **Problem 652: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(d+e x)\,(f+g x)^{3/2}\,\sqrt{a+c x^2}}\,dx$$

Optimal (type 4, 387 leaves, 10 steps):

$$\frac{2 g^2 \sqrt{a+c x^2}}{(e f-d g)\,(c f^2+a g^2)\,\sqrt{f+g x}}+\frac{2 \sqrt{-a}\,\sqrt{c}\,g \sqrt{f+g x}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\,\sqrt{c}\,f-a g}\right]}{(e f-d g)\,(c f^2+a g^2)\,\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+c x^2}}-$$

$$\frac{2 e \sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{c x^2}{a}}\,\text{EllipticPi}\left[\frac{2 e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e},\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right],\frac{2 \sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right]}{\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)\,(e f-d g)\,\sqrt{f+g x}\,\sqrt{a+c x^2}}$$

Result (type 4, 468 leaves):

$$\left(2 i \sqrt{\frac{g\left(\frac{i \sqrt{a}}{\sqrt{c}}+x\right)}{f+g x}}\,\sqrt{-\frac{\frac{i \sqrt{a}\,g}{\sqrt{c}}-g x}{f+g x}}\,(f+g x)\left(\sqrt{c}\,(e f-d g)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i \sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c}\,f-i \sqrt{a}\,g}{\sqrt{c}\,f+i \sqrt{a}\,g}\right]+\right.\right.$$

$$\left(i \sqrt{a}\,e g+\sqrt{c}\,(-2 e f+d g)\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i \sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c}\,f-i \sqrt{a}\,g}{\sqrt{c}\,f+i \sqrt{a}\,g}\right]+$$

$$\left.\left.e\left(\sqrt{c}\,f-i \sqrt{a}\,g\right)\text{EllipticPi}\left[\frac{\sqrt{c}\,(e f-d g)}{e\left(\sqrt{c}\,f+i \sqrt{a}\,g\right)},i\,\text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i \sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+g x}}\right],\frac{\sqrt{c}\,f-i \sqrt{a}\,g}{\sqrt{c}\,f+i \sqrt{a}\,g}\right]\right)\right)\Big/$$

$$\left(\left(\sqrt{c}\,f-i \sqrt{a}\,g\right)\sqrt{-f-\frac{i \sqrt{a}\,g}{\sqrt{c}}}\,(e f-d g)^2 \sqrt{a+c x^2}\right)$$

- **Problem 653: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(d+e x)(f+g x)^{5/2}\sqrt{a+c x^2}}\,dx$$

Optimal (type 4, 818 leaves, 17 steps):

$$\frac{2 g^2 \sqrt{a+c x^2}}{3 (e f-d g)\left(c f^2+a g^2\right)(f+g x)^{3/2}}+\frac{8 c f g^2 \sqrt{a+c x^2}}{3 (e f-d g)\left(c f^2+a g^2\right)^2 \sqrt{f+g x}}+$$

$$\frac{2 e g^2 \sqrt{a+c x^2}}{(e f-d g)^2\left(c f^2+a g^2\right)\sqrt{f+g x}}+\frac{8 \sqrt{-a}\, c^{3/2} f g \sqrt{f+g x}\sqrt{1+\frac{c x^2}{a}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\sqrt{c}\, f-a g}\right]}{3 (e f-d g)\left(c f^2+a g^2\right)^2\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\, f+\sqrt{-a}\, g}}\sqrt{a+c x^2}}+$$

$$\frac{2 \sqrt{-a}\sqrt{c}\, e g \sqrt{f+g x}\sqrt{1+\frac{c x^2}{a}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\sqrt{c}\, f-a g}\right]}{(e f-d g)^2\left(c f^2+a g^2\right)\sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\, f+\sqrt{-a}\, g}}\sqrt{a+c x^2}}-$$

$$\frac{2 \sqrt{-a}\sqrt{c}\, g \sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\, f+\sqrt{-a}\, g}}\sqrt{1+\frac{c x^2}{a}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],-\frac{2 a g}{\sqrt{-a}\sqrt{c}\, f-a g}\right]}{3 (e f-d g)\left(c f^2+a g^2\right)\sqrt{f+g x}\sqrt{a+c x^2}}-$$

$$\frac{2 e^2 \sqrt{\frac{\sqrt{c}\,(f+g x)}{\sqrt{c}\, f+\sqrt{-a}\, g}}\sqrt{1+\frac{c x^2}{a}}\, \text{EllipticPi}\left[\frac{2 e}{\frac{\sqrt{c}\, d}{\sqrt{-a}}+e},\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{c}\, x}{\sqrt{-a}}}}{\sqrt{2}}\right],\frac{2\sqrt{-a}\, g}{\sqrt{c}\, f+\sqrt{-a}\, g}\right]}{\left(\frac{\sqrt{c}\, d}{\sqrt{-a}}+e\right)(e f-d g)^2\sqrt{f+g x}\sqrt{a+c x^2}}$$

Result (type 4, 6926 leaves):

$$\sqrt{f+g x}\sqrt{a+c x^2}\left(\frac{2 g^2}{3 (e f-d g)\left(c f^2+a g^2\right)(f+g x)^2}+\frac{2 g^2\left(7 c e f^2-4 c d f g+3 a e g^2\right)}{3 (e f-d g)^2\left(c f^2+a g^2\right)^2 (f+g x)}\right)-\frac{1}{3 (-e f+d g)^2\left(c f^2+a g^2\right)^2}$$

$$2\left(\frac{\left(7\,c\,e\,f^2-4\,c\,d\,f\,g+3\,a\,e\,g^2\right)(f+g\,x)^{3/2}\left(c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}\right)}{\sqrt{a+\frac{c\,(f+g\,x)^2\left(-1+\frac{f}{f+g\,x}\right)^2}{g^2}}}+\frac{1}{\sqrt{a+\frac{c\,(f+g\,x)^2\left(-1+\frac{f}{f+g\,x}\right)^2}{g^2}}}\,(e\,f-d\,g)\left(c\,f^2+a\,g^2\right)(f+g\,x)\right.$$

$$\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\left(\left(-7\,i\,c\,e\,f^2\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\right.\right.$$

$$\left.\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]\right)\right)\right/$$

$$\left((e\,f-d\,g)\left(c\,f^2+a\,g^2\right)\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)+$$

$$\left(4\,i\,c\,d\,f\,g\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]\right)\right)\right/$$

$$\left((e\,f-d\,g)\left(c\,f^2+a\,g^2\right)\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(3\,i\,a\,e\,g^2\left(c\,f+i\sqrt{a}\sqrt{c}\,g\right)\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\sqrt{c}\,g\right)(f+g\,x)}}\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\sqrt{c}\,g\right)(f+g\,x)}}\right.$$

$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\sqrt{c}\,g}{c\,f+i\sqrt{a}\sqrt{c}\,g}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\sqrt{c}\,g}{c\,f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\right)\Big/$$

$$\left((e\,f-d\,g)\left(c\,f^2+a\,g^2\right)\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)+$$

$$\left(7\,i\,c\,e^2\,f^2\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\sqrt{c}\,g\right)(f+g\,x)}}\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\sqrt{c}\,g\right)(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\frac{c\,f-i\sqrt{a}\sqrt{c}\,g}{c\,f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\Big/\left((e\,f-d\,g)^2\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(4\,i\,c\,d\,e\,f\,g\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\sqrt{c}\,g\right)(f+g\,x)}}\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\sqrt{c}\,g\right)(f+g\,x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\right.\right.$$

$$\left.\left.\frac{c\,f-i\sqrt{a}\sqrt{c}\,g}{c\,f+i\sqrt{a}\sqrt{c}\,g}\right]\right)\Big/\left((e\,f-d\,g)^2\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)+$$

$$\left(3\,i\,a\,e^2\,g^2\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\right.\right.$$

$$\left.\left.\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left((e\,f-d\,g)^2\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-$$

$$\left(5\,i\,c\,e\,f\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\right.\right.$$

$$\left.\left.\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left((e\,f-d\,g)\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)+$$

$$\left(i\,c\,d\,g\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\right],\right.\right.$$

$$\left.\left.\frac{c\,f-i\sqrt{a}\,\sqrt{c}\,g}{c\,f+i\sqrt{a}\,\sqrt{c}\,g}\right]\right)\Big/\left((e\,f-d\,g)\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-i\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2+a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\right)-\frac{1}{(e\,f-d\,g)^3}$$

$$7\,c\,e^3\,f^2\left(\left(\left(i\,f\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+i\sqrt{a}\,\sqrt{c}\,g\right)(f+g\,x)}}\,\text{EllipticPi}\left[\frac{\left(c\,f-i\sqrt{a}\,\sqrt{c}\,g\right)(e\,f-d\,g)}{e\left(c\,f^2+a\,g^2\right)},\right.\right.\right.\right.$$

$$\left.\left.\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}{c f+\mathbb{i}\sqrt{a}\sqrt{c}\,g}\right]\right)\Big/\left(\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(\mathbb{i}\,d\,g\sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathbb{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\,\text{EllipticPi}\left[\frac{\left(c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\mathbb{i}\,\text{ArcSinh}\left[\right.\right.\right.$$

$$\left.\left.\left.\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}{c f+\mathbb{i}\sqrt{a}\sqrt{c}\,g}\right]\right)\Big/\left(e\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\Bigg)+\frac{1}{(e f-d g)^3}$$

$$4 c d e^2 f g\left(\left(\mathbb{i}\,f\sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathbb{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\,\text{EllipticPi}\left[\frac{\left(c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\right.\right.\right.$$

$$\left.\left.\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}{c f+\mathbb{i}\sqrt{a}\sqrt{c}\,g}\right]\right)\Big/\left(\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)-$$

$$\left(\mathbb{i}\,d\,g\sqrt{1-\frac{c f^2+a g^2}{\left(c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\sqrt{1-\frac{c f^2+a g^2}{\left(c f+\mathbb{i}\sqrt{a}\sqrt{c}\,g\right)(f+g x)}}\,\text{EllipticPi}\left[\frac{\left(c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g\right)(e f-d g)}{e\left(c f^2+a g^2\right)},\mathbb{i}\,\text{ArcSinh}\left[\right.\right.\right.$$

$$\left.\left.\left.\frac{\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}}}{\sqrt{f+g x}}\right],\frac{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}{c f+\mathbb{i}\sqrt{a}\sqrt{c}\,g}\right]\right)\Big/\left(e\sqrt{-\frac{c f^2+a g^2}{c f-\mathbb{i}\sqrt{a}\sqrt{c}\,g}}\sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\Bigg)-\frac{1}{(e f-d g)^3}$$

$$3\,a\,e^3\,g^2\,\Bigg(\Bigg(\mathrm{i}\,f\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\Big[\frac{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},$$

$$\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\Big],\,\frac{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}\Big]\Bigg)\Bigg/\Bigg(\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\Bigg)-$$

$$\Bigg(\mathrm{i}\,d\,g\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\Big[\frac{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\,\mathrm{i}\,\text{ArcSinh}\Big[$$

$$\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\Big],\,\frac{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}\Big]\Bigg)\Bigg/\Bigg(e\,\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\Bigg)\Bigg)+\frac{1}{(e\,f-d\,g)^2}$$

$$5\,c\,e^2\,f\,\Bigg(\Bigg(\mathrm{i}\,f\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\Big[\frac{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},$$

$$\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}}{\sqrt{f+g\,x}}\Big],\,\frac{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}{c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}\Big]\Bigg)\Bigg/\Bigg(\sqrt{-\frac{c\,f^2+a\,g^2}{c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}}\Bigg)-$$

$$\Bigg(\mathrm{i}\,d\,g\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\sqrt{1-\frac{c\,f^2+a\,g^2}{\left(c\,f+\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(f+g\,x)}}\,\text{EllipticPi}\Big[\frac{\left(c\,f-\mathrm{i}\,\sqrt{a}\,\sqrt{c}\,g\right)\,(e\,f-d\,g)}{e\,\left(c\,f^2+a\,g^2\right)},\,\mathrm{i}\,\text{ArcSinh}\Big[$$

$$\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\Big], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\Big]\Bigg) \Bigg/ \left(e \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\Bigg) - \frac{1}{(e f-d g)^2}$$

$$c d e g \Bigg(\Bigg(i f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \text{EllipticPi}\Big[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)},$$

$$i \text{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\Big], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\Big]\Bigg) \Bigg/ \left(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right) -$$

$$\Bigg(i d g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \text{EllipticPi}\Big[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)},$$

$$i \text{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\Big], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\Big]\Bigg) \Bigg/ \left(e \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\Bigg) - \frac{1}{e f-d g}$$

$$c e \Bigg(\Bigg(i f \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c} g\right) (f+g x)}} \text{EllipticPi}\Big[\frac{\left(c f-i \sqrt{a} \sqrt{c} g\right) (e f-d g)}{e \left(c f^2+a g^2\right)},$$

$$i \text{ArcSinh}\Big[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}}}{\sqrt{f+g x}}\Big], \frac{c f-i \sqrt{a} \sqrt{c} g}{c f+i \sqrt{a} \sqrt{c} g}\Big]\Bigg) \Bigg/ \left(\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c} g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right) -$$

$$\left(i\, d\, g \sqrt{1-\frac{c f^2+a g^2}{\left(c f-i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}} \sqrt{1-\frac{c f^2+a g^2}{\left(c f+i \sqrt{a} \sqrt{c}\, g\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{\left(c f-i \sqrt{a} \sqrt{c}\, g\right)(e f-d g)}{e \left(c f^2+a g^2\right)},\right.\right.$$

$$\left.\left.\left. i\, \text{ArcSinh}\left[\frac{\sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}}}{\sqrt{f+g x}}\right], \frac{c f-i \sqrt{a} \sqrt{c}\, g}{c f+i \sqrt{a} \sqrt{c}\, g}\right]\right) \middle/ \left(e \sqrt{-\frac{c f^2+a g^2}{c f-i \sqrt{a} \sqrt{c}\, g}} \sqrt{c+\frac{c f^2}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}}\right)\right)\right)$$

- **Problem 654: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(d+e x) \sqrt{f+g x} \sqrt{1+c x^2}}\, dx$$

Optimal (type 4, 110 leaves, 4 steps):

$$-\frac{2 \sqrt{\frac{\sqrt{-c}\, (f+g x)}{\sqrt{-c}\, f+g}}\ \text{EllipticPi}\left[\frac{2 e}{\sqrt{-c}\, d+e}, \text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c}\, x}}{\sqrt{2}}\right], \frac{2 g}{\sqrt{-c}\, f+g}\right]}{\left(\sqrt{-c}\, d+e\right) \sqrt{f+g x}}$$

Result (type 4, 261 leaves):

$$-\left(2 i \sqrt{\frac{g \left(\frac{i}{\sqrt{c}}+x\right)}{f+g x}} \sqrt{-\frac{\frac{i g}{\sqrt{c}}-g x}{f+g x}}\ (f+g x) \left(\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i g}{\sqrt{c}}}}{\sqrt{f+g x}}\right], \frac{\sqrt{c}\, f-i g}{\sqrt{c}\, f+i g}\right]-\right.\right.$$

$$\left.\left.\text{EllipticPi}\left[\frac{\sqrt{c}\, (e f-d g)}{e \left(\sqrt{c}\, f+i g\right)}, i\, \text{ArcSinh}\left[\frac{\sqrt{-f-\frac{i g}{\sqrt{c}}}}{\sqrt{f+g x}}\right], \frac{\sqrt{c}\, f-i g}{\sqrt{c}\, f+i g}\right]\right)\right) \middle/ \left(\sqrt{-f-\frac{i g}{\sqrt{c}}}\ (e f-d g) \sqrt{1+c x^2}\right)$$

- **Problem 655: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{d+e x} \sqrt{f+g x} \sqrt{a+c x^2}}\, dx$$

Optimal (type 4, 454 leaves, 2 steps):

$$-\left(\left(c f^2+a g^2\right)^{1/4}(d+e x)\sqrt{\frac{(e f-d g)^2\left(a+c x^2\right)}{\left(c f^2+a g^2\right)(d+e x)^2}}\left(1+\frac{\sqrt{c d^2+a e^2}\,(f+g x)}{\sqrt{c f^2+a g^2}\,(d+e x)}\right)\right.$$

$$\left.\sqrt{\frac{1-\frac{2(c d f+a e g)(f+g x)}{\left(c f^2+a g^2\right)(d+e x)}+\frac{\left(c d^2+a e^2\right)(f+g x)^2}{\left(c f^2+a g^2\right)(d+e x)^2}}{\left(1+\frac{\sqrt{c d^2+a e^2}\,(f+g x)}{\sqrt{c f^2+a g^2}\,(d+e x)}\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\left(c d^2+a e^2\right)^{1/4}\sqrt{f+g x}}{\left(c f^2+a g^2\right)^{1/4}\sqrt{d+e x}}\right],\frac{1}{2}\left(1+\frac{c d f+a e g}{\sqrt{c d^2+a e^2}\,\sqrt{c f^2+a g^2}}\right)\right]\right)\Big/$$

$$\left(\left(c d^2+a e^2\right)^{1/4}(e f-d g)\sqrt{a+c x^2}\sqrt{1-\frac{2(c d f+a e g)(f+g x)}{\left(c f^2+a g^2\right)(d+e x)}+\frac{\left(c d^2+a e^2\right)(f+g x)^2}{\left(c f^2+a g^2\right)(d+e x)^2}}\right)$$

Result (type 4, 344 leaves) :

$$\left(\sqrt{2}\left(i\sqrt{a}+\sqrt{c}\,x\right)\sqrt{d+e x}\sqrt{\frac{d-\frac{i\sqrt{a}\,e}{\sqrt{c}}+\frac{i\sqrt{c}\,d x}{\sqrt{a}}+e x}{d+e x}}\right.$$

$$\left.\sqrt{\frac{\left(i\sqrt{c}\,d+\sqrt{a}\,e\right)(f+g x)}{\left(i\sqrt{c}\,f+\sqrt{a}\,g\right)(d+e x)}}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(e f-d g)\left(i\sqrt{a}+\sqrt{c}\,x\right)}{\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)(d+e x)}}\right],-\frac{\frac{i\sqrt{c}\,d f}{\sqrt{a}}-e f+d g+\frac{i\sqrt{a}\,e g}{\sqrt{c}}}{2 e f-2 d g}\right]\right)\Big/$$

$$\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\sqrt{\frac{(e f-d g)\left(i\sqrt{a}+\sqrt{c}\,x\right)}{\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)(d+e x)}}\sqrt{f+g x}\sqrt{a+c x^2}\right)$$

- **Problem 656: Result more than twice size of optimal antiderivative.**

$$\int\frac{1}{\sqrt{-1+x}\,\sqrt{1+x}\,\sqrt{-1+2x^2}}\,dx$$

Optimal (type 4, 52 leaves, 4 steps) :

$$\frac{\sqrt{1-2x^2}\,\sqrt{1-x^2}\ \text{EllipticF}[\text{ArcSin}[x],2]}{\sqrt{-1+x}\,\sqrt{1+x}\,\sqrt{-1+2x^2}}$$

Result (type 4, 107 leaves):

$$-\frac{2\,(-1+x)^{3/2}\,\sqrt{\frac{1+x}{1-x}}\,\sqrt{\frac{1-2\,x^2}{(-1+x)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2+\sqrt{2}+\frac{1}{-1+x}}}{2^{3/4}}\right],\,4\,\left(-4+3\,\sqrt{2}\right)\right]}{\sqrt{3+2\,\sqrt{2}}\,\sqrt{1+x}\,\sqrt{-1+2\,x^2}}$$

- **Problem 773: Unable to integrate problem.**

$$\int \frac{(d+e\,x)^m\,\left(a\,d\,e+\left(c\,d^2+a\,e^2\right)\,x+c\,d\,e\,x^2\right)^{-m}}{(f+g\,x)^2}\,dx$$

Optimal (type 5, 101 leaves, 2 steps):

$$\frac{1}{(c\,d\,f-a\,e\,g)^2\,(1-m)}c\,d\,(a\,e+c\,d\,x)\,(d+e\,x)^m\,\left(a\,d\,e+\left(c\,d^2+a\,e^2\right)\,x+c\,d\,e\,x^2\right)^{-m}\,\text{Hypergeometric2F1}\left[2,\,1-m,\,2-m,\,-\frac{g\,(a\,e+c\,d\,x)}{c\,d\,f-a\,e\,g}\right]$$

Result (type 8, 46 leaves):

$$\int \frac{(d+e\,x)^m\,\left(a\,d\,e+\left(c\,d^2+a\,e^2\right)\,x+c\,d\,e\,x^2\right)^{-m}}{(f+g\,x)^2}\,dx$$

- **Problem 774: Unable to integrate problem.**

$$\int \frac{(d+e\,x)^m\,\left(a\,d\,e+\left(c\,d^2+a\,e^2\right)\,x+c\,d\,e\,x^2\right)^{-m}}{(f+g\,x)^3}\,dx$$

Optimal (type 5, 105 leaves, 2 steps):

$$\frac{1}{(c\,d\,f-a\,e\,g)^3\,(1-m)}c^2\,d^2\,(a\,e+c\,d\,x)\,(d+e\,x)^m\,\left(a\,d\,e+\left(c\,d^2+a\,e^2\right)\,x+c\,d\,e\,x^2\right)^{-m}\,\text{Hypergeometric2F1}\left[3,\,1-m,\,2-m,\,-\frac{g\,(a\,e+c\,d\,x)}{c\,d\,f-a\,e\,g}\right]$$

Result (type 8, 46 leaves):

$$\int \frac{(d+e\,x)^m\,\left(a\,d\,e+\left(c\,d^2+a\,e^2\right)\,x+c\,d\,e\,x^2\right)^{-m}}{(f+g\,x)^3}\,dx$$

- **Problem 775: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (d+e\,x)^m\,(f+g\,x)^{3/2}\,\left(a\,d\,e+\left(c\,d^2+a\,e^2\right)\,x+c\,d\,e\,x^2\right)^{-m}\,dx$$

Optimal (type 5, 105 leaves, 3 steps):

$$\frac{1}{5\,g}2\,\left(-\frac{g\,(a\,e+c\,d\,x)}{c\,d\,f-a\,e\,g}\right)^m\,(d+e\,x)^m\,(f+g\,x)^{5/2}\,\left(a\,d\,e+\left(c\,d^2+a\,e^2\right)\,x+c\,d\,e\,x^2\right)^{-m}\,\text{Hypergeometric2F1}\left[\frac{5}{2},\,m,\,\frac{7}{2},\,\frac{c\,d\,(f+g\,x)}{c\,d\,f-a\,e\,g}\right]$$

Result (type 6, 234 leaves):

$$\frac{1}{3g} f (d+e x)^m ((a e+c d x)(d+e x))^{-m} \sqrt{f+g x}$$

$$\left(\left(9 a e g^2 x^2 \text{AppellF1}\left[2, m, -\frac{1}{2}, 3, -\frac{c d x}{a e}, -\frac{g x}{f}\right]\right) \Big/ \left(6 a e f \text{AppellF1}\left[2, m, -\frac{1}{2}, 3, -\frac{c d x}{a e}, -\frac{g x}{f}\right] + a e g x \text{AppellF1}\left[3, m, \frac{1}{2}, 4, -\frac{c d x}{a e}, -\frac{g x}{f}\right] - 2 c d f m x \text{AppellF1}\left[3, 1+m, -\frac{1}{2}, 4, -\frac{c d x}{a e}, -\frac{g x}{f}\right]\right) + 2 \left(\frac{g (a e+c d x)}{-c d f+a e g}\right)^m (f+g x) \text{Hypergeometric2F1}\left[\frac{3}{2}, m, \frac{5}{2}, \frac{c d (f+g x)}{c d f-a e g}\right]\right)$$

Problem 802: Result more than twice size of optimal antiderivative.

$$\int (1-e x)^m (1+e x)^m \left(a+c x^2\right)^p dx$$

Optimal (type 6, 54 leaves, 3 steps):

$$x \left(a+c x^2\right)^p \left(1+\frac{c x^2}{a}\right)^{-p} \text{AppellF1}\left[\frac{1}{2}, -p, -m, \frac{3}{2}, -\frac{c x^2}{a}, e^2 x^2\right]$$

Result (type 6, 167 leaves):

$$\left(3 a x \left(a+c x^2\right)^p \left(1-e^2 x^2\right)^m \text{AppellF1}\left[\frac{1}{2}, -p, -m, \frac{3}{2}, -\frac{c x^2}{a}, e^2 x^2\right]\right) \Big/ \left(3 a \text{AppellF1}\left[\frac{1}{2}, -p, -m, \frac{3}{2}, -\frac{c x^2}{a}, e^2 x^2\right] + 2 x^2 \left(c p \text{AppellF1}\left[\frac{3}{2}, 1-p, -m, \frac{5}{2}, -\frac{c x^2}{a}, e^2 x^2\right] - a e^2 m \text{AppellF1}\left[\frac{3}{2}, -p, 1-m, \frac{5}{2}, -\frac{c x^2}{a}, e^2 x^2\right]\right)\right)$$

Problem 803: Unable to integrate problem.

$$\int (d-e x)^m (d+e x)^m \left(a+c x^2\right)^p dx$$

Optimal (type 6, 89 leaves, 4 steps):

$$x (d-e x)^m (d+e x)^m \left(a+c x^2\right)^p \left(1+\frac{c x^2}{a}\right)^{-p} \left(1-\frac{e^2 x^2}{d^2}\right)^{-m} \text{AppellF1}\left[\frac{1}{2}, -p, -m, \frac{3}{2}, -\frac{c x^2}{a}, \frac{e^2 x^2}{d^2}\right]$$

Result (type 8, 27 leaves):

$$\int (d-e x)^m (d+e x)^m \left(a+c x^2\right)^p dx$$

Problem 804: Unable to integrate problem.

$$\int (d+e x)^m (d f-e f x)^m \left(a+c x^2\right)^p dx$$

Optimal (type 6, 92 leaves, 4 steps):

$$x (d+e x)^m (d f-e f x)^m \left(a+c x^2\right)^p \left(1+\frac{c x^2}{a}\right)^{-p} \left(1-\frac{e^2 x^2}{d^2}\right)^{-m} \text{AppellF1}\left[\frac{1}{2}, -p, -m, \frac{3}{2}, -\frac{c x^2}{a}, \frac{e^2 x^2}{d^2}\right]$$

Result (type 8, 30 leaves):

$$\int (d+e x)^m (d f-e f x)^m \left(a+c x^2\right)^p \, dx$$

- **Problem 805: Result more than twice size of optimal antiderivative.**

$$\int (d+e x)^3 (f+g x)^n \left(a+2 c d x+c e x^2\right) \, dx$$

Optimal (type 3, 275 leaves, 2 steps):

$$-\frac{(e f-d g)^3 \left(a g^2+c f (e f-2 d g)\right) (f+g x)^{1+n}}{g^6 (1+n)}+\frac{(e f-d g)^2 \left(3 a e g^2+c \left(5 e^2 f^2-10 d e f g+2 d^2 g^2\right)\right) (f+g x)^{2+n}}{g^6 (2+n)}-\frac{e (e f-d g) \left(3 a e g^2+c \left(10 e^2 f^2-20 d e f g+7 d^2 g^2\right)\right) (f+g x)^{3+n}}{g^6 (3+n)}+\frac{e^2 \left(a e g^2+c \left(10 e^2 f^2-20 d e f g+9 d^2 g^2\right)\right) (f+g x)^{4+n}}{g^6 (4+n)}-\frac{5 c e^3 (e f-d g) (f+g x)^{5+n}}{g^6 (5+n)}+\frac{c e^4 (f+g x)^{6+n}}{g^6 (6+n)}$$

Result (type 3, 577 leaves):

$$\frac{1}{g^6 (1+n) (2+n) (3+n) (4+n) (5+n) (6+n)}(f+g x)^{1+n} \left(a g^2 \left(30+11 n+n^2\right) \left(d^3 g^3 \left(24+26 n+9 n^2+n^3\right)+3 d^2 e g^2 \left(12+7 n+n^2\right) (-f+g (1+n) x)+3 d e^2 g (4+n) \left(2 f^2-2 f g (1+n) x+g^2 \left(2+3 n+n^2\right) x^2\right)+e^3 \left(-6 f^3+6 f^2 g (1+n) x-3 f g^2 \left(2+3 n+n^2\right) x^2+g^3 \left(6+11 n+6 n^2+n^3\right) x^3\right)\right)+c \left(2 d^4 g^4 \left(360+342 n+119 n^2+18 n^3+n^4\right) (-f+g (1+n) x)+7 d^3 e g^3 \left(120+74 n+15 n^2+n^3\right) \left(2 f^2-2 f g (1+n) x+g^2 \left(2+3 n+n^2\right) x^2\right)+9 d^2 e^2 g^2 \left(30+11 n+n^2\right) \left(-6 f^3+6 f^2 g (1+n) x-3 f g^2 \left(2+3 n+n^2\right) x^2+g^3 \left(6+11 n+6 n^2+n^3\right) x^3\right)+5 d e^3 g (6+n) \left(24 f^4-24 f^3 g (1+n) x+12 f^2 g^2 \left(2+3 n+n^2\right) x^2-4 f g^3 \left(6+11 n+6 n^2+n^3\right) x^3+g^4 \left(24+50 n+35 n^2+10 n^3+n^4\right) x^4\right)-e^4 \left(120 f^5-120 f^4 g (1+n) x+60 f^3 g^2 \left(2+3 n+n^2\right) x^2-20 f^2 g^3 \left(6+11 n+6 n^2+n^3\right) x^3+5 f g^4 \left(24+50 n+35 n^2+10 n^3+n^4\right) x^4-g^5 \left(120+274 n+225 n^2+85 n^3+15 n^4+n^5\right) x^5\right)\right)\right)$$

- **Problem 810: Unable to integrate problem.**

$$\int \frac{(f+g x)^n \left(a+2 c d x+c e x^2\right)}{(d+e x)^2} \, dx$$

Optimal (type 5, 88 leaves, 3 steps):

$$\frac{c (f+g x)^{1+n}}{e g (1+n)}-\frac{\left(c d^2-a e\right) g (f+g x)^{1+n} \text{Hypergeometric2F1}\left[2,1+n,2+n,\frac{e (f+g x)}{e f-d g}\right]}{e (e f-d g)^2 (1+n)}$$

Result (type 8, 30 leaves):

$$\int \frac{(f+g x)^n \left(a+2 c d x+c e x^2\right)}{(d+e x)^2} \, dx$$

Problem 811: Unable to integrate problem.

$$\int \frac{(f+g x)^n \left(a+2 c d x+c e x^2\right)}{(d+e x)^3} \, dx$$

Optimal (type 5, 193 leaves, 3 steps):

$$-\frac{\left(a-\frac{c d^2}{e}\right) (f+g x)^{1+n}}{2 (e f-d g) (d+e x)^2}-\frac{\left(c d^2-a e\right) g (1-n) (f+g x)^{1+n}}{2 e (e f-d g)^2 (d+e x)}+\frac{1}{2 e (e f-d g)^3 (1+n)}$$
$$\left(a e g^2 (1-n) n-c \left(2 e^2 f^2-4 d e f g+d^2 g^2 \left(2+n-n^2\right)\right)\right) (f+g x)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{e (f+g x)}{e f-d g}\right]$$

Result (type 8, 30 leaves):

$$\int \frac{(f+g x)^n \left(a+2 c d x+c e x^2\right)}{(d+e x)^3} \, dx$$

Problem 812: Unable to integrate problem.

$$\int \frac{(f+g x)^n \left(a+2 c d x+c e x^2\right)}{(d+e x)^4} \, dx$$

Optimal (type 5, 197 leaves, 3 steps):

$$-\frac{\left(a-\frac{c d^2}{e}\right) (f+g x)^{1+n}}{3 (e f-d g) (d+e x)^3}-\frac{\left(c d^2-a e\right) g (2-n) (f+g x)^{1+n}}{6 e (e f-d g)^2 (d+e x)^2}+\frac{1}{6 e (e f-d g)^4 (1+n)}$$
$$g \left(a e g^2 \left(2-3 n+n^2\right)+c \left(6 e^2 f^2-12 d e f g+d^2 g^2 \left(4+3 n-n^2\right)\right)\right) (f+g x)^{1+n} \text{Hypergeometric2F1}\left[2, 1+n, 2+n, \frac{e (f+g x)}{e f-d g}\right]$$

Result (type 8, 30 leaves):

$$\int \frac{(f+g x)^n \left(a+2 c d x+c e x^2\right)}{(d+e x)^4} \, dx$$

Problem 813: Result unnecessarily involves higher level functions.

$$\int (d+e x)^m (f+g x)^n \left(a+2 c d x+c e x^2\right) \, dx$$

Optimal (type 5, 231 leaves, 4 steps):

$$-\frac{c\,(e f-d g)\,(2+m)\,(d+e x)^{1+m}\,(f+g x)^{1+n}}{e g^2\,(2+m+n)\,(3+m+n)}+\frac{c\,(d+e x)^{2+m}\,(f+g x)^{1+n}}{e g\,(3+m+n)}+$$
$$\Big((c\,(e f-d g)\,(2+m)\,(e f\,(1+m)+d g\,(1+n))+g\,(2+m+n)\,(a e g\,(3+m+n)-c d\,(e f\,(2+m)+d g\,(1+n))))\,(d+e x)^{1+m}$$
$$(f+g x)^n\left(\frac{e\,(f+g x)}{e f-d g}\right)^{-n}\text{Hypergeometric2F1}\left[1+m,-n,2+m,-\frac{g\,(d+e x)}{e f-d g}\right]\Big)\Big/\left(e^2 g^2\,(1+m)\,(2+m+n)\,(3+m+n)\right)$$

Result (type 6, 328 leaves):

$$\frac{1}{3}\,(d+e x)^m\,(f+g x)^n\left(\left(9 c d^2 f x^2\,\text{AppellF1}\left[2,-m,-n,3,-\frac{e x}{d},-\frac{g x}{f}\right]\right)\Big/\left(3 d f\,\text{AppellF1}\left[2,-m,-n,3,-\frac{e x}{d},-\frac{g x}{f}\right]+\right.\right.$$
$$\left.e f m x\,\text{AppellF1}\left[3,1-m,-n,4,-\frac{e x}{d},-\frac{g x}{f}\right]+d g n x\,\text{AppellF1}\left[3,-m,1-n,4,-\frac{e x}{d},-\frac{g x}{f}\right]\right)+$$
$$\left(4 c d e f x^3\,\text{AppellF1}\left[3,-m,-n,4,-\frac{e x}{d},-\frac{g x}{f}\right]\right)\Big/\left(4 d f\,\text{AppellF1}\left[3,-m,-n,4,-\frac{e x}{d},-\frac{g x}{f}\right]+\right.$$
$$\left.e f m x\,\text{AppellF1}\left[4,1-m,-n,5,-\frac{e x}{d},-\frac{g x}{f}\right]+d g n x\,\text{AppellF1}\left[4,-m,1-n,5,-\frac{e x}{d},-\frac{g x}{f}\right]\right)+$$
$$\left.\frac{3 a\left(\frac{g\,(d+e x)}{-e f+d g}\right)^{-m}(f+g x)\,\text{Hypergeometric2F1}\left[-m,1+n,2+n,\frac{e\,(f+g x)}{e f-d g}\right]}{g\,(1+n)}\right)$$

Problem 850: Result more than twice size of optimal antiderivative.

$$\int\frac{(d+e x)^{3/2}}{\sqrt{f+g x}\,\left(a+b x+c x^2\right)}\,dx$$

Optimal (type 3, 417 leaves, 11 steps):

$$\frac{2 e^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{g}\,\sqrt{d+e x}}{\sqrt{e}\,\sqrt{f+g x}}\right]}{c\sqrt{g}}-\frac{2\left(e\,(2 c d-b e)+\frac{2 c^2 d^2+b^2 e^2-2 c e\,(b d+a e)}{\sqrt{b^2-4 a c}}\right)\text{ArcTanh}\left[\frac{\sqrt{2 c f-\left(b-\sqrt{b^2-4 a c}\right) g}\,\sqrt{d+e x}}{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\,\sqrt{f+g x}}\right]}{c\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\,\sqrt{2 c f-\left(b-\sqrt{b^2-4 a c}\right) g}}-$$

$$\frac{2\left(e\,(2 c d-b e)-\frac{2 c^2 d^2+b^2 e^2-2 c e\,(b d+a e)}{\sqrt{b^2-4 a c}}\right)\text{ArcTanh}\left[\frac{\sqrt{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\,\sqrt{d+e x}}{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\,\sqrt{f+g x}}\right]}{c\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\,\sqrt{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}}$$

Result (type 3, 1164 leaves):

$$\frac{1}{2\,c}\Bigg(\Big(\sqrt{2}\,\big(2\,c^2\,d^2+b\,\big(b-\sqrt{b^2-4\,a\,c}\,\big)\,e^2-2\,c\,e\,\big(b\,d-\sqrt{b^2-4\,a\,c}\,\,d+a\,e\big)\big)\,\text{Log}\big[-b+\sqrt{b^2-4\,a\,c}\,-2\,c\,x\big]\Big)\Big/$$

$$\Big(\sqrt{b^2-4\,a\,c}\,\sqrt{2\,c^2\,d\,f+b\,\big(b-\sqrt{b^2-4\,a\,c}\,\big)\,e\,g+c\,\big(\sqrt{b^2-4\,a\,c}\,\,e\,f+\sqrt{b^2-4\,a\,c}\,\,d\,g-2\,a\,e\,g-b\,(e\,f+d\,g)\big)}\,\Big)-$$

$$\Big(\sqrt{2}\,\big(2\,c^2\,d^2+b\,\big(b+\sqrt{b^2-4\,a\,c}\,\big)\,e^2-2\,c\,e\,\big(b\,d+\sqrt{b^2-4\,a\,c}\,\,d+a\,e\big)\big)\,\text{Log}\big[b+\sqrt{b^2-4\,a\,c}\,+2\,c\,x\big]\Big)\Big/$$

$$\Big(\sqrt{b^2-4\,a\,c}\,\sqrt{2\,c^2\,d\,f+b\,\big(b+\sqrt{b^2-4\,a\,c}\,\big)\,e\,g-c\,\big(b\,e\,f+\sqrt{b^2-4\,a\,c}\,\,e\,f+b\,d\,g+\sqrt{b^2-4\,a\,c}\,\,d\,g+2\,a\,e\,g\big)}\,\Big)+$$

$$\frac{2\,e^{3/2}\,\text{Log}\big[e\,f+d\,g+2\,e\,g\,x+2\,\sqrt{e}\,\sqrt{g}\,\sqrt{d+e\,x}\,\sqrt{f+g\,x}\,\big]}{\sqrt{g}}+\Big(\sqrt{2}\,\big(-2\,c^2\,d^2+b\,\big(-b+\sqrt{b^2-4\,a\,c}\,\big)\,e^2+2\,c\,e\,\big(b\,d-\sqrt{b^2-4\,a\,c}\,\,d+a\,e\big)\big)$$

$$\text{Log}\Big[2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\sqrt{2\,c^2\,d\,f+b\,\big(b-\sqrt{b^2-4\,a\,c}\,\big)\,e\,g+c\,\big(\sqrt{b^2-4\,a\,c}\,\,e\,f+\sqrt{b^2-4\,a\,c}\,\,d\,g-2\,a\,e\,g-b\,(e\,f+d\,g)\big)}\,\sqrt{d+e\,x}\,\sqrt{f+g\,x}\,+$$

$$b^2\,(d\,g+e\,(f+2\,g\,x))-b\,\sqrt{b^2-4\,a\,c}\,\,(d\,g+e\,(f+2\,g\,x))+$$

$$2\,c\,\big(\sqrt{b^2-4\,a\,c}\,\,e\,f\,x-2\,a\,e\,(f+2\,g\,x)+d\,\big(2\,\sqrt{b^2-4\,a\,c}\,\,f-2\,a\,g+\sqrt{b^2-4\,a\,c}\,\,g\,x\big)\big)\Big]\Big)\Big/$$

$$\Big(\sqrt{b^2-4\,a\,c}\,\sqrt{2\,c^2\,d\,f+b\,\big(b-\sqrt{b^2-4\,a\,c}\,\big)\,e\,g+c\,\big(\sqrt{b^2-4\,a\,c}\,\,e\,f+\sqrt{b^2-4\,a\,c}\,\,d\,g-2\,a\,e\,g-b\,(e\,f+d\,g)\big)}\,\Big)+$$

$$\Big(\sqrt{2}\,\big(2\,c^2\,d^2+b\,\big(b+\sqrt{b^2-4\,a\,c}\,\big)\,e^2-2\,c\,e\,\big(b\,d+\sqrt{b^2-4\,a\,c}\,\,d+a\,e\big)\big)\,\text{Log}\Big[2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}$$

$$\sqrt{2\,c^2\,d\,f+b\,\big(b+\sqrt{b^2-4\,a\,c}\,\big)\,e\,g-c\,\big(b\,e\,f+\sqrt{b^2-4\,a\,c}\,\,e\,f+b\,d\,g+\sqrt{b^2-4\,a\,c}\,\,d\,g+2\,a\,e\,g\big)}\,\sqrt{d+e\,x}\,\sqrt{f+g\,x}\,-b^2\,(d\,g+e\,(f+2\,g\,x))-$$

$$b\,\sqrt{b^2-4\,a\,c}\,\,(d\,g+e\,(f+2\,g\,x))+2\,c\,\big(\sqrt{b^2-4\,a\,c}\,\,e\,f\,x+2\,a\,e\,(f+2\,g\,x)+d\,\big(2\,\sqrt{b^2-4\,a\,c}\,\,f+2\,a\,g+\sqrt{b^2-4\,a\,c}\,\,g\,x\big)\big)\Big]\Big)\Big/$$

$$\Big(\sqrt{b^2-4\,a\,c}\,\sqrt{2\,c^2\,d\,f+b\,\big(b+\sqrt{b^2-4\,a\,c}\,\big)\,e\,g-c\,\big(b\,e\,f+\sqrt{b^2-4\,a\,c}\,\,e\,f+b\,d\,g+\sqrt{b^2-4\,a\,c}\,\,d\,g+2\,a\,e\,g\big)}\,\Big)\Bigg)$$

- **Problem 851: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{d+e\,x}}{\sqrt{f+g\,x}\,\left(a+b\,x+c\,x^2\right)}\,dx$$

Optimal (type 3, 285 leaves, 6 steps):

$$-\frac{2\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\ \text{ArcTanh}\left[\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\ \sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\ \sqrt{f+gx}}\right]}{\sqrt{b^2-4ac}\ \sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}+\frac{2\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\ \text{ArcTanh}\left[\frac{\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\ \sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\ \sqrt{f+gx}}\right]}{\sqrt{b^2-4ac}\ \sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}$$

Result (type 3, 925 leaves):

$$\frac{1}{\sqrt{2}\,\sqrt{b^2-4ac}}\left(\frac{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\operatorname{Log}\left[-b+\sqrt{b^2-4ac}-2cx\right]}{\sqrt{2c^2df+b\left(b-\sqrt{b^2-4ac}\right)eg+c\left(\sqrt{b^2-4ac}\,ef+\sqrt{b^2-4ac}\,dg-2aeg-b\,(ef+dg)\right)}}-\right.$$

$$\frac{\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\operatorname{Log}\left[b+\sqrt{b^2-4ac}+2cx\right]}{\sqrt{2c^2df+b\left(b+\sqrt{b^2-4ac}\right)eg-c\left(bef+\sqrt{b^2-4ac}\,ef+bdg+\sqrt{b^2-4ac}\,dg+2aeg\right)}}-\left(\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\right.$$

$$\operatorname{Log}\Big[2\sqrt{2}\,\sqrt{b^2-4ac}\,\sqrt{2c^2df+b\left(b-\sqrt{b^2-4ac}\right)eg+c\left(\sqrt{b^2-4ac}\,ef+\sqrt{b^2-4ac}\,dg-2aeg-b\,(ef+dg)\right)}\,\sqrt{d+ex}\,\sqrt{f+gx}+$$

$$b^2\,(dg+e\,(f+2gx))-b\sqrt{b^2-4ac}\,(dg+e\,(f+2gx))+$$

$$\left.2c\left(\sqrt{b^2-4ac}\,efx-2ae\,(f+2gx)+d\left(2\sqrt{b^2-4ac}\,f-2ag+\sqrt{b^2-4ac}\,gx\right)\right)\Big]\right)\Big/$$

$$\left(\sqrt{2c^2df+b\left(b-\sqrt{b^2-4ac}\right)eg+c\left(\sqrt{b^2-4ac}\,ef+\sqrt{b^2-4ac}\,dg-2aeg-b\,(ef+dg)\right)}\right)+$$

$$\left(\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\operatorname{Log}\Big[2\sqrt{2}\,\sqrt{b^2-4ac}\right.$$

$$\sqrt{2c^2df+b\left(b+\sqrt{b^2-4ac}\right)eg-c\left(bef+\sqrt{b^2-4ac}\,ef+bdg+\sqrt{b^2-4ac}\,dg+2aeg\right)}\,\sqrt{d+ex}\,\sqrt{f+gx}-b^2\,(dg+e\,(f+2gx))-$$

$$\left.b\sqrt{b^2-4ac}\,(dg+e\,(f+2gx))+2c\left(\sqrt{b^2-4ac}\,efx+2ae\,(f+2gx)+d\left(2\sqrt{b^2-4ac}\,f+2ag+\sqrt{b^2-4ac}\,gx\right)\right)\Big]\right)\Big/$$

$$\left.\left(\sqrt{2c^2df+b\left(b+\sqrt{b^2-4ac}\right)eg-c\left(bef+\sqrt{b^2-4ac}\,ef+bdg+\sqrt{b^2-4ac}\,dg+2aeg\right)}\right)\right)$$

- **Problem 852: Result more than twice size of optimal antiderivative.**

$$\int\frac{1}{\sqrt{d+ex}\,\sqrt{f+gx}\,\left(a+bx+cx^2\right)}\,dx$$

Optimal (type 3, 287 leaves, 6 steps) :

$$-\frac{4\,c\,\text{ArcTanh}\left[\frac{\sqrt{2\,c\,f-\left(b-\sqrt{b^2-4\,a\,c}\right)g}\,\sqrt{d+e\,x}}{\sqrt{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)e}\,\sqrt{f+g\,x}}\right]}{\sqrt{b^2-4\,a\,c}\,\sqrt{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)e}\,\sqrt{2\,c\,f-\left(b-\sqrt{b^2-4\,a\,c}\right)g}}+\frac{4\,c\,\text{ArcTanh}\left[\frac{\sqrt{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)g}\,\sqrt{d+e\,x}}{\sqrt{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\,\sqrt{f+g\,x}}\right]}{\sqrt{b^2-4\,a\,c}\,\sqrt{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\,\sqrt{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)g}}$$

Result (type 3, 836 leaves) :

$$\frac{1}{\sqrt{b^2-4\,a\,c}}\sqrt{2}\,c\left(\frac{\text{Log}\left[-b+\sqrt{b^2-4\,a\,c}-2\,c\,x\right]}{\sqrt{2\,c^2\,d\,f+b\left(b-\sqrt{b^2-4\,a\,c}\right)e\,g+c\left(\sqrt{b^2-4\,a\,c}\,e\,f+\sqrt{b^2-4\,a\,c}\,d\,g-2\,a\,e\,g-b\,(e\,f+d\,g)\right)}}-\right.$$

$$\frac{\text{Log}\left[b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right]}{\sqrt{2\,c^2\,d\,f+b\left(b+\sqrt{b^2-4\,a\,c}\right)e\,g-c\left(b\,e\,f+\sqrt{b^2-4\,a\,c}\,e\,f+b\,d\,g+\sqrt{b^2-4\,a\,c}\,d\,g+2\,a\,e\,g\right)}}-$$

$$\text{Log}\Big[2\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\sqrt{2\,c^2\,d\,f+b\left(b-\sqrt{b^2-4\,a\,c}\right)e\,g+c\left(\sqrt{b^2-4\,a\,c}\,e\,f+\sqrt{b^2-4\,a\,c}\,d\,g-2\,a\,e\,g-b\,(e\,f+d\,g)\right)}\,\sqrt{d+e\,x}\,\sqrt{f+g\,x}+$$
$$b^2\,(d\,g+e\,(f+2\,g\,x))-b\sqrt{b^2-4\,a\,c}\,(d\,g+e\,(f+2\,g\,x))+$$
$$2\,c\left(\sqrt{b^2-4\,a\,c}\,e\,f\,x-2\,a\,e\,(f+2\,g\,x)+d\left(2\sqrt{b^2-4\,a\,c}\,f-2\,a\,g+\sqrt{b^2-4\,a\,c}\,g\,x\right)\right)\Big]\Big/$$
$$\left(\sqrt{2\,c^2\,d\,f+b\left(b-\sqrt{b^2-4\,a\,c}\right)e\,g+c\left(\sqrt{b^2-4\,a\,c}\,e\,f+\sqrt{b^2-4\,a\,c}\,d\,g-2\,a\,e\,g-b\,(e\,f+d\,g)\right)}\right)+$$

$$\text{Log}\Big[2\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\sqrt{2\,c^2\,d\,f+b\left(b+\sqrt{b^2-4\,a\,c}\right)e\,g-c\left(b\,e\,f+\sqrt{b^2-4\,a\,c}\,e\,f+b\,d\,g+\sqrt{b^2-4\,a\,c}\,d\,g+2\,a\,e\,g\right)}\,\sqrt{d+e\,x}\,\sqrt{f+g\,x}-$$
$$b^2\,(d\,g+e\,(f+2\,g\,x))-b\sqrt{b^2-4\,a\,c}\,(d\,g+e\,(f+2\,g\,x))+$$
$$2\,c\left(\sqrt{b^2-4\,a\,c}\,e\,f\,x+2\,a\,e\,(f+2\,g\,x)+d\left(2\sqrt{b^2-4\,a\,c}\,f+2\,a\,g+\sqrt{b^2-4\,a\,c}\,g\,x\right)\right)\Big]\Big/$$
$$\left.\left(\sqrt{2\,c^2\,d\,f+b\left(b+\sqrt{b^2-4\,a\,c}\right)e\,g-c\left(b\,e\,f+\sqrt{b^2-4\,a\,c}\,e\,f+b\,d\,g+\sqrt{b^2-4\,a\,c}\,d\,g+2\,a\,e\,g\right)}\right)\right)$$

Problem 853: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(d+e x)^{3/2} \sqrt{f+g x} \left(a+b x+c x^2\right)} \, dx$$

Optimal (type 3, 429 leaves, 8 steps):

$$\frac{4 c e \sqrt{f+g x}}{\sqrt{b^2-4 a c} \left(2 c d-\left(b-\sqrt{b^2-4 a c}\right) e\right) (e f-d g) \sqrt{d+e x}}-\frac{4 c e \sqrt{f+g x}}{\sqrt{b^2-4 a c} \left(2 c d-\left(b+\sqrt{b^2-4 a c}\right) e\right) (e f-d g) \sqrt{d+e x}}-$$

$$\frac{8 c^2 \operatorname{ArcTanh}\left[\frac{\sqrt{2 c f-\left(b-\sqrt{b^2-4 a c}\right) g} \sqrt{d+e x}}{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e} \sqrt{f+g x}}\right]}{\sqrt{b^2-4 a c} \left(2 c d-\left(b-\sqrt{b^2-4 a c}\right) e\right)^{3/2} \sqrt{2 c f-\left(b-\sqrt{b^2-4 a c}\right) g}}+\frac{8 c^2 \operatorname{ArcTanh}\left[\frac{\sqrt{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g} \sqrt{d+e x}}{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e} \sqrt{f+g x}}\right]}{\sqrt{b^2-4 a c} \left(2 c d-\left(b+\sqrt{b^2-4 a c}\right) e\right)^{3/2} \sqrt{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}}$$

Result (type 3, 1011 leaves):

$$-\frac{2 e^2 \sqrt{f+g x}}{\left(c d^2+e (-b d+a e)\right) (e f-d g) \sqrt{d+e x}}+\left(c \left(-2 c d+\left(b+\sqrt{b^2-4 a c}\right) e\right) \text{Log}\left[-b+\sqrt{b^2-4 a c}-2 c x\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{b^2-4 a c} \left(-c d^2+e (b d-a e)\right) \sqrt{2 c^2 d f+b \left(b-\sqrt{b^2-4 a c}\right) e g+c \left(\sqrt{b^2-4 a c} e f+\sqrt{b^2-4 a c} d g-2 a e g-b (e f+d g)\right)}\right)+$$

$$\left(c \left(2 c d+\left(-b+\sqrt{b^2-4 a c}\right) e\right) \text{Log}\left[b+\sqrt{b^2-4 a c}+2 c x\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{b^2-4 a c} \left(-c d^2+e (b d-a e)\right) \sqrt{2 c^2 d f+b \left(b+\sqrt{b^2-4 a c}\right) e g-c \left(b e f+\sqrt{b^2-4 a c} e f+b d g+\sqrt{b^2-4 a c} d g+2 a e g\right)}\right)-$$

$$\Bigg(c \left(2 c d+\left(-b+\sqrt{b^2-4 a c}\right) e\right) \text{Log}\Big[b e f+\sqrt{b^2-4 a c} e f+b d g+\sqrt{b^2-4 a c} d g+2 b e g x+2 \sqrt{b^2-4 a c} e g x-$$

$$2 \sqrt{4 c^2 d f+2 b \left(b+\sqrt{b^2-4 a c}\right) e g-2 c \left(b e f+\sqrt{b^2-4 a c} e f+b d g+\sqrt{b^2-4 a c} d g+2 a e g\right)} \sqrt{d+e x} \sqrt{f+g x}-$$

$$2 c (2 d f+e f x+d g x)\Big]\Bigg) \Big/$$

$$\left(\sqrt{2} \sqrt{b^2-4 a c} \left(-c d^2+e (b d-a e)\right) \sqrt{2 c^2 d f+b \left(b+\sqrt{b^2-4 a c}\right) e g-c \left(b e f+\sqrt{b^2-4 a c} e f+b d g+\sqrt{b^2-4 a c} d g+2 a e g\right)}\right)-$$

$$\Bigg(c \left(-2 c d+\left(b+\sqrt{b^2-4 a c}\right) e\right) \text{Log}\Big[-b e f+\sqrt{b^2-4 a c} e f-b d g+\sqrt{b^2-4 a c} d g-2 b e g x+2 \sqrt{b^2-4 a c} e g x+$$

$$2 \sqrt{2} \sqrt{2 c^2 d f+b \left(b-\sqrt{b^2-4 a c}\right) e g+c \left(\sqrt{b^2-4 a c} e f+\sqrt{b^2-4 a c} d g-2 a e g-b (e f+d g)\right)} \sqrt{d+e x} \sqrt{f+g x}+$$

$$2 c (2 d f+e f x+d g x)\Big]\Bigg) \Big/$$

$$\left(\sqrt{2} \sqrt{b^2-4 a c} \left(-c d^2+e (b d-a e)\right) \sqrt{2 c^2 d f+b \left(b-\sqrt{b^2-4 a c}\right) e g+c \left(\sqrt{b^2-4 a c} e f+\sqrt{b^2-4 a c} d g-2 a e g-b (e f+d g)\right)}\right)$$

- **Problem 886: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (d+e x)^3 \sqrt{f+g x} \sqrt{a+b x+c x^2}\, dx$$

Optimal (type 4, 1551 leaves, 10 steps) :

$$-\frac{1}{3465 c^4 e g^4} 2 \left(64 b^4 e^4 g^4 + 4 b^2 c e^3 g^3 (7 b e f - 66 b d g - 69 a e g) + c^4 \left(187 e^4 f^4 - 732 d e^3 f^3 g + 1098 d^2 e^2 f^2 g^2 - 798 d^3 e f g^3 + 315 d^4 g^4\right) + 3 c^2 e^2 g^2 \left(50 a^2 e^2 g^2 - a b e g (29 e f - 297 d g) + 3 b^2 \left(e^2 f^2 - 11 d e f g + 44 d^2 g^2\right)\right) - c^3 e g \left(6 a e g \left(2 e^2 f^2 - 33 d e f g + 165 d^2 g^2\right) + b \left(8 e^3 f^3 - 99 d^2 e f g^2 + 231 d^3 g^3\right)\right)\right) \sqrt{f+g x} \sqrt{a+b x+c x^2} +$$

$$\frac{2 (d+e x)^4 \sqrt{f+g x} \sqrt{a+b x+c x^2}}{11 e} + \frac{1}{3465 c^3 g^4} 2 \left(48 b^3 e^3 g^3 + b c e^2 g^2 (67 b e f - 198 b d g - 157 a e g) + c^3 \left(233 e^3 f^3 - 843 d e^2 f^2 g + 1107 d^2 e f g^2 - 567 d^3 g^3\right) - c^2 e g \left(2 a e g (74 e f - 231 d g) - 3 b \left(24 e^2 f^2 - 88 d e f g + 99 d^2 g^2\right)\right)\right) (f+g x)^{3/2} \sqrt{a+b x+c x^2} - \frac{1}{693 c^2 g^4} 2 e \left(8 b^2 e^2 g^2 + c e g (19 b e f - 33 b d g - 18 a e g) + c^2 \left(29 e^2 f^2 - 96 d e f g + 81 d^2 g^2\right)\right) (f+g x)^{5/2} \sqrt{a+b x+c x^2} +$$

$$\frac{2 e^2 (c e f - 3 c d g + b e g) (f+g x)^{7/2} \sqrt{a+b x+c x^2}}{99 c g^4} + \frac{1}{3465 c^5 g^5 \sqrt{\frac{c (f+g x)}{2 c f - \left(b+\sqrt{b^2-4 a c}\right) g}} \sqrt{a+b x+c x^2}} \sqrt{2} \sqrt{b^2-4 a c} \left(128 b^5 e^3 g^5 - 8 b^3 c e^2 g^4 (7 b e f + 66 b d g + 87 a e g) + 2 c^5 f^2 \left(64 e^3 f^3 - 264 d e^2 f^2 g + 396 d^2 e f g^2 - 231 d^3 g^3\right) + b c^2 e g^3 \left(771 a^2 e^2 g^2 + 6 a b e g (43 e f + 396 d g) - b^2 \left(37 e^2 f^2 - 264 d e f g - 792 d^2 g^2\right)\right) - c^4 g \left(b f \left(56 e^3 f^3 - 264 d e^2 f^2 g + 495 d^2 e f g^2 - 462 d^3 g^3\right) - 18 a g \left(6 e^3 f^3 - 33 d e^2 f^2 g + 88 d^2 e f g^2 + 77 d^3 g^3\right)\right) - c^3 g^2 \left(6 a^2 e^2 g^2 (26 e f + 231 d g) - 9 a b e g \left(15 e^2 f^2 - 110 d e f g - 319 d^2 g^2\right) + b^2 \left(37 e^3 f^3 - 198 d e^2 f^2 g + 495 d^2 e f g^2 + 462 d^3 g^3\right)\right)\right) \sqrt{f+g x} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2-4 a c}\, g}{2 c f - \left(b+\sqrt{b^2-4 a c}\right) g}\right] +$$

$$\frac{1}{3465 c^5 g^5 \sqrt{f+g x} \sqrt{a+b x+c x^2}} 2 \sqrt{2} \sqrt{b^2-4 a c} \left(c f^2 - b f g + a g^2\right) \left(64 b^4 e^3 g^4 + 4 b^2 c e^2 g^3 (7 b e f - 66 b d g - 69 a e g) - 2 c^4 f \left(64 e^3 f^3 - 264 d e^2 f^2 g + 396 d^2 e f g^2 - 231 d^3 g^3\right) + 3 c^2 e g^2 \left(50 a^2 e^2 g^2 - a b e g (29 e f - 297 d g) + 3 b^2 \left(e^2 f^2 - 11 d e f g + 44 d^2 g^2\right)\right) - c^3 g \left(6 a e g \left(2 e^2 f^2 - 33 d e f g + 165 d^2 g^2\right) + b \left(8 e^3 f^3 - 99 d^2 e f g^2 + 231 d^3 g^3\right)\right)\right) \sqrt{\frac{c (f+g x)}{2 c f - \left(b+\sqrt{b^2-4 a c}\right) g}} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2-4 a c}\, g}{2 c f - \left(b+\sqrt{b^2-4 a c}\right) g}\right]$$

Result (type 4, 26 600 leaves) : Display of huge result suppressed !

- **Problem 887: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (d+e x)^2 \sqrt{f+g x} \sqrt{a+b x+c x^2}\, dx$$

Optimal (type 4, 1015 leaves, 9 steps):

$$\frac{1}{315 c^3 e g^3}2 \left(8 b^3 e^3 g^3+3 b c e^2 g^2 (b e f-8 b d g-9 a e g)+c^3 \left(19 e^3 f^3-57 d e^2 f^2 g+63 d^2 e f g^2-35 d^3 g^3\right)-3 c^2 e g^2 (2 a e (e f-10 d g)+b d (2 e f-7 d g))\right) \sqrt{f+g x} \sqrt{a+b x+c x^2}+\frac{2 (d+e x)^3 \sqrt{f+g x} \sqrt{a+b x+c x^2}}{9 e}-\frac{1}{315 c^2 g^3}4 \left(3 b^2 e^2 g^2+c e g (4 b e f-9 b d g-7 a e g)+c^2 \left(8 e^2 f^2-24 d e f g+21 d^2 g^2\right)\right) (f+g x)^{3/2} \sqrt{a+b x+c x^2}+\frac{2 e (c e f-3 c d g+b e g) (f+g x)^{5/2} \sqrt{a+b x+c x^2}}{63 c g^3}-\frac{1}{315 c^4 g^4 \sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}} \sqrt{a+b x+c x^2}}2 \sqrt{2} \sqrt{b^2-4 a c} \left(8 b^4 e^2 g^4-4 b^2 c e g^3 (b e f+6 b d g+9 a e g)+c^4 f^2 \left(8 e^2 f^2-24 d e f g+21 d^2 g^2\right)+3 c^2 g^2 \left(7 a^2 e^2 g^2+a b e g (5 e f+29 d g)-b^2 \left(e^2 f^2-5 d e f g-7 d^2 g^2\right)\right)+c^3 g \left(3 a g \left(3 e^2 f^2-16 d e f g-21 d^2 g^2\right)-b f \left(4 e^2 f^2-15 d e f g+21 d^2 g^2\right)\right)\right) \sqrt{f+g x} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]-$$

$$\left(2 \sqrt{2} \sqrt{b^2-4 a c} \left(c f^2-b f g+a g^2\right) \left(8 b^3 e^2 g^3+3 b c e g^2 (b e f-8 b d g-9 a e g)-2 c^3 f \left(8 e^2 f^2-24 d e f g+21 d^2 g^2\right)-3 c^2 g^2 (2 a e (e f-10 d g)+b d (2 e f-7 d g))\right) \sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right]\Bigg/\left(315c^4g^4\sqrt{f+gx}\sqrt{a+bx+cx^2}\right)$$

Result (type 4, 15 781 leaves) :

$$\sqrt{f+gx}\left(\frac{1}{315c^3g^3}2\left(8c^3e^2f^3-24c^3def^2g-3bc^2e^2f^2g+21c^3d^2fg^2+12bc^2defg^2-3b^2ce^2fg^2+8ac^2e^2fg^2+21bc^2d^2g^3-24b^2cdeg^3+60ac^2deg^3+8b^3e^2g^3-27abce^2g^3\right)+\frac{2\left(-6c^2e^2f^2+18c^2defg+2bce^2fg+63c^2d^2g^2+18bcdeg^2-6b^2e^2g^2+14ace^2g^2\right)x}{315c^2g^2}+\frac{2e(cef+18cdg+beg)x^2}{63cg}+\frac{2e^2x^3}{9}\right)\sqrt{a+x(b+cx)}-\frac{1}{315c^3g^5\sqrt{a+bx+cx^2}}2\sqrt{a+x(b+cx)}$$

$$\left(\frac{1}{c\sqrt{\frac{(f+gx)^2\left(c\left(-1+\frac{f}{f+gx}\right)^2+\frac{g\left(b-\frac{bf}{f+gx}+\frac{ag}{f+gx}\right)}{f+gx}\right)}{g^2}}}2\left(8c^4e^2f^4-24c^4def^3g-4bc^3e^2f^3g+21c^4d^2f^2g^2+15bc^3def^2g^2-3b^2c^2e^2f^2g^2+9ac^3e^2f^2g^2-21bc^3d^2fg^3+15b^2c^2defg^3-48ac^3defg^3-4b^3ce^2fg^3+15abc^2e^2fg^3+21b^2c^2d^2g^4-63ac^3d^2g^4-24b^3cdeg^4+87abc^2deg^4+8b^4e^2g^4-36ab^2ce^2g^4+21a^2c^2e^2g^4\right)(f+gx)^{3/2}\left(c+\frac{cf^2}{(f+gx)^2}-\frac{bfg}{(f+gx)^2}+\frac{ag^2}{(f+gx)^2}-\frac{2cf}{f+gx}+\frac{bg}{f+gx}\right)-\right.$$

$$\frac{1}{c\sqrt{\frac{(f+gx)^2\left(c\left(-1+\frac{f}{f+gx}\right)^2+\frac{g\left(b-\frac{bf}{f+gx}+\frac{ag}{f+gx}\right)}{f+gx}\right)}{g^2}}}\left(cf^2-bfg+ag^2\right)(f+gx)\sqrt{c+\frac{cf^2}{(f+gx)^2}-\frac{bfg}{(f+gx)^2}+\frac{ag^2}{(f+gx)^2}-\frac{2cf}{f+gx}+\frac{bg}{f+gx}}$$

$$\Bigg(\Bigg(4\,i\,\sqrt{2}\,c^4 e^2 f^4 \left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Bigg)\Bigg/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\Bigg(12\,i\,\sqrt{2}\,c^4 d\,e\,f^3 g\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(2\,i\sqrt{2}\,b\,c^3 e^2 f^3 g\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(21\, i\, c^4 d^2 f^2 g^2 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2} \left(c f^2 - b f g + a g^2\right) \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}} \sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right) +$$

$$\left(15\, i\, b\, c^3 d\, e\, f^2 g^2 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(3\,i\,b^2 c^2 e^2 f^2 g^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(9\, i\, a\, c^3 e^2 f^2 g^2 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2} \left(c f^2 - b f g + a g^2\right) \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}} \sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right) -$$

$$\left(21\, i\, b\, c^3 d^2 f\, g^3 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\Bigg(15\,i\,b^2 c^2 d e f g^3\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(24\, i \sqrt{2}\, a c^3 d e f g^3 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}}\right.$$

$$\sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\right)\right) \Big/$$

$$\left(\left(c f^2 - b f g + a g^2\right) \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}} \sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right) -$$

$$\left(2\, i \sqrt{2}\, b^3 c e^2 f g^3 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}}\right.$$

$$\sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\Bigg(\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\Bigg)+$$

$$\Bigg(15\,\mathbb{i}\,a\,b\,c^2\,e^2\,f\,g^3\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\,\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\Bigg(\sqrt{2}\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\Bigg)+$$

$$\left(21\,i\,b^2 c^2 d^2 g^4 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2} \left(c f^2 - b f g + a g^2\right) \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}} \sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right) -$$

$$\left(63\,i\,a c^3 d^2 g^4 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] -\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\Bigg(12\,i\sqrt{2}\,b^3 c d e g^4\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(87\,i\,a\,b\,c^2\,d\,e\,g^4\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(4\,i\sqrt{2}\,b^4\,e^2\,g^4\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(18\,i\sqrt{2}\,a b^2 c e^2 g^4\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(21\,i\,a^2 c^2 e^2 g^4 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2}\left(c f^2 - b f g + a g^2\right) \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}} \sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right) +$$

$$\left(8\,i\sqrt{2}\,c^4 e^2 f^3 \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}} \sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right) -$$

$$\left(24 \, i \sqrt{2} \, c^4 d e f^2 g \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\left.\text{EllipticF}\left[i \, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}} \sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right) +$$

$$\left(21 \, i \sqrt{2} \, c^4 d^2 f g^2 \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\left.\text{EllipticF}\left[i \, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}} \sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right) +$$

$$\left(3\,i\sqrt{2}\,b\,c^3\,d\,e\,f\,g^2\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)-$$

$$\left(3\,i\,b^2\,c^2\,e^2\,f\,g^2\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(3\,i\,\sqrt{2}\,a\,c^3\,e^2\,f\,g^2\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)-$$

$$\left(21\,i\,b\,c^3\,d^2\,g^3\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(12\, i \sqrt{2}\, b^2 c^2 d e g^3 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(30\, i \sqrt{2}\, a c^3 d e g^3 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(4\,i\,\sqrt{2}\,b^3\,c\,e^2\,g^3\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(27\,i\,a\,b\,c^2\,e^2\,g^3\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Bigg/$$

$$\left.\left.\left(\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)\right)\right)$$

Problem 888: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (d + e x) \sqrt{f + g x} \sqrt{a + b x + c x^2} \, dx$$

Optimal (type 4, 652 leaves, 7 steps) :

$$-\frac{1}{105 c^2 g^2} 2 \sqrt{f+g x}\left(4 b^2 e g^2+c^2 f (4 e f-7 d g)-c g (2 b e f+7 b d g-5 a e g)-3 c g (c e f+7 c d g-4 b e g) x\right) \sqrt{a+b x+c x^2}+$$

$$\frac{2 e \sqrt{f+g x}\left(a+b x+c x^2\right)^{3/2}}{7 c}+$$

$$\left(\sqrt{2} \sqrt{b^2-4 a c}\left((c e f+7 c d g-4 b e g)\left(8 c^2 f^2-2 b^2 g^2-3 c g (b f-2 a g)\right)-5 c g (2 c f-b g)(7 c d f-e (3 b f+a g))\right)\right.$$

$$\left.\sqrt{f+g x} \sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]\right) \Big/$$

$$\left(105 c^3 g^3 \sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}} \sqrt{a+b x+c x^2}\right)+$$

$$\left(2 \sqrt{2} \sqrt{b^2-4 a c}\left(c f^2-b f g+a g^2\right)\left(4 b^2 e g^2-2 c^2 f (4 e f-7 d g)+c g (b e f-7 b d g-10 a e g)\right) \sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}}\right.$$

$$\left.\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]\right) \Big/ \left(105 c^3 g^3 \sqrt{f+g x} \sqrt{a+b x+c x^2}\right)$$

Result (type 4, 8432 leaves) :

$$\sqrt{f+g\,x}\left(\frac{2\left(-4\,c^2\,e\,f^2+7\,c^2\,d\,f\,g+2\,b\,c\,e\,f\,g+7\,b\,c\,d\,g^2-4\,b^2\,e\,g^2+10\,a\,c\,e\,g^2\right)}{105\,c^2\,g^2}+\frac{2\,(c\,e\,f+7\,c\,d\,g+b\,e\,g)\,x}{35\,c\,g}+\frac{2\,e\,x^2}{7}\right)\sqrt{a+x\,(b+c\,x)}+$$

$$\frac{1}{105\,c^2\,g^4\,\sqrt{a+b\,x+c\,x^2}}\sqrt{a+x\,(b+c\,x)}$$

$$\left(\left(2\left(8\,c^3\,e\,f^3-14\,c^3\,d\,f^2\,g-5\,b\,c^2\,e\,f^2\,g+14\,b\,c^2\,d\,f\,g^2-5\,b^2\,c\,e\,f\,g^2+16\,a\,c^2\,e\,f\,g^2-14\,b^2\,c\,d\,g^3+42\,a\,c^2\,d\,g^3+8\,b^3\,e\,g^3-29\,a\,b\,c\,e\,g^3\right)\right.\right.$$

$$\left.\left.(f+g\,x)^{3/2}\left(c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}\right)\right)\middle/\left(c\sqrt{\frac{(f+g\,x)^2\left(c\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}\right)-\right.$$

$$\frac{1}{c\sqrt{\frac{(f+g\,x)^2\left(c\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}}\,2\left(c\,f^2-b\,f\,g+a\,g^2\right)(f+g\,x)\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}}$$

$$\left(\left(\left(2\,i\,\sqrt{2}\,c^3\,e\,f^3\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.\right.\right.$$

$$\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\ \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(7\,\mathbb{i}\,c^3 d f^2 g\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\ \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(5\,i\,b\,c^2\,e\,f^2\,g\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(7\,i\,b\,c^2\,d\,f\,g^2\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\Bigg(5\,i\,b^2 c e f g^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\,\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(4\,i\,\sqrt{2}\,a\,c^2\,e\,f\,g^2\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)-$$

$$\left(7\,i\,b^2\,c\,d\,g^3\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\left.\left.\mathrm{EllipticF}\left[\mathbb{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\ \frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(21\,\mathbb{i}\,a\,c^2\,d\,g^3\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\left(\mathrm{EllipticE}\left[\mathbb{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\ \frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\left.\left.\mathrm{EllipticF}\left[\mathbb{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\ \frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(2\,i\,\sqrt{2}\,b^3\,e\,g^3\,\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,\sqrt{1-\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\right.$$

$$\sqrt{1-\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\,\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,f^2-b\,f\,g+a\,g^2\right)\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)-$$

$$\left(29\,i\,a\,b\,c\,e\,g^3\,\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,\sqrt{1-\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\right.$$

$$\sqrt{1-\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\,\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\Bigg(4\,i\sqrt{2}\,c^3 e f^2\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\Bigg(7\,i\sqrt{2}\,c^3 d f g\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(i\,b\,c^2 e f g\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(7\,i\,b\,c^2 d g^2\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(2\,i\sqrt{2}\,b^2 c e g^2\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(5\,i\sqrt{2}\,a c^2 e g^2\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)\Bigg)\Bigg)$$

- **Problem 889: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\sqrt{f+g x}\,\sqrt{a+b x+c x^2}\,dx$$

Optimal (type 4, 513 leaves, 7 steps) :

$$-\frac{2\,(2\,c\,f-b\,g)\,\sqrt{f+g\,x}\,\sqrt{a+b\,x+c\,x^2}}{15\,c\,g}+\frac{2\,(f+g\,x)^{3/2}\,\sqrt{a+b\,x+c\,x^2}}{5\,g}-$$

$$\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(c^2\,f^2+b^2\,g^2-c\,g\,(b\,f+3\,a\,g)\right)\,\sqrt{f+g\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,g}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)\,g}\right]\right)\Bigg/\left(15\,c^2\,g^2\,\sqrt{\frac{c\,(f+g\,x)}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)\,g}}\,\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,(2\,c\,f-b\,g)\,\left(c\,f^2-b\,f\,g+a\,g^2\right)\,\sqrt{\frac{c\,(f+g\,x)}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)\,g}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,g}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)\,g}\right]\right)\Bigg/\left(15\,c^2\,g^2\,\sqrt{f+g\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 3384 leaves) :

$$\left(\frac{2\,(c\,f+b\,g)}{15\,c\,g}+\frac{2\,x}{5}\right)\,\sqrt{f+g\,x}\,\sqrt{a+x\,(b+c\,x)}\,+$$

$$\frac{1}{15\,c\,g^3\,\sqrt{a+b\,x+c\,x^2}}\,\sqrt{a+x\,(b+c\,x)}\,\Bigg(-\frac{4\,\left(c^2\,f^2-b\,c\,f\,g+b^2\,g^2-3\,a\,c\,g^2\right)\,(f+g\,x)^{3/2}\,\left(c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}\right)}{c\,\sqrt{\frac{(f+g\,x)^2\,\left(c\,\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\,\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}}+$$

$$\frac{1}{c\,\sqrt{\frac{(f+g\,x)^2\,\left(c\,\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\,\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}}\,2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)\,(f+g\,x)\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}}$$

$$\Bigg(\Bigg(\mathbb{i}\,c^2\,f^2\,\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,\sqrt{1-\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\,\sqrt{1-\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}$$

$$\Bigg(\text{EllipticE}\Big[\mathbb{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\Big],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\Big]-$$

$$\text{EllipticF}\Big[\mathbb{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\Big],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\Big]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\left(c\,f^2-b\,f\,g+a\,g^2\right)\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)-$$

$$\Bigg(\mathbb{i}\, b\, c\, f\, g \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] -$$

$$\text{EllipticF}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\Bigg)\Bigg) \Bigg/$$

$$\left(\sqrt{2} \left(c f^2 - b f g + a g^2\right) \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}} \sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right) +$$

$$\Bigg(\mathbb{i}\, b^2 g^2 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] -$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\Bigg(3\,i\,a c g^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(i \sqrt{2}\, c^2 f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right) -$$

$$\left(i\, b c g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left.\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)\right)\right)$$

Problem 890: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{f+g x} \sqrt{a+b x+c x^2}}{d+e x} \, dx$$

Optimal (type 4, 764 leaves, 15 steps) :

$$\frac{2\sqrt{f+g x}\sqrt{a+b x+c x^2}}{3 e}+$$

$$\left(\sqrt{2}\sqrt{b^2-4 a c}\,(c e f-3 c d g+b e g)\sqrt{f+g x}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]\right)\Bigg/$$

$$\left(3 c e^2 g\sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}}\sqrt{a+b x+c x^2}\right)+$$

$$\left(2\sqrt{2}\sqrt{b^2-4 a c}\left(e g (b e f-3 b d g+2 a e g)+c\left(-e^2 f^2+3 d^2 g^2\right)\right)\sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}}\sqrt{\frac{c (a+x (b+c x))}{-b^2+4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],\frac{2\sqrt{b^2-4 a c}\,g}{-2 c f+\left(b+\sqrt{b^2-4 a c}\right) g}\right]\right)\Bigg/\left(3 c e^3 g\sqrt{f+g x}\sqrt{a+x (b+c x)}\right)-\frac{1}{\sqrt{c}\,e^3\sqrt{a+b x+c x^2}}$$

$$\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{2 c f-\left(b-\sqrt{b^2-4 a c}\right) g}\sqrt{1-\frac{2 c (f+g x)}{2 c f-\left(b-\sqrt{b^2-4 a c}\right) g}}\sqrt{1-\frac{2 c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}}$$

$$\text{EllipticPi}\left[\frac{e\left(2 c f-b g+\sqrt{b^2-4 a c}\,g\right)}{2 c (e f-d g)},\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{c}\sqrt{f+g x}}{\sqrt{2 c f-\left(b-\sqrt{b^2-4 a c}\right) g}}\right],\frac{b-\sqrt{b^2-4 a c}-\frac{2 c f}{g}}{b+\sqrt{b^2-4 a c}-\frac{2 c f}{g}}\right]$$

Result (type 4, 35 245 leaves) : Display of huge result suppressed!

- **Problem 891: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{f+g x}\ \sqrt{a+b x+c x^2}}{(d+e x)^2}\, dx$$

Optimal (type 4, 743 leaves, 15 steps):

$$-\frac{\sqrt{f+g x}\ \sqrt{a+b x+c x^2}}{e\,(d+e x)}+\frac{3\sqrt{b^2-4 a c}\ \sqrt{f+g x}\ \sqrt{-\frac{c\,(a+b x+c x^2)}{b^2-4 a c}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\ g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]}{\sqrt{2}\ e^2\sqrt{\frac{c\,(f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}}\ \sqrt{a+b x+c x^2}}+$$

$$\left(\sqrt{2}\ \sqrt{b^2-4 a c}\ (2 b e g-c\,(e f+3 d g))\sqrt{\frac{c\,(f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}}\ \sqrt{\frac{c\,(a+x\,(b+c x))}{-b^2+4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],\frac{2\sqrt{b^2-4 a c}\ g}{-2 c f+\left(b+\sqrt{b^2-4 a c}\right) g}\right]\right)\Big/\left(c\,e^3\sqrt{f+g x}\ \sqrt{a+x\,(b+c x)}\right)+$$

$$\left(\sqrt{2 c f-\left(b-\sqrt{b^2-4 a c}\right) g}\ (c d\,(2 e f-3 d g)-e\,(b e f-2 b d g+a e g))\sqrt{1-\frac{2 c\,(f+g x)}{2 c f-\left(b-\sqrt{b^2-4 a c}\right) g}}\ \sqrt{1-\frac{2 c\,(f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{e\left(2 c f-b g+\sqrt{b^2-4 a c}\ g\right)}{2 c\,(e f-d g)},\text{ArcSin}\left[\frac{\sqrt{2}\ \sqrt{c}\ \sqrt{f+g x}}{\sqrt{2 c f-\left(b-\sqrt{b^2-4 a c}\right) g}}\right],\frac{b-\sqrt{b^2-4 a c}-\frac{2 c f}{g}}{b+\sqrt{b^2-4 a c}-\frac{2 c f}{g}}\right]\right)\Big/\left(\sqrt{2}\right.$$

$$\left.\sqrt{c}\ e^3\,(e f-d g)\sqrt{a+b x+c x^2}\right)$$

Result (type 4, 16 573 leaves) :

$$-\frac{\sqrt{f+g x}\,\sqrt{a+x\,(b+c x)}}{e\,(d+e x)}-\frac{1}{e g\sqrt{a+b x+c x^2}}\sqrt{a+x\,(b+c x)}\left(-\frac{3\,(f+g x)^{3/2}\left(c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}\right)}{e\sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}}-\right.$$

$$\frac{1}{e\sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}}(f+g x)\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}$$

$$\left(-\left(3\,\mathrm{i}\,c f^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\right.\right.$$

$$\left.\left.\left.\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Bigg/$$

$$\left(2\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(3\,i\,b\,f\,g\left(2\,c\,f-b\,g+\sqrt{b^2 g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2 g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2 g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2 g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2 g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2 g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2 g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2 g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2 g^2-4\,a\,c\,g^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2 g^2-4\,a\,c\,g^2}}}\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)-$$

$$\left(3\,i\,a\,g^2\left(2\,c\,f-b\,g+\sqrt{b^2 g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2 g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2 g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2 g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2 g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2 g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\Bigg(3\,i\,c\,e^2 f^3\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,(e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\Bigg(3\,i\,c\,d\,e\,f^2 g\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,(e f-d g)^2\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(3\,\mathrm{i}\,b\,e^2 f^2 g\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,(e f-d g)^2\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(3\,\mathrm{i}\,b\,d\,e\,f\,g^2\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left(\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,(e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(3\,i\,a e^2 f g^2\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,(e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(3\,i\,a d e g^3\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\ \frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\Bigg)\Big/$$

$$\left(\sqrt{2}\,(e\,f-d\,g)^2\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)-$$

$$\left(2\,i\,\sqrt{2}\,c\,e\,f^2\,\sqrt{1-\frac{2\,(c\,f^2-b\,f\,g+a\,g^2)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\,\sqrt{1-\frac{2\,(c\,f^2-b\,f\,g+a\,g^2)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\ \frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\Bigg)\Big/$$

$$\left((e\,f-d\,g)\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(2\,i\,\sqrt{2}\,b\,e\,f\,g\,\sqrt{1-\frac{2\,(c\,f^2-b\,f\,g+a\,g^2)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\,\sqrt{1-\frac{2\,(c\,f^2-b\,f\,g+a\,g^2)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Bigg/$$

$$\left((e f-d g)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\Bigg(2\,i\sqrt{2}\,a e g^2\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg/$$

$$\left((e f-d g)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\frac{1}{(e f-d g)^3}3 c e^3 f^3\Bigg(\Bigg(i f\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\;\text{EllipticPi}\Big[$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\Bigg(i\,d\,g\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\Big[$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\, e\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg)+$$

$$\frac{1}{(e f-d g)^3}\,3\,c\,d\,e^2 f^2 g\Bigg(\Bigg(i\,f\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\Big[$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\, i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\, \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg) \Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\Bigg(i\, d\, g\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\Big[$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\, i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\, \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg) \Bigg/$$

$$\left(\sqrt{2}\, e\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg)+$$

$$\frac{1}{(e f-d g)^3}\, 3 b e^3 f^2 g\left(\left(i\, f\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\Big[\right.\right.$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathrm{i}\, \mathrm{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\Bigg]\Bigg) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\Bigg(\mathrm{i}\, d g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \mathrm{EllipticPi}\Big[$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathrm{i}\, \mathrm{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\Bigg]\Bigg) \Big/$$

$$\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg)-$$

$$\frac{1}{(e f-d g)^3}\, 3 b d e^2 f g^2 \Bigg(\Bigg(\mathrm{i}\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \mathrm{EllipticPi}\Big[$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\, i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\, \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\left(i\, d\, g\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\, \text{EllipticPi}\left[\right.\right.$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\, i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\, \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Bigg/$$

$$\left.\left(\sqrt{2}\, e\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right)-$$

$$\frac{1}{(e f-d g)^3} 3 a e^3 f g^2\left(\left(i\, f\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\, \text{EllipticPi}\left[\right.\right.\right.$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg) \Bigg/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\left(i d g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \text{ EllipticPi}\left[\right.\right.$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg) \Bigg/$$

$$\left.\left(\sqrt{2} e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right)+$$

$$\frac{1}{(e f-d g)^3} 3 a d e^2 g^3 \left(\left(\left(i f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \text{ EllipticPi}\left[\right.\right.\right.\right.$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\Bigg]\Bigg) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\Bigg(\text{i} d g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \operatorname{EllipticPi}\Bigg[$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\Bigg]\Bigg) \Big/$$

$$\left(\left(\sqrt{2} e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right)+$$

$$\frac{1}{(e f-d g)^2} 4 c e^2 f^2 \Bigg(\Bigg(\text{i} f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \operatorname{EllipticPi}\Bigg[$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathrm{i} \operatorname{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right) -$$

$$\Bigg(\mathrm{i}\, d g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \operatorname{EllipticPi}\Big[$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathrm{i} \operatorname{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg) \Big/$$

$$\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg) -$$

$$\frac{1}{(e f-d g)^2} 4 b e^2 f g \Bigg(\Bigg(\mathrm{i}\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \operatorname{EllipticPi}\Big[$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \text{i ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\Bigg]\Bigg) \Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\Bigg(\text{i } d g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \text{ EllipticPi}\Big[$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \text{i ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\Bigg]\Bigg) \Big/$$

$$\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg)+$$

$$\frac{1}{(e f-d g)^2} 4 a e^2 g^2\Bigg(\Bigg(\text{i } f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \text{ EllipticPi}\Big[$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\Bigg]\Bigg) \Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\Bigg(i d g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \operatorname{EllipticPi}\Bigg[$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\Bigg]\Bigg) \Bigg/$$

$$\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg)-$$

$$\frac{1}{e f-d g} c e f \Bigg(\Bigg(i f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \operatorname{EllipticPi}\Bigg[$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg) \Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right) -$$

$$\Bigg(i d g\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \text{EllipticPi}\Bigg[$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg) \Bigg/$$

$$\left(\sqrt{2}\, e\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg) -$$

$$\frac{1}{e f-d g} 3 c d g\Bigg(\Bigg(i f\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \text{EllipticPi}\Bigg[$$

$$\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\left(i d g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \operatorname{EllipticPi}\left[\right.\right.$$

$$\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left.\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right)+$$

$$\frac{1}{e f-d g} 2 b e g\left(\left(\left(i f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \operatorname{EllipticPi}\left[\right.\right.\right.\right.$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\Bigg]\Bigg) \Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\Bigg(i d g\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \operatorname{EllipticPi}\Bigg[$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\Bigg]\Bigg) \Big/$$

$$\left(\sqrt{2}\, e\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg)\Bigg)\Bigg)$$

- **Problem 892: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{f+g x}\sqrt{a+b x+c x^2}}{(d+e x)^3}\, dx$$

Optimal (type 4, 1034 leaves, 25 steps) :

$$-\frac{\sqrt{f+g x}\,\sqrt{a+b x+c x^2}}{2 e (d+e x)^2}+\frac{(c d (2 e f-3 d g)-e (b e f-2 b d g+a e g))\sqrt{f+g x}\,\sqrt{a+b x+c x^2}}{4 e \left(c d^2-b d e+a e^2\right)(e f-d g)(d+e x)}-$$

$$\left(\sqrt{b^2-4 a c}\,(c d (2 e f-3 d g)-e (b e f-2 b d g+a e g))\sqrt{f+g x}\right.$$

$$\left.\sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\ g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]\right)\Bigg/$$

$$\left(4 \sqrt{2}\ e^2 \left(c d^2-b d e+a e^2\right)(e f-d g)\sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}}\ \sqrt{a+b x+c x^2}\right)-$$

$$\left(\sqrt{b^2-4 a c}\,(-c d (2 e f+3 d g)+e (b e f+4 b d g-5 a e g))\sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}}\ \sqrt{\frac{c (a+x (b+c x))}{-b^2+4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],\frac{2 \sqrt{b^2-4 a c}\ g}{-2 c f+\left(b+\sqrt{b^2-4 a c}\right) g}\right]\right)\Bigg/\left(2 \sqrt{2}\ e^3 \left(c d^2+e (-b d+a e)\right)\sqrt{f+g x}\ \sqrt{a+x (b+c x)}\right)+$$

$$\left(\sqrt{2 c f-b g+\sqrt{b^2-4 a c}\; g}\,\left(b^2 e^4 f^2+a^2 e^4 g^2+c^2 d^3 g\,(4 e f-3 d g)-2 a c e^2\left(2 e^2 f^2-6 d e f g+3 d^2 g^2\right)-2 b e g\left(a e^3 f+c d^2\,(3 e f-2 d g)\right)\right)\right.$$

$$\sqrt{\frac{g\left(-b+\sqrt{b^2-4 a c}-2 c x\right)}{2 c f+\left(-b+\sqrt{b^2-4 a c}\right) g}}\;\sqrt{\frac{g\left(b+\sqrt{b^2-4 a c}+2 c x\right)}{-2 c f+\left(b+\sqrt{b^2-4 a c}\right) g}}\;\text{EllipticPi}\left[\frac{2 c e f-b e g+\sqrt{b^2-4 a c}\; e g}{2 c e f-2 c d g},\right.$$

$$\left.\left.\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{f+g x}}{\sqrt{2 c f-b g+\sqrt{b^2-4 a c}\; g}}\right],\frac{2 c f+\left(-b+\sqrt{b^2-4 a c}\right) g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]\right)\Big/\left(4\sqrt{2}\,\sqrt{c}\, e^3\left(c d^2+e\,(-b d+a e)\right)(e f-d g)^2\sqrt{a+x\,(b+c x)}\right)$$

Result (type 4, 33 765 leaves) : Display of huge result suppressed !

- **Problem 893: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(d+e x)^3\sqrt{a+b x+c x^2}}{\sqrt{f+g x}}\,dx$$

Optimal (type 4, 1098 leaves, 9 steps) :

$$\frac{1}{315\,c^3\,g^4}2\left(8\,b^3\,e^3\,g^3+3\,b\,c\,e^2\,g^2\,(5\,b\,e\,f-12\,b\,d\,g-9\,a\,e\,g)-c^3\left(152\,e^3\,f^3-408\,d\,e^2\,f^2\,g+336\,d^2\,e\,f\,g^2-70\,d^3\,g^3\right)-\right.$$

$$\left.3\,c^2\,e\,g\left(6\,a\,e\,g\,(2\,e\,f-5\,d\,g)-b\left(8\,e^2\,f^2-24\,d\,e\,f\,g+21\,d^2\,g^2\right)\right)\right)\sqrt{f+g\,x}\,\sqrt{a+b\,x+c\,x^2}+\frac{2\,(d+e\,x)^3\,\sqrt{f+g\,x}\,\sqrt{a+b\,x+c\,x^2}}{9\,g}-$$

$$\frac{1}{315\,c^2\,g^4}2\,e\left(6\,b^2\,e^2\,g^2+c\,e\,g\,(17\,b\,e\,f-27\,b\,d\,g-14\,a\,e\,g)-2\,c^2\left(64\,e^2\,f^2-111\,d\,e\,f\,g+42\,d^2\,g^2\right)\right)(f+g\,x)^{3/2}\,\sqrt{a+b\,x+c\,x^2}-$$

$$\frac{2\,e^2\,(8\,c\,e\,f-6\,c\,d\,g-b\,e\,g)\,(f+g\,x)^{5/2}\,\sqrt{a+b\,x+c\,x^2}}{63\,c\,g^4}-$$

$$\frac{1}{315\,c^4\,g^5\,\sqrt{\frac{c\,(f+g\,x)}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)g}}\,\sqrt{a+b\,x+c\,x^2}}\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(16\,b^4\,e^3\,g^4+8\,b^2\,c\,e^2\,g^3\,(2\,b\,e\,f-9\,b\,d\,g-9\,a\,e\,g)-\right.$$

$$2\,c^4\,f\left(64\,e^3\,f^3-216\,d\,e^2\,f^2\,g+252\,d^2\,e\,f\,g^2-105\,d^3\,g^3\right)+3\,c^2\,e\,g^2\left(14\,a^2\,e^2\,g^2-a\,b\,e\,g\,(19\,e\,f-87\,d\,g)+b^2\left(7\,e^2\,f^2-27\,d\,e\,f\,g+42\,d^2\,g^2\right)\right)-$$
$$\left.c^3\,g\left(6\,a\,e\,g\left(10\,e^2\,f^2-39\,d\,e\,f\,g+63\,d^2\,g^2\right)-b\left(40\,e^3\,f^3-144\,d\,e^2\,f^2\,g+189\,d^2\,e\,f\,g^2-105\,d^3\,g^3\right)\right)\right)$$

$$\sqrt{f+g\,x}\,\sqrt{-\frac{c\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\;g}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)g}\right]-$$

$$\frac{1}{315\,c^4\,g^5\,\sqrt{f+g\,x}\,\sqrt{a+b\,x+c\,x^2}}2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(c\,f^2-b\,f\,g+a\,g^2\right)\left(8\,b^3\,e^3\,g^3+3\,b\,c\,e^2\,g^2\,(5\,b\,e\,f-12\,b\,d\,g-9\,a\,e\,g)+\right.$$

$$\left.2\,c^3\left(64\,e^3\,f^3-216\,d\,e^2\,f^2\,g+252\,d^2\,e\,f\,g^2-105\,d^3\,g^3\right)-3\,c^2\,e\,g\left(6\,a\,e\,g\,(2\,e\,f-5\,d\,g)-b\left(8\,e^2\,f^2-24\,d\,e\,f\,g+21\,d^2\,g^2\right)\right)\right)$$

$$\sqrt{\frac{c\,(f+g\,x)}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)g}}\,\sqrt{-\frac{c\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\;g}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)g}\right]$$

Result (type 4, 17 771 leaves) :

$$\sqrt{f+g\,x}\,\left(\frac{1}{315\,c^3\,g^4}2\left(-64\,c^3\,e^3\,f^3+216\,c^3\,d\,e^2\,f^2\,g+12\,b\,c^2\,e^3\,f^2\,g-252\,c^3\,d^2\,e\,f\,g^2-45\,b\,c^2\,d\,e^2\,f\,g^2+\right.\right.$$

$$\left.9\,b^2\,c\,e^3\,f\,g^2-22\,a\,c^2\,e^3\,f\,g^2+105\,c^3\,d^3\,g^3+63\,b\,c^2\,d^2\,e\,g^3-36\,b^2\,c\,d\,e^2\,g^3+90\,a\,c^2\,d\,e^2\,g^3+8\,b^3\,e^3\,g^3-27\,a\,b\,c\,e^3\,g^3\right)+$$

$$\frac{2\,e\left(48\,c^2\,e^2\,f^2-162\,c^2\,d\,e\,f\,g-7\,b\,c\,e^2\,f\,g+189\,c^2\,d^2\,g^2+27\,b\,c\,d\,e\,g^2-6\,b^2\,e^2\,g^2+14\,a\,c\,e^2\,g^2\right)x}{315\,c^2\,g^3}+$$

$$\frac{2 e^2 (-8 c e f + 27 c d g + b e g) x^2}{63 c g^2} + \frac{2 e^3 x^3}{9 g}\Bigg) \sqrt{a + x (b + c x)} - \frac{1}{315 c^3 g^6 \sqrt{a + b x + c x^2}}$$

$$2 \sqrt{a + x (b + c x)} \Bigg(\frac{1}{c \sqrt{\frac{(f+g x)^2 \left(c \left(-1+\frac{f}{f+g x}\right)^2+\frac{g \left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}} \big(-128 c^4 e^3 f^4 + 432 c^4 d e^2 f^3 g + 40 b c^3 e^3 f^3 g - 504 c^4 d^2 e f^2 g^2 - 144 b c^3 d e^2 f^2 g^2 +$$

$$21 b^2 c^2 e^3 f^2 g^2 - 60 a c^3 e^3 f^2 g^2 + 210 c^4 d^3 f g^3 + 189 b c^3 d^2 e f g^3 - 81 b^2 c^2 d e^2 f g^3 + 234 a c^3 d e^2 f g^3 + 16 b^3 c e^3 f g^3 - 57 a b c^2 e^3 f g^3 -$$
$$105 b c^3 d^3 g^4 + 126 b^2 c^2 d^2 e g^4 - 378 a c^3 d^2 e g^4 - 72 b^3 c d e^2 g^4 + 261 a b c^2 d e^2 g^4 + 16 b^4 e^3 g^4 - 72 a b^2 c e^3 g^4 + 42 a^2 c^2 e^3 g^4\big)$$

$$(f + g x)^{3/2} \left(c + \frac{c f^2}{(f + g x)^2} - \frac{b f g}{(f + g x)^2} + \frac{a g^2}{(f + g x)^2} - \frac{2 c f}{f + g x} + \frac{b g}{f + g x}\right) +$$

$$\frac{1}{c \sqrt{\frac{(f+g x)^2 \left(c \left(-1+\frac{f}{f+g x}\right)^2+\frac{g \left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}} \left(c f^2 - b f g + a g^2\right) (f + g x) \sqrt{c + \frac{c f^2}{(f + g x)^2} - \frac{b f g}{(f + g x)^2} + \frac{a g^2}{(f + g x)^2} - \frac{2 c f}{f + g x} + \frac{b g}{f + g x}}$$

$$\Bigg(\Bigg(32 \, i \sqrt{2}\, c^4 e^3 f^4 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}$$

$$\sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \Bigg(\text{EllipticE}\Big[i \,\text{ArcSinh}\Big[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\Big], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\Big] -$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\Bigg(108\,i\sqrt{2}\,c^4 d e^2 f^3 g\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(10\,i\,\sqrt{2}\,b\,c^3\,e^3\,f^3\,g\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(126\,i\,\sqrt{2}\,c^4\,d^2\,e\,f^2\,g^2\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(36\,i\sqrt{2}\,b c^3 d e^2 f^2 g^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(21\,i\,b^2 c^2 e^3 f^2 g^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(15\,i\sqrt{2}\,a c^3 e^3 f^2 g^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\Bigg(105\,i\,c^4 d^3 f g^3\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(189\, i\, b\, c^3 d^2 e\, f\, g^3 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\right)\right) \Big/$$

$$\left(2 \sqrt{2} \left(c f^2 - b f g + a g^2\right) \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}} \sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right) +$$

$$\left(81\, i\, b^2 c^2 d\, e^2 f\, g^3 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(2\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\Bigg(117\,i\,a c^3 d e^2 f g^3\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(4\, i \sqrt{2}\, b^3 c\, e^3 f\, g^3 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\right)\right) \Big/$$

$$\left(\left(c f^2 - b f g + a g^2\right) \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}} \sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right) +$$

$$\left(57\, i\, a\, b\, c^2 e^3 f\, g^3 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(105\,i\,b\,c^3 d^3 g^4\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(63\, i\, b^2 c^2 d^2 e\, g^4 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}}\right.$$

$$\sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2}\left(c f^2 - b f g + a g^2\right) \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}} \sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right) +$$

$$\left(189\, i\, a\, c^3 d^2 e\, g^4 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}}\right.$$

$$\sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(18\,i\sqrt{2}\,b^3 c d e^2 g^4\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(261\, i\, a\, b\, c^2\, d\, e^2\, g^4 \left(2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right) \sqrt{1 - \frac{2\left(c\, f^2 - b\, f\, g + a\, g^2\right)}{\left(2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right)(f + g\, x)}}\right.$$

$$\sqrt{1 - \frac{2\left(c\, f^2 - b\, f\, g + a\, g^2\right)}{\left(2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right)(f + g\, x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\, f^2 - b\, f\, g + a\, g^2}{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}}}{\sqrt{f + g\, x}}\right], \frac{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}{2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\, f^2 - b\, f\, g + a\, g^2}{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}}}{\sqrt{f + g\, x}}\right], \frac{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}{2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}\right]\right)\right) \Big/$$

$$\left(2\sqrt{2}\left(c\, f^2 - b\, f\, g + a\, g^2\right)\sqrt{-\frac{c\, f^2 - b\, f\, g + a\, g^2}{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}} \sqrt{c + \frac{c\, f^2 - b\, f\, g + a\, g^2}{(f + g\, x)^2} + \frac{-2\, c\, f + b\, g}{f + g\, x}}\right) -$$

$$\left(4\, i\sqrt{2}\, b^4\, e^3\, g^4 \left(2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right) \sqrt{1 - \frac{2\left(c\, f^2 - b\, f\, g + a\, g^2\right)}{\left(2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right)(f + g\, x)}}\right.$$

$$\sqrt{1 - \frac{2\left(c\, f^2 - b\, f\, g + a\, g^2\right)}{\left(2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right)(f + g\, x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\, f^2 - b\, f\, g + a\, g^2}{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}}}{\sqrt{f + g\, x}}\right], \frac{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}{2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}\right] - \right.$$

$$\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(18\,\text{i}\sqrt{2}\,a b^2 c e^3 g^4\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(21\, i\, a^2 c^2 e^3 g^4 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}}\right.$$

$$\sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2}\left(c f^2 - b f g + a g^2\right) \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}} \sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right) +$$

$$\left(64\, i \sqrt{2}\, c^4 e^3 f^3 \sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}} \sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(216\,\mathrm{i}\sqrt{2}\,c^4 d e^2 f^2 g\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(12\,\mathrm{i}\sqrt{2}\,b c^3 e^3 f^2 g\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(252\,\mathbb{i}\,\sqrt{2}\,c^4\,d^2\,e\,f\,g^2\,\sqrt{1-\frac{2\,(c\,f^2-b\,f\,g+a\,g^2)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\,\sqrt{1-\frac{2\,(c\,f^2-b\,f\,g+a\,g^2)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)-$$

$$\left(36\,\mathbb{i}\,\sqrt{2}\,b\,c^3\,d\,e^2\,f\,g^2\,\sqrt{1-\frac{2\,(c\,f^2-b\,f\,g+a\,g^2)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\,\sqrt{1-\frac{2\,(c\,f^2-b\,f\,g+a\,g^2)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(15\,i\,b^2 c^2 e^3 f g^2 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(18\,i\sqrt{2}\,a c^3 e^3 f g^2 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(105\,i\,\sqrt{2}\,c^4\,d^3\,g^3\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(63\,i\,b\,c^3\,d^2\,e\,g^3\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)-$$

$$\left(18\,i\,\sqrt{2}\,b^2 c^2 d\,e^2 g^3 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\,\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(45\,i\,\sqrt{2}\,a\,c^3 d\,e^2 g^3 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\,\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(4\,i\,\sqrt{2}\,b^3\,c\,e^3\,g^3\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)-$$

$$\left(27\,i\,a\,b\,c^2\,e^3\,g^3\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left.\left.\left(\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)\right)\right)$$

Problem 894: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(d+e x)^2 \sqrt{a+b x+c x^2}}{\sqrt{f+g x}} \, dx$$

Optimal (type 4, 755 leaves, 8 steps) :

$$-\frac{1}{105\,c^2 g^3}4\left(2 b^2 e^2 g^2+c e g\,(4 b e f-7 b d g-5 a e g)-c^2\left(21 e^2 f^2-34 d e f g+10 d^2 g^2\right)\right)\sqrt{f+g x}\,\sqrt{a+b x+c x^2}+$$

$$\frac{2\,(d+e x)^2\sqrt{f+g x}\,\sqrt{a+b x+c x^2}}{7 g}-\frac{2 e\,(6 c e f-4 c d g-b e g)\,(f+g x)^{3/2}\sqrt{a+b x+c x^2}}{35 c g^3}+$$

$$\Bigg(\sqrt{2}\,\sqrt{b^2-4 a c}\,\left(8 b^3 e^2 g^3+b c e g^2\,(9 b e f-28 b d g-29 a e g)-2 c^3 f\left(24 e^2 f^2-56 d e f g+35 d^2 g^2\right)-\right.$$

$$\left.c^2 g\left(2 a e g\,(13 e f-42 d g)-b\left(16 e^2 f^2-42 d e f g+35 d^2 g^2\right)\right)\right)\sqrt{f+g x}\,\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}\right]\Bigg)\Bigg/\left(105 c^3 g^4\sqrt{\frac{c\,(f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}}\,\sqrt{a+b x+c x^2}\right)+$$

$$\Bigg(4\sqrt{2}\,\sqrt{b^2-4 a c}\,\left(c f^2-b f g+a g^2\right)\left(2 b^2 e^2 g^2+c e g\,(4 b e f-7 b d g-5 a e g)+c^2\left(24 e^2 f^2-56 d e f g+35 d^2 g^2\right)\right)\sqrt{\frac{c\,(f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}}$$

$$\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}\right]\Bigg)\Bigg/\left(105 c^3 g^4\sqrt{f+g x}\,\sqrt{a+b x+c x^2}\right)$$

Result (type 4, 10 030 leaves) :

$$\sqrt{f+g x}\left(\frac{2\left(24 c^2 e^2 f^2-56 c^2 d e f g-5 b c e^2 f g+35 c^2 d^2 g^2+14 b c d e g^2-4 b^2 e^2 g^2+10 a c e^2 g^2\right)}{105 c^2 g^3}+\frac{2 e\,(-6 c e f+14 c d g+b e g)\,x}{35 c g^2}+\frac{2 e^2 x^2}{7 g}\right)$$

$$\sqrt{a+x\,(b+c\,x)}\; +$$

$$\frac{1}{105\,c^2\,g^5\,\sqrt{a+b\,x+c\,x^2}}\,2\,\sqrt{a+x\,(b+c\,x)}\,\Bigg(\Bigg(\Big(-48\,c^3\,e^2\,f^3+112\,c^3\,d\,e\,f^2\,g+16\,b\,c^2\,e^2\,f^2\,g-70\,c^3\,d^2\,f\,g^2-42\,b\,c^2\,d\,e\,f\,g^2+9\,b^2\,c\,e^2\,f\,g^2-$$

$$26\,a\,c^2\,e^2\,f\,g^2+35\,b\,c^2\,d^2\,g^3-28\,b^2\,c\,d\,e\,g^3+84\,a\,c^2\,d\,e\,g^3+8\,b^3\,e^2\,g^3-29\,a\,b\,c\,e^2\,g^3\Big)\,(f+g\,x)^{3/2}$$

$$\left(c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}\right)\Bigg)\Bigg/\left(c\,\sqrt{\frac{(f+g\,x)^2\left(c\,\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\,\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}\right)+$$

$$\frac{1}{c\,\sqrt{\frac{(f+g\,x)^2\left(c\,\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\,\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}}\left(c\,f^2-b\,f\,g+a\,g^2\right)\,(f+g\,x)\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}}$$

$$\Bigg(\Bigg(12\,i\,\sqrt{2}\,c^3\,e^2\,f^3\,\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,\sqrt{1-\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}$$

$$\sqrt{1-\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\,\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(28\,\mathrm{i}\,\sqrt{2}\,c^3 d e f^2 g\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(4\,i\,\sqrt{2}\;b\,c^2\,e^2\,f^2\,g\,\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\right.$$

$$\sqrt{1-\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\;\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(35\,i\,c^3\,d^2\,f\,g^2\,\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\right.$$

$$\sqrt{1-\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\Bigg(21\,i\,b\,c^2 d e f g^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(9\, i\, b^2\, c\, e^2\, f\, g^2 \left(2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right) \sqrt{1 - \frac{2 \left(c\, f^2 - b\, f\, g + a\, g^2\right)}{\left(2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right) (f + g\, x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c\, f^2 - b\, f\, g + a\, g^2\right)}{\left(2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right) (f + g\, x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c\, f^2 - b\, f\, g + a\, g^2}{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}}}{\sqrt{f + g\, x}}\right], \frac{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}{2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c\, f^2 - b\, f\, g + a\, g^2}{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}}}{\sqrt{f + g\, x}}\right], \frac{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}{2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}\right]\right)\right) \Big/$$

$$\left(2 \sqrt{2} \left(c\, f^2 - b\, f\, g + a\, g^2\right) \sqrt{-\frac{c\, f^2 - b\, f\, g + a\, g^2}{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}} \sqrt{c + \frac{c\, f^2 - b\, f\, g + a\, g^2}{(f + g\, x)^2} + \frac{-2\, c\, f + b\, g}{f + g\, x}}\right) +$$

$$\left(13\, i\, a\, c^2\, e^2\, f\, g^2 \left(2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right) \sqrt{1 - \frac{2 \left(c\, f^2 - b\, f\, g + a\, g^2\right)}{\left(2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right) (f + g\, x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c\, f^2 - b\, f\, g + a\, g^2\right)}{\left(2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right) (f + g\, x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c\, f^2 - b\, f\, g + a\, g^2}{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}}}{\sqrt{f + g\, x}}\right], \frac{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}{2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}\right] - \right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\ \frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)-$$

$$\Bigg(35\,i\,b\,c^2\,d^2\,g^3\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\ \Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\ \frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\ \frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\,\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(7\,i\,\sqrt{2}\,b^2\,c\,d\,e\,g^3\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\right)\Bigg/$$

$$\left(\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)-$$

$$\left(21\,i\,\sqrt{2}\,a\,c^2\,d\,e\,g^3\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Bigg/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(2\,\mathbb{i}\,\sqrt{2}\,b^3 e^2 g^3\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Bigg/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(29\, i\, a\, b\, c\, e^2\, g^3 \left(2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right) \sqrt{1 - \frac{2\left(c\, f^2 - b\, f\, g + a\, g^2\right)}{\left(2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right)(f + g\, x)}}\right.$$

$$\sqrt{1 - \frac{2\left(c\, f^2 - b\, f\, g + a\, g^2\right)}{\left(2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right)(f + g\, x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\, \sqrt{-\frac{c\, f^2 - b\, f\, g + a\, g^2}{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}}}{\sqrt{f + g\, x}}\right], \frac{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}{2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\, \sqrt{-\frac{c\, f^2 - b\, f\, g + a\, g^2}{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}}}{\sqrt{f + g\, x}}\right], \frac{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}{2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}\right]\right)\right) \Big/$$

$$\left(2\sqrt{2}\left(c\, f^2 - b\, f\, g + a\, g^2\right) \sqrt{-\frac{c\, f^2 - b\, f\, g + a\, g^2}{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}}\, \sqrt{c + \frac{c\, f^2 - b\, f\, g + a\, g^2}{(f + g\, x)^2} + \frac{-2\, c\, f + b\, g}{f + g\, x}}\right) +$$

$$\left(24\, i\, \sqrt{2}\, c^3\, e^2\, f^2\, \sqrt{1 - \frac{2\left(c\, f^2 - b\, f\, g + a\, g^2\right)}{\left(2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right)(f + g\, x)}}\, \sqrt{1 - \frac{2\left(c\, f^2 - b\, f\, g + a\, g^2\right)}{\left(2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right)(f + g\, x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\, \sqrt{-\frac{c\, f^2 - b\, f\, g + a\, g^2}{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}}}{\sqrt{f + g\, x}}\right], \frac{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}{2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(56 \, i \sqrt{2} \, c^3 d e f g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i \, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(4 \, i \sqrt{2} \, b c^2 e^2 f g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i \, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(35\,i\,\sqrt{2}\,c^3 d^2 g^2 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(7\,i\,\sqrt{2}\,b c^2 d e g^2 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(2\,\mathbb{i}\,\sqrt{2}\,b^2\,c\,e^2\,g^2\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)-$$

$$\left(5\,\mathbb{i}\,\sqrt{2}\,a\,c^2\,e^2\,g^2\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left.\left(\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)\right)\right)$$

Problem 895: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(d+e x) \sqrt{a+b x+c x^2}}{\sqrt{f+g x}} \, dx$$

Optimal (type 4, 519 leaves, 6 steps):

$$-\frac{2 \sqrt{f+g x} \, (4 c e f-5 c d g-b e g-3 c e g x) \sqrt{a+b x+c x^2}}{15 c g^2}-$$

$$\left(\sqrt{2} \sqrt{b^2-4 a c} \left(2 b^2 e g^2-2 c^2 f (4 e f-5 d g)+c g (3 b e f-5 b d g-6 a e g)\right) \sqrt{f+g x} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2-4 a c} \, g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]\right) \Bigg/ \left(15 c^2 g^3 \sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}} \sqrt{a+b x+c x^2}\right)-$$

$$\left(2 \sqrt{2} \sqrt{b^2-4 a c} \, (8 c e f-10 c d g+b e g) \left(c f^2-b f g+a g^2\right) \sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2-4 a c} \, g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]\right) \Bigg/ \left(15 c^2 g^3 \sqrt{f+g x} \sqrt{a+b x+c x^2}\right)$$

Result (type 4, 4921 leaves):

$$\left(\frac{2 (-4 c e f+5 c d g+b e g)}{15 c g^2}+\frac{2 e x}{5 g}\right) \sqrt{f+g x} \sqrt{a+x (b+c x)} +$$

$$\frac{1}{15\,c\,g^4\sqrt{a+b\,x+c\,x^2}}\,2\sqrt{a+x\,(b+c\,x)}\,\Bigg(\Bigg(\left(8\,c^2\,e\,f^2-10\,c^2\,d\,f\,g-3\,b\,c\,e\,f\,g+5\,b\,c\,d\,g^2-2\,b^2\,e\,g^2+6\,a\,c\,e\,g^2\right)\,(f+g\,x)^{3/2}$$

$$\left(c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}\right)\Bigg)\Bigg/\left(c\sqrt{\frac{(f+g\,x)^2\left(c\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}\right)-$$

$$\frac{1}{c\sqrt{\frac{(f+g\,x)^2\left(c\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}}\left(c\,f^2-b\,f\,g+a\,g^2\right)\,(f+g\,x)\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}}$$

$$\Bigg(\Bigg(2\,\mathbb{i}\sqrt{2}\,c^2\,e\,f^2\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\,\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\ \frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\Bigg(5\,\text{i}\,c^2 d f g\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\Bigg(\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-$$

$$\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(3\,\mathrm{i}\,b\,c\,e\,f\,g\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\;\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\right)\Big/$$

$$\left(2\,\sqrt{2}\,\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\;\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(5\,\mathrm{i}\,b\,c\,d\,g^2\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\;\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\Bigg(i\,b^2 e g^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(3\,i\,a\,c\,e\,g^2\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(4\,i\sqrt{2}\,c^2\,e\,f\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)-$$

$$\left(5\,i\,\sqrt{2}\,c^2\,d\,g\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(i\,b\,c\,e\,g\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\ \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}\ \sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right)\Bigg)\Bigg)$$

- **Problem 896: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + b x + c x^2}}{\sqrt{f + g x}}\, dx$$

Optimal (type 4, 444 leaves, 6 steps):

$$\frac{2 \sqrt{f + g x}\ \sqrt{a + b x + c x^2}}{3 g} - \left(\sqrt{2}\ \sqrt{b^2 - 4 a c}\ (2 c f - b g)\ \sqrt{f + g x}\ \sqrt{-\frac{c\,(a + b x + c x^2)}{b^2 - 4 a c}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x}{\sqrt{b^2 - 4 a c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2 - 4 a c}\ g}{2 c f - \left(b + \sqrt{b^2 - 4 a c}\right) g}\right]\right) \Bigg/$$

$$\left(3 c g^2 \sqrt{\frac{c\,(f + g x)}{2 c f - \left(b + \sqrt{b^2 - 4 a c}\right) g}}\ \sqrt{a + b x + c x^2}\right) + \Bigg(4 \sqrt{2}\ \sqrt{b^2 - 4 a c}\ (c f^2 - b f g + a g^2)\ \sqrt{\frac{c\,(f + g x)}{2 c f - \left(b + \sqrt{b^2 - 4 a c}\right) g}}$$

$$\sqrt{-\frac{c\,(a + b x + c x^2)}{b^2 - 4 a c}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x}{\sqrt{b^2 - 4 a c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2 - 4 a c}\ g}{2 c f - \left(b + \sqrt{b^2 - 4 a c}\right) g}\right]\Bigg) \Bigg/ \left(3 c g^2 \sqrt{f + g x}\ \sqrt{a + b x + c x^2}\right)$$

Result (type 4, 1847 leaves):

$$\frac{2\sqrt{f+g\,x}\,\sqrt{a+x\,(b+c\,x)}}{3\,g}+\frac{1}{3\,g^3\,\sqrt{a+b\,x+c\,x^2}}\sqrt{a+x\,(b+c\,x)}\left(-\frac{2\,(2\,c\,f-b\,g)\,(f+g\,x)^{3/2}\left(c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}\right)}{c\sqrt{\frac{(f+g\,x)^2\left(c\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}}+\right.$$

$$\frac{1}{c\sqrt{\frac{(f+g\,x)^2\left(c\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}}\,2\left(c\,f^2-b\,f\,g+a\,g^2\right)\,(f+g\,x)\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}}$$

$$\left(\left(i\,c\,f\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\left.\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\right)\right/$$

$$\left(\sqrt{2}\,\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)-$$

$$\left(\mathbb{i}\, b\, g \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\right)\right) \Big/$$

$$\left(2 \sqrt{2} \left(c f^2 - b f g + a g^2\right) \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}} \sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right) +$$

$$\left(\mathbb{i} \sqrt{2}\, c \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\right) \Big/$$

$$\left.\left.\left(\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}\ \sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right)\right)\right)$$

- **Problem 897: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + b x + c x^2}}{(d + e x) \sqrt{f + g x}}\, dx$$

Optimal (type 4, 700 leaves, 11 steps):

$$\frac{\sqrt{2}\,\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],\,-\frac{2\sqrt{b^2-4ac}\;g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right]}{e\,g\,\sqrt{\frac{c\,(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{a+bx+cx^2}}\;-$$

$$\left(2\sqrt{2}\,\sqrt{b^2-4ac}\,(cef+cdg-beg)\,\sqrt{\frac{c\,(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],\,-\frac{2\sqrt{b^2-4ac}\;g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right]\right)\Big/\left(c\,e^2\,g\,\sqrt{f+gx}\,\sqrt{a+bx+cx^2}\right)\;-$$

$$\left(\sqrt{2}\,\left(cd^2-bde+ae^2\right)\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,\sqrt{1-\frac{2c\,(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\,\sqrt{1-\frac{2c\,(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\;\text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{e\left(2cf-bg+\sqrt{b^2-4ac}\;g\right)}{2c\,(ef-dg)},\,\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{f+gx}}{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\right],\,\frac{b-\sqrt{b^2-4ac}-\frac{2cf}{g}}{b+\sqrt{b^2-4ac}-\frac{2cf}{g}}\right]\right)\Big/\left(\sqrt{c}\,e^2\,(ef-dg)\,\sqrt{a+bx+cx^2}\right)$$

Result (type 4, 16 471 leaves) :

$$-\frac{1}{\sqrt{a+bx+cx^2}}\,2\sqrt{a+x\,(b+cx)}$$

$$\left(-\frac{\sqrt{\frac{(f+g\,x)^2\left(c\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}}{e\sqrt{f+g\,x}}-\frac{1}{e\,(f+g\,x)\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}}}\sqrt{\frac{(f+g\,x)^2\left(c\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}\right.$$

$$\left(\left(-\mathbb{i}\,c\,f^2\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\text{EllipticF}\left[\right.\right.$$

$$\left.\left.\left.\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(\mathbb{i}\,b\,f\,g\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(\mathbb{i}\,a g^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(\mathbb{i}\, c\, e^2 f^3 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,(e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(\mathbb{i}\, c\, d\, e\, f^2 g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,(e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(\mathbb{i}\, b\, e^2 f^2 g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,(e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\ \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(\mathbb{i}\, b\, d\, e\, f\, g^2 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,(e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\ \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(\mathrm{i}\, a\, e^2 f g^2 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,(e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\ \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(\mathrm{i}\, a\, d\, e\, g^3 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,(e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\ \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(i\sqrt{2}\,c\,e\,f^2\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left((e f-d g)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(i\sqrt{2}\,b\,e\,f\,g\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left((e f-d g)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(\mathbb{i}\sqrt{2}\,a\,e\,g^2\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left((e f-d g)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\frac{1}{(e f-d g)^3}c\,e^3 f^3\left(\left(\mathbb{i}\,f\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\text{EllipticPi}\right[\right.\right.$$

$$\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\left(i\,d\,g\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\,i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\,\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left.\left(\sqrt{2}\,e\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right)+$$

$$\frac{1}{(e f-d g)^3}\,c\,d\,e^2 f^2 g\left(\left(i\,f\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\text{EllipticPi}\left[\right.\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\,i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\,\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\left(\mathbb{i}\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ \mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left.\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right)+$$

$$\frac{1}{(e f-d g)^3}\, b\, e^3 f^2 g\left(\left(\mathbb{i}\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ \mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\left(\mathrm{i}\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg) -$$

$$\frac{1}{(e f-d g)^3}\, b\, d\, e^2 f g^2 \left(\left(\left(\mathrm{i}\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right) -$$

$$\left(i\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Bigg/$$

$$\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right) -$$

$$\frac{1}{(e f-d g)^3} a e^3 f g^2 \left(\left(i\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Bigg/\right.$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right) -$$

$$\left(i\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left.\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right)+$$

$$\frac{1}{(e f-d g)^3} a\, d\, e^2 g^3 \left(\left(i\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \text{EllipticPi}\left[\right.\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\left(i\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right)+$$

$$\frac{1}{(e f-d g)^2} 2 c e^2 f^2 \left(\left(i\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/\right.$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\left(i\,d\,g\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\,\text{EllipticPi}\left[\frac{(e\,f-d\,g)\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)}{2\,e\left(c\,f^2-b\,f\,g+a\,g^2\right)},\,i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,e\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}}\right)\Bigg)-$$

$$\frac{1}{(e\,f-d\,g)^2}\,2\,b\,e^2\,f\,g\left(\left(i\,f\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\,\text{EllipticPi}\left[\frac{(e\,f-d\,g)\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)}{2\,e\left(c\,f^2-b\,f\,g+a\,g^2\right)},\,i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/\right.$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}}\right)-$$

$$\left(i\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg)+$$

$$\frac{1}{(e f-d g)^2} 2 a e^2 g^2 \left(\left(i\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\left(i\,d\,g\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\;\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\;\text{EllipticPi}\Big[\right.$$

$$\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\; i\,\text{ArcSinh}\Big[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\Big],\; \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\Big]\right)\Big/$$

$$\left(\sqrt{2}\,e\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\;\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg)-$$

$$\frac{1}{e f-d g}\,c\,e\,f\left(\left(i\,f\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\;\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\;\text{EllipticPi}\Big[\right.\right.$$

$$\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\; i\,\text{ArcSinh}\Big[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\Big],\; \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\Big]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\;\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\left(i\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right) -$$

$$\frac{1}{e f-d g} c\, d\, g \left(\left(i\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right) -$$

$$\left(\mathrm{i}\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathrm{i} \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg) +$$

$$\frac{1}{e f-d g} b\, e\, g \left(\left(\mathrm{i}\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathrm{i} \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/\right.$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right) -$$

$$\left(i\, d\, g\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.$$

$$\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Bigg/$$

$$\left.\left.\left.\left(\sqrt{2}\, e\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right)\right)\right)$$

- **Problem 898: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b x+c x^2}}{(d+e x)^2\sqrt{f+g x}}\, dx$$

Optimal (type 4, 736 leaves, 15 steps) :

$$-\frac{\sqrt{f+g\,x}\,\sqrt{a+b\,x+c\,x^2}}{(e\,f-d\,g)\,(d+e\,x)}+\frac{\sqrt{b^2-4\,a\,c}\,\sqrt{f+g\,x}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,g}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)\,g}\right]}{\sqrt{2}\,e\,(e\,f-d\,g)\,\sqrt{\frac{c\,(f+g\,x)}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)\,g}}\,\sqrt{a+b\,x+c\,x^2}}+$$

$$\left(\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\sqrt{\frac{c\,(f+g\,x)}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)\,g}}\,\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\frac{2\,\sqrt{b^2-4\,a\,c}\,g}{-2\,c\,f+\left(b+\sqrt{b^2-4\,a\,c}\right)\,g}\right]\right)\Big/$$

$$\left(e^2\,\sqrt{f+g\,x}\,\sqrt{a+x\,(b+c\,x)}\right)-$$

$$\left(\sqrt{2\,c\,f-\left(b-\sqrt{b^2-4\,a\,c}\right)\,g}\,\left(e^2\,(b\,f-a\,g)-c\,d\,(2\,e\,f-d\,g)\right)\,\sqrt{1-\frac{2\,c\,(f+g\,x)}{2\,c\,f-\left(b-\sqrt{b^2-4\,a\,c}\right)\,g}}\,\sqrt{1-\frac{2\,c\,(f+g\,x)}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)\,g}}\,\text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{e\left(2\,c\,f-b\,g+\sqrt{b^2-4\,a\,c}\,g\right)}{2\,c\,(e\,f-d\,g)},\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{f+g\,x}}{\sqrt{2\,c\,f-\left(b-\sqrt{b^2-4\,a\,c}\right)\,g}}\right],\frac{b-\sqrt{b^2-4\,a\,c}-\frac{2\,c\,f}{g}}{b+\sqrt{b^2-4\,a\,c}-\frac{2\,c\,f}{g}}\right]\right)\Big/\left(\sqrt{2}\,\sqrt{c}\,e^2\,(e\,f-d\,g)^2\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 6911 leaves) :

$$\frac{\sqrt{f+g\,x}\,\sqrt{a+x\,(b+c\,x)}}{(-e\,f+d\,g)\,(d+e\,x)}+\frac{1}{g\,(-e\,f+d\,g)\,\sqrt{a+b\,x+c\,x^2}}\sqrt{a+x\,(b+c\,x)}\left(-\frac{(f+g\,x)^{3/2}\left(c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}\right)}{e\,\sqrt{\frac{(f+g\,x)^2\left(c\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}}-\right.$$

$$\frac{1}{e\sqrt{\frac{(f+g\,x)^2\left(c\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}}\,(e\,f-d\,g)\,(f+g\,x)\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}}$$

$$\left(-\left(\mathbb{i}\,c\,f^2\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.\right.$$

$$\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\right.\right.$$

$$\left.\left.\left.\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,(e\,f-d\,g)\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(\mathbb{i}\,b\,f\,g\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,(e f-d g)\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(i\,a g^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,(e f-d g)\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(i\, c\, e\, f^2\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,(e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(i\, b\, e\, f\, g\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,(e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(\mathrm{i}\, a\, e\, g^2 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2}\,(e f-d g)^2 \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right) -$$

$$\frac{1}{(e f-d g)^3} c\, e^2 f^2 \left(\left(\mathrm{i}\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right) -$$

$$\left(\mathrm{i}\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Bigg/$$

$$\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg) +$$

$$\frac{1}{(e f-d g)^3} b\, e^2 f\, g \left(\left(\mathrm{i}\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Bigg/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right) -$$

$$\left(\mathrm{i}\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right) -$$

$$\frac{1}{(e f-d g)^3} a e^2 g^2 \left(\left(\mathrm{i}\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/\right.$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right) -$$

$$\left(\mathbb{i}\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\Big[\right.$$

$$\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathbb{i}\, \text{ArcSinh}\Big[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\Big], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\Big]\right) \Big/$$

$$\left(\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right)+$$

$$\frac{1}{e f-d g} c \left(\left(\mathbb{i}\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\Big[\right.\right.$$

$$\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathbb{i}\, \text{ArcSinh}\Big[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\Big], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\Big]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\left(\mathrm{i}\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ \mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Bigg/$$

$$\left.\left.\left.\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right)\right)\right)$$

- **Problem 899: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b x+c x^2}}{(d+e x)^3 \sqrt{f+g x}}\, dx$$

Optimal (type 4, 1049 leaves, 25 steps):

$$-\frac{\sqrt{f+g x}\sqrt{a+b x+c x^2}}{2(e f-d g)(d+e x)^2}+\frac{(c d(2 e f+d g)-e(b e f+2 b d g-3 a e g))\sqrt{f+g x}\sqrt{a+b x+c x^2}}{4\left(c d^2-b d e+a e^2\right)(e f-d g)^2(d+e x)}-$$

$$\left(\sqrt{b^2-4 a c}\,(c d(2 e f+d g)-e(b e f+2 b d g-3 a e g))\sqrt{f+g x}\right.$$

$$\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\ g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]\Bigg/$$

$$\left(4 \sqrt{2}\ e\left(c d^2-b d e+a e^2\right)(e f-d g)^2 \sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}} \sqrt{a+b x+c x^2}\right)-$$

$$\left(\sqrt{b^2-4 a c}\left(e^2 (b f-a g)+c d(-2 e f+d g)\right) \sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}} \sqrt{\frac{c (a+x (b+c x))}{-b^2+4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],\frac{2 \sqrt{b^2-4 a c}\ g}{-2 c f+\left(b+\sqrt{b^2-4 a c}\right) g}\right]\right)\Bigg/\left(2 \sqrt{2}\ e^2\left(c d^2+e(-b d+a e)\right)(e f-d g) \sqrt{f+g x} \sqrt{a+x (b+c x)}\right)-$$

$$\left(\sqrt{2 c f-b g+\sqrt{b^2-4 a c}\ g}\left(3 a^2 e^4 g^2+c^2 d^3 g(4 e f-d g)+b^2 e^3 f(-e f+4 d g)+2 a c e^2\left(2 e^2 f^2-2 d e f g+3 d^2 g^2\right)-\right.\right.$$

$$\left.\left.2 b e^2 g\left(3 c d^2 f+a e(e f+2 d g)\right)\right) \sqrt{\frac{g\left(-b+\sqrt{b^2-4 a c}-2 c x\right)}{2 c f+\left(-b+\sqrt{b^2-4 a c}\right) g}} \sqrt{\frac{g\left(b+\sqrt{b^2-4 a c}+2 c x\right)}{-2 c f+\left(b+\sqrt{b^2-4 a c}\right) g}}\right.$$

$$\text{EllipticPi}\left[\frac{2 c e f - b e g + \sqrt{b^2 - 4 a c}\; e g}{2 c e f - 2 c d g}, \text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{f + g x}}{\sqrt{2 c f - b g + \sqrt{b^2 - 4 a c}\; g}}\right], \frac{2 c f + \left(-b + \sqrt{b^2 - 4 a c}\right) g}{2 c f - \left(b + \sqrt{b^2 - 4 a c}\right) g}\right]\Bigg) \Big/$$

$$\left(4 \sqrt{2}\,\sqrt{c}\; e^2 \left(c d^2 + e\,(-b d + a e)\right)\,(e f - d g)^3 \sqrt{a + x\,(b + c x)}\right)$$

Result (type 4, 36 617 leaves) : Display of huge result suppressed !

- **Problem 900: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(d + e x)^3 \sqrt{f + g x}}{\sqrt{a + b x + c x^2}}\, dx$$

Optimal (type 4, 774 leaves, 8 steps) :

$$\frac{1}{105\,c^3\,g^2}2\,e\,\left(24\,b^2\,e^2\,g^2+c\,e\,g\,(13\,b\,e\,f-84\,b\,d\,g-25\,a\,e\,g)-c^2\,\left(7\,e^2\,f^2+12\,d\,e\,f\,g-90\,d^2\,g^2\right)\right)\sqrt{f+g\,x}\,\sqrt{a+b\,x+c\,x^2}+$$

$$\frac{2\,e\,(d+e\,x)^2\,\sqrt{f+g\,x}\,\sqrt{a+b\,x+c\,x^2}}{7\,c}+\frac{2\,e^2\,(c\,e\,f+11\,c\,d\,g-6\,b\,e\,g)\,(f+g\,x)^{3/2}\,\sqrt{a+b\,x+c\,x^2}}{35\,c^2\,g^2}-$$

$$\left(\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(48\,b^3\,e^3\,g^3-8\,b\,c\,e^2\,g^2\,(2\,b\,e\,f+21\,b\,d\,g+13\,a\,e\,g)-c^3\,\left(8\,e^3\,f^3-42\,d\,e^2\,f^2\,g+105\,d^2\,e\,f\,g^2+105\,d^3\,g^3\right)+\right.\right.$$

$$\left.c^2\,e\,g\,\left(a\,e\,g\,(19\,e\,f+189\,d\,g)-b\,\left(9\,e^2\,f^2-63\,d\,e\,f\,g-210\,d^2\,g^2\right)\right)\right)\sqrt{f+g\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,g}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)\,g}\right]\right)\Bigg/\left(105\,c^4\,g^3\,\sqrt{\frac{c\,(f+g\,x)}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)\,g}}\,\sqrt{a+b\,x+c\,x^2}\right)-$$

$$\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\,e\,\left(c\,f^2-b\,f\,g+a\,g^2\right)\,\left(24\,b^2\,e^2\,g^2+c\,e\,g\,(13\,b\,e\,f-84\,b\,d\,g-25\,a\,e\,g)+c^2\,\left(8\,e^2\,f^2-42\,d\,e\,f\,g+105\,d^2\,g^2\right)\right)\right.$$

$$\sqrt{\frac{c\,(f+g\,x)}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)\,g}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,g}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)\,g}\right]\right)\Bigg/\left(105\,c^4\,g^3\,\sqrt{f+g\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 10 649 leaves) :

$$\frac{1}{\sqrt{a+x\,(b+c\,x)}}\sqrt{f+g\,x}\,\left(a+b\,x+c\,x^2\right)$$

$$\left(-\frac{2\,e\,\left(4\,c^2\,e^2\,f^2-21\,c^2\,d\,e\,f\,g+5\,b\,c\,e^2\,f\,g-105\,c^2\,d^2\,g^2+84\,b\,c\,d\,e\,g^2-24\,b^2\,e^2\,g^2+25\,a\,c\,e^2\,g^2\right)}{105\,c^3\,g^2}-\frac{2\,e^2\,(-c\,e\,f-21\,c\,d\,g+6\,b\,e\,g)\,x}{35\,c^2\,g}+\frac{2\,e^3\,x^2}{7\,c}\right)-$$

$$\frac{1}{105\,c^3\,g^4\,\sqrt{a+x\,(b+c\,x)}}\,2\,\sqrt{a+b\,x+c\,x^2}\,\left(\left(\left(-8\,c^3\,e^3\,f^3+42\,c^3\,d\,e^2\,f^2\,g-9\,b\,c^2\,e^3\,f^2\,g-105\,c^3\,d^2\,e\,f\,g^2+63\,b\,c^2\,d\,e^2\,f\,g^2-\right.\right.\right.$$

$$\left.16\,b^2\,c\,e^3\,f\,g^2+19\,a\,c^2\,e^3\,f\,g^2-105\,c^3\,d^3\,g^3+210\,b\,c^2\,d^2\,e\,g^3-168\,b^2\,c\,d\,e^2\,g^3+189\,a\,c^2\,d\,e^2\,g^3+48\,b^3\,e^3\,g^3-104\,a\,b\,c\,e^3\,g^3\right)$$

$$\left.(f+g\,x)^{3/2}\left(c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}\right)\right)\Bigg/\left(c\,\sqrt{\frac{(f+g\,x)^2\left(c\,\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\,\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}\right)+$$

$$\frac{1}{c\,\sqrt{\frac{(f+g\,x)^2\left(c\,\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\,\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}}\left(c\,f^2-b\,f\,g+a\,g^2\right)\,(f+g\,x)\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}}$$

$$\left(\left(\left(2\,i\,\sqrt{2}\,c^3\,e^3\,f^3\,\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\right.\right.\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(21 i c^3 d e^2 f^2 g\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(9\,\mathbb{i}\,b c^2 e^3 f^2 g\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(105\,\mathbb{i}\,c^3 d^2 e f g^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(63\,i\,b\,c^2 d\,e^2 f g^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(4\,i\sqrt{2}\,b^2 c e^3 f g^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Bigg/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(19\,i\,a c^2 e^3 f g^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(105\,i\,c^3 d^3 g^3\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(105\,\mathbb{i}\,b c^2 d^2 e g^3\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(42\,\mathbb{i}\sqrt{2}\,b^2 c d e^2 g^3\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(189\, i\, a c^2 d e^2 g^3\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(12\,\mathbb{i}\sqrt{2}\,b^3 e^3 g^3\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(26\,\mathbb{i}\sqrt{2}\,a b c e^3 g^3\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(4 i\sqrt{2}\,c^3 e^3 f^2\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(21\,i\,\sqrt{2}\,c^3\,d\,e^2\,f\,g\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(13\,i\,b\,c^2\,e^3\,f\,g\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(105\, i\, c^3 d^2 e\, g^2 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(42\, i \sqrt{2}\, b\, c^2 d\, e^2 g^2 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(12\,i\,\sqrt{2}\,b^2\,c\,e^3\,g^2\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)-$$

$$\left(25\,i\,a\,c^2\,e^3\,g^2\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left.\left.\left(\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)\right)\right)$$

Problem 901: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(d+e x)^2 \sqrt{f+g x}}{\sqrt{a+b x+c x^2}}\, dx$$

Optimal (type 4, 567 leaves, 7 steps):

$$\frac{2 e (c e f+7 c d g-4 b e g) \sqrt{f+g x} \sqrt{a+b x+c x^2}}{15 c^2 g}+\frac{2 e (d+e x) \sqrt{f+g x} \sqrt{a+b x+c x^2}}{5 c}+$$

$$\left(\sqrt{2} \sqrt{b^2-4 a c} \left(8 b^2 e^2 g^2-c e g (3 b e f+20 b d g+9 a e g)-c^2 \left(2 e^2 f^2-10 d e f g-15 d^2 g^2\right)\right) \sqrt{f+g x} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]\right) \Big/ \left(15 c^3 g^2 \sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}} \sqrt{a+b x+c x^2}\right)+$$

$$\left(4 \sqrt{2} \sqrt{b^2-4 a c}\, e (c e f-5 c d g+2 b e g) \left(c f^2-b f g+a g^2\right) \sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]\right) \Big/ \left(15 c^3 g^2 \sqrt{f+g x} \sqrt{a+b x+c x^2}\right)$$

Result (type 4, 5536 leaves):

$$\frac{\left(-\frac{2 e (-c e f-10 c d g+4 b e g)}{15 c^2 g}+\frac{2 e^2 x}{5 c}\right) \sqrt{f+g x}\, \left(a+b x+c x^2\right)}{\sqrt{a+x (b+c x)}}-$$

$$\frac{1}{15 c^2 g^3 \sqrt{a+x (b+c x)}} 2 \sqrt{a+b x+c x^2} \left(\left(\left(2 c^2 e^2 f^2-10 c^2 d e f g+3 b c e^2 f g-15 c^2 d^2 g^2+20 b c d e g^2-8 b^2 e^2 g^2+9 a c e^2 g^2\right)\right.\right.$$

$$(f+g x)^{3/2} \left(c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}\right)\Bigg) \Bigg/ \left(c \sqrt{\frac{(f+g x)^2 \left(c \left(-1+\frac{f}{f+g x}\right)^2+\frac{g \left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}\right)-$$

$$\frac{1}{c \sqrt{\frac{(f+g x)^2 \left(c \left(-1+\frac{f}{f+g x}\right)^2+\frac{g \left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}} \left(c f^2-b f g+a g^2\right) (f+g x) \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}$$

$$\left(\left(\mathrm{i}\, c^2 e^2 f^2 \left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right) \sqrt{1-\frac{2 \left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right) (f+g x)}} \sqrt{1-\frac{2 \left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right) (f+g x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(5\,\mathrm{i}\,c^2 d e f g\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(3\, i\, b\, c\, e^2\, f\, g\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right)\sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}}\right.$$

$$\sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}}\left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\right)\right) \Big/$$

$$\left(2\sqrt{2}\left(c f^2 - b f g + a g^2\right)\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}\sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right) -$$

$$\left(15\, i\, c^2\, d^2\, g^2\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right)\sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}}\right.$$

$$\sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}}\left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(2\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(5\,i\sqrt{2}\,b c d e g^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(2\,i\,\sqrt{2}\,b^2 e^2 g^2 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}}\right.$$

$$\sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}} \left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\right)\right) \Big/$$

$$\left(\left(c f^2 - b f g + a g^2\right) \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}} \sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right) +$$

$$\left(9\,i\,a\,c\,e^2 g^2 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}} \sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\Bigg(i\sqrt{2}\,c^2 e^2 f\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\Bigg(5\,i\sqrt{2}\,c^2 d e g\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(2 i \sqrt{2}\, b c e^2 g\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg/$$

$$\left.\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)\right)$$

- **Problem 902: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(d+e x)\sqrt{f+g x}}{\sqrt{a+b x+c x^2}}\,dx$$

Optimal (type 4, 452 leaves, 6 steps) :

$$\frac{2\,e\,\sqrt{f+g\,x}\,\sqrt{a+b\,x+c\,x^2}}{3\,c}+$$

$$\left(\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(c\,e\,f+3\,c\,d\,g-2\,b\,e\,g\right)\sqrt{f+g\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,g}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)g}\right]\right)\Big/$$

$$\left(3\,c^2\,g\,\sqrt{\frac{c\,(f+g\,x)}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)g}}\,\sqrt{a+b\,x+c\,x^2}\right)-\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,e\,\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{\frac{c\,(f+g\,x)}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)g}}\right.$$

$$\left.\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,g}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)g}\right]\right)\Big/\left(3\,c^2\,g\,\sqrt{f+g\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 766 leaves) :

$$\frac{2 e \sqrt{f+g x}\,(a+b x+c x^2)}{3 c \sqrt{a+x (b+c x)}}+\frac{1}{3 c^2 g^2 \sqrt{a+x (b+c x)}\sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}}$$

$$2 (f+g x)^{3/2} \sqrt{a+b x+c x^2}\left((c e f+3 c d g-2 b e g)\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)+\frac{1}{2\sqrt{2}\sqrt{\frac{c f^2+g(-b f+a g)}{-2 c f+b g+\sqrt{(b^2-4 a c) g^2}}}\sqrt{f+g x}}\right.$$

$$i \sqrt{1-\frac{2 (c f^2+g (-b f+a g))}{\left(2 c f-b g+\sqrt{(b^2-4 a c) g^2}\right)(f+g x)}}\sqrt{1+\frac{2 (c f^2+g (-b f+a g))}{\left(-2 c f+b g+\sqrt{(b^2-4 a c) g^2}\right)(f+g x)}}\left(\left(2 c f-b g+\sqrt{(b^2-4 a c) g^2}\right)\right.$$

$$(2 b e g-c (e f+3 d g))\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c f^2-b f g+a g^2}{-2 c f+b g+\sqrt{(b^2-4 a c) g^2}}}}{\sqrt{f+g x}}\right],-\frac{-2 c f+b g+\sqrt{(b^2-4 a c) g^2}}{2 c f-b g+\sqrt{(b^2-4 a c) g^2}}\right]+$$

$$\left(6 c^2 d f g+2 b e g\left(b g-\sqrt{(b^2-4 a c) g^2}\right)+c\left(-2 a e g^2-3 b g (e f+d g)+\sqrt{(b^2-4 a c) g^2}\,(e f+3 d g)\right)\right)$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c f^2-b f g+a g^2}{-2 c f+b g+\sqrt{(b^2-4 a c) g^2}}}}{\sqrt{f+g x}}\right],-\frac{-2 c f+b g+\sqrt{(b^2-4 a c) g^2}}{2 c f-b g+\sqrt{(b^2-4 a c) g^2}}\right]\right)\right)$$

- **Problem 903: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{f+g x}}{\sqrt{a+b x+c x^2}}\,dx$$

Optimal (type 4, 188 leaves, 2 steps) :

$$\frac{\sqrt{2}\,\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],\,-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right]}{c\,\sqrt{\frac{c\,(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{a+bx+cx^2}}$$

Result (type 4, 365 leaves) :

$$\left(i\left(2cf+\left(-b+\sqrt{b^2-4ac}\right)g\right)\sqrt{\frac{g\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{1-\frac{2c\,(f+gx)}{2cf+\left(-b+\sqrt{b^2-4ac}\right)g}}\right.$$
$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{f+gx}\right],\,\frac{2cf-\left(b+\sqrt{b^2-4ac}\right)g}{2cf+\left(-b+\sqrt{b^2-4ac}\right)g}\right]-\right.$$
$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{f+gx}\right],\,\frac{2cf-\left(b+\sqrt{b^2-4ac}\right)g}{2cf+\left(-b+\sqrt{b^2-4ac}\right)g}\right]\right)\right)\Big/$$
$$\left(\sqrt{2}\,c\,g\,\sqrt{\frac{c}{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{a+x\,(b+cx)}\right)$$

- **Problem 904: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{f+gx}}{(d+ex)\,\sqrt{a+bx+cx^2}}\,dx$$

Optimal (type 4, 467 leaves, 8 steps) :

$$\left(2\sqrt{2}\,\sqrt{b^2-4ac}\,g\,\sqrt{\frac{c\,(f+g\,x)}{2\,c\,f-\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4ac}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2\,c\,x}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,g}{2\,c\,f-\left(b+\sqrt{b^2-4ac}\right)g}\right]\right)\Big/$$

$$\left(c\,e\,\sqrt{f+g\,x}\,\sqrt{a+b\,x+c\,x^2}\right)-\frac{1}{\sqrt{c}\,e\,\sqrt{a+b\,x+c\,x^2}}\sqrt{2}\,\sqrt{2\,c\,f-\left(b-\sqrt{b^2-4ac}\right)g}\,\sqrt{1-\frac{2\,c\,(f+g\,x)}{2\,c\,f-\left(b-\sqrt{b^2-4ac}\right)g}}$$

$$\sqrt{1-\frac{2\,c\,(f+g\,x)}{2\,c\,f-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticPi}\left[\frac{e\left(2\,c\,f-b\,g+\sqrt{b^2-4ac}\,g\right)}{2\,c\,(e\,f-d\,g)},\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{f+g\,x}}{\sqrt{2\,c\,f-\left(b-\sqrt{b^2-4ac}\right)g}}\right],\frac{b-\sqrt{b^2-4ac}-\frac{2\,c\,f}{g}}{b+\sqrt{b^2-4ac}-\frac{2\,c\,f}{g}}\right]$$

Result (type 4, 379 leaves):

$$-\left(i\sqrt{2}\,\sqrt{\frac{g\left(b+\sqrt{b^2-4ac}+2\,c\,x\right)}{-2\,c\,f+\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{1-\frac{2\,c\,(f+g\,x)}{2\,c\,f+\left(-b+\sqrt{b^2-4ac}\right)g}}\right.$$

$$\left(\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{-2\,c\,f+\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{f+g\,x}\right],\frac{2\,c\,f-\left(b+\sqrt{b^2-4ac}\right)g}{2\,c\,f+\left(-b+\sqrt{b^2-4ac}\right)g}\right]-\right.$$

$$\left.\left.\text{EllipticPi}\left[\frac{e\left(2\,c\,f-\left(b+\sqrt{b^2-4ac}\right)g\right)}{2\,c\,(e\,f-d\,g)},i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{-2\,c\,f+\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{f+g\,x}\right],\frac{2\,c\,f-\left(b+\sqrt{b^2-4ac}\right)g}{2\,c\,f+\left(-b+\sqrt{b^2-4ac}\right)g}\right]\right)\right)\Big/$$

$$\left(e\,\sqrt{\frac{c}{-2\,c\,f+\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{a+x\,(b+c\,x)}\right)$$

- **Problem 905: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{f+g\,x}}{(d+e\,x)^2\,\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 994 leaves, 15 steps) :

$$-\frac{e\sqrt{f+g x}\,\sqrt{a+b x+c x^2}}{\left(c d^2-b d e+a e^2\right)(d+e x)}+\frac{\sqrt{b^2-4 a c}\,\sqrt{f+g x}\,\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}\right]}{\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{\frac{c(f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}}\,\sqrt{a+b x+c x^2}}-$$

$$\left(\sqrt{2}\,\sqrt{b^2-4 a c}\,f\sqrt{\frac{c(f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}}\,\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}\right]\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{f+g x}\,\sqrt{a+b x+c x^2}\right)+$$

$$\left(\sqrt{2}\,\sqrt{b^2-4 a c}\,d g\sqrt{\frac{c(f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}}\,\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}\right]\right)\Big/$$

$$\left(e\left(c d^2-b d e+a e^2\right)\sqrt{f+g x}\,\sqrt{a+b x+c x^2}\right)+$$

$$\left(\sqrt{2 c f-\left(b-\sqrt{b^2-4 a c}\right)g}\,\left(e^2(b f-a g)-c d(2 e f-d g)\right)\sqrt{1-\frac{2 c(f+g x)}{2 c f-\left(b-\sqrt{b^2-4 a c}\right)g}}\,\sqrt{1-\frac{2 c(f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{e\left(2 c f-b g+\sqrt{b^2-4 a c}\,g\right)}{2 c(e f-d g)},\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{f+g x}}{\sqrt{2 c f-\left(b-\sqrt{b^2-4 a c}\right)g}}\right],\frac{b-\sqrt{b^2-4 a c}-\frac{2 c f}{g}}{b+\sqrt{b^2-4 a c}-\frac{2 c f}{g}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{c}\,e\left(c d^2-b d e+a e^2\right)(e f-d g)\sqrt{a+b x+c x^2}\right)$$

Result (type 4, 18563 leaves):

$$-\frac{e\sqrt{f+g\,x}\,\left(a+b\,x+c\,x^2\right)}{\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)\sqrt{a+x\,(b+c\,x)}}-$$

$$\frac{1}{\left(c\,d^2-b\,d\,e+a\,e^2\right)g\sqrt{a+x\,(b+c\,x)}}\sqrt{a+b\,x+c\,x^2}\left(-\frac{(f+g\,x)^{3/2}\left(c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}\right)}{\sqrt{\frac{(f+g\,x)^2\left(c\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}}+\right.$$

$$\left(i\,c\,f^2\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}}\right.$$

$$\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\left.\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\right)\right/\left(2\sqrt{2}\left(c\,f^2-b\,f\,g+a\,g^2\right)\right.$$

$$\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\;\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\;\sqrt{\frac{(f+g\,x)^2\left(c\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}\;\Bigg) -$$

$$\Bigg(\mathbb{i}\,b\,f\,g\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}}$$

$$\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\;\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\Bigg)\Bigg)\Bigg/\Bigg(2\,\sqrt{2}\,\left(c\,f^2-b\,f\,g+a\,g^2\right)$$

$$\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\;\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\;\sqrt{\frac{(f+g\,x)^2\left(c\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}\;\Bigg) +$$

$$\Bigg(\mathrm{i}\, a\, g^2 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x) \sqrt{c + \frac{c f^2}{(f + g x)^2} - \frac{b f g}{(f + g x)^2} + \frac{a g^2}{(f + g x)^2} - \frac{2 c f}{f + g x} + \frac{b g}{f + g x}}$$

$$\sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}$$

$$\Bigg(\text{EllipticE}\Bigg[\mathrm{i}\, \text{ArcSinh}\Bigg[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\Bigg], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\Bigg] -$$

$$\text{EllipticF}\Bigg[\mathrm{i}\, \text{ArcSinh}\Bigg[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\Bigg], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\Bigg]\Bigg)\Bigg) \Bigg/ \Bigg(2 \sqrt{2} \left(c f^2 - b f g + a g^2\right)$$

$$\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}} \sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}} \sqrt{\frac{(f + g x)^2 \left(c \left(-1 + \frac{f}{f + g x}\right)^2 + \frac{g \left(b - \frac{b f}{f + g x} + \frac{a g}{f + g x}\right)}{f + g x}\right)}{g^2}}\Bigg) -$$

$$\Bigg(\mathrm{i}\, c\, e^2 f^3 (f + g x) \sqrt{c + \frac{c f^2}{(f + g x)^2} - \frac{b f g}{(f + g x)^2} + \frac{a g^2}{(f + g x)^2} - \frac{2 c f}{f + g x} + \frac{b g}{f + g x}} \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg/$$

$$\left(\sqrt{2}\,(e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}\right)+$$

$$\left(i\,c\,d\,e\,f^2 g\,(f+g x)\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,(e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}\right)+$$

$$\left(i\,b\,e^2 f^2 g\,(f+g x)\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg/$$

$$\left(\sqrt{2}\,(e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}\right)-$$

$$\left(i\,b\,d\,e\,f\,g^2(f+g x)\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,(e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}\right)-$$

$$\left(i\,a\,e^2 f g^2(f+g x)\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,(e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}\right)+$$

$$\Bigg(i\,a\,d\,e\,g^3(f+g x)\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,(e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}\right)+$$

$$\Bigg(i\sqrt{2}\,c\,e f^2(f+g x)\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg) \Bigg/$$

$$\left((e f-d g)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}\right)-$$

$$\Bigg(i\sqrt{2}\,b e f g\,(f+g x)\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg) \Bigg/$$

$$\left((e f-d g)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}\right)+$$

$$\Bigg(i\sqrt{2}\,a e g^2\,(f+g x)\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg/$$

$$\left((e f-d g)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}\right)+$$

$$\frac{1}{(e f-d g)^3\sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}}\,c\,e^3 f^3 (f+g x)\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}$$

$$\Bigg(\Bigg(i f\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\left(i\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left.\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right) -$$

$$\frac{1}{(e f-d g)^3 \sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}}\, c\, d\, e^2 f^2 g\, (f+g x) \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}$$

$$\left(\left(i\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.\right.$$

$$\text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\Bigg(i\, d g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg) \Big/$$

$$\left(\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right)-$$

$$\frac{1}{(e f-d g)^3 \sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}}\, b e^3 f^2 g (f+g x) \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}$$

$$\Bigg(\Bigg(i\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right) -$$

$$\Bigg(i\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg) +$$

$$\frac{1}{(e\,f-d\,g)^{3}\sqrt{\frac{(f+g\,x)^{2}\left(c\left(-1+\frac{f}{f+g\,x}\right)^{2}+\frac{g\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^{2}}}}\,b\,d\,e^{2}\,f\,g^{2}\,(f+g\,x)\sqrt{c+\frac{c\,f^{2}}{(f+g\,x)^{2}}-\frac{b\,f\,g}{(f+g\,x)^{2}}+\frac{a\,g^{2}}{(f+g\,x)^{2}}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}}$$

$$\Bigg(\Bigg(\mathbb{i}\,f\sqrt{1-\frac{2\left(c\,f^{2}-b\,f\,g+a\,g^{2}\right)}{\left(2\,c\,f-b\,g-\sqrt{b^{2}\,g^{2}-4\,a\,c\,g^{2}}\right)(f+g\,x)}}\sqrt{1-\frac{2\left(c\,f^{2}-b\,f\,g+a\,g^{2}\right)}{\left(2\,c\,f-b\,g+\sqrt{b^{2}\,g^{2}-4\,a\,c\,g^{2}}\right)(f+g\,x)}}$$

$$\text{EllipticPi}\left[\frac{(e\,f-d\,g)\left(2\,c\,f-b\,g-\sqrt{b^{2}\,g^{2}-4\,a\,c\,g^{2}}\right)}{2\,e\left(c\,f^{2}-b\,f\,g+a\,g^{2}\right)},\ \mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^{2}-b\,f\,g+a\,g^{2}}{2\,c\,f-b\,g-\sqrt{b^{2}\,g^{2}-4\,a\,c\,g^{2}}}}}{\sqrt{f+g\,x}}\right],\ \frac{2\,c\,f-b\,g-\sqrt{b^{2}\,g^{2}-4\,a\,c\,g^{2}}}{2\,c\,f-b\,g+\sqrt{b^{2}\,g^{2}-4\,a\,c\,g^{2}}}\right]\Bigg)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c\,f^{2}-b\,f\,g+a\,g^{2}}{2\,c\,f-b\,g-\sqrt{b^{2}\,g^{2}-4\,a\,c\,g^{2}}}}\sqrt{c+\frac{c\,f^{2}}{(f+g\,x)^{2}}-\frac{b\,f\,g}{(f+g\,x)^{2}}+\frac{a\,g^{2}}{(f+g\,x)^{2}}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}}\right)-$$

$$\Bigg(\mathbb{i}\,d\,g\sqrt{1-\frac{2\left(c\,f^{2}-b\,f\,g+a\,g^{2}\right)}{\left(2\,c\,f-b\,g-\sqrt{b^{2}\,g^{2}-4\,a\,c\,g^{2}}\right)(f+g\,x)}}\sqrt{1-\frac{2\left(c\,f^{2}-b\,f\,g+a\,g^{2}\right)}{\left(2\,c\,f-b\,g+\sqrt{b^{2}\,g^{2}-4\,a\,c\,g^{2}}\right)(f+g\,x)}}$$

$$\text{EllipticPi}\left[\frac{(e\,f-d\,g)\left(2\,c\,f-b\,g-\sqrt{b^{2}\,g^{2}-4\,a\,c\,g^{2}}\right)}{2\,e\left(c\,f^{2}-b\,f\,g+a\,g^{2}\right)},\ \mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^{2}-b\,f\,g+a\,g^{2}}{2\,c\,f-b\,g-\sqrt{b^{2}\,g^{2}-4\,a\,c\,g^{2}}}}}{\sqrt{f+g\,x}}\right],\ \frac{2\,c\,f-b\,g-\sqrt{b^{2}\,g^{2}-4\,a\,c\,g^{2}}}{2\,c\,f-b\,g+\sqrt{b^{2}\,g^{2}-4\,a\,c\,g^{2}}}\right]\Bigg)\Big/$$

$$\left(\sqrt{2}\ e \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}} \sqrt{c + \frac{c f^2}{(f + g x)^2} - \frac{b f g}{(f + g x)^2} + \frac{a g^2}{(f + g x)^2} - \frac{2 c f}{f + g x} + \frac{b g}{f + g x}}\right)\Bigg) +$$

$$\frac{1}{(e f - d g)^3 \sqrt{\frac{(f+g x)^2 \left(c \left(-1+\frac{f}{f+g x}\right)^2+\frac{g \left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}}\ a e^3 f g^2 (f + g x) \sqrt{c + \frac{c f^2}{(f + g x)^2} - \frac{b f g}{(f + g x)^2} + \frac{a g^2}{(f + g x)^2} - \frac{2 c f}{f + g x} + \frac{b g}{f + g x}}$$

$$\Bigg(\Bigg(\mathbb{i}\, f \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}$$

$$\text{EllipticPi}\left[\frac{(e f - d g) \left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right)}{2 e \left(c f^2 - b f g + a g^2\right)}, \mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\Bigg) \Bigg/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}} \sqrt{c + \frac{c f^2}{(f + g x)^2} - \frac{b f g}{(f + g x)^2} + \frac{a g^2}{(f + g x)^2} - \frac{2 c f}{f + g x} + \frac{b g}{f + g x}}\right) -$$

$$\Bigg(\mathbb{i}\, d g \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}$$

$$\text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg/$$

$$\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg) -$$

$$\frac{1}{(e f-d g)^3 \sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}}\, a d e^2 g^3 (f+g x) \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}$$

$$\Bigg(\Bigg(i f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right) -$$

$$\left(\mathbb{i}\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ \mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left.\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right)-$$

$$\frac{1}{(e f-d g)^2 \sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}}\, 2 c e^2 f^2 (f+g x) \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}$$

$$\left(\left(\mathbb{i}\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.\right.$$

$$\text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\left(i\,d g\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg/$$

$$\left.\left(\sqrt{2}\, e\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right)+$$

$$\frac{1}{(e f-d g)^2\sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}}\, 2 b e^2 f g\,(f+g x)\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}$$

$$\Bigg(\Bigg(\mathrm{i}\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\text{EllipticPi}\Big[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathrm{i}\, \text{ArcSinh}\Big[\frac{\sqrt{2}\ \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\Big], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\Big]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\ \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\ \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\Bigg(\mathrm{i}\, d g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\text{EllipticPi}\Big[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathrm{i}\, \text{ArcSinh}\Big[\frac{\sqrt{2}\ \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\Big], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\Big]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\ e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\ \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg)-$$

$$\frac{1}{(e f-d g)^2\sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}}\, 2 a e^2 g^2 (f+g x)\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}$$

$$\Bigg(\Bigg(\mathrm{i}\, f\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\Bigg(\mathrm{i}\, d g\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Big/$$

$$\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg) +$$

$$\frac{1}{(e f-d g)\sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}}\, c\, e\, f\, (f+g x) \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}$$

$$\Bigg(\Bigg(i f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right) -$$

$$\Bigg(i\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\text{EllipticPi}\left[\frac{(e\,f-d\,g)\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)}{2\,e\left(c\,f^2-b\,f\,g+a\,g^2\right)},\ i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\ \frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\Bigg) \Bigg/$$

$$\left(\sqrt{2}\,e\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}}\right)\Bigg) -$$

$$\frac{1}{(e\,f-d\,g)\sqrt{\frac{(f+g\,x)^2\left(c\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}}\,c\,d\,g\,(f+g\,x)\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}}$$

$$\Bigg(\Bigg(i\,f\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}$$

$$\text{EllipticPi}\left[\frac{(e\,f-d\,g)\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)}{2\,e\left(c\,f^2-b\,f\,g+a\,g^2\right)},\ i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\ \frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\Bigg) \Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}}\right) -$$

$$\left(i\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right)\right)$$

- **Problem 906: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{f+g x}}{(d+e x)^3 \sqrt{a+b x+c x^2}}\, dx$$

Optimal (type 4, 1786 leaves, 25 steps):

$$-\frac{e\sqrt{f+g x}\sqrt{a+b x+c x^2}}{2\left(c d^2-b d e+a e^2\right)(d+e x)^2}-\frac{e\,(c d\,(6 e f-5 d g)-e\,(3 b e f-2 b d g-a e g))\sqrt{f+g x}\sqrt{a+b x+c x^2}}{4\left(c d^2-b d e+a e^2\right)^2(e f-d g)(d+e x)}+$$

$$\left(\sqrt{b^2-4 a c}\,(c d\,(6 e f-5 d g)-e\,(3 b e f-2 b d g-a e g))\sqrt{f+g x}\right.$$

$$\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2-4 a c}\ g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]\Bigg) \Big/$$

$$\left(4 \sqrt{2}\ \left(c d^2-b d e+a e^2\right)^2 (e f-d g) \sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}}\ \sqrt{a+b x+c x^2}\right)-$$

$$\left(\sqrt{b^2-4 a c}\ g \sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}}\ \sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2-4 a c}\ g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]\right) \Big/$$

$$\left(\sqrt{2}\ e\left(c d^2-b d e+a e^2\right) \sqrt{f+g x}\ \sqrt{a+b x+c x^2}\right)-$$

$$\Bigg(\sqrt{b^2-4 a c}\ f (c d (6 e f-5 d g)-e (3 b e f-2 b d g-a e g)) \sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}}\ \sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2-4 a c}\ g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]\Bigg) \Big/ \left(2 \sqrt{2}\ \left(c d^2-b d e+a e^2\right)^2 (e f-d g) \sqrt{f+g x}\ \sqrt{a+b x+c x^2}\right)+$$

$$\left(\sqrt{b^2-4ac}\; d\, g\, (c\, d\, (6\, e\, f-5\, d\, g)-e\, (3\, b\, e\, f-2\, b\, d\, g-a\, e\, g))\, \sqrt{\frac{c\, (f+g\, x)}{2\, c\, f-\left(b+\sqrt{b^2-4ac}\right) g}}\, \sqrt{-\frac{c\left(a+b\, x+c\, x^2\right)}{b^2-4ac}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],\, -\frac{2\sqrt{b^2-4ac}\; g}{2\, c\, f-\left(b+\sqrt{b^2-4ac}\right) g}\right]\right) \Big/ \left(2\sqrt{2}\; e\left(c\, d^2-b\, d\, e+a\, e^2\right)^2 (e\, f-d\, g)\, \sqrt{f+g\, x}\, \sqrt{a+b\, x+c\, x^2}\right)+$$

$$\left(\sqrt{2\, c\, f-\left(b-\sqrt{b^2-4ac}\right) g}\; (c\, e\, f-3\, c\, d\, g+b\, e\, g)\, \sqrt{1-\frac{2\, c\, (f+g\, x)}{2\, c\, f-\left(b-\sqrt{b^2-4ac}\right) g}}\, \sqrt{1-\frac{2\, c\, (f+g\, x)}{2\, c\, f-\left(b+\sqrt{b^2-4ac}\right) g}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{e\left(2\, c\, f-b\, g+\sqrt{b^2-4ac}\; g\right)}{2\, c\, (e\, f-d\, g)},\, \text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{f+g\, x}}{\sqrt{2\, c\, f-\left(b-\sqrt{b^2-4ac}\right) g}}\right],\, \frac{b-\sqrt{b^2-4ac}-\frac{2cf}{g}}{b+\sqrt{b^2-4ac}-\frac{2cf}{g}}\right]\right) \Big/$$

$$\left(\sqrt{2}\,\sqrt{c}\; e\left(c\, d^2-b\, d\, e+a\, e^2\right)(e\, f-d\, g)\, \sqrt{a+b\, x+c\, x^2}\right)-\left(\sqrt{2\, c\, f-\left(b-\sqrt{b^2-4ac}\right) g}\; (c\, d\, (6\, e\, f-5\, d\, g)-e\, (3\, b\, e\, f-2\, b\, d\, g-a\, e\, g))\right.$$

$$(c\, d\, (2\, e\, f-3\, d\, g)-e\, (b\, e\, f-2\, b\, d\, g+a\, e\, g))\, \sqrt{1-\frac{2\, c\, (f+g\, x)}{2\, c\, f-\left(b-\sqrt{b^2-4ac}\right) g}}\, \sqrt{1-\frac{2\, c\, (f+g\, x)}{2\, c\, f-\left(b+\sqrt{b^2-4ac}\right) g}}$$

$$\left.\text{EllipticPi}\left[\frac{e\left(2\, c\, f-b\, g+\sqrt{b^2-4ac}\; g\right)}{2\, c\, (e\, f-d\, g)},\, \text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{f+g\, x}}{\sqrt{2\, c\, f-\left(b-\sqrt{b^2-4ac}\right) g}}\right],\, \frac{b-\sqrt{b^2-4ac}-\frac{2cf}{g}}{b+\sqrt{b^2-4ac}-\frac{2cf}{g}}\right]\right) \Big/$$

$$\left(4 \sqrt{2} \sqrt{c}\, e \left(c d^2 - b d e + a e^2\right)^2 (e f - d g)^2 \sqrt{a + b x + c x^2}\right)$$

Result (type 4, 36 634 leaves): Display of huge result suppressed!

- **Problem 907: Result more than twice size of optimal antiderivative.**

$$\int \frac{(f + g x)^{3/2}}{(d + e x) \sqrt{a + b x + c x^2}}\, dx$$

Optimal (type 4, 675 leaves, 11 steps):

$$\frac{\sqrt{2} \sqrt{b^2 - 4 a c}\, g \sqrt{f + g x} \sqrt{-\frac{c (a+b x+c x^2)}{b^2-4 a c}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2-4 a c}\, g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]}{c e \sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}} \sqrt{a + b x + c x^2}} +$$

$$\left(2 \sqrt{2} \sqrt{b^2 - 4 a c}\, g (e f - d g) \sqrt{\frac{c (f + g x)}{2 c f - \left(b + \sqrt{b^2 - 4 a c}\right) g}} \sqrt{-\frac{c \left(a + b x + c x^2\right)}{b^2 - 4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2 - 4 a c}\, g}{2 c f - \left(b + \sqrt{b^2 - 4 a c}\right) g}\right]\right) \Bigg/ \left(c e^2 \sqrt{f + g x} \sqrt{a + b x + c x^2}\right) -$$

$$\frac{1}{\sqrt{c}\, e^2 \sqrt{a + b x + c x^2}} \sqrt{2} \sqrt{2 c f - \left(b - \sqrt{b^2 - 4 a c}\right) g}\, (e f - d g) \sqrt{1 - \frac{2 c (f + g x)}{2 c f - \left(b - \sqrt{b^2 - 4 a c}\right) g}} \sqrt{1 - \frac{2 c (f + g x)}{2 c f - \left(b + \sqrt{b^2 - 4 a c}\right) g}}$$

$$\text{EllipticPi}\left[\frac{e \left(2 c f - b g + \sqrt{b^2 - 4 a c}\, g\right)}{2 c (e f - d g)}, \text{ArcSin}\left[\frac{\sqrt{2} \sqrt{c} \sqrt{f + g x}}{\sqrt{2 c f - \left(b - \sqrt{b^2 - 4 a c}\right) g}}\right], \frac{b - \sqrt{b^2 - 4 a c} - \frac{2 c f}{g}}{b + \sqrt{b^2 - 4 a c} - \frac{2 c f}{g}}\right]$$

Result (type 4, 2358 leaves) :

$$\frac{1}{\sqrt{f+g x}\,\sqrt{a+x\,(b+c\,x)}}\sqrt{(f+g x)\,\left(a+b x+c x^2\right)}\left(\left(\left(4 f g\sqrt{\frac{-\frac{-b-\sqrt{b^2-4 a c}}{2 c}+x}{-\frac{-b-\sqrt{b^2-4 a c}}{2 c}+\frac{-b+\sqrt{b^2-4 a c}}{2 c}}}\left(-\frac{-b+\sqrt{b^2-4 a c}}{2 c}+x\right)\sqrt{\frac{\frac{f}{g}+x}{\frac{-b+\sqrt{b^2-4 a c}}{2 c}+\frac{f}{g}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{-b+\sqrt{b^2-4 a c}-2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],\right.\right.\right.\right.$$

$$\left.\left.\left.\frac{2\sqrt{b^2-4 a c}\;g}{2 c f-b g+\sqrt{b^2-4 a c}\;g}\right]\right)\Bigg/\left(e\sqrt{\frac{-\frac{-b+\sqrt{b^2-4 a c}}{2 c}+x}{\frac{-b-\sqrt{b^2-4 a c}}{2 c}-\frac{-b+\sqrt{b^2-4 a c}}{2 c}}}\sqrt{(f+g x)\,\left(a+b x+c x^2\right)}\right)-\left(2 d g^2\sqrt{\frac{-\frac{-b-\sqrt{b^2-4 a c}}{2 c}+x}{-\frac{-b-\sqrt{b^2-4 a c}}{2 c}+\frac{-b+\sqrt{b^2-4 a c}}{2 c}}}\right.$$

$$\left.\left(-\frac{-b+\sqrt{b^2-4 a c}}{2 c}+x\right)\sqrt{\frac{\frac{f}{g}+x}{\frac{-b+\sqrt{b^2-4 a c}}{2 c}+\frac{f}{g}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{-b+\sqrt{b^2-4 a c}-2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],\frac{2\sqrt{b^2-4 a c}\;g}{2 c f-b g+\sqrt{b^2-4 a c}\;g}\right]\right)\Bigg/$$

$$\left(e^2\sqrt{\frac{-\frac{-b+\sqrt{b^2-4 a c}}{2 c}+x}{\frac{-b-\sqrt{b^2-4 a c}}{2 c}-\frac{-b+\sqrt{b^2-4 a c}}{2 c}}}\sqrt{(f+g x)\,\left(a+b x+c x^2\right)}\right)+\left(2\sqrt{2}\;g^2\sqrt{-\frac{c g\left(-\frac{-b-\sqrt{b^2-4 a c}}{2 c}+x\right)}{2 c f-b g-\sqrt{b^2-4 a c}\;g}}\left(-\frac{-b+\sqrt{b^2-4 a c}}{2 c}+x\right)\right.$$

$$\sqrt{\frac{\frac{f}{g}+x}{\frac{-b+\sqrt{b^2-4 a c}}{2 c}+\frac{f}{g}}}\left(\frac{\left(2 c f-b g-\sqrt{b^2-4 a c}\;g\right)\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{2}\sqrt{\frac{c\,(f+g x)}{2 c f-b g+\sqrt{b^2-4 a c}\;g}}\right],\frac{2 c f-b g+\sqrt{b^2-4 a c}\;g}{2 c f-b g-\sqrt{b^2-4 a c}\;g}\right]}{2 c g}-\right.$$

$$\left.\left.\frac{\left(-b-\sqrt{b^2-4ac}\right)\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{2}\sqrt{\frac{c\,(f+g\,x)}{2\,c\,f-b\,g+\sqrt{b^2-4ac}\;g}}\right],\frac{2\,c\,f-b\,g+\sqrt{b^2-4ac}\;g}{2\,c\,f-b\,g-\sqrt{b^2-4ac}\;g}\right]}{2\,c}\right)\right)\Big/$$

$$\left(e\sqrt{\frac{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x}{-\frac{-b+\sqrt{b^2-4ac}}{2c}-\frac{f}{g}}}\sqrt{(f+g\,x)\left(a+b\,x+c\,x^2\right)}\right)+\left(2\left(-\frac{-b-\sqrt{b^2-4ac}}{2c}+\frac{-b+\sqrt{b^2-4ac}}{2c}\right)f^2\sqrt{-\frac{\left(-\frac{-b-\sqrt{b^2-4ac}}{2c}+x\right)\left(-\frac{-b+\sqrt{b^2-4ac}}{2c}+x\right)}{\left(-\frac{-b-\sqrt{b^2-4ac}}{2c}+\frac{-b+\sqrt{b^2-4ac}}{2c}\right)^2}}\right.$$

$$\left.\sqrt{\frac{\frac{f}{g}+x}{\frac{-b+\sqrt{b^2-4ac}}{2c}+\frac{f}{g}}}\;\text{EllipticPi}\left[\frac{2\sqrt{b^2-4ac}\;e}{2\,c\,d-b\,e+\sqrt{b^2-4ac}\;e},\text{ArcSin}\left[\frac{\sqrt{\frac{-b+\sqrt{b^2-4ac}-2\,c\,x}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],\frac{2\sqrt{b^2-4ac}\;g}{2\,c\,f-b\,g+\sqrt{b^2-4ac}\;g}\right]\right)\Big/$$

$$\left(\left(-d-\frac{\left(-b+\sqrt{b^2-4ac}\right)e}{2c}\right)\sqrt{(f+g\,x)\left(a+b\,x+c\,x^2\right)}\right)-$$

$$\left(4\left(-\frac{-b-\sqrt{b^2-4ac}}{2c}+\frac{-b+\sqrt{b^2-4ac}}{2c}\right)d\,f\,g\sqrt{-\frac{\left(-\frac{-b-\sqrt{b^2-4ac}}{2c}+x\right)\left(-\frac{-b+\sqrt{b^2-4ac}}{2c}+x\right)}{\left(-\frac{-b-\sqrt{b^2-4ac}}{2c}+\frac{-b+\sqrt{b^2-4ac}}{2c}\right)^2}}\sqrt{\frac{\frac{f}{g}+x}{\frac{-b+\sqrt{b^2-4ac}}{2c}+\frac{f}{g}}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{2\sqrt{b^2-4ac}\;e}{2\,c\,d-b\,e+\sqrt{b^2-4ac}\;e},\text{ArcSin}\left[\frac{\sqrt{\frac{-b+\sqrt{b^2-4ac}-2\,c\,x}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],\frac{2\sqrt{b^2-4ac}\;g}{2\,c\,f-b\,g+\sqrt{b^2-4ac}\;g}\right]\right)\Big/$$

$$\left(e\left(-d-\frac{\left(-b+\sqrt{b^2-4\,a\,c}\right)e}{2\,c}\right)\sqrt{(f+g\,x)\,\left(a+b\,x+c\,x^2\right)}\right)+$$

$$\left(2\left(-\frac{-b-\sqrt{b^2-4\,a\,c}}{2\,c}+\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c}\right)d^2\,g^2\,\sqrt{-\frac{\left(-\frac{-b-\sqrt{b^2-4\,a\,c}}{2\,c}+x\right)\left(-\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c}+x\right)}{\left(-\frac{-b-\sqrt{b^2-4\,a\,c}}{2\,c}+\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c}\right)^2}}\,\sqrt{\frac{\frac{f}{g}+x}{\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c}+\frac{f}{g}}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{2\sqrt{b^2-4\,a\,c}\;e}{2\,c\,d-b\,e+\sqrt{b^2-4\,a\,c}\;e},\,\text{ArcSin}\left[\frac{\sqrt{\frac{-b+\sqrt{b^2-4\,a\,c}-2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,\frac{2\sqrt{b^2-4\,a\,c}\;g}{2\,c\,f-b\,g+\sqrt{b^2-4\,a\,c}\;g}\right]\right)\Big/$$

$$\left(e^2\left(-d-\frac{\left(-b+\sqrt{b^2-4\,a\,c}\right)e}{2\,c}\right)\sqrt{(f+g\,x)\,\left(a+b\,x+c\,x^2\right)}\right)$$

- **Problem 908: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{(f+g\,x)^{5/2}}{(d+e\,x)\,\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 1138 leaves, 17 steps):

$$\frac{2\,g^2\,\sqrt{f+g\,x}\,\sqrt{a+b\,x+c\,x^2}}{3\,c\,e}+$$

$$\left(2\sqrt{2}\,\sqrt{b^2-4\,a\,c}\;g\,(2\,c\,f-b\,g)\,\sqrt{f+g\,x}\,\sqrt{-\frac{c\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\sqrt{b^2-4\,a\,c}\;g}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)g}\right]\right)\Big/$$

$$\left(3 c^2 e \sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}} \sqrt{a+b x+c x^2}\right)+$$

$$\left(\sqrt{2} \sqrt{b^2-4 a c}\ g (e f-d g) \sqrt{f+g x} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\ g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]\right)\Big/$$

$$\left(c e^2 \sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}} \sqrt{a+b x+c x^2}\right)+\left(2 \sqrt{2} \sqrt{b^2-4 a c}\ g (e f-d g)^2 \sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}}\right.$$

$$\left.\sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\ g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]\right)\Big/\left(c e^3 \sqrt{f+g x} \sqrt{a+b x+c x^2}\right)-$$

$$\left(2 \sqrt{2} \sqrt{b^2-4 a c}\ g \left(c f^2-b f g+a g^2\right) \sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\ g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]\right)\Big/\left(3 c^2 e \sqrt{f+g x} \sqrt{a+b x+c x^2}\right)-$$

$$\frac{1}{\sqrt{c}\, e^3 \sqrt{a+b x+c x^2}} \sqrt{2} \sqrt{2 c f-\left(b-\sqrt{b^2-4 a c}\right) g}\ (e f-d g)^2 \sqrt{1-\frac{2 c (f+g x)}{2 c f-\left(b-\sqrt{b^2-4 a c}\right) g}} \sqrt{1-\frac{2 c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}}$$

$$\text{EllipticPi}\left[\frac{e \left(2 c f-b g+\sqrt{b^2-4 a c}\ g\right)}{2 c (e f-d g)}, \text{ArcSin}\left[\frac{\sqrt{2} \sqrt{c} \sqrt{f+g x}}{\sqrt{2 c f-\left(b-\sqrt{b^2-4 a c}\right) g}}\right], \frac{b-\sqrt{b^2-4 a c}-\frac{2 c f}{g}}{b+\sqrt{b^2-4 a c}-\frac{2 c f}{g}}\right]$$

Result (type 4, 37 137 leaves) : Display of huge result suppressed !

- **Problem 909: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(d+e x)^3}{\sqrt{f+g x} \sqrt{a+b x+c x^2}} \, dx$$

Optimal (type 4, 631 leaves, 7 steps) :

$$-\frac{8 e^2 (c e f - 3 c d g + b e g) \sqrt{f + g x} \sqrt{a + b x + c x^2}}{15 c^2 g^2} + \frac{2 e^2 (d + e x) \sqrt{f + g x} \sqrt{a + b x + c x^2}}{5 c g} +$$

$$\left(\sqrt{2} \sqrt{b^2 - 4 a c}\; e \left(8 b^2 e^2 g^2 + c e g (7 b e f - 30 b d g - 9 a e g) + c^2 \left(8 e^2 f^2 - 30 d e f g + 45 d^2 g^2\right)\right) \sqrt{f + g x} \sqrt{-\frac{c \left(a + b x + c x^2\right)}{b^2 - 4 a c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x}{\sqrt{b^2 - 4 a c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2 - 4 a c}\; g}{2 c f - \left(b + \sqrt{b^2 - 4 a c}\right) g}\right]\right) \Big/ \left(15 c^3 g^3 \sqrt{\frac{c (f + g x)}{2 c f - \left(b + \sqrt{b^2 - 4 a c}\right) g}} \sqrt{a + b x + c x^2}\right) -$$

$$\left(2 \sqrt{2} \sqrt{b^2 - 4 a c} \left(4 b e^3 g^2 (b f - a g) + c^2 \left(8 e^3 f^3 - 30 d e^2 f^2 g + 45 d^2 e f g^2 - 15 d^3 g^3\right) - c e^2 g (a g (7 e f - 15 d g) - 3 b f (e f - 5 d g))\right)\right.$$

$$\sqrt{\frac{c (f + g x)}{2 c f - \left(b + \sqrt{b^2 - 4 a c}\right) g}} \sqrt{-\frac{c \left(a + b x + c x^2\right)}{b^2 - 4 a c}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x}{\sqrt{b^2 - 4 a c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2 - 4 a c}\; g}{2 c f - \left(b + \sqrt{b^2 - 4 a c}\right) g}\right]\right) \Big/ \left(15 c^3 g^3 \sqrt{f + g x} \sqrt{a + b x + c x^2}\right)$$

Result (type 4, 12 746 leaves) :

$$\frac{\left(-\frac{2 e^2 (4 c e f - 15 c d g + 4 b e g)}{15 c^2 g^2} + \frac{2 e^3 x}{5 c g}\right) \sqrt{f + g x} \left(a + b x + c x^2\right)}{\sqrt{a + x (b + c x)}} -$$

$$\frac{1}{15\,c^2\,g^4\,\sqrt{a+x\,(b+c\,x)}}\,2\,\sqrt{a+b\,x+c\,x^2}\,\Bigg(-\Big(e\,\big(8\,c^2\,e^2\,f^2-30\,c^2\,d\,e\,f\,g+7\,b\,c\,e^2\,f\,g+45\,c^2\,d^2\,g^2-30\,b\,c\,d\,e\,g^2+8\,b^2\,e^2\,g^2-9\,a\,c\,e^2\,g^2\big)$$

$$(f+g\,x)^{3/2}\left(c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}\right)\Big)\Bigg/\left(c\,\sqrt{\frac{(f+g\,x)^2\left(c\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}\right)+$$

$$\frac{1}{c\,\sqrt{\frac{(f+g\,x)^2\left(c\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}}\,(f+g\,x)\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}}$$

$$\Bigg(\Bigg(2\,i\,\sqrt{2}\,c^3\,e^3\,f^4\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}$$

$$\sqrt{1-\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\,\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\Bigg(15\,i\,c^3 d e^2 f^3 g\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(i\, b\, c^2 e^3 f^3 g \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\right)\right) \Big/$$

$$\left(2 \sqrt{2} \left(c f^2 - b f g + a g^2\right) \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}} \sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right) +$$

$$\left(45\, i\, c^3 d^2 e f^2 g^2 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\,\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\Bigg(i\,b^2 c\,e^3 f^2 g^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\,\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(i\, a\, c^2\, e^3\, f^2\, g^2 \left(2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right) \sqrt{1 - \frac{2\,(c\, f^2 - b\, f\, g + a\, g^2)}{\left(2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right)(f + g\, x)}}\right.$$

$$\sqrt{1 - \frac{2\,(c\, f^2 - b\, f\, g + a\, g^2)}{\left(2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right)(f + g\, x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\, \sqrt{-\frac{c\, f^2 - b\, f\, g + a\, g^2}{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}}}{\sqrt{f + g\, x}}\right], \frac{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}{2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\, \sqrt{-\frac{c\, f^2 - b\, f\, g + a\, g^2}{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}}}{\sqrt{f + g\, x}}\right], \frac{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}{2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}\right]\right)\right) \Big/$$

$$\left(2\, \sqrt{2}\, (c\, f^2 - b\, f\, g + a\, g^2) \sqrt{-\frac{c\, f^2 - b\, f\, g + a\, g^2}{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}} \sqrt{c + \frac{c\, f^2 - b\, f\, g + a\, g^2}{(f + g\, x)^2} + \frac{-2\, c\, f + b\, g}{f + g\, x}}\right) -$$

$$\left(45\, i\, b\, c^2\, d^2\, e\, f\, g^3 \left(2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right) \sqrt{1 - \frac{2\,(c\, f^2 - b\, f\, g + a\, g^2)}{\left(2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right)(f + g\, x)}}\right.$$

$$\sqrt{1 - \frac{2\,(c\, f^2 - b\, f\, g + a\, g^2)}{\left(2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right)(f + g\, x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\, \sqrt{-\frac{c\, f^2 - b\, f\, g + a\, g^2}{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}}}{\sqrt{f + g\, x}}\right], \frac{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}{2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}\right] -\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\,\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\Bigg(15\,i\,b^2 c d e^2 f g^3\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(15\, i\, a\, c^2\, d\, e^2\, f\, g^3 \left(2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right) \sqrt{1 - \frac{2 \left(c\, f^2 - b\, f\, g + a\, g^2\right)}{\left(2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right) (f + g\, x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c\, f^2 - b\, f\, g + a\, g^2\right)}{\left(2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right) (f + g\, x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c\, f^2 - b\, f\, g + a\, g^2}{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}}}{\sqrt{f + g\, x}}\right], \frac{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}{2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c\, f^2 - b\, f\, g + a\, g^2}{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}}}{\sqrt{f + g\, x}}\right], \frac{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}{2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2} \left(c\, f^2 - b\, f\, g + a\, g^2\right) \sqrt{-\frac{c\, f^2 - b\, f\, g + a\, g^2}{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}} \sqrt{c + \frac{c\, f^2 - b\, f\, g + a\, g^2}{(f + g\, x)^2} + \frac{-2\, c\, f + b\, g}{f + g\, x}}\right) -$$

$$\left(2\, i \sqrt{2}\, b^3\, e^3\, f\, g^3 \left(2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right) \sqrt{1 - \frac{2 \left(c\, f^2 - b\, f\, g + a\, g^2\right)}{\left(2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right) (f + g\, x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c\, f^2 - b\, f\, g + a\, g^2\right)}{\left(2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}\right) (f + g\, x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c\, f^2 - b\, f\, g + a\, g^2}{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}}}{\sqrt{f + g\, x}}\right], \frac{2\, c\, f - b\, g - \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}{2\, c\, f - b\, g + \sqrt{b^2\, g^2 - 4\, a\, c\, g^2}}\right] - \right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(4\,\mathbb{i}\sqrt{2}\,a b c e^3 f g^3\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(45\,i\,a\,c^2\,d^2\,e\,g^4\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\right)\Bigg/$$

$$\left(2\sqrt{2}\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)-$$

$$\left(15\,i\,a\,b\,c\,d\,e^2\,g^4\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\Bigg(2\,i\sqrt{2}\,a b^2 e^3 g^4\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(9\, i\, a^2 c\, e^3 g^4 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\right)\right) \Big/$$

$$\left(2 \sqrt{2} \left(c f^2 - b f g + a g^2\right) \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}} \sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right) +$$

$$\left(4\, i \sqrt{2}\, c^3 e^3 f^3 \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(15\, i \sqrt{2}\, c^3 d e^2 f^2 g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(3\, i\, b c^2 e^3 f^2 g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(45\, i\, c^3 d^2 e f g^2 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\; \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\; \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right) -$$

$$\left(15\, i\, b\, c^2 d\, e^2 f g^2 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\; \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\; \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right) +$$

$$\left(2\,i\,\sqrt{2}\,b^2\,c\,e^3\,f\,g^2\,\sqrt{1-\frac{2\,(c\,f^2-b\,f\,g+a\,g^2)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\,\sqrt{1-\frac{2\,(c\,f^2-b\,f\,g+a\,g^2)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)-$$

$$\left(7\,i\,a\,c^2\,e^3\,f\,g^2\,\sqrt{1-\frac{2\,(c\,f^2-b\,f\,g+a\,g^2)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\,\sqrt{1-\frac{2\,(c\,f^2-b\,f\,g+a\,g^2)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\,(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)-$$

$$\left(15\,i\,c^3 d^3 g^3 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(15\,i\,a\,c^2 d\,e^2 g^3 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(2\,i\,\sqrt{2}\,a\,b\,c\,e^3\,g^3\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\bigg/$$

$$\left(\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)\Bigg)$$

- **Problem 910: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{(d+e\,x)^2}{\sqrt{f+g\,x}\,\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 479 leaves, 7 steps) :

$$\frac{2 e^2 \sqrt{f+g x} \sqrt{a+b x+c x^2}}{3 c g}-$$

$$\left(2 \sqrt{2} \sqrt{b^2-4 a c}\, e\, (c e f-3 c d g+b e g) \sqrt{f+g x} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]\right) \Big/$$

$$\left(3 c^2 g^2 \sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}} \sqrt{a+b x+c x^2}\right)+$$

$$\left(2 \sqrt{2} \sqrt{b^2-4 a c} \left(e^2 g (b f-a g)+c \left(2 e^2 f^2-6 d e f g+3 d^2 g^2\right)\right) \sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c}\, g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]\right) \Big/ \left(3 c^2 g^2 \sqrt{f+g x} \sqrt{a+b x+c x^2}\right)$$

Result (type 4, 6194 leaves) :

$$\frac{2 e^2 \sqrt{f+g x} \left(a+b x+c x^2\right)}{3 c g \sqrt{a+x (b+c x)}}+$$

$$\frac{1}{3\,c\,g^3\,\sqrt{a+x\,(b+c\,x)}}\,2\,\sqrt{a+b\,x+c\,x^2}\,\left(-\frac{2\,e\,(c\,e\,f-3\,c\,d\,g+b\,e\,g)\,(f+g\,x)^{3/2}\left(c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}\right)}{c\,\sqrt{\frac{(f+g\,x)^2\left(c\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}}+\right.$$

$$\frac{1}{c\,\sqrt{\frac{(f+g\,x)^2\left(c\left(-1+\frac{f}{f+g\,x}\right)^2+\frac{g\left(b-\frac{b\,f}{f+g\,x}+\frac{a\,g}{f+g\,x}\right)}{f+g\,x}\right)}{g^2}}}\,(f+g\,x)\,\sqrt{c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}}$$

$$\left(\left(\mathbb{i}\,c^2\,e^2\,f^3\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\left.\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\right)\right/$$

$$\left(\sqrt{2}\,\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)-$$

$$\left(3\,i\,c^2\,d\,e\,f^2\,g\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\right)\Bigg/$$

$$\left(\sqrt{2}\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(3\,i\,b\,c\,d\,e\,f\,g^2\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\Bigg(i\,b^2 e^2 f g^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(i\, a\, c\, e^2 f\, g^2 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2} \left(c f^2 - b f g + a g^2\right) \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}} \sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right) -$$

$$\left(3\, i\, a\, c\, d\, e\, g^3 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) (f + g x)}} \left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\right], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(\text{i}\,a\,b\,e^2\,g^3\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\,\sqrt{1-\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{\left(2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}\right)(f+g\,x)}}\right.$$

$$\left(\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}}{\sqrt{f+g\,x}}\right],\,\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,f^2-b\,f\,g+a\,g^2\right)\sqrt{-\frac{c\,f^2-b\,f\,g+a\,g^2}{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}}\,\sqrt{c+\frac{c\,f^2-b\,f\,g+a\,g^2}{(f+g\,x)^2}+\frac{-2\,c\,f+b\,g}{f+g\,x}}\right)+$$

$$\left(i\sqrt{2}\,c^2 e^2 f^2 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(3\,i\sqrt{2}\,c^2 d e f g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(\mathrm{i}\, b\, c\, e^2 f\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(3\, \mathrm{i}\, c^2 d^2 g^2 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(i\, a\, c\, e^2 g^2 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\ \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)\right)$$

- **Problem 911: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{d+e x}{\sqrt{f+g x}\ \sqrt{a+b x+c x^2}}\, dx$$

Optimal (type 4, 393 leaves, 5 steps) :

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\,e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right]}{cg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}-$$

$$\left(2\sqrt{2}\sqrt{b^2-4ac}\,(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right]\right)\Big/\left(cg\sqrt{f+gx}\sqrt{a+bx+cx^2}\right)$$

Result (type 4, 2732 leaves) :

$$-\frac{1}{g^2\sqrt{a+x(b+cx)}}2\sqrt{a+bx+cx^2}\left(-\frac{e(f+gx)^{3/2}\left(c+\frac{cf^2}{(f+gx)^2}-\frac{bfg}{(f+gx)^2}+\frac{ag^2}{(f+gx)^2}-\frac{2cf}{f+gx}+\frac{bg}{f+gx}\right)}{c\sqrt{\frac{(f+gx)^2\left(c\left(-1+\frac{f}{f+gx}\right)^2+\frac{g\left(b-\frac{bf}{f+gx}+\frac{ag}{f+gx}\right)}{f+gx}\right)}{g^2}}}+\right.$$

$$\frac{1}{c\sqrt{\frac{(f+gx)^2\left(c\left(-1+\frac{f}{f+gx}\right)^2+\frac{g\left(b-\frac{bf}{f+gx}+\frac{ag}{f+gx}\right)}{f+gx}\right)}{g^2}}}(f+gx)\sqrt{c+\frac{cf^2}{(f+gx)^2}-\frac{bfg}{(f+gx)^2}+\frac{ag^2}{(f+gx)^2}-\frac{2cf}{f+gx}+\frac{bg}{f+gx}}$$

$$\Bigg(\Bigg(i\, c\, e\, f^2 \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}} \sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}}$$

$$\Bigg(\text{EllipticE}\Big[i\, \text{ArcSinh}\Big[\frac{\sqrt{2}\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\Big], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\Big] -$$

$$\text{EllipticF}\Big[i\, \text{ArcSinh}\Big[\frac{\sqrt{2}\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\Big], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\Big]\Bigg)\Bigg) \Big/$$

$$\left(2\sqrt{2}\left(c f^2 - b f g + a g^2\right)\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}\sqrt{c + \frac{c f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{f + g x}}\right) -$$

$$\Bigg(i\, b\, e\, f\, g \left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right) \sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}} \sqrt{1 - \frac{2\left(c f^2 - b f g + a g^2\right)}{\left(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}\right)(f + g x)}}$$

$$\Bigg(\text{EllipticE}\Big[i\, \text{ArcSinh}\Big[\frac{\sqrt{2}\sqrt{-\frac{c f^2 - b f g + a g^2}{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}}}{\sqrt{f + g x}}\Big], \frac{2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}}{2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}}\Big] -$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\Bigg(i\,a\,e\,g^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(\mathbb{i}\, c\, e\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right) -$$

$$\left(\mathbb{i}\, c\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left.\left.\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)\right)\right)$$

Problem 912: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{f+g x}\,\sqrt{a+b x+c x^2}}\,dx$$

Optimal (type 4, 189 leaves, 2 steps):

$$\frac{1}{c\sqrt{f+g x}\,\sqrt{a+b x+c x^2}} 2\sqrt{2}\,\sqrt{b^2-4 a c}\,\sqrt{\frac{c\,(f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}}\,\sqrt{-\frac{c\,\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]$$

Result (type 4, 308 leaves):

$$\left(i\,(f+g x)\,\sqrt{2-\frac{4\left(c f^2+g\,(-b f+a g)\right)}{\left(2 c f-b g+\sqrt{\left(b^2-4 a c\right) g^2}\right)(f+g x)}}\,\sqrt{1+\frac{2\left(c f^2+g\,(-b f+a g)\right)}{\left(-2 c f+b g+\sqrt{\left(b^2-4 a c\right) g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{\frac{c f^2-b f g+a g^2}{-2 c f+b g+\sqrt{\left(b^2-4 a c\right) g^2}}}}{\sqrt{f+g x}}\right],-\frac{-2 c f+b g+\sqrt{\left(b^2-4 a c\right) g^2}}{2 c f-b g+\sqrt{\left(b^2-4 a c\right) g^2}}\right]\right)\Big/\left(g\sqrt{\frac{c f^2+g\,(-b f+a g)}{-2 c f+b g+\sqrt{\left(b^2-4 a c\right) g^2}}}\,\sqrt{a+x\,(b+c x)}\right)$$

- **Problem 913: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(d+e x)\sqrt{f+g x}\,\sqrt{a+b x+c x^2}}\,dx$$

Optimal (type 4, 280 leaves, 5 steps):

$$-\left(\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{e\left(2cf-bg+\sqrt{b^2-4ac}\,g\right)}{2c(ef-dg)},\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{c}\sqrt{f+gx}}{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\right],\frac{b-\sqrt{b^2-4ac}-\frac{2cf}{g}}{b+\sqrt{b^2-4ac}-\frac{2cf}{g}}\right]\right)\Big/\left(\sqrt{c}\,(ef-dg)\sqrt{a+bx+cx^2}\right)$$

Result (type 4, 499 leaves) :

$$\left(i(f+gx)\sqrt{2-\frac{4\left(cf^2+g(-bf+ag)\right)}{\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\sqrt{1+\frac{2\left(cf^2+g(-bf+ag)\right)}{\left(-2cf+bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\right.$$

$$\left(\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{cf^2-bfg+ag^2}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}}{\sqrt{f+gx}}\right],-\frac{-2cf+bg+\sqrt{(b^2-4ac)g^2}}{2cf-bg+\sqrt{(b^2-4ac)g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticPi}\left[\frac{(ef-dg)\left(2cf-bg-\sqrt{(b^2-4ac)g^2}\right)}{2e\left(cf^2+g(-bf+ag)\right)},i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{cf^2-bfg+ag^2}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}}{\sqrt{f+gx}}\right],-\frac{-2cf+bg+\sqrt{(b^2-4ac)g^2}}{2cf-bg+\sqrt{(b^2-4ac)g^2}}\right]\right)\right)\Big/$$

$$\left((-ef+dg)\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}\sqrt{a+x(b+cx)}\right)$$

- **Problem 914: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

Optimal (type 4, 1037 leaves, 15 steps) :

$$-\frac{e^2\sqrt{f+g x}\,\sqrt{a+b x+c x^2}}{\left(c d^2-b d e+a e^2\right)(e f-d g)(d+e x)}+\frac{\sqrt{b^2-4 a c}\;e\sqrt{f+g x}\,\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\;g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}\right]}{\sqrt{2}\left(c d^2-b d e+a e^2\right)(e f-d g)\sqrt{\frac{c(f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}}\,\sqrt{a+b x+c x^2}}-$$

$$\left(\sqrt{2}\sqrt{b^2-4 a c}\;e f\sqrt{\frac{c(f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}}\,\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\;g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}\right]\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)(e f-d g)\sqrt{f+g x}\,\sqrt{a+b x+c x^2}\right)+$$

$$\left(\sqrt{2}\sqrt{b^2-4 a c}\;d g\sqrt{\frac{c(f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}}\,\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\;g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}\right]\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)(e f-d g)\sqrt{f+g x}\,\sqrt{a+b x+c x^2}\right)-$$

$$\left(\sqrt{2 c f-\left(b-\sqrt{b^2-4 a c}\right)g}\;\left(c d(2 e f-3 d g)-e(b e f-2 b d g+a e g)\right)\sqrt{1-\frac{2 c(f+g x)}{2 c f-\left(b-\sqrt{b^2-4 a c}\right)g}}\,\sqrt{1-\frac{2 c(f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{e\left(2 c f-b g+\sqrt{b^2-4 a c}\;g\right)}{2 c(e f-d g)},\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{c}\sqrt{f+g x}}{\sqrt{2 c f-\left(b-\sqrt{b^2-4 a c}\right)g}}\right],\frac{b-\sqrt{b^2-4 a c}-\frac{2 c f}{g}}{b+\sqrt{b^2-4 a c}-\frac{2 c f}{g}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{c}\left(c d^2-b d e+a e^2\right)(e f-d g)^2\sqrt{a+b x+c x^2}\right)$$

Result (type 4, 10 881 leaves) :

$$-\frac{e^2\sqrt{f+g x}\,\left(a+b x+c x^2\right)}{\left(c d^2-b d e+a e^2\right)(e f-d g)(d+e x)\sqrt{a+x(b+c x)}}+$$

$$\frac{1}{2\left(c d^2-b d e+a e^2\right) g(-e f+d g)\sqrt{a+x(b+c x)}}\sqrt{a+b x+c x^2}\left(-\frac{2 e (f+g x)^{3/2}\left(c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}\right)}{\sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}}-\right.$$

$$\frac{1}{\sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}}\,2(e f-d g)(f+g x)\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}$$

$$\left(\left(-\mathrm{i}\, c e f^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\right.\right.$$

$$\left.\left.\left.\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,(e f-d g)\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(\mathrm{i}\, b e f g\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,(e f-d g)\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(\mathrm{i}\, a e g^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,(e f-d g)\,\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(i\,c\,e^2 f^2\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,(e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(i\, b\, e^2 f g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\, \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2}\,(e f-d g)^2 \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\, \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right) +$$

$$\left(i\, a\, e^2 g^2 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\, \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2}\,(e f-d g)^2 \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\, \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right) -$$

$$\left(i\sqrt{2}\, c\, e\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\, \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left((e f-d g)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\, \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right) -$$

$$\left(i\sqrt{2}\, c\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\, \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left((e f-d g)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\, \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right) +$$

$$\left(i\sqrt{2}\,b\,e\,g\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left((e f-d g)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\frac{1}{(e f-d g)^3}c\,e^3 f^2\left(\left(i f\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\text{EllipticPi}\left[\right.\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\left(i\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left.\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right)+$$

$$\frac{1}{(e f-d g)^3}\, b\, e^3 f g \left(\left(i\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\left(i\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg) -$$

$$\frac{1}{(e f-d g)^3} a e^3 g^2 \left(\left(\left(i f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \text{EllipticPi}\left[\right.\right.\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right) -$$

$$\left(i\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg)+$$

$$\frac{1}{(e f-d g)^2}\, 2 c e^2 f \left(\left(i\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\left(i\,d\,g\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\;\text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,e\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right)+$$

$$\frac{1}{(e f-d g)^2}\,2 c d e g\left(\left(\left(i\,f\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\;\text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/\right.\right.$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\left(i\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right)-$$

$$\frac{1}{(e f-d g)^2} 2 b e^2 g \left(\left(i\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\Bigg(i\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\Big[$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ i\,\text{ArcSinh}\Big[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\Big],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\Big]\Bigg)\Big/$$

$$\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg) -$$

$$\frac{1}{e f-d g}\, c\, e\Bigg(\Bigg(i\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\Big[$$

$$\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ i\,\text{ArcSinh}\Big[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\Big],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\Big]\Bigg)\Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right) -$$

$$\left(\mathrm{i}\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ \mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Bigg/ \left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg)\Bigg)\Bigg)$$

Problem 915: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(d+e x)^3 \sqrt{f+g x} \sqrt{a+b x+c x^2}}\, dx$$

Optimal (type 4, 1114 leaves, 25 steps):

$$-\frac{e^2 \sqrt{f+g x} \sqrt{a+b x+c x^2}}{2\left(c d^2-b d e+a e^2\right)(e f-d g)(d+e x)^2}-\frac{3 e^2 (c d (2 e f-3 d g)-e (b e f-2 b d g+a e g)) \sqrt{f+g x} \sqrt{a+b x+c x^2}}{4\left(c d^2-b d e+a e^2\right)^2 (e f-d g)^2 (d+e x)}+$$

$$\left(3 \sqrt{b^2-4 a c}\ e\ (c d (2 e f-3 d g)-e (b e f-2 b d g+a e g)) \sqrt{f+g x}\right.$$

$$\sqrt{-\frac{c\,(a+b\,x+c\,x^2)}{b^2-4\,a\,c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\;g}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)g}\right]\Bigg)\Bigg/$$

$$\left(4\,\sqrt{2}\;\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,(e\,f-d\,g)^2\,\sqrt{\frac{c\,(f+g\,x)}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)g}}\;\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\Bigg(\sqrt{b^2-4\,a\,c}\;\left(c\,d\,(-6\,e\,f+7\,d\,g)+e\,(3\,b\,e\,f-4\,b\,d\,g+a\,e\,g)\right)\,\sqrt{\frac{c\,(f+g\,x)}{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)g}}$$

$$\sqrt{\frac{c\,(a+x\,(b+c\,x))}{-b^2+4\,a\,c}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,\frac{2\,\sqrt{b^2-4\,a\,c}\;g}{-2\,c\,f+\left(b+\sqrt{b^2-4\,a\,c}\right)g}\right]\Bigg)\Bigg/$$

$$\left(2\,\sqrt{2}\;\left(c\,d^2+e\,(-b\,d+a\,e)\right)^2\,(e\,f-d\,g)\,\sqrt{f+g\,x}\;\sqrt{a+x\,(b+c\,x)}\right)+\frac{1}{4\,\sqrt{2}\;\sqrt{c}\;\left(c\,d^2+e\,(-b\,d+a\,e)\right)^2\,(-e\,f+d\,g)^3\,\sqrt{a+x\,(b+c\,x)}}$$

$$\sqrt{2\,c\,f-b\,g+\sqrt{b^2-4\,a\,c}\;g}\;\left(c^2\,d^2\left(8\,e^2\,f^2-20\,d\,e\,f\,g+15\,d^2\,g^2\right)+2\,c\,e\left(b\,d\left(-4\,e^2\,f^2+11\,d\,e\,f\,g-10\,d^2\,g^2\right)+a\,e\left(-2\,e^2\,f^2+2\,d\,e\,f\,g+3\,d^2\,g^2\right)\right)+\right.$$

$$\left.e^2\left(3\,a^2\,e^2\,g^2+2\,a\,b\,e\,g\,(e\,f-4\,d\,g)+b^2\left(3\,e^2\,f^2-8\,d\,e\,f\,g+8\,d^2\,g^2\right)\right)\right)\sqrt{\frac{g\left(-b+\sqrt{b^2-4\,a\,c}-2\,c\,x\right)}{2\,c\,f+\left(-b+\sqrt{b^2-4\,a\,c}\right)g}}\;\sqrt{\frac{g\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}{-2\,c\,f+\left(b+\sqrt{b^2-4\,a\,c}\right)g}}$$

$$\text{EllipticPi}\left[\frac{2 c e f-b e g+\sqrt{b^2-4 a c}\; e g}{2 c e f-2 c d g},\ \text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{f+g x}}{\sqrt{2 c f-b g+\sqrt{b^2-4 a c}\; g}}\right],\ \frac{2 c f+\left(-b+\sqrt{b^2-4 a c}\right) g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]$$

Result (type 4, 40 396 leaves) : Display of huge result suppressed !

■ **Problem 916: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(d+e x)\,(f+g x)^{3/2}\sqrt{a+b x+c x^2}}\,dx$$

Optimal (type 4, 553 leaves, 11 steps) :

$$\frac{2 g^2\sqrt{a+b x+c x^2}}{(e f-d g)\left(c f^2-b f g+a g^2\right)\sqrt{f+g x}}-\frac{\sqrt{2}\,\sqrt{b^2-4 a c}\; g\sqrt{f+g x}\,\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\; g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}\right]}{(e f-d g)\left(c f^2-b f g+a g^2\right)\sqrt{\frac{c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}}\;\sqrt{a+b x+c x^2}}-$$

$$\left(\sqrt{2}\; e\sqrt{2 c f-\left(b-\sqrt{b^2-4 a c}\right) g}\;\sqrt{1-\frac{2 c (f+g x)}{2 c f-\left(b-\sqrt{b^2-4 a c}\right) g}}\;\sqrt{1-\frac{2 c (f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right) g}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{e\left(2 c f-b g+\sqrt{b^2-4 a c}\; g\right)}{2 c (e f-d g)},\ \text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{f+g x}}{\sqrt{2 c f-\left(b-\sqrt{b^2-4 a c}\right) g}}\right],\ \frac{b-\sqrt{b^2-4 a c}-\frac{2 c f}{g}}{b+\sqrt{b^2-4 a c}-\frac{2 c f}{g}}\right]\right)\Big/\left(\sqrt{c}\,(e f-d g)^2\sqrt{a+b x+c x^2}\right)$$

Result (type 4, 1061 leaves) :

$$\frac{2 g^2\left(a+b x+c x^2\right)}{(e f-d g)\left(c f^2-b f g+a g^2\right)\sqrt{f+g x}\;\sqrt{a+x (b+c x)}}+$$

$$\left(2\,(f+g x)^{3/2}\sqrt{a+b x+c x^2}\left(c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}-\frac{1}{4\,(e f-d g)\sqrt{\frac{c f^2+g\,(-b f+a g)}{-2 c f+b g+\sqrt{(b^2-4 a c)\,g^2}}}\sqrt{f+g x}}\right.\right.$$

$$i\sqrt{1-\frac{2\left(c f^2+g\,(-b f+a g)\right)}{\left(2 c f-b g+\sqrt{(b^2-4 a c)\,g^2}\right)(f+g x)}}\sqrt{2+\frac{4\left(c f^2+g\,(-b f+a g)\right)}{\left(-2 c f+b g+\sqrt{(b^2-4 a c)\,g^2}\right)(f+g x)}}$$

$$\left((e f-d g)\left(2 c f-b g+\sqrt{(b^2-4 a c)\,g^2}\right)\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c f^2-b f g+a g^2}{-2 c f+b g+\sqrt{(b^2-4 a c)\,g^2}}}}{\sqrt{f+g x}}\right],-\frac{-2 c f+b g+\sqrt{(b^2-4 a c)\,g^2}}{2 c f-b g+\sqrt{(b^2-4 a c)\,g^2}}\right]-\right.\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c f^2-b f g+a g^2}{-2 c f+b g+\sqrt{(b^2-4 a c)\,g^2}}}}{\sqrt{f+g x}}\right],-\frac{-2 c f+b g+\sqrt{(b^2-4 a c)\,g^2}}{2 c f-b g+\sqrt{(b^2-4 a c)\,g^2}}\right]\right)-2 e\left(c f^2+g\,(-b f+a g)\right)$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c f^2-b f g+a g^2}{-2 c f+b g+\sqrt{(b^2-4 a c)\,g^2}}}}{\sqrt{f+g x}}\right],-\frac{-2 c f+b g+\sqrt{(b^2-4 a c)\,g^2}}{2 c f-b g+\sqrt{(b^2-4 a c)\,g^2}}\right]+2 e\left(c f^2+g\,(-b f+a g)\right)\text{EllipticPi}\Big[$$

$$\left.\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{(b^2-4 a c)\,g^2}\right)}{2 e\left(c f^2+g\,(-b f+a g)\right)},\,i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{\frac{c f^2-b f g+a g^2}{-2 c f+b g+\sqrt{(b^2-4 a c)\,g^2}}}}{\sqrt{f+g x}}\right],-\frac{-2 c f+b g+\sqrt{(b^2-4 a c)\,g^2}}{2 c f-b g+\sqrt{(b^2-4 a c)\,g^2}}\Big]\right)\right)\right)\Big/$$

$$\left((-e f+d g)\left(c f^2-b f g+a g^2\right)\sqrt{a+x\,(b+c x)}\,\sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}\right)$$

- **Problem 917: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(d+e x)\,(f+g x)^{5/2}\sqrt{a+b x+c x^2}}\,dx$$

Optimal (type 4, 1125 leaves, 18 steps) :

$$\frac{2 g^2\sqrt{a+b x+c x^2}}{3\,(e f-d g)\left(c f^2-b f g+a g^2\right)(f+g x)^{3/2}}+\frac{4 g^2\,(2 c f-b g)\sqrt{a+b x+c x^2}}{3\,(e f-d g)\left(c f^2-b f g+a g^2\right)^2\sqrt{f+g x}}+\frac{2 e g^2\sqrt{a+b x+c x^2}}{(e f-d g)^2\left(c f^2-b f g+a g^2\right)\sqrt{f+g x}}-$$

$$\left(2\sqrt{2}\sqrt{b^2-4 a c}\;g\,(2 c f-b g)\sqrt{f+g x}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\;g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}\right]\right)\Big/$$

$$\left(3\,(e f-d g)\left(c f^2-b f g+a g^2\right)^2\sqrt{\frac{c\,(f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}}\sqrt{a+b x+c x^2}\right)-$$

$$\frac{\sqrt{2}\sqrt{b^2-4 a c}\;e g\sqrt{f+g x}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\;g}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}\right]}{(e f-d g)^2\left(c f^2-b f g+a g^2\right)\sqrt{\frac{c\,(f+g x)}{2 c f-\left(b+\sqrt{b^2-4 a c}\right)g}}\sqrt{a+b x+c x^2}}+$$

$$\left(2\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\text{ EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right]\right)\Big/$$

$$\left(3(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}\sqrt{a+bx+cx^2}\right)-$$

$$\left(\sqrt{2}\,e^2\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{e\left(2cf-bg+\sqrt{b^2-4ac}\,g\right)}{2c(ef-dg)},\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{c}\sqrt{f+gx}}{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\right],\frac{b-\sqrt{b^2-4ac}-\frac{2cf}{g}}{b+\sqrt{b^2-4ac}-\frac{2cf}{g}}\right]\right)\Big/\left(\sqrt{c}\,(ef-dg)^3\sqrt{a+bx+cx^2}\right)$$

Result (type 4, 14 762 leaves) :

$$\frac{\sqrt{f+gx}\,(a+bx+cx^2)\left(\frac{2g^2}{3(ef-dg)(cf^2-bfg+ag^2)(f+gx)^2}+\frac{2g^2(7cef^2-4cdfg-5befg+2bdg^2+3aeg^2)}{3(ef-dg)^2(cf^2-bfg+ag^2)^2(f+gx)}\right)}{\sqrt{a+x(b+cx)}}+\frac{1}{3(-ef+dg)^2\left(cf^2-bfg+ag^2\right)^2\sqrt{a+x(b+cx)}}$$

$$2\sqrt{a+bx+cx^2}\left(\left(\left(-7cef^2+4cdfg+5befg-2bdg^2-3aeg^2\right)(f+gx)^{3/2}\left(c+\frac{cf^2}{(f+gx)^2}-\frac{bfg}{(f+gx)^2}+\frac{ag^2}{(f+gx)^2}-\frac{2cf}{f+gx}+\frac{bg}{f+gx}\right)\right)\Big/\right.$$

$$\left(\sqrt{\frac{(f+gx)^2\left(c\left(-1+\frac{f}{f+gx}\right)^2+\frac{g\left(b-\frac{bf}{f+gx}+\frac{ag}{f+gx}\right)}{f+gx}\right)}{g^2}}\right)-$$

$$\frac{1}{\sqrt{\frac{(f+g x)^2 \left(c \left(-1+\frac{f}{f+g x}\right)^2+\frac{g \left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}} (e f-d g) \left(c f^2-b f g+a g^2\right) (f+g x) \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}$$

$$\left(-\left(7 i c e f^2 \left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right) \sqrt{1-\frac{2 \left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right) (f+g x)}}\right.\right.$$

$$\sqrt{1-\frac{2 \left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right) (f+g x)}} \left(\text{EllipticE}\left[i \, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\right.\right.$$

$$\left.\left.\left.\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\text{EllipticF}\left[i \, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right) \Big/$$

$$\left(2 \sqrt{2} (e f-d g) \left(c f^2-b f g+a g^2\right) \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(i \sqrt{2} c d f g \left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right) \sqrt{1-\frac{2 \left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right) (f+g x)}}\right.$$

$$\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left((e f-d g)\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(5\,i\,b e f g\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,(e f-d g)\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(i\, b\, d\, g^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,(e f-d g)\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(3\, i\, a\, e\, g^2\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,(e f-d g)\left(c f^2-b f g+a g^2\right)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(7\,\mathrm{i}\,c\,e^2 f^2\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,(e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(2\,i\sqrt{2}\,c\,d\,e\,f\,g\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Bigg/$$

$$\left((e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(5\,i\,b\,e^2 f\,g\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{2}\,(e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(i\sqrt{2}\, b\, d\, e\, g^2 \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left((e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(3 i\, a\, e^2 g^2\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,(e f-d g)^2\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\left(5\,\mathbb{i}\,c\,e\,f\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,(e f-d g)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(\mathbb{i}\,c\,d\,g\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,(e f-d g)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)+$$

$$\left(i\sqrt{2}\, b\, e\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left((e f-d g)\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}}\right)-$$

$$\frac{1}{(e f-d g)^3}\, 7\, c\, e^3 f^2 \left(\left(i\, f\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\left(i\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.$$

$$\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left.\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right)+$$

$$\frac{1}{(e f-d g)^3} 4 c d e^2 f g \left(\left(i\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.\right.$$

$$\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\left(\mathrm{i}\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ \mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg) +$$

$$\frac{1}{(e f-d g)^3} 5 b e^3 f g \left(\left(\mathrm{i}\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ \mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right) -$$

$$\left(i\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right) -$$

$$\frac{1}{(e f-d g)^3}\, 2 b d e^2 g^2 \left(\left(\left(i\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right) -$$

$$\left(\left(i\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.\right.$$

$$\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left.\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right)-$$

$$\frac{1}{(e f-d g)^3} 3 a e^3 g^2\left(\left(\left(i\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.\right.\right.$$

$$\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\left(\mathrm{i}\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left.\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right) +$$

$$\frac{1}{(e f-d g)^2} 5 c e^2 f \left(\left(\mathrm{i}\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right) -$$

$$\left(i\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Bigg/$$

$$\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right) -$$

$$\frac{1}{(e f-d g)^2} c\, d\, e\, g \left(\left(\left(i\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Bigg/\right.\right.$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right) -$$

$$\left(\left(i\,d\,g\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\text{EllipticPi}\left[\right.\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left.\left(\sqrt{2}\,e\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right)-$$

$$\frac{1}{(e f-d g)^2}\,2\,b\,e^2\,g\left(\left(i\,f\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\,\text{EllipticPi}\left[\right.\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)},\ i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right],\ \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}\,\sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)-$$

$$\left(\mathrm{i}\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left.\left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\right) -$$

$$\frac{1}{e f-d g} c\, e \left(\left(\mathrm{i}\, f \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\right.\right.\right.$$

$$\left.\left.\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, \mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right) -$$

$$\left(i\, d\, g \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}} \sqrt{1-\frac{2\left(c f^2-b f g+a g^2\right)}{\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)(f+g x)}}\ \text{EllipticPi}\left[\frac{(e f-d g)\left(2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}\right)}{2 e\left(c f^2-b f g+a g^2\right)}, i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}}}{\sqrt{f+g x}}\right], \frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}\right]\right) \Bigg/ \left(\sqrt{2}\, e \sqrt{-\frac{c f^2-b f g+a g^2}{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}} \sqrt{c+\frac{c f^2}{(f+g x)^2}-\frac{b f g}{(f+g x)^2}+\frac{a g^2}{(f+g x)^2}-\frac{2 c f}{f+g x}+\frac{b g}{f+g x}}\right)\Bigg)\Bigg)\Bigg)$$

- **Problem 918: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{d+e x}}{\sqrt{f+g x}\ \sqrt{a+b x+c x^2}}\, dx$$

Optimal (type 4, 475 leaves, 1 step) :

$$\left(\sqrt{2}\,\sqrt{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)g}\,\sqrt{b-\sqrt{b^2-4\,a\,c}+2\,c\,x}\,\sqrt{\frac{(e\,f-d\,g)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}{\left(2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)g\right)(d+e\,x)}}\right.$$

$$\sqrt{\frac{(e\,f-d\,g)\left(2\,a+\left(b+\sqrt{b^2-4\,a\,c}\right)x\right)}{\left(b\,f+\sqrt{b^2-4\,a\,c}\,f-2\,a\,g\right)(d+e\,x)}}\,(d+e\,x)\,\text{EllipticPi}\left[\frac{e\left(2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)g\right)}{\left(2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e\right)g},\right.$$

$$\left.\left.\text{ArcSin}\left[\frac{\sqrt{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\,\sqrt{f+g\,x}}{\sqrt{2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)g}\,\sqrt{d+e\,x}}\right],\ \frac{\left(b\,d+\sqrt{b^2-4\,a\,c}\,d-2\,a\,e\right)\left(2\,c\,f-\left(b+\sqrt{b^2-4\,a\,c}\right)g\right)}{\left(2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e\right)\left(b\,f+\sqrt{b^2-4\,a\,c}\,f-2\,a\,g\right)}\right]\right)\Big/$$

$$\left(\sqrt{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\;g\,\sqrt{\frac{2\,a\,c}{b+\sqrt{b^2-4\,a\,c}}+c\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 2493 leaves) :

$$-\left(\left(2\,\sqrt{f+g\,x}\,\sqrt{a+b\,x+c\,x^2}\,\sqrt{\left(c+\frac{c\,f^2}{(f+g\,x)^2}-\frac{b\,f\,g}{(f+g\,x)^2}+\frac{a\,g^2}{(f+g\,x)^2}-\frac{2\,c\,f}{f+g\,x}+\frac{b\,g}{f+g\,x}\right)\left(e-\frac{e\,f}{f+g\,x}+\frac{d\,g}{f+g\,x}\right)}\,\sqrt{d+\frac{(f+g\,x)\left(e-\frac{e\,f}{f+g\,x}\right)}{g}}\right.\right.$$

$$\left(-\left(e\,f\,\sqrt{\frac{-\frac{e}{e\,f-d\,g}+\frac{1}{f+g\,x}}{-\frac{e}{e\,f-d\,g}+\frac{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,(c\,f^2-b\,f\,g+a\,g^2)}}}\,\sqrt{\frac{-\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,(c\,f^2-b\,f\,g+a\,g^2)}+\frac{1}{f+g\,x}}{-\frac{2\,c\,f-b\,g-\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,(c\,f^2-b\,f\,g+a\,g^2)}+\frac{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,(c\,f^2-b\,f\,g+a\,g^2)}}}\,\left(-\frac{2\,c\,f-b\,g+\sqrt{b^2\,g^2-4\,a\,c\,g^2}}{2\,(c\,f^2-b\,f\,g+a\,g^2)}+\frac{1}{f+g\,x}\right)\text{EllipticF}\left[\right.\right.\right.$$

$$\text{ArcSin}\left[\frac{\sqrt{\frac{2 c f-b g+\sqrt{(b^2-4 a c) g^2}-\frac{2 c f^2}{f+g x}+\frac{2 b f g}{f+g x}-\frac{2 a g^2}{f+g x}}{\sqrt{(b^2-4 a c) g^2}}}}{\sqrt{2}}, \frac{2 \sqrt{(b^2-4 a c) g^2}\,(e f-d g)}{-2 c d f g+b e f g+b d g^2-2 a e g^2+e f \sqrt{(b^2-4 a c) g^2}-d g \sqrt{(b^2-4 a c) g^2}}\right]\Bigg) \Big/$$

$$\left(\sqrt{\frac{-\frac{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}{2 (c f^2-b f g+a g^2)}+\frac{1}{f+g x}}{\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 (c f^2-b f g+a g^2)}-\frac{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}{2 (c f^2-b f g+a g^2)}}} \sqrt{\left(c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}\right)\left(e+\frac{-e f+d g}{f+g x}\right)}\right)+$$

$$\Bigg(d g \sqrt{\frac{-\frac{e}{e f-d g}+\frac{1}{f+g x}}{-\frac{e}{e f-d g}+\frac{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}{2 (c f^2-b f g+a g^2)}}} \sqrt{\frac{-\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 (c f^2-b f g+a g^2)}+\frac{1}{f+g x}}{-\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 (c f^2-b f g+a g^2)}+\frac{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}{2 (c f^2-b f g+a g^2)}}} \left(-\frac{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}{2 (c f^2-b f g+a g^2)}+\frac{1}{f+g x}\right) \text{EllipticF}\Big[$$

$$\text{ArcSin}\left[\frac{\sqrt{\frac{2 c f-b g+\sqrt{(b^2-4 a c) g^2}-\frac{2 c f^2}{f+g x}+\frac{2 b f g}{f+g x}-\frac{2 a g^2}{f+g x}}{\sqrt{(b^2-4 a c) g^2}}}}{\sqrt{2}}, \frac{2 \sqrt{(b^2-4 a c) g^2}\,(e f-d g)}{-2 c d f g+b e f g+b d g^2-2 a e g^2+e f \sqrt{(b^2-4 a c) g^2}-d g \sqrt{(b^2-4 a c) g^2}}\right]\Bigg) \Big/$$

$$\left(\sqrt{\frac{-\frac{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}{2 (c f^2-b f g+a g^2)}+\frac{1}{f+g x}}{\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 (c f^2-b f g+a g^2)}-\frac{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}{2 (c f^2-b f g+a g^2)}}} \sqrt{\left(c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}\right)\left(e+\frac{-e f+d g}{f+g x}\right)}\right)-$$

$$\Bigg(2 e (c f^2-b f g+a g^2)\left(-\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2 (c f^2-b f g+a g^2)}+\frac{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}{2 (c f^2-b f g+a g^2)}\right) \sqrt{\frac{-\frac{e}{e f-d g}+\frac{1}{f+g x}}{-\frac{e}{e f-d g}+\frac{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}{2 (c f^2-b f g+a g^2)}}}$$

$$\sqrt{-\frac{\left(-\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2\left(c f^2-b f g+a g^2\right)}+\frac{1}{f+g x}\right)\left(-\frac{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}{2\left(c f^2-b f g+a g^2\right)}+\frac{1}{f+g x}\right)}{\left(-\frac{2 c f-b g-\sqrt{b^2 g^2-4 a c g^2}}{2\left(c f^2-b f g+a g^2\right)}+\frac{2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}}{2\left(c f^2-b f g+a g^2\right)}\right)^2}}\ \text{EllipticPi}\left[\frac{2\sqrt{\left(b^2-4 a c\right) g^2}}{2 c f-b g+\sqrt{\left(b^2-4 a c\right) g^2}},\right.$$

$$\left.\text{ArcSin}\left[\frac{\sqrt{\frac{2 c f-b g+\sqrt{\left(b^2-4 a c\right) g^2}-\frac{2 c f^2}{f+g x}+\frac{2 b f g}{f+g x}-\frac{2 a g^2}{f+g x}}{\sqrt{\left(b^2-4 a c\right) g^2}}}}{\sqrt{2}}\right],\frac{2\sqrt{\left(b^2-4 a c\right) g^2}\left(e f-d g\right)}{-2 c d f g+b e f g+b d g^2-2 a e g^2+e f\sqrt{\left(b^2-4 a c\right) g^2}-d g\sqrt{\left(b^2-4 a c\right) g^2}}\right]\right)\Big/$$

$$\left(\left(2 c f-b g+\sqrt{b^2 g^2-4 a c g^2}\right)\sqrt{\left(c+\frac{c f^2-b f g+a g^2}{(f+g x)^2}+\frac{-2 c f+b g}{f+g x}\right)\left(e+\frac{-e f+d g}{f+g x}\right)}\right)\Bigg)\Bigg)\Big/$$

$$\left(g\sqrt{a+x(b+c x)}\left(e-\frac{e f}{f+g x}+\frac{d g}{f+g x}\right)\sqrt{\frac{(f+g x)^2\left(c\left(-1+\frac{f}{f+g x}\right)^2+\frac{g\left(b-\frac{b f}{f+g x}+\frac{a g}{f+g x}\right)}{f+g x}\right)}{g^2}}\right)\Bigg)$$

■ Problem 923: Unable to integrate problem.

$$\int\frac{(d+e x)^m\left(a+b x+c x^2\right)}{(f+g x)^2}\,dx$$

Optimal (type 5, 157 leaves, 3 steps):

$$\frac{c\,(d+e x)^{1+m}}{e g^2 (1+m)}+\frac{\left(a+\frac{f (c f-b g)}{g^2}\right)(d+e x)^{1+m}}{(e f-d g)(f+g x)}+\frac{1}{g^2 (e f-d g)^2 (1+m)}$$

$$(c f (2 d g-e f (2+m))-g (a e g m+b (d g-e f (1+m))))\,(d+e x)^{1+m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,-\frac{g (d+e x)}{e f-d g}\right]$$

Result (type 8, 27 leaves):

$$\int \frac{(d+e\,x)^m\,\left(a+b\,x+c\,x^2\right)}{(f+g\,x)^2}\,dx$$

- **Problem 924: Unable to integrate problem.**

$$\int \frac{(d+e\,x)^m\,\left(a+b\,x+c\,x^2\right)}{(f+g\,x)^3}\,dx$$

Optimal (type 5, 245 leaves, 3 steps):

$$\frac{\left(a+\frac{f\,(c\,f-b\,g)}{g^2}\right)\,(d+e\,x)^{1+m}}{2\,(e\,f-d\,g)\,(f+g\,x)^2}+\frac{(c\,f\,(4\,d\,g-e\,f\,(3+m))+g\,(a\,e\,g\,(1-m)-b\,(2\,d\,g-e\,f\,(1+m))))\,(d+e\,x)^{1+m}}{2\,g^2\,(e\,f-d\,g)^2\,(f+g\,x)}+$$
$$\frac{1}{2\,g^2\,(e\,f-d\,g)^3\,(1+m)}\left(c\,\left(2\,d^2\,g^2-4\,d\,e\,f\,g\,(1+m)+e^2\,f^2\,\left(2+3\,m+m^2\right)\right)-e\,g\,m\,(a\,e\,g\,(1-m)-b\,(2\,d\,g-e\,f\,(1+m)))\right)$$
$$(d+e\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\,1+m,\,2+m,\,-\frac{g\,(d+e\,x)}{e\,f-d\,g}\right]$$

Result (type 8, 27 leaves):

$$\int \frac{(d+e\,x)^m\,\left(a+b\,x+c\,x^2\right)}{(f+g\,x)^3}\,dx$$

- **Problem 925: Result more than twice size of optimal antiderivative.**

$$\int (d+e\,x)^m\,(f+g\,x)^2\,\left(a+b\,x+c\,x^2\right)^2\,dx$$

Optimal (type 3, 525 leaves, 2 steps):

$$\frac{\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,(e\,f-d\,g)^2\,(d+e\,x)^{1+m}}{e^7\,(1+m)}-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,(e\,f-d\,g)\,(c\,d\,(2\,e\,f-3\,d\,g)-e\,(b\,e\,f-2\,b\,d\,g+a\,e\,g))\,(d+e\,x)^{2+m}}{e^7\,(2+m)}+$$
$$\frac{1}{e^7\,(3+m)}\left(c^2\,d^2\,\left(6\,e^2\,f^2-20\,d\,e\,f\,g+15\,d^2\,g^2\right)+e^2\,\left(a^2\,e^2\,g^2+2\,a\,b\,e\,g\,(2\,e\,f-3\,d\,g)+b^2\,\left(e^2\,f^2-6\,d\,e\,f\,g+6\,d^2\,g^2\right)\right)+\right.$$
$$\left.2\,c\,e\,\left(a\,e\,\left(e^2\,f^2-6\,d\,e\,f\,g+6\,d^2\,g^2\right)-b\,d\,\left(3\,e^2\,f^2-12\,d\,e\,f\,g+10\,d^2\,g^2\right)\right)\right)\,(d+e\,x)^{3+m}+\frac{1}{e^7\,(4+m)}$$
$$2\,\left(b\,e^2\,g\,(b\,e\,f-2\,b\,d\,g+a\,e\,g)-2\,c^2\,d\,\left(e^2\,f^2-5\,d\,e\,f\,g+5\,d^2\,g^2\right)+c\,e\,\left(2\,a\,e\,g\,(e\,f-2\,d\,g)+b\,\left(e^2\,f^2-8\,d\,e\,f\,g+10\,d^2\,g^2\right)\right)\right)\,(d+e\,x)^{4+m}+$$
$$\frac{\left(b^2\,e^2\,g^2+2\,c\,e\,g\,(2\,b\,e\,f-5\,b\,d\,g+a\,e\,g)+c^2\,\left(e^2\,f^2-10\,d\,e\,f\,g+15\,d^2\,g^2\right)\right)\,(d+e\,x)^{5+m}}{e^7\,(5+m)}+\frac{2\,c\,g\,(c\,e\,f-3\,c\,d\,g+b\,e\,g)\,(d+e\,x)^{6+m}}{e^7\,(6+m)}+\frac{c^2\,g^2\,(d+e\,x)^{7+m}}{e^7\,(7+m)}$$

Result (type 3, 1263 leaves):

$$\frac{1}{e^7 (1+m)(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)}$$
$$(d+e x)^{1+m} \Big(c^2 \big(720 d^6 g^2 - 240 d^5 e g (f (7+m) + 3 g (1+m) x) + 24 d^4 e^2 \left(f^2 (42+13 m+m^2) + 10 f g (7+8 m+m^2) x + 15 g^2 (2+3 m+m^2) x^2\right) -$$
$$24 d^3 e^3 (1+m) x \left(f^2 (42+13 m+m^2) + 5 f g (14+9 m+m^2) x + 5 g^2 (6+5 m+m^2) x^2\right) +$$
$$2 d^2 e^4 (2+3 m+m^2) x^2 \left(6 f^2 (42+13 m+m^2) + 20 f g (21+10 m+m^2) x + 15 g^2 (12+7 m+m^2) x^2\right) -$$
$$2 d e^5 (6+11 m+6 m^2+m^3) x^3 \left(2 f^2 (42+13 m+m^2) + 5 f g (28+11 m+m^2) x + 3 g^2 (20+9 m+m^2) x^2\right) +$$
$$e^6 (24+50 m+35 m^2+10 m^3+m^4) x^4 \left(f^2 (42+13 m+m^2) + 2 f g (35+12 m+m^2) x + g^2 (30+11 m+m^2) x^2\right)\big) +$$
$$e^2 (42+13 m+m^2) \Big(a^2 e^2 (20+9 m+m^2) \left(2 d^2 g^2 - 2 d e g (f (3+m) + g (1+m) x) + e^2 \left(f^2 (6+5 m+m^2) + 2 f g (3+4 m+m^2) x + g^2 (2+3 m+m^2) x^2\right)\right) +$$
$$2 a b e (5+m) \Big(-6 d^3 g^2 + 2 d^2 e g (2 f (4+m) + 3 g (1+m) x) - d e^2 \left(f^2 (12+7 m+m^2) + 4 f g (4+5 m+m^2) x + 3 g^2 (2+3 m+m^2) x^2\right) +$$
$$e^3 (1+m) x \left(f^2 (12+7 m+m^2) + 2 f g (8+6 m+m^2) x + g^2 (6+5 m+m^2) x^2\right)\Big) +$$
$$b^2 \Big(24 d^4 g^2 - 12 d^3 e g (f (5+m) + 2 g (1+m) x) + 2 d^2 e^2 \left(f^2 (20+9 m+m^2) + 6 f g (5+6 m+m^2) x + 6 g^2 (2+3 m+m^2) x^2\right) -$$
$$2 d e^3 (1+m) x \left(f^2 (20+9 m+m^2) + 3 f g (10+7 m+m^2) x + 2 g^2 (6+5 m+m^2) x^2\right) +$$
$$e^4 (2+3 m+m^2) x^2 \left(f^2 (20+9 m+m^2) + 2 f g (15+8 m+m^2) x + g^2 (12+7 m+m^2) x^2\right)\Big)\Big) +$$
$$2 c e (7+m) \Big(a e (6+m) \Big(24 d^4 g^2 - 12 d^3 e g (f (5+m) + 2 g (1+m) x) + 2 d^2 e^2 \left(f^2 (20+9 m+m^2) + 6 f g (5+6 m+m^2) x + 6 g^2 (2+3 m+m^2) x^2\right) -$$
$$2 d e^3 (1+m) x \left(f^2 (20+9 m+m^2) + 3 f g (10+7 m+m^2) x + 2 g^2 (6+5 m+m^2) x^2\right) +$$
$$e^4 (2+3 m+m^2) x^2 \left(f^2 (20+9 m+m^2) + 2 f g (15+8 m+m^2) x + g^2 (12+7 m+m^2) x^2\right)\Big) +$$
$$b \Big(-120 d^5 g^2 + 24 d^4 e g (2 f (6+m) + 5 g (1+m) x) - 6 d^3 e^2 \left(f^2 (30+11 m+m^2) + 8 f g (6+7 m+m^2) x + 10 g^2 (2+3 m+m^2) x^2\right) +$$
$$2 d^2 e^3 (1+m) x \left(3 f^2 (30+11 m+m^2) + 12 f g (12+8 m+m^2) x + 10 g^2 (6+5 m+m^2) x^2\right) -$$
$$d e^4 (2+3 m+m^2) x^2 \left(3 f^2 (30+11 m+m^2) + 8 f g (18+9 m+m^2) x + 5 g^2 (12+7 m+m^2) x^2\right) +$$
$$e^5 (6+11 m+6 m^2+m^3) x^3 \left(f^2 (30+11 m+m^2) + 2 f g (24+10 m+m^2) x + g^2 (20+9 m+m^2) x^2\right)\Big)\Big)\Big)$$

- **Problem 927: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(d+e x)^m \left(a+b x+c x^2\right)^2}{f+g x}\, dx$$

Optimal (type 5, 287 leaves, 4 steps):

$$\frac{(b e g - c (e f + d g)) \left(c \left(e^2 f^2 + d^2 g^2\right) + e g (2 a e g - b (e f + d g))\right) (d+e x)^{1+m}}{e^4 g^4 (1+m)} +$$
$$\frac{\left(b^2 e^2 g^2 + c^2 \left(e^2 f^2 + 2 d e f g + 3 d^2 g^2\right) + 2 c e g (a e g - b (e f + 2 d g))\right) (d+e x)^{2+m}}{e^4 g^3 (2+m)} - \frac{c (c e f + 3 c d g - 2 b e g) (d+e x)^{3+m}}{e^4 g^2 (3+m)} +$$
$$\frac{c^2 (d+e x)^{4+m}}{e^4 g (4+m)} + \frac{\left(c f^2 - b f g + a g^2\right)^2 (d+e x)^{1+m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, -\frac{g (d+e x)}{e f - d g}\right]}{g^4 (e f - d g) (1+m)}$$

Result (type 6, 733 leaves):

$$\left(3\,a\,b\,d\,f\,x^2\,(d+e\,x)^m\,\text{AppellF1}\left[2,-m,1,3,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]\right)\Big/\left((f+g\,x)\left(3\,d\,f\,\text{AppellF1}\left[2,-m,1,3,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]+e\,f\,m\,x\,\text{AppellF1}\left[3,1-m,1,4,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]-d\,g\,x\,\text{AppellF1}\left[3,-m,2,4,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]\right)\right)+$$

$$\left(4\,b^2\,d\,f\,x^3\,(d+e\,x)^m\,\text{AppellF1}\left[3,-m,1,4,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]\right)\Big/\left(3\,(f+g\,x)\left(4\,d\,f\,\text{AppellF1}\left[3,-m,1,4,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]+e\,f\,m\,x\,\text{AppellF1}\left[4,1-m,1,5,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]-d\,g\,x\,\text{AppellF1}\left[4,-m,2,5,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]\right)\right)+$$

$$\left(8\,a\,c\,d\,f\,x^3\,(d+e\,x)^m\,\text{AppellF1}\left[3,-m,1,4,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]\right)\Big/\left(3\,(f+g\,x)\left(4\,d\,f\,\text{AppellF1}\left[3,-m,1,4,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]+e\,f\,m\,x\,\text{AppellF1}\left[4,1-m,1,5,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]-d\,g\,x\,\text{AppellF1}\left[4,-m,2,5,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]\right)\right)+$$

$$\left(5\,b\,c\,d\,f\,x^4\,(d+e\,x)^m\,\text{AppellF1}\left[4,-m,1,5,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]\right)\Big/\left(2\,(f+g\,x)\left(5\,d\,f\,\text{AppellF1}\left[4,-m,1,5,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]+e\,f\,m\,x\,\text{AppellF1}\left[5,1-m,1,6,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]-d\,g\,x\,\text{AppellF1}\left[5,-m,2,6,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]\right)\right)+$$

$$\left(6\,c^2\,d\,f\,x^5\,(d+e\,x)^m\,\text{AppellF1}\left[5,-m,1,6,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]\right)\Big/\left(5\,(f+g\,x)\left(6\,d\,f\,\text{AppellF1}\left[5,-m,1,6,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]+e\,f\,m\,x\,\text{AppellF1}\left[6,1-m,1,7,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]-d\,g\,x\,\text{AppellF1}\left[6,-m,2,7,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]\right)\right)+$$

$$\frac{a^2\left(1+\frac{\left(d-\frac{e\,f}{g}\right)g}{e\,(f+g\,x)}\right)^{-m}\left(d-\frac{e\,f}{g}+\frac{e\,(f+g\,x)}{g}\right)^m\,\text{Hypergeometric2F1}\left[-m,-m,1-m,-\frac{\left(d-\frac{e\,f}{g}\right)g}{e\,(f+g\,x)}\right]}{g\,m}$$

- **Problem 928: Unable to integrate problem.**

$$\int\frac{(d+e\,x)^m\left(a+b\,x+c\,x^2\right)^2}{(f+g\,x)^2}\,dx$$

Optimal (type 5, 298 leaves, 4 steps):

$$\frac{\left(b^2\,e^2\,g^2+c^2\left(3\,e^2\,f^2+2\,d\,e\,f\,g+d^2\,g^2\right)+2\,c\,e\,g\,(a\,e\,g-b\,(2\,e\,f+d\,g))\right)(d+e\,x)^{1+m}}{e^3\,g^4\,(1+m)}-$$

$$\frac{2\,c\,(c\,e\,f+c\,d\,g-b\,e\,g)\,(d+e\,x)^{2+m}}{e^3\,g^3\,(2+m)}+\frac{c^2\,(d+e\,x)^{3+m}}{e^3\,g^2\,(3+m)}+\frac{\left(c\,f^2-b\,f\,g+a\,g^2\right)^2\,(d+e\,x)^{1+m}}{g^4\,(e\,f-d\,g)\,(f+g\,x)}+\frac{1}{g^4\,(e\,f-d\,g)^2\,(1+m)}$$

$$\left(c\,f^2-b\,f\,g+a\,g^2\right)\left(c\,f\,(4\,d\,g-e\,f\,(4+m))-g\,(a\,e\,g\,m+b\,(2\,d\,g-e\,f\,(2+m)))\right)(d+e\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,-\frac{g\,(d+e\,x)}{e\,f-d\,g}\right]$$

Result (type 8, 29 leaves):

$$\int\frac{(d+e\,x)^m\left(a+b\,x+c\,x^2\right)^2}{(f+g\,x)^2}\,dx$$

Problem 929: Unable to integrate problem.

$$\int \frac{(d+e x)^m \left(a+b x+c x^2\right)^2}{(f+g x)^3} \, dx$$

Optimal (type 5, 461 leaves, 5 steps):

$$-\frac{c\,(3 c e f+c d g-2 b e g)\,(d+e x)^{1+m}}{e^2 g^4\,(1+m)}+\frac{c^2\,(d+e x)^{2+m}}{e^2 g^3\,(2+m)}+\frac{\left(c f^2-b f g+a g^2\right)^2\,(d+e x)^{1+m}}{2 g^4\,(e f-d g)\,(f+g x)^2}+$$
$$\frac{\left(c f^2-b f g+a g^2\right)\,(c f\,(8 d g-e f\,(7+m))+g\,(a e g\,(1-m)-b\,(4 d g-e f\,(3+m))))\,(d+e x)^{1+m}}{2 g^4\,(e f-d g)^2\,(f+g x)}+$$
$$\frac{1}{2 g^4\,(e f-d g)^3\,(1+m)}\left(c^2 f^2\left(12 d^2 g^2-8 d e f g\,(3+m)+e^2 f^2\left(12+7 m+m^2\right)\right)-\right.$$
$$g^2\left(a^2 e^2 g^2\,(1-m)\,m-2 a b e g m\,(2 d g-e f\,(1+m))-b^2\left(2 d^2 g^2-4 d e f g\,(1+m)+e^2 f^2\left(2+3 m+m^2\right)\right)\right)+$$
$$\left.2 c g\left(a g\left(2 d^2 g^2-4 d e f g\,(1+m)+e^2 f^2\left(2+3 m+m^2\right)\right)-b f\left(6 d^2 g^2-6 d e f g\,(2+m)+e^2 f^2\left(6+5 m+m^2\right)\right)\right)\right)$$
$$(d+e x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\,1+m,\,2+m,\,-\frac{g\,(d+e x)}{e f-d g}\right]$$

Result (type 8, 29 leaves):

$$\int \frac{(d+e x)^m \left(a+b x+c x^2\right)^2}{(f+g x)^3} \, dx$$

Problem 930: Result more than twice size of optimal antiderivative.

$$\int \frac{(2+3 x)^4\,(1+4 x)^m}{1-5 x+3 x^2} \, dx$$

Optimal (type 5, 183 leaves, 4 steps):

$$\frac{3687\,(1+4 x)^{1+m}}{64\,(1+m)}+\frac{207\,(1+4 x)^{2+m}}{32\,(2+m)}+\frac{27\,(1+4 x)^{3+m}}{64\,(3+m)}-\frac{3\left(5499-1631\sqrt{13}\right)\,(1+4 x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\,1+m,\,2+m,\,\frac{3\,(1+4 x)}{13-2\sqrt{13}}\right]}{26\left(13-2\sqrt{13}\right)\,(1+m)}-$$
$$\frac{3\left(5499+1631\sqrt{13}\right)\,(1+4 x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\,1+m,\,2+m,\,\frac{3\,(1+4 x)}{13+2\sqrt{13}}\right]}{26\left(13+2\sqrt{13}\right)\,(1+m)}$$

Result (type 5, 568 leaves):

$$\frac{48\,(1+4x)^{1+m}}{1+m}+\frac{1}{16\,(1+m)\,(2+m)}$$
$$3^{2-m}\,(-1-4x)^{-m}\,(1+4x)^{m}\,\left(5^{2+m}+39\,(-3-12x)^{m}+192\,(-3-12x)^{m}x+144\,(-3-12x)^{m}x^{2}+12\,m\,(-3-12x)^{m}\,\left(2+11x+12x^{2}\right)\right)+$$
$$\frac{1}{32\,(1+m)\,(2+m)\,(3+m)}3^{-m}\,(-1-4x)^{-m}\,(1+4x)^{m}\,\left(5^{3+m}+387\,(-3-12x)^{m}+2304\,(-3-12x)^{m}x+3456\,(-3-12x)^{m}x^{2}+\right.$$
$$\left.1728\,(-3-12x)^{m}x^{3}+24\,m^{2}\,(-3-12x)^{m}\,(2+3x)^{2}\,(1+4x)+12\,m\,(-3-12x)^{m}\,\left(34+223x+402x^{2}+216x^{3}\right)\right)+$$
$$\frac{1}{\sqrt{13}\,m}17\times 2^{6+m}\,3^{-m}\,(1+4x)^{m}\,\left(-\left(-\frac{1+4x}{5+\sqrt{13}-6x}\right)^{-m}\text{Hypergeometric2F1}\left[-m,\,-m,\,1-m,\,\frac{13+2\sqrt{13}}{2\left(5+\sqrt{13}-6x\right)}\right]+\right.$$
$$\left.\left(\frac{1+4x}{-5+\sqrt{13}+6x}\right)^{-m}\text{Hypergeometric2F1}\left[-m,\,-m,\,1-m,\,\frac{-13+2\sqrt{13}}{2\left(-5+\sqrt{13}+6x\right)}\right]\right)+\frac{1}{\sqrt{13}\,m}$$
$$47\times 2^{-1+2m}\,3^{2-m}\,\left(\frac{1}{2}+2x\right)^{m}\,\left(\left(9+\sqrt{13}\right)\,\left(-\frac{1+4x}{5+\sqrt{13}-6x}\right)^{-m}\text{Hypergeometric2F1}\left[-m,\,-m,\,1-m,\,\frac{13+2\sqrt{13}}{2\left(5+\sqrt{13}-6x\right)}\right]+\right.$$
$$\left.\left(-9+\sqrt{13}\right)\,\left(\frac{1+4x}{-5+\sqrt{13}+6x}\right)^{-m}\text{Hypergeometric2F1}\left[-m,\,-m,\,1-m,\,\frac{-13+2\sqrt{13}}{2\left(-5+\sqrt{13}+6x\right)}\right]\right)$$

- **Problem 931: Result more than twice size of optimal antiderivative.**

$$\int\frac{(2+3x)^{3}\,(1+4x)^{m}}{1-5x+3x^{2}}\,dx$$

Optimal (type 5, 165 leaves, 4 steps):

$$\frac{123\,(1+4x)^{1+m}}{16\,(1+m)}+\frac{9\,(1+4x)^{2+m}}{16\,(2+m)}-\frac{3\left(416-135\sqrt{13}\right)\,(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\,1+m,\,2+m,\,\frac{3\,(1+4x)}{13-2\sqrt{13}}\right]}{13\left(13-2\sqrt{13}\right)\,(1+m)}-$$
$$\frac{3\left(416+135\sqrt{13}\right)\,(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\,1+m,\,2+m,\,\frac{3\,(1+4x)}{13+2\sqrt{13}}\right]}{13\left(13+2\sqrt{13}\right)\,(1+m)}$$

Result (type 5, 410 leaves):

$$\frac{1}{208\, m\, (1+m)\, (2+m)} (1+4x)^m \Bigg(507\, m + 312\, m^2 + 1404\, m\, (2+m) + 325\, m \left(-\frac{3}{5} - \frac{12\, x}{5}\right)^{-m} + 2496\, m\, x + 1716\, m^2\, x + 5616\, m\, (2+m)\, x +$$

$$1872\, m\, x^2 + 1872\, m^2\, x^2 + 13 \times 2^{9+m}\, 3^{-m}\, (1+m)\, (2+m) \left(-\frac{1+4x}{5+\sqrt{13}-6x}\right)^{-m} \text{Hypergeometric2F1}\left[-m, -m, 1-m, \frac{13+2\sqrt{13}}{2\left(5+\sqrt{13}-6x\right)}\right] +$$

$$5 \times 2^{4+m}\, 3^{3-m} \sqrt{13}\, (1+m)\, (2+m) \left(-\frac{1+4x}{5+\sqrt{13}-6x}\right)^{-m} \text{Hypergeometric2F1}\left[-m, -m, 1-m, \frac{13+2\sqrt{13}}{2\left(5+\sqrt{13}-6x\right)}\right] +$$

$$13 \times 2^{9+m}\, 3^{-m}\, (1+m)\, (2+m) \left(\frac{1+4x}{-5+\sqrt{13}+6x}\right)^{-m} \text{Hypergeometric2F1}\left[-m, -m, 1-m, \frac{-13+2\sqrt{13}}{2\left(-5+\sqrt{13}+6x\right)}\right] -$$

$$5 \times 2^{4+m}\, 3^{3-m} \sqrt{13}\, (1+m)\, (2+m) \left(\frac{1+4x}{-5+\sqrt{13}+6x}\right)^{-m} \text{Hypergeometric2F1}\left[-m, -m, 1-m, \frac{-13+2\sqrt{13}}{2\left(-5+\sqrt{13}+6x\right)}\right]\Bigg)$$

- **Problem 937: Unable to integrate problem.**

$$\int \frac{(2+3x)^4\, (1+4x)^m}{\left(1-5x+3x^2\right)^2}\, dx$$

Optimal (type 5, 202 leaves, 5 steps):

$$\frac{9\, (1+4x)^{1+m}}{4\, (1+m)} + \frac{(844-2355x)\, (1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{\left(13689-\sqrt{13}\left(297+4474\, m-1570\sqrt{13}\, m\right)\right) (1+4x)^{1+m}\, \text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{3\,(1+4x)}{13-2\sqrt{13}}\right]}{169\left(13-2\sqrt{13}\right)(1+m)} -$$

$$\frac{\left(13689+\sqrt{13}\left(297+4474\, m+1570\sqrt{13}\, m\right)\right) (1+4x)^{1+m}\, \text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{3\,(1+4x)}{13+2\sqrt{13}}\right]}{169\left(13+2\sqrt{13}\right)(1+m)}$$

Result (type 8, 29 leaves):

$$\int \frac{(2+3x)^4\, (1+4x)^m}{\left(1-5x+3x^2\right)^2}\, dx$$

- **Problem 938: Unable to integrate problem.**

$$\int \frac{(2+3x)^3\, (1+4x)^m}{\left(1-5x+3x^2\right)^2}\, dx$$

Optimal (type 5, 181 leaves, 5 steps):

$$\frac{(209-426\,x)\,(1+4\,x)^{1+m}}{39\,\left(1-5\,x+3\,x^2\right)}-\frac{\left(1521+\sqrt{13}\,\left(1701-1168\,m+568\,\sqrt{13}\,m\right)\right)\,(1+4\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\,1+m,\,2+m,\,\frac{3\,(1+4\,x)}{13-2\,\sqrt{13}}\right]}{338\,\left(13-2\,\sqrt{13}\right)\,(1+m)}+$$

$$\frac{\left(\sqrt{13}\,(1701-1168\,m)-13\,(117+568\,m)\right)\,(1+4\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\,1+m,\,2+m,\,\frac{3\,(1+4\,x)}{13+2\,\sqrt{13}}\right]}{338\,\left(13+2\,\sqrt{13}\right)\,(1+m)}$$

Result (type 8, 29 leaves):

$$\int\frac{(2+3\,x)^3\,(1+4\,x)^m}{\left(1-5\,x+3\,x^2\right)^2}\,dx$$

Problem 939: Unable to integrate problem.

$$\int\frac{(2+3\,x)^2\,(1+4\,x)^m}{\left(1-5\,x+3\,x^2\right)^2}\,dx$$

Optimal (type 5, 179 leaves, 5 steps):

$$\frac{(61-87\,x)\,(1+4\,x)^{1+m}}{39\,\left(1-5\,x+3\,x^2\right)}-\frac{2\,\left(153-\left(23-29\,\sqrt{13}\right)\,m\right)\,(1+4\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\,1+m,\,2+m,\,\frac{3\,(1+4\,x)}{13-2\,\sqrt{13}}\right]}{13\,\sqrt{13}\,\left(13-2\,\sqrt{13}\right)\,(1+m)}+$$

$$\frac{2\,\left(153-\left(23+29\,\sqrt{13}\right)\,m\right)\,(1+4\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\,1+m,\,2+m,\,\frac{3\,(1+4\,x)}{13+2\,\sqrt{13}}\right]}{13\,\sqrt{13}\,\left(13+2\,\sqrt{13}\right)\,(1+m)}$$

Result (type 8, 29 leaves):

$$\int\frac{(2+3\,x)^2\,(1+4\,x)^m}{\left(1-5\,x+3\,x^2\right)^2}\,dx$$

Problem 940: Unable to integrate problem.

$$\int\frac{(2+3\,x)\,(1+4\,x)^m}{\left(1-5\,x+3\,x^2\right)^2}\,dx$$

Optimal (type 5, 179 leaves, 5 steps):

$$\frac{(20-21x)(1+4x)^{1+m}}{39(1-5x+3x^2)} - \frac{\left(81+2\left(5+7\sqrt{13}\right)m\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right]}{13\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)} +$$

$$\frac{\left(81+2\left(5-7\sqrt{13}\right)m\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right]}{13\sqrt{13}\left(13+2\sqrt{13}\right)(1+m)}$$

Result (type 8, 27 leaves) :

$$\int \frac{(2+3x)(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx$$

Problem 941: Unable to integrate problem.

$$\int \frac{(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx$$

Optimal (type 5, 177 leaves, 5 steps) :

$$\frac{(7-6x)(1+4x)^{1+m}}{39(1-5x+3x^2)} - \frac{2\left(9+2\left(2+\sqrt{13}\right)m\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right]}{13\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)} +$$

$$\frac{2\left(9+2\left(2-\sqrt{13}\right)m\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right]}{13\sqrt{13}\left(13+2\sqrt{13}\right)(1+m)}$$

Result (type 8, 22 leaves) :

$$\int \frac{(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx$$

Problem 942: Unable to integrate problem.

$$\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)^2}\,dx$$

Optimal (type 5, 340 leaves, 12 steps) :

$$\frac{(43-33x)(1+4x)^{1+m}}{663(1-5x+3x^2)}+\frac{9(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,-\frac{3}{5}(1+4x)\right]}{1445(1+m)}+$$

$$\frac{9\left(13+9\sqrt{13}\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right]}{7514\left(13-2\sqrt{13}\right)(1+m)}-$$

$$\frac{\left(81+\left(62+22\sqrt{13}\right)m\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right]}{221\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)}+$$

$$\frac{9\left(13-9\sqrt{13}\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right]}{7514\left(13+2\sqrt{13}\right)(1+m)}+$$

$$\frac{\left(81+\left(62-22\sqrt{13}\right)m\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right]}{221\sqrt{13}\left(13+2\sqrt{13}\right)(1+m)}$$

Result (type 8, 29 leaves) :

$$\int\frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)^2}\,dx$$

- **Problem 943: Unable to integrate problem.**

$$\int\frac{(1+4x)^m}{(2+3x)^2\left(1-5x+3x^2\right)^2}\,dx$$

Optimal (type 5, 376 leaves, 13 steps) :

$$\frac{(268-195\,x)\,(1+4\,x)^{1+m}}{11271\,(1-5\,x+3\,x^2)}+\frac{162\,(1+4\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\,1+m,\,2+m,\,-\frac{3}{5}\,(1+4\,x)\right]}{24565\,(1+m)}+$$

$$\frac{9\,\left(117+64\,\sqrt{13}\right)\,(1+4\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\,1+m,\,2+m,\,\frac{3\,(1+4\,x)}{13-2\,\sqrt{13}}\right]}{63869\,\left(13-2\,\sqrt{13}\right)\,(1+m)}-$$

$$\frac{\left(423+2\,\left(211+65\,\sqrt{13}\right)\,m\right)\,(1+4\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\,1+m,\,2+m,\,\frac{3\,(1+4\,x)}{13-2\,\sqrt{13}}\right]}{3757\,\sqrt{13}\,\left(13-2\,\sqrt{13}\right)\,(1+m)}+$$

$$\frac{9\,\left(117-64\,\sqrt{13}\right)\,(1+4\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\,1+m,\,2+m,\,\frac{3\,(1+4\,x)}{13+2\,\sqrt{13}}\right]}{63869\,\left(13+2\,\sqrt{13}\right)\,(1+m)}+$$

$$\frac{\left(423+\left(422-130\,\sqrt{13}\right)\,m\right)\,(1+4\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\,1+m,\,2+m,\,\frac{3\,(1+4\,x)}{13+2\,\sqrt{13}}\right]}{3757\,\sqrt{13}\,\left(13+2\,\sqrt{13}\right)\,(1+m)}+$$

$$\frac{36\,(1+4\,x)^{1+m}\,\text{Hypergeometric2F1}\left[2,\,1+m,\,2+m,\,-\frac{3}{5}\,(1+4\,x)\right]}{7225\,(1+m)}$$

Result (type 8, 29 leaves) :

$$\int\frac{(1+4\,x)^m}{(2+3\,x)^2\,\left(1-5\,x+3\,x^2\right)^2}\,dx$$

- **Problem 945: Unable to integrate problem.**

$$\int(d+e\,x)^m\,(f+g\,x)^2\,\sqrt{a+b\,x+c\,x^2}\,dx$$

Optimal (type 6, 509 leaves, 6 steps) :

$$\frac{g^2 (d+e x)^{1+m} \left(a+b x+c x^2\right)^{3/2}}{c e (4+m)}+\Big(\left(e (b d-a e) g^2 (1+m)+c \left(3 d^2 g^2+e^2 f^2 (4+m)-2 d e f g (4+m)\right)\right)$$

$$(d+e x)^{1+m} \sqrt{a+b x+c x^2} \text{ AppellF1}\left[1+m,-\frac{1}{2},-\frac{1}{2},2+m,\frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e},\frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\Big)\Big/$$

$$\left(c e^3 (1+m) (4+m) \sqrt{1-\frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}} \sqrt{1-\frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\right)-$$

$$\Big(g (b e g (5+2 m)+2 c (3 d g-2 e f (4+m))) (d+e x)^{2+m} \sqrt{a+b x+c x^2}$$

$$\text{AppellF1}\left[2+m,-\frac{1}{2},-\frac{1}{2},3+m,\frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e},\frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\Big)\Big/$$

$$\left(2 c e^3 (2+m) (4+m) \sqrt{1-\frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}} \sqrt{1-\frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\right)$$

Result (type 8, 31 leaves):

$$\int (d+e x)^m (f+g x)^2 \sqrt{a+b x+c x^2} \, dx$$

- **Problem 946: Unable to integrate problem.**

$$\int (d+e x)^m (f+g x) \sqrt{a+b x+c x^2} \, dx$$

Optimal (type 6, 388 leaves, 5 steps):

$$\left((e f - d g)\,(d + e x)^{1+m}\sqrt{a + b x + c x^2}\,\text{AppellF1}\left[1 + m, -\frac{1}{2}, -\frac{1}{2}, 2 + m, \frac{2 c (d + e x)}{2 c d - \left(b - \sqrt{b^2 - 4 a c}\right) e}, \frac{2 c (d + e x)}{2 c d - \left(b + \sqrt{b^2 - 4 a c}\right) e}\right]\right) \Bigg/$$

$$\left(e^2 (1 + m)\sqrt{1 - \frac{2 c (d + e x)}{2 c d - \left(b - \sqrt{b^2 - 4 a c}\right) e}}\sqrt{1 - \frac{2 c (d + e x)}{2 c d - \left(b + \sqrt{b^2 - 4 a c}\right) e}}\right) +$$

$$\frac{g (d + e x)^{2+m}\sqrt{a + b x + c x^2}\,\text{AppellF1}\left[2 + m, -\frac{1}{2}, -\frac{1}{2}, 3 + m, \frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}, \frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]}{e^2 (2 + m)\sqrt{1 - \frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}}\sqrt{1 - \frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}}$$

Result (type 8, 29 leaves) :

$$\int (d + e x)^m (f + g x)\sqrt{a + b x + c x^2}\, dx$$

- **Problem 949: Unable to integrate problem.**

$$\int \frac{(d + e x)^m (f + g x)^2}{\sqrt{a + b x + c x^2}}\, dx$$

Optimal (type 6, 502 leaves, 6 steps) :

$$\frac{g^2 (d+e x)^{1+m} \sqrt{a+b x+c x^2}}{c e (2+m)}+$$

$$\left((e (b d-a e) g^2 (1+m)+c (d^2 g^2+e^2 f^2 (2+m)-2 d e f g (2+m))) (d+e x)^{1+m} \sqrt{1-\frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}} \sqrt{1-\frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\right.$$

$$\left.\text{AppellF1}\left[1+m,\frac{1}{2},\frac{1}{2},2+m,\frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e},\frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right) \Big/ \left(c e^3 (1+m) (2+m) \sqrt{a+b x+c x^2}\right)-$$

$$\left(g (b e g (3+2 m)+c (2 d g-4 e f (2+m))) (d+e x)^{2+m} \sqrt{1-\frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}} \sqrt{1-\frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\right.$$

$$\left.\text{AppellF1}\left[2+m,\frac{1}{2},\frac{1}{2},3+m,\frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e},\frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right) \Big/ \left(2 c e^3 (2+m)^2 \sqrt{a+b x+c x^2}\right)$$

Result (type 8, 31 leaves) :

$$\int \frac{(d+e x)^m (f+g x)^2}{\sqrt{a+b x+c x^2}} \, dx$$

■ Problem 950: Unable to integrate problem.

$$\int \frac{(d+e x)^m (f+g x)}{\sqrt{a+b x+c x^2}} \, dx$$

Optimal (type 6, 388 leaves, 5 steps) :

$$\frac{1}{e^2 (1+m) \sqrt{a+b x+c x^2}} (e f-d g) (d+e x)^{1+m} \sqrt{1-\frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}}$$
$$\sqrt{1-\frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\ \text{AppellF1}\left[1+m, \frac{1}{2}, \frac{1}{2}, 2+m, \frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}, \frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]+$$
$$\frac{1}{e^2 (2+m) \sqrt{a+b x+c x^2}} g (d+e x)^{2+m} \sqrt{1-\frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}} \sqrt{1-\frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}$$
$$\text{AppellF1}\left[2+m, \frac{1}{2}, \frac{1}{2}, 3+m, \frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}, \frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]$$

Result (type 8, 29 leaves) :

$$\int \frac{(d+e x)^m (f+g x)}{\sqrt{a+b x+c x^2}} \, dx$$

- **Problem 953: Result unnecessarily involves higher level functions.**

$$\int (d+e x)^m (f+g x)^n \left(a+b x+c x^2\right) dx$$

Optimal (type 5, 265 leaves, 4 steps) :

$$\frac{(b e g (3+m+n)-c (e f (2+m)+d g (4+m+2 n))) (d+e x)^{1+m} (f+g x)^{1+n}}{e^2 g^2 (2+m+n) (3+m+n)}+\frac{c (d+e x)^{2+m} (f+g x)^{1+n}}{e^2 g (3+m+n)}+$$
$$\left(\left(g (2+m+n) \left(a e^2 g (3+m+n)-c d (e f (2+m)+d g (1+n))\right)-(e f (1+m)+d g (1+n)) (b e g (3+m+n)-c (e f (2+m)+d g (4+m+2 n)))\right)\right.$$
$$\left.(d+e x)^{1+m} (f+g x)^n \left(\frac{e (f+g x)}{e f-d g}\right)^{-n} \text{Hypergeometric2F1}\left[1+m, -n, 2+m, -\frac{g (d+e x)}{e f-d g}\right]\right) \Big/ \left(e^3 g^2 (1+m) (2+m+n) (3+m+n)\right)$$

Result (type 6, 327 leaves) :

$$\frac{1}{3}(d+ex)^m(f+gx)^n\left(\left(9\,b\,d\,f\,x^2\,\text{AppellF1}\left[2,-m,-n,3,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]\right)\Big/\left(6\,d\,f\,\text{AppellF1}\left[2,-m,-n,3,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]+2\,e\,f\,m\,x\,\text{AppellF1}\left[3,1-m,-n,4,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]+2\,d\,g\,n\,x\,\text{AppellF1}\left[3,-m,1-n,4,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]\right)+\left(4\,c\,d\,f\,x^3\,\text{AppellF1}\left[3,-m,-n,4,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]\right)\Big/\left(4\,d\,f\,\text{AppellF1}\left[3,-m,-n,4,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]+e\,f\,m\,x\,\text{AppellF1}\left[4,1-m,-n,5,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]+d\,g\,n\,x\,\text{AppellF1}\left[4,-m,1-n,5,-\frac{e\,x}{d},-\frac{g\,x}{f}\right]\right)+\frac{3\,a\left(\frac{g(d+e\,x)}{-e\,f+d\,g}\right)^{-m}(f+g\,x)\,\text{Hypergeometric2F1}\left[-m,1+n,2+n,\frac{e(f+g\,x)}{e\,f-d\,g}\right]}{g(1+n)}\right)$$

Problem 954: Unable to integrate problem.

$$\int(d+e\,x)^m(f+g\,x)^2\left(a+b\,x+c\,x^2\right)^p\,dx$$

Optimal (type 6, 525 leaves, 6 steps):

$$\frac{g^2(d+e\,x)^{1+m}\left(a+b\,x+c\,x^2\right)^{1+p}}{c\,e\,(3+m+2\,p)}+\frac{1}{c\,e^3(1+m)(3+m+2\,p)}\left(e\,(b\,d-a\,e)\,g^2(1+m)+c\left(2\,d^2g^2(1+p)+e^2f^2(3+m+2\,p)-2\,d\,e\,f\,g\,(3+m+2\,p)\right)\right)(d+e\,x)^{1+m}\left(a+b\,x+c\,x^2\right)^p\left(1-\frac{2\,c\,(d+e\,x)}{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)e}\right)^{-p}\left(1-\frac{2\,c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\right)^{-p}\text{AppellF1}\left[1+m,-p,-p,2+m,\frac{2\,c\,(d+e\,x)}{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)e},\frac{2\,c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\right]-\frac{1}{c\,e^3(2+m)(3+m+2\,p)}g\left(b\,e\,g\,(2+m+p)+2\,c\,(d\,g\,(1+p)-e\,f\,(3+m+2\,p))\right)(d+e\,x)^{2+m}\left(a+b\,x+c\,x^2\right)^p\left(1-\frac{2\,c\,(d+e\,x)}{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)e}\right)^{-p}\left(1-\frac{2\,c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\right)^{-p}\text{AppellF1}\left[2+m,-p,-p,3+m,\frac{2\,c\,(d+e\,x)}{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)e},\frac{2\,c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\right]$$

Result (type 8, 29 leaves):

$$\int(d+e\,x)^m(f+g\,x)^2\left(a+b\,x+c\,x^2\right)^p\,dx$$

- **Problem 955: Unable to integrate problem.**

$$\int (d+e x)^m (f+g x) \left(a+b x+c x^2\right)^p \, dx$$

Optimal (type 6, 384 leaves, 5 steps):

$$\frac{1}{e^2 (1+m)} (e f-d g) (d+e x)^{1+m} \left(a+b x+c x^2\right)^p \left(1-\frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\right)^{-p} \left(1-\frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right)^{-p} \text{AppellF1}\left[1+m,-p,-p,2+m,\frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e},\frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]+\frac{1}{e^2 (2+m)} g (d+e x)^{2+m} \left(a+b x+c x^2\right)^p \left(1-\frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\right)^{-p} \left(1-\frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right)^{-p} \text{AppellF1}\left[2+m,-p,-p,3+m,\frac{2 c (d+e x)}{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e},\frac{2 c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]$$

Result (type 8, 27 leaves):

$$\int (d+e x)^m (f+g x) \left(a+b x+c x^2\right)^p \, dx$$

- **Problem 958: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2 x^2}}\, x^2 \sqrt{d+e x}} \, dx$$

Optimal (type 4, 89 leaves, 5 steps):

$$-\frac{2 \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2}\, \text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right],\frac{2 e}{c d+e}\right]}{\sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}$$

Result (type 4, 188 leaves):

$$-\frac{1}{d\sqrt{-\frac{c\,d+e}{c}}\sqrt{1-\frac{1}{c^2 x^2}}\,x}$$

$$2\,i\sqrt{\frac{e\,(-1+c\,x)}{c\,(d+e\,x)}}\,(d+e\,x)\sqrt{\frac{e+c\,e\,x}{c\,d+c\,e\,x}}\left(\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,d+e}{c}}}{\sqrt{d+e\,x}}\right],\frac{c\,d-e}{c\,d+e}\right]-\text{EllipticPi}\left[\frac{c\,d}{c\,d+e},\,i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{c\,d+e}{c}}}{\sqrt{d+e\,x}}\right],\frac{c\,d-e}{c\,d+e}\right]\right)$$

Test results for the 123 problems in "1.2.1.5 (a+b x+c x^2)^p (d+e x+f x^2)^q.m"

- **Problem 2: Result more than twice size of optimal antiderivative.**

$$\int\frac{1}{\sqrt{d+e\,x+f\,x^2}\left(a+b\,x+\frac{b\,f\,x^2}{e}\right)}\,dx$$

Optimal (type 3, 82 leaves, 2 steps):

$$-\frac{2\sqrt{e}\,\text{ArcTanh}\left[\frac{\sqrt{b\,d-a\,e}\,(e+2\,f\,x)}{\sqrt{e}\,\sqrt{b\,e-4\,a\,f}\,\sqrt{d+e\,x+f\,x^2}}\right]}{\sqrt{b\,d-a\,e}\,\sqrt{b\,e-4\,a\,f}}$$

Result (type 3, 304 leaves):

$$\frac{1}{\sqrt{b\,d-a\,e}\,\sqrt{b\,e-4\,a\,f}}\sqrt{e}\,\Big(-\text{Log}\left[b\,e+\sqrt{b}\,\sqrt{e}\,\sqrt{b\,e-4\,a\,f}+2\,b\,f\,x\right]+\text{Log}\left[-\sqrt{b}\,\sqrt{e}\,\sqrt{b\,e-4\,a\,f}+b\,(e+2\,f\,x)\right]-$$
$$\text{Log}\left[\sqrt{b}\,\sqrt{e}\,\sqrt{b\,e-4\,a\,f}\,\left(e^2-4\,d\,f\right)-b\,e^2\,(e+2\,f\,x)+4\,a\,e\,f\,(e+2\,f\,x)-4\sqrt{e}\,\sqrt{b\,d-a\,e}\,f\sqrt{b\,e-4\,a\,f}\,\sqrt{d+x\,(e+f\,x)}\right]+$$
$$\text{Log}\left[\sqrt{b}\,\sqrt{e}\,\sqrt{b\,e-4\,a\,f}\,\left(e^2-4\,d\,f\right)+b\,e^2\,(e+2\,f\,x)-4\left(a\,e\,f\,(e+2\,f\,x)+\sqrt{e}\,\sqrt{b\,d-a\,e}\,f\sqrt{b\,e-4\,a\,f}\,\sqrt{d+x\,(e+f\,x)}\right)\right]\Big)$$

- **Problem 3: Result more than twice size of optimal antiderivative.**

$$\int\frac{1}{\sqrt{a+b\,x+c\,x^2}\,\left(d+b\,x+c\,x^2\right)}\,dx$$

Optimal (type 3, 66 leaves, 2 steps):

$$-\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a-d}\,(b+2\,c\,x)}{\sqrt{b^2-4\,c\,d}\,\sqrt{a+b\,x+c\,x^2}}\right]}{\sqrt{a-d}\,\sqrt{b^2-4\,c\,d}}$$

Result (type 3, 249 leaves):

$$\frac{1}{\sqrt{a-d}\,\sqrt{b^2-4\,c\,d}}\left(\text{Log}\left[b-\sqrt{b^2-4\,c\,d}+2\,c\,x\right]-\text{Log}\left[b+\sqrt{b^2-4\,c\,d}+2\,c\,x\right]-\right.$$
$$\text{Log}\left[-b^3+4\,b\,c\,d+b^2\left(\sqrt{b^2-4\,c\,d}-2\,c\,x\right)+4\,c\left(-a\,\sqrt{b^2-4\,c\,d}+2\,c\,d\,x-\sqrt{a-d}\,\sqrt{b^2-4\,c\,d}\,\sqrt{a+x\,(b+c\,x)}\right)\right]+$$
$$\left.\text{Log}\left[b^3-4\,b\,c\,d+b^2\left(\sqrt{b^2-4\,c\,d}+2\,c\,x\right)-4\,c\left(a\,\sqrt{b^2-4\,c\,d}+2\,c\,d\,x+\sqrt{a-d}\,\sqrt{b^2-4\,c\,d}\,\sqrt{a+x\,(b+c\,x)}\right)\right]\right)$$

- **Problem 4: Result more than twice size of optimal antiderivative.**

$$\int\frac{1}{\sqrt{a+b\,x+c\,x^2}\,\left(d+b\,x+c\,x^2\right)^2}\,dx$$

Optimal (type 3, 129 leaves, 4 steps):

$$-\frac{(b+2\,c\,x)\,\sqrt{a+b\,x+c\,x^2}}{(a-d)\,\left(b^2-4\,c\,d\right)\,\left(d+b\,x+c\,x^2\right)}+\frac{\left(b^2+4\,c\,(a-2\,d)\right)\,\text{ArcTanh}\left[\frac{\sqrt{a-d}\,(b+2\,c\,x)}{\sqrt{b^2-4\,c\,d}\,\sqrt{a+b\,x+c\,x^2}}\right]}{(a-d)^{3/2}\,\left(b^2-4\,c\,d\right)^{3/2}}$$

Result (type 3, 339 leaves):

$$\frac{1}{2\,(a-d)^{3/2}\,\left(b^2-4\,c\,d\right)^{3/2}\,(d+x\,(b+c\,x))}\left(-2\,\sqrt{a-d}\,\sqrt{b^2-4\,c\,d}\,(b+2\,c\,x)\,\sqrt{a+x\,(b+c\,x)}\right.-$$
$$\left(b^2+4\,c\,(a-2\,d)\right)\,(d+x\,(b+c\,x))\,\text{Log}\left[b-\sqrt{b^2-4\,c\,d}+2\,c\,x\right]+\left(b^2+4\,c\,(a-2\,d)\right)\,(d+x\,(b+c\,x))\,\text{Log}\left[b+\sqrt{b^2-4\,c\,d}+2\,c\,x\right]-$$
$$\left(b^2+4\,c\,(a-2\,d)\right)\,(d+x\,(b+c\,x))\,\text{Log}\left[b^2+b\,\sqrt{b^2-4\,c\,d}+2\,c\left(-2\,a+\sqrt{b^2-4\,c\,d}\,x-2\,\sqrt{a-d}\,\sqrt{a+x\,(b+c\,x)}\right)\right]+$$
$$\left.\left(b^2+4\,c\,(a-2\,d)\right)\,(d+x\,(b+c\,x))\,\text{Log}\left[-b^2+b\,\sqrt{b^2-4\,c\,d}+2\,c\left(2\,a+\sqrt{b^2-4\,c\,d}\,x+2\,\sqrt{a-d}\,\sqrt{a+x\,(b+c\,x)}\right)\right]\right)$$

- **Problem 5: Result more than twice size of optimal antiderivative.**

$$\int\frac{1}{\sqrt{a+b\,x+c\,x^2}\,\left(d+b\,x+c\,x^2\right)^3}\,dx$$

Optimal (type 3, 224 leaves, 5 steps):

$$-\frac{(b+2\,c\,x)\,\sqrt{a+b\,x+c\,x^2}}{2\,(a-d)\,\left(b^2-4\,c\,d\right)\,\left(d+b\,x+c\,x^2\right)^2}+\frac{3\,\left(b^2+4\,c\,(a-2\,d)\right)\,(b+2\,c\,x)\,\sqrt{a+b\,x+c\,x^2}}{4\,(a-d)^2\,\left(b^2-4\,c\,d\right)^2\,\left(d+b\,x+c\,x^2\right)}-$$
$$\frac{\left(3\,b^4+8\,b^2\,c\,(a-4\,d)+16\,c^2\,\left(3\,a^2-8\,a\,d+8\,d^2\right)\right)\,\text{ArcTanh}\left[\frac{\sqrt{a-d}\,(b+2\,c\,x)}{\sqrt{b^2-4\,c\,d}\,\sqrt{a+b\,x+c\,x^2}}\right]}{4\,(a-d)^{5/2}\,\left(b^2-4\,c\,d\right)^{5/2}}$$

Result (type 3, 486 leaves):

$$\frac{1}{8\,(a-d)^{5/2}\,\left(b^2-4\,c\,d\right)^{5/2}\,(d+x\,(b+c\,x))^2}$$
$$\Bigg(-2\,\sqrt{a-d}\,\sqrt{b^2-4\,c\,d}\,(b+2\,c\,x)\,\sqrt{a+x\,(b+c\,x)}\,\left(2\,(a-d)\,\left(b^2-4\,c\,d\right)-3\,\left(b^2+4\,c\,(a-2\,d)\right)\,(d+x\,(b+c\,x))\right)+$$
$$\left(3\,b^4+8\,b^2\,c\,(a-4\,d)+16\,c^2\,\left(3\,a^2-8\,a\,d+8\,d^2\right)\right)\,(d+x\,(b+c\,x))^2\,\text{Log}\left[b-\sqrt{b^2-4\,c\,d}+2\,c\,x\right]-$$
$$\left(3\,b^4+8\,b^2\,c\,(a-4\,d)+16\,c^2\,\left(3\,a^2-8\,a\,d+8\,d^2\right)\right)\,(d+x\,(b+c\,x))^2\,\text{Log}\left[b+\sqrt{b^2-4\,c\,d}+2\,c\,x\right]+$$
$$\left(3\,b^4+8\,b^2\,c\,(a-4\,d)+16\,c^2\,\left(3\,a^2-8\,a\,d+8\,d^2\right)\right)\,(d+x\,(b+c\,x))^2\,\text{Log}\left[b^2+b\,\sqrt{b^2-4\,c\,d}+2\,c\,\left(-2\,a+\sqrt{b^2-4\,c\,d}\,x-2\,\sqrt{a-d}\,\sqrt{a+x\,(b+c\,x)}\right)\right]-$$
$$\left(3\,b^4+8\,b^2\,c\,(a-4\,d)+16\,c^2\,\left(3\,a^2-8\,a\,d+8\,d^2\right)\right)\,(d+x\,(b+c\,x))^2\,\text{Log}\left[-b^2+b\,\sqrt{b^2-4\,c\,d}+2\,c\,\left(2\,a+\sqrt{b^2-4\,c\,d}\,x+2\,\sqrt{a-d}\,\sqrt{a+x\,(b+c\,x)}\right)\right]\Bigg)$$

- **Problem 6: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{a+b\,x+c\,x^2}\,\left(d+b\,x+c\,x^2\right)^4}\,dx$$

Optimal (type 3, 328 leaves, 6 steps):

$$-\frac{(b+2\,c\,x)\,\sqrt{a+b\,x+c\,x^2}}{3\,(a-d)\,\left(b^2-4\,c\,d\right)\,\left(d+b\,x+c\,x^2\right)^3}+\frac{5\,\left(b^2+4\,c\,(a-2\,d)\right)\,(b+2\,c\,x)\,\sqrt{a+b\,x+c\,x^2}}{12\,(a-d)^2\,\left(b^2-4\,c\,d\right)^2\,\left(d+b\,x+c\,x^2\right)^2}-$$
$$\frac{\left(15\,b^4+8\,b^2\,c\,(7\,a-22\,d)+16\,c^2\,\left(15\,a^2-44\,a\,d+44\,d^2\right)\right)\,(b+2\,c\,x)\,\sqrt{a+b\,x+c\,x^2}}{24\,(a-d)^3\,\left(b^2-4\,c\,d\right)^3\,\left(d+b\,x+c\,x^2\right)}+$$
$$\frac{\left(b^2+4\,c\,(a-2\,d)\right)\,\left(5\,b^4-8\,b^2\,c\,(a+4\,d)+16\,c^2\,\left(5\,a^2-8\,a\,d+8\,d^2\right)\right)\,\text{ArcTanh}\left[\frac{\sqrt{a-d}\,(b+2\,c\,x)}{\sqrt{b^2-4\,c\,d}\,\sqrt{a+b\,x+c\,x^2}}\right]}{8\,(a-d)^{7/2}\,\left(b^2-4\,c\,d\right)^{7/2}}$$

Result (type 3, 901 leaves):

$$\frac{1}{\sqrt{a+x\,(b+c\,x)}}\left(a+b\,x+c\,x^2\right)\left(-\frac{-b-2\,c\,x}{3\,(a-d)\,\left(-b^2+4\,c\,d\right)\,\left(d+b\,x+c\,x^2\right)^3}+\frac{5\,\left(b^3+4\,a\,b\,c-8\,b\,c\,d+2\,b^2\,c\,x+8\,a\,c^2\,x-16\,c^2\,d\,x\right)}{12\,(a-d)^2\,\left(-b^2+4\,c\,d\right)^2\,\left(d+b\,x+c\,x^2\right)^2}+\left(15\,b^5+56\,a\,b^3\,c+240\,a^2\,b\,c^2-176\,b^3\,c\,d-704\,a\,b\,c^2\,d+704\,b\,c^2\,d^2+30\,b^4\,c\,x+112\,a\,b^2\,c^2\,x+480\,a^2\,c^3\,x-352\,b^2\,c^2\,d\,x-1408\,a\,c^3\,d\,x+1408\,c^3\,d^2\,x\right)\Big/\left(24\,(a-d)^3\,\left(-b^2+4\,c\,d\right)^3\,\left(d+b\,x+c\,x^2\right)\right)\right)+$$

$$\left(\left(b^2+4\,a\,c-8\,c\,d\right)\,\left(5\,b^4-8\,a\,b^2\,c+80\,a^2\,c^2-32\,b^2\,c\,d-128\,a\,c^2\,d+128\,c^2\,d^2\right)\,\sqrt{a+b\,x+c\,x^2}\,\mathrm{Log}\left[b-\sqrt{b^2-4\,c\,d}+2\,c\,x\right]\right)\Big/\left(16\,\sqrt{a-d}\,(-a+d)^3\,\left(b^2-4\,c\,d\right)^{7/2}\,\sqrt{a+x\,(b+c\,x)}\right)-$$

$$\left(\left(b^2+4\,a\,c-8\,c\,d\right)\,\left(5\,b^4-8\,a\,b^2\,c+80\,a^2\,c^2-32\,b^2\,c\,d-128\,a\,c^2\,d+128\,c^2\,d^2\right)\,\sqrt{a+b\,x+c\,x^2}\,\mathrm{Log}\left[b+\sqrt{b^2-4\,c\,d}+2\,c\,x\right]\right)\Big/\left(16\,\sqrt{a-d}\,(-a+d)^3\,\left(b^2-4\,c\,d\right)^{7/2}\,\sqrt{a+x\,(b+c\,x)}\right)+$$

$$\left(\left(b^2+4\,a\,c-8\,c\,d\right)\,\left(5\,b^4-8\,a\,b^2\,c+80\,a^2\,c^2-32\,b^2\,c\,d-128\,a\,c^2\,d+128\,c^2\,d^2\right)\,\sqrt{a+b\,x+c\,x^2}\,\mathrm{Log}\left[b^2-4\,a\,c+b\,\sqrt{b^2-4\,c\,d}+2\,c\,\sqrt{b^2-4\,c\,d}\,x-4\,c\,\sqrt{a-d}\,\sqrt{a+b\,x+c\,x^2}\right]\right)\Big/\left(16\,\sqrt{a-d}\,(-a+d)^3\,\left(b^2-4\,c\,d\right)^{7/2}\,\sqrt{a+x\,(b+c\,x)}\right)-$$

$$\left(\left(b^2+4\,a\,c-8\,c\,d\right)\,\left(5\,b^4-8\,a\,b^2\,c+80\,a^2\,c^2-32\,b^2\,c\,d-128\,a\,c^2\,d+128\,c^2\,d^2\right)\,\sqrt{a+b\,x+c\,x^2}\,\mathrm{Log}\left[-b^2+4\,a\,c+b\,\sqrt{b^2-4\,c\,d}+2\,c\,\sqrt{b^2-4\,c\,d}\,x+4\,c\,\sqrt{a-d}\,\sqrt{a+b\,x+c\,x^2}\right]\right)\Big/\left(16\,\sqrt{a-d}\,(-a+d)^3\,\left(b^2-4\,c\,d\right)^{7/2}\,\sqrt{a+x\,(b+c\,x)}\right)$$

- **Problem 7: Result more than twice size of optimal antiderivative.**

$$\int\frac{1}{\sqrt{d+e\,x+f\,x^2}\,\left(a\,e+b\,e\,x+b\,f\,x^2\right)^2}\,dx$$

Optimal (type 3, 162 leaves, 4 steps):

$$-\frac{b\,(e+2\,f\,x)\,\sqrt{d+e\,x+f\,x^2}}{e\,(b\,d-a\,e)\,(b\,e-4\,a\,f)\,\left(a\,e+b\,e\,x+b\,f\,x^2\right)}-\frac{\left(8\,a\,e\,f-b\,\left(e^2+4\,d\,f\right)\right)\,\mathrm{ArcTanh}\left[\frac{\sqrt{b\,d-a\,e}\,(e+2\,f\,x)}{\sqrt{e}\,\sqrt{b\,e-4\,a\,f}\,\sqrt{d+e\,x+f\,x^2}}\right]}{e^{3/2}\,(b\,d-a\,e)^{3/2}\,(b\,e-4\,a\,f)^{3/2}}$$

Result (type 3, 463 leaves):

$$-\frac{1}{2 e^{3/2} (b d-a e)^{3/2} (b e-4 a f)^{3/2} (a e+b x (e+f x))} \Big(2 b \sqrt{e} \sqrt{b d-a e} \sqrt{b e-4 a f} (e+2 f x) \sqrt{d+x (e+f x)} +$$

$$\left(-8 a e f+b \left(e^2+4 d f\right)\right) (a e+b x (e+f x)) \operatorname{Log}\left[-\sqrt{b} \sqrt{e} \sqrt{b e-4 a f}+b (e+2 f x)\right] -$$

$$\left(-8 a e f+b \left(e^2+4 d f\right)\right) (a e+b x (e+f x)) \operatorname{Log}\left[\sqrt{b} \sqrt{e} \sqrt{b e-4 a f}+b (e+2 f x)\right] + \left(-8 a e f+b \left(e^2+4 d f\right)\right) (a e+b x (e+f x))$$

$$\operatorname{Log}\left[\sqrt{b} \left(e^{3/2} \sqrt{b e-4 a f}+\sqrt{b} \left(e^2-4 d f\right)+2 \sqrt{e} f \sqrt{b e-4 a f} x-4 \sqrt{b d-a e} f \sqrt{d+x (e+f x)}\right)\right] - \left(-8 a e f+b \left(e^2+4 d f\right)\right)$$

$$(a e+b x (e+f x)) \operatorname{Log}\left[\sqrt{b} \left(e^{3/2} \sqrt{b e-4 a f}-\sqrt{b} \left(e^2-4 d f\right)+2 \sqrt{e} f \sqrt{b e-4 a f} x+4 \sqrt{b d-a e} f \sqrt{d+x (e+f x)}\right)\right]\Big)$$

- **Problem 13: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{a+b x+c x^2} \sqrt{d+f x^2}} \, dx$$

Optimal (type 4, 1077 leaves, 3 steps):

$$-\Bigg(\Bigg(\left(b^2 d+b \sqrt{b^2-4 a c}\, d-2 a (c d-a f)\right)^{1/4} \left(b+\sqrt{b^2-4 a c}+2 c x\right)^{3/2} \sqrt{2 a+\left(b+\sqrt{b^2-4 a c}\right) x}$$

$$\sqrt{\frac{\left(4 a c-\left(b+\sqrt{b^2-4 a c}\right)^2\right)^2 \left(d+f x^2\right)}{\left(\left(b+\sqrt{b^2-4 a c}\right)^2 d+4 a^2 f\right) \left(b+\sqrt{b^2-4 a c}+2 c x\right)^2}} \left(1+\frac{\sqrt{2 c^2 d-2 a c f+b \left(b+\sqrt{b^2-4 a c}\right) f} \left(2 a+\left(b+\sqrt{b^2-4 a c}\right) x\right)}{\sqrt{b^2 d+b \sqrt{b^2-4 a c}\, d-2 a (c d-a f)} \left(b+\sqrt{b^2-4 a c}+2 c x\right)}\right)$$

$$\sqrt{\frac{1-\frac{4 \left(b+\sqrt{b^2-4 a c}\right) (c d+a f) \left(2 a+\left(b+\sqrt{b^2-4 a c}\right) x\right)}{\left(\left(b+\sqrt{b^2-4 a c}\right)^2 d+4 a^2 f\right) \left(b+\sqrt{b^2-4 a c}+2 c x\right)}+\frac{\left(4 c^2 d+\left(b+\sqrt{b^2-4 a c}\right)^2 f\right) \left(2 a+\left(b+\sqrt{b^2-4 a c}\right) x\right)^2}{\left(\left(b+\sqrt{b^2-4 a c}\right)^2 d+4 a^2 f\right) \left(b+\sqrt{b^2-4 a c}+2 c x\right)^2}}{\left(1+\frac{\sqrt{2 c^2 d-2 a c f+b \left(b+\sqrt{b^2-4 a c}\right) f} \left(2 a+\left(b+\sqrt{b^2-4 a c}\right) x\right)}{\sqrt{b^2 d+b \sqrt{b^2-4 a c}\, d-2 a (c d-a f)} \left(b+\sqrt{b^2-4 a c}+2 c x\right)}\right)^2}}$$

$$\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\left(2\,c^2\,d-2\,a\,c\,f+b\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,f\right)^{1/4}\,\sqrt{2\,a+\left(b+\sqrt{b^2-4\,a\,c}\right)\,x}}{\left(b^2\,d+b\,\sqrt{b^2-4\,a\,c}\,d-2\,a\,(c\,d-a\,f)\right)^{1/4}\,\sqrt{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}}\right],\right.$$

$$\left.\frac{1}{2}\left(1+\frac{\left(b+\sqrt{b^2-4\,a\,c}\right)\,(c\,d+a\,f)}{\sqrt{2\,c^2\,d-2\,a\,c\,f+b\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,f}\,\sqrt{b^2\,d+b\,\sqrt{b^2-4\,a\,c}\,d-2\,a\,(c\,d-a\,f)}}\right)\right]\Bigg/$$

$$\left(\left(4\,a\,c-\left(b+\sqrt{b^2-4\,a\,c}\right)^2\right)\,\left(2\,c^2\,d-2\,a\,c\,f+b\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,f\right)^{1/4}\,\sqrt{a+b\,x+c\,x^2}\,\sqrt{d+f\,x^2}\right.$$

$$\left.\sqrt{\left(1-\frac{4\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,(c\,d+a\,f)\,\left(2\,a+\left(b+\sqrt{b^2-4\,a\,c}\right)\,x\right)}{\left(\left(b+\sqrt{b^2-4\,a\,c}\right)^2\,d+4\,a^2\,f\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}+\frac{\left(4\,c^2\,d+\left(b+\sqrt{b^2-4\,a\,c}\right)^2\,f\right)\,\left(2\,a+\left(b+\sqrt{b^2-4\,a\,c}\right)\,x\right)^2}{\left(\left(b+\sqrt{b^2-4\,a\,c}\right)^2\,d+4\,a^2\,f\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)^2}\right)}\right)$$

Result (type 4, 600 leaves) :

$$-\left(2\sqrt{2}\left(-b+\sqrt{b^2-4ac}-2cx\right)\left(-i\sqrt{d}+\sqrt{f}\,x\right)\sqrt{-\frac{c\sqrt{b^2-4ac}\left(i\sqrt{d}+\sqrt{f}\,x\right)}{\left(-2ic\sqrt{d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{f}\right)\left(-b+\sqrt{b^2-4ac}-2cx\right)}}\right.$$

$$\sqrt{\frac{c\left(-i\sqrt{d}\left(\sqrt{b^2-4ac}+2cx\right)+\sqrt{f}\left(-2a+\sqrt{b^2-4ac}\,x\right)+b\left(-i\sqrt{d}-\sqrt{f}\,x\right)\right)}{\left(2ic\sqrt{d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{f}\right)\left(-b+\sqrt{b^2-4ac}-2cx\right)}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-2ic\sqrt{d}+\left(-b+\sqrt{b^2-4ac}\right)\sqrt{f}\right)\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2ic\sqrt{d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{f}\right)\left(-b+\sqrt{b^2-4ac}-2cx\right)}}\right],\frac{cd-i\sqrt{b^2-4ac}\sqrt{d}\sqrt{f}+af}{cd+i\sqrt{b^2-4ac}\sqrt{d}\sqrt{f}+af}\right]\right)\Bigg/$$

$$\left(\left(-2ic\sqrt{d}+\left(-b+\sqrt{b^2-4ac}\right)\sqrt{f}\right)\sqrt{\frac{ic\sqrt{b^2-4ac}\left(\sqrt{d}+i\sqrt{f}\,x\right)}{\left(2ic\sqrt{d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{f}\right)\left(-b+\sqrt{b^2-4ac}-2cx\right)}}\sqrt{d+fx^2}\sqrt{a+x(b+cx)}\right)$$

- **Problem 14: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2}\,dx$$

Optimal (type 3, 98 leaves, 16 steps) :

$$-\frac{1}{2}\text{ArcSin}[2+x]-\frac{\text{ArcTan}\left[\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]}{\sqrt{2}}+\frac{\text{ArcTan}\left[\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]}{\sqrt{2}}-\frac{1}{2}\text{ArcTanh}\left[\frac{x}{\sqrt{-3-4x-x^2}}\right]$$

Result (type 3, 982 leaves) :

$$\frac{1}{8}\left(-4\,\text{ArcSin}[2+x]+\frac{1}{\sqrt{1-2\,i\,\sqrt{2}}}2\,i\,\left(i+2\,\sqrt{2}\right)\right.$$

$$\text{ArcTan}\Big[\Big(60+51\,i\,\sqrt{2}+\left(-16+6\,i\,\sqrt{2}\right)x^4+54\,i\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}+x\left(68+176\,i\,\sqrt{2}+99\,i\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+$$

$$2\,i\,x^3\left(34\left(i+\sqrt{2}\right)+9\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+i\,x^2\left(44\,i+185\,\sqrt{2}+72\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\Big)\Big/$$

$$\left(93\,i+150\,\sqrt{2}+20\left(17\,i+22\,\sqrt{2}\right)x+\left(493\,i+466\,\sqrt{2}\right)x^2+16\left(19\,i+13\,\sqrt{2}\right)x^3+\left(66\,i+32\,\sqrt{2}\right)x^4\right)\Big]+2\,\sqrt{1+2\,i\,\sqrt{2}}$$

$$\text{ArcTan}\Big[\Big(-60+51\,i\,\sqrt{2}+2\left(8+3\,i\,\sqrt{2}\right)x^4+54\,i\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}+2\,x^3\left(34+34\,i\,\sqrt{2}+9\,i\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+$$

$$x^2\left(44+185\,i\,\sqrt{2}+72\,i\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+i\,x\left(68\,i+176\,\sqrt{2}+99\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\Big)\Big/$$

$$\left(-93\,i+150\,\sqrt{2}+20\left(-17\,i+22\,\sqrt{2}\right)x+\left(-493\,i+466\,\sqrt{2}\right)x^2+16\left(-19\,i+13\,\sqrt{2}\right)x^3+\left(-66\,i+32\,\sqrt{2}\right)x^4\right)\Big]-$$

$$\frac{\left(-i+2\,\sqrt{2}\right)\text{Log}\left[4\left(3+4\,x+2\,x^2\right)^2\right]}{\sqrt{1+2\,i\,\sqrt{2}}}-\frac{\left(i+2\,\sqrt{2}\right)\text{Log}\left[4\left(3+4\,x+2\,x^2\right)^2\right]}{\sqrt{1-2\,i\,\sqrt{2}}}+\frac{1}{\sqrt{1-2\,i\,\sqrt{2}}}$$

$$\left(i+2\,\sqrt{2}\right)\text{Log}\Big[\left(3+4\,x+2\,x^2\right)\left(3+6\,i\,\sqrt{2}+\left(2+2\,i\,\sqrt{2}\right)x^2-2\,\sqrt{2-4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}+x\left(4+8\,i\,\sqrt{2}-2\,\sqrt{2-4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\right)\Big]+$$

$$\frac{1}{\sqrt{1+2\,i\,\sqrt{2}}}\left(-i+2\,\sqrt{2}\right)$$

$$\left.\text{Log}\Big[\left(3+4\,x+2\,x^2\right)\left(3-6\,i\,\sqrt{2}+\left(2-2\,i\,\sqrt{2}\right)x^2-2\,\sqrt{2+4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}-2\,x\left(-2+4\,i\,\sqrt{2}+\sqrt{2+4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\right)\Big]\right)$$

- **Problem 62: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{3-x+2\,x^2}}{2+3\,x+5\,x^2}\,dx$$

Optimal (type 3, 174 leaves, 8 steps) :

$$-\frac{1}{5}\sqrt{2}\,\text{ArcSinh}\left[\frac{1-4x}{\sqrt{23}}\right]+\frac{1}{5}\sqrt{\frac{11}{31}\left(13+10\sqrt{2}\right)}\,\text{ArcTan}\left[\frac{\sqrt{\frac{11}{62\left(13+10\sqrt{2}\right)}}\left(6+7\sqrt{2}+\left(20+13\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]-$$

$$\frac{1}{5}\sqrt{\frac{11}{31}\left(-13+10\sqrt{2}\right)}\,\text{ArcTanh}\left[\frac{\sqrt{\frac{11}{62\left(-13+10\sqrt{2}\right)}}\left(6-7\sqrt{2}+\left(20-13\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]$$

Result (type 3, 1133 leaves) :

$$\frac{1}{5}\sqrt{2}\,\text{ArcSinh}\left[\frac{-1+4x}{\sqrt{23}}\right]-\frac{1}{5\sqrt{\frac{62}{11}\left(13+i\sqrt{31}\right)}}i\left(-13i+\sqrt{31}\right)$$

$$\text{ArcTan}\Big[\left(31\left(7588i+4224\sqrt{31}-27836ix+3872\sqrt{31}\,x+4347ix^2+2706\sqrt{31}\,x^2-31860ix^3+2970\sqrt{31}\,x^3-8675ix^4+1100\sqrt{31}\,x^4\right)\right)\Big/$$

$$\Big(65472+35044i\sqrt{31}+1083016x-46668i\sqrt{31}\,x+340318x^2-308889i\sqrt{31}\,x^2+514910x^3-143180i\sqrt{31}\,x^3+$$

$$443300x^4-262775i\sqrt{31}\,x^4-1000i\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+2500i\sqrt{682\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}+$$

$$3500i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^2\sqrt{3-x+2x^2}+10000i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\Big)\Big]-\frac{1}{5\sqrt{\frac{62}{11}\left(-13+i\sqrt{31}\right)}}$$

$$i\left(13i+\sqrt{31}\right)\text{ArcTanh}\Big[\Big(-65472i-35044\sqrt{31}-1083016ix+46668\sqrt{31}\,x-340318ix^2+308889\sqrt{31}\,x^2-514910ix^3+$$

$$143180\sqrt{31}\,x^3-443300ix^4+262775\sqrt{31}\,x^4-63000\sqrt{22\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-72500\sqrt{22\left(-13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}-$$

$$124500\sqrt{22\left(-13+i\sqrt{31}\right)}\,x^2\sqrt{3-x+2x^2}+55000\sqrt{22\left(-13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\Big)\Big/\Big(1764772i+130944\sqrt{31}+$$

$$2352916ix+120032\sqrt{31}\,x+3090243ix^2+83886\sqrt{31}\,x^2-2052340ix^3+92070\sqrt{31}\,x^3+1493925ix^4+34100\sqrt{31}\,x^4\Big)\Big]-$$

$$\frac{\left(-13i+\sqrt{31}\right)\text{Log}\left[\left(-3i+\sqrt{31}-10ix\right)^2\left(3i+\sqrt{31}+10ix\right)^2\right]}{10\sqrt{\frac{62}{11}\left(13+i\sqrt{31}\right)}}+\frac{i\left(13i+\sqrt{31}\right)\text{Log}\left[\left(-3i+\sqrt{31}-10ix\right)^2\left(3i+\sqrt{31}+10ix\right)^2\right]}{10\sqrt{\frac{62}{11}\left(-13+i\sqrt{31}\right)}}-$$

$$\frac{1}{10\sqrt{\frac{62}{11}\left(-13+i\sqrt{31}\right)}}\, i\left(13\,i+\sqrt{31}\right)$$

$$\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-142\,i+66\sqrt{31}+469\,i\,x-22\sqrt{31}\,x+327\,i\,x^2+$$

$$44\sqrt{31}\,x^2+i\sqrt{682\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-4\,i\sqrt{682\left(-13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\Big)\Big]+$$

$$\frac{1}{10\sqrt{\frac{62}{11}\left(13+i\sqrt{31}\right)}}\left(-13\,i+\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-1858\,i+66\sqrt{31}+1041\,i\,x-22\sqrt{31}\,x-817\,i\,x^2+$$

$$44\sqrt{31}\,x^2-63\,i\sqrt{22\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+22\,i\sqrt{22\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\Big)\Big]$$

- **Problem 63: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^2}\,dx$$

Optimal (type 3, 188 leaves, 6 steps) :

$$\frac{(3+10x)\sqrt{3-x+2x^2}}{31\left(2+3x+5x^2\right)}+\frac{1}{62}\sqrt{\frac{1}{682}\left(70517+49942\sqrt{2}\right)}\,\text{ArcTan}\Big[\frac{\sqrt{\frac{11}{31\left(70517+49942\sqrt{2}\right)}}\left(419+277\sqrt{2}+\left(973+696\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\Big]-$$

$$\frac{1}{62}\sqrt{\frac{1}{682}\left(-70517+49942\sqrt{2}\right)}\,\text{ArcTanh}\Big[\frac{\sqrt{\frac{11}{31\left(-70517+49942\sqrt{2}\right)}}\left(419-277\sqrt{2}+\left(973-696\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\Big]$$

Result (type 3, 1066 leaves) :

$$\frac{(3+10x)\sqrt{3-x+2x^2}}{31\left(2+3x+5x^2\right)}-\frac{1}{31\sqrt{682\left(13+i\sqrt{31}\right)}}$$

$$i\left(-348\,i+11\sqrt{31}\right)\text{ArcTan}\Big[\Big(31\left(-5587181+4790313\,i\sqrt{31}+\left(27549757+1169289\,i\sqrt{31}\right)x+\left(-32828614+2670822\,i\sqrt{31}\right)x^2+\right.$$

$$20\left(1416861+85547\,i\sqrt{31}\right)x^3+50\,i\left(261413\,i+5324\sqrt{31}\right)x^4\Big)\Big)\Big/\Big(274003389\,i-48486603\sqrt{31}+34100\left(7656\,i+7013\sqrt{31}\right)x^4+$$

$$1248550\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+x^3\left(826454420\,i+92760910\sqrt{31}-12485500\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$x^2\left(95778716\,i+264613118\sqrt{31}-4369925\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$x\left(1344149367\,i+112716791\sqrt{31}-3121375\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\Big)\Big]+$$

$$\frac{1}{31\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}}\left(348-11\,i\sqrt{31}\right)\text{ArcTanh}\Big[\Big(11\Big(9\left(23473711\,i+1499997\sqrt{31}\right)+3\left(82253999\,i+1098423\sqrt{31}\right)x+$$

$$\left(273535156\,i+7526862\sqrt{31}\right)x^2+220\left(-1205429\,i+21917\sqrt{31}\right)x^3+2200\left(46458\,i+341\sqrt{31}\right)x^4\Big)\Big)\Big/$$

$$\Big(34100\left(-7656\,i+7013\sqrt{31}\right)x^4+x^2\left(-95778716\,i+264613118\sqrt{31}-155444475\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$x\left(-1344149367\,i+112716791\sqrt{31}-90519875\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$110x^3\left(-7513222\,i+843281\sqrt{31}+624275\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)-$$

$$3\left(91334463\,i+16162201\sqrt{31}+26219550\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\Big)\Big]-$$

$$\frac{\left(-348\,i+11\sqrt{31}\right)\text{Log}\left[400\left(2+3x+5x^2\right)^2\right]}{62\sqrt{682\left(13+i\sqrt{31}\right)}}+\frac{i\left(348\,i+11\sqrt{31}\right)\text{Log}\left[400\left(2+3x+5x^2\right)^2\right]}{62\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}}+$$

$$\frac{1}{62\sqrt{682\left(13+i\sqrt{31}\right)}}$$

$$\left(-348\,i+11\sqrt{31}\right)$$

$$\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-1858\,i+66\sqrt{31}+\left(-817\,i+44\sqrt{31}\right)x^2-$$

$$63\,i\sqrt{286+22\,i\sqrt{31}}\sqrt{3-x+2x^2}+x\left(1041\,i-22\sqrt{31}+22\,i\sqrt{286+22\,i\sqrt{31}}\sqrt{3-x+2x^2}\right)\Big)\Big]+$$

$$\frac{1}{62\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}}\left(348-11\,i\sqrt{31}\right)\mathrm{Log}\Big[\left(2+3x+5x^2\right)\Big(-142\,i+66\sqrt{31}+\left(327\,i+44\sqrt{31}\right)x^2+$$

$$i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}+x\left(469\,i-22\sqrt{31}-4\,i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\Big)\Big]$$

- **Problem 64: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^3}\,dx$$

Optimal (type 3, 223 leaves, 7 steps):

$$\frac{(3+10x)\sqrt{3-x+2x^2}}{62\left(2+3x+5x^2\right)^2}+\frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568\left(2+3x+5x^2\right)}+\frac{1}{169136}\sqrt{\frac{1}{682}\left(112285869463+79399380740\sqrt{2}\right)}$$

$$\mathrm{ArcTan}\left[\frac{\sqrt{\frac{11}{31\left(112285869463+79399380740\sqrt{2}\right)}}\left(509587+362788\sqrt{2}+\left(1235163+872375\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]-\frac{1}{169136}$$

$$\sqrt{\frac{1}{682}\left(-112285869463+79399380740\sqrt{2}\right)}\;\mathrm{ArcTanh}\left[\frac{\sqrt{\frac{11}{31\left(-112285869463+79399380740\sqrt{2}\right)}}\left(509587-362788\sqrt{2}+\left(1235163-872375\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]$$

Result (type 3, 1170 leaves):

$$\sqrt{3-x+2x^2}\left(\frac{3+10x}{62\left(2+3x+5x^2\right)^2}+\frac{3464+13665x}{84568\left(2+3x+5x^2\right)}\right)-\frac{1}{169136\sqrt{682\left(13+i\sqrt{31}\right)}}5\,i\left(-174475\,i+6521\sqrt{31}\right)$$

$$\mathrm{ArcTan}\Big[\Big(31\Big(779181710662\,i+621237299826\sqrt{31}-3659080865574\,i\,x+210477093398\sqrt{31}\,x+3786698475623\,i\,x^2+$$

$$345136479754\sqrt{31}\,x^2-3744647381480\,i\,x^3+254982903010\sqrt{31}\,x^3+1313174142725\,i\,x^4+46775785100\sqrt{31}\,x^4\Big)\Big)\Big/$$

$$\Big(31886584896738+6160809644426\,i\sqrt{31}+173254405285214\,x-13553199916122\,i\sqrt{31}\,x+18159288904922\,x^2-$$

$$36221356993731\,i\sqrt{31}\,x^2+103190181962890\,x^3-13468529326720\,i\sqrt{31}\,x^3+38797325297500\,x^4-32372991877825\,i\sqrt{31}\,x^4-$$

$$158\,798\,761\,480\,i\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+396\,996\,903\,700\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}+$$

$$555\,795\,665\,180\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^2\sqrt{3-x+2x^2}+1\,587\,987\,614\,800\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\Bigg)\Bigg]-$$

$$\frac{1}{169\,136\sqrt{682\left(-13+i\sqrt{31}\right)}}5\,i\left(174\,475\,i+6521\sqrt{31}\right)\text{ArcTanh}\Big[$$

$$\Big(-31\,886\,584\,896\,738\,i-6\,160\,809\,644\,426\sqrt{31}-173\,254\,405\,285\,214\,i\,x+13\,553\,199\,916\,122\sqrt{31}\,x-18\,159\,288\,904\,922\,i\,x^2+$$

$$36\,221\,356\,993\,731\sqrt{31}\,x^2-103\,190\,181\,962\,890\,i\,x^3+13\,468\,529\,326\,720\sqrt{31}\,x^3-38\,797\,325\,297\,500\,i\,x^4+32\,372\,991\,877\,825\sqrt{31}\,x^4-$$

$$10\,004\,321\,973\,240\sqrt{22\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-11\,512\,910\,207\,300\sqrt{22\left(-13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}-$$

$$19\,770\,445\,804\,260\sqrt{22\left(-13+i\sqrt{31}\right)}\,x^2\sqrt{3-x+2x^2}+8\,733\,931\,881\,400\sqrt{22\left(-13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\Big)\Big/$$

$$\Big(293\,442\,889\,929\,478\,i+19\,258\,356\,294\,606\sqrt{31}+350\,041\,661\,437\,994\,i\,x+6\,524\,789\,895\,338\sqrt{31}\,x+394\,738\,353\,028\,687\,i\,x^2+$$

$$10\,699\,230\,872\,374\sqrt{31}\,x^2-366\,664\,166\,073\,320\,i\,x^3+7\,904\,469\,993\,310\sqrt{31}\,x^3+153\,820\,084\,388\,525\,i\,x^4+1\,450\,049\,338\,100\sqrt{31}\,x^4\Big)\Big]-$$

$$\frac{5\left(-174\,475\,i+6521\sqrt{31}\right)\text{Log}\left[\left(-3\,i+\sqrt{31}-10\,i\,x\right)^2\left(3\,i+\sqrt{31}+10\,i\,x\right)^2\right]}{338\,272\sqrt{682\left(13+i\sqrt{31}\right)}}+$$

$$\frac{5\,i\left(174\,475\,i+6521\sqrt{31}\right)\text{Log}\left[\left(-3\,i+\sqrt{31}-10\,i\,x\right)^2\left(3\,i+\sqrt{31}+10\,i\,x\right)^2\right]}{338\,272\sqrt{682\left(-13+i\sqrt{31}\right)}}-$$

$$\Big(5\,i\left(174\,475\,i+6521\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-142\,i+66\sqrt{31}+469\,i\,x-22\sqrt{31}\,x+327\,i\,x^2+44\sqrt{31}\,x^2+$$

$$i\sqrt{682\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-4\,i\sqrt{682\left(-13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/\left(338\,272\sqrt{682\left(-13+i\sqrt{31}\right)}\right)+$$

$$\Big(5\left(-174\,475\,i+6521\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-1858\,i+66\sqrt{31}+1041\,i\,x-22\sqrt{31}\,x-817\,i\,x^2+44\sqrt{31}\,x^2-$$

$$63\,i\sqrt{22\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+22\,i\sqrt{22\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/\left(338\,272\sqrt{682\left(13+i\sqrt{31}\right)}\right)$$

Problem 69: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\left(3-x+2 x^2\right)^{3/2}}{2+3 x+5 x^2} \, dx$$

Optimal (type 3, 197 leaves, 9 steps):

$$-\frac{1}{100} (49-20 x) \sqrt{3-x+2 x^2}-\frac{2203 \operatorname{ArcSinh}\left[\frac{1-4 x}{\sqrt{23}}\right]}{1000 \sqrt{2}}+\frac{11}{125} \sqrt{\frac{11}{31} \left(247+500 \sqrt{2}\right)} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{11}{62 \left(247+500 \sqrt{2}\right)}} \left(8+61 \sqrt{2}+\left(130+69 \sqrt{2}\right) x\right)}{\sqrt{3-x+2 x^2}}\right]-$$
$$\frac{11}{125} \sqrt{\frac{11}{31} \left(-247+500 \sqrt{2}\right)} \operatorname{ArcTanh}\left[\frac{\sqrt{\frac{11}{62 \left(-247+500 \sqrt{2}\right)}} \left(8-61 \sqrt{2}+\left(130-69 \sqrt{2}\right) x\right)}{\sqrt{3-x+2 x^2}}\right]$$

Result (type 3, 1175 leaves):

$$\left(-\frac{49}{100}+\frac{x}{5}\right) \sqrt{3-x+2 x^2}+\frac{2203 \operatorname{ArcSinh}\left[\frac{-1+4 x}{\sqrt{23}}\right]}{1000 \sqrt{2}}+\frac{1}{125 \sqrt{\frac{62}{11} \left(-13+i \sqrt{31}\right)}} 11 \left(69 i+13 \sqrt{31}\right)$$
$$\operatorname{ArcTan}\Big[\Big(10\,827\,432+603\,036 i \sqrt{31}-28\,693\,104 x+2\,334\,908 i \sqrt{31} x-30\,301\,942 x^2-15\,923\,341 i \sqrt{31} x^2-1\,428\,790 x^3-9\,329\,420 i \sqrt{31} x^3-$$
$$30\,587\,700 x^4-12\,631\,475 i \sqrt{31} x^4+3\,150\,000 i \sqrt{22 \left(-13+i \sqrt{31}\right)} \sqrt{3-x+2 x^2}+3\,625\,000 i \sqrt{22 \left(-13+i \sqrt{31}\right)} x \sqrt{3-x+2 x^2}+$$
$$6\,225\,000 i \sqrt{22 \left(-13+i \sqrt{31}\right)} x^2 \sqrt{3-x+2 x^2}-2\,750\,000 i \sqrt{22 \left(-13+i \sqrt{31}\right)} x^3 \sqrt{3-x+2 x^2}\Big)\Big/$$
$$\Big(82\,622\,268 i+5\,966\,136 \sqrt{31}+117\,642\,204 i x+12\,374\,208 \sqrt{31} x+229\,312\,267 i x^2+7\,834\,134 \sqrt{31} x^2-$$
$$63\,298\,460 i x^3+6\,693\,830 \sqrt{31} x^3+136\,148\,325 i x^4+5\,762\,900 \sqrt{31} x^4\Big)\Big]-\frac{1}{125 \sqrt{\frac{62}{11} \left(13+i \sqrt{31}\right)}}$$
$$11 i \left(-69 i+13 \sqrt{31}\right) \operatorname{ArcTan}\Big[\Big(31 \Big(560\,572 i+192\,456 \sqrt{31}-1\,391\,684 i x+399\,168 \sqrt{31} x-2\,195\,557 i x^2+$$
$$252\,714 \sqrt{31} x^2-2\,861\,340 i x^3+215\,930 \sqrt{31} x^3-2\,416\,075 i x^4+185\,900 \sqrt{31} x^4\Big)\Big)\Big/$$
$$\Big(-10\,827\,432+603\,036 i \sqrt{31}+28\,693\,104 x+2\,334\,908 i \sqrt{31} x+30\,301\,942 x^2-15\,923\,341 i \sqrt{31} x^2+1\,428\,790 x^3-9\,329\,420 i \sqrt{31} x^3+$$

$$30\,587\,700\,x^4 - 12\,631\,475\,i\sqrt{31}\,x^4 - 50\,000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,\sqrt{3-x+2x^2} + 125\,000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2} +$$

$$175\,000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^2\sqrt{3-x+2x^2} + 500\,000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\Bigg)\Bigg] -$$

$$\frac{11\left(-69\,i+13\sqrt{31}\right)\mathrm{Log}\left[\left(-3\,i+\sqrt{31}-10\,i\,x\right)^2\left(3\,i+\sqrt{31}+10\,i\,x\right)^2\right]}{250\sqrt{\frac{62}{11}\left(13+i\sqrt{31}\right)}} +$$

$$\frac{11\,i\left(69\,i+13\sqrt{31}\right)\mathrm{Log}\left[\left(-3\,i+\sqrt{31}-10\,i\,x\right)^2\left(3\,i+\sqrt{31}+10\,i\,x\right)^2\right]}{250\sqrt{\frac{62}{11}\left(-13+i\sqrt{31}\right)}} -$$

$$\frac{1}{250\sqrt{\frac{62}{11}\left(-13+i\sqrt{31}\right)}}$$

$$11\,i\left(69\,i+13\sqrt{31}\right)$$

$$\mathrm{Log}\Big[\left(2+3x+5x^2\right)\Big(-142\,i+66\sqrt{31}+469\,i\,x-22\sqrt{31}\,x+327\,i\,x^2+$$

$$44\sqrt{31}\,x^2+i\sqrt{682\left(-13+i\sqrt{31}\right)}\,\sqrt{3-x+2x^2}-4\,i\sqrt{682\left(-13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\Big)\Big] +$$

$$\frac{1}{250\sqrt{\frac{62}{11}\left(13+i\sqrt{31}\right)}}11\left(-69\,i+13\sqrt{31}\right)\mathrm{Log}\Big[\left(2+3x+5x^2\right)\Big(-1858\,i+66\sqrt{31}+1041\,i\,x-22\sqrt{31}\,x-817\,i\,x^2+$$

$$44\sqrt{31}\,x^2-63\,i\sqrt{22\left(13+i\sqrt{31}\right)}\,\sqrt{3-x+2x^2}+22\,i\sqrt{22\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\Big)\Big]$$

- **Problem 70: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\left(3-x+2x^2\right)^{3/2}}{\left(2+3x+5x^2\right)^2}\,dx$$

Optimal (type 3, 232 leaves, 10 steps) :

$$\frac{4}{155}(4-5x)\sqrt{3-x+2x^2}+\frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{31\left(2+3x+5x^2\right)}-\frac{2}{25}\sqrt{2}\,\text{ArcSinh}\left[\frac{1-4x}{\sqrt{23}}\right]+$$

$$\frac{\sqrt{\frac{11}{31}\left(3169333+2265350\sqrt{2}\right)}\;\text{ArcTan}\left[\frac{\sqrt{\frac{11}{62\left(3169333+2265350\sqrt{2}\right)}}\left(3514+2963\sqrt{2}+\left(9440+6477\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]}{1550}-$$

$$\frac{\sqrt{\frac{11}{31}\left(-3169333+2265350\sqrt{2}\right)}\;\text{ArcTanh}\left[\frac{\sqrt{\frac{11}{62\left(-3169333+2265350\sqrt{2}\right)}}\left(3514-2963\sqrt{2}+\left(9440-6477\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]}{1550}$$

Result (type 3, 1088 leaves) :

$$\frac{1}{192200}\left(\frac{13640\,(7+13x)\sqrt{3-x+2x^2}}{2+3x+5x^2}+15376\sqrt{2}\,\text{ArcSinh}\left[\frac{-1+4x}{\sqrt{23}}\right]-\right.$$

$$\frac{1}{\sqrt{\frac{1}{682}\left(13+i\sqrt{31}\right)}}\,2\,i\left(-6477\,i+329\sqrt{31}\right)\text{ArcTan}\left[\left(31\left(-1332489508+919236384\,i\sqrt{31}+\left(5674354076+503954352\,i\sqrt{31}\right)x+\right.\right.\right.$$

$$\left.\left.\left(-3996168827+521299746\,i\sqrt{31}\right)x^2+10\left(589405626+48071177\,i\sqrt{31}\right)x^3+25\,i\left(25228373\,i+4762604\sqrt{31}\right)x^4\right)\right)\Big/$$

$$\left(775\left(93761052\,i+66916121\sqrt{31}\right)x^4+x^3\left(138879039310\,i+24348414380\sqrt{31}-2265350000\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+\right.$$

$$x^2\left(44889007438\,i+59243175649\sqrt{31}-792872500\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$x\left(251068416456\,i+16524047788\sqrt{31}-566337500\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$\left.\left.4\left(8811565488\,i-2160968001\sqrt{31}+56633750\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\right)\right]+$$

$$\frac{1}{\sqrt{\frac{1}{682}\,i\left(13\,i+\sqrt{31}\right)}}\,2\left(6477-329\,i\sqrt{31}\right)\text{ArcTanh}\left[\left(775\left(-93761052\,i+66916121\sqrt{31}\right)x^4+\right.\right.$$

$$x^2 \left(-44\,889\,007\,438\,i + 59\,243\,175\,649\sqrt{31} - 28\,203\,607\,500\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right) +$$

$$4x\left(-62\,767\,104\,114\,i + 4\,131\,011\,947\sqrt{31} - 4\,105\,946\,875\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right) -$$

$$12\left(2\,937\,188\,496\,i + 720\,322\,667\sqrt{31} + 1\,189\,308\,750\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right) +$$

$$10x^3\left(-13\,887\,903\,931\,i + 2\,434\,841\,438\sqrt{31} + 1\,245\,942\,500\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\Bigg)\Bigg/$$

$$\Big(36\left(11\,437\,856\,257\,i + 791\,564\,664\sqrt{31}\right) + 12\left(42\,786\,843\,863\,i + 1\,301\,882\,076\sqrt{31}\right)x + \left(606\,694\,141\,363\,i + 16\,160\,292\,126\sqrt{31}\right)x^2 +$$

$$10\left(-50\,595\,065\,594\,i + 1\,490\,206\,487\sqrt{31}\right)x^3 + 25\left(10\,318\,135\,437\,i + 147\,640\,724\sqrt{31}\right)x^4\Big)\Big] -$$

$$\frac{\left(-6477\,i + 329\sqrt{31}\right)\text{Log}\left[400\left(2+3x+5x^2\right)^2\right]}{\sqrt{\frac{1}{682}\left(13+i\sqrt{31}\right)}} + \frac{i\left(6477\,i + 329\sqrt{31}\right)\text{Log}\left[400\left(2+3x+5x^2\right)^2\right]}{\sqrt{\frac{1}{682}\,i\left(13\,i+\sqrt{31}\right)}} +$$

$$\frac{1}{\sqrt{\frac{1}{682}\left(13+i\sqrt{31}\right)}}$$

$$\left(-6477\,i + 329\sqrt{31}\right)$$

$$\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-1858\,i + 66\sqrt{31} + \left(-817\,i + 44\sqrt{31}\right)x^2 -$$

$$63\,i\sqrt{286+22\,i\sqrt{31}}\sqrt{3-x+2x^2} + x\left(1041\,i - 22\sqrt{31} + 22\,i\sqrt{286+22\,i\sqrt{31}}\sqrt{3-x+2x^2}\right)\Big)\Big] +$$

$$\frac{1}{\sqrt{\frac{1}{682}\,i\left(13\,i+\sqrt{31}\right)}}\left(6477 - 329\,i\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-142\,i + 66\sqrt{31} + \left(327\,i + 44\sqrt{31}\right)x^2 +$$

$$i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2} + x\left(469\,i - 22\sqrt{31} - 4\,i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\Big)\Big]\Bigg)$$

- **Problem 71: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\left(3-x+2\,x^2\right)^{3/2}}{\left(2+3\,x+5\,x^2\right)^3}\,dx$$

Optimal (type 3, 223 leaves, 7 steps) :

$$\frac{(3+10\,x)\,\left(3-x+2\,x^2\right)^{3/2}}{62\,\left(2+3\,x+5\,x^2\right)^2}+\frac{3\,(277+696\,x)\,\sqrt{3-x+2\,x^2}}{3844\,\left(2+3\,x+5\,x^2\right)}+\frac{1}{7688}$$

$$3\,\sqrt{\frac{1}{682}\,\left(366\,990\,269+259\,509\,026\,\sqrt{2}\right)}\,\text{ArcTan}\left[\frac{\sqrt{\frac{11}{31\,\left(366\,990\,269+259\,509\,026\,\sqrt{2}\right)}}\,\left(29\,367+20\,575\,\sqrt{2}+\left(70\,517+49\,942\,\sqrt{2}\right)\,x\right)}{\sqrt{3-x+2\,x^2}}\right]-\frac{1}{7688}$$

$$3\,\sqrt{\frac{1}{682}\,\left(-366\,990\,269+259\,509\,026\,\sqrt{2}\right)}\,\text{ArcTanh}\left[\frac{\sqrt{\frac{11}{31\,\left(-366\,990\,269+259\,509\,026\,\sqrt{2}\right)}}\,\left(29\,367-20\,575\,\sqrt{2}+\left(70\,517-49\,942\,\sqrt{2}\right)\,x\right)}{\sqrt{3-x+2\,x^2}}\right]$$

Result (type 3, 1171 leaves) :

$$\sqrt{3-x+2\,x^2}\,\left(\frac{11\,(7+13\,x)}{310\,\left(2+3\,x+5\,x^2\right)^2}+\frac{3163+11\,680\,x}{19\,220\,\left(2+3\,x+5\,x^2\right)}\right)-$$

$$\frac{1}{3844\,\sqrt{682\,\left(13+i\,\sqrt{31}\right)}}\,3\,i\,\left(-24\,971\,i+902\,\sqrt{31}\right)\,\text{ArcTan}\Big[\Big(31\,\Big(31\,227\,856\,109\,i+25\,278\,538\,857\,\sqrt{31}-148\,151\,300\,773\,i\,x+8\,050\,492\,021\,\sqrt{31}\,x+$$

$$158\,238\,605\,196\,i\,x^2+14\,045\,028\,558\,\sqrt{31}\,x^2-151\,681\,537\,680\,i\,x^3+10\,089\,483\,360\,\sqrt{31}\,x^3+56\,810\,945\,600\,i\,x^4+1\,789\,928\,800\,\sqrt{31}\,x^4\Big)\Big)\Big/$$

$$\Big(1\,329\,350\,472\,021+251\,835\,138\,467\,i\,\sqrt{31}+7\,060\,303\,464\,863\,x-560\,818\,641\,999\,i\,\sqrt{31}\,x+689\,282\,588\,324\,x^2-$$

$$1\,457\,613\,959\,802\,i\,\sqrt{31}\,x^2+4\,234\,217\,180\,380\,x^3-535\,663\,546\,990\,i\,\sqrt{31}\,x^3+1\,536\,126\,024\,400\,x^4-1\,305\,722\,486\,200\,i\,\sqrt{31}\,x^4-$$

$$6\,487\,725\,650\,i\,\sqrt{682\,\left(13+i\,\sqrt{31}\right)}\,\sqrt{3-x+2\,x^2}+16\,219\,314\,125\,i\,\sqrt{682\,\left(13+i\,\sqrt{31}\right)}\,x\,\sqrt{3-x+2\,x^2}+$$

$$22\,707\,039\,775\,i\,\sqrt{682\,\left(13+i\,\sqrt{31}\right)}\,x^2\,\sqrt{3-x+2\,x^2}+64\,877\,256\,500\,i\,\sqrt{682\,\left(13+i\,\sqrt{31}\right)}\,x^3\,\sqrt{3-x+2\,x^2}\Big)\Big]-$$

$$\frac{1}{3844\,\sqrt{682\,\left(-13+i\,\sqrt{31}\right)}}\,3\,i\,\left(24\,971\,i+902\,\sqrt{31}\right)\,\text{ArcTanh}\Big[$$

$$\Big(11\,\Big(1\,091\,580\,705\,511\,i+71\,239\,518\,597\,\sqrt{31}+1\,296\,309\,231\,133\,i\,x+22\,687\,750\,241\,\sqrt{31}\,x+1\,456\,138\,041\,834\,i\,x^2+$$

$$39\,581\,444\,118\,\sqrt{31}\,x^2-1\,365\,505\,300\,720\,i\,x^3+28\,433\,998\,560\,\sqrt{31}\,x^3+562\,393\,146\,150\,i\,x^4+5\,044\,344\,800\,\sqrt{31}\,x^4\Big)\Big)\Big/$$

$$\Big(-1\,329\,350\,472\,021\,i-251\,835\,138\,467\,\sqrt{31}-7\,060\,303\,464\,863\,i\,x+560\,818\,641\,999\,\sqrt{31}\,x-689\,282\,588\,324\,i\,x^2+$$

$$1\,457\,613\,959\,802\,\sqrt{31}\,x^2-4\,234\,217\,180\,380\,i\,x^3+535\,663\,546\,990\,\sqrt{31}\,x^3-1\,536\,126\,024\,400\,i\,x^4+1\,305\,722\,486\,200\,\sqrt{31}\,x^4-$$

$$408\,726\,715\,950\,\sqrt{22\left(-13+i\sqrt{31}\right)}\,\sqrt{3-x+2x^2}-470\,360\,109\,625\,\sqrt{22\left(-13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}-$$

$$807\,721\,843\,425\,\sqrt{22\left(-13+i\sqrt{31}\right)}\,x^2\sqrt{3-x+2x^2}+356\,824\,910\,750\,\sqrt{22\left(-13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\Big)\Big]-$$

$$\frac{3\left(-24\,971\,i+902\sqrt{31}\right)\text{Log}\left[\left(-3i+\sqrt{31}-10ix\right)^2\left(3i+\sqrt{31}+10ix\right)^2\right]}{7688\sqrt{682\left(13+i\sqrt{31}\right)}}+$$

$$\frac{3i\left(24\,971\,i+902\sqrt{31}\right)\text{Log}\left[\left(-3i+\sqrt{31}-10ix\right)^2\left(3i+\sqrt{31}+10ix\right)^2\right]}{7688\sqrt{682\left(-13+i\sqrt{31}\right)}}-$$

$$\Bigg(3i\left(24\,971\,i+902\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-142\,i+66\sqrt{31}+469\,i\,x-22\sqrt{31}\,x+$$

$$327\,i\,x^2+44\sqrt{31}\,x^2+i\sqrt{682\left(-13+i\sqrt{31}\right)}\,\sqrt{3-x+2x^2}-4i\sqrt{682\left(-13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\Big)\Big]\Bigg)\Big/$$

$$\left(7688\sqrt{682\left(-13+i\sqrt{31}\right)}\right)+\frac{1}{7688\sqrt{682\left(13+i\sqrt{31}\right)}}3\left(-24\,971\,i+902\sqrt{31}\right)$$

$$\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-1858\,i+66\sqrt{31}+1041\,i\,x-22\sqrt{31}\,x-817\,i\,x^2+$$

$$44\sqrt{31}\,x^2-63\,i\sqrt{22\left(13+i\sqrt{31}\right)}\,\sqrt{3-x+2x^2}+22\,i\sqrt{22\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\Big)\Big]$$

- **Problem 76: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\left(3-x+2x^2\right)^{5/2}}{2+3x+5x^2}\,dx$$

Optimal (type 3, 222 leaves, 10 steps) :

$$-\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000}-\frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2}-$$

$$\frac{7216203\,\text{ArcSinh}\left[\frac{1-4x}{\sqrt{23}}\right]}{800000\sqrt{2}}-\frac{121\sqrt{\frac{11}{31}\left(-15457+25000\sqrt{2}\right)}\,\text{ArcTan}\left[\frac{\sqrt{\frac{11}{62\left(-15457+25000\sqrt{2}\right)}}\left(196-443\sqrt{2}-\left(690+247\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]}{3125}+$$

$$\frac{121\sqrt{\frac{11}{31}\left(15457+25000\sqrt{2}\right)}\,\text{ArcTanh}\left[\frac{\sqrt{\frac{11}{62\left(15457+25000\sqrt{2}\right)}}\left(196+443\sqrt{2}-\left(690-247\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]}{3125}$$

Result (type 3, 1189 leaves) :

$$\sqrt{3-x+2x^2}\left(-\frac{267449}{80000}+\frac{20603x}{12000}-\frac{133x^2}{300}+\frac{x^3}{5}\right)+\frac{7216203\,\text{ArcSinh}\left[\frac{-1+4x}{\sqrt{23}}\right]}{800000\sqrt{2}}+$$

$$\frac{1}{3125\sqrt{\frac{62}{11}\left(-13+i\sqrt{31}\right)}}121\left(247i+119\sqrt{31}\right)\text{ArcTan}\Big[\Big(910772808-46000516i\sqrt{31}+727715824x+$$

$$277778652i\sqrt{31}\,x-1240038998x^2-326488029i\sqrt{31}\,x^2+1188688490x^3-285779980i\sqrt{31}\,x^3-1002301300x^4-$$

$$214634275i\sqrt{31}\,x^4+157500000i\sqrt{22\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+181250000i\sqrt{22\left(-13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}+$$

$$311250000i\sqrt{22\left(-13+i\sqrt{31}\right)}\,x^2\sqrt{3-x+2x^2}-137500000i\sqrt{22\left(-13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\Big)\Big/$$

$$\Big(4168906492i+186603384\sqrt{31}+4941322076ix+673090352\sqrt{31}\,x+14142713923ix^2+603640246\sqrt{31}\,x^2-$$

$$1371093740ix^3+248749270\sqrt{31}\,x^3+8825296925ix^4+482890100\sqrt{31}\,x^4\Big)\Big]-$$

$$\frac{1}{3125\sqrt{\frac{62}{11}\left(13+i\sqrt{31}\right)}}121i\left(-247i+119\sqrt{31}\right)\text{ArcTan}\Big[\Big(31\Big(26809468i+6019464\sqrt{31}-39236196ix+21712592\sqrt{31}\,x-$$

$$196135933ix^2+19472266\sqrt{31}\,x^2-200932460ix^3+8024170\sqrt{31}\,x^3-185896675ix^4+15577100\sqrt{31}\,x^4\Big)\Big)\Big/$$

$$\Big(-910772808-46000516i\sqrt{31}-727715824x+277778652i\sqrt{31}\,x+1240038998x^2-326488029i\sqrt{31}\,x^2-$$

$$1188688490x^3-285779980i\sqrt{31}\,x^3+1002301300x^4-214634275i\sqrt{31}\,x^4-$$

$$2\,500\,000\,\mathrm{i}\,\sqrt{682\left(13+\mathrm{i}\sqrt{31}\right)}\,\sqrt{3-x+2x^2}+6\,250\,000\,\mathrm{i}\,\sqrt{682\left(13+\mathrm{i}\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}+$$

$$8\,750\,000\,\mathrm{i}\,\sqrt{682\left(13+\mathrm{i}\sqrt{31}\right)}\,x^2\sqrt{3-x+2x^2}+25\,000\,000\,\mathrm{i}\,\sqrt{682\left(13+\mathrm{i}\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\Bigg)\Bigg]-$$

$$\frac{121\left(-247\,\mathrm{i}+119\sqrt{31}\right)\mathrm{Log}\left[\left(-3\,\mathrm{i}+\sqrt{31}-10\,\mathrm{i}\,x\right)^2\left(3\,\mathrm{i}+\sqrt{31}+10\,\mathrm{i}\,x\right)^2\right]}{6250\sqrt{\frac{62}{11}\left(13+\mathrm{i}\sqrt{31}\right)}}+$$

$$\frac{121\,\mathrm{i}\left(247\,\mathrm{i}+119\sqrt{31}\right)\mathrm{Log}\left[\left(-3\,\mathrm{i}+\sqrt{31}-10\,\mathrm{i}\,x\right)^2\left(3\,\mathrm{i}+\sqrt{31}+10\,\mathrm{i}\,x\right)^2\right]}{6250\sqrt{\frac{62}{11}\left(-13+\mathrm{i}\sqrt{31}\right)}}-$$

$$\frac{1}{6250\sqrt{\frac{62}{11}\left(-13+\mathrm{i}\sqrt{31}\right)}}$$

$$121\,\mathrm{i}\left(247\,\mathrm{i}+119\sqrt{31}\right)$$

$$\mathrm{Log}\Big[\left(2+3x+5x^2\right)\Big(-142\,\mathrm{i}+66\sqrt{31}+469\,\mathrm{i}\,x-22\sqrt{31}\,x+327\,\mathrm{i}\,x^2+$$

$$44\sqrt{31}\,x^2+\mathrm{i}\sqrt{682\left(-13+\mathrm{i}\sqrt{31}\right)}\,\sqrt{3-x+2x^2}-4\,\mathrm{i}\sqrt{682\left(-13+\mathrm{i}\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\Big)\Big]+$$

$$\frac{1}{6250\sqrt{\frac{62}{11}\left(13+\mathrm{i}\sqrt{31}\right)}}121\left(-247\,\mathrm{i}+119\sqrt{31}\right)\mathrm{Log}\Big[\left(2+3x+5x^2\right)\Big(-1858\,\mathrm{i}+66\sqrt{31}+1041\,\mathrm{i}\,x-22\sqrt{31}\,x-817\,\mathrm{i}\,x^2+$$

$$44\sqrt{31}\,x^2-63\,\mathrm{i}\sqrt{22\left(13+\mathrm{i}\sqrt{31}\right)}\,\sqrt{3-x+2x^2}+22\,\mathrm{i}\sqrt{22\left(13+\mathrm{i}\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\Big)\Big]$$

- **Problem 77: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\left(3-x+2x^2\right)^{5/2}}{\left(2+3x+5x^2\right)^2}\,dx$$

Optimal (type 3, 255 leaves, 11 steps) :

$$-\frac{(1277+2240\,x)\sqrt{3-x+2\,x^2}}{7750}+\frac{4}{155}\,(4-5\,x)\left(3-x+2\,x^2\right)^{3/2}+\frac{(3+10\,x)\left(3-x+2\,x^2\right)^{5/2}}{31\left(2+3\,x+5\,x^2\right)}-\frac{4799\,\text{ArcSinh}\left[\frac{1-4\,x}{\sqrt{23}}\right]}{2500\sqrt{2}}+\frac{1}{38\,750}$$

$$11\sqrt{\frac{11}{31}\left(224\,510\,383+194\,487\,500\sqrt{2}\right)}\;\text{ArcTan}\left[\frac{\sqrt{\frac{11}{62\left(224\,510\,383+194\,487\,500\sqrt{2}\right)}}\left(21\,136+33\,287\sqrt{2}+\left(87\,710+54\,423\sqrt{2}\right)x\right)}{\sqrt{3-x+2\,x^2}}\right]-\frac{1}{38\,750}$$

$$11\sqrt{\frac{11}{31}\left(-224\,510\,383+194\,487\,500\sqrt{2}\right)}\;\text{ArcTanh}\left[\frac{\sqrt{\frac{11}{62\left(-224\,510\,383+194\,487\,500\sqrt{2}\right)}}\left(21\,136-33\,287\sqrt{2}+\left(87\,710-54\,423\sqrt{2}\right)x\right)}{\sqrt{3-x+2\,x^2}}\right]$$

Result (type 3, 1110 leaves) :

$$\frac{1}{4\,805\,000}$$

$$\left(\frac{620\sqrt{3-x+2\,x^2}\left(8996+9289\,x-12\,555\,x^2+3100\,x^3\right)}{2+3\,x+5\,x^2}+4\,611\,839\sqrt{2}\;\text{ArcSinh}\left[\frac{-1+4\,x}{\sqrt{23}}\right]-\frac{1}{\sqrt{\frac{1}{682}\left(13+\mathbb{i}\sqrt{31}\right)}}\,22\,\mathbb{i}\left(-54\,423\,\mathbb{i}+5471\sqrt{31}\right)\right.$$

$$\text{ArcTan}\Big[\Big(31\left(-171\,942\,569\,308+82\,792\,691\,784\,\mathbb{i}\sqrt{31}+4\left(141\,772\,726\,169+25\,072\,968\,888\,\mathbb{i}\sqrt{31}\right)x+7\left(21\,854\,082\,139+8\,850\,407\,478\,\mathbb{i}\sqrt{31}\right)\right.$$

$$\left.x^2+10\left(73\,391\,640\,726+6\,879\,711\,377\,\mathbb{i}\sqrt{31}\right)x^3+25\left(14\,752\,730\,827+1\,317\,001\,004\,\mathbb{i}\sqrt{31}\right)x^4\right)\Big)\Big/$$

$$\Big(775\left(13\,100\,922\,252\,\mathbb{i}+6\,966\,216\,221\sqrt{31}\right)x^4+x^3\Big(7\,951\,179\,150\,310\,\mathbb{i}+3\,217\,382\,742\,380\sqrt{31}-194\,487\,500\,000\sqrt{682\left(13+\mathbb{i}\sqrt{31}\right)}$$

$$\sqrt{3-x+2\,x^2}\Big)+x^2\left(8\,562\,978\,915\,238\,\mathbb{i}+6\,467\,393\,362\,549\sqrt{31}-68\,070\,625\,000\sqrt{682\left(13+\mathbb{i}\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)+$$

$$x\left(19\,618\,154\,755\,056\,\mathbb{i}+442\,968\,415\,588\sqrt{31}-48\,621\,875\,000\sqrt{682\left(13+\mathbb{i}\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)+$$

$$4\left(-57\,356\,227\,962\,\mathbb{i}-149\,533\,752\,351\sqrt{31}+4\,862\,187\,500\sqrt{682\left(13+\mathbb{i}\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)\Big)\Big]+$$

$$\frac{1}{\sqrt{\frac{1}{682}\,\mathbb{i}\left(13\,\mathbb{i}+\sqrt{31}\right)}}\,22\left(54\,423\,\mathbb{i}+5471\sqrt{31}\right)\text{ArcTan}\Big[\Big(-775\,\mathbb{i}\left(-13\,100\,922\,252\,\mathbb{i}+6\,966\,216\,221\sqrt{31}\right)x^4-$$

$$4\,i\,x\left(-4904538688764\,i+110742103897\sqrt{31}-352508593750\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$12\left(19118742654+49844584117\,i\sqrt{31}+102105937500\,i\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$x^2\left(-8562978915238-6467393362549\,i\sqrt{31}+2421369375000\,i\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)-$$

$$10\,i\,x^3\left(-795117915031\,i+321738274238\sqrt{31}+106968125000\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\Big)\Big/$$

$$\left(36\left(932424454207\,i+71293706814\sqrt{31}\right)+12\left(3879871295413\,i+259087345176\sqrt{31}\right)x+\left(67464554574163\,i+1920538422726\sqrt{31}\right)x^2+10\left(-3637279137494\,i+213271052687\sqrt{31}\right)x^3+25\left(1410323405637\,i+40827031124\sqrt{31}\right)x^4\right)\Big]-$$

$$\frac{11\left(-54423\,i+5471\sqrt{31}\right)\text{Log}\left[400\left(2+3x+5x^2\right)^2\right]}{\sqrt{\frac{1}{682}\left(13+i\sqrt{31}\right)}}+\frac{11\,i\left(54423\,i+5471\sqrt{31}\right)\text{Log}\left[400\left(2+3x+5x^2\right)^2\right]}{\sqrt{\frac{1}{682}\,i\left(13\,i+\sqrt{31}\right)}}+$$

$$\frac{1}{\sqrt{\frac{1}{682}\left(13+i\sqrt{31}\right)}}11\left(-54423\,i+5471\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-1858\,i+66\sqrt{31}+\left(-817\,i+44\sqrt{31}\right)x^2-63\,i\sqrt{286+22\,i\sqrt{31}}\sqrt{3-x+2x^2}+x\left(1041\,i-22\sqrt{31}+22\,i\sqrt{286+22\,i\sqrt{31}}\sqrt{3-x+2x^2}\right)\Big)\Big]+$$

$$\frac{1}{\sqrt{\frac{1}{682}\,i\left(13\,i+\sqrt{31}\right)}}11\left(54423-5471\,i\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-142\,i+66\sqrt{31}+\left(327\,i+44\sqrt{31}\right)x^2+i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}+x\left(469\,i-22\sqrt{31}-4\,i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\Big)\Big]\Bigg)$$

- **Problem 78: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\left(3-x+2x^2\right)^{5/2}}{\left(2+3x+5x^2\right)^3}\,dx$$

Optimal (type 3, 281 leaves, 11 steps):

$$\frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050}+\frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2}+\frac{(769+2336x)\left(3-x+2x^2\right)^{3/2}}{3844\left(2+3x+5x^2\right)}-\frac{4}{125}\sqrt{2}\,\text{ArcSinh}\left[\frac{1-4x}{\sqrt{23}}\right]+\frac{1}{29791000}\sqrt{11\left(1+4\sqrt{2}\right)}\left(2937349+1978861\sqrt{2}\right)\text{ArcTan}\left[\frac{\sqrt{\frac{11}{62\left(3531015707557+2498852071250\sqrt{2}\right)}}\left(3957722+2937349\sqrt{2}+\left(9832420+6895071\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]-\frac{1}{29791000}\left(2937349-1978861\sqrt{2}\right)\sqrt{11\left(-1+4\sqrt{2}\right)}\,\text{ArcTanh}\left[\frac{\sqrt{\frac{11}{62\left(-3531015707557+2498852071250\sqrt{2}\right)}}\left(3957722-2937349\sqrt{2}+\left(9832420-6895071\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]$$

Result (type 3, 1203 leaves):

$$\sqrt{3-x+2x^2}\left(\frac{121(61+69x)}{7750\left(2+3x+5x^2\right)^2}+\frac{11(35579+97155x)}{480500\left(2+3x+5x^2\right)}\right)+\frac{4}{125}\sqrt{2}\,\text{ArcSinh}\left[\frac{-1+4x}{\sqrt{23}}\right]-\frac{1}{961000\sqrt{\frac{62}{11}\left(13+i\sqrt{31}\right)}}\,i\left(-6895071\,i+280267\sqrt{31}\right)\text{ArcTan}\Big[\Big(31\Big(1286646864280132\,i+987421307406336\sqrt{31}-5888947864615004\,i\,x+386335744679808\sqrt{31}\,x+5595672650742083\,i\,x^2+549395637070434\sqrt{31}\,x^2-6029547074679540\,i\,x^3+433781845112330\sqrt{31}\,x^3+1742846817367925\,i\,x^4+86404550417900\sqrt{31}\,x^4\Big)\Big)\Big/\Big(47470658398910208+9672976872245316\,i\sqrt{31}+274205806118598024\,x-20598732824854252\,i\sqrt{31}\,x+33816025817929102\,x^2-59172316611299521\,i\sqrt{31}\,x^2+160404448215022990\,x^3-22636449983151020\,i\sqrt{31}\,x^3+65896915460933700\,x^4-52587956640176975\,i\sqrt{31}\,x^4-249885207125000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+624713017812500\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}+874598224937500\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^2\sqrt{3-x+2x^2}+2498852071250000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\Big)\Big]-$$

$$\frac{1}{961000\sqrt{\frac{62}{11}\left(-13+i\sqrt{31}\right)}}\, i\left(6895071\, i+280267\sqrt{31}\right)\text{ArcTanh}\Big[\Big(-47470658398910208\, i-9672976872245316\sqrt{31}-$$

$$274205806118598024\, i\, x+20598732824854252\sqrt{31}\, x-33816025817929102\, i\, x^2+59172316611299521\sqrt{31}\, x^2-$$

$$160404448215022990\, i\, x^3+22636449983151020\sqrt{31}\, x^3-65896915460933700\, i\, x^4+52587956640176975\sqrt{31}\, x^4-$$

$$15742768048875000\sqrt{22\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-18116677516562500\sqrt{22\left(-13+i\sqrt{31}\right)}\, x\sqrt{3-x+2x^2}-$$

$$31110708287062500\sqrt{22\left(-13+i\sqrt{31}\right)}\, x^2\sqrt{3-x+2x^2}+13743686391875000\sqrt{22\left(-13+i\sqrt{31}\right)}\, x^3\sqrt{3-x+2x^2}\Big)\Big/$$

$$\Big(459884361457315908\, i+30610060529596416\sqrt{31}+554886342419315124\, i\, x+11976408085074048\sqrt{31}\, x+$$

$$632413940805120427\, i\, x^2+17031264749183454\sqrt{31}\, x^2-572735070344934260\, i\, x^3+$$

$$13447237198482230\sqrt{31}\, x^3+252081127389719325\, i\, x^4+2678541062954900\sqrt{31}\, x^4\Big)\Big]-$$

$$\frac{\left(-6895071\, i+280267\sqrt{31}\right)\text{Log}\left[\left(-3\, i+\sqrt{31}-10\, i\, x\right)^2\left(3\, i+\sqrt{31}+10\, i\, x\right)^2\right]}{1922000\sqrt{\frac{62}{11}\left(13+i\sqrt{31}\right)}}+$$

$$\frac{i\left(6895071\, i+280267\sqrt{31}\right)\text{Log}\left[\left(-3\, i+\sqrt{31}-10\, i\, x\right)^2\left(3\, i+\sqrt{31}+10\, i\, x\right)^2\right]}{1922000\sqrt{\frac{62}{11}\left(-13+i\sqrt{31}\right)}}-$$

$$\Big(i\left(6895071\, i+280267\sqrt{31}\right)$$

$$\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-142\, i+66\sqrt{31}+469\, i\, x-22\sqrt{31}\, x+327\, i\, x^2+44\sqrt{31}\, x^2+$$

$$i\sqrt{682\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-4\, i\sqrt{682\left(-13+i\sqrt{31}\right)}\, x\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/\left(1922000\sqrt{\frac{62}{11}\left(-13+i\sqrt{31}\right)}\right)+$$

$$\Big(\left(-6895071\, i+280267\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-1858\, i+66\sqrt{31}+1041\, i\, x-22\sqrt{31}\, x-817\, i\, x^2+44\sqrt{31}\, x^2-$$

$$63\, i\sqrt{22\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+22\, i\sqrt{22\left(13+i\sqrt{31}\right)}\, x\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/\left(1922000\sqrt{\frac{62}{11}\left(13+i\sqrt{31}\right)}\right)$$

Problem 83: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{3-x+2x^2}\,\left(2+3x+5x^2\right)}\,dx$$

Optimal (type 3, 148 leaves, 5 steps) :

$$\sqrt{\frac{1}{682}\left(13+10\sqrt{2}\right)}\ \text{ArcTan}\left[\frac{\sqrt{\frac{11}{31\left(13+10\sqrt{2}\right)}}\ \left(7+3\sqrt{2}+\left(13+10\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]-$$

$$\sqrt{\frac{1}{682}\left(-13+10\sqrt{2}\right)}\ \text{ArcTanh}\left[\frac{\sqrt{\frac{11}{31\left(-13+10\sqrt{2}\right)}}\ \left(7-3\sqrt{2}+\left(13-10\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]$$

Result (type 3, 874 leaves) :

$$\frac{1}{4\sqrt{341}}\left(2\,i\sqrt{-13+i\sqrt{31}}\;\text{ArcTan}\Big[\left(31\left(-7+11\,i\sqrt{31}+50\,x-100\,x^2\right)\left(3-x+2\,x^2\right)\right)\Big/\right.$$

$$\left(3069\,i-363\sqrt{31}+1100\sqrt{31}\,x^4+10\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2\,x^2}+x^3\left(110\left(62\,i+\sqrt{31}\right)-100\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)+\right.$$

$$\left.x^2\left(22\left(-62\,i+49\sqrt{31}\right)-35\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)+x\left(9207\,i+1111\sqrt{31}-25\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)\right)\Big]-$$

$$2\sqrt{13+i\sqrt{31}}\;\text{ArcTan}\Big[\left(11\left(-1759+93\,i\sqrt{31}+\left(-1797-31\,i\sqrt{31}\right)x+\left(-1906+62\,i\sqrt{31}\right)x^2+2200\,x^3-550\,x^4\right)\right)\Big/\left(1100\sqrt{31}\,x^4+\right.$$

$$x^2\left(22\left(62\,i+49\sqrt{31}\right)-1245\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)+x\left(-9207\,i+1111\sqrt{31}-725\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)+$$

$$\left.110\,x^3\left(-62\,i+\sqrt{31}+5\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)-3\left(1023\,i+121\sqrt{31}+210\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)\right)\Big]+$$

$$\sqrt{-13+i\sqrt{31}}\;\text{Log}\left[400\left(2+3\,x+5\,x^2\right)^2\right]+i\sqrt{13+i\sqrt{31}}\;\text{Log}\left[400\left(2+3\,x+5\,x^2\right)^2\right]-$$

$$\sqrt{-13+i\sqrt{31}}\;\text{Log}\Big[\left(2+3\,x+5\,x^2\right)\left(-1858\,i+66\sqrt{31}+\left(-817\,i+44\sqrt{31}\right)x^2-\right.$$

$$\left.63\,i\sqrt{286+22\,i\sqrt{31}}\sqrt{3-x+2\,x^2}+x\left(1041\,i-22\sqrt{31}+22\,i\sqrt{286+22\,i\sqrt{31}}\sqrt{3-x+2\,x^2}\right)\right)\Big]-$$

$$i\sqrt{13+i\sqrt{31}}\;\text{Log}\Big[\left(2+3\,x+5\,x^2\right)\left(-142\,i+66\sqrt{31}+\left(327\,i+44\sqrt{31}\right)x^2+i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2\,x^2}+\right.$$

$$\left.\left.x\left(469\,i-22\sqrt{31}-4\,i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)\right)\Big]\right)$$

- **Problem 84: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{1}{\sqrt{3-x+2\,x^2}\,\left(2+3\,x+5\,x^2\right)^2}\,dx$$

Optimal (type 3, 188 leaves, 6 steps):

$$\frac{(4+65x)\sqrt{3-x+2x^2}}{682\left(2+3x+5x^2\right)}+\frac{\sqrt{\frac{1}{682}\left(2343727+1678700\sqrt{2}\right)}\,\text{ArcTan}\left[\frac{\sqrt{\frac{11}{31\left(2343727+1678700\sqrt{2}\right)}}\left(2119+1816\sqrt{2}+\left(5751+3935\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]}{1364}-$$

$$\frac{\sqrt{\frac{1}{682}\left(-2343727+1678700\sqrt{2}\right)}\,\text{ArcTanh}\left[\frac{\sqrt{\frac{11}{31\left(-2343727+1678700\sqrt{2}\right)}}\left(2119-1816\sqrt{2}+\left(5751-3935\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]}{1364}$$

Result (type 3, 1063 leaves) :

$$\frac{1}{1860496}\left(\frac{2728\,(4+65x)\sqrt{3-x+2x^2}}{2+3x+5x^2}-\frac{1}{\sqrt{\frac{1}{682}\left(13+i\sqrt{31}\right)}}\right.$$

$$10\,i\left(-787\,i+41\sqrt{31}\right)\text{ArcTan}\Big[\Big(31\left(-802246+546546\,i\sqrt{31}+10\left(338727+31031\,i\sqrt{31}\right)x+\left(-2284079+311146\,i\sqrt{31}\right)x^2+\right.$$

$$\left.\left.\left(3529208+291346\,i\sqrt{31}\right)x^3+\left(-299597+73964\,i\sqrt{31}\right)x^4\right)\right)\Big/\Big(20294274\,i-5110826\sqrt{31}+31\left(1419748\,i+1001071\sqrt{31}\right)x^4+$$

$$134296\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+x^3\left(81775210\,i+14709760\sqrt{31}-1342960\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$x^2\left(27657146\,i+35512659\sqrt{31}-470036\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$x\left(148907198\,i+9626874\sqrt{31}-335740\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\Big)\Big]+\frac{1}{\sqrt{\frac{1}{682}\,i\left(13\,i+\sqrt{31}\right)}}10\left(787-41\,i\sqrt{31}\right)$$

$$\text{ArcTanh}\Big[\Big(31\left(-1419748\,i+1001071\sqrt{31}\right)x^4+x^2\left(-27657146\,i+35512659\sqrt{31}-16719852\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$2x\left(-74453599\,i+4813437\sqrt{31}-4868230\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)-$$

$$14\left(1449591\,i+365059\sqrt{31}+604332\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$10x^3\left(-8177521\,i+1470976\sqrt{31}+738628\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\Big)\Big/$$

$$\left(98\left(2486963\,i+172887\sqrt{31}\right)+70\left(4358663\,i+137423\sqrt{31}\right)x+\left(362298151\,i+9645526\sqrt{31}\right)x^2+\left(-298854392\,i+9031726\sqrt{31}\right)x^3+\left(155225093\,i+2292884\sqrt{31}\right)x^4\right)\Big]-$$

$$\frac{5\left(-787\,i+41\sqrt{31}\right)\mathrm{Log}\left[400\left(2+3x+5x^2\right)^2\right]}{\sqrt{\frac{1}{682}\left(13+i\sqrt{31}\right)}}+\frac{5\,i\left(787\,i+41\sqrt{31}\right)\mathrm{Log}\left[400\left(2+3x+5x^2\right)^2\right]}{\sqrt{\frac{1}{682}\,i\left(13\,i+\sqrt{31}\right)}}+$$

$$\frac{1}{\sqrt{\frac{1}{682}\left(13+i\sqrt{31}\right)}}$$

$$5\left(-787\,i+41\sqrt{31}\right)$$

$$\mathrm{Log}\Big[\left(2+3x+5x^2\right)\Big(-1858\,i+66\sqrt{31}+\left(-817\,i+44\sqrt{31}\right)x^2-63\,i\sqrt{286+22\,i\sqrt{31}}\sqrt{3-x+2x^2}+x\left(1041\,i-22\sqrt{31}+22\,i\sqrt{286+22\,i\sqrt{31}}\sqrt{3-x+2x^2}\right)\Big)\Big]+\frac{1}{\sqrt{\frac{1}{682}\,i\left(13\,i+\sqrt{31}\right)}}$$

$$5\left(787-41\,i\sqrt{31}\right)\mathrm{Log}\Big[\left(2+3x+5x^2\right)\Big(-142\,i+66\sqrt{31}+\left(327\,i+44\sqrt{31}\right)x^2+i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}+x\left(469\,i-22\sqrt{31}-4\,i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\Big)\Big]\Bigg)$$

- **Problem 85: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3}\,dx$$

Optimal (type 3, 223 leaves, 7 steps) :

$$\frac{(4+65\,x)\,\sqrt{3-x+2\,x^2}}{1364\,\left(2+3\,x+5\,x^2\right)^2}+\frac{(26\,794+86\,265\,x)\,\sqrt{3-x+2\,x^2}}{1\,860\,496\,\left(2+3\,x+5\,x^2\right)}+\frac{1}{3\,720\,992}$$

$$25\,\sqrt{\frac{1}{682}\left(6\,414\,867\,847+4\,536\,374\,600\,\sqrt{2}\right)}\;\text{ArcTan}\left[\frac{\sqrt{\frac{11}{31\,\left(6\,414\,867\,847+4\,536\,374\,600\,\sqrt{2}\right)}}\,\left(123\,161+85\,754\,\sqrt{2}+\left(294\,669+208\,915\,\sqrt{2}\right)\,x\right)}{\sqrt{3-x+2\,x^2}}\right]-\frac{1}{3\,720\,992}$$

$$25\,\sqrt{\frac{1}{682}\left(-6\,414\,867\,847+4\,536\,374\,600\,\sqrt{2}\right)}\;\text{ArcTanh}\left[\frac{\sqrt{\frac{11}{31\,\left(-6\,414\,867\,847+4\,536\,374\,600\,\sqrt{2}\right)}}\,\left(123\,161-85\,754\,\sqrt{2}+\left(294\,669-208\,915\,\sqrt{2}\right)\,x\right)}{\sqrt{3-x+2\,x^2}}\right]$$

Result (type 3, 1170 leaves):

$$\sqrt{3-x+2\,x^2}\,\left(\frac{4+65\,x}{1364\,\left(2+3\,x+5\,x^2\right)^2}+\frac{26\,794+86\,265\,x}{1\,860\,496\,\left(2+3\,x+5\,x^2\right)}\right)-\frac{1}{3\,720\,992\,\sqrt{682\,\left(13+\mathrm{i}\,\sqrt{31}\right)}}$$

$$125\,\mathrm{i}\,\left(-41\,783\,\mathrm{i}+1489\,\sqrt{31}\right)\,\text{ArcTan}\Big[\Big(31\,\big(1\,733\,669\,734\,\mathrm{i}+1\,411\,781\,250\,\sqrt{31}-8\,257\,920\,150\,\mathrm{i}\,x+438\,440\,750\,\sqrt{31}\,x+$$
$$8\,927\,431\,079\,\mathrm{i}\,x^2+784\,505\,986\,\sqrt{31}\,x^2-8\,456\,927\,744\,\mathrm{i}\,x^3+557\,246\,338\,\sqrt{31}\,x^3+3\,245\,899\,757\,\mathrm{i}\,x^4+97\,553\,324\,\sqrt{31}\,x^4\big)\Big)\Big/$$
$$\Big(74\,935\,517\,250+14\,089\,391\,258\,\mathrm{i}\,\sqrt{31}+394\,528\,763\,486\,x-31\,523\,713\,098\,\mathrm{i}\,\sqrt{31}\,x+37\,412\,913\,890\,x^2-81\,049\,798\,431\,\mathrm{i}\,\sqrt{31}\,x^2+$$
$$237\,240\,959\,890\,x^3-29\,645\,645\,200\,\mathrm{i}\,\sqrt{31}\,x^3+84\,861\,105\,868\,x^4-72\,669\,503\,461\,\mathrm{i}\,\sqrt{31}\,x^4-$$
$$362\,909\,968\,\mathrm{i}\,\sqrt{682\,\left(13+\mathrm{i}\,\sqrt{31}\right)}\,\sqrt{3-x+2\,x^2}+907\,274\,920\,\mathrm{i}\,\sqrt{682\,\left(13+\mathrm{i}\,\sqrt{31}\right)}\,x\,\sqrt{3-x+2\,x^2}+$$
$$1\,270\,184\,888\,\mathrm{i}\,\sqrt{682\,\left(13+\mathrm{i}\,\sqrt{31}\right)}\,x^2\,\sqrt{3-x+2\,x^2}+3\,629\,099\,680\,\mathrm{i}\,\sqrt{682\,\left(13+\mathrm{i}\,\sqrt{31}\right)}\,x^3\,\sqrt{3-x+2\,x^2}\Big)\Big]-$$

$$\frac{1}{3\,720\,992\,\sqrt{682\,\left(-13+\mathrm{i}\,\sqrt{31}\right)}}\,125\,\mathrm{i}\,\left(41\,783\,\mathrm{i}+1489\,\sqrt{31}\right)\,\text{ArcTanh}\Big[\Big(-74\,935\,517\,250\,\mathrm{i}-14\,089\,391\,258\,\sqrt{31}-394\,528\,763\,486\,\mathrm{i}\,x+$$
$$31\,523\,713\,098\,\sqrt{31}\,x-37\,412\,913\,890\,\mathrm{i}\,x^2+81\,049\,798\,431\,\sqrt{31}\,x^2-237\,240\,959\,890\,\mathrm{i}\,x^3+29\,645\,645\,200\,\sqrt{31}\,x^3-84\,861\,105\,868\,\mathrm{i}\,x^4+$$
$$72\,669\,503\,461\,\sqrt{31}\,x^4-22\,863\,327\,984\,\sqrt{22\,\left(-13+\mathrm{i}\,\sqrt{31}\right)}\,\sqrt{3-x+2\,x^2}-26\,310\,972\,680\,\sqrt{22\,\left(-13+\mathrm{i}\,\sqrt{31}\right)}\,x\,\sqrt{3-x+2\,x^2}-$$
$$45\,182\,291\,016\,\sqrt{22\,\left(-13+\mathrm{i}\,\sqrt{31}\right)}\,x^2\,\sqrt{3-x+2\,x^2}+19\,960\,048\,240\,\sqrt{22\,\left(-13+\mathrm{i}\,\sqrt{31}\right)}\,x^3\,\sqrt{3-x+2\,x^2}\Big)\Big/$$

$$\left(672\,076\,174\,246\,i + 43\,765\,218\,750\sqrt{31} + 796\,731\,376\,970\,i\,x + 13\,591\,663\,250\sqrt{31}\,x + 893\,634\,283\,351\,i\,x^2 + 24\,319\,685\,566\sqrt{31}\,x^2 - 841\,081\,542\,656\,i\,x^3 + 17\,274\,636\,478\sqrt{31}\,x^3 + 343\,941\,818\,333\,i\,x^4 + 3\,024\,153\,044\sqrt{31}\,x^4\right)\Big] -$$

$$\frac{125\left(-41\,783\,i + 1489\sqrt{31}\right)\text{Log}\left[\left(-3\,i + \sqrt{31} - 10\,i\,x\right)^2\left(3\,i + \sqrt{31} + 10\,i\,x\right)^2\right]}{7\,441\,984\sqrt{682\left(13 + i\sqrt{31}\right)}} +$$

$$\frac{125\,i\left(41\,783\,i + 1489\sqrt{31}\right)\text{Log}\left[\left(-3\,i + \sqrt{31} - 10\,i\,x\right)^2\left(3\,i + \sqrt{31} + 10\,i\,x\right)^2\right]}{7\,441\,984\sqrt{682\left(-13 + i\sqrt{31}\right)}} -$$

$$\left(125\,i\left(41\,783\,i + 1489\sqrt{31}\right)\text{Log}\left[\left(2 + 3x + 5x^2\right)\left(-142\,i + 66\sqrt{31} + 469\,i\,x - 22\sqrt{31}\,x + 327\,i\,x^2 + 44\sqrt{31}\,x^2 + i\sqrt{682\left(-13 + i\sqrt{31}\right)}\sqrt{3 - x + 2x^2} - 4\,i\sqrt{682\left(-13 + i\sqrt{31}\right)}\,x\sqrt{3 - x + 2x^2}\right)\right]\right) \Big/ \left(7\,441\,984\sqrt{682\left(-13 + i\sqrt{31}\right)}\right) +$$

$$\left(125\left(-41\,783\,i + 1489\sqrt{31}\right)\text{Log}\left[\left(2 + 3x + 5x^2\right)\left(-1858\,i + 66\sqrt{31} + 1041\,i\,x - 22\sqrt{31}\,x - 817\,i\,x^2 + 44\sqrt{31}\,x^2 - 63\,i\sqrt{22\left(13 + i\sqrt{31}\right)}\sqrt{3 - x + 2x^2} + 22\,i\sqrt{22\left(13 + i\sqrt{31}\right)}\,x\sqrt{3 - x + 2x^2}\right)\right]\right) \Big/ \left(7\,441\,984\sqrt{682\left(13 + i\sqrt{31}\right)}\right)$$

- **Problem 90: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left(3 - x + 2x^2\right)^{3/2}\left(2 + 3x + 5x^2\right)}\,dx$$

Optimal (type 3, 176 leaves, 6 steps):

$$\frac{13 - 6x}{253\sqrt{3 - x + 2x^2}} + \frac{1}{22}\sqrt{\frac{1}{682}\left(247 + 500\sqrt{2}\right)}\,\text{ArcTan}\left[\frac{\sqrt{\frac{11}{31\left(247 + 500\sqrt{2}\right)}}\left(61 + 4\sqrt{2} + \left(69 + 65\sqrt{2}\right)x\right)}{\sqrt{3 - x + 2x^2}}\right] -$$

$$\frac{1}{22}\sqrt{\frac{1}{682}\left(-247 + 500\sqrt{2}\right)}\,\text{ArcTanh}\left[\frac{\sqrt{\frac{11}{31\left(-247 + 500\sqrt{2}\right)}}\left(61 - 4\sqrt{2} + \left(69 - 65\sqrt{2}\right)x\right)}{\sqrt{3 - x + 2x^2}}\right]$$

Result (type 3, 1044 leaves):

$$\frac{13-6x}{253\sqrt{3-x+2x^2}}+\frac{1}{22\sqrt{682\left(13+i\sqrt{31}\right)}}$$

$$5i\left(13i+\sqrt{31}\right)\text{ArcTan}\Big[\Big(31\left(-74-66i\sqrt{31}+14\left(3+11i\sqrt{31}\right)x+7\left(185-22i\sqrt{31}\right)x^2+\left(-1160+110i\sqrt{31}\right)x^3+\left(797-44i\sqrt{31}\right)x^4\right)\Big)\Big/$$

$$\Big(-14322i+602\sqrt{31}+\left(17732i+4439\sqrt{31}\right)x^4-40\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+$$

$$10x^3\left(-1705i+512\sqrt{31}+40\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+2x\left(-3751i-2133\sqrt{31}+50\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$x^2\left(21142i+6405\sqrt{31}+140\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\Big)\Big]-\frac{1}{22\sqrt{682i\left(13i+\sqrt{31}\right)}}$$

$$5\left(-13i+\sqrt{31}\right)\text{ArcTan}\Big[\Big(\left(17732+4439i\sqrt{31}\right)x^4+10ix^3\left(1705i+512\sqrt{31}-220\sqrt{22i\left(13i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$x\left(-7502-4266i\sqrt{31}+2900i\sqrt{22i\left(13i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$x^2\left(21142+6405i\sqrt{31}+4980i\sqrt{22i\left(13i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+14i\left(1023i+43\sqrt{31}+180\sqrt{22i\left(13i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\Big)\Big/$$

$$\left(82294i+2046\sqrt{31}+\left(58298i-4774\sqrt{31}\right)x+\left(88855i+4774\sqrt{31}\right)x^2-10\left(8564i+341\sqrt{31}\right)x^3+\left(24293i+1364\sqrt{31}\right)x^4\right)\Big]-$$

$$\frac{5i\left(-13i+\sqrt{31}\right)\text{Log}\left[400\left(2+3x+5x^2\right)^2\right]}{44\sqrt{682i\left(13i+\sqrt{31}\right)}}+\frac{5\left(13i+\sqrt{31}\right)\text{Log}\left[400\left(2+3x+5x^2\right)^2\right]}{44\sqrt{682\left(13+i\sqrt{31}\right)}}-$$

$$\frac{1}{44\sqrt{682\left(13+i\sqrt{31}\right)}}$$

$$5\left(13i+\sqrt{31}\right)$$

$$\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-1858i+66\sqrt{31}+\left(-817i+44\sqrt{31}\right)x^2-63i\sqrt{286+22i\sqrt{31}}\sqrt{3-x+2x^2}+$$

$$x\left(1041i-22\sqrt{31}+22i\sqrt{286+22i\sqrt{31}}\sqrt{3-x+2x^2}\right)\Big)\Big]+\frac{1}{44\sqrt{682i\left(13i+\sqrt{31}\right)}}5\left(13+i\sqrt{31}\right)\text{Log}\left[\left(2+3x+5x^2\right)\right.$$

$$\left(-142\,i+66\sqrt{31}+\left(327\,i+44\sqrt{31}\right)x^2+i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}+x\left(469\,i-22\sqrt{31}-4\,i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\right)\Big]$$

- **Problem 91: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2}\,dx$$

Optimal (type 3, 211 leaves, 7 steps):

$$-\frac{6315-2306x}{345092\sqrt{3-x+2x^2}}+\frac{4+65x}{682\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}+\frac{1}{30008}$$

$$\sqrt{\frac{1}{682}\left(129694447+103775000\sqrt{2}\right)}\,\mathrm{ArcTan}\Big[\frac{\sqrt{\frac{11}{31\left(129694447+103775000\sqrt{2}\right)}}\left(12611+16454\sqrt{2}+\left(45519+29065\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\Big]-\frac{1}{30008}$$

$$\sqrt{\frac{1}{682}\left(-129694447+103775000\sqrt{2}\right)}\,\mathrm{ArcTanh}\Big[\frac{\sqrt{\frac{11}{31\left(-129694447+103775000\sqrt{2}\right)}}\left(12611-16454\sqrt{2}+\left(45519-29065\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\Big]$$

Result (type 3, 1170 leaves):

$$\sqrt{3-x+2x^2}\left(\frac{-31-14x}{5566\left(3-x+2x^2\right)}+\frac{-98+345x}{15004\left(2+3x+5x^2\right)}\right)-$$

$$\frac{1}{30008\sqrt{682\left(13+i\sqrt{31}\right)}}5\,i\left(-5813\,i+499\sqrt{31}\right)\mathrm{ArcTan}\Big[\Big(31\left(67211446\,i+35267826\sqrt{31}-236270118\,i\,x+36393566\sqrt{31}\,x-\right.$$

$$\left.\left.2553985\,i\,x^2+23896114\sqrt{31}\,x^2-282686240\,i\,x^3+26621650\sqrt{31}\,x^3-104765803\,i\,x^4+10956044\sqrt{31}\,x^4\right)\right)\Big/$$

$$\Big(294638322+278507402\,i\sqrt{31}+8796989102\,x-311643066\,i\sqrt{31}\,x+3166163858\,x^2-2655130695\,i\sqrt{31}\,x^2+$$

$$3951866050\,x^3-1267524880\,i\sqrt{31}\,x^3+3956537068\,x^4-2241477661\,i\sqrt{31}\,x^4-$$

$$8302000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+20755000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}+$$

$$29057000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^2\sqrt{3-x+2x^2}+83020000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\Big)\Big]-$$

$$\frac{1}{30\,008\sqrt{682\left(-13+i\sqrt{31}\right)}} 5 i \left(5813 i+499\sqrt{31}\right) \text{ArcTanh}\Big[\Big(-294\,638\,322 i-278\,507\,402\sqrt{31}-8\,796\,989\,102 i x+ 311\,643\,066\sqrt{31} x-3\,166\,163\,858 i x^2+2\,655\,130\,695\sqrt{31} x^2-3\,951\,866\,050 i x^3+1\,267\,524\,880\sqrt{31} x^3-3\,956\,537\,068 i x^4+ 2\,241\,477\,661\sqrt{31} x^4-523\,026\,000\sqrt{22\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-601\,895\,000\sqrt{22\left(-13+i\sqrt{31}\right)} x\sqrt{3-x+2x^2}- 1\,033\,599\,000\sqrt{22\left(-13+i\sqrt{31}\right)} x^2\sqrt{3-x+2x^2}+456\,610\,000\sqrt{22\left(-13+i\sqrt{31}\right)} x^3\sqrt{3-x+2x^2}\Big)\Big/ \left(14\,520\,445\,174 i+1\,093\,302\,606\sqrt{31}+19\,694\,353\,658 i x+1\,128\,200\,546\sqrt{31} x+26\,853\,123\,535 i x^2+740\,779\,534\sqrt{31} x^2- 16\,474\,806\,560 i x^3+825\,271\,150\sqrt{31} x^3+13\,417\,689\,893 i x^4+339\,637\,364\sqrt{31} x^4\right)\Big]-$$

$$\frac{5\left(-5813 i+499\sqrt{31}\right)\text{Log}\left[\left(-3 i+\sqrt{31}-10 i x\right)^2\left(3 i+\sqrt{31}+10 i x\right)^2\right]}{60\,016\sqrt{682\left(13+i\sqrt{31}\right)}}+$$

$$\frac{5 i\left(5813 i+499\sqrt{31}\right)\text{Log}\left[\left(-3 i+\sqrt{31}-10 i x\right)^2\left(3 i+\sqrt{31}+10 i x\right)^2\right]}{60\,016\sqrt{682\left(-13+i\sqrt{31}\right)}}-$$

$$\Big(5 i\left(5813 i+499\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-142 i+66\sqrt{31}+469 i x-22\sqrt{31} x+327 i x^2+44\sqrt{31} x^2+ i\sqrt{682\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-4 i\sqrt{682\left(-13+i\sqrt{31}\right)} x\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/\left(60\,016\sqrt{682\left(-13+i\sqrt{31}\right)}\right)+$$

$$\Big(5\left(-5813 i+499\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-1858 i+66\sqrt{31}+1041 i x-22\sqrt{31} x-817 i x^2+44\sqrt{31} x^2- 63 i\sqrt{22\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+22 i\sqrt{22\left(13+i\sqrt{31}\right)} x\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/\left(60\,016\sqrt{682\left(13+i\sqrt{31}\right)}\right)$$

- **Problem 92: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3}\, dx$$

Optimal (type 3, 246 leaves, 8 steps) :

$$-\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}}+\frac{4+65x}{1364\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2}+$$

$$\frac{5(7318+17315x)}{1860496\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}+\frac{1}{81861824}3\sqrt{\frac{1}{682}\left(13874275807943+9819738650000\sqrt{2}\right)}$$

$$\text{ArcTan}\left[\frac{\sqrt{\frac{11}{31\left(13874275807943+9819738650000\sqrt{2}\right)}}\left(5538393+4123702\sqrt{2}+\left(13785797+9662095\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]-$$

$$\frac{1}{81861824}3\sqrt{\frac{1}{682}\left(-13874275807943+9819738650000\sqrt{2}\right)}$$

$$\text{ArcTanh}\left[\frac{\sqrt{\frac{11}{31\left(-13874275807943+9819738650000\sqrt{2}\right)}}\left(5538393-4123702\sqrt{2}+\left(13785797-9662095\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]$$

Result (type 3, 1191 leaves) :

$$\sqrt{3-x+2x^2}\left(\frac{-11+90x}{122452\left(3-x+2x^2\right)}+\frac{-98+345x}{30008\left(2+3x+5x^2\right)^2}+\frac{231418+632255x}{40930912\left(2+3x+5x^2\right)}\right)-$$

$$\frac{1}{81861824\sqrt{682\left(13+i\sqrt{31}\right)}}15\,i\left(-1932419\,i+79037\sqrt{31}\right)$$

$$\text{ArcTan}\Big[\Big(31\Big(4059546477574\,i+3106527877794\sqrt{31}-18544569435542\,i\,x+1227936189854\sqrt{31}\,x+17501774027535\,i\,x^2+$$
$$1728828684066\sqrt{31}\,x^2-18989790004560\,i\,x^3+1371533012850\sqrt{31}\,x^3+5399410180693\,i\,x^4+274861284236\sqrt{31}\,x^4\Big)\Big)\Big/$$
$$\Big(148573472722818+30402744893338\,i\sqrt{31}+862374952340638\,x-64577765937354\,i\sqrt{31}\,x+107573401361602\,x^2-$$
$$186540875521455\,i\sqrt{31}\,x^2+503769328622450\,x^3-71509340960720\,i\sqrt{31}\,x^3+208327267086092\,x^4-165714245597909\,i\sqrt{31}\,x^4-$$
$$785579092000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+1963947730000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}+$$
$$2749526822000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^2\sqrt{3-x+2x^2}+7855790920000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\Big)\Big]-$$

$$\frac{1}{81861824\sqrt{682\left(-13+i\sqrt{31}\right)}}15\,i\left(1932419\,i+79037\sqrt{31}\right)\text{ArcTanh}\Big[\left(-148573472722818\,i-30402744893338\sqrt{31}-862374952340638\,i\,x+64577765937354\sqrt{31}\,x-107573401361602\,i\,x^2+186540875521455\sqrt{31}\,x^2-503769328622450\,i\,x^3+71509340960720\sqrt{31}\,x^3-208327267086092\,i\,x^4+165714245597909\sqrt{31}\,x^4-49491482796000\sqrt{22\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-56954484170000\sqrt{22\left(-13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}-97804596954000\sqrt{22\left(-13+i\sqrt{31}\right)}\,x^2\sqrt{3-x+2x^2}+43206850060000\sqrt{22\left(-13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\right)\Big/\left(1445312243195206\,i+96302364211614\sqrt{31}+1745394499581802\,i\,x+38066021885474\sqrt{31}\,x+1990937576846415\,i\,x^2+53593689206046\sqrt{31}\,x^2-1799476949538640\,i\,x^3+42517523398350\sqrt{31}\,x^3+794952672098517\,i\,x^4+8520699811316\sqrt{31}\,x^4\right)\Big]-$$

$$\frac{15\left(-1932419\,i+79037\sqrt{31}\right)\text{Log}\left[\left(-3\,i+\sqrt{31}-10\,i\,x\right)^2\left(3\,i+\sqrt{31}+10\,i\,x\right)^2\right]}{163723648\sqrt{682\left(13+i\sqrt{31}\right)}}+$$

$$\frac{15\,i\left(1932419\,i+79037\sqrt{31}\right)\text{Log}\left[\left(-3\,i+\sqrt{31}-10\,i\,x\right)^2\left(3\,i+\sqrt{31}+10\,i\,x\right)^2\right]}{163723648\sqrt{682\left(-13+i\sqrt{31}\right)}}-$$

$$\left(15\,i\left(1932419\,i+79037\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\left(-142\,i+66\sqrt{31}+469\,i\,x-22\sqrt{31}\,x+327\,i\,x^2+44\sqrt{31}\,x^2+i\sqrt{682\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-4\,i\sqrt{682\left(-13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\right)\Big]\right)\Big/\left(163723648\sqrt{682\left(-13+i\sqrt{31}\right)}\right)+$$

$$\left(15\left(-1932419\,i+79037\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\left(-1858\,i+66\sqrt{31}+1041\,i\,x-22\sqrt{31}\,x-817\,i\,x^2+44\sqrt{31}\,x^2-63\,i\sqrt{22\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+22\,i\sqrt{22\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\right)\Big]\right)\Big/\left(163723648\sqrt{682\left(13+i\sqrt{31}\right)}\right)$$

- **Problem 97: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)}\,dx$$

Optimal (type 3, 199 leaves, 7 steps) :

$$\frac{13-6x}{759\left(3-x+2x^2\right)^{3/2}}+\frac{3603-658x}{128018\sqrt{3-x+2x^2}}+\frac{1}{484}\sqrt{\frac{1}{682}\left(-15457+25000\sqrt{2}\right)}\,\text{ArcTan}\left[\frac{\sqrt{\frac{11}{31\left(-15457+25000\sqrt{2}\right)}}\left(443-98\sqrt{2}+\left(247+345\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]-\frac{1}{484}\sqrt{\frac{1}{682}\left(15457+25000\sqrt{2}\right)}\,\text{ArcTanh}\left[\frac{\sqrt{\frac{11}{31\left(15457+25000\sqrt{2}\right)}}\left(443+98\sqrt{2}+\left(247-345\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]$$

Result (type 3, 1080 leaves) :

$$\frac{13-6x}{759\left(3-x+2x^2\right)^{3/2}}+\frac{3603-658x}{128018\sqrt{3-x+2x^2}}+\frac{1}{484\sqrt{682\left(13+i\sqrt{31}\right)}}5\,i\left(69\,i+13\sqrt{31}\right)\text{ArcTan}\Big[$$
$$\left(31\left(-3626-594\,i\sqrt{31}+\left(24058+5346\,i\sqrt{31}\right)x+\left(-10465-13266\,i\sqrt{31}\right)x^2+\left(-106560+7150\,i\sqrt{31}\right)x^3+\left(-17707-7436\,i\sqrt{31}\right)x^4\right)\right)\Big/$$
$$\Big(-186\left(2013\,i+167\sqrt{31}\right)+\left(1223508\,i+526291\sqrt{31}\right)x^4-2000\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+10x^3$$
$$\left(18755\,i+37528\sqrt{31}+2000\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+2x\left(661881\,i-36077\sqrt{31}+2500\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$
$$x^2\left(1185998\,i+657545\sqrt{31}+7000\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\Big)\Big]-$$
$$\Big(5\left(-69\,i+13\sqrt{31}\right)\text{ArcTan}\Big[\Big(\left(1223508+526291\,i\sqrt{31}\right)x^4+10x^3\left(18755+37528\,i\sqrt{31}-11000\,i\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$
$$6\left(-62403-5177\,i\sqrt{31}+21000\,i\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$
$$2x\left(661881-36077\,i\sqrt{31}+72500\,i\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$
$$x^2\left(1185998+657545\,i\sqrt{31}+249000\,i\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\Big)\Big/\Big(4112406\,i+18414\sqrt{31}-$$
$$6\left(-372367\,i+27621\sqrt{31}\right)x+\left(6774415\,i+411246\sqrt{31}\right)x^2-10\left(277664\,i+22165\sqrt{31}\right)x^3+\left(2998917\,i+230516\sqrt{31}\right)x^4\Big)\Big]\Big)\Big/$$

$$\left(484\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\right)-\frac{5\,i\left(-69\,i+13\sqrt{31}\right)\mathrm{Log}\left[400\left(2+3x+5x^2\right)^2\right]}{968\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}}+$$

$$\frac{5\left(69\,i+13\sqrt{31}\right)\mathrm{Log}\left[400\left(2+3x+5x^2\right)^2\right]}{968\sqrt{682\left(13+i\sqrt{31}\right)}}-$$

$$\frac{1}{968\sqrt{682\left(13+i\sqrt{31}\right)}}$$

$$5\left(69\,i+13\sqrt{31}\right)\mathrm{Log}\Big[\left(2+3x+5x^2\right)\Big(-1858\,i+66\sqrt{31}+\left(-817\,i+44\sqrt{31}\right)x^2-63\,i\sqrt{286+22\,i\sqrt{31}}\sqrt{3-x+2x^2}+x\left(1041\,i-22\sqrt{31}+22\,i\sqrt{286+22\,i\sqrt{31}}\sqrt{3-x+2x^2}\right)\Big)\Big]+$$

$$\Big(5\left(69+13\,i\sqrt{31}\right)\mathrm{Log}\Big[\left(2+3x+5x^2\right)\Big(-142\,i+66\sqrt{31}+\left(327\,i+44\sqrt{31}\right)x^2+i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}+x\left(469\,i-22\sqrt{31}-4\,i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\Big)\Big]\Big)\Big/\left(968\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\right)$$

- **Problem 98: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^2}\,dx$$

Optimal (type 3, 234 leaves, 8 steps) :

$$-\frac{15101-8654x}{1035276\left(3-x+2x^2\right)^{3/2}}-\frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}}+\frac{4+65x}{682\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)}+$$

$$\frac{625\sqrt{\frac{1}{682}\left(30463+23600\sqrt{2}\right)}\ \text{ArcTan}\left[\frac{\sqrt{\frac{11}{31\left(30463+23600\sqrt{2}\right)}}\left(203+242\sqrt{2}+\left(687+445\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]}{660176}-$$

$$\frac{625\sqrt{\frac{1}{682}\left(-30463+23600\sqrt{2}\right)}\ \text{ArcTanh}\left[\frac{\sqrt{\frac{11}{31\left(-30463+23600\sqrt{2}\right)}}\left(203-242\sqrt{2}+\left(687-445\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]}{660176}$$

Result (type 3, 1191 leaves) :

$$\sqrt{3-x+2x^2}\left(\frac{-31-14x}{16698\left(3-x+2x^2\right)^2}+\frac{-10769-17230x}{4224594\left(3-x+2x^2\right)}+\frac{-1474+1235x}{330088\left(2+3x+5x^2\right)}\right)-\Bigg(3125\,i\left(-89\,i+7\sqrt{31}\right)$$

$$\text{ArcTan}\Big[\Big(31\left(14518\,i+7986\sqrt{31}-52806\,i\,x+7502\sqrt{31}\,x+6503\,i\,x^2+5170\sqrt{31}\,x^2-60944\,i\,x^3+5698\sqrt{31}\,x^3-17827\,i\,x^4+2156\sqrt{31}\,x^4\right)\Big)\Big/$$

$$\Big(112530+65642\,i\sqrt{31}+2037134x-84762\,i\sqrt{31}\,x+658130x^2-587559\,i\sqrt{31}\,x^2+958210x^3-274000\,i\sqrt{31}\,x^3+$$

$$849772x^4-499069\,i\sqrt{31}\,x^4-1888\,i\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+4720\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}+$$

$$6608\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^2\sqrt{3-x+2x^2}+18880\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\Big)\Big]\Bigg)\Bigg/\left(660176\sqrt{682\left(13+i\sqrt{31}\right)}\right)-$$

$$\Bigg(3125\,i\left(89\,i+7\sqrt{31}\right)\text{ArcTanh}\Big[\Big(-112530\,i-65642\sqrt{31}-2037134\,i\,x+84762\sqrt{31}\,x-658130\,i\,x^2+587559\sqrt{31}\,x^2-958210\,i\,x^3+$$

$$274000\sqrt{31}\,x^3-849772\,i\,x^4+499069\sqrt{31}\,x^4-118944\sqrt{22\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-136880\sqrt{22\left(-13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}-$$

$$235056\sqrt{22\left(-13+i\sqrt{31}\right)}\,x^2\sqrt{3-x+2x^2}+103840\sqrt{22\left(-13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\Big)\Big/\Big(3325942\,i+247566\sqrt{31}+$$

$$4450106\,i\,x+232562\sqrt{31}\,x+5887207\,i\,x^2+160270\sqrt{31}\,x^2-3850256\,i\,x^3+176638\sqrt{31}\,x^3+2865437\,i\,x^4+66836\sqrt{31}\,x^4\Big)\Big]\Bigg)\Bigg/$$

$$\left(660176\sqrt{682\left(-13+i\sqrt{31}\right)}\right)-\frac{3125\left(-89i+7\sqrt{31}\right)\text{Log}\left[\left(-3i+\sqrt{31}-10ix\right)^2\left(3i+\sqrt{31}+10ix\right)^2\right]}{1320352\sqrt{682\left(13+i\sqrt{31}\right)}}+$$

$$\frac{3125i\left(89i+7\sqrt{31}\right)\text{Log}\left[\left(-3i+\sqrt{31}-10ix\right)^2\left(3i+\sqrt{31}+10ix\right)^2\right]}{1320352\sqrt{682\left(-13+i\sqrt{31}\right)}}-$$

$$\Big(3125i\left(89i+7\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-142i+66\sqrt{31}+469ix-22\sqrt{31}x+327ix^2+44\sqrt{31}x^2+$$

$$i\sqrt{682\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-4i\sqrt{682\left(-13+i\sqrt{31}\right)}x\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/\left(1320352\sqrt{682\left(-13+i\sqrt{31}\right)}\right)+$$

$$\Big(3125\left(-89i+7\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-1858i+66\sqrt{31}+1041ix-22\sqrt{31}x-817ix^2+44\sqrt{31}x^2-$$

$$63i\sqrt{22\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+22i\sqrt{22\left(13+i\sqrt{31}\right)}x\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/\left(1320352\sqrt{682\left(13+i\sqrt{31}\right)}\right)$$

- **Problem 99: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^3}\,dx$$

Optimal (type 3, 269 leaves, 9 steps) :

$$-\frac{12280939-19536786\,x}{2824232928\,\left(3-x+2\,x^2\right)^{3/2}}-\frac{1134826571-1504660754\,x}{476353953856\,\sqrt{3-x+2\,x^2}}+\frac{4+65\,x}{1364\,\left(3-x+2\,x^2\right)^{3/2}\,\left(2+3\,x+5\,x^2\right)^2}+$$

$$\frac{46386+86885\,x}{1860496\,\left(3-x+2\,x^2\right)^{3/2}\,\left(2+3\,x+5\,x^2\right)}+\frac{1}{1800960128}35\,\sqrt{\frac{1}{682}\left(2243059557247+2011748500000\,\sqrt{2}\right)}$$

$$\mathrm{ArcTan}\left[\frac{\sqrt{\frac{11}{31\,\left(2243059557247+2011748500000\,\sqrt{2}\right)}}\,\left(1432939+2428746\,\sqrt{2}+\left(6290431+3861685\,\sqrt{2}\right)\,x\right)}{\sqrt{3-x+2\,x^2}}\right]-$$

$$\frac{1}{1800960128}35\,\sqrt{\frac{1}{682}\left(-2243059557247+2011748500000\,\sqrt{2}\right)}$$

$$\mathrm{ArcTanh}\left[\frac{\sqrt{\frac{11}{31\,\left(-2243059557247+2011748500000\,\sqrt{2}\right)}}\,\left(1432939-2428746\,\sqrt{2}+\left(6290431-3861685\,\sqrt{2}\right)\,x\right)}{\sqrt{3-x+2\,x^2}}\right]$$

Result (type 3, 1218 leaves) :

$$\sqrt{3-x+2\,x^2}\,\left(\frac{-11+90\,x}{367356\,\left(3-x+2\,x^2\right)^2}+\frac{-39095+53754\,x}{61960712\,\left(3-x+2\,x^2\right)}+\frac{-1474+1235\,x}{660176\,\left(2+3\,x+5\,x^2\right)^2}+\frac{157362+468895\,x}{81861824\,\left(2+3\,x+5\,x^2\right)}\right)+$$

$$\frac{1}{1800960128\,\sqrt{682\,\left(-13+i\,\sqrt{31}\right)}}175\,\left(772337\,i+81951\,\sqrt{31}\right)$$

$$\mathrm{ArcTan}\Big[\Big(4655364448878+4766043812202\,i\,\sqrt{31}-158699364373902\,x-2787485821466\,i\,\sqrt{31}\,x-74012991583058\,x^2-$$
$$54042219198695\,i\,\sqrt{31}\,x^2-61598686386050\,x^3-27260449836880\,i\,\sqrt{31}\,x^3-86332728860268\,x^4-44936737584061\,i\,\sqrt{31}\,x^4+$$
$$10139212440000\,i\,\sqrt{22\,\left(-13+i\,\sqrt{31}\right)}\,\sqrt{3-x+2\,x^2}+11668141300000\,i\,\sqrt{22\,\left(-13+i\,\sqrt{31}\right)}\,x\,\sqrt{3-x+2\,x^2}+$$
$$20037015060000\,i\,\sqrt{22\,\left(-13+i\,\sqrt{31}\right)}\,x^2\,\sqrt{3-x+2\,x^2}-8851693400000\,i\,\sqrt{22\,\left(-13+i\,\sqrt{31}\right)}\,x^3\,\sqrt{3-x+2\,x^2}\Big)\Big/$$
$$\Big(276508696366774\,i+21211104525006\,\sqrt{31}+386113686180858\,i\,x+27073970836946\,\sqrt{31}\,x+572257780896535\,i\,x^2+$$
$$16500157269134\,\sqrt{31}\,x^2-293982300056560\,i\,x^3+18182603589150\,\sqrt{31}\,x^3+303413457358093\,i\,x^4+9160578170964\,\sqrt{31}\,x^4\Big)\Big]-$$

$$\frac{1}{1\,800\,960\,128\sqrt{682\left(13+i\sqrt{31}\right)}}\,175\,i\left(-772\,337\,i+81\,951\sqrt{31}\right)$$

$$\text{ArcTan}\Big[\Big(31\,\big(1\,463\,582\,697\,846\,i+684\,229\,178\,226\sqrt{31}-4\,719\,782\,741\,318\,i\,x+873\,353\,897\,966\sqrt{31}\,x-1\,716\,989\,286\,985\,i\,x^2+$$
$$532\,263\,137\,714\sqrt{31}\,x^2-6\,299\,191\,456\,240\,i\,x^3+586\,535\,599\,650\sqrt{31}\,x^3-3\,427\,809\,818\,003\,i\,x^4+295\,502\,521\,644\sqrt{31}\,x^4\big)\Big)\Big/$$
$$\Big(-4\,655\,364\,448\,878+4\,766\,043\,812\,202\,i\sqrt{31}+158\,699\,364\,373\,902\,x-2\,787\,485\,821\,466\,i\sqrt{31}\,x+74\,012\,991\,583\,058\,x^2-$$
$$54\,042\,219\,198\,695\,i\sqrt{31}\,x^2+61\,598\,686\,386\,050\,x^3-27\,260\,449\,836\,880\,i\sqrt{31}\,x^3+86\,332\,728\,860\,268\,x^4-44\,936\,737\,584\,061\,i\sqrt{31}\,x^4-$$
$$160\,939\,880\,000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+402\,349\,700\,000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}+$$
$$563\,289\,580\,000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^2\sqrt{3-x+2x^2}+1\,609\,398\,800\,000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\Big)\Big]-$$

$$\frac{175\left(-772\,337\,i+81\,951\sqrt{31}\right)\text{Log}\left[\left(-3\,i+\sqrt{31}-10\,i\,x\right)^2\left(3\,i+\sqrt{31}+10\,i\,x\right)^2\right]}{3\,601\,920\,256\sqrt{682\left(13+i\sqrt{31}\right)}}+$$

$$\frac{175\,i\left(772\,337\,i+81\,951\sqrt{31}\right)\text{Log}\left[\left(-3\,i+\sqrt{31}-10\,i\,x\right)^2\left(3\,i+\sqrt{31}+10\,i\,x\right)^2\right]}{3\,601\,920\,256\sqrt{682\left(-13+i\sqrt{31}\right)}}-$$

$$\Big(175\,i\left(772\,337\,i+81\,951\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-142\,i+66\sqrt{31}+469\,i\,x-22\sqrt{31}\,x+327\,i\,x^2+44\sqrt{31}\,x^2+$$
$$i\sqrt{682\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-4\,i\sqrt{682\left(-13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/\left(3\,601\,920\,256\sqrt{682\left(-13+i\sqrt{31}\right)}\right)+$$

$$\Big(175\left(-772\,337\,i+81\,951\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-1858\,i+66\sqrt{31}+1041\,i\,x-22\sqrt{31}\,x-817\,i\,x^2+44\sqrt{31}\,x^2-$$
$$63\,i\sqrt{22\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+22\,i\sqrt{22\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/\left(3\,601\,920\,256\sqrt{682\left(13+i\sqrt{31}\right)}\right)$$

- **Problem 106: Result more than twice size of optimal antiderivative.**

$$\int\frac{\left(a+b\,x+c\,x^2\right)^{3/2}}{d+e\,x+f\,x^2}\,dx$$

Optimal (type 3, 679 leaves, 9 steps) :

$$-\frac{(4 c e-5 b f-2 c f x) \sqrt{a+b x+c x^2}}{4 f^2}+\frac{\left(3 b^2 f^2-12 c f (b e-a f)+8 c^2 \left(e^2-d f\right)\right) \text{ArcTanh}\left[\frac{b+2 c x}{2 \sqrt{c} \sqrt{a+b x+c x^2}}\right]}{8 \sqrt{c} f^3}+$$

$$\left(\left((c e-b f) \left(e-\sqrt{e^2-4 d f}\right) \left(f (b e-2 a f)-c \left(e^2-2 d f\right)\right)-2 f \left(2 c d f (b e-a f)-f^2 \left(b^2 d-a^2 f\right)-c^2 d \left(e^2-d f\right)\right)\right)\right.$$

$$\left.\text{ArcTanh}\left[\frac{4 a f-b \left(e-\sqrt{e^2-4 d f}\right)+2 \left(b f-c \left(e-\sqrt{e^2-4 d f}\right)\right) x}{2 \sqrt{2} \sqrt{c e^2-2 c d f-b e f+2 a f^2-(c e-b f) \sqrt{e^2-4 d f}} \sqrt{a+b x+c x^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{2} f^3 \sqrt{e^2-4 d f} \sqrt{c e^2-2 c d f-b e f+2 a f^2-(c e-b f) \sqrt{e^2-4 d f}}\right)-$$

$$\left(\left((c e-b f) \left(e+\sqrt{e^2-4 d f}\right) \left(f (b e-2 a f)-c \left(e^2-2 d f\right)\right)-2 f \left(2 c d f (b e-a f)-f^2 \left(b^2 d-a^2 f\right)-c^2 d \left(e^2-d f\right)\right)\right)\right.$$

$$\left.\text{ArcTanh}\left[\frac{4 a f-b \left(e+\sqrt{e^2-4 d f}\right)+2 \left(b f-c \left(e+\sqrt{e^2-4 d f}\right)\right) x}{2 \sqrt{2} \sqrt{c e^2-2 c d f-b e f+2 a f^2+(c e-b f) \sqrt{e^2-4 d f}} \sqrt{a+b x+c x^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{2} f^3 \sqrt{e^2-4 d f} \sqrt{c e^2-2 c d f-b e f+2 a f^2+(c e-b f) \sqrt{e^2-4 d f}}\right)$$

Result (type 3, 1934 leaves) :

$$\frac{\left(\frac{-4 c e+5 b f}{4 f^2}+\frac{c x}{2 f}\right) (a+x (b+c x))^{3/2}}{a+b x+c x^2}-$$

$$\left(\left(-c^2 e^4+4 c^2 d e^2 f+2 b c e^3 f-2 c^2 d^2 f^2-6 b c d e f^2-b^2 e^2 f^2-2 a c e^2 f^2+2 b^2 d f^3+4 a c d f^3+2 a b e f^3-2 a^2 f^4+\right.\right.$$

$$c^2 e^3 \sqrt{e^2-4 d f}-2 c^2 d e f \sqrt{e^2-4 d f}-2 b c e^2 f \sqrt{e^2-4 d f}+2 b c d f^2 \sqrt{e^2-4 d f}+b^2 e f^2 \sqrt{e^2-4 d f}+$$

$$2\,a\,c\,e\,f^2\,\sqrt{e^2-4\,d\,f}\,-2\,a\,b\,f^3\,\sqrt{e^2-4\,d\,f}\,\Big)\,\left(a+x\,(b+c\,x)\right)^{3/2}\,\mathrm{Log}\left[-e+\sqrt{e^2-4\,d\,f}\,-2\,f\,x\right]\Big)\Big/$$

$$\left(\sqrt{2}\,f^3\,\sqrt{e^2-4\,d\,f}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2-c\,e\,\sqrt{e^2-4\,d\,f}\,+b\,f\,\sqrt{e^2-4\,d\,f}}\,\left(a+b\,x+c\,x^2\right)^{3/2}\right)-$$

$$\Big(\Big(c^2\,e^4-4\,c^2\,d\,e^2\,f-2\,b\,c\,e^3\,f+2\,c^2\,d^2\,f^2+6\,b\,c\,d\,e\,f^2+b^2\,e^2\,f^2+2\,a\,c\,e^2\,f^2-2\,b^2\,d\,f^3-4\,a\,c\,d\,f^3-$$

$$2\,a\,b\,e\,f^3+2\,a^2\,f^4+c^2\,e^3\,\sqrt{e^2-4\,d\,f}\,-2\,c^2\,d\,e\,f\,\sqrt{e^2-4\,d\,f}\,-2\,b\,c\,e^2\,f\,\sqrt{e^2-4\,d\,f}\,+2\,b\,c\,d\,f^2\,\sqrt{e^2-4\,d\,f}\,+$$

$$b^2\,e\,f^2\,\sqrt{e^2-4\,d\,f}\,+2\,a\,c\,e\,f^2\,\sqrt{e^2-4\,d\,f}\,-2\,a\,b\,f^3\,\sqrt{e^2-4\,d\,f}\,\Big)\,\left(a+x\,(b+c\,x)\right)^{3/2}\,\mathrm{Log}\left[e+\sqrt{e^2-4\,d\,f}\,+2\,f\,x\right]\Big)\Big/$$

$$\left(\sqrt{2}\,f^3\,\sqrt{e^2-4\,d\,f}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2+c\,e\,\sqrt{e^2-4\,d\,f}\,-b\,f\,\sqrt{e^2-4\,d\,f}}\,\left(a+b\,x+c\,x^2\right)^{3/2}\right)+$$

$$\frac{\left(8\,c^2\,e^2-8\,c^2\,d\,f-12\,b\,c\,e\,f+3\,b^2\,f^2+12\,a\,c\,f^2\right)\,\left(a+x\,(b+c\,x)\right)^{3/2}\,\mathrm{Log}\left[b+2\,c\,x+2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}\right]}{8\,\sqrt{c}\,f^3\,\left(a+b\,x+c\,x^2\right)^{3/2}}+$$

$$\Big(\Big(c^2\,e^4-4\,c^2\,d\,e^2\,f-2\,b\,c\,e^3\,f+2\,c^2\,d^2\,f^2+6\,b\,c\,d\,e\,f^2+b^2\,e^2\,f^2+2\,a\,c\,e^2\,f^2-2\,b^2\,d\,f^3-4\,a\,c\,d\,f^3-2\,a\,b\,e\,f^3+2\,a^2\,f^4+c^2\,e^3\,\sqrt{e^2-4\,d\,f}\,-$$

$$2\,c^2\,d\,e\,f\,\sqrt{e^2-4\,d\,f}\,-2\,b\,c\,e^2\,f\,\sqrt{e^2-4\,d\,f}\,+2\,b\,c\,d\,f^2\,\sqrt{e^2-4\,d\,f}\,+b^2\,e\,f^2\,\sqrt{e^2-4\,d\,f}\,+2\,a\,c\,e\,f^2\,\sqrt{e^2-4\,d\,f}\,-2\,a\,b\,f^3\,\sqrt{e^2-4\,d\,f}\,\Big)$$

$$\left(a+x\,(b+c\,x)\right)^{3/2}\,\mathrm{Log}\Big[-b\,e^2+4\,b\,d\,f-b\,e\,\sqrt{e^2-4\,d\,f}\,+4\,a\,f\,\sqrt{e^2-4\,d\,f}\,-2\,c\,e^2\,x+8\,c\,d\,f\,x-2\,c\,e\,\sqrt{e^2-4\,d\,f}\,x+$$

$$2\,b\,f\,\sqrt{e^2-4\,d\,f}\,x+2\,\sqrt{2}\,\sqrt{e^2-4\,d\,f}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2+c\,e\,\sqrt{e^2-4\,d\,f}\,-b\,f\,\sqrt{e^2-4\,d\,f}}\,\sqrt{a+b\,x+c\,x^2}\,\Big]\Big)\Big/$$

$$\left(\sqrt{2}\,f^3\,\sqrt{e^2-4\,d\,f}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2+c\,e\,\sqrt{e^2-4\,d\,f}\,-b\,f\,\sqrt{e^2-4\,d\,f}}\,\left(a+b\,x+c\,x^2\right)^{3/2}\right)+$$

$$\Big(\Big(-c^2\,e^4+4\,c^2\,d\,e^2\,f+2\,b\,c\,e^3\,f-2\,c^2\,d^2\,f^2-6\,b\,c\,d\,e\,f^2-b^2\,e^2\,f^2-2\,a\,c\,e^2\,f^2+2\,b^2\,d\,f^3+4\,a\,c\,d\,f^3+2\,a\,b\,e\,f^3-2\,a^2\,f^4+c^2\,e^3\,\sqrt{e^2-4\,d\,f}\,-$$

$$2\,c^2\,d\,e\,f\,\sqrt{e^2-4\,d\,f}\,-2\,b\,c\,e^2\,f\,\sqrt{e^2-4\,d\,f}\,+2\,b\,c\,d\,f^2\,\sqrt{e^2-4\,d\,f}\,+b^2\,e\,f^2\,\sqrt{e^2-4\,d\,f}\,+2\,a\,c\,e\,f^2\,\sqrt{e^2-4\,d\,f}\,-2\,a\,b\,f^3\,\sqrt{e^2-4\,d\,f}\,\Big)$$

$$\left(a+x\,(b+c\,x)\right)^{3/2}\,\mathrm{Log}\Big[b\,e^2-4\,b\,d\,f-b\,e\,\sqrt{e^2-4\,d\,f}\,+4\,a\,f\,\sqrt{e^2-4\,d\,f}\,+2\,c\,e^2\,x-8\,c\,d\,f\,x-2\,c\,e\,\sqrt{e^2-4\,d\,f}\,x+$$

$$2\,b\,f\,\sqrt{e^2-4\,d\,f}\,x+2\,\sqrt{2}\,\sqrt{e^2-4\,d\,f}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2-c\,e\,\sqrt{e^2-4\,d\,f}\,+b\,f\,\sqrt{e^2-4\,d\,f}}\,\sqrt{a+b\,x+c\,x^2}\,\Big]\Big)\Big/$$

$$\left(\sqrt{2}\, f^3 \sqrt{e^2-4 d f}\, \sqrt{c e^2-2 c d f-b e f+2 a f^2-c e \sqrt{e^2-4 d f}+b f \sqrt{e^2-4 d f}}\, \left(a+b x+c x^2\right)^{3/2}\right)$$

- **Problem 107: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+b x+c x^2\right)^{3/2}}{\left(d+e x+f x^2\right)^2}\, dx$$

Optimal (type 3, 704 leaves, 10 steps) :

$$-\frac{\left(c e-2 b f-2 c f x\right)\sqrt{a+b x+c x^2}}{f\left(e^2-4 d f\right)}-\frac{\left(e+2 f x\right)\left(a+b x+c x^2\right)^{3/2}}{\left(e^2-4 d f\right)\left(d+e x+f x^2\right)}+\frac{c^{3/2}\,\text{ArcTanh}\left[\frac{b+2 c x}{2\sqrt{c}\,\sqrt{a+b x+c x^2}}\right]}{f^2}-$$

$$\left(\left((c e-b f)\left(f\,(b e-2 a f)+2 c\left(e^2-5 d f\right)\right)\left(e-\sqrt{e^2-4 d f}\right)-2 f\left(2 c^2 d\left(e^2-4 d f\right)+f\left(2 b^2 d f+4 a f\,(c d+a f)-b e\,(c d+3 a f)\right)\right)\right)\right.$$

$$\left.\text{ArcTanh}\left[\frac{4 a f-b\left(e-\sqrt{e^2-4 d f}\right)+2\left(b f-c\left(e-\sqrt{e^2-4 d f}\right)\right)x}{2\sqrt{2}\sqrt{c e^2-2 c d f-b e f+2 a f^2-(c e-b f)\sqrt{e^2-4 d f}}\,\sqrt{a+b x+c x^2}}\right]\right)\Big/$$

$$\left(2\sqrt{2}\,f^2\left(e^2-4 d f\right)^{3/2}\sqrt{c e^2-2 c d f-b e f+2 a f^2-(c e-b f)\sqrt{e^2-4 d f}}\right)+$$

$$\left(\left((c e-b f)\left(f\,(b e-2 a f)+2 c\left(e^2-5 d f\right)\right)\left(e+\sqrt{e^2-4 d f}\right)-2 f\left(2 c^2 d\left(e^2-4 d f\right)+f\left(2 b^2 d f+4 a f\,(c d+a f)-b e\,(c d+3 a f)\right)\right)\right)\right.$$

$$\left.\text{ArcTanh}\left[\frac{4 a f-b\left(e+\sqrt{e^2-4 d f}\right)+2\left(b f-c\left(e+\sqrt{e^2-4 d f}\right)\right)x}{2\sqrt{2}\sqrt{c e^2-2 c d f-b e f+2 a f^2+(c e-b f)\sqrt{e^2-4 d f}}\,\sqrt{a+b x+c x^2}}\right]\right)\Big/$$

$$\left(2\sqrt{2}\,f^2\left(e^2-4 d f\right)^{3/2}\sqrt{c e^2-2 c d f-b e f+2 a f^2+(c e-b f)\sqrt{e^2-4 d f}}\right)$$

Result (type 3, 1844 leaves) :

$$\frac{\left(c d e-2 b d f+a e f+c e^2 x-2 c d f x-b e f x+2 a f^2 x\right)\left(a+x\,(b+c x)\right)^{3/2}}{f\left(-e^2+4 d f\right)\left(a+b x+c x^2\right)\left(d+e x+f x^2\right)}-$$

$$\left(\left(-2 c^2 e^4+14 c^2 d e^2 f+b c e^3 f-16 c^2 d^2 f^2-12 b c d e f^2+b^2 e^2 f^2+2 a c e^2 f^2+4 b^2 d f^3+8 a c d f^3-\right.\right.$$

$$8 a b e f^3+8 a^2 f^4+2 c^2 e^3\sqrt{e^2-4 d f}-10 c^2 d e f\sqrt{e^2-4 d f}-b c e^2 f\sqrt{e^2-4 d f}+10 b c d f^2\sqrt{e^2-4 d f}-$$

$$b^2 e f^2 \sqrt{e^2-4 d f} - 2 a c e f^2 \sqrt{e^2-4 d f} + 2 a b f^3 \sqrt{e^2-4 d f}\Big) (a+x (b+c x))^{3/2} \text{Log}\left[-e+\sqrt{e^2-4 d f} - 2 f x\right]\Big) \Big/$$

$$\left(2 \sqrt{2} f^2 \left(e^2-4 d f\right)^{3/2} \sqrt{c e^2-2 c d f-b e f+2 a f^2-c e \sqrt{e^2-4 d f} + b f \sqrt{e^2-4 d f}} \left(a+b x+c x^2\right)^{3/2}\right) -$$

$$\Big(\Big(2 c^2 e^4-14 c^2 d e^2 f-b c e^3 f+16 c^2 d^2 f^2+12 b c d e f^2-b^2 e^2 f^2-2 a c e^2 f^2-4 b^2 d f^3-8 a c d f^3+$$
$$8 a b e f^3-8 a^2 f^4+2 c^2 e^3 \sqrt{e^2-4 d f} - 10 c^2 d e f \sqrt{e^2-4 d f} - b c e^2 f \sqrt{e^2-4 d f} + 10 b c d f^2 \sqrt{e^2-4 d f} -$$
$$b^2 e f^2 \sqrt{e^2-4 d f} - 2 a c e f^2 \sqrt{e^2-4 d f} + 2 a b f^3 \sqrt{e^2-4 d f}\Big) (a+x (b+c x))^{3/2} \text{Log}\left[e+\sqrt{e^2-4 d f} + 2 f x\right]\Big) \Big/$$

$$\left(2 \sqrt{2} f^2 \left(e^2-4 d f\right)^{3/2} \sqrt{c e^2-2 c d f-b e f+2 a f^2+c e \sqrt{e^2-4 d f} - b f \sqrt{e^2-4 d f}} \left(a+b x+c x^2\right)^{3/2}\right) +$$

$$\frac{c^{3/2} (a+x (b+c x))^{3/2} \text{Log}\left[b+2 c x+2 \sqrt{c} \sqrt{a+b x+c x^2}\right]}{f^2 \left(a+b x+c x^2\right)^{3/2}} +$$

$$\Big(\Big(2 c^2 e^4-14 c^2 d e^2 f-b c e^3 f+16 c^2 d^2 f^2+12 b c d e f^2-b^2 e^2 f^2-2 a c e^2 f^2-4 b^2 d f^3-8 a c d f^3+8 a b e f^3-8 a^2 f^4+2 c^2 e^3 \sqrt{e^2-4 d f} -$$
$$10 c^2 d e f \sqrt{e^2-4 d f} - b c e^2 f \sqrt{e^2-4 d f} + 10 b c d f^2 \sqrt{e^2-4 d f} - b^2 e f^2 \sqrt{e^2-4 d f} - 2 a c e f^2 \sqrt{e^2-4 d f} + 2 a b f^3 \sqrt{e^2-4 d f}\Big)$$
$$(a+x (b+c x))^{3/2} \text{Log}\Big[b e-4 a f+b \sqrt{e^2-4 d f} + 2 c e x-2 b f x+2 c \sqrt{e^2-4 d f}\, x -$$
$$2 \sqrt{2} \sqrt{c e^2-2 c d f-b e f+2 a f^2+c e \sqrt{e^2-4 d f} - b f \sqrt{e^2-4 d f}} \sqrt{a+b x+c x^2}\Big]\Big) \Big/$$

$$\left(2 \sqrt{2} f^2 \left(e^2-4 d f\right)^{3/2} \sqrt{c e^2-2 c d f-b e f+2 a f^2+c e \sqrt{e^2-4 d f} - b f \sqrt{e^2-4 d f}} \left(a+b x+c x^2\right)^{3/2}\right) +$$

$$\Big(\Big(-2 c^2 e^4+14 c^2 d e^2 f+b c e^3 f-16 c^2 d^2 f^2-12 b c d e f^2+b^2 e^2 f^2+2 a c e^2 f^2+4 b^2 d f^3+8 a c d f^3-8 a b e f^3+8 a^2 f^4+2 c^2 e^3 \sqrt{e^2-4 d f} -$$
$$10 c^2 d e f \sqrt{e^2-4 d f} - b c e^2 f \sqrt{e^2-4 d f} + 10 b c d f^2 \sqrt{e^2-4 d f} - b^2 e f^2 \sqrt{e^2-4 d f} - 2 a c e f^2 \sqrt{e^2-4 d f} + 2 a b f^3 \sqrt{e^2-4 d f}\Big)$$
$$(a+x (b+c x))^{3/2} \text{Log}\Big[-b e+4 a f+b \sqrt{e^2-4 d f} - 2 c e x+2 b f x+2 c \sqrt{e^2-4 d f}\, x +$$
$$2 \sqrt{2} \sqrt{c e^2-2 c d f-b e f+2 a f^2-c e \sqrt{e^2-4 d f} + b f \sqrt{e^2-4 d f}} \sqrt{a+b x+c x^2}\Big]\Big) \Big/$$

$$\left(2\sqrt{2}\, f^2\left(e^2-4df\right)^{3/2}\sqrt{c e^2-2cdf-bef+2af^2-ce\sqrt{e^2-4df}+bf\sqrt{e^2-4df}}\,\left(a+bx+cx^2\right)^{3/2}\right)$$

- **Problem 108: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\left(d+ex+fx^2\right)^3}\,dx$$

Optimal (type 3, 671 leaves, 7 steps):

$$-\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2}+\frac{3\left(4cde+4aef-b\left(e^2+4df\right)+2\left(ce^2-2bef+4af^2\right)x\right)\sqrt{a+bx+cx^2}}{4\left(e^2-4df\right)^2\left(d+ex+fx^2\right)}-$$

$$\left(3\left(2\,(2cd-be+2af)\,(ce-bf)\left(e-\sqrt{e^2-4df}\right)-f\left(4be\,(cd+3af)-b^2\left(e^2+4df\right)-4a\left(ce^2+4af^2\right)\right)\right)\right.$$

$$\left.\text{ArcTanh}\left[\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\,\sqrt{a+bx+cx^2}}\right]\right)\Big/$$

$$\left(4\sqrt{2}\left(e^2-4df\right)^{5/2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\right)+$$

$$\left(3\left(2\,(2cd-be+2af)\,(ce-bf)\left(e+\sqrt{e^2-4df}\right)-f\left(4be\,(cd+3af)-b^2\left(e^2+4df\right)-4a\left(ce^2+4af^2\right)\right)\right)\right.$$

$$\left.\text{ArcTanh}\left[\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\,\sqrt{a+bx+cx^2}}\right]\right)\Big/$$

$$\left(4\sqrt{2}\left(e^2-4df\right)^{5/2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\right)$$

Result (type 3, 1621 leaves) :

$$\frac{1}{a+b x+c x^2}(a+x(b+c x))^{3/2}\left(\frac{c d e-2 b d f+a e f+c e^2 x-2 c d f x-b e f x+2 a f^2 x}{2 f\left(-e^2+4 d f\right)\left(d+e x+f x^2\right)^2}+\frac{2 c e^3+4 c d e f-7 b e^2 f+4 b d f^2+12 a e f^2+2 c e^2 f x+16 c d f^2 x-12 b e f^2 x+24 a f^3 x}{4 f\left(-e^2+4 d f\right)^2\left(d+e x+f x^2\right)}\right)+$$

$$\left(3\left(4 c^2 d e^2-2 b c e^3-8 b c d e f+3 b^2 e^2 f+8 a c e^2 f+4 b^2 d f^2-16 a b e f^2+16 a^2 f^3-4 c^2 d e\sqrt{e^2-4 d f}+2 b c e^2\sqrt{e^2-4 d f}+4 b c d f\sqrt{e^2-4 d f}-2 b^2 e f\sqrt{e^2-4 d f}-4 a c e f\sqrt{e^2-4 d f}+4 a b f^2\sqrt{e^2-4 d f}\right)(a+x(b+c x))^{3/2}\operatorname{Log}\left[-e+\sqrt{e^2-4 d f}-2 f x\right]\right)\Big/$$

$$\left(4\sqrt{2}\left(e^2-4 d f\right)^{5/2}\sqrt{c e^2-2 c d f-b e f+2 a f^2-c e\sqrt{e^2-4 d f}+b f\sqrt{e^2-4 d f}}\left(a+b x+c x^2\right)^{3/2}\right)+$$

$$\left(3\left(-4 c^2 d e^2+2 b c e^3+8 b c d e f-3 b^2 e^2 f-8 a c e^2 f-4 b^2 d f^2+16 a b e f^2-16 a^2 f^3-4 c^2 d e\sqrt{e^2-4 d f}+2 b c e^2\sqrt{e^2-4 d f}+4 b c d f\sqrt{e^2-4 d f}-2 b^2 e f\sqrt{e^2-4 d f}-4 a c e f\sqrt{e^2-4 d f}+4 a b f^2\sqrt{e^2-4 d f}\right)(a+x(b+c x))^{3/2}\operatorname{Log}\left[e+\sqrt{e^2-4 d f}+2 f x\right]\right)\Big/$$

$$\left(4\sqrt{2}\left(e^2-4 d f\right)^{5/2}\sqrt{c e^2-2 c d f-b e f+2 a f^2+c e\sqrt{e^2-4 d f}-b f\sqrt{e^2-4 d f}}\left(a+b x+c x^2\right)^{3/2}\right)-$$

$$\left(3\left(-4 c^2 d e^2+2 b c e^3+8 b c d e f-3 b^2 e^2 f-8 a c e^2 f-4 b^2 d f^2+16 a b e f^2-16 a^2 f^3-4 c^2 d e\sqrt{e^2-4 d f}+2 b c e^2\sqrt{e^2-4 d f}+4 b c d f\sqrt{e^2-4 d f}-2 b^2 e f\sqrt{e^2-4 d f}-4 a c e f\sqrt{e^2-4 d f}+4 a b f^2\sqrt{e^2-4 d f}\right)(a+x(b+c x))^{3/2}\operatorname{Log}\left[b e-4 a f+b\sqrt{e^2-4 d f}+2 c e x-2 b f x+2 c\sqrt{e^2-4 d f}\,x-2\sqrt{2}\sqrt{c e^2-2 c d f-b e f+2 a f^2+c e\sqrt{e^2-4 d f}-b f\sqrt{e^2-4 d f}}\sqrt{a+b x+c x^2}\right]\right)\Big/$$

$$\left(4\sqrt{2}\left(e^2-4 d f\right)^{5/2}\sqrt{c e^2-2 c d f-b e f+2 a f^2+c e\sqrt{e^2-4 d f}-b f\sqrt{e^2-4 d f}}\left(a+b x+c x^2\right)^{3/2}\right)-$$

$$\left(3\left(4 c^2 d e^2-2 b c e^3-8 b c d e f+3 b^2 e^2 f+8 a c e^2 f+4 b^2 d f^2-16 a b e f^2+16 a^2 f^3-4 c^2 d e\sqrt{e^2-4 d f}+2 b c e^2\sqrt{e^2-4 d f}+4 b c d f\sqrt{e^2-4 d f}-2 b^2 e f\sqrt{e^2-4 d f}-4 a c e f\sqrt{e^2-4 d f}+4 a b f^2\sqrt{e^2-4 d f}\right)(a+x(b+c x))^{3/2}\operatorname{Log}\left[-b e+4 a f+b\sqrt{e^2-4 d f}-\right.\right.$$

$$2\,c\,e\,x+2\,b\,f\,x+2\,c\,\sqrt{e^2-4\,d\,f}\;x+2\,\sqrt{2}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2-c\,e\,\sqrt{e^2-4\,d\,f}+b\,f\,\sqrt{e^2-4\,d\,f}}\;\sqrt{a+b\,x+c\,x^2}\,\Big]\Bigg)\Big/$$

$$\left(4\,\sqrt{2}\,\left(e^2-4\,d\,f\right)^{5/2}\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2-c\,e\,\sqrt{e^2-4\,d\,f}+b\,f\,\sqrt{e^2-4\,d\,f}}\;\left(a+b\,x+c\,x^2\right)^{3/2}\right)$$

- **Problem 113: Result more than twice size of optimal antiderivative.**

$$\int\frac{1}{\sqrt{a+b\,x+c\,x^2}\;\left(d+e\,x+f\,x^2\right)^2}\,dx$$

Optimal (type 3, 789 leaves, 6 steps) :

$$\frac{\left(f\left(b e^2-2 b d f-a e f\right)-c\left(e^3-3 d e f\right)+f\left(f\left(b e-2 a f\right)-c\left(e^2-2 d f\right)\right)x\right)\sqrt{a+b x+c x^2}}{\left(e^2-4 d f\right)\left((c d-a f)^2-(b d-a e)(c e-b f)\right)\left(d+e x+f x^2\right)}+$$

$$\left(\left(f(2 c d-b e+2 a f)(c e-b f)\left(e-\sqrt{e^2-4 d f}\right)-2 f\right.\right.$$

$$\left.\left(2 c^2 d\left(e^2-4 d f\right)+f\left(3 a b e f-4 a^2 f^2+b^2\left(e^2-6 d f\right)\right)-c\left(4 a f\left(e^2-3 d f\right)+b\left(e^3-5 d e f\right)\right)\right)\right)$$

$$\left.\text{ArcTanh}\left[\frac{4 a f-b\left(e-\sqrt{e^2-4 d f}\right)+2\left(b f-c\left(e-\sqrt{e^2-4 d f}\right)\right)x}{2\sqrt{2}\sqrt{c e^2-2 c d f-b e f+2 a f^2-(c e-b f)\sqrt{e^2-4 d f}}\sqrt{a+b x+c x^2}}\right]\right)\Big/$$

$$\left(2\sqrt{2}\left(e^2-4 d f\right)^{3/2}\left((c d-a f)^2-(b d-a e)(c e-b f)\right)\sqrt{c e^2-2 c d f-b e f+2 a f^2-(c e-b f)\sqrt{e^2-4 d f}}\right)-$$

$$\left(\left(f(2 c d-b e+2 a f)(c e-b f)\left(e+\sqrt{e^2-4 d f}\right)-\right.\right.$$

$$\left.2 f\left(2 c^2 d\left(e^2-4 d f\right)+f\left(3 a b e f-4 a^2 f^2+b^2\left(e^2-6 d f\right)\right)-c\left(4 a f\left(e^2-3 d f\right)+b\left(e^3-5 d e f\right)\right)\right)\right)$$

$$\left.\text{ArcTanh}\left[\frac{4 a f-b\left(e+\sqrt{e^2-4 d f}\right)+2\left(b f-c\left(e+\sqrt{e^2-4 d f}\right)\right)x}{2\sqrt{2}\sqrt{c e^2-2 c d f-b e f+2 a f^2+(c e-b f)\sqrt{e^2-4 d f}}\sqrt{a+b x+c x^2}}\right]\right)\Big/$$

$$\left(2\sqrt{2}\left(e^2-4 d f\right)^{3/2}\left((c d-a f)^2-(b d-a e)(c e-b f)\right)\sqrt{c e^2-2 c d f-b e f+2 a f^2+(c e-b f)\sqrt{e^2-4 d f}}\right)$$

Result (type 3, 1836 leaves) :

$$\frac{\left(-c e^3+3 c d e f+b e^2 f-2 b d f^2-a e f^2-c e^2 f x+2 c d f^2 x+b e f^2 x-2 a f^3 x\right)\left(a+b x+c x^2\right)}{\left(e^2-4 d f\right)\left(c^2 d^2-b c d e+a c e^2+b^2 d f-2 a c d f-a b e f+a^2 f^2\right)\left(d+e x+f x^2\right)\sqrt{a+x(b+c x)}}-$$

$$\left(f\left(-2c^2de^2+bce^3+16c^2d^2f-12bcdef-b^2e^2f+10ace^2f+12b^2df^2-24acdf^2-8abef^2+8a^2f^3-2c^2de\sqrt{e^2-4df}+bce^2\sqrt{e^2-4df}+2bcdf\sqrt{e^2-4df}-b^2ef\sqrt{e^2-4df}-2acef\sqrt{e^2-4df}+2abf^2\sqrt{e^2-4df}\right)\sqrt{a+bx+cx^2}\,\text{Log}\left[-e+\sqrt{e^2-4df}-2fx\right]\right)\Big/\left(2\sqrt{2}\left(e^2-4df\right)^{3/2}\left(c^2d^2-bcde+ace^2+b^2df-2acdf-abef+a^2f^2\right)\sqrt{ce^2-2cdf-bef+2af^2-ce\sqrt{e^2-4df}+bf\sqrt{e^2-4df}}\sqrt{a+x(b+cx)}\right)-$$

$$\left(f\left(2c^2de^2-bce^3-16c^2d^2f+12bcdef+b^2e^2f-10ace^2f-12b^2df^2+24acdf^2+8abef^2-8a^2f^3-2c^2de\sqrt{e^2-4df}+bce^2\sqrt{e^2-4df}+2bcdf\sqrt{e^2-4df}-b^2ef\sqrt{e^2-4df}-2acef\sqrt{e^2-4df}+2abf^2\sqrt{e^2-4df}\right)\sqrt{a+bx+cx^2}\,\text{Log}\left[e+\sqrt{e^2-4df}+2fx\right]\right)\Big/\left(2\sqrt{2}\left(e^2-4df\right)^{3/2}\left(c^2d^2-bcde+ace^2+b^2df-2acdf-abef+a^2f^2\right)\sqrt{ce^2-2cdf-bef+2af^2+ce\sqrt{e^2-4df}-bf\sqrt{e^2-4df}}\sqrt{a+x(b+cx)}\right)+$$

$$\left(f\left(2c^2de^2-bce^3-16c^2d^2f+12bcdef+b^2e^2f-10ace^2f-12b^2df^2+24acdf^2+8abef^2-8a^2f^3-2c^2de\sqrt{e^2-4df}+bce^2\sqrt{e^2-4df}+2bcdf\sqrt{e^2-4df}-b^2ef\sqrt{e^2-4df}-2acef\sqrt{e^2-4df}+2abf^2\sqrt{e^2-4df}\right)\sqrt{a+bx+cx^2}\,\text{Log}\left[be-4af+b\sqrt{e^2-4df}+2cex-2bfx+2c\sqrt{e^2-4df}\,x-2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+ce\sqrt{e^2-4df}-bf\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}\right]\right)\Big/\left(2\sqrt{2}\left(e^2-4df\right)^{3/2}\left(c^2d^2-bcde+ace^2+b^2df-2acdf-abef+a^2f^2\right)\sqrt{ce^2-2cdf-bef+2af^2+ce\sqrt{e^2-4df}-bf\sqrt{e^2-4df}}\sqrt{a+x(b+cx)}\right)+$$

$$\left(f\left(-2c^2de^2+bce^3+16c^2d^2f-12bcdef-b^2e^2f+10ace^2f+12b^2df^2-24acdf^2-8abef^2+8a^2f^3-2c^2de\sqrt{e^2-4df}+bce^2\sqrt{e^2-4df}+2bcdf\sqrt{e^2-4df}-b^2ef\sqrt{e^2-4df}-2acef\sqrt{e^2-4df}+2abf^2\sqrt{e^2-4df}\right)\sqrt{a+bx+cx^2}\,\text{Log}\left[-be+4af+\right.\right.$$

$$b\sqrt{e^2-4df}-2cex+2bfx+2c\sqrt{e^2-4df}\,x+2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-ce\sqrt{e^2-4df}+bf\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}\Big]\Big)\Big/$$

$$\Big(2\sqrt{2}\left(e^2-4df\right)^{3/2}\left(c^2d^2-bcde+ace^2+b^2df-2acdf-abef+a^2f^2\right)$$

$$\sqrt{ce^2-2cdf-bef+2af^2-ce\sqrt{e^2-4df}+bf\sqrt{e^2-4df}}\sqrt{a+x(b+cx)}\Big)$$

- **Problem 121: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{-7+2x+5x^2}\left(8+12x+5x^2\right)}\,dx$$

Optimal (type 3, 51 leaves, 5 steps):

$$\frac{1}{10}\text{ArcTan}\left[\frac{5(2+x)}{2\sqrt{-7+2x+5x^2}}\right]+\frac{1}{5}\text{ArcTanh}\left[\frac{5(1+x)}{\sqrt{-7+2x+5x^2}}\right]$$

Result (type 3, 193 leaves):

$$\left(\frac{1}{20}+\frac{i}{10}\right)\text{ArcTan}\left[\frac{5(2+x)}{2\sqrt{-7+2x+5x^2}}\right]-\left(\frac{1}{20}-\frac{i}{10}\right)\text{ArcTan}\left[\frac{2\sqrt{-7+2x+5x^2}}{5(2+x)}\right]-\frac{1}{20}i\,\text{Log}[((2+2i)+(1+2i)x)((2+2i)+(2+i)x)]-$$

$$\left(\frac{1}{20}-\frac{i}{40}\right)\text{Log}\left[9+26x+15x^2-5\sqrt{-7+2x+5x^2}-5x\sqrt{-7+2x+5x^2}\right]+\left(\frac{1}{20}+\frac{i}{40}\right)\text{Log}\left[9+26x+15x^2+5\sqrt{-7+2x+5x^2}+5x\sqrt{-7+2x+5x^2}\right]$$

Test results for the 143 problems in "1.2.1.6 (g+h x)^m (a+b x+c x^2)^p (d+e x+f x^2)^q.m"

- **Problem 9: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)^{5/2}\left(d-fx^2\right)}\,dx$$

Optimal (type 3, 797 leaves, 7 steps):

$$-\frac{2\left(a B\left(2 c^2 d-b^2 f+2 a c f\right)+A\left(b^3 f-b c\,(c d+3 a f)\right)+c\left(A b^2 f+b B\,(c d-a f)-2 A c\,(c d+a f)\right) x\right)}{3\left(b^2-4 a c\right)\left(b^2 d f-(c d+a f)^2\right)\left(a+b x+c x^2\right)^{3/2}}-$$

$$\frac{1}{3\left(b^2-4 a c\right)^2\left(c^2 d^2+2 a c d f-f\left(b^2 d-a^2 f\right)\right)^2\sqrt{a+b x+c x^2}}$$

$$2\left(3 b^6 B d f^2+24 a^2 B c^2 f\,(c d+a f)^2-A b^5 f^2\,(7 c d+6 a f)-b^4 B f\left(7 c^2 d^2+14 a c d f-3 a^2 f^2\right)+A b^3 c f\left(15 c^2 d^2+46 a c d f+43 a^2 f^2\right)+\right.$$
$$2 b^2 B c\left(2 c^3 d^3+5 a c^2 d^2 f+4 a^2 c d f^2-11 a^3 f^3\right)-4 A b c^2\left(2 c^3 d^3+9 a c^2 d^2 f+24 a^2 c d f^2+17 a^3 f^3\right)+$$
$$c\left(3 b^5 B d f^2-2 A b^4 f^2\,(4 c d+3 a f)-8 A c^2\,(c d+a f)^2\,(2 c d+5 a f)-b^3 B f\left(17 c^2 d^2+10 a c d f-3 a^2 f^2\right)+\right.$$
$$\left.\left.2 A b^2 c f\left(15 c^2 d^2+22 a c d f+19 a^2 f^2\right)+4 b B c\left(2 c^3 d^3+11 a c^2 d^2 f+4 a^2 c d f^2-5 a^3 f^3\right)\right) x\right)-$$

$$\frac{\left(B\sqrt{d}-A\sqrt{f}\right) f^{3/2}\,\text{ArcTanh}\left[\frac{b\sqrt{d}-2 a\sqrt{f}+\left(2 c\sqrt{d}-b\sqrt{f}\right) x}{2\sqrt{c d-b\sqrt{d}\sqrt{f}+a f}\sqrt{a+b x+c x^2}}\right]}{2\sqrt{d}\left(c d-b\sqrt{d}\sqrt{f}+a f\right)^{5/2}}+\frac{\left(B\sqrt{d}+A\sqrt{f}\right) f^{3/2}\,\text{ArcTanh}\left[\frac{b\sqrt{d}+2 a\sqrt{f}+\left(2 c\sqrt{d}+b\sqrt{f}\right) x}{2\sqrt{c d+b\sqrt{d}\sqrt{f}+a f}\sqrt{a+b x+c x^2}}\right]}{2\sqrt{d}\left(c d+b\sqrt{d}\sqrt{f}+a f\right)^{5/2}}$$

Result (type 3, 1847 leaves) :

$$\frac{1}{(a+x\,(b+c x))^{5/2}}$$

$$\left(a+b x+c x^2\right)^3\left(-\left(2\left(-A b c^2 d+2 a B c^2 d+A b^3 f-a b^2 B f-3 a A b c f+2 a^2 B c f+b B c^2 d x-2 A c^3 d x+A b^2 c f x-a b B c f x-2 a A c^2 f x\right)\right)\Big/\right.$$
$$\left(3\left(b^2-4 a c\right)\left(-c^2 d^2+b^2 d f-2 a c d f-a^2 f^2\right)\left(a+b x+c x^2\right)^2\right)-$$
$$\frac{1}{3\left(b^2-4 a c\right)^2\left(-c^2 d^2+b^2 d f-2 a c d f-a^2 f^2\right)^2\left(a+b x+c x^2\right)}\,2\left(4 b^2 B c^4 d^3-8 A b c^5 d^3-7 b^4 B c^2 d^2 f+15 A b^3 c^3 d^2 f+10 a b^2 B c^3 d^2 f-\right.$$
$$36 a A b c^4 d^2 f+24 a^2 B c^4 d^2 f+3 b^6 B d f^2-7 A b^5 c d f^2-14 a b^4 B c d f^2+46 a A b^3 c^2 d f^2+8 a^2 b^2 B c^2 d f^2-96 a^2 A b c^3 d f^2+$$
$$48 a^3 B c^3 d f^2-6 a A b^5 f^3+3 a^2 b^4 B f^3+43 a^2 A b^3 c f^3-22 a^3 b^2 B c f^3-68 a^3 A b c^2 f^3+24 a^4 B c^2 f^3+8 b B c^5 d^3 x-16 A c^6 d^3 x-$$
$$17 b^3 B c^3 d^2 f x+30 A b^2 c^4 d^2 f x+44 a b B c^4 d^2 f x-72 a A c^5 d^2 f x+3 b^5 B c d f^2 x-8 A b^4 c^2 d f^2 x-10 a b^3 B c^2 d f^2 x+44 a A b^2 c^3 d f^2 x+$$
$$\left.\left.16 a^2 b B c^3 d f^2 x-96 a^2 A c^4 d f^2 x-6 a A b^4 c f^3 x+3 a^2 b^3 B c f^3 x+38 a^2 A b^2 c^2 f^3 x-20 a^3 b B c^2 f^3 x-40 a^3 A c^3 f^3 x\right)\right)-$$

$$\left(f\left(B c^2 d^{5/2}\sqrt{f}-2 b B c d^2 f+A c^2 d^2 f+b^2 B d^{3/2} f^{3/2}-2 A b c d^{3/2} f^{3/2}+2 a B c d^{3/2} f^{3/2}+A b^2 d f^2-2 a b B d f^2+\right.\right.$$
$$\left.\left.2 a A c d f^2-2 a A b\sqrt{d}\,f^{5/2}+a^2 B\sqrt{d}\,f^{5/2}+a^2 A f^3\right)\left(a+b x+c x^2\right)^{5/2}\text{Log}\left[\sqrt{d}\sqrt{f}-f x\right]\right)\Big/$$
$$\left(2\sqrt{d}\sqrt{c d+b\sqrt{d}\sqrt{f}+a f}\left(c^2 d^2-b^2 d f+2 a c d f+a^2 f^2\right)^2(a+x\,(b+c x))^{5/2}\right)+$$

$$\left(f\left(-B c^2 d^{5/2}\sqrt{f}-2 b B c d^2 f+A c^2 d^2 f-b^2 B d^{3/2} f^{3/2}+2 A b c d^{3/2} f^{3/2}-2 a B c d^{3/2} f^{3/2}+A b^2 d f^2-\right.\right.$$
$$\left.\left.2 a b B d f^2+2 a A c d f^2+2 a A b\sqrt{d}\,f^{5/2}-a^2 B\sqrt{d}\,f^{5/2}+a^2 A f^3\right)\left(a+b x+c x^2\right)^{5/2}\text{Log}\left[\sqrt{d}\sqrt{f}+f x\right]\right)\Big/$$
$$\left(2\sqrt{d}\sqrt{c d-b\sqrt{d}\sqrt{f}+a f}\left(c^2 d^2-b^2 d f+2 a c d f+a^2 f^2\right)^2(a+x\,(b+c x))^{5/2}\right)-$$

$$\left(f\left(-B c^2 d^{5/2}\sqrt{f}-2 b B c d^2 f+A c^2 d^2 f-b^2 B d^{3/2} f^{3/2}+2 A b c d^{3/2} f^{3/2}-2 a B c d^{3/2} f^{3/2}+A b^2 d f^2-2 a b B d f^2+2 a A c d f^2+2 a A b\sqrt{d} f^{5/2}-a^2 B\sqrt{d} f^{5/2}+a^2 A f^3\right)\left(a+b x+c x^2\right)^{5/2}\mathrm{Log}\left[-b d+2 a\sqrt{d}\sqrt{f}-2 c d x+b\sqrt{d}\sqrt{f} x+2\sqrt{d}\sqrt{c d-b\sqrt{d}\sqrt{f}+a f}\sqrt{a+b x+c x^2}\right]\right)\Big/\left(2\sqrt{d}\sqrt{c d-b\sqrt{d}\sqrt{f}+a f}\left(c^2 d^2-b^2 d f+2 a c d f+a^2 f^2\right)^2\left(a+x\left(b+c x\right)\right)^{5/2}\right)+$$

$$\left(f\left(B c^2 d^{5/2}\sqrt{f}-2 b B c d^2 f+A c^2 d^2 f+b^2 B d^{3/2} f^{3/2}-2 A b c d^{3/2} f^{3/2}+2 a B c d^{3/2} f^{3/2}+A b^2 d f^2-2 a b B d f^2+2 a A c d f^2-2 a A b\sqrt{d} f^{5/2}+a^2 B\sqrt{d} f^{5/2}+a^2 A f^3\right)\left(a+b x+c x^2\right)^{5/2}\mathrm{Log}\left[b d+2 a\sqrt{d}\sqrt{f}+2 c d x+b\sqrt{d}\sqrt{f} x+2\sqrt{d}\sqrt{c d+b\sqrt{d}\sqrt{f}+a f}\sqrt{a+b x+c x^2}\right]\right)\Big/\left(2\sqrt{d}\sqrt{c d+b\sqrt{d}\sqrt{f}+a f}\left(c^2 d^2-b^2 d f+2 a c d f+a^2 f^2\right)^2\left(a+x\left(b+c x\right)\right)^{5/2}\right)$$

- **Problem 11: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{1+2 x}{\left(1+x^2\right)\sqrt{-1+x+x^2}}\,dx$$

Optimal (type 3, 117 leaves, 5 steps):

$$-\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)}\,\mathrm{ArcTan}\left[\frac{5+2\sqrt{5}-\sqrt{5}\,x}{\sqrt{10\left(2+\sqrt{5}\right)}\sqrt{-1+x+x^2}}\right]+\sqrt{\frac{1}{2}\left(-2+\sqrt{5}\right)}\,\mathrm{ArcTanh}\left[\frac{5-2\sqrt{5}+\sqrt{5}\,x}{\sqrt{10\left(-2+\sqrt{5}\right)}\sqrt{-1+x+x^2}}\right]$$

Result (type 3, 394 leaves):

$$\frac{1}{4}\left(2\sqrt{2-i}\,\text{ArcTan}\left[\frac{-8+8\,i\,x^3+\frac{20\sqrt{-1+x+x^2}}{\sqrt{2-i}}+x^2\left(2-(2-4\,i)\sqrt{2-i}\,\sqrt{-1+x+x^2}\right)-2\,i\,x\left(1+5\sqrt{2-i}\,\sqrt{-1+x+x^2}\right)}{(14+5\,i)-(15+14\,i)\,x-(6-5\,i)\,x^2+(5+6\,i)\,x^3}\right]+\right.$$

$$2\sqrt{2+i}\,\text{ArcTan}\left[\frac{2\left(4\,i+x-4\,x^3+(2+4\,i)\sqrt{2+i}\,\sqrt{-1+x+x^2}-5\sqrt{2+i}\,x\sqrt{-1+x+x^2}+x^2\left(-i+\frac{5\sqrt{-1+x+x^2}}{\sqrt{2+i}}\right)\right)}{(5+14\,i)-(14+15\,i)\,x+(5-6\,i)\,x^2+(6+5\,i)\,x^3}\right]+$$

$$i\left(\left(\sqrt{2-i}+\sqrt{2+i}\right)\text{Log}\left[1+x^2\right]-\sqrt{2-i}\,\text{Log}\left[(3-4\,i)-(8-4\,i)\,x-(13-4\,i)\,x^2+4\sqrt{2-i}\,\sqrt{-1+x+x^2}+8\sqrt{2-i}\,x\sqrt{-1+x+x^2}\right]-\right.$$

$$\left.\left.\sqrt{2+i}\,\text{Log}\left[(-3-4\,i)+(8+4\,i)\,x+(13+4\,i)\,x^2+4\sqrt{2+i}\,\sqrt{-1+x+x^2}+8\sqrt{2+i}\,x\sqrt{-1+x+x^2}\right]\right)\right)$$

Problem 12: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a-c+b\,x}{\left(1+x^2\right)\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 3, 484 leaves, 5 steps) :

$$-\left(\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2\,a\,c+c^2}\right)-a\left(2\,c-\sqrt{a^2+b^2-2\,a\,c+c^2}\right)}\right.$$

$$\text{ArcTan}\left[\left(b\sqrt{a^2+b^2-2\,a\,c+c^2}-\left(b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-2\,a\,c+c^2}\right)\right)x\right)\Big/\right.$$

$$\left.\left.\left(\sqrt{2}\left(a^2+b^2-2\,a\,c+c^2\right)^{1/4}\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2\,a\,c+c^2}\right)-a\left(2\,c-\sqrt{a^2+b^2-2\,a\,c+c^2}\right)}\,\sqrt{a+b\,x+c\,x^2}\right)\right]\right)\Big/$$

$$\left(\sqrt{2}\left(a^2+b^2-2\,a\,c+c^2\right)^{1/4}\right)-\left(\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2\,a\,c+c^2}\right)-a\left(2\,c+\sqrt{a^2+b^2-2\,a\,c+c^2}\right)}\right.$$

$$\text{ArcTanh}\left[\left(b\sqrt{a^2+b^2-2\,a\,c+c^2}+\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2\,a\,c+c^2}\right)\right)x\right)\Big/\left(\sqrt{2}\left(a^2+b^2-2\,a\,c+c^2\right)^{1/4}\right.\right.$$

$$\left.\left.\left.\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2\,a\,c+c^2}\right)-a\left(2\,c+\sqrt{a^2+b^2-2\,a\,c+c^2}\right)}\,\sqrt{a+b\,x+c\,x^2}\right)\right]\right)\Big/\left(\sqrt{2}\left(a^2+b^2-2\,a\,c+c^2\right)^{1/4}\right)$$

Result (type 3, 182 leaves) :

$$\frac{1}{2} i \left(-\sqrt{a-i b-c}\; \text{Log}\left[-\frac{2 i \left(2 a-2 i c x+b (-i+x)+2 \sqrt{a-i b-c} \sqrt{a+x (b+c x)}\right)}{(a-i b-c)^{3/2} (i+x)}\right]+\right.$$
$$\left.\sqrt{a+i b-c}\; \text{Log}\left[\frac{2 i \left(2 a+2 i c x+b (i+x)+2 \sqrt{a+i b-c} \sqrt{a+x (b+c x)}\right)}{(a+i b-c)^{3/2} (-i+x)}\right]\right)$$

- **Problem 19: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+B x) \sqrt{a+b x+c x^2}}{d+e x+f x^2}\, dx$$

Optimal (type 3, 617 leaves, 9 steps) :

$$\frac{B\sqrt{a+bx+cx^2}}{f}-\frac{(2Bce-bBf-2Acf)\,\text{ArcTanh}\left[\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right]}{2\sqrt{c}\,f^2}+$$

$$\left(\left(2f\,(Af\,(cd-af)-Bd\,(ce-bf))-\left(e-\sqrt{e^2-4df}\right)\left(Af\,(ce-bf)+B\left(f\,(be-af)-c\left(e^2-df\right)\right)\right)\right)\right.$$

$$\left.\text{ArcTanh}\left[\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,f^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\right)-$$

$$\left(\left(2f\,(Af\,(cd-af)-Bd\,(ce-bf))-\left(e+\sqrt{e^2-4df}\right)\left(Af\,(ce-bf)+B\left(f\,(be-af)-c\left(e^2-df\right)\right)\right)\right)\right.$$

$$\left.\text{ArcTanh}\left[\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,f^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\right)$$

Result (type 3, 1344 leaves) :

$$\frac{1}{2 f^2}\left(2 B f \sqrt{a+x (b+c x)}+\left(\sqrt{2} \left(A f \left(f \left(-b e+2 a f+b \sqrt{e^2-4 d f}\right)+c \left(e^2-2 d f-e \sqrt{e^2-4 d f}\right)\right)+\right.\right.\right.$$
$$B \left(c \left(-e^3+3 d e f+e^2 \sqrt{e^2-4 d f}-d f \sqrt{e^2-4 d f}\right)+f \left(a f \left(-e+\sqrt{e^2-4 d f}\right)+b \left(e^2-2 d f-e \sqrt{e^2-4 d f}\right)\right)\right)\Big)$$
$$\text{Log}\left[-e+\sqrt{e^2-4 d f}-2 f x\right]\Big)\Big/\left(\sqrt{e^2-4 d f} \sqrt{c \left(e^2-2 d f-e \sqrt{e^2-4 d f}\right)+f \left(2 a f+b \left(-e+\sqrt{e^2-4 d f}\right)\right)}\right)+$$
$$\left(\sqrt{2} \left(A f \left(-c \left(e^2-2 d f+e \sqrt{e^2-4 d f}\right)+f \left(-2 a f+b \left(e+\sqrt{e^2-4 d f}\right)\right)\right)+\right.\right.$$
$$B \left(c \left(e^3-3 d e f+e^2 \sqrt{e^2-4 d f}-d f \sqrt{e^2-4 d f}\right)+f \left(a f \left(e+\sqrt{e^2-4 d f}\right)-b \left(e^2-2 d f+e \sqrt{e^2-4 d f}\right)\right)\right)\Big)$$
$$\text{Log}\left[e+\sqrt{e^2-4 d f}+2 f x\right]\Big)\Big/\left(\sqrt{e^2-4 d f} \sqrt{c \left(e^2-2 d f+e \sqrt{e^2-4 d f}\right)+f \left(2 a f-b \left(e+\sqrt{e^2-4 d f}\right)\right)}\right)-$$
$$\frac{(2 B c e-b B f-2 A c f) \text{Log}\left[b+2 c x+2 \sqrt{c} \sqrt{a+x (b+c x)}\right]}{\sqrt{c}}-\left(\sqrt{2} \left(A f \left(f \left(-b e+2 a f+b \sqrt{e^2-4 d f}\right)+c \left(e^2-2 d f-e \sqrt{e^2-4 d f}\right)\right)+\right.\right.$$
$$B \left(c \left(-e^3+3 d e f+e^2 \sqrt{e^2-4 d f}-d f \sqrt{e^2-4 d f}\right)+f \left(a f \left(-e+\sqrt{e^2-4 d f}\right)+b \left(e^2-2 d f-e \sqrt{e^2-4 d f}\right)\right)\right)\Big)$$
$$\text{Log}\Big[4 a f \sqrt{e^2-4 d f}+2 c e^2 x-8 c d f x-2 c e \sqrt{e^2-4 d f}\, x+b \left(e^2-4 d f-e \sqrt{e^2-4 d f}+2 f \sqrt{e^2-4 d f}\, x\right)+$$
$$2 \sqrt{2} \sqrt{e^2-4 d f} \sqrt{f \left(-b e+2 a f+b \sqrt{e^2-4 d f}\right)+c \left(e^2-2 d f-e \sqrt{e^2-4 d f}\right)} \sqrt{a+x (b+c x)}\Big]\Big)\Big/$$
$$\left(\sqrt{e^2-4 d f} \sqrt{c \left(e^2-2 d f-e \sqrt{e^2-4 d f}\right)+f \left(2 a f+b \left(-e+\sqrt{e^2-4 d f}\right)\right)}\right)-$$
$$\left(\sqrt{2} \left(A f \left(-c \left(e^2-2 d f+e \sqrt{e^2-4 d f}\right)+f \left(-2 a f+b \left(e+\sqrt{e^2-4 d f}\right)\right)\right)+\right.\right.$$
$$B \left(c \left(e^3-3 d e f+e^2 \sqrt{e^2-4 d f}-d f \sqrt{e^2-4 d f}\right)+f \left(a f \left(e+\sqrt{e^2-4 d f}\right)-b \left(e^2-2 d f+e \sqrt{e^2-4 d f}\right)\right)\right)\Big)$$
$$\text{Log}\Big[4 a f \sqrt{e^2-4 d f}-2 c e^2 x+8 c d f x-2 c e \sqrt{e^2-4 d f}\, x+2 \sqrt{2} \sqrt{e^2-4 d f}$$
$$\sqrt{c \left(e^2-2 d f+e \sqrt{e^2-4 d f}\right)+f \left(2 a f-b \left(e+\sqrt{e^2-4 d f}\right)\right)} \sqrt{a+x (b+c x)}-b \left(e^2+e \sqrt{e^2-4 d f}-2 f \left(2 d+\sqrt{e^2-4 d f}\, x\right)\right)\Big]\Big)\Big/$$
$$\left.\left(\sqrt{e^2-4 d f} \sqrt{c \left(e^2-2 d f+e \sqrt{e^2-4 d f}\right)+f \left(2 a f-b \left(e+\sqrt{e^2-4 d f}\right)\right)}\right)\right)$$

- **Problem 20: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B x) \left(a + b x + c x^2\right)^{3/2}}{d + e x + f x^2} \, dx$$

Optimal (type 3, 1092 leaves, 10 steps) :

$$-\frac{1}{8 c f^3}\left(2 A c f (4 c e-5 b f)-B \left(b^2 f^2-2 c f (5 b e-4 a f)+8 c^2 \left(e^2-d f\right)\right)+2 c f (2 B c e-b B f-2 A c f) x\right) \sqrt{a+b x+c x^2}+\frac{B \left(a+b x+c x^2\right)^{3/2}}{3 f}+$$

$$\frac{1}{16 c^{3/2} f^4}\left(2 A c f \left(3 b^2 f^2-12 c f (b e-a f)+8 c^2 \left(e^2-d f\right)\right)-B \left(b^3 f^3+6 b c f^2 (b e-2 a f)-24 c^2 f \left(b e^2-b d f-a e f\right)+16 c^3 \left(e^3-2 d e f\right)\right)\right)$$

$$\text{ArcTanh}\left[\frac{b+2 c x}{2 \sqrt{c} \sqrt{a+b x+c x^2}}\right]-$$

$$\left(\left(2 c f \left(B d (c e-b f) \left(c e^2-2 c d f-b e f+2 a f^2\right)+A f \left(2 c d f (b e-a f)-f^2 \left(b^2 d-a^2 f\right)-c^2 d \left(e^2-d f\right)\right)\right)-c \left(e-\sqrt{e^2-4 d f}\right) \left(A f (c e-b f)\right.\right.\right.$$

$$\left.\left.\left.\left(f (b e-2 a f)-c \left(e^2-2 d f\right)\right)+B \left(c^2 \left(e^4-3 d e^2 f+d^2 f^2\right)-f^2 \left(2 a b e f-a^2 f^2-b^2 \left(e^2-d f\right)\right)+2 c f \left(a f \left(e^2-d f\right)-b \left(e^3-2 d e f\right)\right)\right)\right)\right)$$

$$\left.\text{ArcTanh}\left[\frac{4 a f-b \left(e-\sqrt{e^2-4 d f}\right)+2 \left(b f-c \left(e-\sqrt{e^2-4 d f}\right)\right) x}{2 \sqrt{2} \sqrt{c e^2-2 c d f-b e f+2 a f^2-(c e-b f) \sqrt{e^2-4 d f}} \sqrt{a+b x+c x^2}}\right]\right)\Big/$$

$$\left(\sqrt{2} c f^4 \sqrt{e^2-4 d f} \sqrt{c e^2-2 c d f-b e f+2 a f^2-(c e-b f) \sqrt{e^2-4 d f}}\right)+$$

$$\left(\left(2 f \left(B d (c e-b f) \left(c e^2-2 c d f-b e f+2 a f^2\right)+A f \left(2 c d f (b e-a f)-f^2 \left(b^2 d-a^2 f\right)-c^2 d \left(e^2-d f\right)\right)\right)-\left(e+\sqrt{e^2-4 d f}\right) \left(A f (c e-b f)\right.\right.\right.$$

$$\left.\left.\left.\left(f (b e-2 a f)-c \left(e^2-2 d f\right)\right)+B \left(c^2 \left(e^4-3 d e^2 f+d^2 f^2\right)-f^2 \left(2 a b e f-a^2 f^2-b^2 \left(e^2-d f\right)\right)+2 c f \left(a f \left(e^2-d f\right)-b \left(e^3-2 d e f\right)\right)\right)\right)\right)$$

$$\left.\text{ArcTanh}\left[\frac{4 a f-b \left(e+\sqrt{e^2-4 d f}\right)+2 \left(b f-c \left(e+\sqrt{e^2-4 d f}\right)\right) x}{2 \sqrt{2} \sqrt{c e^2-2 c d f-b e f+2 a f^2+(c e-b f) \sqrt{e^2-4 d f}} \sqrt{a+b x+c x^2}}\right]\right)\Big/$$

$$\left(\sqrt{2} f^4 \sqrt{e^2-4 d f} \sqrt{c e^2-2 c d f-b e f+2 a f^2+(c e-b f) \sqrt{e^2-4 d f}}\right)$$

Result (type 3, 3733 leaves) :

$$\frac{1}{a+b\,x+c\,x^2}$$

$$\left(-\frac{-24\,B\,c^2\,e^2+24\,B\,c^2\,d\,f+30\,b\,B\,c\,e\,f+24\,A\,c^2\,e\,f-3\,b^2\,B\,f^2-30\,A\,b\,c\,f^2-32\,a\,B\,c\,f^2}{24\,c\,f^3}+\frac{(-6\,B\,c\,e+7\,b\,B\,f+6\,A\,c\,f)\,x}{12\,f^2}+\frac{B\,c\,x^2}{3\,f}\right)\,(a+x\,(b+c\,x))^{3/2}+$$

$$\Big(\Big(-B\,c^2\,e^5+5\,B\,c^2\,d\,e^3\,f+2\,b\,B\,c\,e^4\,f+A\,c^2\,e^4\,f-5\,B\,c^2\,d^2\,e\,f^2-8\,b\,B\,c\,d\,e^2\,f^2-4\,A\,c^2\,d\,e^2\,f^2-b^2\,B\,e^3\,f^2-2\,A\,b\,c\,e^3\,f^2-2\,a\,B\,c\,e^3\,f^2+4\,b\,B\,c\,d^2\,f^3+$$

$$2\,A\,c^2\,d^2\,f^3+3\,b^2\,B\,d\,e\,f^3+6\,A\,b\,c\,d\,e\,f^3+6\,a\,B\,c\,d\,e\,f^3+A\,b^2\,e^2\,f^3+2\,a\,b\,B\,e^2\,f^3+2\,a\,A\,c\,e^2\,f^3-2\,A\,b^2\,d\,f^4-4\,a\,b\,B\,d\,f^4-4\,a\,A\,c\,d\,f^4-2\,a\,A\,b\,e\,f^4-$$

$$a^2\,B\,e\,f^4+2\,a^2\,A\,f^5+B\,c^2\,e^4\,\sqrt{e^2-4\,d\,f}-3\,B\,c^2\,d\,e^2\,f\,\sqrt{e^2-4\,d\,f}-2\,b\,B\,c\,e^3\,f\,\sqrt{e^2-4\,d\,f}-A\,c^2\,e^3\,f\,\sqrt{e^2-4\,d\,f}+B\,c^2\,d^2\,f^2\,\sqrt{e^2-4\,d\,f}+$$

$$4\,b\,B\,c\,d\,e\,f^2\,\sqrt{e^2-4\,d\,f}+2\,A\,c^2\,d\,e\,f^2\,\sqrt{e^2-4\,d\,f}+b^2\,B\,e^2\,f^2\,\sqrt{e^2-4\,d\,f}+2\,A\,b\,c\,e^2\,f^2\,\sqrt{e^2-4\,d\,f}+2\,a\,B\,c\,e^2\,f^2\,\sqrt{e^2-4\,d\,f}-$$

$$b^2\,B\,d\,f^3\,\sqrt{e^2-4\,d\,f}-2\,A\,b\,c\,d\,f^3\,\sqrt{e^2-4\,d\,f}-2\,a\,B\,c\,d\,f^3\,\sqrt{e^2-4\,d\,f}-A\,b^2\,e\,f^3\,\sqrt{e^2-4\,d\,f}-2\,a\,b\,B\,e\,f^3\,\sqrt{e^2-4\,d\,f}-$$

$$2\,a\,A\,c\,e\,f^3\,\sqrt{e^2-4\,d\,f}+2\,a\,A\,b\,f^4\,\sqrt{e^2-4\,d\,f}+a^2\,B\,f^4\,\sqrt{e^2-4\,d\,f}\Big)\,(a+x\,(b+c\,x))^{3/2}\,\mathrm{Log}\Big[-e+\sqrt{e^2-4\,d\,f}-2\,f\,x\Big]\Big)\Big/$$

$$\left(\sqrt{2}\,f^4\,\sqrt{e^2-4\,d\,f}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2-c\,e\,\sqrt{e^2-4\,d\,f}+b\,f\,\sqrt{e^2-4\,d\,f}}\,\left(a+b\,x+c\,x^2\right)^{3/2}\right)+$$

$$\Big(\Big(B\,c^2\,e^5-5\,B\,c^2\,d\,e^3\,f-2\,b\,B\,c\,e^4\,f-A\,c^2\,e^4\,f+5\,B\,c^2\,d^2\,e\,f^2+8\,b\,B\,c\,d\,e^2\,f^2+4\,A\,c^2\,d\,e^2\,f^2+b^2\,B\,e^3\,f^2+2\,A\,b\,c\,e^3\,f^2+2\,a\,B\,c\,e^3\,f^2-4\,b\,B\,c\,d^2\,f^3-$$

$$2\,A\,c^2\,d^2\,f^3-3\,b^2\,B\,d\,e\,f^3-6\,A\,b\,c\,d\,e\,f^3-6\,a\,B\,c\,d\,e\,f^3-A\,b^2\,e^2\,f^3-2\,a\,b\,B\,e^2\,f^3-2\,a\,A\,c\,e^2\,f^3+2\,A\,b^2\,d\,f^4+4\,a\,b\,B\,d\,f^4+4\,a\,A\,c\,d\,f^4+$$

$$2\,a\,A\,b\,e\,f^4+a^2\,B\,e\,f^4-2\,a^2\,A\,f^5+B\,c^2\,e^4\,\sqrt{e^2-4\,d\,f}-3\,B\,c^2\,d\,e^2\,f\,\sqrt{e^2-4\,d\,f}-2\,b\,B\,c\,e^3\,f\,\sqrt{e^2-4\,d\,f}-A\,c^2\,e^3\,f\,\sqrt{e^2-4\,d\,f}+$$

$$B\,c^2\,d^2\,f^2\,\sqrt{e^2-4\,d\,f}+4\,b\,B\,c\,d\,e\,f^2\,\sqrt{e^2-4\,d\,f}+2\,A\,c^2\,d\,e\,f^2\,\sqrt{e^2-4\,d\,f}+b^2\,B\,e^2\,f^2\,\sqrt{e^2-4\,d\,f}+2\,A\,b\,c\,e^2\,f^2\,\sqrt{e^2-4\,d\,f}+$$

$$2\,a\,B\,c\,e^2\,f^2\,\sqrt{e^2-4\,d\,f}-b^2\,B\,d\,f^3\,\sqrt{e^2-4\,d\,f}-2\,A\,b\,c\,d\,f^3\,\sqrt{e^2-4\,d\,f}-2\,a\,B\,c\,d\,f^3\,\sqrt{e^2-4\,d\,f}-A\,b^2\,e\,f^3\,\sqrt{e^2-4\,d\,f}-$$

$$2\,a\,b\,B\,e\,f^3\,\sqrt{e^2-4\,d\,f}-2\,a\,A\,c\,e\,f^3\,\sqrt{e^2-4\,d\,f}+2\,a\,A\,b\,f^4\,\sqrt{e^2-4\,d\,f}+a^2\,B\,f^4\,\sqrt{e^2-4\,d\,f}\Big)\,(a+x\,(b+c\,x))^{3/2}\,\mathrm{Log}\Big[e+\sqrt{e^2-4\,d\,f}+2\,f\,x\Big]\Big)\Big/$$

$$\left(\sqrt{2}\,f^4\,\sqrt{e^2-4\,d\,f}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2+c\,e\,\sqrt{e^2-4\,d\,f}-b\,f\,\sqrt{e^2-4\,d\,f}}\,\left(a+b\,x+c\,x^2\right)^{3/2}\right)-$$

$$\Big(\Big(16\,B\,c^3\,e^3-32\,B\,c^3\,d\,e\,f-24\,b\,B\,c^2\,e^2\,f-16\,A\,c^3\,e^2\,f+24\,b\,B\,c^2\,d\,f^2+16\,A\,c^3\,d\,f^2+6\,b^2\,B\,c\,e\,f^2+24\,A\,b\,c^2\,e\,f^2+24\,a\,B\,c^2\,e\,f^2+b^3\,B\,f^3-$$

$$6\,A\,b^2\,c\,f^3-12\,a\,b\,B\,c\,f^3-24\,a\,A\,c^2\,f^3\Big)\,(a+x\,(b+c\,x))^{3/2}\,\mathrm{Log}\Big[b+2\,c\,x+2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}\Big]\Big)\Big/\left(16\,c^{3/2}\,f^4\,\left(a+b\,x+c\,x^2\right)^{3/2}\right)-$$

$$\Big(\Big(B\,c^2\,e^5-5\,B\,c^2\,d\,e^3\,f-2\,b\,B\,c\,e^4\,f-A\,c^2\,e^4\,f+5\,B\,c^2\,d^2\,e\,f^2+8\,b\,B\,c\,d\,e^2\,f^2+4\,A\,c^2\,d\,e^2\,f^2+b^2\,B\,e^3\,f^2+2\,A\,b\,c\,e^3\,f^2+2\,a\,B\,c\,e^3\,f^2-$$

$$4\,b\,B\,c\,d^2\,f^3-2\,A\,c^2\,d^2\,f^3-3\,b^2\,B\,d\,e\,f^3-6\,A\,b\,c\,d\,e\,f^3-6\,a\,B\,c\,d\,e\,f^3-A\,b^2\,e^2\,f^3-2\,a\,b\,B\,e^2\,f^3-2\,a\,A\,c\,e^2\,f^3+2\,A\,b^2\,d\,f^4+$$

$$4\,a\,b\,B\,d\,f^4+4\,a\,A\,c\,d\,f^4+2\,a\,A\,b\,e\,f^4+a^2\,B\,e\,f^4-2\,a^2\,A\,f^5+B\,c^2\,e^4\,\sqrt{e^2-4\,d\,f}-3\,B\,c^2\,d\,e^2\,f\,\sqrt{e^2-4\,d\,f}-2\,b\,B\,c\,e^3\,f\,\sqrt{e^2-4\,d\,f}-$$

$$A\,c^2\,e^3\,f\,\sqrt{e^2-4\,d\,f}+B\,c^2\,d^2\,f^2\,\sqrt{e^2-4\,d\,f}+4\,b\,B\,c\,d\,e\,f^2\,\sqrt{e^2-4\,d\,f}+2\,A\,c^2\,d\,e\,f^2\,\sqrt{e^2-4\,d\,f}+b^2\,B\,e^2\,f^2\,\sqrt{e^2-4\,d\,f}+$$

$$2\,A\,b\,c\,e^2\,f^2\,\sqrt{e^2-4\,d\,f}+2\,a\,B\,c\,e^2\,f^2\,\sqrt{e^2-4\,d\,f}-b^2\,B\,d\,f^3\,\sqrt{e^2-4\,d\,f}-2\,A\,b\,c\,d\,f^3\,\sqrt{e^2-4\,d\,f}-2\,a\,B\,c\,d\,f^3\,\sqrt{e^2-4\,d\,f}-$$

$$A b^2 e f^3 \sqrt{e^2-4 d f} - 2 a b B e f^3 \sqrt{e^2-4 d f} - 2 a A c e f^3 \sqrt{e^2-4 d f} + 2 a A b f^4 \sqrt{e^2-4 d f} + a^2 B f^4 \sqrt{e^2-4 d f}\Big)$$

$$(a + x (b + c x))^{3/2} \operatorname{Log}\Big[-b e^2 + 4 b d f - b e \sqrt{e^2-4 d f} + 4 a f \sqrt{e^2-4 d f} - 2 c e^2 x + 8 c d f x - 2 c e \sqrt{e^2-4 d f}\, x +$$

$$2 b f \sqrt{e^2-4 d f}\, x + 2 \sqrt{2} \sqrt{e^2-4 d f} \sqrt{c e^2 - 2 c d f - b e f + 2 a f^2 + c e \sqrt{e^2-4 d f} - b f \sqrt{e^2-4 d f}} \sqrt{a + b x + c x^2}\Big]\Big) \Big/$$

$$\left(\sqrt{2} f^4 \sqrt{e^2-4 d f} \sqrt{c e^2 - 2 c d f - b e f + 2 a f^2 + c e \sqrt{e^2-4 d f} - b f \sqrt{e^2-4 d f}} \left(a + b x + c x^2\right)^{3/2}\right) -$$

$$\Big(\Big(-B c^2 e^5 + 5 B c^2 d e^3 f + 2 b B c e^4 f + A c^2 e^4 f - 5 B c^2 d^2 e f^2 - 8 b B c d e^2 f^2 - 4 A c^2 d e^2 f^2 - b^2 B e^3 f^2 - 2 A b c e^3 f^2 - 2 a B c e^3 f^2 +$$

$$4 b B c d^2 f^3 + 2 A c^2 d^2 f^3 + 3 b^2 B d e f^3 + 6 A b c d e f^3 + 6 a B c d e f^3 + A b^2 e^2 f^3 + 2 a b B e^2 f^3 + 2 a A c e^2 f^3 - 2 A b^2 d f^4 -$$

$$4 a b B d f^4 - 4 a A c d f^4 - 2 a A b e f^4 - a^2 B e f^4 + 2 a^2 A f^5 + B c^2 e^4 \sqrt{e^2-4 d f} - 3 B c^2 d e^2 f \sqrt{e^2-4 d f} - 2 b B c e^3 f \sqrt{e^2-4 d f} -$$

$$A c^2 e^3 f \sqrt{e^2-4 d f} + B c^2 d^2 f^2 \sqrt{e^2-4 d f} + 4 b B c d e f^2 \sqrt{e^2-4 d f} + 2 A c^2 d e f^2 \sqrt{e^2-4 d f} + b^2 B e^2 f^2 \sqrt{e^2-4 d f} +$$

$$2 A b c e^2 f^2 \sqrt{e^2-4 d f} + 2 a B c e^2 f^2 \sqrt{e^2-4 d f} - b^2 B d f^3 \sqrt{e^2-4 d f} - 2 A b c d f^3 \sqrt{e^2-4 d f} - 2 a B c d f^3 \sqrt{e^2-4 d f} -$$

$$A b^2 e f^3 \sqrt{e^2-4 d f} - 2 a b B e f^3 \sqrt{e^2-4 d f} - 2 a A c e f^3 \sqrt{e^2-4 d f} + 2 a A b f^4 \sqrt{e^2-4 d f} + a^2 B f^4 \sqrt{e^2-4 d f}\Big)$$

$$(a + x (b + c x))^{3/2} \operatorname{Log}\Big[b e^2 - 4 b d f - b e \sqrt{e^2-4 d f} + 4 a f \sqrt{e^2-4 d f} + 2 c e^2 x - 8 c d f x - 2 c e \sqrt{e^2-4 d f}\, x +$$

$$2 b f \sqrt{e^2-4 d f}\, x + 2 \sqrt{2} \sqrt{e^2-4 d f} \sqrt{c e^2 - 2 c d f - b e f + 2 a f^2 - c e \sqrt{e^2-4 d f} + b f \sqrt{e^2-4 d f}} \sqrt{a + b x + c x^2}\Big]\Big) \Big/$$

$$\left(\sqrt{2} f^4 \sqrt{e^2-4 d f} \sqrt{c e^2 - 2 c d f - b e f + 2 a f^2 - c e \sqrt{e^2-4 d f} + b f \sqrt{e^2-4 d f}} \left(a + b x + c x^2\right)^{3/2}\right)$$

- **Problem 22: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B x}{\left(a + c x^2\right) \sqrt{d + e x + f x^2}} \, dx$$

Optimal (type 3, 780 leaves, 5 steps):

$$\left(\sqrt{a B e+A \left(c d-a f-\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)} \sqrt{-A c e+B \left(c d-a f+\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)}\right.$$

$$\text{ArcTanh}\left[\left(\sqrt{e} \left(a \left(A c e-B \left(c d-a f+\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)\right)-c \left(a B e+A \left(c d-a f-\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)\right) x\right)\right)\Big/\right.$$

$$\left(\sqrt{2} \sqrt{a} \sqrt{c} \sqrt{a B e+A \left(c d-a f-\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)} \sqrt{-A c e+B \left(c d-a f+\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)}\right.$$

$$\left.\left.\left.\sqrt{d+e x+f x^2}\right)\right]\right)\Big/\left(\sqrt{2} \sqrt{a} \sqrt{c} \sqrt{e} \sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)-$$

$$\left(\sqrt{-A c e+B \left(c d-a f-\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)} \sqrt{a B e+A \left(c d-a f+\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)}\right.$$

$$\text{ArcTanh}\left[\left(\sqrt{e} \left(a \left(A c e-B \left(c d-a f-\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)\right)-c \left(a B e+A \left(c d-a f+\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)\right) x\right)\right)\Big/\right.$$

$$\left(\sqrt{2} \sqrt{a} \sqrt{c} \sqrt{-A c e+B \left(c d-a f-\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)} \sqrt{a B e+A \left(c d-a f+\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)}\right.$$

$$\left.\left.\left.\sqrt{d+e x+f x^2}\right)\right]\right)\Big/\left(\sqrt{2} \sqrt{a} \sqrt{c} \sqrt{e} \sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)$$

Result (type 3, 411 leaves) :

$$\frac{1}{2 \sqrt{a} \sqrt{c}}\left(-\frac{\left(\sqrt{a} B+i A \sqrt{c}\right) \text{Log}\left[-\frac{\sqrt{a} \sqrt{c} \left(i \sqrt{c} (2 d+e x)+\sqrt{a} (e+2 f x)+2 i \sqrt{c d-i \sqrt{a} \sqrt{c} e-a f} \sqrt{d+x (e+f x)}\right)}{\left(\sqrt{a} B+i A \sqrt{c}\right) \sqrt{c d-i \sqrt{a} \sqrt{c} e-a f} \left(\sqrt{a}-i \sqrt{c} x\right)}\right]}{\sqrt{c d-i \sqrt{a} \sqrt{c} e-a f}}+\right.$$

$$\left.\frac{\left(-\sqrt{a} B+i A \sqrt{c}\right) \text{Log}\left[\frac{i \sqrt{a} \sqrt{c} \left(\sqrt{c} (2 d+e x)+i \sqrt{a} (e+2 f x)+2 \sqrt{c d+i \sqrt{a} \sqrt{c} e-a f} \sqrt{d+x (e+f x)}\right)}{\left(\sqrt{a} B-i A \sqrt{c}\right) \sqrt{c d+i \sqrt{a} \sqrt{c} e-a f} \left(\sqrt{a}+i \sqrt{c} x\right)}\right]}{\sqrt{c d+i \sqrt{a} \sqrt{c} e-a f}}\right)$$

- **Problem 24: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A+B x}{\left(a+c x^2\right) \sqrt{d+f x^2}} \, dx$$

Optimal (type 3, 101 leaves, 6 steps) :

$$\frac{A\,\text{ArcTan}\left[\frac{\sqrt{c\,d-a\,f}\;x}{\sqrt{a}\;\sqrt{d+f\,x^2}}\right]}{\sqrt{a}\;\sqrt{c\,d-a\,f}}-\frac{B\,\text{ArcTanh}\left[\frac{\sqrt{c}\;\sqrt{d+f\,x^2}}{\sqrt{c\,d-a\,f}}\right]}{\sqrt{c}\;\sqrt{c\,d-a\,f}}$$

Result (type 3, 282 leaves) :

$$\frac{1}{2\sqrt{a}\;\sqrt{c}\;\sqrt{c\,d-a\,f}}\left(-\left(\sqrt{a}\;B+i\,A\sqrt{c}\right)\text{Log}\left[\frac{2\sqrt{a}\;\sqrt{c}\;\left(\sqrt{c}\;d-i\sqrt{a}\;f\,x+\sqrt{c\,d-a\,f}\;\sqrt{d+f\,x^2}\right)}{\left(\sqrt{a}\;B+i\,A\sqrt{c}\right)\sqrt{c\,d-a\,f}\;\left(i\sqrt{a}+\sqrt{c}\;x\right)}\right]+\left(-\sqrt{a}\;B+i\,A\sqrt{c}\right)\text{Log}\left[\frac{2\,i\sqrt{a}\;\sqrt{c}\;\left(\sqrt{c}\;d+i\sqrt{a}\;f\,x+\sqrt{c\,d-a\,f}\;\sqrt{d+f\,x^2}\right)}{\left(\sqrt{a}\;B-i\,A\sqrt{c}\right)\sqrt{c\,d-a\,f}\;\left(\sqrt{a}+i\sqrt{c}\;x\right)}\right]\right)$$

- **Problem 27: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{2+x}{\left(2+4\,x-3\,x^2\right)\left(1+3\,x-2\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 193 leaves, 7 steps) :

$$-\frac{2\,(15+14\,x)}{51\left(1+3\,x-2\,x^2\right)^{3/2}}-\frac{2\,(291+4814\,x)}{867\sqrt{1+3\,x-2\,x^2}}+\frac{9}{2}\sqrt{\frac{1}{5}\left(-53+17\sqrt{10}\right)}\;\text{ArcTan}\left[\frac{3\left(4-\sqrt{10}\right)+\left(1+4\sqrt{10}\right)x}{2\sqrt{1+\sqrt{10}}\;\sqrt{1+3\,x-2\,x^2}}\right]+\frac{9}{2}\sqrt{\frac{1}{5}\left(53+17\sqrt{10}\right)}\;\text{ArcTanh}\left[\frac{3\left(4+\sqrt{10}\right)+\left(1-4\sqrt{10}\right)x}{2\sqrt{-1+\sqrt{10}}\;\sqrt{1+3\,x-2\,x^2}}\right]$$

Result (type 3, 304 leaves) :

$$-\frac{2\left(546+5925 x+13860 x^2-9628 x^3\right)}{867\left(1+3 x-2 x^2\right)^{3/2}}-\frac{27\left(-4+\sqrt{10}\right) \text{ArcTan}\left[\frac{12-3\sqrt{10}+x+4\sqrt{10}\, x}{2\sqrt{1+\sqrt{10}}\,\sqrt{1+3x-2x^2}}\right]}{2\sqrt{10\left(1+\sqrt{10}\right)}}-\frac{27\left(4+\sqrt{10}\right) \text{Log}\left[2+\sqrt{10}-3x\right]}{2\sqrt{10\left(-1+\sqrt{10}\right)}}-$$

$$\frac{27\, i\left(-4+\sqrt{10}\right) \text{Log}\left[\left(-2+\sqrt{10}+3x\right)^2\right]}{4\sqrt{10\left(1+\sqrt{10}\right)}}+\frac{27\, i\left(-4+\sqrt{10}\right) \text{Log}\left[14-4\sqrt{10}+6\left(-2+\sqrt{10}\right)x+9x^2\right]}{4\sqrt{10\left(1+\sqrt{10}\right)}}+$$

$$\frac{27\left(4+\sqrt{10}\right) \text{Log}\left[30+12\sqrt{10}-40x+\sqrt{10}\,x+2\sqrt{10\left(-1+\sqrt{10}\right)}\,\sqrt{1+3x-2x^2}\right]}{2\sqrt{10\left(-1+\sqrt{10}\right)}}$$

- **Problem 31: Result more than twice size of optimal antiderivative.**

$$\int \frac{1+x}{\left(4+2x+x^2\right)\sqrt{5+2x+x^2}}\, dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$-\text{ArcTanh}\left[\sqrt{5+2x+x^2}\right]$$

Result (type 3, 41 leaves):

$$\frac{1}{2}\text{Log}\left[1-\sqrt{5+2x+x^2}\right]-\frac{1}{2}\text{Log}\left[1+\sqrt{5+2x+x^2}\right]$$

- **Problem 32: Result more than twice size of optimal antiderivative.**

$$\int \frac{4+x}{\left(4+2x+x^2\right)\sqrt{5+2x+x^2}}\, dx$$

Optimal (type 3, 44 leaves, 5 steps):

$$\sqrt{3}\,\text{ArcTan}\left[\frac{1+x}{\sqrt{3}\,\sqrt{5+2x+x^2}}\right]-\text{ArcTanh}\left[\sqrt{5+2x+x^2}\right]$$

Result (type 3, 109 leaves):

$$\frac{1}{2}\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{\sqrt{3}\left(4+x^2+\sqrt{5+2x+x^2}+x\left(2+\sqrt{5+2x+x^2}\right)\right)}{11+4x+2x^2}\right]+\text{Log}\left[\left(4+2x+x^2\right)^2\right]-\text{Log}\left[\left(4+2x+x^2\right)\left(6+2x+x^2+2\sqrt{5+2x+x^2}\right)\right]\right)$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\left(3+x+x^2\right)\sqrt{5+x+x^2}}\,dx$$

Optimal (type 3, 56 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{\frac{2}{11}}\,(1+2x)}{\sqrt{5+x+x^2}}\right]}{\sqrt{22}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{5+x+x^2}}{\sqrt{2}}\right]}{\sqrt{2}}$$

Result (type 3, 126 leaves):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{11}\,\left(-3+7x+7x^2\right)}{-57-19x^2+12\sqrt{2}\,\sqrt{5+x+x^2}+x\left(-19+24\sqrt{2}\,\sqrt{5+x+x^2}\right)}\right]}{\sqrt{22}}+\frac{\text{Log}[16]+\text{Log}\left[\left(3+x+x^2\right)^2\right]-\text{Log}\left[\left(3+x+x^2\right)\left(7+x+x^2+2\sqrt{2}\,\sqrt{5+x+x^2}\right)\right]}{2\sqrt{2}}$$

Problem 35: Result more than twice size of optimal antiderivative.

$$\int \frac{A+B x}{\sqrt{d+e x+f x^2}\,\left(a e+b e x+b f x^2\right)^2}\,dx$$

Optimal (type 3, 249 leaves, 6 steps):

$$-\frac{((A b-2 a B) e-b (B e-2 A f) x)\sqrt{d+e x+f x^2}}{e (b d-a e) (b e-4 a f)\left(a e+b e x+b f x^2\right)}+$$
$$\frac{(B e-2 A f)\left(8 a e f-b\left(e^2+4 d f\right)\right)\text{ArcTanh}\left[\frac{\sqrt{b d-a e}\,(e+2 f x)}{\sqrt{e}\,\sqrt{b e-4 a f}\,\sqrt{d+e x+f x^2}}\right]}{2 e^{3/2} (b d-a e)^{3/2} f (b e-4 a f)^{3/2}}+\frac{B\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\sqrt{d+e x+f x^2}}{\sqrt{b d-a e}}\right]}{2\sqrt{b}\,(b d-a e)^{3/2} f}$$

Result (type 3, 767 leaves):

$$-\frac{1}{4\,b\,e^{3/2}\,(b\,d-a\,e)^{3/2}\,f\,(b\,e-4\,a\,f)^{3/2}\,(a\,e+b\,x\,(e+f\,x))}$$
$$\Big(4\,b\,\sqrt{e}\,\sqrt{b\,d-a\,e}\,f\,\sqrt{b\,e-4\,a\,f}\,\sqrt{d+x\,(e+f\,x)}\,(-B\,e\,(2\,a+b\,x)+A\,b\,(e+2\,f\,x))-\Big(-b^{3/2}\,B\,e^{5/2}\,\sqrt{b\,e-4\,a\,f}+4\,a\,\sqrt{b}\,B\,e^{3/2}\,f\,\sqrt{b\,e-4\,a\,f}-$$
$$8\,a\,b\,e\,f\,(B\,e-2\,A\,f)+b^2\,(B\,e-2\,A\,f)\,\big(e^2+4\,d\,f\big)\Big)\,(a\,e+b\,x\,(e+f\,x))\,\mathrm{Log}\Big[-\sqrt{b}\,\sqrt{e}\,\sqrt{b\,e-4\,a\,f}+b\,(e+2\,f\,x)\Big]+$$
$$\Big(b^{3/2}\,B\,e^{5/2}\,\sqrt{b\,e-4\,a\,f}-4\,a\,\sqrt{b}\,B\,e^{3/2}\,f\,\sqrt{b\,e-4\,a\,f}-8\,a\,b\,e\,f\,(B\,e-2\,A\,f)+b^2\,(B\,e-2\,A\,f)\,\big(e^2+4\,d\,f\big)\Big)$$
$$(a\,e+b\,x\,(e+f\,x))\,\mathrm{Log}\Big[\sqrt{b}\,\sqrt{e}\,\sqrt{b\,e-4\,a\,f}+b\,(e+2\,f\,x)\Big]-$$
$$\Big(b^{3/2}\,B\,e^{5/2}\,\sqrt{b\,e-4\,a\,f}-4\,a\,\sqrt{b}\,B\,e^{3/2}\,f\,\sqrt{b\,e-4\,a\,f}-8\,a\,b\,e\,f\,(B\,e-2\,A\,f)+b^2\,(B\,e-2\,A\,f)\,\big(e^2+4\,d\,f\big)\Big)\,(a\,e+b\,x\,(e+f\,x))$$
$$\mathrm{Log}\Big[\sqrt{b}\,\Big(e^{3/2}\,\sqrt{b\,e-4\,a\,f}+\sqrt{b}\,\big(e^2-4\,d\,f\big)+2\,\sqrt{e}\,f\,\sqrt{b\,e-4\,a\,f}\,x-4\,\sqrt{b\,d-a\,e}\,f\,\sqrt{d+x\,(e+f\,x)}\Big)\Big]+$$
$$\Big(-b^{3/2}\,B\,e^{5/2}\,\sqrt{b\,e-4\,a\,f}+4\,a\,\sqrt{b}\,B\,e^{3/2}\,f\,\sqrt{b\,e-4\,a\,f}-8\,a\,b\,e\,f\,(B\,e-2\,A\,f)+b^2\,(B\,e-2\,A\,f)\,\big(e^2+4\,d\,f\big)\Big)\,(a\,e+b\,x\,(e+f\,x))$$
$$\mathrm{Log}\Big[\sqrt{b}\,\Big(e^{3/2}\,\sqrt{b\,e-4\,a\,f}-\sqrt{b}\,\big(e^2-4\,d\,f\big)+2\,\sqrt{e}\,f\,\sqrt{b\,e-4\,a\,f}\,x+4\,\sqrt{b\,d-a\,e}\,f\,\sqrt{d+x\,(e+f\,x)}\Big)\Big]\Big)$$

- **Problem 37: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{3+2\,x}{\sqrt{-3-4\,x-x^2}\,\big(3+4\,x+2\,x^2\big)}\,dx$$

Optimal (type 3, 17 leaves, 2 steps):

$$\mathrm{ArcTanh}\Big[\frac{x}{\sqrt{-3-4\,x-x^2}}\Big]$$

Result (type 3, 873 leaves):

$$\frac{1}{4}\Bigg(-\frac{1}{\sqrt{1-2\,i\,\sqrt{2}}}2\,i\,\left(-i+\sqrt{2}\right)\,\text{ArcTan}\Big[\Big((2+x)\,\Big(12-6\,i\,\sqrt{2}+\left(8+6\,i\,\sqrt{2}\right)\,x^3+9\,i\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}+$$

$$x^2\,\left(36+8\,i\,\sqrt{2}+6\,i\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+x\,\left(40-5\,i\,\sqrt{2}+12\,i\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\Big)\Big)\Big/$$

$$\left(21\,i+6\,\sqrt{2}+4\,\left(7\,i+8\,\sqrt{2}\right)\,x+\left(19\,i+58\,\sqrt{2}\right)\,x^2+8\,\left(2\,i+5\,\sqrt{2}\right)\,x^3+\left(6\,i+8\,\sqrt{2}\right)\,x^4\right)\Big]+\frac{1}{\sqrt{1+2\,i\,\sqrt{2}}}$$

$$2\,\left(i+\sqrt{2}\right)\,\text{ArcTanh}\Big[\Big((2+x)\,\Big(12\,i-6\,\sqrt{2}+\left(8\,i+6\,\sqrt{2}\right)\,x^3+9\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}+$$

$$x^2\,\left(36\,i+8\,\sqrt{2}+6\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+x\,\left(40\,i-5\,\sqrt{2}+12\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\Big)\Big)\Big/$$

$$\left(-21\,i+6\,\sqrt{2}+4\,\left(-7\,i+8\,\sqrt{2}\right)\,x+\left(-19\,i+58\,\sqrt{2}\right)\,x^2+8\,\left(-2\,i+5\,\sqrt{2}\right)\,x^3+\left(-6\,i+8\,\sqrt{2}\right)\,x^4\right)\Big]+$$

$$\frac{\left(-i+\sqrt{2}\right)\,\text{Log}\left[4\,\left(3+4\,x+2\,x^2\right)^2\right]}{\sqrt{1-2\,i\,\sqrt{2}}}+\frac{\left(i+\sqrt{2}\right)\,\text{Log}\left[4\,\left(3+4\,x+2\,x^2\right)^2\right]}{\sqrt{1+2\,i\,\sqrt{2}}}-$$

$$\frac{1}{\sqrt{1-2\,i\,\sqrt{2}}}$$

$$\left(-i+\sqrt{2}\right)\,\text{Log}\Big[\left(3+4\,x+2\,x^2\right)\,\Big(3+6\,i\,\sqrt{2}+\left(2+2\,i\,\sqrt{2}\right)\,x^2-2\,\sqrt{2-4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}+x\,\left(4+8\,i\,\sqrt{2}-2\,\sqrt{2-4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\Big)\Big]-$$

$$\frac{1}{\sqrt{1+2\,i\,\sqrt{2}}}$$

$$\left(i+\sqrt{2}\right)\,\text{Log}\Big[\left(3+4\,x+2\,x^2\right)\,\Big(3-6\,i\,\sqrt{2}+\left(2-2\,i\,\sqrt{2}\right)\,x^2-2\,\sqrt{2+4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}-2\,x\,\left(-2+4\,i\,\sqrt{2}+\sqrt{2+4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\Big)\Big]\Bigg)$$

- **Problem 38: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{3+4\,x}{\sqrt{-3-4\,x-x^2}\,\left(3+4\,x+2\,x^2\right)}\,dx$$

Optimal (type 3, 86 leaves, 13 steps):

$$\sqrt{2}\,\text{ArcTan}\left[\frac{1-\frac{3+x}{\sqrt{-3-4\,x-x^2}}}{\sqrt{2}}\right]-\sqrt{2}\,\text{ArcTan}\left[\frac{1+\frac{3+x}{\sqrt{-3-4\,x-x^2}}}{\sqrt{2}}\right]+\text{ArcTanh}\left[\frac{x}{\sqrt{-3-4\,x-x^2}}\right]$$

Result (type 3, 976 leaves):

$$\frac{1}{4}\left(2\sqrt{1-2\,i\,\sqrt{2}}\right.$$

$$\text{ArcTan}\Big[\Big(60+51\,i\,\sqrt{2}+\left(-16+6\,i\,\sqrt{2}\right)x^4+54\,i\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}+x\left(68+176\,i\,\sqrt{2}+99\,i\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+$$

$$2\,i\,x^3\left(34\left(i+\sqrt{2}\right)+9\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+i\,x^2\left(44\,i+185\sqrt{2}+72\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\Big)\Big/$$

$$\left(93\,i+150\sqrt{2}+20\left(17\,i+22\sqrt{2}\right)x+\left(493\,i+466\sqrt{2}\right)x^2+16\left(19\,i+13\sqrt{2}\right)x^3+\left(66\,i+32\sqrt{2}\right)x^4\right)\Big]-\frac{1}{\sqrt{1+2\,i\,\sqrt{2}}}2\,i\left(-i+2\sqrt{2}\right)$$

$$\text{ArcTan}\Big[\Big(-60+51\,i\,\sqrt{2}+2\left(8+3\,i\,\sqrt{2}\right)x^4+54\,i\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}+2\,x^3\left(34+34\,i\,\sqrt{2}+9\,i\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+$$

$$x^2\left(44+185\,i\,\sqrt{2}+72\,i\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+i\,x\left(68\,i+176\sqrt{2}+99\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\Big)\Big/$$

$$\left(-93\,i+150\sqrt{2}+20\left(-17\,i+22\sqrt{2}\right)x+\left(-493\,i+466\sqrt{2}\right)x^2+16\left(-19\,i+13\sqrt{2}\right)x^3+\left(-66\,i+32\sqrt{2}\right)x^4\right)\Big]+$$

$$\frac{\left(-i+2\sqrt{2}\right)\text{Log}\left[4\left(3+4\,x+2\,x^2\right)^2\right]}{\sqrt{1+2\,i\,\sqrt{2}}}+\frac{\left(i+2\sqrt{2}\right)\text{Log}\left[4\left(3+4\,x+2\,x^2\right)^2\right]}{\sqrt{1-2\,i\,\sqrt{2}}}-\frac{1}{\sqrt{1-2\,i\,\sqrt{2}}}$$

$$\left(i+2\sqrt{2}\right)\text{Log}\Big[\left(3+4\,x+2\,x^2\right)\left(3+6\,i\,\sqrt{2}+\left(2+2\,i\,\sqrt{2}\right)x^2-2\sqrt{2-4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}+x\left(4+8\,i\,\sqrt{2}-2\sqrt{2-4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\right)\Big]-$$

$$\frac{1}{\sqrt{1+2\,i\,\sqrt{2}}}\left(-i+2\sqrt{2}\right)$$

$$\left.\text{Log}\Big[\left(3+4\,x+2\,x^2\right)\left(3-6\,i\,\sqrt{2}+\left(2-2\,i\,\sqrt{2}\right)x^2-2\sqrt{2+4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}-2\,x\left(-2+4\,i\,\sqrt{2}+\sqrt{2+4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\right)\Big]\right)$$

- **Problem 59: Result more than twice size of optimal antiderivative.**

$$\int\frac{x\left(a+c\,x^2\right)^{3/2}}{d+e\,x+f\,x^2}\,dx$$

Optimal (type 3, 553 leaves, 10 steps):

$$\frac{\left(2\left(a f^2+c\left(e^2-d f\right)\right)-c e f x\right)\sqrt{a+c x^2}}{2 f^3}+\frac{\left(a+c x^2\right)^{3/2}}{3 f}-\frac{\sqrt{c}\, e\left(3 a f^2+2 c\left(e^2-2 d f\right)\right)\text{ArcTanh}\left[\frac{\sqrt{c}\, x}{\sqrt{a+c x^2}}\right]}{2 f^4}-$$

$$\left(\left(2 c d e f\left(2 a f^2+c\left(e^2-2 d f\right)\right)-\left(e-\sqrt{e^2-4 d f}\right)\left(a^2 f^4+2 a c f^2\left(e^2-d f\right)+c^2\left(e^4-3 d e^2 f+d^2 f^2\right)\right)\right)\right.$$

$$\left.\text{ArcTanh}\left[\frac{2 a f-c\left(e-\sqrt{e^2-4 d f}\right) x}{\sqrt{2}\sqrt{2 a f^2+c\left(e^2-2 d f-e\sqrt{e^2-4 d f}\right)}\sqrt{a+c x^2}}\right]\right)\Big/\left(\sqrt{2}\, f^4\sqrt{e^2-4 d f}\sqrt{2 a f^2+c\left(e^2-2 d f-e\sqrt{e^2-4 d f}\right)}\right)+$$

$$\left(\left(2 c d e f\left(2 a f^2+c\left(e^2-2 d f\right)\right)-\left(e+\sqrt{e^2-4 d f}\right)\left(a^2 f^4+2 a c f^2\left(e^2-d f\right)+c^2\left(e^4-3 d e^2 f+d^2 f^2\right)\right)\right)\right.$$

$$\left.\text{ArcTanh}\left[\frac{2 a f-c\left(e+\sqrt{e^2-4 d f}\right) x}{\sqrt{2}\sqrt{2 a f^2+c\left(e^2-2 d f+e\sqrt{e^2-4 d f}\right)}\sqrt{a+c x^2}}\right]\right)\Big/\left(\sqrt{2}\, f^4\sqrt{e^2-4 d f}\sqrt{2 a f^2+c\left(e^2-2 d f+e\sqrt{e^2-4 d f}\right)}\right)$$

Result (type 3, 1176 leaves) :

$$\frac{1}{6 f^4}\Bigg(f\sqrt{a+c x^2}\left(8 a f^2+c\left(6 e^2-3 e f x+2 f\left(-3 d+f x^2\right)\right)\right)+\Bigg(3\sqrt{2}\left(a^2 f^4\left(-e+\sqrt{e^2-4 d f}\right)-2 a c f^2\left(e^3-3 d e f-e^2\sqrt{e^2-4 d f}+d f\sqrt{e^2-4 d f}\right)+\right.$$

$$\left.c^2\left(-e^5+5 d e^3 f-5 d^2 e f^2+e^4\sqrt{e^2-4 d f}-3 d e^2 f\sqrt{e^2-4 d f}+d^2 f^2\sqrt{e^2-4 d f}\right)\right)\text{Log}\left[-e+\sqrt{e^2-4 d f}-2 f x\right]\Bigg)\Big/$$

$$\left(\sqrt{e^2-4 d f}\sqrt{2 a f^2+c\left(e^2-2 d f-e\sqrt{e^2-4 d f}\right)}\right)+\Bigg(3\sqrt{2}\left(a^2 f^4\left(e+\sqrt{e^2-4 d f}\right)+2 a c f^2\left(e^3-3 d e f+e^2\sqrt{e^2-4 d f}-d f\sqrt{e^2-4 d f}\right)+\right.$$

$$\left.c^2\left(e^5-5 d e^3 f+5 d^2 e f^2+e^4\sqrt{e^2-4 d f}-3 d e^2 f\sqrt{e^2-4 d f}+d^2 f^2\sqrt{e^2-4 d f}\right)\right)\text{Log}\left[e+\sqrt{e^2-4 d f}+2 f x\right]\Bigg)\Big/$$

$$\left(\sqrt{e^2-4 d f}\sqrt{2 a f^2+c\left(e^2-2 d f+e\sqrt{e^2-4 d f}\right)}\right)-3\sqrt{c}\,e\left(3 a f^2+2 c\left(e^2-2 d f\right)\right)\text{Log}\left[c x+\sqrt{c}\sqrt{a+c x^2}\right]-$$

$$\frac{1}{\sqrt{e^2-4 d f}\sqrt{2 a f^2+c\left(e^2-2 d f-e\sqrt{e^2-4 d f}\right)}}3\sqrt{2}\left(a^2 f^4\left(-e+\sqrt{e^2-4 d f}\right)-2 a c f^2\left(e^3-3 d e f-e^2\sqrt{e^2-4 d f}+d f\sqrt{e^2-4 d f}\right)+\right.$$

$$\left.c^2\left(-e^5+5 d e^3 f-5 d^2 e f^2+e^4\sqrt{e^2-4 d f}-3 d e^2 f\sqrt{e^2-4 d f}+d^2 f^2\sqrt{e^2-4 d f}\right)\right)$$

$$\text{Log}\left[2 a f\sqrt{e^2-4 d f}+c\left(e^2-4 d f-e\sqrt{e^2-4 d f}\right)x+\sqrt{2}\sqrt{e^2-4 d f}\sqrt{2 a f^2+c\left(e^2-2 d f-e\sqrt{e^2-4 d f}\right)}\sqrt{a+c x^2}\right]-$$

$$\frac{1}{\sqrt{e^2-4 d f}\sqrt{2 a f^2+c\left(e^2-2 d f+e\sqrt{e^2-4 d f}\right)}}3\sqrt{2}\left(a^2 f^4\left(e+\sqrt{e^2-4 d f}\right)+2 a c f^2\left(e^3-3 d e f+e^2\sqrt{e^2-4 d f}-d f\sqrt{e^2-4 d f}\right)+\right.$$

$$\left.c^2\left(e^5-5 d e^3 f+5 d^2 e f^2+e^4\sqrt{e^2-4 d f}-3 d e^2 f\sqrt{e^2-4 d f}+d^2 f^2\sqrt{e^2-4 d f}\right)\right)$$

$$\text{Log}\left[2 a f\sqrt{e^2-4 d f}-c\left(e^2-4 d f+e\sqrt{e^2-4 d f}\right)x+\sqrt{2}\sqrt{e^2-4 d f}\sqrt{2 a f^2+c\left(e^2-2 d f+e\sqrt{e^2-4 d f}\right)}\sqrt{a+c x^2}\right]\Bigg)$$

- **Problem 93: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(x+x^2\right)^{3/2}}{1+x^2}\,dx$$

Optimal (type 3, 130 leaves, 10 steps):

$$\frac{1}{4}(5+2x)\sqrt{x+x^2}+\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left[\frac{1+\sqrt{2}-x}{\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+x^2}}\right]-\sqrt{-1+\sqrt{2}}\,\text{ArcTanh}\left[\frac{1-\sqrt{2}-x}{\sqrt{2\left(-1+\sqrt{2}\right)}\sqrt{x+x^2}}\right]-\frac{5}{4}\text{ArcTanh}\left[\frac{x}{\sqrt{x+x^2}}\right]$$

Result (type 3, 124 leaves):

$$\frac{1}{4\sqrt{x(1+x)}}\sqrt{x}\sqrt{1+x}\left(5\sqrt{x}\sqrt{1+x}+2x^{3/2}\sqrt{1+x}-5\,\text{ArcSinh}\left[\sqrt{x}\right]-4\sqrt{2-2\,i}\,\text{ArcTan}\left[(1-i)^{3/2}\sqrt{\frac{x}{2+2x}}\right]-4\sqrt{2+2\,i}\,\text{ArcTan}\left[(1+i)^{3/2}\sqrt{\frac{x}{2+2x}}\right]\right)$$

- **Problem 110: Result more than twice size of optimal antiderivative.**

$$\int\frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

Optimal (type 3, 549 leaves, 9 steps):

$$\frac{\sqrt{a+b\,x+c\,x^2}}{f}-\frac{(2\,c\,e-b\,f)\,\text{ArcTanh}\left[\frac{b+2\,c\,x}{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}\right]}{2\,\sqrt{c}\,f^2}-\left(\left(2\,d\,f\,(c\,e-b\,f)+\left(e-\sqrt{e^2-4\,d\,f}\right)\,\left(f\,(b\,e-a\,f)-c\,\left(e^2-d\,f\right)\right)\right)\right.$$

$$\left.\text{ArcTanh}\left[\frac{4\,a\,f-b\,\left(e-\sqrt{e^2-4\,d\,f}\right)+2\,\left(b\,f-c\,\left(e-\sqrt{e^2-4\,d\,f}\right)\right)\,x}{2\,\sqrt{2}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2-(c\,e-b\,f)\,\sqrt{e^2-4\,d\,f}}\,\sqrt{a+b\,x+c\,x^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,f^2\,\sqrt{e^2-4\,d\,f}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2-(c\,e-b\,f)\,\sqrt{e^2-4\,d\,f}}\right)+\left(\left(2\,d\,f\,(c\,e-b\,f)+\left(e+\sqrt{e^2-4\,d\,f}\right)\,\left(f\,(b\,e-a\,f)-c\,\left(e^2-d\,f\right)\right)\right)\right.$$

$$\left.\text{ArcTanh}\left[\frac{4\,a\,f-b\,\left(e+\sqrt{e^2-4\,d\,f}\right)+2\,\left(b\,f-c\,\left(e+\sqrt{e^2-4\,d\,f}\right)\right)\,x}{2\,\sqrt{2}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2+(c\,e-b\,f)\,\sqrt{e^2-4\,d\,f}}\,\sqrt{a+b\,x+c\,x^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,f^2\,\sqrt{e^2-4\,d\,f}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2+(c\,e-b\,f)\,\sqrt{e^2-4\,d\,f}}\right)$$

Result (type 3, 1112 leaves) :

$$\frac{1}{2 f^2} \Bigg(2 f \sqrt{a+x (b+c x)} + \Bigg(\sqrt{2} \Big(c \Big(-e^3+3 d e f+e^2 \sqrt{e^2-4 d f} - d f \sqrt{e^2-4 d f}\Big) + f \Big(a f \Big(-e+\sqrt{e^2-4 d f}\Big) + b \Big(e^2-2 d f-e \sqrt{e^2-4 d f}\Big)\Big)\Big)$$

$$\text{Log}\Big[-e+\sqrt{e^2-4 d f} - 2 f x\Big]\Bigg) \Bigg/ \Bigg(\sqrt{e^2-4 d f} \sqrt{c \Big(e^2-2 d f-e \sqrt{e^2-4 d f}\Big) + f \Big(2 a f+b \Big(-e+\sqrt{e^2-4 d f}\Big)\Big)}\Bigg) +$$

$$\Bigg(\sqrt{2} \Big(c \Big(e^3-3 d e f+e^2 \sqrt{e^2-4 d f} - d f \sqrt{e^2-4 d f}\Big) + f \Big(a f \Big(e+\sqrt{e^2-4 d f}\Big) - b \Big(e^2-2 d f+e \sqrt{e^2-4 d f}\Big)\Big)\Big) \text{Log}\Big[e+\sqrt{e^2-4 d f} + 2 f x\Big]\Bigg) \Bigg/$$

$$\Bigg(\sqrt{e^2-4 d f} \sqrt{c \Big(e^2-2 d f+e \sqrt{e^2-4 d f}\Big) + f \Big(2 a f-b \Big(e+\sqrt{e^2-4 d f}\Big)\Big)}\Bigg) - \frac{(2 c e-b f) \text{Log}\Big[b+2 c x+2 \sqrt{c} \sqrt{a+x (b+c x)}\Big]}{\sqrt{c}} -$$

$$\Bigg(\sqrt{2} \Big(c \Big(-e^3+3 d e f+e^2 \sqrt{e^2-4 d f} - d f \sqrt{e^2-4 d f}\Big) + f \Big(a f \Big(-e+\sqrt{e^2-4 d f}\Big) + b \Big(e^2-2 d f-e \sqrt{e^2-4 d f}\Big)\Big)\Big)$$

$$\text{Log}\Big[4 a f \sqrt{e^2-4 d f} + 2 c e^2 x-8 c d f x-2 c e \sqrt{e^2-4 d f}\, x+b \Big(e^2-4 d f-e \sqrt{e^2-4 d f} + 2 f \sqrt{e^2-4 d f}\, x\Big) +$$

$$2 \sqrt{2} \sqrt{e^2-4 d f} \sqrt{f \Big(-b e+2 a f+b \sqrt{e^2-4 d f}\Big) + c \Big(e^2-2 d f-e \sqrt{e^2-4 d f}\Big)} \sqrt{a+x (b+c x)}\Big]\Bigg) \Bigg/$$

$$\Bigg(\sqrt{e^2-4 d f} \sqrt{c \Big(e^2-2 d f-e \sqrt{e^2-4 d f}\Big) + f \Big(2 a f+b \Big(-e+\sqrt{e^2-4 d f}\Big)\Big)}\Bigg) -$$

$$\Bigg(\sqrt{2} \Big(c \Big(e^3-3 d e f+e^2 \sqrt{e^2-4 d f} - d f \sqrt{e^2-4 d f}\Big) + f \Big(a f \Big(e+\sqrt{e^2-4 d f}\Big) - b \Big(e^2-2 d f+e \sqrt{e^2-4 d f}\Big)\Big)\Big) \text{Log}\Big[4 a f \sqrt{e^2-4 d f} -$$

$$2 c e^2 x+8 c d f x-2 c e \sqrt{e^2-4 d f}\, x+2 \sqrt{2} \sqrt{e^2-4 d f} \sqrt{c \Big(e^2-2 d f+e \sqrt{e^2-4 d f}\Big) + f \Big(2 a f-b \Big(e+\sqrt{e^2-4 d f}\Big)\Big)} \sqrt{a+x (b+c x)} -$$

$$b \Big(e^2+e \sqrt{e^2-4 d f} - 2 f \Big(2 d+\sqrt{e^2-4 d f}\, x\Big)\Big)\Big]\Bigg) \Bigg/ \Bigg(\sqrt{e^2-4 d f} \sqrt{c \Big(e^2-2 d f+e \sqrt{e^2-4 d f}\Big) + f \Big(2 a f-b \Big(e+\sqrt{e^2-4 d f}\Big)\Big)}\Bigg)\Bigg)$$

- **Problem 118: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{x \sqrt{a+b x+c x^2} \left(d+e x+f x^2\right)} \, dx$$

Optimal (type 3, 451 leaves, 9 steps) :

$$-\frac{\text{ArcTanh}\left[\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right]}{\sqrt{a}\,d}+\frac{f\left(e+\sqrt{e^2-4df}\right)\text{ArcTanh}\left[\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right]}{\sqrt{2}\,d\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}}-$$

$$\frac{f\left(e-\sqrt{e^2-4df}\right)\text{ArcTanh}\left[\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right]}{\sqrt{2}\,d\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

Result (type 3, 994 leaves) :

$$\frac{\sqrt{a+b\,x+c\,x^2}\;\text{Log}[x]}{\sqrt{a}\;d\,\sqrt{a+x\,(b+c\,x)}}-\frac{f\left(e+\sqrt{e^2-4\,d\,f}\right)\sqrt{a+b\,x+c\,x^2}\;\text{Log}\left[-e+\sqrt{e^2-4\,d\,f}-2\,f\,x\right]}{\sqrt{2}\;d\,\sqrt{e^2-4\,d\,f}\;\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2-c\,e\,\sqrt{e^2-4\,d\,f}+b\,f\,\sqrt{e^2-4\,d\,f}}\;\sqrt{a+x\,(b+c\,x)}}-$$

$$\frac{f\left(-e+\sqrt{e^2-4\,d\,f}\right)\sqrt{a+b\,x+c\,x^2}\;\text{Log}\left[e+\sqrt{e^2-4\,d\,f}+2\,f\,x\right]}{\sqrt{2}\;d\,\sqrt{e^2-4\,d\,f}\;\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2+c\,e\,\sqrt{e^2-4\,d\,f}-b\,f\,\sqrt{e^2-4\,d\,f}}\;\sqrt{a+x\,(b+c\,x)}}-$$

$$\frac{\sqrt{a+b\,x+c\,x^2}\;\text{Log}\left[2\,a+b\,x+2\,\sqrt{a}\;\sqrt{a+b\,x+c\,x^2}\right]}{\sqrt{a}\;d\,\sqrt{a+x\,(b+c\,x)}}+$$

$$\Big(f\left(-e+\sqrt{e^2-4\,d\,f}\right)\sqrt{a+b\,x+c\,x^2}\;\text{Log}\Big[-b\,e^2+4\,b\,d\,f-b\,e\,\sqrt{e^2-4\,d\,f}+4\,a\,f\,\sqrt{e^2-4\,d\,f}-2\,c\,e^2\,x+8\,c\,d\,f\,x-2\,c\,e\,\sqrt{e^2-4\,d\,f}\;x+$$
$$2\,b\,f\,\sqrt{e^2-4\,d\,f}\;x+2\,\sqrt{2}\;\sqrt{e^2-4\,d\,f}\;\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2+c\,e\,\sqrt{e^2-4\,d\,f}-b\,f\,\sqrt{e^2-4\,d\,f}}\;\sqrt{a+b\,x+c\,x^2}\Big]\Big)\Big/$$
$$\left(\sqrt{2}\;d\,\sqrt{e^2-4\,d\,f}\;\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2+c\,e\,\sqrt{e^2-4\,d\,f}-b\,f\,\sqrt{e^2-4\,d\,f}}\;\sqrt{a+x\,(b+c\,x)}\right)+$$

$$\Big(f\left(e+\sqrt{e^2-4\,d\,f}\right)\sqrt{a+b\,x+c\,x^2}\;\text{Log}\Big[b\,e^2-4\,b\,d\,f-b\,e\,\sqrt{e^2-4\,d\,f}+4\,a\,f\,\sqrt{e^2-4\,d\,f}+2\,c\,e^2\,x-8\,c\,d\,f\,x-2\,c\,e\,\sqrt{e^2-4\,d\,f}\;x+$$
$$2\,b\,f\,\sqrt{e^2-4\,d\,f}\;x+2\,\sqrt{2}\;\sqrt{e^2-4\,d\,f}\;\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2-c\,e\,\sqrt{e^2-4\,d\,f}+b\,f\,\sqrt{e^2-4\,d\,f}}\;\sqrt{a+b\,x+c\,x^2}\Big]\Big)\Big/$$
$$\left(\sqrt{2}\;d\,\sqrt{e^2-4\,d\,f}\;\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2-c\,e\,\sqrt{e^2-4\,d\,f}+b\,f\,\sqrt{e^2-4\,d\,f}}\;\sqrt{a+x\,(b+c\,x)}\right)$$

- **Problem 126: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{x^4}{\sqrt{-3-4\,x-x^2}\;\left(3+4\,x+2\,x^2\right)}\,dx$$

Optimal (type 3, 140 leaves, 24 steps):

$$\frac{5}{2}\sqrt{-3-4x-x^2}-\frac{1}{4}x\sqrt{-3-4x-x^2}+\frac{11}{2}\text{ArcSin}[2+x]+\frac{\text{ArcTan}\left[\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]}{2\sqrt{2}}-\frac{\text{ArcTan}\left[\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]}{2\sqrt{2}}-\frac{5}{4}\text{ArcTanh}\left[\frac{x}{\sqrt{-3-4x-x^2}}\right]$$

Result (type 3, 1000 leaves) :

$$\frac{1}{16}\left(-4\,(-10+x)\sqrt{-3-4x-x^2}+88\,\text{ArcSin}[2+x]+\frac{1}{\sqrt{1-2\,i\sqrt{2}}}2\left(7+4\,i\sqrt{2}\right)\right.$$

$$\text{ArcTan}\left[\left(132-471\,i\sqrt{2}+\left(224+78\,i\sqrt{2}\right)x^4+486\,i\sqrt{1-2\,i\sqrt{2}}\sqrt{-3-4x-x^2}+2x^3\left(638+10\,i\sqrt{2}+81\,i\sqrt{1-2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+\right.\right.$$

$$\left.x^2\left(2236-727\,i\sqrt{2}+648\,i\sqrt{1-2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+x\left(1316-1168\,i\sqrt{2}+891\,i\sqrt{1-2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\Big/$$

$$\left.\left(885\,i+6\sqrt{2}+4\left(349\,i+26\sqrt{2}\right)x+\left(685\,i+514\sqrt{2}\right)x^2+16\left(13\,i+34\sqrt{2}\right)x^3+2\left(33\,i+64\sqrt{2}\right)x^4\right)\right]-\frac{1}{\sqrt{1+2\,i\sqrt{2}}}2\left(7\,i+4\sqrt{2}\right)$$

$$\text{ArcTanh}\left[\left(132\,i-471\sqrt{2}+\left(224\,i+78\sqrt{2}\right)x^4+486\sqrt{1+2\,i\sqrt{2}}\sqrt{-3-4x-x^2}+2x^3\left(638\,i+10\sqrt{2}+81\sqrt{1+2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+\right.\right.$$

$$\left.x^2\left(2236\,i-727\sqrt{2}+648\sqrt{1+2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+x\left(1316\,i-1168\sqrt{2}+891\sqrt{1+2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\Big/$$

$$\left.\left(-885\,i+6\sqrt{2}+4\left(-349\,i+26\sqrt{2}\right)x+\left(-685\,i+514\sqrt{2}\right)x^2+16\left(-13\,i+34\sqrt{2}\right)x^3+2\left(-33\,i+64\sqrt{2}\right)x^4\right)\right]-$$

$$\frac{\left(-7\,i+4\sqrt{2}\right)\text{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1-2\,i\sqrt{2}}}-\frac{\left(7\,i+4\sqrt{2}\right)\text{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1+2\,i\sqrt{2}}}+\frac{1}{\sqrt{1-2\,i\sqrt{2}}}\left(-7\,i+4\sqrt{2}\right)$$

$$\text{Log}\left[\left(3+4x+2x^2\right)\left(3+6\,i\sqrt{2}+\left(2+2\,i\sqrt{2}\right)x^2-2\sqrt{2-4\,i\sqrt{2}}\sqrt{-3-4x-x^2}+x\left(4+8\,i\sqrt{2}-2\sqrt{2-4\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\right]+$$

$$\frac{1}{\sqrt{1+2\,i\sqrt{2}}}\left(7\,i+4\sqrt{2}\right)$$

$$\left.\text{Log}\left[\left(3+4x+2x^2\right)\left(3-6\,i\sqrt{2}+\left(2-2\,i\sqrt{2}\right)x^2-2\sqrt{2+4\,i\sqrt{2}}\sqrt{-3-4x-x^2}-2x\left(-2+4\,i\sqrt{2}+\sqrt{2+4\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\right]\right)$$

- **Problem 127: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{x^3}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$$

Optimal (type 3, 115 leaves, 20 steps) :

$$-\frac{1}{2}\sqrt{-3-4x-x^2}-2\,\text{ArcSin}[2+x]+\frac{\text{ArcTan}\left[\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]}{2\sqrt{2}}-\frac{\text{ArcTan}\left[\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]}{2\sqrt{2}}+\text{ArcTanh}\left[\frac{x}{\sqrt{-3-4x-x^2}}\right]$$

Result (type 3, 1001 leaves) :

$$\frac{1}{16}\left(-8\sqrt{-3-4x-x^2}-32\,\text{ArcSin}[2+x]-\frac{1}{\sqrt{1-2\,i\sqrt{2}}}2\,i\left(-2\,i+5\sqrt{2}\right)\right.$$
$$\text{ArcTan}\left[\left(\left(40+66\,i\sqrt{2}\right)x^4+6\left(56-i\sqrt{2}+27\,i\sqrt{1-2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+x^3\left(332+316\,i\sqrt{2}+54\,i\sqrt{1-2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+\right.\right.$$
$$\left.x^2\left(920+469\,i\sqrt{2}+216\,i\sqrt{1-2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+x\left(964+208\,i\sqrt{2}+297\,i\sqrt{1-2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\Big/$$
$$\left.\left(132\,i+192\sqrt{2}+4\left(71\,i+184\sqrt{2}\right)x+\left(455\,i+1004\sqrt{2}\right)x^2+56\left(7\,i+10\sqrt{2}\right)x^3+2\left(57\,i+50\sqrt{2}\right)x^4\right)\right]+\frac{1}{\sqrt{1+2\,i\sqrt{2}}}2\left(2\,i+5\sqrt{2}\right)$$
$$\text{ArcTanh}\left[\left(\left(40\,i+66\sqrt{2}\right)x^4-6\left(-56\,i+\sqrt{2}-27\sqrt{1+2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+x^3\left(332\,i+316\sqrt{2}+54\sqrt{1+2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+\right.\right.$$
$$\left.x^2\left(920\,i+469\sqrt{2}+216\sqrt{1+2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+x\left(964\,i+208\sqrt{2}+297\sqrt{1+2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\Big/$$
$$\left.\left(-132\,i+192\sqrt{2}+4\left(-71\,i+184\sqrt{2}\right)x+\left(-455\,i+1004\sqrt{2}\right)x^2+56\left(-7\,i+10\sqrt{2}\right)x^3+2\left(-57\,i+50\sqrt{2}\right)x^4\right)\right]+$$
$$\frac{\left(-2\,i+5\sqrt{2}\right)\text{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1-2\,i\sqrt{2}}}+\frac{\left(2\,i+5\sqrt{2}\right)\text{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1+2\,i\sqrt{2}}}-\frac{1}{\sqrt{1-2\,i\sqrt{2}}}\left(-2\,i+5\sqrt{2}\right)$$
$$\text{Log}\left[\left(3+4x+2x^2\right)\left(3+6\,i\sqrt{2}+\left(2+2\,i\sqrt{2}\right)x^2-2\sqrt{2-4\,i\sqrt{2}}\sqrt{-3-4x-x^2}+x\left(4+8\,i\sqrt{2}-2\sqrt{2-4\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\right]-$$
$$\frac{1}{\sqrt{1+2\,i\sqrt{2}}}\left(2\,i+5\sqrt{2}\right)$$
$$\left.\text{Log}\left[\left(3+4x+2x^2\right)\left(3-6\,i\sqrt{2}+\left(2-2\,i\sqrt{2}\right)x^2-2\sqrt{2+4\,i\sqrt{2}}\sqrt{-3-4x-x^2}-2x\left(-2+4\,i\sqrt{2}+\sqrt{2+4\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\right]\right)$$

- **Problem 128: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{x^2}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$$

Optimal (type 3, 98 leaves, 16 steps) :

$$\frac{1}{2}\,\text{ArcSin}[2+x]-\frac{\text{ArcTan}\left[\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]}{\sqrt{2}}+\frac{\text{ArcTan}\left[\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]}{\sqrt{2}}-\frac{1}{2}\,\text{ArcTanh}\left[\frac{x}{\sqrt{-3-4x-x^2}}\right]$$

Result (type 3, 982 leaves) :

$$\frac{1}{8}\left(4\,\text{ArcSin}[2+x]+\frac{1}{\sqrt{1-2\,i\sqrt{2}}}2\,i\left(i+2\sqrt{2}\right)\right.$$

$$\text{ArcTan}\Big[\Big(60+51\,i\sqrt{2}+\left(-16+6\,i\sqrt{2}\right)x^4+54\,i\sqrt{1-2\,i\sqrt{2}}\,\sqrt{-3-4x-x^2}+x\left(68+176\,i\sqrt{2}+99\,i\sqrt{1-2\,i\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)+$$

$$2\,i\,x^3\left(34\left(i+\sqrt{2}\right)+9\sqrt{1-2\,i\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)+i\,x^2\left(44\,i+185\sqrt{2}+72\sqrt{1-2\,i\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)\Big)\Big/$$

$$\left(93\,i+150\sqrt{2}+20\left(17\,i+22\sqrt{2}\right)x+\left(493\,i+466\sqrt{2}\right)x^2+16\left(19\,i+13\sqrt{2}\right)x^3+\left(66\,i+32\sqrt{2}\right)x^4\right)\Big]+2\sqrt{1+2\,i\sqrt{2}}$$

$$\text{ArcTan}\Big[\Big(-60+51\,i\sqrt{2}+2\left(8+3\,i\sqrt{2}\right)x^4+54\,i\sqrt{1+2\,i\sqrt{2}}\,\sqrt{-3-4x-x^2}+2x^3\left(34+34\,i\sqrt{2}+9\,i\sqrt{1+2\,i\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)+$$

$$x^2\left(44+185\,i\sqrt{2}+72\,i\sqrt{1+2\,i\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)+i\,x\left(68\,i+176\sqrt{2}+99\sqrt{1+2\,i\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)\Big)\Big/$$

$$\left(-93\,i+150\sqrt{2}+20\left(-17\,i+22\sqrt{2}\right)x+\left(-493\,i+466\sqrt{2}\right)x^2+16\left(-19\,i+13\sqrt{2}\right)x^3+\left(-66\,i+32\sqrt{2}\right)x^4\right)\Big]-$$

$$\frac{\left(-i+2\sqrt{2}\right)\text{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1+2\,i\sqrt{2}}}-\frac{\left(i+2\sqrt{2}\right)\text{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1-2\,i\sqrt{2}}}+\frac{1}{\sqrt{1-2\,i\sqrt{2}}}$$

$$\left(i+2\sqrt{2}\right)\text{Log}\left[\left(3+4x+2x^2\right)\left(3+6\,i\sqrt{2}+\left(2+2\,i\sqrt{2}\right)x^2-2\sqrt{2-4\,i\sqrt{2}}\,\sqrt{-3-4x-x^2}+x\left(4+8\,i\sqrt{2}-2\sqrt{2-4\,i\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)\right)\right]+$$

$$\frac{1}{\sqrt{1+2\,i\sqrt{2}}}\left(-i+2\sqrt{2}\right)$$

$$\left.\text{Log}\left[\left(3+4x+2x^2\right)\left(3-6\,i\sqrt{2}+\left(2-2\,i\sqrt{2}\right)x^2-2\sqrt{2+4\,i\sqrt{2}}\,\sqrt{-3-4x-x^2}-2x\left(-2+4\,i\sqrt{2}+\sqrt{2+4\,i\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)\right)\right]\right)$$

- **Problem 129: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{x}{\sqrt{-3-4x-x^2}\,\left(3+4x+2x^2\right)}\,dx$$

Optimal (type 3, 68 leaves, 6 steps) :

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right]}{\sqrt{2}}+\frac{\text{ArcTan}\left[\frac{1+\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right]}{\sqrt{2}}$$

Result (type 3, 814 leaves) :

$$\frac{1}{8}\left(\frac{1}{\sqrt{1+2\,i\,\sqrt{2}}}2\left(2+i\,\sqrt{2}\right)\right.$$
$$\text{ArcTan}\left[\left((2+x)\left(3\left(5+4\,i\,\sqrt{2}\right)+16\left(2+i\,\sqrt{2}\right)x+2\left(9+2\,i\,\sqrt{2}\right)x^2\right)\right)\Big/\left(12\,i-6\,\sqrt{2}+\left(8\,i+6\,\sqrt{2}\right)x^3-9\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}+\right.\right.$$
$$\left.\left.x\left(40\,i-5\,\sqrt{2}-12\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+x^2\left(36\,i+8\,\sqrt{2}-6\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\right)\right]-\frac{1}{\sqrt{1-2\,i\,\sqrt{2}}}$$
$$2\left(2\,i+\sqrt{2}\right)\text{ArcTanh}\left[\left((2+x)\left(3\left(5\,i+4\,\sqrt{2}\right)+16\left(2\,i+\sqrt{2}\right)x+2\left(9\,i+2\,\sqrt{2}\right)x^2\right)\right)\Big/\left(-5\left(8\,i+\sqrt{2}\right)x+\left(-8\,i+6\,\sqrt{2}\right)x^3-\right.\right.$$
$$\left.\left.12\,\sqrt{1-2\,i\,\sqrt{2}}\,x\,\sqrt{-3-4\,x-x^2}+x^2\left(-36\,i+8\,\sqrt{2}-6\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)-3\left(4\,i+2\,\sqrt{2}+3\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\right)\right]+$$
$$\frac{\left(-2\,i+\sqrt{2}\right)\text{Log}\left[4\left(3+4\,x+2\,x^2\right)^2\right]}{\sqrt{1+2\,i\,\sqrt{2}}}+\frac{\left(2\,i+\sqrt{2}\right)\text{Log}\left[4\left(3+4\,x+2\,x^2\right)^2\right]}{\sqrt{1-2\,i\,\sqrt{2}}}-\frac{1}{\sqrt{1-2\,i\,\sqrt{2}}}$$
$$\left(2\,i+\sqrt{2}\right)\text{Log}\left[\left(3+4\,x+2\,x^2\right)\left(3+6\,i\,\sqrt{2}+\left(2+2\,i\,\sqrt{2}\right)x^2-2\,\sqrt{2-4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}+x\left(4+8\,i\,\sqrt{2}-2\,\sqrt{2-4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\right)\right]-$$
$$\frac{1}{\sqrt{1+2\,i\,\sqrt{2}}}\left(-2\,i+\sqrt{2}\right)$$
$$\left.\text{Log}\left[\left(3+4\,x+2\,x^2\right)\left(3-6\,i\,\sqrt{2}+\left(2-2\,i\,\sqrt{2}\right)x^2-2\,\sqrt{2+4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}-2\,x\left(-2+4\,i\,\sqrt{2}+\sqrt{2+4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\right)\right]\right)$$

- **Problem 130: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{1}{\sqrt{-3-4\,x-x^2}\,\left(3+4\,x+2\,x^2\right)}\,dx$$

Optimal (type 3, 95 leaves, 10 steps) :

$$-\frac{1}{3}\sqrt{2}\,\text{ArcTan}\left[\frac{1-\frac{3+x}{\sqrt{-3-4\,x-x^2}}}{\sqrt{2}}\right]+\frac{1}{3}\sqrt{2}\,\text{ArcTan}\left[\frac{1+\frac{3+x}{\sqrt{-3-4\,x-x^2}}}{\sqrt{2}}\right]+\frac{1}{3}\text{ArcTanh}\left[\frac{x}{\sqrt{-3-4\,x-x^2}}\right]$$

Result (type 3, 800 leaves) :

$$\frac{1}{12}\left(-2\sqrt{1-2\,i\,\sqrt{2}}\ \text{ArcTan}\Big[\left(\left(3+4\,x+x^2\right)\left(7+2\,i\,\sqrt{2}+8\,x+2\,x^2\right)\right)\Big/\left(2\,\sqrt{2}\,x^4+x\left(28\,i+16\,\sqrt{2}-11\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+x^2\left(20\,i+23\,\sqrt{2}-8\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+3\left(4\,i+\sqrt{2}-2\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+2\,x^3\left(2\,i+6\,\sqrt{2}-\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\right)\Big]+2\,i\,\sqrt{1+2\,i\,\sqrt{2}}\ \text{ArcTanh}\Big[\left(\left(7\,i+2\,\sqrt{2}+8\,i\,x+2\,i\,x^2\right)\left(3+4\,x+x^2\right)\right)\Big/\left(2\,\sqrt{2}\,x^4+x\left(-28\,i+16\,\sqrt{2}-11\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+x^2\left(-20\,i+23\,\sqrt{2}-8\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+3\left(-4\,i+\sqrt{2}-2\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+2\,x^3\left(-2\,i+6\,\sqrt{2}-\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\right)\Big]+i\left(\left(\sqrt{1-2\,i\,\sqrt{2}}-\sqrt{1+2\,i\,\sqrt{2}}\right)\text{Log}\left[4\left(3+4\,x+2\,x^2\right)^2\right]+\sqrt{1+2\,i\,\sqrt{2}}\ \text{Log}\Big[\left(3+4\,x+2\,x^2\right)\left(3+6\,i\,\sqrt{2}+\left(2+2\,i\,\sqrt{2}\right)x^2-2\,\sqrt{2-4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}+x\left(4+8\,i\,\sqrt{2}-2\,\sqrt{2-4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\right)\Big]-\sqrt{1-2\,i\,\sqrt{2}}\ \text{Log}\Big[\left(3+4\,x+2\,x^2\right)\left(3-6\,i\,\sqrt{2}+\left(2-2\,i\,\sqrt{2}\right)x^2-2\,\sqrt{2+4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}-2\,x\left(-2+4\,i\,\sqrt{2}+\sqrt{2+4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\right)\Big]\right)\right)$$

- **Problem 131: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x\,\sqrt{-3-4\,x-x^2}\,\left(3+4\,x+2\,x^2\right)}\,dx$$

Optimal (type 3, 130 leaves, 17 steps) :

$$-\frac{\text{ArcTan}\left[\frac{3+2\,x}{\sqrt{3}\,\sqrt{-3-4\,x-x^2}}\right]}{3\,\sqrt{3}}+\frac{1}{9}\,\sqrt{2}\ \text{ArcTan}\left[\frac{1-\frac{3+x}{\sqrt{-3-4\,x-x^2}}}{\sqrt{2}}\right]-\frac{1}{9}\,\sqrt{2}\ \text{ArcTan}\left[\frac{1+\frac{3+x}{\sqrt{-3-4\,x-x^2}}}{\sqrt{2}}\right]-\frac{4}{9}\,\text{ArcTanh}\left[\frac{x}{\sqrt{-3-4\,x-x^2}}\right]$$

Result (type 3, 959 leaves) :

$$\frac{1}{36}\left(-4\sqrt{3}\,\text{ArcTan}\left[\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right]+\frac{1}{\sqrt{1-2i\sqrt{2}}}\right.$$

$$6\left(2+i\sqrt{2}\right)\text{ArcTan}\left[\left(\left(8+2i\sqrt{2}\right)x^4-18i\left(\sqrt{2}-\sqrt{1-2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+x^3\left(44-4i\sqrt{2}+6i\sqrt{1-2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+\right.\right.$$

$$\left.x^2\left(72-35i\sqrt{2}+24i\sqrt{1-2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+x\left(36-48i\sqrt{2}+33i\sqrt{1-2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\Big/$$

$$\left.\left(36i+60ix+\left(31i+12\sqrt{2}\right)x^2+8\left(i+2\sqrt{2}\right)x^3+\left(2i+4\sqrt{2}\right)x^4\right)\right]-\frac{1}{\sqrt{1+2i\sqrt{2}}}$$

$$6\left(2i+\sqrt{2}\right)\text{ArcTanh}\left[\left(2\left(4i+\sqrt{2}\right)x^4-18\left(\sqrt{2}-\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+x^3\left(44i-4\sqrt{2}+6\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+\right.\right.$$

$$\left.x^2\left(72i-35\sqrt{2}+24\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+x\left(36i-48\sqrt{2}+33\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\Big/$$

$$\left.\left(-36i-60ix+\left(-31i+12\sqrt{2}\right)x^2+8\left(-i+2\sqrt{2}\right)x^3+\left(-2i+4\sqrt{2}\right)x^4\right)\right]-$$

$$\frac{3\left(-2i+\sqrt{2}\right)\text{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1-2i\sqrt{2}}}-\frac{3\left(2i+\sqrt{2}\right)\text{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1+2i\sqrt{2}}}+\frac{1}{\sqrt{1-2i\sqrt{2}}}3\left(-2i+\sqrt{2}\right)$$

$$\text{Log}\left[\left(3+4x+2x^2\right)\left(3+6i\sqrt{2}+\left(2+2i\sqrt{2}\right)x^2-2\sqrt{2-4i\sqrt{2}}\sqrt{-3-4x-x^2}+x\left(4+8i\sqrt{2}-2\sqrt{2-4i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\right]+$$

$$\frac{1}{\sqrt{1+2i\sqrt{2}}}3\left(2i+\sqrt{2}\right)$$

$$\left.\text{Log}\left[\left(3+4x+2x^2\right)\left(3-6i\sqrt{2}+\left(2-2i\sqrt{2}\right)x^2-2\sqrt{2+4i\sqrt{2}}\sqrt{-3-4x-x^2}-2x\left(-2+4i\sqrt{2}+\sqrt{2+4i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\right]\right)$$

- **Problem 132: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{1}{x^2\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$$

Optimal (type 3, 151 leaves, 20 steps):

$$\frac{\sqrt{-3-4x-x^2}}{9x}+\frac{2\,\text{ArcTan}\left[\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right]}{3\sqrt{3}}+\frac{2}{27}\sqrt{2}\,\text{ArcTan}\left[\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]-\frac{2}{27}\sqrt{2}\,\text{ArcTan}\left[\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]+\frac{10}{27}\text{ArcTanh}\left[\frac{x}{\sqrt{-3-4x-x^2}}\right]$$

Result (type 3, 1039 leaves):

$$\frac{1}{18}\left(\frac{2\sqrt{-3-4x-x^2}}{x}+4\sqrt{3}\,\text{ArcTan}\left[\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right]-\frac{1}{\sqrt{1-2\,i\sqrt{2}}}2\,i\left(-i+2\sqrt{2}\right)\right.$$

$$\text{ArcTan}\left[\left(2\left(8+11\,i\sqrt{2}\right)x^4+9\left(12-i\sqrt{2}+6\,i\sqrt{1-2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+2x^3\left(62+50\,i\sqrt{2}+9\,i\sqrt{1-2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+\right.\right.$$

$$\left.x^2\left(324+137\,i\sqrt{2}+72\,i\sqrt{1-2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+x\left(324+48\,i\sqrt{2}+99\,i\sqrt{1-2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\Big/$$

$$\left.\left(9\left(5\,i+6\sqrt{2}\right)+12\left(7\,i+18\sqrt{2}\right)x+\left(125\,i+306\sqrt{2}\right)x^2+16\left(7\,i+11\sqrt{2}\right)x^3+\left(34\,i+32\sqrt{2}\right)x^4\right)\right]+\frac{1}{\sqrt{1+2\,i\sqrt{2}}}2\left(i+2\sqrt{2}\right)$$

$$\text{ArcTanh}\left[\left(2\left(8\,i+11\sqrt{2}\right)x^4-9\left(-12\,i+\sqrt{2}-6\sqrt{1+2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+2x^3\left(62\,i+50\sqrt{2}+9\sqrt{1+2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+\right.\right.$$

$$\left.x^2\left(324\,i+137\sqrt{2}+72\sqrt{1+2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+x\left(324\,i+48\sqrt{2}+99\sqrt{1+2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\Big/$$

$$\left.\left(9\left(-5\,i+6\sqrt{2}\right)+12\left(-7\,i+18\sqrt{2}\right)x+\left(-125\,i+306\sqrt{2}\right)x^2+16\left(-7\,i+11\sqrt{2}\right)x^3+\left(-34\,i+32\sqrt{2}\right)x^4\right)\right]+$$

$$\frac{\left(-i+2\sqrt{2}\right)\text{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1-2\,i\sqrt{2}}}+\frac{\left(i+2\sqrt{2}\right)\text{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1+2\,i\sqrt{2}}}-\frac{1}{\sqrt{1-2\,i\sqrt{2}}}$$

$$\left(-i+2\sqrt{2}\right)\text{Log}\left[\left(3+4x+2x^2\right)\left(3+6\,i\sqrt{2}+\left(2+2\,i\sqrt{2}\right)x^2-2\sqrt{2-4\,i\sqrt{2}}\sqrt{-3-4x-x^2}+x\left(4+8\,i\sqrt{2}-2\sqrt{2-4\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\right]-$$

$$\frac{1}{\sqrt{1+2\,i\sqrt{2}}}$$

$$\left.\left(i+2\sqrt{2}\right)\text{Log}\left[\left(3+4x+2x^2\right)\left(3-6\,i\sqrt{2}+\left(2-2\,i\sqrt{2}\right)x^2-2\sqrt{2+4\,i\sqrt{2}}\sqrt{-3-4x-x^2}-2x\left(-2+4\,i\sqrt{2}+\sqrt{2+4\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\right]\right)$$

- **Problem 142: Result unnecessarily involves higher level functions.**

$$\int\frac{g+h\,x}{\left(-\frac{c\,g^2}{h^2}+9\,c\,x^2\right)^{1/3}\left(g^2+3\,h^2x^2\right)}\,dx$$

Optimal (type 3, 242 leaves, 2 steps):

$$\frac{\left(1-\frac{9h^2x^2}{g^2}\right)^{1/3}\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3hx}{g}\right)^{2/3}}{\sqrt{3}\left(1+\frac{3hx}{g}\right)^{1/3}}\right]}{2^{2/3}\sqrt{3}\,h\left(-\frac{c\,g^2}{h^2}+9\,c\,x^2\right)^{1/3}}+\frac{\left(1-\frac{9h^2x^2}{g^2}\right)^{1/3}\text{Log}\left[g^2+3h^2x^2\right]}{6\times2^{2/3}\,h\left(-\frac{c\,g^2}{h^2}+9\,c\,x^2\right)^{1/3}}-\frac{\left(1-\frac{9h^2x^2}{g^2}\right)^{1/3}\text{Log}\left[\left(1-\frac{3hx}{g}\right)^{2/3}+2^{1/3}\left(1+\frac{3hx}{g}\right)^{1/3}\right]}{2\times2^{2/3}\,h\left(-\frac{c\,g^2}{h^2}+9\,c\,x^2\right)^{1/3}}$$

Result (type 6, 331 leaves):

$$\left(g^2 x \left(\left(g\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right]\right)\Big/\left(g^2\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right]+2h^2x^2\left(-\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right]\right)\right)-\left(h\,x\,\text{AppellF1}\left[1,\frac{1}{3},1,2,\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right]\right)\Big/\left(-2g^2\,\text{AppellF1}\left[1,\frac{1}{3},1,2,\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right]+3h^2x^2\left(\text{AppellF1}\left[2,\frac{1}{3},2,3,\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right]-\text{AppellF1}\left[2,\frac{4}{3},1,3,\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right]\right)\right)\right)\right)\Big/\left(\left(c\left(-\frac{g^2}{h^2}+9x^2\right)\right)^{1/3}\left(g^2+3h^2x^2\right)\right)$$

- **Problem 143: Unable to integrate problem.**

$$\int \frac{g+h x}{\left(\frac{-c^2 g^2+b c g h+2 b^2 h^2}{9 c h^2}+b x+c x^2\right)^{1/3}\left(\frac{f\left(b^2-\frac{-c^2 g^2+b c g h+2 b^2 h^2}{3 h^2}\right)}{c^2}+\frac{b f x}{c}+f x^2\right)}\,dx$$

Optimal (type 3, 488 leaves, 2 steps):

$$\frac{3\times 3^{1/6} h\left(\frac{c h^2\left(\frac{(c g-2 b h)(c g+b h)}{c h^2}-9 b x-9 c x^2\right)}{(2 c g-b h)^2}\right)^{1/3}\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3 h (b+2 c x)}{2 c g-b h}\right)^{2/3}}{\sqrt{3}\left(1+\frac{3 h (b+2 c x)}{2 c g-b h}\right)^{1/3}}\right]}{f\left(-\frac{(c g-2 b h)(c g+b h)}{c h^2}+9 b x+9 c x^2\right)^{1/3}}+\frac{3^{2/3} h\left(\frac{c h^2\left(\frac{(c g-2 b h)(c g+b h)}{c h^2}-9 b x-9 c x^2\right)}{(2 c g-b h)^2}\right)^{1/3}\text{Log}\left[\frac{f\left(c^2 g^2-b c g h+b^2 h^2\right)}{3 c^2 h^2}+\frac{b f x}{c}+f x^2\right]}{2 f\left(-\frac{(c g-2 b h)(c g+b h)}{c h^2}+9 b x+9 c x^2\right)^{1/3}}-$$

$$\frac{3\times 3^{2/3} h\left(\frac{c h^2\left(\frac{(c g-2 b h)(c g+b h)}{c h^2}-9 b x-9 c x^2\right)}{(2 c g-b h)^2}\right)^{1/3}\text{Log}\left[\left(1-\frac{3 h (b+2 c x)}{2 c g-b h}\right)^{2/3}+2^{1/3}\left(1+\frac{3 h (b+2 c x)}{2 c g-b h}\right)^{1/3}\right]}{2 f\left(-\frac{(c g-2 b h)(c g+b h)}{c h^2}+9 b x+9 c x^2\right)^{1/3}}$$

Result (type 8, 106 leaves):

$$\int \frac{g+h x}{\left(\frac{-c^2 g^2+b c g h+2 b^2 h^2}{9 c h^2}+b x+c x^2\right)^{1/3}\left(\frac{f\left(b^2-\frac{-c^2 g^2+b c g h+2 b^2 h^2}{3 h^2}\right)}{c^2}+\frac{b f x}{c}+f x^2\right)}\,dx$$

Test results for the 400 problems in "1.2.1.9 P(x) (d+e x)^m (a+b x+c x^2)^p.m"

- **Problem 39: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+b x^2\right)\left(-a d+4 b c x+3 b d x^2\right)}{(c+d x)^2}\,dx$$

Optimal (type 1, 17 leaves, 1 step):

$$\frac{\left(a+b x^2\right)^2}{c+d x}$$

Result (type 1, 62 leaves):

$$\frac{a^2 d^4+2 a b d^2 \left(c^2+c d x+d^2 x^2\right)+b^2 \left(c^4+c^3 d x+d^4 x^4\right)}{d^4 (c+d x)}$$

- **Problem 40: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+b x^2\right) (-a d+b x (4 c+3 d x))}{(c+d x)^2} \, dx$$

Optimal (type 1, 17 leaves, 1 step):

$$\frac{\left(a+b x^2\right)^2}{c+d x}$$

Result (type 1, 62 leaves):

$$\frac{a^2 d^4+2 a b d^2 \left(c^2+c d x+d^2 x^2\right)+b^2 \left(c^4+c^3 d x+d^4 x^4\right)}{d^4 (c+d x)}$$

- **Problem 41: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+b x^2\right)^2 \left(-a d+6 b c x+5 b d x^2\right)}{(c+d x)^2} \, dx$$

Optimal (type 1, 17 leaves, 1 step):

$$\frac{\left(a+b x^2\right)^3}{c+d x}$$

Result (type 1, 90 leaves):

$$\frac{a^3 d^6+3 a^2 b d^4 \left(c^2+c d x+d^2 x^2\right)+3 a b^2 d^2 \left(c^4+c^3 d x+d^4 x^4\right)+b^3 \left(c^6+c^5 d x+d^6 x^6\right)}{d^6 (c+d x)}$$

- **Problem 42: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+b x^2\right)^2 (-a d+b x (6 c+5 d x))}{(c+d x)^2} \, dx$$

Optimal (type 1, 17 leaves, 1 step):

$$\frac{\left(a+b x^2\right)^3}{c+d x}$$

Result (type 1, 90 leaves):

$$\frac{a^3 d^6 + 3 a^2 b d^4 \left(c^2 + c d x + d^2 x^2\right) + 3 a b^2 d^2 \left(c^4 + c^3 d x + d^4 x^4\right) + b^3 \left(c^6 + c^5 d x + d^6 x^6\right)}{d^6 (c + d x)}$$

Problem 136: Unable to integrate problem.

$$\int (g + h x)^m \left(a + c x^2\right)^p \left(d + e x + f x^2\right) dx$$

Optimal (type 6, 420 leaves, 6 steps):

$$\frac{f (g + h x)^{1+m} \left(a + c x^2\right)^{1+p}}{c h (3 + m + 2 p)} - \frac{1}{c h^3 (1 + m) (3 + m + 2 p)} \left(a f h^2 (1 + m) - c \left(2 f g^2 (1 + p) - h (e g - d h) (3 + m + 2 p)\right)\right) (g + h x)^{1+m} \left(a + c x^2\right)^p \left(1 - \frac{g + h x}{g - \frac{\sqrt{-a}\, h}{\sqrt{c}}}\right)^{-p} \left(1 - \frac{g + h x}{g + \frac{\sqrt{-a}\, h}{\sqrt{c}}}\right)^{-p} \text{AppellF1}\left[1 + m, -p, -p, 2 + m, \frac{g + h x}{g - \frac{\sqrt{-a}\, h}{\sqrt{c}}}, \frac{g + h x}{g + \frac{\sqrt{-a}\, h}{\sqrt{c}}}\right] - \frac{1}{h^3 (2 + m) (3 + m + 2 p)} (2 f g (1 + p) - e h (3 + m + 2 p)) (g + h x)^{2+m} \left(a + c x^2\right)^p \left(1 - \frac{g + h x}{g - \frac{\sqrt{-a}\, h}{\sqrt{c}}}\right)^{-p} \left(1 - \frac{g + h x}{g + \frac{\sqrt{-a}\, h}{\sqrt{c}}}\right)^{-p} \text{AppellF1}\left[2 + m, -p, -p, 3 + m, \frac{g + h x}{g - \frac{\sqrt{-a}\, h}{\sqrt{c}}}, \frac{g + h x}{g + \frac{\sqrt{-a}\, h}{\sqrt{c}}}\right]$$

Result (type 8, 29 leaves):

$$\int (g + h x)^m \left(a + c x^2\right)^p \left(d + e x + f x^2\right) dx$$

Problem 137: Unable to integrate problem.

$$\int (g + h x)^m \sqrt{a + c x^2} \left(d + e x + f x^2\right) dx$$

Optimal (type 6, 403 leaves, 6 steps):

$$\frac{f (g + h x)^{1+m} \left(a + c x^2\right)^{3/2}}{c h (4 + m)} - \left(\left(a f h^2 (1 + m) - c \left(3 f g^2 - h (e g - d h) (4 + m)\right)\right) (g + h x)^{1+m} \sqrt{a + c x^2}\, \text{AppellF1}\left[1 + m, -\frac{1}{2}, -\frac{1}{2}, 2 + m, \frac{g + h x}{g - \frac{\sqrt{-a}\, h}{\sqrt{c}}}, \frac{g + h x}{g + \frac{\sqrt{-a}\, h}{\sqrt{c}}}\right]\right) \Bigg/ \left(c h^3 (1 + m) (4 + m) \sqrt{1 - \frac{g + h x}{g - \frac{\sqrt{-a}\, h}{\sqrt{c}}}} \sqrt{1 - \frac{g + h x}{g + \frac{\sqrt{-a}\, h}{\sqrt{c}}}}\right) - \frac{(3 f g - e h (4 + m)) (g + h x)^{2+m} \sqrt{a + c x^2}\, \text{AppellF1}\left[2 + m, -\frac{1}{2}, -\frac{1}{2}, 3 + m, \frac{g+h x}{g-\frac{\sqrt{-a}\, h}{\sqrt{c}}}, \frac{g+h x}{g+\frac{\sqrt{-a}\, h}{\sqrt{c}}}\right]}{h^3 (2 + m) (4 + m) \sqrt{1 - \frac{g+h x}{g-\frac{\sqrt{-a}\, h}{\sqrt{c}}}} \sqrt{1 - \frac{g+h x}{g+\frac{\sqrt{-a}\, h}{\sqrt{c}}}}}$$

Result (type 8, 31 leaves):

$$\int (g+h\,x)^m \sqrt{a+c\,x^2}\,\left(d+e\,x+f\,x^2\right)\,dx$$

- **Problem 138: Unable to integrate problem.**

$$\int (g+h\,x)^{-3-2\,p}\,\left(a+c\,x^2\right)^p\,\left(d+e\,x+f\,x^2\right)\,dx$$

Optimal (type 6, 474 leaves, 5 steps) :

$$-\frac{\left(f\,g^2-e\,g\,h+d\,h^2\right)\,(g+h\,x)^{-2\,(1+p)}\,\left(a+c\,x^2\right)^{1+p}}{2\,h\,\left(c\,g^2+a\,h^2\right)\,(1+p)}-\frac{1}{2\,h^3\,p}$$

$$f\,(g+h\,x)^{-2\,p}\,\left(a+c\,x^2\right)^p\,\left(1-\frac{g+h\,x}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right)^{-p}\,\left(1-\frac{g+h\,x}{g+\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right)^{-p}\,\text{AppellF1}\left[-2\,p,\,-p,\,-p,\,1-2\,p,\,\frac{g+h\,x}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}},\,\frac{g+h\,x}{g+\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right]+$$

$$\left(\left(a\,h^2\,(2\,f\,g-e\,h)+c\,\left(f\,g^3-d\,g\,h^2\right)\right)\,\left(\sqrt{-a}-\sqrt{c}\,x\right)\,\left(-\frac{\left(\sqrt{c}\,g+\sqrt{-a}\,h\right)\,\left(\sqrt{-a}+\sqrt{c}\,x\right)}{\left(\sqrt{c}\,g-\sqrt{-a}\,h\right)\,\left(\sqrt{-a}-\sqrt{c}\,x\right)}\right)^{-p}\,(g+h\,x)^{-1-2\,p}\,\left(a+c\,x^2\right)^p\right.$$

$$\left.\text{Hypergeometric2F1}\left[-1-2\,p,\,-p,\,-2\,p,\,\frac{2\,\sqrt{-a}\,\sqrt{c}\,(g+h\,x)}{\left(\sqrt{c}\,g-\sqrt{-a}\,h\right)\,\left(\sqrt{-a}-\sqrt{c}\,x\right)}\right]\right)\Big/\left(h^2\,\left(\sqrt{c}\,g+\sqrt{-a}\,h\right)\,\left(c\,g^2+a\,h^2\right)\,(1+2\,p)\right)$$

Result (type 8, 33 leaves) :

$$\int (g+h\,x)^{-3-2\,p}\,\left(a+c\,x^2\right)^p\,\left(d+e\,x+f\,x^2\right)\,dx$$

- **Problem 139: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (d+e\,x)^m\,\left(-c\,d^2+b\,d\,e+b\,e^2\,x+c\,e^2\,x^2\right)^p\,\left(-(c\,d-b\,e)\,f+(c\,e\,f-c\,d\,g+b\,e\,g)\,x+c\,e\,g\,x^2\right)\,dx$$

Optimal (type 5, 222 leaves, 6 steps) :

$$\frac{g\,(d+e\,x)^{-1+m}\,\left(-d\,(c\,d-b\,e)+b\,e^2\,x+c\,e^2\,x^2\right)^{2+p}}{c\,e^2\,(3+m+2\,p)}-\frac{1}{c^2\,e^2\,(2+p)\,(3+m+2\,p)}(b\,e\,g\,(1+m+p)+c\,(d\,g\,(1-m)-e\,f\,(3+m+2\,p)))\,(d+e\,x)^m$$

$$\left(\frac{c\,(d+e\,x)}{2\,c\,d-b\,e}\right)^{-m-p}\,(c\,d-b\,e-c\,e\,x)^2\,\left(-d\,(c\,d-b\,e)+b\,e^2\,x+c\,e^2\,x^2\right)^p\,\text{Hypergeometric2F1}\left[-m-p,\,2+p,\,3+p,\,\frac{c\,d-b\,e-c\,e\,x}{2\,c\,d-b\,e}\right]$$

Result (type 6, 527 leaves) :

$$\frac{1}{3}(d+e x)^m\,(-(d+e x)\,(-b e+c\,(d-e x)))^p\left(\left(9 d\,(c d-b e)\,(-c e f+c d g-b e g)\,x^2\,\text{AppellF1}\left[2,-m-p,-p,3,-\frac{e x}{d},\frac{c e x}{c d-b e}\right]\right)\Big/\right.$$
$$\left(2\left(3 d\,(-c d+b e)\,\text{AppellF1}\left[2,-m-p,-p,3,-\frac{e x}{d},\frac{c e x}{c d-b e}\right]+\right.\right.$$
$$\left.\left.e x\left(c d p\,\text{AppellF1}\left[3,-m-p,1-p,4,-\frac{e x}{d},\frac{c e x}{c d-b e}\right]-(c d-b e)\,(m+p)\,\text{AppellF1}\left[3,1-m-p,-p,4,-\frac{e x}{d},\frac{c e x}{c d-b e}\right]\right)\right)\right)+$$
$$\left(4 c d e\,(-c d+b e)\,g x^3\,\text{AppellF1}\left[3,-m-p,-p,4,-\frac{e x}{d},\frac{c e x}{c d-b e}\right]\right)\Big/\left(4 d\,(-c d+b e)\,\text{AppellF1}\left[3,-m-p,-p,4,-\frac{e x}{d},\frac{c e x}{c d-b e}\right]+\right.$$
$$\left.e x\left(c d p\,\text{AppellF1}\left[4,-m-p,1-p,5,-\frac{e x}{d},\frac{c e x}{c d-b e}\right]-(c d-b e)\,(m+p)\,\text{AppellF1}\left[4,1-m-p,-p,5,-\frac{e x}{d},\frac{c e x}{c d-b e}\right]\right)\right)-$$
$$\left.\frac{3\,(c d-b e)\,f\left(\frac{c\,(d+e x)}{2 c d-b e}\right)^{-m-p}(-c d+b e+c e x)\,\text{Hypergeometric2F1}\left[-m-p,1+p,2+p,\frac{-c d+b e+c e x}{-2 c d+b e}\right]}{c e\,(1+p)}\right)$$

- **Problem 206: Result more than twice size of optimal antiderivative.**

$$\int\frac{\left(a+b x+c x^2\right)^{3/2}\left(d+e x+f x^2\right)}{(g+h x)^7}\,dx$$

Optimal (type 3, 657 leaves, 6 steps):

$$-\left(\left(b^2-4 a c\right)\left(24 c^2 d g^2+24 a^2 f h^2-12 a b h\,(2 f g+e h)+b^2\left(7 f g^2+5 e g h+7 d h^2\right)-4 c\left(3 b g\,(e g+2 d h)+a\left(f g^2-7 e g h+d h^2\right)\right)\right)\right.$$
$$\left.(b g-2 a h+(2 c g-b h)\,x)\sqrt{a+b x+c x^2}\right)\Big/\left(512\left(c g^2-b g h+a h^2\right)^4(g+h x)^2\right)+$$
$$\left(\left(24 c^2 d g^2+24 a^2 f h^2-12 a b h\,(2 f g+e h)+b^2\left(7 f g^2+5 e g h+7 d h^2\right)-4 c\left(3 b g\,(e g+2 d h)+a\left(f g^2-7 e g h+d h^2\right)\right)\right)\right.$$
$$\left.(b g-2 a h+(2 c g-b h)\,x)\left(a+b x+c x^2\right)^{3/2}\right)\Big/\left(192\left(c g^2-b g h+a h^2\right)^3(g+h x)^4\right)-\frac{\left(f g^2-h\,(e g-d h)\right)\left(a+b x+c x^2\right)^{5/2}}{6 h\left(c g^2-b g h+a h^2\right)(g+h x)^6}+$$
$$\frac{\left(2 c g\left(5 f g^2+h\,(e g-7 d h)\right)+h\left(12 a h\,(2 f g-e h)-b\left(17 f g^2-5 e g h-7 d h^2\right)\right)\right)\left(a+b x+c x^2\right)^{5/2}}{60 h\left(c g^2-b g h+a h^2\right)^2(g+h x)^5}+$$
$$\left(\left(b^2-4 a c\right)^2\left(24 c^2 d g^2+24 a^2 f h^2-12 a b h\,(2 f g+e h)+b^2\left(7 f g^2+5 e g h+7 d h^2\right)-4 c\left(3 b g\,(e g+2 d h)+a\left(f g^2-7 e g h+d h^2\right)\right)\right)\right.$$
$$\left.\text{ArcTanh}\left[\frac{b g-2 a h+(2 c g-b h)\,x}{2\sqrt{c g^2-b g h+a h^2}\,\sqrt{a+b x+c x^2}}\right]\right)\Big/\left(1024\left(c g^2-b g h+a h^2\right)^{9/2}\right)$$

Result (type 3, 2022 leaves):

$$\frac{1}{a+b\,x+c\,x^2}\,(a+x\,(b+c\,x))^{3/2}$$

$$\left(-\frac{\left(c\,g^2-b\,g\,h+a\,h^2\right)\,\left(f\,g^2-e\,g\,h+d\,h^2\right)}{6\,h^5\,(g+h\,x)^6}+\frac{50\,c\,f\,g^3-38\,c\,e\,g^2\,h-37\,b\,f\,g^2\,h+26\,c\,d\,g\,h^2+25\,b\,e\,g\,h^2+24\,a\,f\,g\,h^2-13\,b\,d\,h^3-12\,a\,e\,h^3}{60\,h^5\,(g+h\,x)^5}+\right.$$

$$\left(-800\,c^2\,f\,g^4+416\,c^2\,e\,g^3\,h+1184\,b\,c\,f\,g^3\,h-152\,c^2\,d\,g^2\,h^2-548\,b\,c\,e\,g^2\,h^2-387\,b^2\,f\,g^2\,h^2-908\,a\,c\,f\,g^2\,h^2+152\,b\,c\,d\,g\,h^3+135\,b^2\,e\,g\,h^3+\right.$$
$$\left.404\,a\,c\,e\,g\,h^3+504\,a\,b\,f\,g\,h^3-3\,b^2\,d\,h^4-140\,a\,c\,d\,h^4-132\,a\,b\,e\,h^4-120\,a^2\,f\,h^4\right)\Big/\left(480\,h^5\,\left(c\,g^2-b\,g\,h+a\,h^2\right)\,(g+h\,x)^4\right)+$$

$$\frac{1}{960\,h^5\,\left(c\,g^2-b\,g\,h+a\,h^2\right)^2\,(g+h\,x)^3}\,\left(1600\,c^3\,f\,g^5-448\,c^3\,e\,g^4\,h-3552\,b\,c^2\,f\,g^4\,h+16\,c^3\,d\,g^3\,h^2+888\,b\,c^2\,e\,g^3\,h^2+2322\,b^2\,c\,f\,g^3\,h^2+\right.$$
$$3144\,a\,c^2\,f\,g^3\,h^2-24\,b\,c^2\,d\,g^2\,h^3-438\,b^2\,c\,e\,g^2\,h^3-888\,a\,c^2\,e\,g^2\,h^3-377\,b^3\,f\,g^2\,h^3-3828\,a\,b\,c\,f\,g^2\,h^3-6\,b^2\,c\,d\,g\,h^4+72\,a\,c^2\,d\,g\,h^4+$$
$$\left.5\,b^3\,e\,g\,h^4+852\,a\,b\,c\,e\,g\,h^4+744\,a\,b^2\,f\,g\,h^4+1488\,a^2\,c\,f\,g\,h^4+7\,b^3\,d\,h^5-36\,a\,b\,c\,d\,h^5-12\,a\,b^2\,e\,h^5-384\,a^2\,c\,e\,h^5-360\,a^2\,b\,f\,h^5\right)+$$

$$\frac{1}{3840\,h^5\,\left(c\,g^2-b\,g\,h+a\,h^2\right)^3\,(g+h\,x)^2}\,\left(-3200\,c^4\,f\,g^6+128\,c^4\,e\,g^5\,h+9472\,b\,c^3\,f\,g^5\,h+64\,c^4\,d\,g^4\,h^2-352\,b\,c^3\,e\,g^4\,h^2-\right.$$
$$9288\,b^2\,c^2\,f\,g^4\,h^2-9504\,a\,c^3\,f\,g^4\,h^2-128\,b\,c^3\,d\,g^3\,h^3+264\,b^2\,c^2\,e\,g^3\,h^3+480\,a\,c^3\,e\,g^3\,h^3+3016\,b^3\,c\,f\,g^3\,h^3+18\,528\,a\,b\,c^2\,f\,g^3\,h^3+$$
$$384\,a\,c^3\,d\,g^2\,h^4+20\,b^3\,c\,e\,g^2\,h^4-912\,a\,b\,c^2\,e\,g^2\,h^4-35\,b^4\,f\,g^2\,h^4-8808\,a\,b^2\,c\,f\,g^2\,h^4-9264\,a^2\,c^2\,f\,g^2\,h^4+64\,b^3\,c\,d\,g\,h^5-$$
$$384\,a\,b\,c^2\,d\,g\,h^5-25\,b^4\,e\,g\,h^5+96\,a\,b^2\,c\,e\,g\,h^5+912\,a^2\,c^2\,e\,g\,h^5+120\,a\,b^3\,f\,g\,h^5+8352\,a^2\,b\,c\,f\,g\,h^5-35\,b^4\,d\,h^6+216\,a\,b^2\,c\,d\,h^6-$$
$$\left.240\,a^2\,c^2\,d\,h^6+60\,a\,b^3\,e\,h^6-336\,a^2\,b\,c\,e\,h^6-120\,a^2\,b^2\,f\,h^6-2400\,a^3\,c\,f\,h^6\right)+\frac{1}{7680\,h^5\,\left(c\,g^2-b\,g\,h+a\,h^2\right)^4\,(g+h\,x)}$$
$$\left(1280\,c^5\,f\,g^7+256\,c^5\,e\,g^6\,h-4736\,b\,c^4\,f\,g^6\,h+128\,c^5\,d\,g^5\,h^2-832\,b\,c^4\,e\,g^5\,h^2+6192\,b^2\,c^3\,f\,g^5\,h^2+5312\,a\,c^4\,f\,g^5\,h^2-320\,b\,c^4\,d\,g^4\,h^3+\right.$$
$$816\,b^2\,c^3\,e\,g^4\,h^3+1216\,a\,c^4\,e\,g^4\,h^3-3016\,b^3\,c^2\,f\,g^4\,h^3-14\,496\,a\,b\,c^3\,f\,g^4\,h^3+96\,b^2\,c^3\,d\,g^3\,h^4+896\,a\,c^4\,d\,g^3\,h^4-80\,b^3\,c^2\,e\,g^3\,h^4-$$
$$2880\,a\,b\,c^3\,e\,g^3\,h^4+70\,b^4\,c\,f\,g^3\,h^4+11\,664\,a\,b^2\,c^2\,f\,g^3\,h^4+8544\,a^2\,c^3\,f\,g^3\,h^4+176\,b^3\,c^2\,d\,g^2\,h^5-1344\,a\,b\,c^3\,d\,g^2\,h^5-130\,b^4\,c\,e\,g^2\,h^5+$$
$$1104\,a\,b^2\,c^2\,e\,g^2\,h^5+2784\,a^2\,c^3\,e\,g^2\,h^5+105\,b^5\,f\,g^2\,h^5-1000\,a\,b^3\,c\,f\,g^2\,h^5-15\,600\,a^2\,b\,c^2\,f\,g^2\,h^5-290\,b^4\,c\,d\,g\,h^6+1968\,a\,b^2\,c^2\,d\,g\,h^6-$$
$$2592\,a^2\,c^3\,d\,g\,h^6+75\,b^5\,e\,g\,h^6-200\,a\,b^3\,c\,e\,g\,h^6-1488\,a^2\,b\,c^2\,e\,g\,h^6-360\,a\,b^4\,f\,g\,h^6+2640\,a^2\,b^2\,c\,f\,g\,h^6+7872\,a^3\,c^2\,f\,g\,h^6+$$
$$\left.\left.105\,b^5\,d\,h^7-760\,a\,b^3\,c\,d\,h^7+1296\,a^2\,b\,c^2\,d\,h^7-180\,a\,b^4\,e\,h^7+1200\,a^2\,b^2\,c\,e\,h^7-1536\,a^3\,c^2\,e\,h^7+360\,a^2\,b^3\,f\,h^7-2400\,a^3\,b\,c\,f\,h^7\right)\right)+$$

$$\left(\left(b^2-4\,a\,c\right)^2\,\left(24\,c^2\,d\,g^2-12\,b\,c\,e\,g^2+7\,b^2\,f\,g^2-4\,a\,c\,f\,g^2-24\,b\,c\,d\,g\,h+5\,b^2\,e\,g\,h+28\,a\,c\,e\,g\,h-24\,a\,b\,f\,g\,h+\right.\right.$$
$$\left.7\,b^2\,d\,h^2-4\,a\,c\,d\,h^2-12\,a\,b\,e\,h^2+24\,a^2\,f\,h^2\right)$$
$$\left.(a+x\,(b+c\,x))^{3/2}\,\mathrm{Log}[g+h\,x]\right)\Big/\left(1024\,\left(c\,g^2-b\,g\,h+a\,h^2\right)^{9/2}\,\left(a+b\,x+c\,x^2\right)^{3/2}\right)-$$

$$\left(\left(b^2-4\,a\,c\right)^2\,\left(24\,c^2\,d\,g^2-12\,b\,c\,e\,g^2+7\,b^2\,f\,g^2-4\,a\,c\,f\,g^2-24\,b\,c\,d\,g\,h+5\,b^2\,e\,g\,h+28\,a\,c\,e\,g\,h-\right.\right.$$
$$\left.24\,a\,b\,f\,g\,h+7\,b^2\,d\,h^2-4\,a\,c\,d\,h^2-12\,a\,b\,e\,h^2+24\,a^2\,f\,h^2\right)$$
$$\left.(a+x\,(b+c\,x))^{3/2}\,\mathrm{Log}\left[-b\,g+2\,a\,h-2\,c\,g\,x+b\,h\,x+2\,\sqrt{c\,g^2-b\,g\,h+a\,h^2}\,\sqrt{a+b\,x+c\,x^2}\right]\right)\Big/$$
$$\left(1024\,\left(c\,g^2-b\,g\,h+a\,h^2\right)^{9/2}\,\left(a+b\,x+c\,x^2\right)^{3/2}\right)$$

- **Problem 207: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+b x+c x^2\right)^{3/2} \left(d+e x+f x^2\right)}{(g+h x)^8} \, dx$$

Optimal (type 3, 1062 leaves, 7 steps):

$$-\left(\left(b^2-4 a c\right) \left(48 c^3 d g^3-8 c^2 g \left(3 b g (e g+3 d h)+a \left(f g^2-8 e g h+3 d h^2\right)\right)-b h \left(24 a^2 f h^2-2 a b h (10 f g+7 e h)+b^2 \left(5 f g^2+5 e g h+9 d h^2\right)\right)+2 c \left(4 a^2 h^2 (8 f g-e h)-2 a b h \left(13 f g^2+13 e g h-3 d h^2\right)+b^2 g \left(7 f g^2+10 e g h+21 d h^2\right)\right)\right) (b g-2 a h+(2 c g-b h) x) \sqrt{a+b x+c x^2}\right) \Big/ \left(1024 \left(c g^2-b g h+a h^2\right)^5 (g+h x)^2\right)+$$
$$\left(\left(48 c^3 d g^3-8 c^2 g \left(3 b g (e g+3 d h)+a \left(f g^2-8 e g h+3 d h^2\right)\right)-b h \left(24 a^2 f h^2-2 a b h (10 f g+7 e h)+b^2 \left(5 f g^2+5 e g h+9 d h^2\right)\right)+2 c \left(4 a^2 h^2 (8 f g-e h)-2 a b h \left(13 f g^2+13 e g h-3 d h^2\right)+b^2 g \left(7 f g^2+10 e g h+21 d h^2\right)\right)\right) (b g-2 a h+(2 c g-b h) x) \left(a+b x+c x^2\right)^{3/2}\right) \Big/ \left(384 \left(c g^2-b g h+a h^2\right)^4 (g+h x)^4\right)-\frac{\left(f g^2-h (e g-d h)\right) \left(a+b x+c x^2\right)^{5/2}}{7 h \left(c g^2-b g h+a h^2\right) (g+h x)^7}+$$
$$\frac{\left(2 c g \left(5 f g^2+h (2 e g-9 d h)\right)+h \left(14 a h (2 f g-e h)-b \left(19 f g^2-5 e g h-9 d h^2\right)\right)\right) \left(a+b x+c x^2\right)^{5/2}}{84 h \left(c g^2-b g h+a h^2\right)^2 (g+h x)^6}+$$
$$\left(\left(4 c^2 g^2 \left(5 f g^2+h (2 e g-51 d h)\right)-7 h^2 \left(24 a^2 f h^2-2 a b h (10 f g+7 e h)+b^2 \left(5 f g^2+5 e g h+9 d h^2\right)\right)-2 c h \left(3 b g \left(8 f g^2-15 e g h-34 d h^2\right)-2 a h \left(26 f g^2-61 e g h+12 d h^2\right)\right)\right) \left(a+b x+c x^2\right)^{5/2}\right) \Big/ \left(840 h \left(c g^2-b g h+a h^2\right)^3 (g+h x)^5\right)+$$
$$\left(\left(b^2-4 a c\right)^2 \left(48 c^3 d g^3-8 c^2 g \left(3 b g (e g+3 d h)+a \left(f g^2-8 e g h+3 d h^2\right)\right)-b h \left(24 a^2 f h^2-2 a b h (10 f g+7 e h)+b^2 \left(5 f g^2+5 e g h+9 d h^2\right)\right)+2 c \left(4 a^2 h^2 (8 f g-e h)-2 a b h \left(13 f g^2+13 e g h-3 d h^2\right)+b^2 g \left(7 f g^2+10 e g h+21 d h^2\right)\right)\right) \text{ArcTanh}\left[\frac{b g-2 a h+(2 c g-b h) x}{2 \sqrt{c g^2-b g h+a h^2} \sqrt{a+b x+c x^2}}\right]\right) \Big/ \left(2048 \left(c g^2-b g h+a h^2\right)^{11/2}\right)$$

Result (type 3, 3059 leaves):

$$\frac{1}{a+b x+c x^2} (a+x (b+c x))^{3/2} \left(-\frac{\left(c g^2-b g h+a h^2\right) \left(f g^2-e g h+d h^2\right)}{7 h^5 (g+h x)^7}+\frac{58 c f g^3-44 c e g^2 h-43 b f g^2 h+30 c d g h^2+29 b e g h^2+28 a f g h^2-15 b d h^3-14 a e h^3}{84 h^5 (g+h x)^6}+\right.$$
$$\left(-1100 c^2 f g^4+568 c^2 e g^3 h+1632 b c f g^3 h-204 c^2 d g^2 h^2-750 b c e g^2 h^2-535 b^2 f g^2 h^2-1256 a c f g^2 h^2+204 b c d g h^3+185 b^2 e g h^3+556 a c e g h^3+700 a b f g h^3-3 b^2 d h^4-192 a c d h^4-182 a b e h^4-168 a^2 f h^4\right) \Big/ \left(840 h^5 \left(c g^2-b g h+a h^2\right) (g+h x)^5\right)+$$
$$\frac{1}{6720 h^5 \left(c g^2-b g h+a h^2\right)^2 (g+h x)^4} \left(8000 c^3 f g^5-2176 c^3 e g^4 h-17824 b c^2 f g^4 h+48 c^3 d g^3 h^2+4328 b c^2 e g^3 h^2+11702 b^2 c f g^3 h^2+\right.$$
$$15832 a c^2 f g^3 h^2-72 b c^2 d g^2 h^3-2140 b^2 c e g^2 h^3-4352 a c^2 e g^2 h^3-1905 b^3 f g^2 h^3-19396 a b c f g^2 h^3-30 b^2 c d g h^4+264 a c^2 d g h^4+$$

$$15 b^3 e g h^4 + 4220 a b c e g h^4 + 3780 a b^2 f g h^4 + 7616 a^2 c f g h^4 + 27 b^3 d h^5 - 132 a b c d h^5 - 42 a b^2 e h^5 - 1960 a^2 c e h^5 - 1848 a^2 b f h^5\Big) +$$

$$\frac{1}{13440 h^5 \left(c g^2 - b g h + a h^2\right)^3 (g + h x)^3} \Big(-6400 c^4 f g^6 + 128 c^4 e g^5 h + 19072 b c^3 f g^5 h + 96 c^4 d g^4 h^2 - 368 b c^3 e g^4 h^2 -$$
$$18852 b^2 c^2 f g^4 h^2 - 19216 a c^3 f g^4 h^2 - 192 b c^3 d g^3 h^3 + 288 b^2 c^2 e g^3 h^3 + 512 a c^3 e g^3 h^3 + 6152 b^3 c f g^3 h^3 + 37920 a b c^2 f g^3 h^3 -$$
$$36 b^2 c^2 d g^2 h^4 + 720 a c^3 d g^2 h^4 + 50 b^3 c e g^2 h^4 - 1128 a b c^2 e g^2 h^4 - 35 b^4 f g^2 h^4 - 18276 a b^2 c f g^2 h^4 - 19200 a^2 c^2 f g^2 h^4 +$$
$$132 b^3 c d g h^5 - 720 a b c^2 d g h^5 - 35 b^4 e g h^5 + 48 a b^2 c e g h^5 + 1392 a^2 c^2 e g h^5 + 140 a b^3 f g h^5 + 17808 a^2 b c f g h^5 - 63 b^4 d h^6 +$$
$$372 a b^2 c d h^6 - 384 a^2 c^2 d h^6 + 98 a b^3 e h^6 - 504 a^2 b c e h^6 - 168 a^2 b^2 f h^6 - 5376 a^3 c f h^6\Big) + \frac{1}{53760 h^5 \left(c g^2 - b g h + a h^2\right)^4 (g + h x)^2}$$
$$\Big(1280 c^5 f g^7 + 512 c^5 e g^6 h - 4992 b c^4 f g^6 h + 384 c^5 d g^5 h^2 - 1728 b c^4 e g^5 h^2 + 6928 b^2 c^3 f g^5 h^2 + 5696 a c^4 f g^5 h^2 - 960 b c^4 d g^4 h^3 +$$
$$1696 b^2 c^3 e g^4 h^3 + 2816 a c^4 e g^4 h^3 - 3496 b^3 c^2 f g^4 h^3 - 17056 a b c^3 f g^4 h^3 + 96 b^2 c^3 d g^3 h^4 + 3456 a c^4 d g^3 h^4 + 80 b^3 c^2 e g^3 h^4 -$$
$$7360 a b c^3 e g^3 h^4 - 210 b^4 c f g^3 h^4 + 15504 a b^2 c^2 f g^3 h^4 + 10464 a^2 c^3 f g^3 h^4 + 816 b^3 c^2 d g^2 h^5 - 5184 a b c^3 d g^2 h^5 - 420 b^4 c e g^2 h^5 +$$
$$2304 a b^2 c^2 e g^2 h^5 + 9024 a^2 c^3 e g^2 h^5 + 175 b^5 f g^2 h^5 - 280 a b^3 c f g^2 h^5 - 24720 a^2 b c^2 f g^2 h^5 - 966 b^4 c d g h^6 + 6096 a b^2 c^2 d g h^6 -$$
$$7008 a^2 c^3 d g h^6 + 175 b^5 e g h^6 + 56 a b^3 c e g h^6 - 5520 a^2 b c^2 e g h^6 - 700 a b^4 f g h^6 + 3024 a^2 b^2 c f g h^6 + 16128 a^3 c^2 f g h^6 +$$
$$315 b^5 d h^7 - 2184 a b^3 c d h^7 + 3504 a^2 b c^2 d h^7 - 490 a b^4 e h^7 + 3024 a^2 b^2 c e h^7 - 3360 a^3 c^2 e h^7 + 840 a^2 b^3 f h^7 - 4704 a^3 b c f h^7\Big) +$$

$$\frac{1}{107520 h^5 \left(c g^2 - b g h + a h^2\right)^5 (g + h x)} \Big(2560 c^6 f g^8 + 1024 c^6 e g^7 h - 11264 b c^5 f g^7 h + 768 c^6 d g^6 h^2 - 3968 b c^5 e g^6 h^2 +$$
$$18208 b^2 c^4 f g^6 h^2 + 13952 a c^5 f g^6 h^2 - 2304 b c^5 d g^5 h^3 + 4864 b^2 c^4 e g^5 h^3 + 6656 a c^5 e g^5 h^3 - 11744 b^3 c^3 f g^5 h^3 -$$
$$48512 a b c^4 f g^5 h^3 + 960 b^2 c^4 d g^4 h^4 + 7680 a c^5 d g^4 h^4 - 800 b^3 c^3 e g^4 h^4 - 20480 a b c^4 e g^4 h^4 + 700 b^4 c^2 f g^4 h^4 + 54720 a b^2 c^3 f g^4 h^4 +$$
$$32320 a^2 c^4 f g^4 h^4 + 1920 b^3 c^3 d g^3 h^5 - 15360 a b c^4 d g^3 h^5 - 1400 b^4 c^2 e g^3 h^5 + 12480 a b^2 c^3 e g^3 h^5 + 23680 a^2 c^4 e g^3 h^5 +$$
$$1120 b^5 c f g^3 h^5 - 11200 a b^3 c^2 f g^3 h^5 - 88320 a^2 b c^3 f g^3 h^5 - 5124 b^4 c^2 d g^2 h^6 + 35232 a b^2 c^3 d g^2 h^6 - 47424 a^2 c^4 d g^2 h^6 +$$
$$1750 b^5 c e g^2 h^6 - 8176 a b^3 c^2 e g^2 h^6 - 11808 a^2 b c^3 e g^2 h^6 - 525 b^6 f g^2 h^6 - 560 a b^4 c f g^2 h^6 + 27216 a^2 b^2 c^2 f g^2 h^6 +$$
$$59904 a^3 c^3 f g^2 h^6 + 3780 b^5 c d g h^7 - 27552 a b^3 c^2 d g h^7 + 47424 a^2 b c^3 d g h^7 - 525 b^6 e g h^7 - 980 a b^4 c e g h^7 + 25872 a^2 b^2 c^2 e g h^7 -$$
$$42432 a^3 c^3 e g h^7 + 2100 a b^5 f g h^7 - 8960 a^2 b^3 c f g h^7 - 17472 a^3 b c^2 f g h^7 - 945 b^6 d h^8 + 7560 a b^4 c d h^8 - 16464 a^2 b^2 c^2 d h^8 +$$
$$6144 a^3 c^3 d h^8 + 1470 a b^5 e h^8 - 10640 a^2 b^3 c e h^8 + 18144 a^3 b c^2 e h^8 - 2520 a^2 b^4 f h^8 + 16800 a^3 b^2 c f h^8 - 21504 a^4 c^2 f h^8\Big)\Bigg) -$$

$$\frac{1}{2048 \left(c g^2 - b g h + a h^2\right)^{11/2} \left(a + b x + c x^2\right)^{3/2}} \left(b^2 - 4 a c\right)^2 \Big(-48 c^3 d g^3 + 24 b c^2 e g^3 - 14 b^2 c f g^3 + 8 a c^2 f g^3 +$$
$$72 b c^2 d g^2 h - 20 b^2 c e g^2 h - 64 a c^2 e g^2 h + 5 b^3 f g^2 h + 52 a b c f g^2 h -$$
$$42 b^2 c d g h^2 + 24 a c^2 d g h^2 + 5 b^3 e g h^2 + 52 a b c e g h^2 - 20 a b^2 f g h^2 -$$
$$64 a^2 c f g h^2 + 9 b^3 d h^3 - 12 a b c d h^3 - 14 a b^2 e h^3 + 8 a^2 c e h^3 + 24 a^2 b f h^3\Big)$$

$$(a + x (b + c x))^{3/2} \operatorname{Log}[g + h x] + \frac{1}{2048 \left(c g^2 - b g h + a h^2\right)^{11/2} \left(a + b x + c x^2\right)^{3/2}}$$
$$\left(b^2 - 4 a c\right)^2 \Big(-48 c^3 d g^3 + 24 b c^2 e g^3 - 14 b^2 c f g^3 + 8 a c^2 f g^3 + 72 b c^2 d g^2 h - 20 b^2 c e g^2 h -$$
$$64 a c^2 e g^2 h + 5 b^3 f g^2 h + 52 a b c f g^2 h - 42 b^2 c d g h^2 + 24 a c^2 d g h^2 + 5 b^3 e g h^2 + 52 a b c e g h^2 -$$
$$20 a b^2 f g h^2 - 64 a^2 c f g h^2 + 9 b^3 d h^3 - 12 a b c d h^3 - 14 a b^2 e h^3 + 8 a^2 c e h^3 + 24 a^2 b f h^3\Big)$$
$$(a + x (b + c x))^{3/2} \operatorname{Log}\left[-b g + 2 a h - 2 c g x + b h x + 2 \sqrt{c g^2 - b g h + a h^2} \sqrt{a + b x + c x^2}\right]$$

Problem 259: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\,\left(A+B\,x+C\,x^2\right)\,dx$$

Optimal (type 4, 906 leaves, 8 steps) :

$$\frac{1}{315 c^3 e^3} 2 \sqrt{d+e x} \left(8 b^3 C e^3-3 b c e^2 (b C d+4 b B e-a C e)+c^3 d \left(8 C d^2-3 e (4 B d-7 A e)\right)+3 c^2 e \left(a e (C d-5 B e)-b \left(C d^2-2 B d e-7 A e^2\right)\right)+\right.$$

$$\left.3 c e \left(8 b^2 C e^2-c e (b C d+12 b B e+7 a C e)-c^2 \left(2 C d^2-3 e (B d+7 A e)\right)\right) x\right) \sqrt{a+b x+c x^2}-$$

$$\frac{2 (2 c C d-3 B c e+2 b C e) \sqrt{d+e x} \left(a+b x+c x^2\right)^{3/2}}{21 c^2 e}+\frac{2 C (d+e x)^{3/2} \left(a+b x+c x^2\right)^{3/2}}{9 c e}+\frac{1}{315 c^4 e^4 \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{a+b x+c x^2}}$$

$$\left(\sqrt{2} \sqrt{b^2-4 a c} \left(2 \left(4 c^2 d^2-b^2 e^2-\frac{3}{2} c e (b d-2 a e)\right) \left(8 b^2 C e^2-c e (b C d+12 b B e+7 a C e)-c^2 \left(2 C d^2-3 e (B d+7 A e)\right)\right)-\right.\right.$$

$$\left.5 c e (2 c d-b e) \left(6 b^2 C d e+c e (21 A c d-5 a C d-3 a B e)+b \left(2 a C e^2-c d (C d+9 B e)\right)\right)\right)$$

$$\sqrt{d+e x} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c} e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]-$$

$$\left(2 \sqrt{2} \sqrt{b^2-4 a c} \left(c d^2-b d e+a e^2\right) \left(8 b^3 C e^3-3 c^2 e^2 (b B d+2 a C d-7 A b e-10 a B e)+3 b c e^2 (b C d-4 b B e-9 a C e)-\right.\right.$$

$$\left.2 c^3 d \left(8 C d^2-3 e (4 B d-7 A e)\right)\right) \sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}} \sqrt{-\frac{c \left(a+b x+c x^2\right)}{b^2-4 a c}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2 \sqrt{b^2-4 a c} e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right) \Big/ \left(315 c^4 e^4 \sqrt{d+e x} \sqrt{a+b x+c x^2}\right)$$

Result (type 4, 15 669 leaves):

$$\sqrt{d+e x}$$

$$\left(\frac{1}{315 c^3 e^3} 2 \left(8 c^3 C d^3-12 B c^3 d^2 e-3 b c^2 C d^2 e+6 b B c^2 d e^2+21 A c^3 d e^2-3 b^2 c C d e^2+8 a c^2 C d e^2-12 b^2 B c e^3+21 A b c^2 e^3+30 a B c^2 e^3+\right.\right.$$

$$8 b^3 C e^3 - 27 a b c C e^3\Big) + \frac{2 \left(-6 c^2 C d^2 + 9 B c^2 d e + 2 b c C d e + 9 b B c e^2 + 63 A c^2 e^2 - 6 b^2 C e^2 + 14 a c C e^2\right) x}{315 c^2 e^2} +$$

$$\frac{2 (c C d + 9 B c e + b C e) x^2}{63 c e} + \frac{2 C x^3}{9}\Bigg) \sqrt{a + x (b + c x)} - \frac{1}{315 c^3 e^5 \sqrt{a + b x + c x^2}} 2 \sqrt{a + x (b + c x)}$$

$$\Bigg(\frac{1}{c \sqrt{\frac{(d+e x)^2 \left(c \left(-1+\frac{d}{d+e x}\right)^2+\frac{e \left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}} \left(16 c^4 C d^4 - 24 B c^4 d^3 e - 8 b c^3 C d^3 e + 15 b B c^3 d^2 e^2 + 42 A c^4 d^2 e^2 - 6 b^2 c^2 C d^2 e^2 + 18 a c^3 C d^2 e^2 + \right.$$

$$15 b^2 B c^2 d e^3 - 42 A b c^3 d e^3 - 48 a B c^3 d e^3 - 8 b^3 c C d e^3 + 30 a b c^2 C d e^3 - 24 b^3 B c e^4 + 42 A b^2 c^2 e^4 + 87 a b B c^2 e^4 -$$

$$\left. 126 a A c^3 e^4 + 16 b^4 C e^4 - 72 a b^2 c C e^4 + 42 a^2 c^2 C e^4\right) (d + e x)^{3/2} \left(c + \frac{c d^2}{(d + e x)^2} - \frac{b d e}{(d + e x)^2} + \frac{a e^2}{(d + e x)^2} - \frac{2 c d}{d + e x} + \frac{b e}{d + e x}\right) -$$

$$\frac{1}{c \sqrt{\frac{(d+e x)^2 \left(c \left(-1+\frac{d}{d+e x}\right)^2+\frac{e \left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}} \left(c d^2 - b d e + a e^2\right) (d + e x) \sqrt{c + \frac{c d^2}{(d + e x)^2} - \frac{b d e}{(d + e x)^2} + \frac{a e^2}{(d + e x)^2} - \frac{2 c d}{d + e x} + \frac{b e}{d + e x}}$$

$$\Bigg(\Bigg(4 i \sqrt{2} c^4 C d^4 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}$$

$$\sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\Bigg) \Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(6 i \sqrt{2}\, B c^4 d^3 e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(2 i \sqrt{2}\, b c^3 C d^3 e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left((c d^2 - b d e + a e^2) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\Bigg(15\, i\, b B c^3 d^2 e^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}$$

$$\sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \Bigg(\text{EllipticE}\Big[i\, \text{ArcSinh}\Big[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\Big], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\Big] -$$

$$\text{EllipticF}\Big[i\, \text{ArcSinh}\Big[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\Big], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\Big]\Bigg)\Bigg) \Big/$$

$$\left(2 \sqrt{2} (c d^2 - b d e + a e^2) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\Bigg(21\, i\, A c^4 d^2 e^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}$$

$$\sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \Bigg(\text{EllipticE}\Big[i\, \text{ArcSinh}\Big[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\Big], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\Big] -$$

$$\text{EllipticF}\Big[i\, \text{ArcSinh}\Big[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\Big], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\Big]\Bigg)\Bigg) \Big/$$

$$\left(\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(3\,\mathbb{i}\,b^2 c^2 C d^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(9\,\mathbb{i}\,a c^3 C d^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(15\,\mathrm{i}\,b^2\,B\,c^2\,d\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(21\,\mathrm{i}\,A\,b\,c^3\,d\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(12\,\mathbb{i}\,\sqrt{2}\,a\,B\,c^3\,d\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(2\,\mathbb{i}\,\sqrt{2}\,b^3\,c\,C\,d\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(15 i a b c^2 C d e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(6 i \sqrt{2} b^3 B c e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(21\, \mathrm{i}\, A b^2 c^2 e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(87\, \mathrm{i}\, a b B c^2 e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(63\,\mathrm{i}\,a A c^3 e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(4\,\mathrm{i}\sqrt{2}\,b^4 C e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(18\,\mathrm{i}\,\sqrt{2}\,a\,b^2\,c\,C\,e^4\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(21\,\mathrm{i}\,a^2\,c^2\,C\,e^4\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(8\,\mathbb{i}\,\sqrt{2}\,c^4\,C\,d^3\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(12\,\mathbb{i}\,\sqrt{2}\,B\,c^4\,d^2\,e\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(3\,\mathbb{i}\,b\,B\,c^3\,d\,e^2\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(21\, i \sqrt{2}\, A c^4 d e^2 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(3\, i\, b^2 c^2 C d e^2 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(3\,i\,\sqrt{2}\,a\,c^3\,C\,d\,e^2\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(6\,i\,\sqrt{2}\,b^2\,B\,c^2\,e^3\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(21\,i\,A\,b\,c^3\,e^3\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(15 i \sqrt{2} a B c^3 e^3 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(4 i \sqrt{2} b^3 c C e^3 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(27 i a b c^2 C e^3 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right) \Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\Bigg)\Bigg)$$

- **Problem 260: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b\,x+c\,x^2}\,\left(A+B\,x+C\,x^2\right)}{\sqrt{d+e\,x}}\,dx$$

Optimal (type 4, 668 leaves, 7 steps) :

$$-\frac{1}{105 c^2 e^3}2\sqrt{d+e x}\,(5 c e (3 b C d-7 A c e+a C e)-(4 c d-b e)(6 c C d-7 B c e+4 b C e)+3 c e (6 c C d-7 B c e+4 b C e) x)\sqrt{a+b x+c x^2}+$$

$$\frac{2 C\sqrt{d+e x}\,\left(a+b x+c x^2\right)^{3/2}}{7 c e}+$$

$$\left(\sqrt{2}\sqrt{b^2-4 a c}\,\left(5 c e (2 c d-b e)(3 b C d-7 A c e+a C e)-(6 c C d-7 B c e+4 b C e)\left(8 c^2 d^2-2 b^2 e^2-3 c e (b d-2 a e)\right)\right)\right.$$

$$\left.\sqrt{d+e x}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Bigg/$$

$$\left(105 c^3 e^4\sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\sqrt{a+b x+c x^2}\right)+$$

$$\left(2\sqrt{2}\sqrt{b^2-4 a c}\,\left(c d^2-b d e+a e^2\right)\left(4 b^2 C e^2+c e (8 b C d-7 b B e-10 a C e)+c^2\left(48 C d^2-14 e (4 B d-5 A e)\right)\right)\sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\right.$$

$$\left.\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Bigg/\left(105 c^3 e^4\sqrt{d+e x}\sqrt{a+b x+c x^2}\right)$$

Result (type 4, 9965 leaves) :

$$\sqrt{d+e x}$$

$$\left(\frac{2\left(24 c^2 C d^2-28 B c^2 d e-5 b c C d e+7 b B c e^2+35 A c^2 e^2-4 b^2 C e^2+10 a c C e^2\right)}{105 c^2 e^3}+\frac{2 (-6 c C d+7 B c e+b C e) x}{35 c e^2}+\frac{2 C x^2}{7 e}\right)\sqrt{a+x (b+c x)}+$$

$$\frac{1}{105\,c^2\,e^5\,\sqrt{a+b\,x+c\,x^2}}\,2\,\sqrt{a+x\,(b+c\,x)}\,\Bigg(\Bigg(\Big(-48\,c^3\,C\,d^3+56\,B\,c^3\,d^2\,e+16\,b\,c^2\,C\,d^2\,e-21\,b\,B\,c^2\,d\,e^2-70\,A\,c^3\,d\,e^2+9\,b^2\,c\,C\,d\,e^2-26\,a\,c^2\,C\,d\,e^2-$$

$$14\,b^2\,B\,c\,e^3+35\,A\,b\,c^2\,e^3+42\,a\,B\,c^2\,e^3+8\,b^3\,C\,e^3-29\,a\,b\,c\,C\,e^3\Big)\,(d+e\,x)^{3/2}$$

$$\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)\Bigg)\Bigg/\left(c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)+$$

$$\frac{1}{c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\Bigg(\Bigg(12\,i\,\sqrt{2}\,c^3\,C\,d^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(14\, i \sqrt{2}\, B c^3 d^2 e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(4\, i \sqrt{2}\, b c^2 C d^2 e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left((c d^2 - b d e + a e^2) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(21 i b B c^2 d e^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(2 \sqrt{2} (c d^2 - b d e + a e^2) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(35 i A c^3 d e^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(9\,\mathbb{i}\,b^2 c\, C\, d\, e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(13\,\mathbb{i}\,a\,c^2\, C\, d\, e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(7\,i\,b^2\,B\,c\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(35\,i\,A\,b\,c^2\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(21\, \mathbb{i}\, a B c^2 e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(2\, \mathbb{i}\sqrt{2}\, b^3 C e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(29\,\mathrm{i}\,a\,b\,c\,C\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(24\,\mathrm{i}\sqrt{2}\,c^3\,C\,d^2\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\;\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(28\,i\,\sqrt{2}\,B\,c^3\,d\,e\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(4\,i\,\sqrt{2}\,b\,c^2\,C\,d\,e\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(7\,i\,b\,B\,c^2\,e^2\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(35\,\mathbb{i}\,\sqrt{2}\,A\,c^3\,e^2\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(2\,\mathbb{i}\,\sqrt{2}\,b^2\,c\,C\,e^2\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(5\,\mathbb{i}\,\sqrt{2}\,a\,c^2\,C\,e^2\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\Bigg)\Bigg)$$

- **Problem 261: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{a+b\,x+c\,x^2}\,\left(A+B\,x+C\,x^2\right)}{(d+e\,x)^{3/2}}\,dx$$

Optimal (type 4, 749 leaves, 7 steps) :

$$-\frac{1}{15\,c\,e^3\,(c\,d^2-b\,d\,e+a\,e^2)}2\sqrt{d+e\,x}\,\Bigg(b\,C\,e^2\,(b\,d-a\,e)+c^2\,d\,\big(24\,C\,d^2-5\,e\,(4\,B\,d-3\,A\,e)\big)+c\,e\,\big(a\,e\,(9\,C\,d-5\,B\,e)-5\,b\,\big(5\,C\,d^2-4\,B\,d\,e+3\,A\,e^2\big)\big)+$$

$$3\,c\,e^2\left(5\,B\,c\,d+b\,C\,d-\frac{6\,c\,C\,d^2}{e}-5\,A\,c\,e-a\,C\,e\right)x\Bigg)\sqrt{a+b\,x+c\,x^2}-\frac{2\,\big(C\,d^2-e\,(B\,d-A\,e)\big)\,\big(a+b\,x+c\,x^2\big)^{3/2}}{e\,\big(c\,d^2-b\,d\,e+a\,e^2\big)\,\sqrt{d+e\,x}}-$$

$$\Bigg(\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\Big(2\,b^2\,C\,e^2+c\,e\,(8\,b\,C\,d-5\,b\,B\,e-6\,a\,C\,e)-c^2\,\big(48\,C\,d^2-10\,e\,(4\,B\,d-3\,A\,e)\big)\Big)\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\big(a+b\,x+c\,x^2\big)}{b^2-4\,a\,c}}$$

$$\text{EllipticE}\Bigg[\text{ArcSin}\Bigg[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\Bigg],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\big(b+\sqrt{b^2-4\,a\,c}\big)\,e}\Bigg]\Bigg)\Bigg/\left(15\,c^2\,e^4\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\big(b+\sqrt{b^2-4\,a\,c}\big)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\Bigg(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\Big(b\,C\,e^2\,(b\,d-a\,e)-2\,c^2\,d\,\big(24\,C\,d^2-5\,e\,(4\,B\,d-3\,A\,e)\big)-c\,e\,\big(2\,a\,e\,(9\,C\,d-5\,B\,e)-b\,\big(32\,C\,d^2-5\,e\,(5\,B\,d-3\,A\,e)\big)\big)\Big)$$

$$\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\big(b+\sqrt{b^2-4\,a\,c}\big)\,e}}\,\sqrt{-\frac{c\,\big(a+b\,x+c\,x^2\big)}{b^2-4\,a\,c}}$$

$$\text{EllipticF}\Bigg[\text{ArcSin}\Bigg[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\Bigg],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\big(b+\sqrt{b^2-4\,a\,c}\big)\,e}\Bigg]\Bigg)\Bigg/\left(15\,c^2\,e^4\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 13 240 leaves) :

$$\sqrt{d+e\,x}\,\sqrt{a+x\,(b+c\,x)}\left(\frac{2\,(-9\,c\,C\,d+5\,B\,c\,e+b\,C\,e)}{15\,c\,e^3}+\frac{2\,C\,x}{5\,e^2}-\frac{2\,\left(C\,d^2-B\,d\,e+A\,e^2\right)}{e^3\,(d+e\,x)}\right)+$$

$$\frac{1}{15\,c\,e^5\,\sqrt{a+b\,x+c\,x^2}}\sqrt{a+x\,(b+c\,x)}\left(\left(2\,\left(48\,c^2\,C\,d^2-40\,B\,c^2\,d\,e-8\,b\,c\,C\,d\,e+5\,b\,B\,c\,e^2+30\,A\,c^2\,e^2-2\,b^2\,C\,e^2+6\,a\,c\,C\,e^2\right)\right.\right.$$

$$\left.\left.(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)\right)\Bigg/\left(c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)-\right.$$

$$\frac{1}{c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\,2\,(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\left(\left(12\,i\,\sqrt{2}\,c^3\,C\,d^4\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.\right.$$

$$\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Bigg/$$

$$\left((c d^2 - b d e + a e^2) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\left(10 i \sqrt{2} B c^3 d^3 e \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left((c d^2 - b d e + a e^2) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\left(14 i \sqrt{2} b c^2 C d^3 e \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\sqrt{1 - \frac{2 (c d^2 - b d e + a e^2)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(45\, i\, b B c^2 d^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(15\, i\, A c^3 d^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(3\,\mathbb{i}\,b^2\,c\,C\,d^2\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(27\,\mathbb{i}\,a\,c^2\,C\,d^2\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(5 i b^2 B c d e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(15 i A b c^2 d e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(10\, i \sqrt{2}\, a B c^2 d e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i\, b^3 C d e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(7\,\mathrm{i}\,a\,b\,c\,C\,d\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(5\,\mathrm{i}\,a\,b\,B\,c\,e^4\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(15\,\mathbb{i}\,a A c^2 e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(\mathbb{i}\,a b^2 C e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(3\,i\,a^2\,c\,C\,e^4\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(24\,i\,\sqrt{2}\,c^3\,C\,d^3\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(20\,\mathrm{i}\,\sqrt{2}\,B\,c^3\,d^2\,e\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(16\,\mathrm{i}\,\sqrt{2}\,b\,c^2\,C\,d^2\,e\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(25\,\mathrm{i}\,b\,B\,c^2\,d\,e^2\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(15\,\mathbb{i}\,\sqrt{2}\,A c^3 d e^2\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(\mathbb{i}\,b^2 c C d e^2\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(9\,\mathbb{i}\,\sqrt{2}\,a c^2 C d e^2\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\text{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right) -$$

$$\left(15 i A b c^2 e^3 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right) -$$

$$\left(5 i \sqrt{2}\, a B c^2 e^3 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right) +$$

$$\left(i\, a\, b\, c\, C\, e^3 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left.\left(\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)\right)\right)$$

- **Problem 262: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b x+c x^2}\left(A+B x+C x^2\right)}{(d+e x)^{5/2}}\, dx$$

Optimal (type 4, 712 leaves, 7 steps) :

$$-\left(2\left(e\,(b\,d-a\,e)\,(7\,C\,d-3\,B\,e)-c\,d\left(8\,C\,d^2-e\,(4\,B\,d-A\,e)\right)+e^2\left(B\,c\,d+b\,C\,d-\frac{2\,c\,C\,d^2}{e}-A\,c\,e-a\,C\,e\right)x\right)\sqrt{a+b\,x+c\,x^2}\right)\Big/$$

$$\left(3\,e^3\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{d+e\,x}\right)-\frac{2\left(C\,d^2-e\,(B\,d-A\,e)\right)\left(a+b\,x+c\,x^2\right)^{3/2}}{3\,e\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)^{3/2}}+$$

$$\left(\sqrt{2}\,\sqrt{b^2-4\,a\,c}\left(2\left(4\,c\,d-\frac{b\,e}{2}\right)\left(B\,c\,d+b\,C\,d-\frac{2\,c\,C\,d^2}{e}-A\,c\,e-a\,C\,e\right)+6\,c\,(b\,d\,(C\,d-B\,e)+e\,(A\,c\,d-a\,C\,d+a\,B\,e))\right)\right.$$

$$\left.\sqrt{d+e\,x}\,\sqrt{-\frac{c\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\;e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\right]\right)\Big/$$

$$\left(3\,c\,e^3\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}}\;\sqrt{a+b\,x+c\,x^2}\right)-$$

$$\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(e\,(8\,b\,C\,d-3\,b\,B\,e-2\,a\,C\,e)-2\,c\left(8\,C\,d^2-e\,(4\,B\,d-A\,e)\right)\right)\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}}\;\sqrt{-\frac{c\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\;e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\right]\right)\Big/\left(3\,c\,e^4\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 8456 leaves) :

$$\sqrt{d+e x}\ \sqrt{a+x\,(b+c x)}\ \left(\frac{2\,C}{3\,e^3}-\frac{2\left(C\,d^2-B\,d\,e+A\,e^2\right)}{3\,e^3\,(d+e x)^2}-\frac{2\left(-8\,c\,C\,d^3+5\,B\,c\,d^2\,e+7\,b\,C\,d^2\,e-4\,b\,B\,d\,e^2-2\,A\,c\,d\,e^2-6\,a\,C\,d\,e^2+A\,b\,e^3+3\,a\,B\,e^3\right)}{3\,e^3\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e x)}\right)-$$

$$\frac{1}{3\,e^5\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{a+b\,x+c\,x^2}}$$

$$2\sqrt{a+x\,(b+c x)}\ \Bigg(\Bigg(\left(16\,c^2\,C\,d^3-8\,B\,c^2\,d^2\,e-16\,b\,c\,C\,d^2\,e+7\,b\,B\,c\,d\,e^2+2\,A\,c^2\,d\,e^2+b^2\,C\,d\,e^2+14\,a\,c\,C\,d\,e^2-A\,b\,c\,e^3-6\,a\,B\,c\,e^3-a\,b\,C\,e^3\right)$$

$$(d+e x)^{3/2}\left(c+\frac{c\,d^2}{(d+e x)^2}-\frac{b\,d\,e}{(d+e x)^2}+\frac{a\,e^2}{(d+e x)^2}-\frac{2\,c\,d}{d+e x}+\frac{b\,e}{d+e x}\right)\Bigg)\Bigg/\left(c\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e x}+\frac{a\,e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)-$$

$$\frac{1}{c\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e x}+\frac{a\,e}{d+e x}\right)}{d+e x}\right)}{e^2}}}\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e x)\sqrt{c+\frac{c\,d^2}{(d+e x)^2}-\frac{b\,d\,e}{(d+e x)^2}+\frac{a\,e^2}{(d+e x)^2}-\frac{2\,c\,d}{d+e x}+\frac{b\,e}{d+e x}}$$

$$\Bigg(\Bigg(4\,i\sqrt{2}\,c^2\,C\,d^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e x)}}\ \Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(2\,\mathrm{i}\,\sqrt{2}\,B\,c^2\,d^2\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\ \Bigg(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(4\,\mathrm{i}\,\sqrt{2}\,b\,c\,C\,d^2\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\ \Bigg(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(7\,i\,b\,B\,c\,d\,e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(i\,A\,c^2\,d\,e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(i\,b^2 C d e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(7\,i\,a c C d e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(\mathbb{i}\,A\,b\,c\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(3\,\mathbb{i}\,a\,B\,c\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(i\,a\,b\,C\,e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(8\,i\sqrt{2}\,c^2\,C\,d^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(4\,i\,\sqrt{2}\,B\,c^2\,d\,e\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\;\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(4\,i\,\sqrt{2}\,b\,c\,C\,d\,e\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\;\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(3\,i\,b\,B\,c\,e^2\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\;\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i\sqrt{2}\,A\,c^2 e^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i\sqrt{2}\,a\,c\,C\,e^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\Bigg)$$

- **Problem 263: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{a+b\,x+c\,x^2}\,\left(A+B\,x+C\,x^2\right)}{(d+e\,x)^{7/2}}\,dx$$

Optimal (type 4, 992 leaves, 7 steps):

$$-\frac{1}{15\,e^3\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,(d+e\,x)^{3/2}}2\,\Big(c^2\,d^3\,\left(24\,C\,d^2-e\,(4\,B\,d+A\,e)\right)+$$
$$e^2\,\left(15\,b^2\,C\,d^3+5\,a^2\,e^2\,(C\,d+B\,e)-a\,b\,e\,\left(22\,C\,d^2+3\,B\,d\,e+2\,A\,e^2\right)\right)-c\,d\,e\,\left(b\,d\,\left(41\,C\,d^2-6\,B\,d\,e+A\,e^2\right)-a\,e\,\left(37\,C\,d^2-7\,B\,d\,e+7\,A\,e^2\right)\right)+$$
$$e\,\Big(5\,c^2\,d^2\,\left(6\,C\,d^2-e\,(B\,d+A\,e)\right)+e^2\,\left(15\,a^2\,C\,e^2-5\,a\,b\,e\,(8\,C\,d-B\,e)+b^2\,\left(23\,C\,d^2-3\,B\,d\,e-2\,A\,e^2\right)\right)-$$
$$c\,e\,\left(5\,b\,d\,\left(11\,C\,d^2-2\,B\,d\,e-A\,e^2\right)-a\,e\,\left(53\,C\,d^2-13\,B\,d\,e+3\,A\,e^2\right)\right)\Big)\,x\Big)\,\sqrt{a+b\,x+c\,x^2}-\frac{2\,\left(C\,d^2-e\,(B\,d-A\,e)\right)\,\left(a+b\,x+c\,x^2\right)^{3/2}}{5\,e\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,(d+e\,x)^{5/2}}+$$

$$\Bigg(\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\Big(2\,c^2\,d^2\,\left(24\,C\,d^2-e\,(4\,B\,d+A\,e)\right)+e^2\,\left(30\,a^2\,C\,e^2-5\,a\,b\,e\,(14\,C\,d-B\,e)+b^2\,\left(38\,C\,d^2-3\,B\,d\,e-2\,A\,e^2\right)\right)-$$
$$c\,e\,\left(b\,d\,\left(88\,C\,d^2-13\,B\,d\,e-2\,A\,e^2\right)-2\,a\,e\,\left(43\,C\,d^2-8\,B\,d\,e+3\,A\,e^2\right)\right)\Big)$$
$$\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\Bigg)\Big/$$

$$\left(15 e^4 \left(c d^2 - b d e + a e^2\right)^2 \sqrt{\frac{c (d + e x)}{2 c d - \left(b + \sqrt{b^2 - 4 a c}\right) e}} \sqrt{a + b x + c x^2}\right) -$$

$$\left(2 \sqrt{2} \sqrt{b^2 - 4 a c} \left(15 b C e^2 (b d - a e) + 2 c^2 d \left(24 C d^2 - e (4 B d + A e)\right) + c e \left(10 a e (5 C d - B e) - b \left(64 C d^2 - 9 B d e - A e^2\right)\right)\right)\right.$$

$$\left.\sqrt{\frac{c (d + e x)}{2 c d - \left(b + \sqrt{b^2 - 4 a c}\right) e}} \sqrt{-\frac{c \left(a + b x + c x^2\right)}{b^2 - 4 a c}} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x}{\sqrt{b^2 - 4 a c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2 - 4 a c}\, e}{2 c d - \left(b + \sqrt{b^2 - 4 a c}\right) e}\right]\right) /$$

$$\left(15 c e^4 \left(c d^2 - b d e + a e^2\right) \sqrt{d + e x} \sqrt{a + b x + c x^2}\right)$$

Result (type 4, 12 997 leaves) :

$$\sqrt{d + e x} \sqrt{a + x (b + c x)} \left(-\frac{2 \left(C d^2 - B d e + A e^2\right)}{5 e^3 (d + e x)^3} - \frac{2 \left(-12 c C d^3 + 7 B c d^2 e + 11 b C d^2 e - 6 b B d e^2 - 2 A c d e^2 - 10 a C d e^2 + A b e^3 + 5 a B e^3\right)}{15 e^3 \left(c d^2 - b d e + a e^2\right) (d + e x)^2} -\right.$$

$$\left(2 \left(33 c^2 C d^4 - 8 B c^2 d^3 e - 58 b c C d^3 e + 13 b B c d^2 e^2 - 2 A c^2 d^2 e^2 + 23 b^2 C d^2 e^2 + 56 a c C d^2 e^2 - 3 b^2 B d e^3 + 2 A b c d e^3 -\right.\right.$$

$$\left.\left.\left.16 a B c d e^3 - 40 a b C d e^3 - 2 A b^2 e^4 + 5 a b B e^4 + 6 a A c e^4 + 15 a^2 C e^4\right)\right) / \left(15 e^3 \left(c d^2 - b d e + a e^2\right)^2 (d + e x)\right)\right) -$$

$$\frac{1}{15 e^5 \left(c d^2 - b d e + a e^2\right)^2 \sqrt{a + b x + c x^2}} 2 \sqrt{a + x (b + c x)} \left(\left(\left(-48 c^2 C d^4 + 8 B c^2 d^3 e + 88 b c C d^3 e - 13 b B c d^2 e^2 + 2 A c^2 d^2 e^2 -\right.\right.\right.$$

$$38 b^2 C d^2 e^2 - 86 a c C d^2 e^2 + 3 b^2 B d e^3 - 2 A b c d e^3 + 16 a B c d e^3 + 70 a b C d e^3 + 2 A b^2 e^4 - 5 a b B e^4 - 6 a A c e^4 - 30 a^2 C e^4\Big)$$

$$(d+e x)^{3/2}\left(c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)\Bigg/\left(\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)+$$

$$\frac{1}{\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}\left(c d^2-b d e+a e^2\right)(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\Bigg(\Bigg(12\, i \sqrt{2}\, c^2 C d^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(2\, i \sqrt{2}\, B c^2 d^3 e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(22\,\mathbb{i}\sqrt{2}\,b\,c\,C\,d^3 e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(13\,\mathbb{i}\,b\,B\,c\,d^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Bigg/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(\mathrm{i}\,A c^2 d^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Bigg/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(19\,\mathrm{i}\,b^2 C d^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(43\, i\, a\, c\, C\, d^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(3\, i\, b^2 B\, d\, e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i\,A\,b\,c\,d\,e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(4\,i\sqrt{2}\,a\,B\,c\,d\,e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(35 i a b C d e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(i A b^2 e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(5\,\mathbb{i}\,a\,b\,B\,e^4\,\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(3\,\mathbb{i}\,a\,A\,c\,e^4\,\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(15\,i\,a^2\,C\,e^4\,\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(24\,i\,\sqrt{2}\,c^2\,C\,d^3\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(4\,i\,\sqrt{2}\,B\,c^2\,d^2\,e\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\ \sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(32\,i\,\sqrt{2}\,b\,c\,C\,d^2\,e\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\ \sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(9\,i\,b\,B\,c\,d\,e^2\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(i\sqrt{2}\,A\,c^2\,d\,e^2\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(15\,i\,b^2\,C\,d\,e^2\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(25\,\mathbb{i}\,\sqrt{2}\,a\,c\,C\,d\,e^2\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(\mathbb{i}\,A\,b\,c\,e^3\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(5\,\mathbb{i}\,\sqrt{2}\,a\,B\,c\,e^3\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(15\,i\,a\,b\,C\,e^3\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left.\left(\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)\right)\right)$$

- **Problem 264: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{a+b x+c x^2}\left(A+B x+C x^2\right)}{(d+e x)^{9/2}}\,dx$$

Optimal (type 4, 1363 leaves, 8 steps) :

$$\frac{1}{105\, e^3 \left(c\, d^2 - b\, d\, e + a\, e^2\right)^3 \sqrt{d + e\, x}}\, 2 \left(2\, c^3\, d^3 \left(24\, C\, d^2 + e\, (4\, B\, d + 3\, A\, e)\right) - b\, e^3 \left(35\, a^2\, C\, e^2 - 14\, a\, b\, e\, (3\, C\, d + B\, e) + b^2 \left(15\, C\, d^2 + 6\, B\, d\, e + 8\, A\, e^2\right)\right) +\right.$$

$$c^2\, d\, e \left(2\, a\, e \left(69\, C\, d^2 + e\, (15\, B\, d - 29\, A\, e)\right) - b\, d \left(128\, C\, d^2 + e\, (19\, B\, d + 9\, A\, e)\right)\right) +$$

$$\left.c\, e^2 \left(14\, a^2\, e^2\, (11\, C\, d - 3\, B\, e) - a\, b\, e \left(237\, C\, d^2 + e\, (B\, d - 29\, A\, e)\right) + b^2\, d \left(103\, C\, d^2 + e\, (9\, B\, d + 19\, A\, e)\right)\right)\right)$$

$$\sqrt{a + b\, x + c\, x^2} - \frac{1}{105\, e^3 \left(c\, d^2 - b\, d\, e + a\, e^2\right)^2 (d + e\, x)^{5/2}}$$

$$2 \left(c^2\, d^3 \left(24\, C\, d^2 + e\, (4\, B\, d + 3\, A\, e)\right) - e^2 \left(7\, a^2\, e^2\, (C\, d - 3\, B\, e) - b^2\, d \left(15\, C\, d^2 + 6\, B\, d\, e + 8\, A\, e^2\right) + a\, b\, e \left(12\, C\, d^2 + 23\, B\, d\, e + 12\, A\, e^2\right)\right) -\right.$$

$$c\, d\, e \left(b\, d \left(43\, C\, d^2 + 6\, B\, d\, e + 15\, A\, e^2\right) - a\, e \left(33\, C\, d^2 + 9\, B\, d\, e + 19\, A\, e^2\right)\right) + e \left(7\, c^2\, d^2 \left(6\, C\, d^2 + e\, (B\, d - 3\, A\, e)\right) +\right.$$

$$\left.\left.e^2 \left(35\, a^2\, C\, e^2 - 7\, a\, b\, e\, (12\, C\, d - B\, e) + b^2 \left(45\, C\, d^2 - 3\, B\, d\, e - 4\, A\, e^2\right)\right) + c\, e \left(a\, e \left(93\, C\, d^2 - 9\, B\, d\, e - 5\, A\, e^2\right) - b \left(91\, C\, d^3 - 21\, A\, d\, e^2\right)\right)\right) x\right)$$

$$\sqrt{a + b\, x + c\, x^2} - \frac{2 \left(C\, d^2 - e\, (B\, d - A\, e)\right) \left(a + b\, x + c\, x^2\right)^{3/2}}{7\, e \left(c\, d^2 - b\, d\, e + a\, e^2\right) (d + e\, x)^{7/2}} - \frac{1}{105\, e^4 \left(c\, d^2 - b\, d\, e + a\, e^2\right)^3 \sqrt{\frac{c\, (d + e\, x)}{2\, c\, d - \left(b + \sqrt{b^2 - 4\, a\, c}\right) e}} \sqrt{a + b\, x + c\, x^2}}$$

$$\sqrt{2} \sqrt{b^2 - 4\, a\, c} \left(2\, c^3\, d^3 \left(24\, C\, d^2 + e\, (4\, B\, d + 3\, A\, e)\right) - b\, e^3 \left(35\, a^2\, C\, e^2 - 14\, a\, b\, e\, (3\, C\, d + B\, e) + b^2 \left(15\, C\, d^2 + 6\, B\, d\, e + 8\, A\, e^2\right)\right) +\right.$$

$$c^2\, d\, e \left(2\, a\, e \left(69\, C\, d^2 + e\, (15\, B\, d - 29\, A\, e)\right) - b\, d \left(128\, C\, d^2 + e\, (19\, B\, d + 9\, A\, e)\right)\right) +$$

$$\left.c\, e^2 \left(14\, a^2\, e^2\, (11\, C\, d - 3\, B\, e) - a\, b\, e \left(237\, C\, d^2 + e\, (B\, d - 29\, A\, e)\right) + b^2\, d \left(103\, C\, d^2 + e\, (9\, B\, d + 19\, A\, e)\right)\right)\right)$$

$$\sqrt{d + e\, x} \sqrt{-\frac{c \left(a + b\, x + c\, x^2\right)}{b^2 - 4\, a\, c}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b + \sqrt{b^2 - 4\, a\, c} + 2\, c\, x}{\sqrt{b^2 - 4\, a\, c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2 - 4\, a\, c}\ e}{2\, c\, d - \left(b + \sqrt{b^2 - 4\, a\, c}\right) e}\right] +$$

$$\frac{1}{105\, e^4 \left(c\, d^2 - b\, d\, e + a\, e^2\right)^2 \sqrt{d + e\, x} \sqrt{a + b\, x + c\, x^2}}\, 2 \sqrt{2} \sqrt{b^2 - 4\, a\, c} \left(2\, c^2\, d^2 \left(24\, C\, d^2 + e\, (4\, B\, d + 3\, A\, e)\right) +\right.$$

$$\left.c\, e \left(2\, a\, e \left(51\, C\, d^2 + e\, (12\, B\, d - 5\, A\, e)\right) - b\, d \left(104\, C\, d^2 + 3\, e\, (5\, B\, d + 2\, A\, e)\right)\right) + e^2 \left(70\, a^2\, C\, e^2 - 7\, a\, b\, e\, (18\, C\, d + B\, e) + b^2 \left(60\, C\, d^2 + e\, (3\, B\, d + 4\, A\, e)\right)\right)\right)$$

$$\sqrt{\frac{c\, (d + e\, x)}{2\, c\, d - \left(b + \sqrt{b^2 - 4\, a\, c}\right) e}} \sqrt{-\frac{c \left(a + b\, x + c\, x^2\right)}{b^2 - 4\, a\, c}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b + \sqrt{b^2 - 4\, a\, c} + 2\, c\, x}{\sqrt{b^2 - 4\, a\, c}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{b^2 - 4\, a\, c}\ e}{2\, c\, d - \left(b + \sqrt{b^2 - 4\, a\, c}\right) e}\right]$$

Result (type 4, 19 853 leaves) :

$$\sqrt{d + e\, x} \sqrt{a + x\, (b + c\, x)} \left(-\frac{2 \left(C\, d^2 - B\, d\, e + A\, e^2\right)}{7\, e^3\, (d + e\, x)^4} - \frac{2 \left(-16\, c\, C\, d^3 + 9\, B\, c\, d^2\, e + 15\, b\, C\, d^2\, e - 8\, b\, B\, d\, e^2 - 2\, A\, c\, d\, e^2 - 14\, a\, C\, d\, e^2 + A\, b\, e^3 + 7\, a\, B\, e^3\right)}{35\, e^3 \left(c\, d^2 - b\, d\, e + a\, e^2\right) (d + e\, x)^3} -\right.$$

$$\left(2 \left(57\, c^2\, C\, d^4 - 8\, B\, c^2\, d^3\, e - 106\, b\, c\, C\, d^3\, e + 15\, b\, B\, c\, d^2\, e^2 - 6\, A\, c^2\, d^2\, e^2 + 45\, b^2\, C\, d^2\, e^2 + 108\, a\, c\, C\, d^2\, e^2 - 3\, b^2\, B\, d\, e^3 + 6\, A\, b\, c\, d\, e^3 -\right.\right.$$

$$24\,a\,B\,c\,d\,e^3-84\,a\,b\,C\,d\,e^3-4\,A\,b^2\,e^4+7\,a\,b\,B\,e^4+10\,a\,A\,c\,e^4+35\,a^2\,C\,e^4\Big)\Big)\Big/\left(105\,e^3\left(c\,d^2-b\,d\,e+a\,e^2\right)^2(d+e\,x)^2\right)-$$

$$\frac{1}{105\,e^3\left(c\,d^2-b\,d\,e+a\,e^2\right)^3(d+e\,x)}\,2\,\big(-48\,c^3\,C\,d^5-8\,B\,c^3\,d^4\,e+128\,b\,c^2\,C\,d^4\,e+19\,b\,B\,c^2\,d^3\,e^2-6\,A\,c^3\,d^3\,e^2-103\,b^2\,c\,C\,d^3\,e^2-138\,a\,c^2\,C\,d^3\,e^2-9\,b^2\,B\,c\,d^2\,e^3+9\,A\,b\,c^2\,d^2\,e^3-30\,a\,B\,c^2\,d^2\,e^3+15\,b^3\,C\,d^2\,e^3+237\,a\,b\,c\,C\,d^2\,e^3+6\,b^3\,B\,d\,e^4-19\,A\,b^2\,c\,d\,e^4+a\,b\,B\,c\,d\,e^4+58\,a\,A\,c^2\,d\,e^4-42\,a\,b^2\,C\,d\,e^4-154\,a^2\,c\,C\,d\,e^4+8\,A\,b^3\,e^5-14\,a\,b^2\,B\,e^5-29\,a\,A\,b\,c\,e^5+42\,a^2\,B\,c\,e^5+35\,a^2\,b\,C\,e^5\big)-$$

$$\frac{1}{105\,e^5\left(c\,d^2-b\,d\,e+a\,e^2\right)^3\sqrt{a+b\,x+c\,x^2}}\,2\,c\,\sqrt{a+x\,(b+c\,x)}\left(\frac{1}{c\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\right.$$

$$\big(48\,c^3\,C\,d^5+8\,B\,c^3\,d^4\,e-128\,b\,c^2\,C\,d^4\,e-19\,b\,B\,c^2\,d^3\,e^2+6\,A\,c^3\,d^3\,e^2+103\,b^2\,c\,C\,d^3\,e^2+138\,a\,c^2\,C\,d^3\,e^2+9\,b^2\,B\,c\,d^2\,e^3-9\,A\,b\,c^2\,d^2\,e^3+30\,a\,B\,c^2\,d^2\,e^3-15\,b^3\,C\,d^2\,e^3-237\,a\,b\,c\,C\,d^2\,e^3-6\,b^3\,B\,d\,e^4+19\,A\,b^2\,c\,d\,e^4-a\,b\,B\,c\,d\,e^4-58\,a\,A\,c^2\,d\,e^4+42\,a\,b^2\,C\,d\,e^4+154\,a^2\,c\,C\,d\,e^4-8\,A\,b^3\,e^5+14\,a\,b^2\,B\,e^5+29\,a\,A\,b\,c\,e^5-42\,a^2\,B\,c\,e^5-35\,a^2\,b\,C\,e^5\big)\,(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)-$$

$$\frac{1}{c\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\left(\left(12\,i\,\sqrt{2}\,c^3\,C\,d^5\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(2\,i\,\sqrt{2}\,B\,c^3\,d^4\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(32\,i\,\sqrt{2}\,b\,c^2\,C\,d^4\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Big],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Big]\Big)\Big)\Big/$$

$$\Big(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\Big)-$$

$$\Big(19\,\mathrm{i}\,b\,B\,c^2\,d^3\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\ \Big(\text{EllipticE}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Big],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Big]-$$

$$\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Big],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Big]\Big)\Big)\Big/$$

$$\Big(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\Big)+$$

$$\Big(3\,\mathrm{i}\,A\,c^3\,d^3\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\ \Big(\text{EllipticE}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Big],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Big]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(103\,i\,b^2 c\,C\,d^3 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(69\,i\,a\,c^2\,C\,d^3 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(9\,i\,b^2 B c d^2 e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(9\,i\,A b c^2 d^2 e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(15\, i\, a B c^2 d^2 e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(15\, i\, b^3 C d^2 e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg) \Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right) -$$

$$\Bigg(237\, i\, a b c C d^2 e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right] -$$

$$\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg) \Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right) -$$

$$\Bigg(3\, i\, b^3 B d e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right] -$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(19\,i\,A b^2 c d e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(i\,a b B c d e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(29\,i\,a\,A\,c^2 d\,e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(21\,i\,a\,b^2 C\,d\,e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(77\,i\,a^2 c\,C\,d\,e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(2\,i\sqrt{2}\,A\,b^3 e^5\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left((c d^2-b d e+a e^2)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(7\,i\,a b^2 B e^5\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2(c d^2-b d e+a e^2)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2(c d^2-b d e+a e^2)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\,(c d^2-b d e+a e^2)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(29\,i\,a A b c e^5\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2(c d^2-b d e+a e^2)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2(c d^2-b d e+a e^2)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(21\,\mathbb{i}\,a^2\,B\,c\,e^5\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(35\,\mathbb{i}\,a^2\,b\,C\,e^5\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(2\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(24\,\mathbb{i}\sqrt{2}\,c^3\,C\,d^4\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(4\,\mathbb{i}\sqrt{2}\,B\,c^3\,d^3\,e\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(52\,i\,\sqrt{2}\,b\,c^2\,C\,d^3\,e\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\text{EllipticF}\Bigg[i\,\text{ArcSinh}\Bigg[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Bigg],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Bigg]\Bigg)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(15\,i\,b\,B\,c^2\,d^2\,e^2\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\text{EllipticF}\Bigg[i\,\text{ArcSinh}\Bigg[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Bigg],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Bigg]\Bigg)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(3\,i\,\sqrt{2}\,A\,c^3\,d^2\,e^2\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\text{EllipticF}\Bigg[i\,\text{ArcSinh}\Bigg[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Bigg],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Bigg]\Bigg)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(30\,i\,\sqrt{2}\,b^2\,c\,C\,d^2\,e^2\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(51\,i\,\sqrt{2}\,a\,c^2\,C\,d^2\,e^2\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(3\,i\,b^2\,B\,c\,d\,e^3\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(3\,i\sqrt{2}\,A b c^2 d e^3\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(12\,i\sqrt{2}\,a B c^2 d e^3\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(63\, i \sqrt{2}\, a\, b\, c\, C\, d\, e^3 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(2\, i \sqrt{2}\, A\, b^2\, c\, e^4 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(7\, i\, a\, b\, B\, c\, e^4 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(5\,i\,\sqrt{2}\,a\,A\,c^2\,e^4\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(35\,i\,\sqrt{2}\,a^2\,c\,C\,e^4\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left.\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)$$

- **Problem 265: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b\,x+c\,x^2}\,\left(A+B\,x+C\,x^2\right)}{(d+e\,x)^{11/2}}\,dx$$

Optimal (type 4, 1904 leaves, 9 steps) :

$$\Big(2\left(2c^3d^3\left(8Cd^2+e\,(4Bd+5Ae)\right)+3c^2de\left(2ae\left(9Cd^2+7Bde-9Ae^2\right)-bd\left(16Cd^2+7Bde+5Ae^2\right)\right)+\right.$$
$$3ce^2\left(2a^2e^2\,(17Cd-5Be)-abe\left(41Cd^2+5Bde-9Ae^2\right)+b^2d\left(15Cd^2+3Bde+7Ae^2\right)\right)-$$
$$\left.be^3\left(21a^2Ce^2-6abe\,(3Cd+2Be)+b^2\left(5Cd^2+4Bde+8Ae^2\right)\right)\right)\sqrt{a+bx+cx^2}\Big)\Big/\left(315e^3\left(cd^2-bde+ae^2\right)^3(d+ex)^{3/2}\right)+$$

$$\frac{1}{315e^3\left(cd^2-bde+ae^2\right)^4\sqrt{d+ex}}\,2\left(2c^4d^4\left(8Cd^2+e\,(4Bd+5Ae)\right)+2b^2e^4\left(21a^2Ce^2-6abe\,(3Cd+2Be)+b^2\left(5Cd^2+4Bde+8Ae^2\right)\right)-\right.$$
$$6c^2e^2\left(abde\left(30Cd^2-5Bde-34Ae^2\right)-a^2e^2\left(30Cd^2-36Bde+7Ae^2\right)-b^2d^2\left(11Cd^2+3Bde+11Ae^2\right)\right)-$$
$$ce^3\left(126a^3Ce^3-3a^2be^2\,(12Cd+29Be)-6ab^2e\left(5Cd^2+7Bde-12Ae^2\right)+b^3d\left(20Cd^2+25Bde+56Ae^2\right)\right)+$$
$$\left.c^3d^2e\left(6ae\left(11Cd^2+8Bde-34Ae^2\right)-bd\left(56Cd^2+5e\,(5Bd+4Ae)\right)\right)\right)\sqrt{a+bx+cx^2}-\frac{1}{105e^3\left(cd^2-bde+ae^2\right)^2(d+ex)^{7/2}}$$

$$2\left(c^2d^3\left(8Cd^2+e\,(4Bd+5Ae)\right)-e^2\left(3a^2e^2\,(3Cd-5Be)-abe\left(2Cd^2-17Bde-10Ae^2\right)-b^2d\left(5Cd^2+4Bde+8Ae^2\right)\right)-\right.$$
$$cde\left(3bd\left(5Cd^2+2Bde+5Ae^2\right)-ae\left(7Cd^2+11Bde+13Ae^2\right)\right)+e\left(3c^2d^2\left(6Cd^2+e\,(3Bd-5Ae)\right)+\right.$$
$$\left.\left.ce\left(ae\left(47Cd^2+Bde-7Ae^2\right)-3bd\left(15Cd^2+2Bde-5Ae^2\right)\right)+e^2\left(21a^2Ce^2-3abe\,(16Cd-Be)+b^2\left(25Cd^2-e\,(Bd+2Ae)\right)\right)\right)x\right)$$

$$\sqrt{a+bx+cx^2}-\frac{2\left(Cd^2-e\,(Bd-Ae)\right)\left(a+bx+cx^2\right)^{3/2}}{9e\left(cd^2-bde+ae^2\right)(d+ex)^{9/2}}-\frac{1}{315e^4\left(cd^2-bde+ae^2\right)^4\sqrt{\frac{c\,(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\,\sqrt{a+bx+cx^2}}$$

$$\sqrt{2}\,\sqrt{b^2-4ac}\,\left(2c^4d^4\left(8Cd^2+e\,(4Bd+5Ae)\right)+2b^2e^4\left(21a^2Ce^2-6abe\,(3Cd+2Be)+b^2\left(5Cd^2+4Bde+8Ae^2\right)\right)-\right.$$
$$6c^2e^2\left(abde\left(30Cd^2-5Bde-34Ae^2\right)-a^2e^2\left(30Cd^2-36Bde+7Ae^2\right)-b^2d^2\left(11Cd^2+3Bde+11Ae^2\right)\right)-$$
$$ce^3\left(126a^3Ce^3-3a^2be^2\,(12Cd+29Be)-6ab^2e\left(5Cd^2+7Bde-12Ae^2\right)+b^3d\left(20Cd^2+25Bde+56Ae^2\right)\right)+$$
$$\left.c^3d^2e\left(6ae\left(11Cd^2+8Bde-34Ae^2\right)-bd\left(56Cd^2+5e\,(5Bd+4Ae)\right)\right)\right)$$

$$\sqrt{d+ex}\,\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],\,-\frac{2\sqrt{b^2-4ac}\;e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right]+$$

$$\frac{1}{315e^4\left(cd^2-bde+ae^2\right)^3\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}$$
$$2\sqrt{2}\,\sqrt{b^2-4ac}\,\left(2c^3d^3\left(8Cd^2+e\,(4Bd+5Ae)\right)+3c^2de\left(2ae\left(9Cd^2+7Bde-9Ae^2\right)-bd\left(16Cd^2+7Bde+5Ae^2\right)\right)+\right.$$
$$3ce^2\left(2a^2e^2\,(17Cd-5Be)-abe\left(41Cd^2+5Bde-9Ae^2\right)+b^2d\left(15Cd^2+3Bde+7Ae^2\right)\right)-$$

$$b\,e^3\left(21\,a^2\,C\,e^2-6\,a\,b\,e\,(3\,C\,d+2\,B\,e)+b^2\left(5\,C\,d^2+4\,B\,d\,e+8\,A\,e^2\right)\right)\Big)$$

$$\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}}\;\sqrt{-\frac{c\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\;e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)e}\right]$$

Result (type 4, 29 140 leaves) : Display of huge result suppressed!

- **Problem 266: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{(d+e\,x)^{3/2}\left(A+B\,x+C\,x^2\right)}{\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 724 leaves, 8 steps) :

$$\frac{2\left(24 b^2 C e^2-c e (15 b C d+28 b B e+25 a C e)-c^2\left(6 C d^2-7 e (3 B d+5 A e)\right)\right)\sqrt{d+e x}\sqrt{a+b x+c x^2}}{105 c^3 e}-$$

$$\frac{2 (2 c C d-7 B c e+6 b C e)(d+e x)^{3/2}\sqrt{a+b x+c x^2}}{35 c^2 e}+\frac{2 C (d+e x)^{5/2}\sqrt{a+b x+c x^2}}{7 c e}-$$

$$\left(\sqrt{2}\sqrt{b^2-4 a c}\left(48 b^3 C e^3-8 b c e^2 (9 b C d+7 b B e+13 a C e)+c^3 d\left(6 C d^2-7 e (3 B d+20 A e)\right)+\right.\right.$$

$$\left.\left.c^2 e\left(a e (82 C d+63 B e)+b\left(12 C d^2+91 B d e+70 A e^2\right)\right)\right)\sqrt{d+e x}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Bigg/\left(105 c^4 e^2\sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\sqrt{a+b x+c x^2}\right)-$$

$$\left(2\sqrt{2}\sqrt{b^2-4 a c}\left(c d^2-b d e+a e^2\right)\left(24 b^2 C e^2-c e (15 b C d+28 b B e+25 a C e)-c^2\left(6 C d^2-7 e (3 B d+5 A e)\right)\right)\sqrt{\frac{c (d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}}\right.$$

$$\left.\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\, e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Bigg/\left(105 c^4 e^2\sqrt{d+e x}\sqrt{a+b x+c x^2}\right)$$

Result (type 4, 9972 leaves) :

$$\frac{1}{\sqrt{a+x (b+c x)}}\sqrt{d+e x}\left(a+b x+c x^2\right)$$

$$\left(\frac{2\left(3 c^2 C d^2+42 B c^2 d e-33 b c C d e-28 b B c e^2+35 A c^2 e^2+24 b^2 C e^2-25 a c C e^2\right)}{105 c^3 e}+\frac{2 (8 c C d+7 B c e-6 b C e) x}{35 c^2}+\frac{2 C e x^2}{7 c}\right)+$$

$$\frac{1}{105 c^3 e^3 \sqrt{a+x (b+c x)}} 2 \sqrt{a+b x+c x^2} \left(\left(\left(-6 c^3 C d^3+21 B c^3 d^2 e-12 b c^2 C d^2 e-91 b B c^2 d e^2+140 A c^3 d e^2+\right.\right.\right.$$

$$\left.72 b^2 c C d e^2-82 a c^2 C d e^2+56 b^2 B c e^3-70 A b c^2 e^3-63 a B c^2 e^3-48 b^3 C e^3+104 a b c C e^3\right) (d+e x)^{3/2}$$

$$\left.\left.\left(c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)\right)\Bigg/\left(c \sqrt{\frac{(d+e x)^2 \left(c \left(-1+\frac{d}{d+e x}\right)^2+\frac{e \left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)+$$

$$\frac{1}{c \sqrt{\frac{(d+e x)^2 \left(c \left(-1+\frac{d}{d+e x}\right)^2+\frac{e \left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}} \left(c d^2-b d e+a e^2\right) (d+e x) \sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\left(\left(3 i c^3 C d^3 \left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}} \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Bigg/$$

$$\left(\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(21\,\mathbb{i}\,B c^3 d^2 e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(3\,\mathbb{i}\sqrt{2}\,b c^2 C d^2 e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(91\, i\, b B c^2 d e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(35\, i\sqrt{2}\, A c^3 d e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\left(18 \, \mathrm{i} \sqrt{2} \, b^2 c C d e^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \left(\text{EllipticE}\left[\mathrm{i} \, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i} \, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\left(c d^2 - b d e + a e^2\right) \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(41 \, \mathrm{i} \, a c^2 C d e^2 \left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \left(\text{EllipticE}\left[\mathrm{i} \, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i} \, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(\sqrt{2}\ \left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(14\,\mathbb{i}\,\sqrt{2}\ b^2\,B\,c\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\ \sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\ \sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(35\,\mathbb{i}\,A\,b\,c^2\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\ \sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\ \sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(63\,\mathbb{i}\,a\,B\,c^2\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(12\,\mathbb{i}\,\sqrt{2}\,b^3\,C\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(26\,\mathbb{i}\,\sqrt{2}\,a b c C e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(3\,\mathbb{i}\,\sqrt{2}\,c^3 C d^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\,\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(21\, i\, B\, c^3\, d\, e \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(15\, i\, b\, c^2\, C\, d\, e \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(14\, i \sqrt{2}\, b\, B\, c^2\, e^2 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\left(35 i A c^3 e^2 \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) -$$

$$\left(12 i \sqrt{2}\, b^2 c C e^2 \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c d^2 - b d e + a e^2}{2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{c + \frac{c d^2 - b d e + a e^2}{(d + e x)^2} + \frac{-2 c d + b e}{d + e x}}\right) +$$

$$\left(25 i a c^2 C e^2 \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}} \sqrt{1 - \frac{2 \left(c d^2 - b d e + a e^2\right)}{\left(2 c d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\Bigg)\Bigg)$$

- **Problem 267: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{d+e\,x}\,\left(A+B\,x+C\,x^2\right)}{\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 557 leaves, 7 steps) :

$$-\frac{2\,(2\,c\,C\,d-5\,B\,c\,e+4\,b\,C\,e)\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}}{15\,c^2\,e}+\frac{2\,C\,(d+e\,x)^{3/2}\,\sqrt{a+b\,x+c\,x^2}}{5\,c\,e}+$$

$$\left(\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(8\,b^2\,C\,e^2-c\,e\,(3\,b\,C\,d+10\,b\,B\,e+9\,a\,C\,e)-c^2\,\left(2\,C\,d^2-5\,e\,(B\,d+3\,A\,e)\right)\right)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/\left(15\,c^3\,e^2\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,(2\,c\,C\,d-5\,B\,c\,e+4\,b\,C\,e)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/\left(15\,c^3\,e^2\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 5505 leaves) :

$$\frac{\left(\frac{2\,(c\,C\,d+5\,B\,c\,e-4\,b\,C\,e)}{15\,c^2\,e}+\frac{2\,C\,x}{5\,c}\right)\,\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)}{\sqrt{a+x\,(b+c\,x)}}-$$

$$\frac{1}{15\,c^2\,e^3\,\sqrt{a+x\,(b+c\,x)}}\,2\,\sqrt{a+b\,x+c\,x^2}\,\Bigg(\Big(\big(2\,c^2\,C\,d^2-5\,B\,c^2\,d\,e+3\,b\,c\,C\,d\,e+10\,b\,B\,c\,e^2-15\,A\,c^2\,e^2-8\,b^2\,C\,e^2+9\,a\,c\,C\,e^2\big)$$

$$(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)\Bigg)\Bigg/\left(c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)-$$

$$\frac{1}{c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\,\big(c\,d^2-b\,d\,e+a\,e^2\big)\,(d+e\,x)\,\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\Bigg(\Bigg(i\,c^2\,C\,d^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\,\big(c\,d^2-b\,d\,e+a\,e^2\big)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\,\big(c\,d^2-b\,d\,e+a\,e^2\big)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Big],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Big]-$$

$$\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Big],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Big]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\big(c\,d^2-b\,d\,e+a\,e^2\big)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(5\,i\,B\,c^2\,d\,e\left(2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(3\,i\,b\,c\,C\,d\,e\left(2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2 e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2 e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(5\,i\,b\,B\,c\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(15\,i\,A\,c^2e^2\left(2\,c\,d-b\,e+\sqrt{b^2e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(2\,\mathbb{i}\,\sqrt{2}\,b^2\,C\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(9\,\mathbb{i}\,a\,c\,C\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(i\sqrt{2}\,c^2\,C\,d\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(5\,i\,B\,c^2\,e\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(2\,i\sqrt{2}\,b\,c\,C\,e\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left.\left.\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)\right)$$

- **Problem 268: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A+B\,x+C\,x^2}{\sqrt{d+e\,x}\;\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 471 leaves, 6 steps):

$$\frac{2\,C\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}}{3\,c\,e}-$$

$$\left(\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(2\,c\,C\,d-3\,B\,c\,e+2\,b\,C\,e\right)\,\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/$$

$$\left(3\,c^2\,e^2\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(C\,e\,(b\,d-a\,e)+c\,\left(2\,C\,d^2-3\,e\,(B\,d-A\,e)\right)\right)\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/\left(3\,c^2\,e^2\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 6180 leaves) :

$$\frac{2\,C\,\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)}{3\,c\,e\,\sqrt{a+x\,(b+c\,x)}}+$$

$$\frac{1}{3\,c\,e^3\,\sqrt{a+x\,(b+c\,x)}}\sqrt{a+b\,x+c\,x^2}\left(-\frac{2\,(2\,c\,C\,d-3\,B\,c\,e+2\,b\,C\,e)\,(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)}{c\,\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}+\right.$$

$$\frac{1}{c\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\,2\,(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\Bigg(\Bigg(\mathbb{i}\,c^2\,C\,d^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(3\,\mathbb{i}\,B\,c^2\,d^2\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(3\,\mathrm{i}\,b\,B\,c\,d\,e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(\mathrm{i}\,b^2\,C\,d\,e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(\mathbb{i}\,a\,c\,C\,d\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(3\,\mathbb{i}\,a\,B\,c\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(i\,a\,b\,C\,e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(i\sqrt{2}\,c^2\,C\,d^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(3\,\mathbb{i}\,B c^2 d e \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(\mathbb{i}\,b c C d e \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(3\,\mathbb{i}\,A c^2 e^2 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(i\,a\,c\,C\,e^2\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,(c\,d^2-b\,d\,e+a\,e^2)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Bigg/$$

$$\left.\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)$$

- **Problem 269: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A+B\,x+C\,x^2}{(d+e\,x)^{3/2}\,\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 508 leaves, 6 steps) :

$$-\frac{2\left(C d^2-e\,(B d-A e)\right)\sqrt{a+b x+c x^2}}{e\left(c d^2-b d e+a e^2\right)\sqrt{d+e x}}-\left(\sqrt{2}\,\sqrt{b^2-4 a c}\,\left(C e\,(b d-a e)-c\left(2 C d^2-e\,(B d-A e)\right)\right)\right.$$

$$\left.\sqrt{d+e x}\,\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}\right]\right)\Big/$$

$$\left(c e^2\left(c d^2-b d e+a e^2\right)\sqrt{\frac{c\,(d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}}\,\sqrt{a+b x+c x^2}\right)-\left(2\sqrt{2}\,\sqrt{b^2-4 a c}\,(2 C d-B e)\sqrt{\frac{c\,(d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}}\right.$$

$$\left.\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\,e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}\right]\right)\Big/\left(c e^2\sqrt{d+e x}\,\sqrt{a+b x+c x^2}\right)$$

Result (type 4, 3987 leaves) :

$$-\frac{2\left(C d^2-B d e+A e^2\right)\left(a+b x+c x^2\right)}{e\left(c d^2-b d e+a e^2\right)\sqrt{d+e x}\,\sqrt{a+x\,(b+c x)}}-\frac{1}{e^3\left(c d^2-b d e+a e^2\right)\sqrt{a+x\,(b+c x)}}$$

$$2\sqrt{a+b x+c x^2}\left(\frac{\left(-2 c C d^2+B c d e+b C d e-A c e^2-a C e^2\right)(d+e x)^{3/2}\left(c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)}{c\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}+\right.$$

$$\frac{1}{c\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\Bigg(\Bigg(\mathbb{i}\,c\,C\,d^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(\mathbb{i}\,B\,c\,d\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(\mathrm{i}\,b\,C\,d\,e\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(\mathrm{i}\,A\,c\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(i\,a\,C\,e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(i\sqrt{2}\,c\,C\,d\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(i\,B\,c\,e\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\;\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Bigg/$$

$$\left.\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)\right)\right)$$

- **Problem 270: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{A+B\,x+C\,x^2}{(d+e\,x)^{5/2}\,\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 4, 684 leaves, 7 steps):

$$-\frac{2\left(C d^2-e\,(B d-A e)\right)\sqrt{a+b x+c x^2}}{3 e\left(c d^2-b d e+a e^2\right)(d+e x)^{3/2}}+\frac{2\left(c d\left(2 C d^2+e\,(B d-4 A e)\right)+e\left(3 a e\,(2 C d-B e)-b\left(4 C d^2-B d e-2 A e^2\right)\right)\right)\sqrt{a+b x+c x^2}}{3 e\left(c d^2-b d e+a e^2\right)^2\sqrt{d+e x}}-$$

$$\left(\sqrt{2}\sqrt{b^2-4 a c}\left(c d\left(2 C d^2+e\,(B d-4 A e)\right)+e\left(3 a e\,(2 C d-B e)-b\left(4 C d^2-B d e-2 A e^2\right)\right)\right)\right.$$

$$\left.\sqrt{d+e x}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\;e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}\right]\right)\Big/$$

$$\left(3 e^2\left(c d^2-b d e+a e^2\right)^2\sqrt{\frac{c\,(d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}}\sqrt{a+b x+c x^2}\right)-$$

$$\left(2\sqrt{2}\sqrt{b^2-4 a c}\left(3 C e\,(b d-a e)-c\left(2 C d^2+e\,(B d-A e)\right)\right)\sqrt{\frac{c\,(d+e x)}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}}\sqrt{-\frac{c\left(a+b x+c x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\;e}{2 c d-\left(b+\sqrt{b^2-4 a c}\right)e}\right]\right)\Big/\left(3 c e^2\left(c d^2-b d e+a e^2\right)\sqrt{d+e x}\sqrt{a+b x+c x^2}\right)$$

Result (type 4, 6924 leaves) :

$$\frac{1}{\sqrt{a+x\,(b+c x)}}$$

$$\sqrt{d+e x}\,\left(a+b x+c x^2\right)\left(-\frac{2\left(C d^2-B d e+A e^2\right)}{3 e\left(c d^2-b d e+a e^2\right)(d+e x)^2}-\frac{2\left(-2 c C d^3-B c d^2 e+4 b C d^2 e-b B d e^2+4 A c d e^2-6 a C d e^2-2 A b e^3+3 a B e^3\right)}{3 e\left(c d^2-b d e+a e^2\right)^2 (d+e x)}\right)+$$

$$\frac{1}{3 e^3\left(c d^2-b d e+a e^2\right)^2\sqrt{a+x(b+c x)}}$$

$$2\sqrt{a+b x+c x^2}\left(\left(\left(-2 c C d^3-B c d^2 e+4 b C d^2 e-b B d e^2+4 A c d e^2-6 a C d e^2-2 A b e^3+3 a B e^3\right)(d+e x)^{3/2}\right.\right.$$

$$\left.\left.\left(c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}\right)\right)\Big/\left(\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)+\right.$$

$$\frac{1}{\sqrt{\frac{(d+e x)^2\left(c\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{a e}{d+e x}\right)}{d+e x}\right)}{e^2}}}\left(c d^2-b d e+a e^2\right)(d+e x)\sqrt{c+\frac{c d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{a e^2}{(d+e x)^2}-\frac{2 c d}{d+e x}+\frac{b e}{d+e x}}$$

$$\left(\left(i\, c\, C\, d^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i\,B\,c\,d^2 e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(i\sqrt{2}\,b\,C\,d^2 e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\Big],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\Big]\Bigg)\Bigg)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(\mathrm{i}\,b\,B\,d\,e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\Big],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\Big]-$$

$$\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\Big],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\Big]\Bigg)\Bigg)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(\mathrm{i}\sqrt{2}\,A\,c\,d\,e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \Bigg(\text{EllipticE}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\Big],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\Big]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(3\,i\,a\,C\,d\,e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(i\,A\,b\,e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(3\,\mathbb{i}\,a\,B\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\Bigg(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(\mathbb{i}\,\sqrt{2}\,c\,C\,d^2\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i B c d e \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}} \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(3 i b C d e \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}} \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(i A c e^2 \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}} \sqrt{1-\frac{2 \left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(3\,i\,a\,C\,e^2\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left.\left(\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)\right)$$

- **Problem 271: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int\frac{A+B x+C x^2}{(d+e x)^{7/2}\sqrt{a+b x+c x^2}}\,dx$$

Optimal (type 4, 944 leaves, 8 steps):

$$-\frac{2\left(C d^2-e (B d-A e)\right)\sqrt{a+b x+c x^2}}{5 e\left(c d^2-b d e+a e^2\right)(d+e x)^{5/2}}+\frac{2\left(c d\left(2 C d^2+e (3 B d-8 A e)\right)+e\left(5 a e (2 C d-B e)-b\left(6 C d^2-B d e-4 A e^2\right)\right)\right)\sqrt{a+b x+c x^2}}{15 e\left(c d^2-b d e+a e^2\right)^2(d+e x)^{3/2}}+$$

$$\left(2\left(c^2 d^2\left(2 C d^2+e (3 B d-23 A e)\right)-e^2\left(15 a^2 C e^2-10 a b e (C d+B e)+b^2\left(3 C d^2+2 B d e+8 A e^2\right)\right)-\right.\right.$$

$$\left.\left.c e\left(b d\left(7 C d^2-7 B d e-23 A e^2\right)-a e\left(19 C d^2-29 B d e+9 A e^2\right)\right)\right)\sqrt{a+b x+c x^2}\right)\Big/\left(15 e\left(c d^2-b d e+a e^2\right)^3\sqrt{d+e x}\right)-$$

$$\left(\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(c^2\,d^2\,\left(2\,C\,d^2+e\,(3\,B\,d-23\,A\,e)\right)-e^2\,\left(15\,a^2\,C\,e^2-10\,a\,b\,e\,(C\,d+B\,e)+b^2\,\left(3\,C\,d^2+2\,B\,d\,e+8\,A\,e^2\right)\right)-\right.\right.$$

$$\left.c\,e\,\left(b\,d\,\left(7\,C\,d^2-7\,B\,d\,e-23\,A\,e^2\right)-a\,e\,\left(19\,C\,d^2-29\,B\,d\,e+9\,A\,e^2\right)\right)\right)$$

$$\left.\sqrt{d+e\,x}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/$$

$$\left(15\,e^2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^3\,\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{a+b\,x+c\,x^2}\right)+$$

$$\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(c\,d\,\left(2\,C\,d^2+e\,(3\,B\,d-8\,A\,e)\right)+e\,\left(5\,a\,e\,(2\,C\,d-B\,e)-b\,\left(6\,C\,d^2-B\,d\,e-4\,A\,e^2\right)\right)\right)\right.$$

$$\left.\sqrt{\frac{c\,(d+e\,x)}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{c\,\left(a+b\,x+c\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/$$

$$\left(15\,e^2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2\,\sqrt{d+e\,x}\,\sqrt{a+b\,x+c\,x^2}\right)$$

Result (type 4, 12 295 leaves) :

$$\frac{1}{\sqrt{a+x\,(b+c\,x)}}$$

$$\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)\left(-\frac{2\left(C\,d^2-B\,d\,e+A\,e^2\right)}{5\,e\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)^3}-\frac{2\left(-2\,c\,C\,d^3-3\,B\,c\,d^2\,e+6\,b\,C\,d^2\,e-b\,B\,d\,e^2+8\,A\,c\,d\,e^2-10\,a\,C\,d\,e^2-4\,A\,b\,e^3+5\,a\,B\,e^3\right)}{15\,e\left(c\,d^2-b\,d\,e+a\,e^2\right)^2(d+e\,x)^2}-\right.$$

$$\left(2\left(-2\,c^2\,C\,d^4-3\,B\,c^2\,d^3\,e+7\,b\,c\,C\,d^3\,e-7\,b\,B\,c\,d^2\,e^2+23\,A\,c^2\,d^2\,e^2+3\,b^2\,C\,d^2\,e^2-19\,a\,c\,C\,d^2\,e^2+2\,b^2\,B\,d\,e^3-23\,A\,b\,c\,d\,e^3+\right.\right.$$

$$\left.\left.\left.29\,a\,B\,c\,d\,e^3-10\,a\,b\,C\,d\,e^3+8\,A\,b^2\,e^4-10\,a\,b\,B\,e^4-9\,a\,A\,c\,e^4+15\,a^2\,C\,e^4\right)\right)\Big/\left(15\,e\left(c\,d^2-b\,d\,e+a\,e^2\right)^3(d+e\,x)\right)\right)+$$

$$\frac{1}{15\,e^3\left(c\,d^2-b\,d\,e+a\,e^2\right)^3\sqrt{a+x\,(b+c\,x)}}\,2\,c\,\sqrt{a+b\,x+c\,x^2}\left(\left(\left(-2\,c^2\,C\,d^4-3\,B\,c^2\,d^3\,e+7\,b\,c\,C\,d^3\,e-7\,b\,B\,c\,d^2\,e^2+23\,A\,c^2\,d^2\,e^2+\right.\right.\right.$$

$$3\,b^2\,C\,d^2\,e^2-19\,a\,c\,C\,d^2\,e^2+2\,b^2\,B\,d\,e^3-23\,A\,b\,c\,d\,e^3+29\,a\,B\,c\,d\,e^3-10\,a\,b\,C\,d\,e^3+8\,A\,b^2\,e^4-10\,a\,b\,B\,e^4-9\,a\,A\,c\,e^4+15\,a^2\,C\,e^4\Big)$$

$$\left.(d+e\,x)^{3/2}\left(c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)\right)\Bigg/\left(c\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)+$$

$$\frac{1}{c\sqrt{\frac{(d+e\,x)^2\left(c\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{a\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}\left(c\,d^2-b\,d\,e+a\,e^2\right)(d+e\,x)\sqrt{c+\frac{c\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{a\,e^2}{(d+e\,x)^2}-\frac{2\,c\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}$$

$$\left(\left(i\,c^2\,C\,d^4\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(3\,i\,B c^2 d^3 e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(7\,i\,b c C d^3 e\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(7\,\mathrm{i}\,b\,B\,c\,d^2\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\ \Bigg(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(23\,\mathrm{i}\,A\,c^2\,d^2\,e^2\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\ \Bigg(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(3\,i\,b^2 C d^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(19\,i\,a c C d^2 e^2\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(i\,b^2 B d e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(23\,i\,A b c d e^3\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\Bigg(29\,i\,a\,B\,c\,d\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(5\,i\,a\,b\,C\,d\,e^3\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}$$

$$\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\,\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(2\,i\,\sqrt{2}\,A\,b^2 e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(5\,i\,a\,b\,B\,e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\Bigg(9\,i\,a A c e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(2\sqrt{2}\left(c d^2-b d e+a e^2\right)\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\Bigg(15\,i\,a^2 C e^4\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Bigg)\Big/$$

$$\left(2\,\sqrt{2}\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(i\,\sqrt{2}\,c^2\,C\,d^3\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\Bigg(3\,i\,B\,c^2\,d^2\,e\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\,\sqrt{1-\frac{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\Bigg)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(3\, i \sqrt{2}\, b\, c\, C\, d^2 e \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)+$$

$$\left(i\, b\, B\, c\, d\, e^2 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)-$$

$$\left(4\, i \sqrt{2}\, A\, c^2 d\, e^2 \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}} \sqrt{1-\frac{2\left(c d^2-b d e+a e^2\right)}{\left(2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(5\,\mathbb{i}\,\sqrt{2}\,a\,c\,C\,d\,e^2\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(2\,\mathbb{i}\,\sqrt{2}\,A\,b\,c\,e^3\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c\,d^2-b\,d\,e+a\,e^2}{2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{c+\frac{c\,d^2-b\,d\,e+a\,e^2}{(d+e\,x)^2}+\frac{-2\,c\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(5\,\mathbb{i}\,a\,B\,c\,e^3\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\,\sqrt{1-\frac{2\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\left(2\,c\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)(d+e\,x)}}\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 c d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left.\left(\sqrt{2}\sqrt{-\frac{c d^2-b d e+a e^2}{2 c d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{c+\frac{c d^2-b d e+a e^2}{(d+e x)^2}+\frac{-2 c d+b e}{d+e x}}\right)\right)$$

■ **Problem 272: Unable to integrate problem.**

$$\int (g+h x)^m \left(a+b x+c x^2\right)^p \left(d+e x+f x^2\right) dx$$

Optimal (type 6, 510 leaves, 6 steps):

$$\frac{f (g+h x)^{1+m} \left(a+b x+c x^2\right)^{1+p}}{c h (3+m+2 p)}+\frac{1}{c h^3 (1+m) (3+m+2 p)}$$

$$\left(f h (b g-a h) (1+m)+c \left(2 f g^2 (1+p)-h (e g-d h) (3+m+2 p)\right)\right) (g+h x)^{1+m} \left(a+b x+c x^2\right)^p \left(1-\frac{2 c (g+h x)}{2 c g-\left(b-\sqrt{b^2-4 a c}\right) h}\right)^{-p}$$

$$\left(1-\frac{2 c (g+h x)}{2 c g-\left(b+\sqrt{b^2-4 a c}\right) h}\right)^{-p} \text{AppellF1}\left[1+m,-p,-p,2+m,\frac{2 c (g+h x)}{2 c g-\left(b-\sqrt{b^2-4 a c}\right) h},\frac{2 c (g+h x)}{2 c g-\left(b+\sqrt{b^2-4 a c}\right) h}\right]-$$

$$\frac{1}{c h^3 (2+m) (3+m+2 p)}(b f h (2+m+p)+c (2 f g (1+p)-e h (3+m+2 p))) (g+h x)^{2+m} \left(a+b x+c x^2\right)^p \left(1-\frac{2 c (g+h x)}{2 c g-\left(b-\sqrt{b^2-4 a c}\right) h}\right)^{-p}$$

$$\left(1-\frac{2 c (g+h x)}{2 c g-\left(b+\sqrt{b^2-4 a c}\right) h}\right)^{-p} \text{AppellF1}\left[2+m,-p,-p,3+m,\frac{2 c (g+h x)}{2 c g-\left(b-\sqrt{b^2-4 a c}\right) h},\frac{2 c (g+h x)}{2 c g-\left(b+\sqrt{b^2-4 a c}\right) h}\right]$$

Result (type 8, 32 leaves):

$$\int (g+h x)^m \left(a+b x+c x^2\right)^p \left(d+e x+f x^2\right) dx$$

- **Problem 273: Unable to integrate problem.**

$$\int (g+h x)^m \sqrt{a+b x+c x^2} \left(d+e x+f x^2\right) dx$$

Optimal (type 6, 496 leaves, 6 steps):

$$\frac{f (g+h x)^{1+m} \left(a+b x+c x^2\right)^{3/2}}{c h (4+m)}+\left(\left(f h (b g-a h) (1+m)+c \left(3 f g^2-h (e g-d h) (4+m)\right)\right)\right.$$
$$(g+h x)^{1+m} \sqrt{a+b x+c x^2} \,\text{AppellF1}\left[1+m,-\frac{1}{2},-\frac{1}{2},2+m,\frac{2 c (g+h x)}{2 c g-\left(b-\sqrt{b^2-4 a c}\right) h},\frac{2 c (g+h x)}{2 c g-\left(b+\sqrt{b^2-4 a c}\right) h}\right]\Bigg)\Bigg/$$
$$\left(c h^3 (1+m) (4+m) \sqrt{1-\frac{2 c (g+h x)}{2 c g-\left(b-\sqrt{b^2-4 a c}\right) h}} \sqrt{1-\frac{2 c (g+h x)}{2 c g-\left(b+\sqrt{b^2-4 a c}\right) h}}\right)-$$
$$\Bigg((b f h (5+2 m)+c (6 f g-2 e h (4+m))) (g+h x)^{2+m} \sqrt{a+b x+c x^2}$$
$$\text{AppellF1}\left[2+m,-\frac{1}{2},-\frac{1}{2},3+m,\frac{2 c (g+h x)}{2 c g-\left(b-\sqrt{b^2-4 a c}\right) h},\frac{2 c (g+h x)}{2 c g-\left(b+\sqrt{b^2-4 a c}\right) h}\right]\Bigg)\Bigg/$$
$$\left(2 c h^3 (2+m) (4+m) \sqrt{1-\frac{2 c (g+h x)}{2 c g-\left(b-\sqrt{b^2-4 a c}\right) h}} \sqrt{1-\frac{2 c (g+h x)}{2 c g-\left(b+\sqrt{b^2-4 a c}\right) h}}\right)$$

Result (type 8, 34 leaves):

$$\int (g+h x)^m \sqrt{a+b x+c x^2} \left(d+e x+f x^2\right) dx$$

- **Problem 274: Unable to integrate problem.**

$$\int (g+h x)^{-3-2 p} \left(a+b x+c x^2\right)^p \left(d+e x+f x^2\right) dx$$

Optimal (type 6, 590 leaves, 5 steps) :

$$-\frac{\left(f g^2-h (e g-d h)\right) (g+h x)^{-2 (1+p)} \left(a+b x+c x^2\right)^{1+p}}{2 h \left(c g^2-b g h+a h^2\right) (1+p)}-\frac{1}{2 h^3 p}f (g+h x)^{-2 p} \left(a+b x+c x^2\right)^p \left(1-\frac{2 c (g+h x)}{2 c g-\left(b-\sqrt{b^2-4 a c}\right) h}\right)^{-p}$$
$$\left(1-\frac{2 c (g+h x)}{2 c g-\left(b+\sqrt{b^2-4 a c}\right) h}\right)^{-p} \text{AppellF1}\left[-2 p,-p,-p,1-2 p,\frac{2 c (g+h x)}{2 c g-\left(b-\sqrt{b^2-4 a c}\right) h},\frac{2 c (g+h x)}{2 c g-\left(b+\sqrt{b^2-4 a c}\right) h}\right]-$$
$$\left(\left(2 c \left(f g^3-d g h^2\right)+h \left(2 a h (2 f g-e h)-b \left(3 f g^2-e g h-d h^2\right)\right)\right) \left(b-\sqrt{b^2-4 a c}+2 c x\right)\right.$$
$$\left(\frac{\left(2 c g-\left(b-\sqrt{b^2-4 a c}\right) h\right) \left(b+\sqrt{b^2-4 a c}+2 c x\right)}{\left(2 c g-\left(b+\sqrt{b^2-4 a c}\right) h\right) \left(b-\sqrt{b^2-4 a c}+2 c x\right)}\right)^{-p} (g+h x)^{-1-2 p} \left(a+b x+c x^2\right)^p \text{Hypergeometric2F1}\left[-1-2 p,-p,-2 p,\right.$$
$$\left.\left.-\frac{4 c \sqrt{b^2-4 a c} (g+h x)}{\left(2 c g-\left(b+\sqrt{b^2-4 a c}\right) h\right) \left(b-\sqrt{b^2-4 a c}+2 c x\right)}\right]\right)\Big/\left(2 h^2 \left(2 c g-\left(b-\sqrt{b^2-4 a c}\right) h\right) \left(c g^2-b g h+a h^2\right) (1+2 p)\right)$$

Result (type 8, 36 leaves) :

$$\int (g+h x)^{-3-2 p} \left(a+b x+c x^2\right)^p \left(d+e x+f x^2\right) \, dx$$

- **Problem 278: Result more than twice size of optimal antiderivative.**

$$\int (d+e x)^3 \left(a+b x+c x^2\right)^5 \left(d (6 b d+5 a e)+\left(12 c d^2+17 b d e+5 a e^2\right) x+e (29 c d+11 b e) x^2+17 c e^2 x^3\right) \, dx$$

Optimal (type 1, 20 leaves, 2 steps) :

$$(d+e x)^5 \left(a+b x+c x^2\right)^6$$

Result (type 1, 167 leaves) :

$$x \left(6 a^5 (b+c x) (d+e x)^5+15 a^4 x (b+c x)^2 (d+e x)^5+20 a^3 x^2 (b+c x)^3 (d+e x)^5+15 a^2 x^3 (b+c x)^4 (d+e x)^5+\right.$$
$$\left.6 a x^4 (b+c x)^5 (d+e x)^5+x^5 (b+c x)^6 (d+e x)^5+a^6 e \left(5 d^4+10 d^3 e x+10 d^2 e^2 x^2+5 d e^3 x^3+e^4 x^4\right)\right)$$

- **Problem 366: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{f+g x+h x^2+i x^3+j x^4}{\left(a+b x-c x^2\right)^{5/2}} \, dx$$

Optimal (type 3, 353 leaves, 5 steps):

$$\left(2 \left(a b^2 c i+2 a c^2 (c g+a i)+a b^3 j-b c \left(c^2 f-a c h-3 a^2 j\right)+\left(2 c^4 f+c^3 (b g+2 a h)+b^4 j+b^2 c (b i+4 a j)+c^2 \left(b^2 h+3 a b i+2 a^2 j\right)\right) x\right)\right) \Big/ \left(3 c^3 \left(b^2+4 a c\right) \left(a+b x-c x^2\right)^{3/2}\right)-\left(2 \left(b^4 c i+24 a^2 c^3 i+2 b^2 c^2 (2 c g+3 a i)+b^5 j+b^3 c (c h+10 a j)+4 b c^2 \left(2 c^2 f-a c h+8 a^2 j\right)-c \left(16 c^4 f+8 c^3 (b g-a h)-4 b^4 j-b^2 c (b i+28 a j)+2 c^2 \left(b^2 h-6 a b i-16 a^2 j\right)\right) x\right)\right) \Big/ \left(3 c^3 \left(b^2+4 a c\right)^2 \sqrt{a+b x-c x^2}\right)-\frac{j \operatorname{ArcTan}\left[\frac{b-2 c x}{2 \sqrt{c} \sqrt{a+b x-c x^2}}\right]}{c^{5/2}}$$

Result (type 3, 319 leaves):

$$-\frac{1}{3 c^2 \left(b^2+4 a c\right)^2 (a+x (b-c x))^{3/2}} 2 \left(3 b^5 j x^2+b^4 \left(6 a j x-4 c j x^3\right)+b^3 \left(3 a^2 j+18 a c j x^2+c^2 \left(f+3 g x-x^2 (3 h+i x)\right)\right)+8 c^2 \left(2 c^3 f x^3+a^3 (2 i+3 j x)-a c^2 x \left(3 f+h x^2\right)-a^2 c \left(g+x^2 (3 i+4 j x)\right)\right)+4 b c \left(5 a^3 j+2 c^3 x^2 (-3 f+g x)-2 a^2 c (h-3 i x)+3 a c^2 \left(f-x \left(g-h x+i x^2\right)\right)\right)+2 b^2 c \left(21 a^2 j x+c^2 x (3 f+x (-6 g+h x))+a c \left(g+x \left(-6 h+3 i x-14 j x^2\right)\right)\right)\right)+\frac{i j \operatorname{Log}\left[\frac{i (b-2 c x)}{\sqrt{c}}+2 \sqrt{a+x (b-c x)}\right]}{c^{5/2}}$$

Problem 367: Result more than twice size of optimal antiderivative.

$$\int (d+e x)^m \left(3+2 x+5 x^2\right)^3 \left(2+x+3 x^2-5 x^3+4 x^4\right) dx$$

Optimal (type 3, 588 leaves, 2 steps):

$$\frac{\left(5 d^2-2 d e+3 e^2\right)^3 \left(4 d^4+5 d^3 e+3 d^2 e^2-d e^3+2 e^4\right) (d+e x)^{1+m}}{e^{11} (1+m)}-\frac{\left(5 d^2-2 d e+3 e^2\right)^2 \left(200 d^5+169 d^4 e+108 d^3 e^2-20 d^2 e^3+86 d e^4-15 e^5\right) (d+e x)^{2+m}}{e^{11} (2+m)}+\frac{3 \left(5 d^2-2 d e+3 e^2\right) \left(1500 d^6+660 d^5 e+792 d^4 e^2+58 d^3 e^3+547 d^2 e^4-156 d e^5+53 e^6\right) (d+e x)^{3+m}}{e^{11} (3+m)}-\frac{2 \left(30000 d^7+1050 d^6 e+21420 d^5 e^2+1715 d^4 e^3+9990 d^3 e^4-2550 d^2 e^5+2218 d e^6-287 e^7\right) (d+e x)^{4+m}}{e^{11} (4+m)}+\frac{\left(105000 d^6+3150 d^5 e+53550 d^4 e^2+3430 d^3 e^3+14985 d^2 e^4-2550 d e^5+1109 e^6\right) (d+e x)^{5+m}}{e^{11} (5+m)}-\frac{6 \left(21000 d^5+525 d^4 e+7140 d^3 e^2+343 d^2 e^3+999 d e^4-85 e^5\right) (d+e x)^{6+m}}{e^{11} (6+m)}+\frac{\left(105000 d^4+2100 d^3 e+21420 d^2 e^2+686 d e^3+999 e^4\right) (d+e x)^{7+m}}{e^{11} (7+m)}-\frac{2 \left(30000 d^3+450 d^2 e+3060 d e^2+49 e^3\right) (d+e x)^{8+m}}{e^{11} (8+m)}+\frac{45 \left(500 d^2+5 d e+17 e^2\right) (d+e x)^{9+m}}{e^{11} (9+m)}-\frac{25 (200 d+e) (d+e x)^{10+m}}{e^{11} (10+m)}+\frac{500 (d+e x)^{11+m}}{e^{11} (11+m)}$$

Result (type 3, 2576 leaves) :

$$\Big(\left(d+e\,x\right)^{1+m}\,\Big(1814400000\,d^{10}-9072000\,d^{9}\,e\,\left(-11+200\,x+m\,\left(-1+200\,x\right)\right)+$$

$$1814400\,d^{8}\,e^{2}\,\left(5\,\left(374-11\,x+200\,x^{2}\right)+3\,m\,\left(119-20\,x+500\,x^{2}\right)+m^{2}\,\left(17-5\,x+500\,x^{2}\right)\right)-$$

$$10080\,d^{7}\,e^{3}\,\left(90\,\left(-539+3740\,x-110\,x^{2}+2000\,x^{3}\right)+30\,m^{2}\,\left(-49+2244\,x-210\,x^{2}+6000\,x^{3}\right)+m^{3}\,\left(-49+3060\,x-450\,x^{2}+30000\,x^{3}\right)+\right.$$

$$\left.m\,\left(-14651+400860\,x-15750\,x^{2}+330000\,x^{3}\right)\right)+720\,d^{6}\,e^{4}\,\left(180\,\left(43956-3773\,x+26180\,x^{2}-770\,x^{3}+14000\,x^{4}\right)+\right.$$

$$49\,m^{2}\,\left(10989-4606\,x+76500\,x^{2}-3300\,x^{3}+75000\,x^{4}\right)+m^{4}\,\left(999-686\,x+21420\,x^{2}-2100\,x^{3}+105000\,x^{4}\right)+$$

$$\left.2\,m^{3}\,\left(18981-10633\,x+257040\,x^{2}-17850\,x^{3}+525000\,x^{4}\right)+m\,\left(3378618-884254\,x+7968240\,x^{2}-266700\,x^{3}+5250000\,x^{4}\right)\right)-$$

$$720\,d^{5}\,e^{5}\,\left(180\,\left(26180+43956\,x-3773\,x^{2}+26180\,x^{3}-770\,x^{4}+14000\,x^{5}\right)+m^{5}\,\left(85+999\,x-343\,x^{2}+7140\,x^{3}-525\,x^{4}+21000\,x^{5}\right)+\right.$$

$$3\,m^{4}\,\left(1275+12987\,x-3773\,x^{2}+64260\,x^{3}-3675\,x^{4}+105000\,x^{5}\right)+$$

$$3\,m^{2}\,\left(202725+1305693\,x-222607\,x^{2}+2134860\,x^{3}-76125\,x^{4}+1575000\,x^{5}\right)+m^{3}\,\left(68425+576423\,x-134113\,x^{2}+\right.$$

$$\left.\left.1763580\,x^{3}-76125\,x^{4}+1785000\,x^{5}\right)+2\,m\,\left(1342745+5645349\,x-611912\,x^{2}+4769520\,x^{3}-150675\,x^{4}+2877000\,x^{5}\right)\right)+$$

$$24\,d^{4}\,e^{6}\,\left(1080\,\left(341572+130900\,x+219780\,x^{2}-18865\,x^{3}+130900\,x^{4}-3850\,x^{5}+70000\,x^{6}\right)+\right.$$

$$m^{6}\,\left(1109+2550\,x+14985\,x^{2}-3430\,x^{3}+53550\,x^{4}-3150\,x^{5}+105000\,x^{6}\right)+25\,m^{4}\,\left(47687+86700\,x+392607\,x^{2}-67228\,x^{3}+\right.$$

$$\left.760410\,x^{4}-31500\,x^{5}+735000\,x^{6}\right)+3\,m^{5}\,\left(18853+39100\,x+204795\,x^{2}-41160\,x^{3}+553350\,x^{4}-27300\,x^{5}+735000\,x^{6}\right)+$$

$$15\,m^{3}\,\left(886091+1353200\,x+5069925\,x^{2}-713440\,x^{3}+6729450\,x^{4}-243600\,x^{5}+5145000\,x^{6}\right)+$$

$$12\,m\,\left(22642453+18494725\,x+38116845\,x^{2}-3625510\,x^{3}+26792850\,x^{4}-822675\,x^{5}+15435000\,x^{6}\right)+$$

$$\left.2\,m^{2}\,\left(41323558+49404975\,x+143436420\,x^{2}-16136435\,x^{3}+131840100\,x^{4}-4329675\,x^{5}+85260000\,x^{6}\right)\right)-$$

$$12\,d^{3}\,e^{7}\,\left(m^{7}\,\left(287+2218\,x+2550\,x^{2}+9990\,x^{3}-1715\,x^{4}+21420\,x^{5}-1050\,x^{6}+30000\,x^{7}\right)+\right.$$

$$2160\,\left(220990+341572\,x+130900\,x^{2}+219780\,x^{3}-18865\,x^{4}+130900\,x^{5}-3850\,x^{6}+70000\,x^{7}\right)+$$

$$8\,m^{6}\,\left(2009+14417\,x+15300\,x^{2}+54945\,x^{3}-8575\,x^{4}+96390\,x^{5}-4200\,x^{6}+105000\,x^{7}\right)+$$

$$40\,m^{4}\,\left(124558+724177\,x+615825\,x^{2}+1758240\,x^{3}-217805\,x^{4}+1959930\,x^{5}-69825\,x^{6}+1470000\,x^{7}\right)+$$

$$2\,m^{5}\,\left(190855+1248734\,x+1201050\,x^{2}+3886110\,x^{3}-543655\,x^{4}+5462100\,x^{5}-213150\,x^{6}+4830000\,x^{7}\right)+$$

$$12\,m\,\left(37254035+106767866\,x+48770450\,x^{2}+89420490\,x^{3}-8099945\,x^{4}+58298100\,x^{5}-1760850\,x^{6}+32670000\,x^{7}\right)+$$

$$8\,m^{2}\,\left(22157261+88589138\,x+52444575\,x^{2}+109835055\,x^{3}-10787350\,x^{4}+81995760\,x^{5}-2576175\,x^{6}+49245000\,x^{7}\right)+$$

$$\left.m^{3}\,\left(38586863+191876962\,x+139405950\,x^{2}+343346310\,x^{3}-37539635\,x^{4}+307355580\,x^{5}-10194450\,x^{6}+203070000\,x^{7}\right)\right)+$$

$$6\,d^{2}\,e^{8}\,\left(m^{8}\,\left(159+574\,x+2218\,x^{2}+1700\,x^{3}+4995\,x^{4}-686\,x^{5}+7140\,x^{6}-300\,x^{7}+7500\,x^{8}\right)+\right.$$

$$6\,m^{7}\,\left(1590+5453\,x+19962\,x^{2}+14450\,x^{3}+39960\,x^{4}-5145\,x^{5}+49980\,x^{6}-1950\,x^{7}+45000\,x^{8}\right)+$$

$$4320\,\left(244860+220990\,x+341572\,x^{2}+130900\,x^{3}+219780\,x^{4}-18865\,x^{5}+130900\,x^{6}-3850\,x^{7}+70000\,x^{8}\right)+$$

$$6\,m^{6}\,\left(41181+132594\,x+454690\,x^{2}+307700\,x^{3}+794205\,x^{4}-95354\,x^{5}+863940\,x^{6}-31500\,x^{7}+682500\,x^{8}\right)+$$

$$12\,m^{5}\,\left(300510+894005\,x+2830168\,x^{2}+1768850\,x^{3}+4225770\,x^{4}-471625\,x^{5}+3998400\,x^{6}-137550\,x^{7}+2835000\,x^{8}\right)+$$

$$24\,m\,\left(52296690+77032235\,x+137509346\,x^{2}+56624450\,x^{3}+99310590\,x^{4}-8779085\,x^{5}+62225100\,x^{6}-1859850\,x^{7}+34245000\,x^{8}\right)+$$

$$3\,m^{4}\,\left(10806117+29046122\,x+83270374\,x^{2}+47401100\,x^{3}+104110785\,x^{4}-10813418\,x^{5}+86415420\,x^{6}-2832900\,x^{7}+56122500\,x^{8}\right)+$$

$$6\,m^{3}\,\left(30618630+71948317\,x+182077838\,x^{2}+93086050\,x^{3}+187672140\,x^{4}-18266465\,x^{5}+138894420\,x^{6}-4379550\,x^{7}+84105000\,x^{8}\right)+$$

$$4\,m^{2}\,\left(160119201+312153254\,x+674660150\,x^{2}+307319200\,x^{3}+573470955\,x^{4}-52869334\,x^{5}+386281140\,x^{6}-\right.$$

$$\left.\left.11814000\,x^{7}+221482500\,x^{8}\right)\right)-d\,e^{9}\,\left(m^{9}\,\left(3+2\,x+5\,x^{2}\right)^{2}\,\left(15+86\,x+20\,x^{2}+108\,x^{3}-169\,x^{4}+200\,x^{5}\right)+\right.$$

$$25920\,\left(103950+244860\,x+220990\,x^{2}+341572\,x^{3}+130900\,x^{4}+219780\,x^{5}-18865\,x^{6}+130900\,x^{7}-3850\,x^{8}+70000\,x^{9}\right)+$$

$$3\,m^{8}\,\left(2835+19398\,x+33866\,x^{2}+84284\,x^{3}+46750\,x^{4}+105894\,x^{5}-11662\,x^{6}+99960\,x^{7}-3525\,x^{8}+75000\,x^{9}\right)+$$

$$6\,m^{7}\,\left(39015+256626\,x+430500\,x^{2}+1029152\,x^{3}+548250\,x^{4}+1192806\,x^{5}-126224\,x^{6}+1040400\,x^{7}-35325\,x^{8}+725000\,x^{9}\right)+$$

$$6\,m^6\,(615\,195+3\,853\,206\,x+6\,159\,594\,x^2+14\,048\,812\,x^3+7\,152\,750\,x^4+14\,907\,078\,x^5-1\,515\,374\,x^6+12\,038\,040\,x^7-395\,325\,x^8+7\,875\,000\,x^9)+$$
$$144\,m\,(28\,438\,425+96\,371\,490\,x+96\,921\,335\,x^2+158\,003\,666\,x^3+62\,514\,950\,x^4+107\,222\,670\,x^5-9\,345\,035\,x^6+65\,591\,100\,x^7-$$
$$1\,946\,475\,x^8+35\,645\,000\,x^9)+3\,m^5\,(12\,236\,805+72\,048\,942\,x+108\,594\,486\,x^2+234\,464\,780\,x^3+113\,554\,050\,x^4+226\,351\,422\,x^5-$$
$$22\,132\,418\,x^6+170\,031\,960\,x^7-5\,425\,875\,x^8+105\,455\,000\,x^9)+3\,m^4\,(79\,518\,915+432\,260\,262\,x+605\,966\,634\,x^2+$$
$$1\,227\,933\,596\,x^3+563\,664\,750\,x^4+1\,075\,077\,846\,x^5-101\,413\,438\,x^6+756\,597\,240\,x^7-23\,566\,725\,x^8+448\,875\,000\,x^9)+$$
$$12\,m^2\,(224\,755\,965+947\,798\,682\,x+1\,086\,499\,918\,x^2+1\,899\,357\,684\,x^3+784\,511\,750\,x^4+1\,385\,287\,326\,x^5-123\,296\,838\,x^6+$$
$$879\,233\,880\,x^7-26\,417\,775\,x^8+488\,625\,000\,x^9)+4\,m^3\,(252\,936\,540+1\,236\,282\,876\,x+1\,583\,994\,615\,x^2+2\,988\,020\,842\,x^3+$$
$$1\,298\,753\,250\,x^4+2\,377\,214\,406\,x^5-217\,267\,519\,x^6+1\,581\,147\,900\,x^7-48\,276\,450\,x^8+904\,600\,000\,x^9))+$$
$$e^{10}\,(m^{10}\,(3+2\,x+5\,x^2)^3\,(2+x+3\,x^2-5\,x^3+4\,x^4)+m^9\,(3+2\,x+5\,x^2)^2\,(390+440\,x+1279\,x^2-280\,x^3+997\,x^4-936\,x^5+1100\,x^6)+$$
$$25\,920\,(83\,160+103\,950\,x+244\,860\,x^2+220\,990\,x^3+341\,572\,x^4+130\,900\,x^5+219\,780\,x^6-18\,865\,x^7+130\,900\,x^8-3850\,x^9+70\,000\,x^{10})+$$
$$3\,m^8\,(33\,480+80\,865\,x+276\,024\,x^2+320\,866\,x^3+598\,860\,x^4+266\,050\,x^5+503\,496\,x^6-47\,726\,x^7+360\,060\,x^8-11\,375\,x^9+220\,000\,x^{10})+$$
$$6\,m^7\,(277\,290+654\,210\,x+2\,183\,229\,x^2+2\,483\,698\,x^3+4\,541\,355\,x^4+1\,978\,800\,x^5+3\,677\,319\,x^6-342\,706\,x^7+2\,545\,155\,x^8-$$
$$79\,250\,x^9+1\,512\,500\,x^{10})+3\,m^6\,(5\,879\,034+13\,467\,195\,x+43\,730\,883\,x^2+48\,517\,350\,x^3+86\,713\,819\,x^4+37\,016\,310\,x^5+$$
$$67\,539\,393\,x^6-6\,192\,522\,x^7+45\,330\,075\,x^8-1\,393\,525\,x^9+26\,295\,500\,x^{10})+144\,m\,(30\,235\,140+47\,149\,425\,x+118\,408\,890\,x^2+$$
$$110\,180\,735\,x^3+173\,374\,406\,x^4+67\,227\,350\,x^5+113\,816\,070\,x^6-9\,830\,135\,x^7+68\,536\,350\,x^8-2\,023\,475\,x^9+36\,905\,000\,x^{10})+$$
$$3\,m^5\,(41\,597\,010+91\,755\,720\,x+288\,179\,073\,x^2+310\,583\,364\,x^3+541\,448\,179\,x^4+226\,287\,000\,x^5+405\,531\,063\,x^6-36\,620\,052\,x^7+$$
$$264\,606\,615\,x^8-8\,044\,400\,x^9+150\,342\,500\,x^{10})+12\,m^2\,(316\,309\,212+566\,017\,065\,x+1\,526\,027\,622\,x^2+1\,474\,185\,258\,x^3+$$
$$2\,373\,368\,682\,x^4+934\,547\,630\,x^5+1\,599\,732\,666\,x^6-139\,316\,898\,x^7+977\,620\,530\,x^8-29\,013\,075\,x^9+531\,399\,000\,x^{10})+$$
$$4\,m^3\,(472\,056\,390+927\,204\,435\,x+2\,657\,980\,899\,x^2+2\,670\,494\,533\,x^3+4\,412\,539\,105\,x^4+1\,769\,460\,300\,x^5+3\,069\,858\,069\,x^6-$$
$$270\,109\,021\,x^7+1\,910\,860\,605\,x^8-57\,082\,375\,x^9+1\,051\,187\,500\,x^{10})+m^4\,(595\,543\,860+1\,250\,302\,905\,x+3\,769\,346\,538\,x^2+3\,929\,892\,722\,x^3+$$
$$6\,671\,821\,630\,x^4+2\,729\,996\,850\,x^5+4\,810\,043\,142\,x^6-428\,393\,182\,x^7+3\,060\,365\,670\,x^8-92\,156\,375\,x^9+1\,708\,465\,000\,x^{10}))))\Big/$$
$$(e^{11}\,(1+m)\,(2+m)\,(3+m)\,(4+m)\,(5+m)\,(6+m)\,(7+m)\,(8+m)\,(9+m)$$
$$(10+m)$$
$$(11+m))$$

- **Problem 368: Result more than twice size of optimal antiderivative.**

$$\int (d+e\,x)^m\,(3+2\,x+5\,x^2)^2\,(2+x+3\,x^2-5\,x^3+4\,x^4)\,dx$$

Optimal (type 3, 432 leaves, 2 steps):

$$\frac{\left(5 d^2-2 d e+3 e^2\right)^2 \left(4 d^4+5 d^3 e+3 d^2 e^2-d e^3+2 e^4\right) (d+e x)^{1+m}}{e^9 (1+m)}-\frac{\left(5 d^2-2 d e+3 e^2\right) \left(160 d^5+127 d^4 e+88 d^3 e^2-4 d^2 e^3+64 d e^4-11 e^5\right) (d+e x)^{2+m}}{e^9 (2+m)}+\frac{\left(2800 d^6+945 d^5 e+1665 d^4 e^2+370 d^3 e^3+888 d^2 e^4-195 d e^5+107 e^6\right) (d+e x)^{3+m}}{e^9 (3+m)}-\frac{\left(5600 d^5+1575 d^4 e+2220 d^3 e^2+370 d^2 e^3+592 d e^4-65 e^5\right) (d+e x)^{4+m}}{e^9 (4+m)}+\frac{\left(7000 d^4+1575 d^3 e+1665 d^2 e^2+185 d e^3+148 e^4\right) (d+e x)^{5+m}}{e^9 (5+m)}-\frac{\left(5600 d^3+945 d^2 e+666 d e^2+37 e^3\right) (d+e x)^{6+m}}{e^9 (6+m)}+\frac{\left(2800 d^2+315 d e+111 e^2\right) (d+e x)^{7+m}}{e^9 (7+m)}-\frac{5 (160 d+9 e) (d+e x)^{8+m}}{e^9 (8+m)}+\frac{100 (d+e x)^{9+m}}{e^9 (9+m)}$$

Result (type 3, 1476 leaves) :

$$\frac{1}{e^9\,(1+m)\,(2+m)\,(3+m)\,(4+m)\,(5+m)\,(6+m)\,(7+m)\,(8+m)\,(9+m)}\,(d+e\,x)^{1+m}\,\Big(4\,032\,000\,d^8-$$

$$25\,200\,d^7\,e\,(-81+160\,x+m\,(-9+160\,x))+720\,d^6\,e^2\,\big(7992-2835\,x+5600\,x^2+3\,m\,\big(629-1050\,x+2800\,x^2\big)+m^2\,\big(111-315\,x+2800\,x^2\big)\big)-$$

$$120\,d^5\,e^3\,\big(12\,m^2\,\big(-74+999\,x-945\,x^2+2800\,x^3\big)+6\,\big(-3108+7992\,x-2835\,x^2+5600\,x^3\big)+m^3\,\big(-37+666\,x-945\,x^2+5600\,x^3\big)+$$

$$m\,\big(-7067+59\,274\,x-27\,405\,x^2+61\,600\,x^3\big)\big)+24\,d^4\,e^4\,\big(m^4\,\big(148-185\,x+1665\,x^2-1575\,x^3+7000\,x^4\big)+$$

$$25\,m\,\big(9768-5143\,x+16\,650\,x^2-6615\,x^3+14\,000\,x^4\big)+5\,m^3\,\big(888-925\,x+6660\,x^2-4725\,x^3+14\,000\,x^4\big)+$$

$$6\,\big(74\,592-15\,540\,x+39\,960\,x^2-14\,175\,x^3+28\,000\,x^4\big)+5\,m^2\,\big(9916-7955\,x+41\,625\,x^2-20\,475\,x^3+49\,000\,x^4\big)\big)-$$

$$6\,d^3\,e^5\,\big(m^5\,\big(65+592\,x-370\,x^2+2220\,x^3-1575\,x^4+5600\,x^5\big)+24\,\big(40\,950+74\,592\,x-15\,540\,x^2+39\,960\,x^3-14\,175\,x^4+28\,000\,x^5\big)+$$

$$m^4\,\big(2275+18\,352\,x-9990\,x^2+51\,060\,x^3-29\,925\,x^4+84\,000\,x^5\big)+5\,m^3\,\big(6305+43\,216\,x-19\,610\,x^2+82\,140\,x^3-39\,375\,x^4+95\,200\,x^5\big)+$$

$$5\,m^2\,\big(43\,225+235\,024\,x-83\,250\,x^2+277\,500\,x^3-114\,975\,x^4+252\,000\,x^5\big)+$$

$$2\,m\,\big(366\,405+1\,383\,504\,x-350\,390\,x^2+992\,340\,x^3-373\,275\,x^4+767\,200\,x^5\big)\big)+$$

$$2\,d^2\,e^6\,\big(m^6\,\big(107+195\,x+888\,x^2-370\,x^3+1665\,x^4-945\,x^5+2800\,x^6\big)+3\,m^5\,\big(1391+2340\,x+9768\,x^2-3700\,x^3+14\,985\,x^4-7560\,x^5+19\,600\,x^6\big)+$$

$$72\,\big(89\,880+40\,950\,x+74\,592\,x^2-15\,540\,x^3+39\,960\,x^4-14\,175\,x^5+28\,000\,x^6\big)+$$

$$15\,m^3\,\big(37\,557+49\,530\,x+160\,728\,x^2-47\,360\,x^3+151\,515\,x^4-62\,370\,x^5+137\,200\,x^6\big)+$$

$$m^4\,\big(66\,875+101\,400\,x+379\,176\,x^2-128\,020\,x^3+461\,205\,x^4-207\,900\,x^5+490\,000\,x^6\big)+$$

$$6\,m\,\big(1\,073\,852+857\,805\,x+1\,831\,056\,x^2-412\,550\,x^3+1\,112\,220\,x^4-407\,295\,x^5+823\,200\,x^6\big)+$$

$$m^2\,\big(2\,629\,418+2\,846\,805\,x+7\,675\,872\,x^2-1\,949\,530\,x^3+5\,651\,010\,x^4-2\,172\,555\,x^5+4\,547\,200\,x^6\big)\big)-$$

$$d\,e^7\,\big(m^7\,\big(33+214\,x+195\,x^2+592\,x^3-185\,x^4+666\,x^5-315\,x^6+800\,x^7\big)+2\,m^6\,\big(693+4280\,x+3705\,x^2+10\,656\,x^3-3145\,x^4+$$

$$10\,656\,x^5-4725\,x^6+11\,200\,x^7\big)+144\,\big(41\,580+89\,880\,x+40\,950\,x^2+74\,592\,x^3-15\,540\,x^4+39\,960\,x^5-14\,175\,x^6+28\,000\,x^7\big)+$$

$$2\,m^5\,\big(12\,243+71\,048\,x+57\,720\,x^2+155\,696\,x^3-43\,105\,x^4+137\,196\,x^5-57\,330\,x^6+128\,800\,x^7\big)+$$

$$2\,m^4\,\big(117\,810+630\,230\,x+472\,875\,x^2+1\,182\,816\,x^3-305\,620\,x^4+915\,750\,x^5-363\,825\,x^6+784\,000\,x^7\big)+$$

$$12\,m\,\big(663\,102+2\,152\,412\,x+1\,103\,505\,x^2+2\,129\,424\,x^3-459\,170\,x^4+1\,208\,124\,x^5-435\,645\,x^6+871\,200\,x^7\big)+$$

$$2\,m^2\,\big(2\,209\,977+9\,072\,530\,x+5\,420\,220\,x^2+11\,337\,984\,x^3-2\,568\,355\,x^4+6\,985\,674\,x^5-2\,579\,850\,x^6+5\,252\,800\,x^7\big)+$$

$$m^3\,\big(1\,332\,177+6\,385\,546\,x+4\,332\,705\,x^2+9\,939\,088\,x^3-2\,395\,565\,x^4+6\,805\,854\,x^5-2\,595\,285\,x^6+5\,415\,200\,x^7\big)\big)+$$

$$e^8\,\big(m^8\,\big(3+2\,x+5\,x^2\big)^2\,\big(2+x+3\,x^2-5\,x^3+4\,x^4\big)+m^7\,\big(792+1419\,x+4494\,x^2+2665\,x^3+5920\,x^4-1443\,x^5+4218\,x^6-1665\,x^7+3600\,x^8\big)+$$

$$2\,m^6\,\big(7434+12\,936\,x+39\,804\,x^2+22\,945\,x^3+49\,580\,x^4-11\,766\,x^5+33\,522\,x^6-12\,915\,x^7+27\,300\,x^8\big)+$$

$$144\,\big(45\,360+41\,580\,x+89\,880\,x^2+40\,950\,x^3+74\,592\,x^4-15\,540\,x^5+39\,960\,x^6-14\,175\,x^7+28\,000\,x^8\big)+$$

$$2\,m^5\,\big(77\,616+130\,053\,x+386\,163\,x^2+215\,345\,x^3+451\,400\,x^4-104\,229\,x^5+289\,821\,x^6-109\,305\,x^7+226\,800\,x^8\big)+$$

$$12\,m\,\big(995\,544+1\,162\,062\,x+2\,691\,692\,x^2+1\,267\,305\,x^3+2\,353\,200\,x^4-496\,466\,x^5+1\,288\,044\,x^6-459\,945\,x^7+913\,200\,x^8\big)+$$

$$m^4\,\big(983\,682+1\,567\,797\,x+4\,453\,233\,x^2+2\,389\,985\,x^3+4\,850\,404\,x^4-1\,090\,353\,x^5+2\,965\,809\,x^6-1\,098\,405\,x^7+2\,244\,900\,x^8\big)+$$

$$2\,m^2\,\big(4\,581\,036+6\,188\,589\,x+15\,529\,766\,x^2+7\,627\,230\,x^3+14\,532\,120\,x^4-3\,119\,359\,x^5+8\,193\,798\,x^6-2\,953\,260\,x^7+5\,906\,200\,x^8\big)+$$

$$m^3\,\big(3\,864\,168+5\,752\,131\,x+15\,458\,076\,x^2+7\,946\,185\,x^3+15\,608\,080\,x^4-3\,422\,907\,x^5+9\,134\,412\,x^6-3\,332\,385\,x^7+6\,728\,400\,x^8\big)\big)\Big)$$

- **Problem 369: Result more than twice size of optimal antiderivative.**

$$\int (d+e\,x)^m\,\big(3+2\,x+5\,x^2\big)\,\big(2+x+3\,x^2-5\,x^3+4\,x^4\big)\,dx$$

Optimal (type 3, 292 leaves, 2 steps) :

$$\frac{\left(5 d^2-2 d e+3 e^2\right) \left(4 d^4+5 d^3 e+3 d^2 e^2-d e^3+2 e^4\right) (d+e x)^{1+m}}{e^7 (1+m)}-$$

$$\frac{\left(120 d^5+85 d^4 e+68 d^3 e^2+12 d^2 e^3+42 d e^4-7 e^5\right) (d+e x)^{2+m}}{e^7 (2+m)}+\frac{\left(300 d^4+170 d^3 e+102 d^2 e^2+12 d e^3+21 e^4\right) (d+e x)^{3+m}}{e^7 (3+m)}-$$

$$\frac{2 \left(200 d^3+85 d^2 e+34 d e^2+2 e^3\right) (d+e x)^{4+m}}{e^7 (4+m)}+\frac{\left(300 d^2+85 d e+17 e^2\right) (d+e x)^{5+m}}{e^7 (5+m)}-\frac{(120 d+17 e) (d+e x)^{6+m}}{e^7 (6+m)}+\frac{20 (d+e x)^{7+m}}{e^7 (7+m)}$$

Result (type 3, 743 leaves) :

$$\frac{1}{e^7 (1+m) (2+m) (3+m) (4+m) (5+m) (6+m) (7+m)} (d+e x)^{1+m}$$

$$\left(14400 d^6-120 d^5 e (-119+120 x+m (-17+120 x))+24 d^4 e^2 \left(714-595 x+600 x^2+m^2 \left(17-85 x+300 x^2\right)+m \left(221-680 x+900 x^2\right)\right)-12 d^3 e^3\right.$$

$$\left(m^3 \left(-2+34 x-85 x^2+200 x^3\right)+2 \left(-210+714 x-595 x^2+600 x^3\right)+2 m^2 \left(-18+238 x-425 x^2+600 x^3\right)+m \left(-214+1870 x-1955 x^2+2200 x^3\right)\right)+$$

$$2 d^2 e^4 \left(m^4 \left(21-12 x+102 x^2-170 x^3+300 x^4\right)+12 \left(1470-210 x+714 x^2-595 x^3+600 x^4\right)+2 m^3 \left(231-114 x+816 x^2-1105 x^3+1500 x^4\right)+\right.$$

$$\left.2 m \left(6699-1902 x+7752 x^2-7055 x^3+7500 x^4\right)+m^2 \left(3759-1500 x+8466 x^2-9010 x^3+10500 x^4\right)\right)-$$

$$d e^5 \left(m^5 \left(7+42 x-12 x^2+68 x^3-85 x^4+120 x^5\right)+24 \left(735+1470 x-210 x^2+714 x^3-595 x^4+600 x^5\right)+\right.$$

$$m^4 \left(175+966 x-252 x^2+1292 x^3-1445 x^4+1800 x^5\right)+m^3 \left(1715+8442 x-1956 x^2+8908 x^3-8925 x^4+10200 x^5\right)+$$

$$\left.2 m \left(9639+31038 x-5064 x^2+18360 x^3-15895 x^4+16440 x^5\right)+m^2 \left(8225+34314 x-6804 x^2+27268 x^3-25075 x^4+27000 x^5\right)\right)+$$

$$e^6 \left(m^6 \left(6+7 x+21 x^2-4 x^3+17 x^4-17 x^5+20 x^6\right)+m^5 \left(162+182 x+525 x^2-96 x^3+391 x^4-374 x^5+420 x^6\right)+\right.$$

$$24 \left(1260+735 x+1470 x^2-210 x^3+714 x^4-595 x^5+600 x^6\right)+m^4 \left(1770+1890 x+5187 x^2-904 x^3+3519 x^4-3230 x^5+3500 x^6\right)+$$

$$m^3 \left(9990+9940 x+25599 x^2-4224 x^3+15725 x^4-13940 x^5+14700 x^6\right)+$$

$$2 m \left(24084+18459 x+39858 x^2-5904 x^3+20502 x^4-17323 x^5+17640 x^6\right)+$$

$$\left.\left.m^2 \left(30624+27503 x+65352 x^2-10180 x^3+36448 x^4-31433 x^5+32480 x^6\right)\right)\right)$$

- **Problem 370: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d+e x)^m \left(2+x+3 x^2-5 x^3+4 x^4\right)}{3+2 x+5 x^2} \, dx$$

Optimal (type 5, 255 leaves, 4 steps) :

$$\frac{\left(100 d^2+165 d e+81 e^2\right) (d+e x)^{1+m}}{125 e^3 (1+m)}-\frac{(40 d+33 e) (d+e x)^{2+m}}{25 e^3 (2+m)}+$$

$$\frac{4 (d+e x)^{3+m}}{5 e^3 (3+m)}-\frac{\left(6412 i-423 \sqrt{14}\right) (d+e x)^{1+m} \text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{5 (d+e x)}{5 d-e+i \sqrt{14} e}\right]}{3500 \left(5 i d-\left(i+\sqrt{14}\right) e\right) (1+m)}-$$

$$\frac{\left(6412 i+423 \sqrt{14}\right) (d+e x)^{1+m} \text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{5 (d+e x)}{5 d-\left(1+i \sqrt{14}\right) e}\right]}{3500 \left(5 i d-\left(i-\sqrt{14}\right) e\right) (1+m)}$$

Result (type 5, 621 leaves) :

$$\frac{1}{17500\, e^3\, m\, (1+m)\, (2+m)\, (3+m)} (d+e\,x)^m \Bigg(28000\, d^3\, m + 23100\, d^2\, e\, m\, (3+m) + 11340\, d\, e^2\, m\, (2+m)\, (3+m) - 28000\, d^2\, e\, m^2\, x - 23100\, d\, e^2\, m^2\, (3+m)\, x + 11340\, e^3\, m\, (2+m)\, (3+m)\, x +$$
$$14000\, d\, e^2\, m^2\, x^2 + 14000\, d\, e^2\, m^3\, x^2 - 23100\, e^3\, m\, (3+m)\, x^2 - 23100\, e^3\, m^2\, (3+m)\, x^2 + 28000\, e^3\, m\, x^3 +$$
$$42000\, e^3\, m^2\, x^3 + 14000\, e^3\, m^3\, x^3 - 28000\, d^3\, m \left(1+\frac{e\,x}{d}\right)^{-m} - 23100\, d^2\, e\, m\, (3+m) \left(1+\frac{e\,x}{d}\right)^{-m} +$$
$$6412\times 5^{-m}\, e^3\, (1+m)\, (2+m)\, (3+m) \left(\frac{d+e\,x}{e\left(1+i\sqrt{14}+5\,x\right)}\right)^{-m} \text{Hypergeometric2F1}\left[-m, -m, 1-m, \frac{5\,i\,d + \left(-i+\sqrt{14}\right) e}{e\left(-i+\sqrt{14}-5\,i\,x\right)}\right] -$$
$$423\, i\, 5^{-m} \sqrt{14}\, e^3\, (1+m)\, (2+m)\, (3+m) \left(\frac{d+e\,x}{e\left(1+i\sqrt{14}+5\,x\right)}\right)^{-m} \text{Hypergeometric2F1}\left[-m, -m, 1-m, \frac{5\,i\,d + \left(-i+\sqrt{14}\right) e}{e\left(-i+\sqrt{14}-5\,i\,x\right)}\right] +$$
$$6412\times 5^{-m}\, e^3\, (1+m)\, (2+m)\, (3+m) \left(\frac{d+e\,x}{e\left(1-i\sqrt{14}+5\,x\right)}\right)^{-m} \text{Hypergeometric2F1}\left[-m, -m, 1-m, \frac{-5\,i\,d + \left(i+\sqrt{14}\right) e}{e\left(i+\sqrt{14}+5\,i\,x\right)}\right] +$$
$$423\, i\, 5^{-m} \sqrt{14}\, e^3\, (1+m)\, (2+m)\, (3+m) \left(\frac{d+e\,x}{e\left(1-i\sqrt{14}+5\,x\right)}\right)^{-m} \text{Hypergeometric2F1}\left[-m, -m, 1-m, \frac{-5\,i\,d + \left(i+\sqrt{14}\right) e}{e\left(i+\sqrt{14}+5\,i\,x\right)}\right]\Bigg)$$

- **Problem 371: Unable to integrate problem.**

$$\int \frac{(d+e\,x)^m \left(2+x+3\,x^2-5\,x^3+4\,x^4\right)}{\left(3+2\,x+5\,x^2\right)^2}\, dx$$

Optimal (type 5, 377 leaves, 5 steps) :

$$\frac{4\,(d+e\,x)^{1+m}}{25\,e\,(1+m)}-\frac{(1367\,d-293\,e+(423\,d-1367\,e)\,x)\,(d+e\,x)^{1+m}}{700\,\left(5\,d^2-2\,d\,e+3\,e^2\right)\,\left(3+2\,x+5\,x^2\right)}+$$

$$\left(\left(80\,360\,d^2-32\,144\,d\,e+48\,216\,e^2+i\,\sqrt{14}\,\left(6565\,d^2-2\,d\,e\,(1313-3206\,m)+e^2\,(3939-98\,m)\right)-5922\,d\,e\,m+19\,138\,e^2\,m\right)\,(d+e\,x)^{1+m}\right.$$

$$\left.\text{Hypergeometric2F1}\left[1,\,1+m,\,2+m,\,\frac{5\,(d+e\,x)}{5\,d-e+i\,\sqrt{14}\,e}\right]\right)\Big/\left(19\,600\,\left(5\,d+i\,\left(i+\sqrt{14}\right)\,e\right)\,\left(5\,d^2-2\,d\,e+3\,e^2\right)\,(1+m)\right)+$$

$$\left(\left(80\,360\,d^2-32\,144\,d\,e+48\,216\,e^2-i\,\sqrt{14}\,\left(6565\,d^2-2\,d\,e\,(1313-3206\,m)+e^2\,(3939-98\,m)\right)-5922\,d\,e\,m+19\,138\,e^2\,m\right)\,(d+e\,x)^{1+m}\right.$$

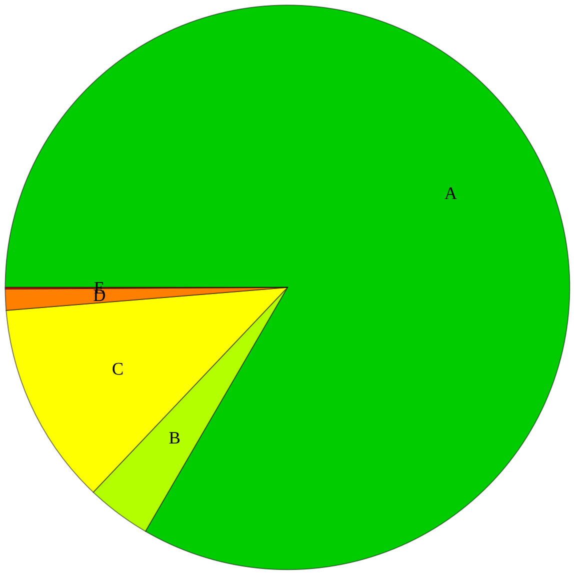
$$\left.\text{Hypergeometric2F1}\left[1,\,1+m,\,2+m,\,\frac{5\,(d+e\,x)}{5\,d-\left(1+i\,\sqrt{14}\right)\,e}\right]\right)\Big/\left(19\,600\,\left(5\,d-\left(1+i\,\sqrt{14}\right)\,e\right)\,\left(5\,d^2-2\,d\,e+3\,e^2\right)\,(1+m)\right)$$

Result (type 8, 40 leaves):

$$\int\frac{(d+e\,x)^m\,\left(2+x+3\,x^2-5\,x^3+4\,x^4\right)}{\left(3+2\,x+5\,x^2\right)^2}\,dx$$

Summary of Integration Test Results

7003 integration problems



A - 5840 optimal antiderivatives

B - 258 more than twice size of optimal antiderivatives

C - 814 unnecessarily complex antiderivatives

D - 85 unable to integrate problems

E - 6 integration timeouts